Time Series Analysis – Quiz I

1. **Answer the following: (10 Marks)**

Give expressions for the **theoretical** ACF of a random process that obeys $v[k] - 0.8v[k-1] + 0.15v[k-2] = e[k]$ for lags $l = 0, 1, 2$. Further compute ACF of $v[k]$ at lags $l = 3, 4$ using the difference equation relationships for the ACF.

In the above model e[k] is i.i.d sequence N(0, σ^2) (Normally distributed sequence with 0 mean and variance as σ^2).

2. **Answer the following: (20 Marks, 10 each)**

Section A

Consider a random process $X(t)$ defined by

$$X(t) = A\cos(\omega t + \Theta) \qquad -\infty < t < \infty$$

Here A and w are constants while the phase θ is random over the interval (-π π). Is this process stationary?

Section B

The correlation of two sequences is an operation defined by the relation

$$x(n) \star h(n) = \sum_{k=-\infty}^{\infty} x(k)h(n+k)$$

Note that we use a star $\star$ to denote correlation and an asterisk $*$ to denote convolution.

(*a*) Find the correlation between the sequence $x(n) = u(n) - u(n-6)$ and $h(n) = u(n-2) - u(n-5)$.

(*b*) Find the correlation of $x(n) = \alpha^n u(n)$ with itself (i.e., $h(n) = x(n)$). This is known as the *autocorrelation* of $x(n)$. Assume that $|\alpha| < 1$.

3. **Answer the following: (10 Marks)**
   Explain: Ensemble mean, time average mean, strict stationarity, wide sense stationarity and ergodicity (Use mathematical equations to explain).

4 **Answer the following: (30 marks, 15 each)**

Part A

Consider the discrete-time sequence

$$x(n) = \cos\left(\frac{n\pi}{8}\right)$$

Find two different continuous-time signals that would produce this sequence when sampled at a frequency of $f_s = 10$ Hz.

Part B

If the Nyquist rate for $x_a(t)$ is Ω_s, what is the Nyquist rate for each of the following signals that are derived from $x_a(t)$?

(a) $\dfrac{dx_a(t)}{dt}$

(b) $x_a(2t)$

(c) $x_a^2(t)$

(d) $x_a(t)\cos(\Omega_0 t)$

5. Answer the following: (30 Marks, 15 each)

Part A

Consider the binary communication channel shown in Fig. 1-15. The channel input symbol X may assume the state 0 or the state 1, and, similarly, the channel output symbol Y may assume either the state 0 or the state 1. Because of the channel noise, an input 0 may convert to an output 1 and vice versa. The channel is characterized by the channel transition probabilities p_0, q_0, p_1, and q_1, defined by

$$p_0 = P(y_1 \mid x_0) \quad \text{and} \quad p_1 = P(y_0 \mid x_1)$$
$$q_0 = P(y_0 \mid x_0) \quad \text{and} \quad q_1 = P(y_1 \mid x_1)$$

where x_0 and x_1 denote the events $(X = 0)$ and $(X = 1)$, respectively, and y_0 and y_1 denote the events $(Y = 0)$ and $(Y = 1)$, respectively. Note that $p_0 + q_0 = 1 = p_1 + q_1$. Let $P(x_0) = 0.5$, $p_0 = 0.1$, and $p_1 = 0.2$.

(a) Find $P(y_0)$ and $P(y_1)$.

(b) If a 0 was observed at the output, what is the probability that a 0 was the input state?

(c) If a 1 was observed at the output, what is the probability that a 1 was the input state?

(d) Calculate the probability of error P_e.

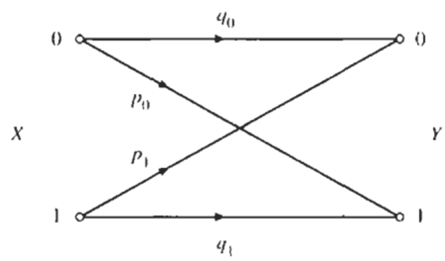

Fig. 1-15

Part B

Consider the binary communication channel shown in Fig. 3-4 (Prob. 1.52). Let (X, Y) be a bivariate r.v., where X is the input to the channel and Y is the output of the channel. Let $P(X = 0) = 0.5$, $P(Y = 1 \mid X = 0) = 0.1$, and $P(Y = 0 \mid X = 1) = 0.2$.

(a) Find the joint pmf's of (X, Y).

(b) Find the marginal pmf's of X and Y.

(c) Are X and Y independent?

Figure is given above for this part B.