

Bayesian Causal Inference in Stan

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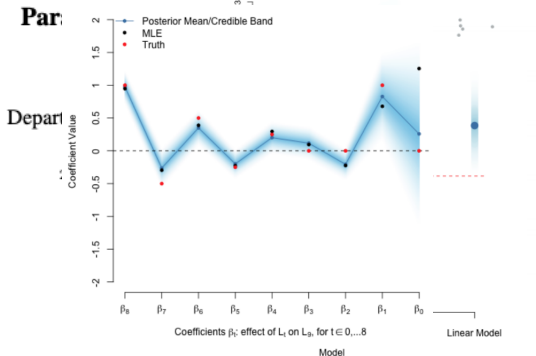
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1/1

REVIEW / TUTORIAL PAPER

A Practical Introduction to Partial Identification of Causal Effects:

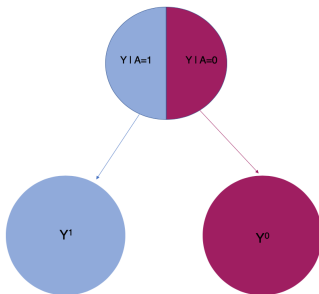


2/1

WHAT IS CAUSAL INFERENCE?

What **would have happened** had everyone in the target population if ...

- ▶ ... everyone took treatment 1 versus treatment 0?
- ▶ ... were vaccinated ?
- ▶ ... were enrolled in a job training program?



IDENTIFICATION VIA THE g-FORMULA

$$D = \{Y_i, A_i, L_i, V_i\}_{1:n}.$$

Define potential outcomes Y^a for $a \in \{0, 1\}$

$$\Psi(v) = E[Y^1 - Y^0 \mid V = v]$$

Under some **identification assumptions**

$$E[Y^a \mid V = v] = \int_{\mathcal{L}} \underbrace{E[Y \mid A = a, V = v, L]}_{\text{Regression, } \mu(a, v, l)} \cdot \underbrace{dP(L \mid V = v)}_{\text{Confounder } P_v(L)}$$

REGRESSION MODELING

- ▶ Parametric Approaches:

$$\mu(A, V, L) = g^{-1}(\beta_0 + \beta_1 A + \beta_2 V + \beta_3' L)$$

Need priors on β s.

- ▶ Nonparametric Approaches:

$$\mu(A, V, L) = g^{-1}(f(A, V, L))$$

Need prior for f .

WHY BAYES?

- ▶ Priors can help us compute causal effects under sparsity.
- ▶ Avoid *ad hoc* approaches.
- ▶ Powerful suite of nonparametric models (BART, DP, GP, etc).
- ▶ Probabilistic sensitivity analyses.

LOGISTIC MODEL FOR CATEs

Suppose Y is **binary** and $V \in \{1, 2, 3, 4, 5\}$ (e.g., race/ethnicity)

$$Y \mid A, V, L \sim \text{Ber}(\mu(A, V))$$

Specify **logistic regression**

$$\mu(A, V, L) = g^{-1}(\beta_v + \beta'_L L + \theta_v A)$$

with parameters $\omega = (\beta_1 \dots, \beta_5, \beta_L, \theta_1, \dots, \theta_5)$

PRIOR FOR RACE EFFECTS

$$\mu(A, V, L) = g^{-1}(\beta_v + \beta'_L L + \theta_v A)$$

- ▶ Consider “partial pooling” prior:

$$\theta_v \mid \theta^* \sim N(\theta^*, \phi)$$

- ▶ **Shrinkage**: shrinkage race effects towards common effect.
- ▶ **Belief**: the race effects shouldn't be *that* different.
- ▶ **Causal** intuition: small ϕ shrinks towards homogeneity.
- ▶ Note: implies

$$\theta_4 - \theta_5 \sim N(0, 2\phi)$$

As opposed to setting $\phi \approx 0$

$$\theta_4 - \theta_5 \sim \delta_0$$

WE HAVE A DATA MODEL...NOW WHAT?

Suppose we want to compute **Causal Odds Ratio**:

$$\Psi(v) = \frac{E(Y^1 | v) / [1 - E(Y^1 | v)]}{E(Y^0 | v) / [1 - E(Y^0 | v)]}$$

Using g -computation,

$$E(Y^a | v) = \int_{\mathcal{L}} \mu(a, v, L) dP_v(L)$$

But what about model for $P(L)$?

THE BAYESIAN BOOTSTRAP

Let $S_v = \{i : V_i = v\}$ with size n_v

- Frequentist estimate:

$$\hat{P}_v(l) = \sum_{j \in S_v} \frac{1}{n_v} \cdot \delta_{L_j}(l)$$

- Bayesian model:

$$P_v(l \mid p_v) = \sum_{j \in S_v} p_{vj} \cdot \delta_{L_j}(l)$$

- Unknown weight vector: $p_v = \{p_{vj} : j \in S_v\}$.
- Prior: $p_v \sim \text{Dirichlet}(0_{n_v})$
- Posterior: $p_v \mid L \sim \text{Dirichlet}(1_{n_v})$

$$E[p_{vj} \mid L] = 1/n_v$$

FULL MCMC INFERENCE

Inference on stratum v :

1. Obtain m^{th} set of posterior draws $\omega^{(m)}$ and for each $A = a$, for $j \in S_v$

$$\mu^{(m)}(a, v, L_j) = g^{-1}(\beta_v^{(m)} + \beta_L^{(m)} L_j + \theta_v^{(m)} a)$$

2. Draw Bayesian Bootstrap weights from posterior:

$$p_v^{(m)} \sim \text{Dirichlet}(1_{n_v})$$

3. Integrate of confounder distribution

$$E^{(m)}(Y^a | v) = \int_{\mathcal{L}} \mu(a, v, L) dP_v(L) \approx \sum_{j \in S_v} \mu^{(m)}(a, v, L_j) \cdot p_{vj}^{(m)}$$

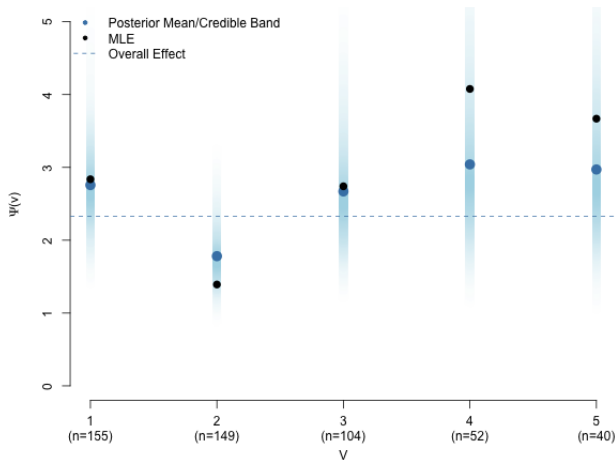
4. Compute draw of causal odds ratio:

$$\Psi^{(m)}(v) = \frac{E^{(m)}(Y^1 | v) / [1 - E^{(m)}(Y^1 | v)]}{E^{(m)}(Y^0 | v) / [1 - E^{(m)}(Y^1 | v)]}$$

IMPLEMENTATION IN STAN

```
for(v in 1:Pv){  
  ...  
  // compute conditional means.  
  cond_mean_y1 = inv_logit( Wv*beta_w + beta_v[v] + theta[v] );  
  cond_mean_y0 = inv_logit( Wv*beta_w + beta_v[v] );  
  
  // Bayesian bootstrap weights  
  bb_weights = dirichlet_rng( rep_vector(1, nv) ) ;  
  
  // taking average over bayesian bootstrap weights  
  marg_mean_y1 = bb_weights' * cond_mean_y1;  
  marg_mean_y0 = bb_weights' * cond_mean_y0;  
  
  // compute odds ratio  
  odds_1 = (marg_mean_y1/(1 - marg_mean_y1));  
  odds_0 = (marg_mean_y0/(1 - marg_mean_y0));  
  odds_ratio[v] = odds_1/odds_0;  
  
  ...  
}
```

SYNTHETIC EXAMPLE



SENSITIVITY ANALYSIS

Identification requires **conditional ignorability**

$$Y^a \perp A \mid L, V = v$$

But, what if ignorability is violated?

$$E[Y^a \mid A = 1, L, v] \neq E[Y^a \mid A = 0, L, v]$$

CONSEQUENCE OF VIOLATION

Define,

$$\Delta^a(L) = E[Y^a \mid A = 1, L, v] - E[Y^a \mid A = 0, L, v]$$

$$\int \mu(1, v, L) - \mu(0, v, L) dP_v(L) = E[Y^1 - Y^0 \mid v] + \xi$$

Estimate of **risk difference** is biased by ξ .

FORM OF VIOLATION

Trade-offs involved in sensitivity analyses

$$\xi = \int \Delta^1(L)(1 - \pi_v(L)) + \Delta^0(L)\pi_v(L) dP_v(L)$$

Where $\pi_v = P(A = 1 \mid L, v)$ is the propensity score.

Simplify $\Delta := \Delta^1 = \Delta^0$ and $\Delta \perp L, V$.

Then,

$$\xi = \Delta$$

Now we can specify priors over Δ .

PRIORS OVER BIAS

Note that $-1 < E[Y^1 - Y^0 \mid v] < 1$ and recall:

$$\Delta = E[Y^a \mid A = 1, L, v] - E[Y^a \mid A = 0, L, v]$$

- ▶ Treated patients **systematically worse**:

$$\Delta \sim U(0, 1)$$

- ▶ Treated patients **systematically better**:

$$\Delta \sim U(-1, 0)$$

- ▶ Biased with **uncertain direction**:

$$\Delta \sim U(-1, 1)$$

MODIFIED MCMC INFERENCE

In Step 3 at m^{th} iteration:

Draw $\Delta^{(m)}$ from the prior and compute,

$$E^{(m)}(Y^1 | v) - E^{(m)}(Y^0 | v) = \sum_{j \in S_v} \left\{ \mu^{(m)}(1, v, L_j) - \mu^{(m)}(0, v, L_j) \right\} \cdot p_{vj}^{(m)}$$

Subtract off bias from prior,

$$\left\{ E^{(m)}(Y^1 | v) - E^{(m)}(Y^0 | v) \right\} - \Delta^{(m)}$$

IMPLEMENTATION IN STAN

- ▶ Could specify prior for Δ in “model” block. Manipulate in “generated quantities”.
- ▶ Could draw Δ from specified distribution in “generated quantities” block.

SOME RESOURCES

- ▶ A Practical Introduction to Bayesian Estimation of Causal Effects: Parametric and Nonparametric Approaches
<https://arxiv.org/pdf/2004.07375.pdf>
- ▶ Companion GitHub repo for paper: https://github.com/stablemarkets/intro_bayesian_causal
- ▶ GitHub Repo for this talk: https://github.com/stablemarkets/StanCon2020_BayesCausal

THANK YOU!

