lab2

March 24, 2025

[1]:
```
import pint
import matplotlib.pyplot as plt
import numpy as np
```

1 Explanation

Following is the analytic solution to the advection-diffusion equasion. In its most fundamental form, it states that:

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} = D\frac{\partial^2 c}{\partial x^2},$$

where:

$c(x,t)$ - concentration of substance c as the function of space x and time t

u - is the advection constant velocity (how fast is the medium carring substance)

D - diffusion coefficient, quantifying how quickly the substance spreads out.

Analytic solution is derived by assuming instantaneous release or continiuous flux of mass.

$$c(x,t) = \frac{M}{A} \cdot \frac{1}{2\sqrt{\pi D}} \cdot \frac{x}{t^{3/2}} \exp\left(-\frac{(ut-x)^2}{4Dt}\right),$$

where:

M - mass

A - area.

Following 2D plot is generated by assuming constant value of x. The values were chosen by brute-force in order to find the ones generating plot most resembling one from the lecture.

[2]:
```
si = pint.UnitRegistry()
si.setup_matplotlib()

t = np.linspace(1, 100) * si.s
M = 1 * si.kg
A = 1 * (si.m ** 3) / si.s
D = 0.1 * (si.m ** 2) / si.s
```

```
u = 0.3 * (si.m / si.s)
x = 15 * si.m

R = (M / A) * (1 / (2 * np.sqrt(np.pi * D)))
R = 1 * si.kg * (si.s ** 1.5) / (si.m ** 4)

c = R * (x / (t ** (3/2))) * np.exp(-(((u * t - x) ** 2) / (4 * D * t)))
plt.plot(t, c)
```

[2]: [<matplotlib.lines.Line2D at 0x7352ecaa12d0>]