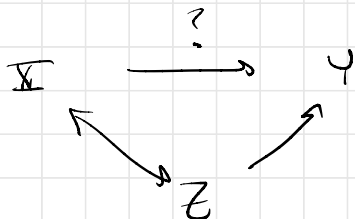
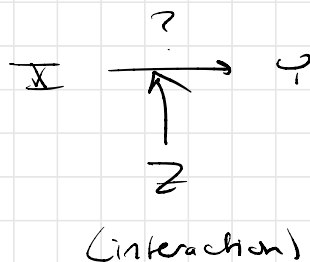


Confounders (Z)Effect modifier (Z)Hosmer-Lemeshow Test: (Sec. 5.2.3)

Goodness-of-fit test

where we have ungrouped
binary (0/1) observations
→ continuous predictor.

$$\left[\begin{array}{l} H_0: \text{our model} \\ H_a: \text{perfect model} \end{array} \right]$$

- ① Fit the model (logistic regression)

$$\log\left(\frac{\hat{\pi}_i}{1-\hat{\pi}_i}\right) = \beta_0 + \beta_1 x_i \quad i=1, 2, 3, \dots, n$$

- ② Generate fitted probabilities $\hat{\pi}_i$

- ③ Order $\hat{\pi}_i$ from smallest to largest.

- ④ Group the data by quantiles of the fitted probabilities.
→ # of groups should allow ≥ 5
expected events in each group.

- ⑤ For each group, compute the observed & expected
of successes:

$$\text{expected \#} = \text{Sum of fitted probs.}$$

⑥ Compare observed counts to expected via a chi-squared test stat:

$$\chi^2_{HL} = \sum_{j=1}^g \frac{(\text{obs}_j - \text{exp}_j)^2}{\text{exp}_j \left(1 - \frac{\text{exp}_j}{n_j}\right)} \stackrel{H_0}{\sim} \chi^2_{(g-2)}$$

$g = \# \text{ of groups}$

\nearrow
of obs.
in group j

Model in matrix terms:

$$\underset{n \times 1}{g(\underline{Y})} = \underset{n \times p}{X} \underset{p \times 1}{\beta}$$