Chapter 3: Common Families of Distributions (Sections 3.1-3.3)

3.1 Introduction

Statistical distributions are used to model populations. Therefore, we usually deal with families of distributions, where each family is indexed by one or more parameters that allow us to vary certain characteristics of the distribution, such as shape and/or spread, while staying with one functional form.

There are many common discrete and continuous distributions, and we'll discuss a few of them, along with their interrelationships and common applications, in the following sections.

3.2 Discrete Distributions

A random variable X is said to have a discrete distribution if its support, $\mathcal{X}$, is countable. In most discrete distributions, the random variable has integer-valued outcomes. In this section, we'll discuss the following discrete distributions: 1) Discrete Uniform; 2) Binomial; 3) Negative Binomial; 4) Geometric; 5) Hypergeometric; and 6) Poisson.

Discrete Uniform Distribution

When Used?	This distribution puts equal mass on each of the outcomes $1,2,\ldots,N.$ That is, each of the N outcomes has equal probability of being observed.
PMF:	A random variable X has a **discrete uniform** $(1,N)$ distribution if $$f_X(x\mid N)=P(X=x\mid N)=\begin{cases}\frac{1}{N} & x=1,2,\ldots,N\\ 0 & else\end{cases}$$ where N is a specified integer.
MGF:	$M_X(t)=\dfrac{e^t\left(1-e^{Nt}\right)}{N\left(1-e^t\right)}$
Mean:	$E(X)=\dfrac{N+1}{2}$
Variance:	$Var(X)=\dfrac{(N+1)(N-1)}{12}$
Example:	The German Tank Problem is a well-known example that uses a discrete uniform distribution to model the serial number of a tank captured during World War II in order to estimate $N,$ the total number of tanks produced.
Fun Fact:	This distribution can be generalized so that the sample space is any range of integers, $N_0, N_0+1,\ldots,N_1,$ with pmf $P(X=x\mid N_0,N_1)=\dfrac{1}{(N_1-N_0+1)}.$

Bernoulli Distribution

When Used?	When an experiment has only two possible outcomes: "success" or "failure." A Bernoulli random variable X is defined in the following manner: $X = \begin{cases} 1 & if\ \ success \\ 0 & if\ \ failure \end{cases}$ where $P(X=1)=p$ and $P(X=0)=1-p.$
PMF:	A random variable X has a **Bernoulli** (p) distribution if $f_X(x \mid p) = \begin{cases} p^x (1-p)^{1-x} & x=0,1;\ 0 \le p \le 1 \\ 0 & else \end{cases}$ Note: The notation $f_X(x \mid p)$ implies the value of the function is dependent on the value p. A value such as p, required for the calculation of any probability, is known as a **distributional parameter.**
MGF:	$M_X(t) = pe^t + (1-p)$
Mean:	$E(X) = p$
Variance:	$Var(X) = p(1-p)$
Examples:	Outcome of one coin flip; Whether a randomly selected student is infected with a disease; etc.
Fun Facts:	A Bernoulli trial is named for James Bernoulli, one of the founding fathers of probability theory. A Bernoulli distribution is a special case of the Binomial distribution where $n=1$.
In R:	To find P(X = x) use: `dbinom(x, n=1, p)` To find P(X ≤ x) use: `pbinom(x, n=1, p)` To find smallest x* such that P(X ≤ x*) ≥ c, use: `qbinom(c, n=1, p)` To simulate *m* random draws, use: `rbinom(m, n=1, p)`

Binomial Distribution

When Used?	When a random variable is a result of a sequence of independent Bernoulli trials, and we are interested in the number of successes observed during a fixed number of trials. This requires: 1. A fixed number of n identical trials. 2. Each trial results in one of two possible outcomes: "success" or "failure." 3. The probability of success (denoted by p) remains the same for each trial. The probability of failure is denoted as $(1-p)$. 4. The trials are independent. 5. The random variable, X, is defined as the number of successes observed during the n trials.
PMF:	A random variable X has a **binomial** (n, p) distribution if $f_X(x \mid n, p) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x} & x = 0,1,2,...,n;\ 0 \le p \le 1 \\ 0 & else \end{cases}$ Note: The notation $f_X(x \mid n, p)$ implies the value of the function is dependent on the values of n and p. The values n and p are the **parameters** of the distribution.
MGF:	$M_X(t) = \left[pe^t + (1-p)\right]^n$
Mean:	$E(X) = np$
Variance:	$Var(X) = np(1-p)$
Fun Fact:	Recursive Relationship: $P(X = x) = \left(\frac{n-x+1}{x}\right)\left(\frac{p}{1-p}\right)P(X = x-1)\ \ for\ x = 1,2,...$
Examples:	Number of heads observed in n coin flips; Number of n randomly selected voters who voted for a particular candidate; Number of n randomly selected students who are infected with a disease; etc.
In R:	To find P(X = x) use: `dbinom(x, n, p)` To find P(X ≤ x) use: `pbinom(x, n, p)` To find smallest x* such that P(X ≤ x*) ≥ c, use: `qbinom(c, n, p)` To simulate *m* random draws, use: `rbinom(m, n, p)`

Geometric Distribution

When Used?	When a random variable is a result of a sequence of independent Bernoulli trials, and we are interested in the trial on which the first success occurs. This requires: 1. Identical and independent trials. 2. Each trial results in one of two possible outcomes: "success" or "failure." 3. The probability of success (denoted by p) remains the same for each trial. The probability of failure is denoted as $(1-p)$. 4. The random variable, X, is defined as the trial on which the first success occurs.
PMF:	A random variable X has a **Geometric** (p) distribution if $f_X(x \mid p) = \begin{cases} p(1-p)^{x-1} & x = 1,2,3,....;\ 0 \le p \le 1 \\ 0 & else \end{cases}$ Note: The notation $f_X(x \mid p)$ implies the value of the function is dependent on the value p. The value p is the **parameter** of the distribution.
MGF:	$M_X(t) = \dfrac{pe^t}{1-(1-p)e^t},\ t < -\ln(1-p)$
Mean:	$E(X) = \dfrac{1}{p}$
Variance:	$Var(X) = \dfrac{(1-p)}{p^2}$
Examples:	Number of coin flips to observe first head flipped; Number of students randomly chosen until one infected with a disease is found; etc.
Fun Facts:	The geometric distribution is a special case of the negative binomial distribution where $r=1$. The geometric distribution has what's known as the "memoryless property." (see pg. 97) The geometric distribution can also be re-parameterized to represent the number of failures before the 1^{st} success occurs.

In R:

	Geom(p) – # Trials	Geom*(p) – # Failures
To find P(X = x):	`dgeom(x-1, p)`	`dgeom(x, p)`
To find P(X ≤ x):	`pgeom(x-1, p)`	`pgeom(x, p)`
To find smallest x* such that P(X ≤ x*) ≥ c:	`qgeom(c, p) + 1`	`qgeom(c, p)`
To simulate *m* random draws:	`rgeom(m, p) + 1`	`rgeom(m, p)`

Example: Suppose a baseball player has a batting average of .300. Assuming times at bat are independent, let the random variable X represent the number of times at bat since the season started when he gets his first hit. If he has had no hits in his first 20 at bats, what is the probability it takes him more than 25 at bats to get his first hit of the season?

Negative Binomial Distribution

When Used?	When a random variable is a result of a sequence of independent Bernoulli trials, and we are interested in the trial on which the r^{th} success occurs $(r = 1, 2, 3, 4, etc.)$. This requires: 1. Identical and independent trials. 2. Each trial results in one of two possible outcomes: "success" or "failure." 3. The probability of success (denoted by p) remains the same for each trial. The probability of failure is denoted as $(1-p)$. 4. The random variable, X, is defined as the trial on which the r^{th} success occurs $(r = 1, 2, 3, 4, etc.)$.
PMF:	A random variable X has a **Negative Binomial** (r, p) distribution if $$f_X(x \mid r,p) = \begin{cases} \binom{x-1}{r-1} p^r (1-p)^{x-r} & x = r, r+1, r+2, ...;\ 0 \le p \le 1 \\ 0 & else \end{cases}$$ Note: The notation $f_X(x \mid r,p)$ implies the value of the function is dependent on the values r and p. The values r and p are the **parameters** of the distribution.
MGF:	$M_X(t) = \left[\dfrac{pe^t}{1-(1-p)e^t}\right]^r, \ t < -\ln(1-p)$
Mean:	$E(X) = \dfrac{r}{p}$
Variance:	$Var(X) = \dfrac{r(1-p)}{p^2}$
Examples:	Number of coin flips to observe 7 heads; Number of students randomly chosen until 100 infected with a disease are found; etc.
Fun Facts:	The negative binomial distribution can be re-parameterized to represent the number of failures before the r^{th} success occurs $(r = 1, 2, 3, 4, etc.)$.

In R:

	NBin(r,p) – # Trials	NBin*(r,p) – # Failures
To find $P(X = x)$:	`dnbinom(x-r, r, p)`	`dnbinom(x, r, p)`
To find $P(X \leq x)$:	`pnbinom(x-r, r, p)`	`pnbinom(x, r, p)`
To find smallest x^* such that $P(X \leq x^*) \geq c$:	`qnbinom(c, r, p) + r`	`qnbinom(c, r, p)`
To simulate *m* random draws:	`rnbinom(m, r, p) + r`	`rnbinom(m, r, p)`

What are the differences and similarities between Binomial and Negative Binomial random variables?

Hypergeometric Distribution

When Used?	When a random variable represents the number of items with a certain characteristic observed in a sample of size n, from a finite population where there are M total items with that characteristic and $N-M$ total items without the characteristic $(M = 0, 1, 2, 3, etc.)$. The following conditions are also required: 1. Population contains a finite number of elements, N, and the sample size, n, is large relative to the population size. 2. Each item in the population possesses one of two characteristics. 3. The random variable, X, is defined as the number of items with a certain characteristic observed in a sample of size n.
PMF:	A random variable X has a **Hypergeometric** (N,M,n) distribution if $f_X(\mathrm{x} \mid N,M,n) = \begin{cases} \dfrac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}} & x = 0,1,...,b;\ b = \min(n,M) \text{ and } n+M-N \le x \le b \\ 0 & otherwise \end{cases}$ Note: The notation $f_X(x \mid N,M,n)$ implies the value of the function is dependent on the values N, M, and n. The values N, M, and n are the **parameters** of the distribution.
MGF:	Not useful
Mean:	$E(X) = \dfrac{nM}{N}$
Variance:	$Var(X) = \dfrac{nM}{N}\left(\dfrac{(N-M)(N-n)}{N(N-1)}\right)$
Example:	Number of defective machine parts observed in 4 parts randomly sampled from a shipment of 25 total parts; etc.
Fun Facts:	As $N \to \infty$ and $M \to \infty$ with $p = M/N$, the Hypergeometric distribution approaches a Binomial (n, p) distribution.
In R:	To find P(X = x) use: `dhyper(x, M, N-M, n)` To find P(X ≤ x) use: `phyper(x, M, N-M, n)` To find smallest x* such that P(X ≤ x*) ≥ c, use: `qhyper(c, M, N-M, n)` To simulate *m* random draws, use: `rhyper(m, M, N-M, n)`

Example: A shipment of 50 refurbished smartphones were sent to a Bozeman distributor, and a family purchases six of them. Suppose 15 of the refurbished phones are still malfunctioning. What is the probability the family received at least one malfunctioning phone?

Poisson Distribution

When Used?	When a random variable represents a number of occurrences over a certain amount of time or space.
PMF:	A random variable X has a **Poisson** (λ) distribution if $$f_X(x \mid \lambda) = \begin{cases} \dfrac{e^{-\lambda}\lambda^x}{x!} & x = 0,1,2,\ldots;\ \lambda > 0 \\ 0 & else \end{cases}$$ Note: The notation $f_X(x \mid \lambda)$ implies the value of the function is dependent on the value λ. The value λ is the **parameter** of the distribution.
MGF:	$M_X(t) = \exp\left[\lambda\left(e^t - 1\right)\right]$
Mean:	$E(X) = \lambda$
Variance:	$Var(X) = \lambda$
Examples:	The number of accidents at an intersection in a week; the number of hits to a website each minute; the number of plants of a particular species found in a 1-m^2 area; etc.
Fun Facts:	Recursive Relationship: $P(X = x) = \frac{\lambda}{x} P(X = x-1)\ \ for\ x = 1,2,\ldots$ As $n \to \infty$ and $p \to 0$ with $np = \lambda$, the Binomial distribution approaches a Poisson distribution. If Y_1 is a $Poisson(\lambda_1)$ random variable and Y_2 is a $Poisson(\lambda_2)$ random variable, where Y_1 and Y_2 are independent, then $X = Y_1 + Y_2$ is a $Poisson(\lambda_1 + \lambda_2)$ random variable
In R:	To find P(X = x) use: `dpois(x, λ)` To find P(X ≤ x) use: `ppois(x, λ)` To find smallest x* such that P(X ≤ x*) ≥ c, use: `qpois(c, λ)` To simulate *m* random draws, use: `rpois(m, λ)`

Example: A certain type of tree has seedlings randomly dispersed in a large area, with the mean density of seedlings being approximately five per square yard. Let the random variable X represent the number of such seedlings in 0.25 square yards. What is the probability there are at least four seedlings in 0.25 square yards?

3.3 Continuous Distributions

In this section, we'll discuss some common families of continuous distributions: 1) Uniform; 2) Gamma; 3) Normal; 4) Beta; 5) Cauchy; 6) Lognormal; and 7) Double Exponential. This list of continuous distributions is by no means exhaustive.

Uniform Distribution

When Used?	When a random variable takes on any value between two limits a and b with constant probability.
PDF:	A random variable X has a **Uniform** (a,b) distribution if $$f_X(x\mid a,b)=\begin{cases}\frac{1}{b-a} & a\le x\le b\\ 0 & else\end{cases}$$ Note: The values a and b are the **parameters** of the distribution. The parameter a represents the minimum value the random variable can assume with non-zero probability, and b represents the maximum.
MGF:	$M_X(t)=\dfrac{e^{bt}-e^{at}}{t(b-a)},\ t\neq 0$ and $M_X(t)=1,\ t=0$
Mean:	$E(X)=\dfrac{a+b}{2}$
Variance:	$Var(X)=\dfrac{(b-a)^2}{12}$
Fun Fact:	The Uniform distribution is often used for noninformative priors in Bayesian Statistics.
In R:	To get the density, use: `dunif(x, a, b)` To find P(X ≤ x) use: `punif(x, a, b)` To find x* such that P(X ≤ x*) = c, use: `qunif(c, a, b)` To simulate *m* random draws, use: `runif(m, a, b)`

Gamma Distribution

When Used? This is a large, flexible family of distributions with many uses, such as when a random variable describes the time between events or the time to an event occurring, such as an equipment failure (reliability analysis) or death (survival analysis).

PDF: A random variable X has a **Gamma** (α, β) distribution if

$$f_X(x \mid \alpha, \beta) = \begin{cases} \dfrac{1}{\Gamma(\alpha)\beta^{\alpha}} x^{\alpha-1} e^{-x/\beta} & 0 \le x < \infty;\ \alpha, \beta > 0 \\ 0 & else \end{cases}$$

where $\Gamma(\alpha) = \int_0^{\infty} t^{\alpha-1} e^{-t}\, dt$. For $\alpha > 0$, $\Gamma(\alpha+1) = \alpha\Gamma(\alpha)$.

Note: The values α and β are the **parameters** of the distribution**.** The α parameter is known as the shape parameter, because it most influences the peakedness of the distribution, and the β parameter is called the scale parameter, because most of its influence is on the spread of the distribution. For any integer $n > 0$, $\Gamma(n) = (n-1)!$ Also, $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

MGF: $M_X(t) = (1 - \beta t)^{-\alpha}$, $t < 1/\beta$

Mean: $E(X) = \alpha\beta$

Variance: $Var(X) = \alpha\beta^2$

Fun Facts: If $X \sim$ Gamma (α, β), where α is an integer, then for any x, $P(X \le x) = P(Y \ge \alpha)$, where $Y \sim$ Poisson (x/β).

Special Cases:

Chi-Squared Distribution

The chi-squared distribution is a special case of the gamma distribution where $\alpha = p/2$ and $\beta = 2$, where p is a positive integer (a.k.a. degrees of freedom). The chi-squared distribution plays an important role in statistical inference, especially when sampling from a normal distribution (see Chapter 5).

A random variable X has a χ_p^2 distribution if

$$f_X(x \mid p) = \begin{cases} \dfrac{1}{\Gamma(p/2) 2^{p/2}} x^{p/2 - 1} e^{-x/2} & 0 \le x < \infty \\ 0 & else \end{cases}$$

$M_X(t) = (1 - 2t)^{-p/2}$, $t < 1/2$

$E(X) = p$

$Var(X) = 2p$

Exponential Distribution
The exponential distribution is a special case of the gamma distribution where $\alpha = 1$, and it is often used to model lifetimes. The exponential distribution has the "memoryless" property.

A random variable X has an **Exponential**(β) distribution if

$$f_X(x \mid \beta) = \begin{cases} \frac{1}{\beta} e^{-x/\beta} & 0 \le y < \infty;\ \beta > 0 \\ 0 & else \end{cases}$$

$$M_X(t) = (1-\beta t)^{-1},\ t < 1/\beta$$
$$E(X) = \beta$$
$$Var(X) = \beta^2$$

In R:

	Gamma (α,β)	**Exp** (β)	**Chi-squared** (p)
To get the density, use	`dgamma(x, α , 1/β)`	`dexp(x, 1/β)`	`dchisq(x, p)`
To find P(X ≤ x) use	`pgamma(x, α , 1/β)`	`pexp(x, 1/β)`	`pchisq(x, p)`
To find x* such that P(X ≤ x*) = c, use	`qgamma(c, α , 1/β)`	`qexp(c, 1/β)`	`qchisq(c, p)`
To simulate *m* random draws, use	`rgamma(m, α , 1/β)`	`rexp(m, 1/β)`	`rchisq(m, p)`

Note: R uses the rate parameter $1/\beta$ instead of the scale parameter β for the gamma and exponential distribution functions. These merely make use of another parameterization of these distributions, denoted with the following notation: $X \sim$ Gamma*(α,β) or $X \sim$ Exp*(β).

Derived Distributions:

Weibull Distribution

If $X \sim$ Exponential(β), then $Y = X^{1/\gamma}$ has a **Weibull**(γ,β) distribution. The Weibull distribution is important in modeling failure time data, as well as fatigue and breaking strength of materials. It is also useful in survival analysis for modeling hazard functions, which give the probability that an object survives a little past time t given that the object survives to time t.

A random variable Y has a **Weibull**(γ,β) distribution if

$$f_Y(y \mid \gamma,\beta) = \begin{cases} \frac{\gamma}{\beta} y^{\gamma-1} e^{-y^\gamma/\beta} & 0 \le y < \infty;\ \gamma, \beta > 0 \\ 0 & else \end{cases}$$

Others (see Exercise 3.24)

- If $X \sim$ Exponential(β), then $Y = \left(\frac{2X}{\beta}\right)^{1/2}$ has a **Rayleigh** distribution.
- If $X \sim$ Gamma(α,β), then $Y = \frac{1}{X}$ has an **Inverted Gamma** (α,β) distribution.
- If $X \sim$ Gamma$\left(\frac{3}{2},\beta\right)$, (β), then $Y = \left(\frac{X}{\beta}\right)^{1/2}$ has a **Maxwell** distribution.
- If $X \sim$ Exponential(1), then $Y = \alpha - \gamma \ln(X)$ has a **Gumbel**(α,γ) distribution.

Normal Distribution

When Used? Often! The normal distribution (also called the Gaussian distribution) is the most widely used continuous probability distribution, mainly because it is tractable analytically, it follows the familiar bell shape which fits with a lot of population models, and the central limit theorem says that, with a large enough sample, the normal distribution can be used to approximate a large variety of other distributions (e.g., Normal approximation to the Binomial).

PDF: A random variable X has a **Normal** $\left(\mu,\sigma^2\right)$ distribution if

$$f_X(x \mid \mu,\sigma^2) = \begin{cases} \dfrac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2 / \left(2\sigma^2\right)} \end{cases} \qquad -\infty < x < \infty;\, -\infty < \mu < \infty,\, \sigma^2 > 0$$

Note: The values μ and σ^2 are the **parameters** of the distribution.

MGF: $M_X\left(t\right) = \exp\left(\mu t + \dfrac{t^2\sigma^2}{2}\right)$

Mean: $E\left(X\right) = \mu$

Variance: $Var\left(X\right) = \sigma^2$

Fun Facts: The normal distribution was published by de Moivre in 1733!
The chi-squared, *t* and *F* distributions can be derived from the normal distribution (see Chapter 5).
The normal distribution follows the 68-95-99.7 Rule:

For large n and not extreme p, the distribution of $X \sim$ Binomial$\left(n,p\right)$ can be approximated by a Normal$\left(np, np\left(1-p\right)\right)$ distribution. This approximation can be improved by using a "continuity correction" (see pg. 105).

Special Cases:

Standard Normal Distribution

The standard normal distribution is a special case of the normal distribution, where $\mu = 0$ and $\sigma^2 = 1$. If $Y \sim N\left(\mu,\sigma^2\right)$, then $Z = \dfrac{Y-\mu}{\sigma} \sim N\left(0,1\right)$.

A random variable Z has a **Normal** $(0,1)$ distribution if

$$f_Z(z) = \begin{cases} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \end{cases} \qquad -\infty < z < \infty$$

$$M_Z(t) = \exp\left(t^2/2\right)$$

$$E(Z) = 0$$

$$Var(Z) = 1$$

All normal probabilities may be calculated in terms of the standard normal.

<u>In R</u>: To get the density, use: `dnorm(x, μ, σ)`
To find P(X ≤ x) use: `pnorm(x, μ, σ)`
To find x* such that P(X ≤ x*) = c, use: `qnorm(c, μ, σ)`
To simulate *m* random draws, use: `rnorm(m, μ, σ)`

Beta Distribution

When Used?	When a random variable is defined over the interval $0 \le x \le 1$; typically the beta distribution is used to model proportions, which naturally lie between 0 and 1.
PDF:	A random variable X has a **Beta** (α, β) distribution if $$f_X(x \mid \alpha, \beta) = \begin{cases} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1} & 0 \le x \le 1;\ \alpha, \beta > 0 \\ 0 & else \end{cases}$$ Note: The values α and β are the **parameters** of the distribution. Often, the normalizing constant is expressed as a function of the beta function $B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$.
MGF:	Not useful
Mean:	$E(X) = \frac{\alpha}{\alpha+\beta}$
Variance:	$Var(X) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$
Fun Fact:	If $\alpha = \beta = 1$, the beta distribution is a Uniform $(0,1)$ distribution.
In R:	To get the density, use: `dbeta(x, α , β)` To find P(X ≤ x) use: `pbeta(x, α , β)` To find x* such that P(X ≤ x*) = c, use: `qbeta(c, α , β)` To simulate *m* random draws, use: `rbeta(m, α , β)`

How can the Beta distribution be applied to a random variable defined over the interval $a \le y \le b$, where $a \ne 0$ and $b \ne 1$?

Cauchy Distribution

When Used?	Surprisingly more often than one would expect. Usually this distribution arises when comparing ratios of standard normal random variables.
PDF:	A random variable X has a **Cauchy** (θ) distribution if $$f_X(x\mid\theta)=\begin{cases}\dfrac{1}{\pi\left[1+(x-\theta)^2\right]} & -\infty<x<\infty;\ -\infty<\theta<\infty\\ 0 & else\end{cases}$$ Note: The value θ is the **parameter** of the distribution**.** This parameter is the median of the distribution.
MGF:	Does not exist
Mean:	Does not exist
Variance:	Does not exist
Fun Facts:	The Cauchy distribution is a symmetric, bell-shaped distribution. No moments of the Cauchy distribution exist. The ratio of two standard normal random variables has a Cauchy distribution.
In R:	To get the density, use: `dcauchy(x, θ)` To find P(X ≤ x) use: `pcauchy(x, θ)` To find x* such that P(X ≤ x*) = c, use: `qcauchy(c, θ)` To simulate *m* random draws, use: `rcauchy(m, θ)`
Another Version:	The following is a two-parameter version of a Cauchy (θ,σ) distribution: $$f_X(x\mid\theta,\sigma)=\begin{cases}\dfrac{1}{\sigma\pi\left[1+\left(\dfrac{x-\theta}{\sigma}\right)^2\right]} & -\infty<x<\infty;\ -\infty<\theta<\infty,\ \sigma>0\\ 0 & else\end{cases}$$ Similar to the one-parameter version, the moments and the mgf do not exist for this distribution.

Lognormal Distribution

When Used?	When the variable of interest is skewed to the right, and the natural log (log transformation) of the data is normally distributed, allowing the use of normal-theory statistical procedures. Examples include income, movement data, and electrical measurements.
PDF:	If $Y \sim N\left(\mu,\sigma^2\right)$, the $X = e^Y$ follows a **Lognormal** $\left(\mu,\sigma^2\right)$ distribution: $$f_X(x \mid \mu,\sigma^2) = \begin{cases} \frac{1}{x\sqrt{2\pi\sigma^2}} e^{-\left(\ln(x)-\mu\right)^2 / \left(2\sigma^2\right)} \end{cases} \quad 0 < x < \infty;\ -\infty < \mu < \infty,\ \sigma^2 > 0$$ Note: The values μ and σ^2 are the **parameters** of the distribution.
MGF:	Does not exist
Mean:	$E(X) = \exp\left(\mu + \frac{\sigma^2}{2}\right)$
Variance:	$Var(X) = \mathrm{e}^{2\left(\mu+\sigma^2\right)} - e^{2\mu+\sigma^2}$
In R:	To get the density, use: `dlnorm(x,` μ , σ `)` To find P(X ≤ x) use: `plnorm (x,` μ , σ `)` To find x* such that P(X ≤ x*) = c, use: `qlnorm (c,` μ , σ `)` To simulate *m* random draws, use: `rlnorm (m,` μ , σ `)`

Double Exponential / Laplace Distribution

When Used?	Great question! We should investigate this more.
PDF:	A random variable X has a **Double Exponential** (μ,σ) distribution if $$f_X(x\mid\mu,\sigma)=\begin{cases}\frac{1}{2\sigma}e^{-\lvert x-\mu\rvert/\sigma} & -\infty<x<\infty;\ -\infty<\mu<\infty,\ \sigma>0\end{cases}$$ Note: The values μ and σ are the **parameters** of the distribution.
MGF:	$M_X(t)=\frac{\exp(\mu t)}{1-(\sigma t)^2},\quad \lvert t\rvert<1/\sigma$
Mean:	$E(X)=\mu$
Variance:	$Var(X)=2\sigma^2$
Fun Facts:	The double exponential distribution is also called the Laplace distribution. The double exponential distribution is formed by reflecting the exponential distribution around its mean.