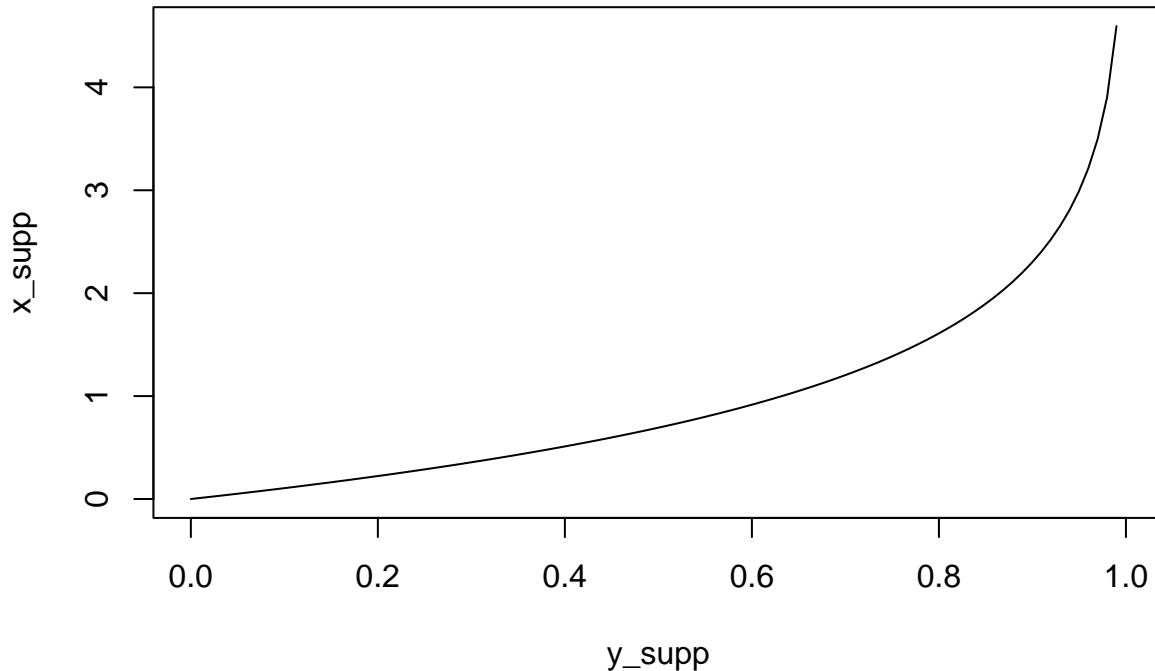


Example: Probability Integral Transform

Generate a random sample from $F_X(x) = [1 - \exp^{-x}]I_{(0,\infty)}(x)$. Note: $X \sim \text{Exp}(\beta = 1)$.

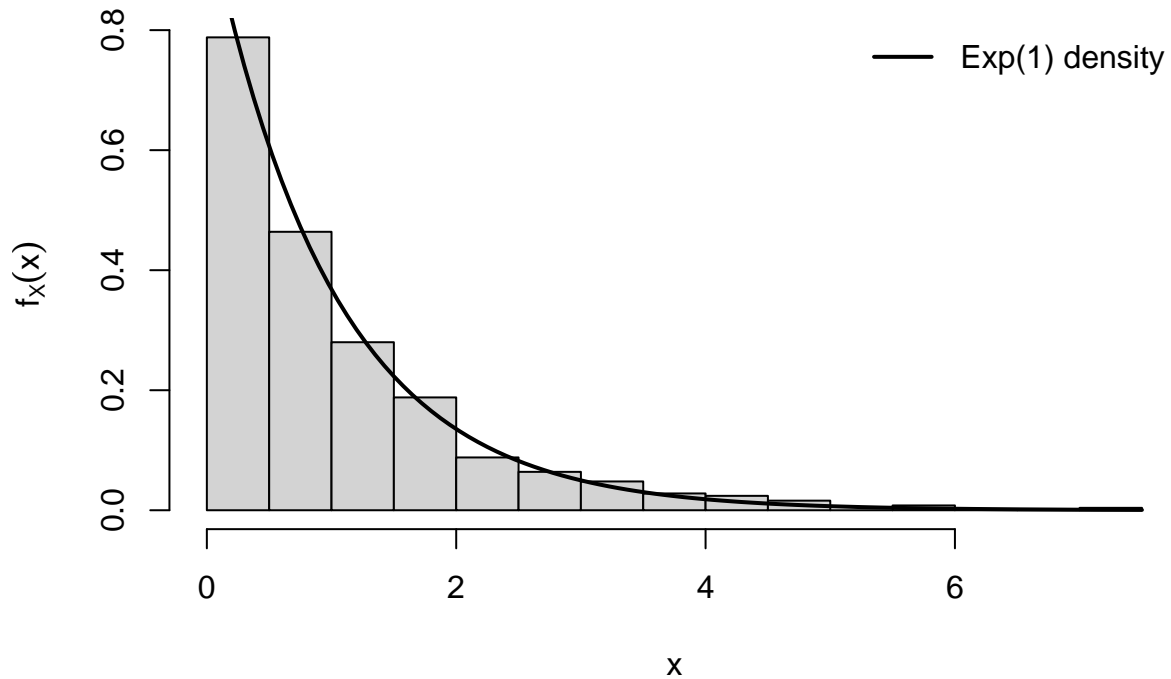
```
# Y = F_X(X) = 1 - exp(-X) <=> X = -ln(1-Y)
# THM 2.1.10 => Y ~ Unif(0,1)
# Plot the transformation on the support
y_supp <- seq(from = 0, to = 1, length = 100)
x_supp <- -log(1 - y_supp)
plot(x_supp ~ y_supp, type = "l",
     main = "Inverse Transformation from Y to X")
```

Inverse Transformation from Y to X



```
# Generate random sample from uniform distribution
y <- runif(n = 500, min = 0, max = 1)
# Probability integral transformation (PIT)
x_samp <- -log(1-y)
# Compare samples to actual exp(1) distribution
hist(x_samp, nclass = 25, freq = F,
     xlab = "x", ylab = expression(f[X](x)),
     main = "500 draws from Exp(1) using PIT")
curve(dexp(x, rate = 1/1), lwd = 2, add = T)
legend("topright", bty = "n", lwd = 2, lty = 1,
     legend = "Exp(1) density")
```

500 draws from Exp(1) using PIT



```
# qqplot to compare theoretical exp(1) dist to samples
qqplot(qexp(ppoints(500), 1), x_samp,
       xlab = "Exp(1) Quantiles", ylab = "Sample using PIT",
       main = "QQ plot for sample using PIT")
abline(a = 0, b = 1)
```

QQ plot for sample using PIT

