

M/STAT 501: Weighted Least Squares Activity

Professor Ratings

Bleske-Rechek and Fritsch (2011) analyzed a data set of the ratings of 366 instructors at one large campus in the Midwest. Each instructor in the data had at least 10 ratings over a several year period. Students provided ratings from 1 (worst) to 5 (best). These data are built into R in the `alr4` library.

```
library(pracma) # eye function  
library(alr4) # data set  
data(Rateprof)
```

Let Y_{ij} be the quality rating of the i th instructor by the j th student, $j = 1, \dots, n_i$, and $\bar{Y}_i = \sum_{j=1}^{n_i} Y_{ij}/n_i$ be the mean quality rating for the i th instructor. Similarly, let x_{1ij} and x_{2ij} be the easiness and helpfulness ratings, respectively, of the i th instructor by the j th student, with mean easiness and mean helpfulness for the i th instructor denoted by \bar{x}_{1i} and \bar{x}_{2i} . Note that the data set only reports mean ratings, not individual student's ratings.

Do in class:

1. Assume $E(Y_{ij}|X) = \beta_0 + \beta_1 x_{1ij} + \beta_2 x_{2ij}$ and $Var(Y_{ij}|X) = \sigma^2$. Derive the expression for $E(\bar{Y}_i|X)$ and $Var(\bar{Y}_i|X)$.
2. If we fit a linear model to $E(\bar{Y}_i|X)$, would it meet the constant variance assumption? Explain why or why not.
3. Let $\mathbf{Y} = (\bar{Y}_1 \ \bar{Y}_2 \ \dots \ \bar{Y}_{366})'$ be the response vector for this data set with variance-covariance matrix $Var(\mathbf{Y}) = \sigma^2 \boldsymbol{\Omega}$. Write out the elements in the first four rows and first four columns of $\boldsymbol{\Omega}$, i.e., report the 4×4 matrix that consists of elements in rows 1-4 and columns 1-4.
4. Generate plots to investigate the distributions and relationships between the three variables of interest.
Add code here
5. Fit the weighted least squares model. Write the equation of the fitted model, and interpret each of the three fitted coefficients in context of the problem.
Add code here
6. Fit the ordinary least squares fit to these data. How do the coefficients change? How does the residual standard error change? Why?
Add code here