9.1

Using that $S = \lambda f.\lambda n.\lambda x.fn(fx)$:

$$
\begin{aligned}
&S(\lambda x.m)(\lambda x.n) = (\lambda f.\lambda n.\lambda x.fn(fx))(\lambda x.m)(\lambda x.n)\\
&= (\lambda f.\lambda x.f((\lambda x.m)fx))(\lambda x.n)\\
&= \lambda x.(\lambda x.m)x((\lambda x.n)x)\\
&= \lambda x.m((\lambda x.n)x) = \lambda x.mn
\end{aligned}
$$

9.2

$$\text{xor} = \lambda b.\lambda c.b(\text{not } c)c$$

It is commutative if xor true false = xor false true
Checking:

$$\text{xor true true } =_\beta \text{ true (not true) true } =_\beta \text{ false}$$

$$\text{xor true false } =_\beta \text{ true (not false) false } =_\beta \text{ true}$$

$$\text{xor false true } =_\beta \text{ false (not true) true } =_\beta \text{ true}$$

$$\text{xor false false } =_\beta \text{ false (not false) false } =_\beta \text{ not true } =_\beta \text{ false}$$

As we can see, it is commutative.

9.3 Problem

$$\text{pred} = \lambda n \lambda f \lambda x.(n(\lambda g(\lambda h.h(gf))))(\lambda u.x)(\lambda u.u)$$

Here we can see that $\lambda u.x$ is $const_x$.
$\lambda u.u$ is the identity function id.
Looking at this part:

$$\lambda g(\lambda h.h(gf))$$

Say that $evaluator_y == \lambda h.hy$

$$F = \lambda g.evaluator_{g(f)}$$

Now, $F^n const_x = evaluator_{f^{n-}(x)}$

pred = $\lambda nfx.F^n const_x id = \lambda nfx.ealuator_{f^{n-1}(x)} id = \lambda nfx.id(f^{n-1}(x)) = \lambda nfx.f^{n-1}(x)$

Now, for the proof:

$$\begin{aligned}
&\text{pred(succ m)) = pred}(\lambda f.\lambda n.\lambda x.fn(fx)m) = \text{pred (m + 1)) =} \\
&= \lambda n \lambda f \lambda x.(n(\lambda g(\lambda h.h(gf))))(\lambda u.x)id(m+1) = \\
&= \lambda f \lambda x.(m+1)(\lambda g(\lambda h.h(gf))))(\lambda u.x)id = \\
&= \lambda f \lambda x.(\lambda h.f(...f(x))id = m
\end{aligned}$$

9.4 Problem

Factorial is defined as follows:

fact n = if (iszro n) 1 mult n (fact (pred n))

$$3 = \lambda sz.sssz$$

Hence,

fact 3 = if (iszro 3) 1 mult 3 (fact (pred 3)) = mult 3 (fact (pred 3)) =

= mult 3 (fact 2) = mult 3 (if (iszro 2) 1 mult 2 (fact (pred 2))) = if (iszro 3) 1 mult 3 (fact (pred 3)) =

= mult 3 (fact (pred 3)) = mult 3 (fact 2) = mult 3 (mult 2 (fact (pred 2))) = mult 3 (mult 2 (fact 1)) =

= mult 3 (mult 2 (if (iszro 1) 1 mult 1 (fact (pred 1)))) = mult 3 (mult 2 (mult 1 (fact (pred 1)))) = mul

= mult 3 (mult 2 (mult 1 (if (iszro 0) 1 mult 0 (fact (pred 0))))) = mult 3 (mult 2 (mult 1 (1)))) =

= mult 3(mult 2 (mult 1 1)) = mult 3 (mult 2 1) = mult 3 2 = 6

9.5 Problem

$$\text{mult m n} = \lambda m.\lambda n.\lambda f.\lambda x.n(mf)x$$

means the same as ”apply s” n times. Thus, when we write m (n s) z, we’re adding n to zero m times (i.e., m times).
Now to prove that 1 is the unit of multiplication:

$$\text{mult 1 n} = \lambda f.\lambda x.(\lambda f x.(f x))(n f)x = n$$