

The stack of tasks

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Introduction

Theoretical foundations

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Outline

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The stack of tasks provides a control framework for real-time redundant manipulator control

- ▶ implementation of a data-flow,
- ▶ control of the graph by python scripting,
- ▶ task-based hierarchical control,
- ▶ portable: tested on HRP-2, Nao, Romeo.

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Rigid body \mathcal{B}

- Configuration represented by an homogeneous matrix

$$M_{\mathcal{B}} = \begin{pmatrix} R_{\mathcal{B}} & \mathbf{t}_{\mathcal{B}} \\ 0 & 1 \end{pmatrix} \in SE(3)$$

$$R_{\mathcal{B}} \in SO(3) \Leftrightarrow R_{\mathcal{B}}^T R_{\mathcal{B}} = I_3$$

Point $\mathbf{x} \in \mathbb{R}^3$ in local frame of \mathcal{B} is moved to $\mathbf{y} \in \mathbb{R}^3$ in global frame:

$$\begin{pmatrix} \mathbf{y} \\ 1 \end{pmatrix} = M_{\mathcal{B}} \begin{pmatrix} \mathbf{x} \\ 1 \end{pmatrix}$$

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Rigid body \mathcal{B}

- ▶ Velocity represented by $(\mathbf{v}_{\mathcal{B}}, \omega_{\mathcal{B}}) \in \mathbb{R}^6$ where

$$\dot{R}_{\mathcal{B}} = \hat{\omega}_{\mathcal{B}} R_{\mathcal{B}}$$

and

$$\hat{\omega} = \begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix}$$

is the matrix corresponding to the cross product operator

- ▶ Velocity of point P on \mathcal{B}

$$\mathbf{v}_p = \dot{\mathbf{t}}_{\mathcal{B}} + \omega_{\mathcal{B}} \times O_{\mathcal{B}} \vec{P}$$

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Configuration space

- ▶ Robot: set of rigid-bodies linked by joints $\mathcal{B}_0, \dots \mathcal{B}_m$.
- ▶ Configuration: position in space of each body.

$$\mathbf{q} = (\mathbf{q}_{waist}, \theta_1, \dots, \theta_{n-6}) \in SE(3) \times \mathbb{R}^{n-6}$$
$$\mathbf{q}_{waist} = (x, y, z, roll, pitch, yaw)$$



- ▶ Position of \mathcal{B}_i depends on \mathbf{q} :

$$M_{\mathcal{B}_i}(\mathbf{q}) \in SE(3)$$

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$$\begin{aligned} \dot{\mathbf{q}} &= (\dot{x}, \dot{y}, \dot{z}, \omega_x, \omega_y, \omega_z, \dot{\theta}_1, \dots, \dot{\theta}_{n-6}) \\ \omega &\in \mathbb{R}^3 \end{aligned}$$

► Velocity of \mathcal{B}_i



$$\begin{pmatrix} \mathbf{v}_{\mathcal{B}_i} \\ \omega_{\mathcal{B}_i} \end{pmatrix} (\mathbf{q}, \dot{\mathbf{q}}) = J_{\mathcal{B}_i}(\mathbf{q}) \cdot \dot{\mathbf{q}} \in \mathbb{R}^6$$

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- ▶ Definition: function of the
 - ▶ robot configuration,
 - ▶ time and
 - ▶ possibly external parameters

that should converge to 0:

$$T \in C^\infty(\mathcal{C} \times \mathbb{R}, \mathbb{R}^m)$$

- ▶ Example: position tracking of an end-effector \mathcal{B}_{ee}
 - ▶ $M(\mathbf{q}) \in SE(3)$ position of the end-effector,
 - ▶ $M^*(t) \in SE(3)$ reference position

$$T(\mathbf{q}, t) = \begin{pmatrix} \mathbf{t}(M^{*-1}(t)M(\mathbf{q})) \\ u_\theta(R^{*-1}(t)R(\mathbf{q})) \end{pmatrix}$$

where

- ▶ $\mathbf{t}()$ is the translation part of an homogeneous matrix,
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Hierarchical task based control

Given

- ▶ a configuration \mathbf{q} ,
- ▶ two tasks of decreasing priorities:
 - ▶ $T_1 \in C^\infty(\mathcal{C} \times \mathbb{R}, \mathbb{R}^{m_1})$,
 - ▶ $T_2 \in C^\infty(\mathcal{C} \times \mathbb{R}, \mathbb{R}^{m_2})$,

compute a control vector $\dot{\mathbf{q}}$

- ▶ that makes T_1 converge toward 0 and
- ▶ that makes T_2 converge toward 0 if possible.

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Hierarchical task based control

Jacobian:

- ▶ we denote

- ▶ $J_i = \frac{\partial T_i}{\partial \mathbf{q}}$ for $i \in \{1, 2\}$

- ▶ then

- ▶ $\forall \mathbf{q} \in \mathcal{C}, \forall t \in \mathbb{R}, \forall \dot{\mathbf{q}} \in \mathbb{R}^n, \dot{T}_i = J_i(\mathbf{q}, t)\dot{\mathbf{q}} + \frac{\partial T_i}{\partial t}(\mathbf{q}, t)$

We try to enforce

- ▶ $\dot{T}_1 = -\lambda_1 T_1 \Rightarrow T_1(t) = e^{-\lambda_1 t} T_1(0) \rightarrow 0$

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- ▶ λ_1 and λ_2 are called the gains associated to T_1 and T_2 .

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Moore Penrose pseudo-inverse

Given a matrix $A \in \mathbb{R}^{m \times n}$, the Moore Penrose pseudo inverse $A^+ \in \mathbb{R}^{n \times m}$ of A is the unique matrix satisfying:

$$\begin{aligned}AA^+A &= A \\A^+AA^+ &= A^+ \\(AA^+)^T &= AA^+ \\(A^+A)^T &= A^+A\end{aligned}$$

Given a linear system:

$$Ax = b, \quad A \in \mathbb{R}^{m \times n}, \quad x \in \mathbb{R}^n, \quad b \in \mathbb{R}^m$$

$x = A^+b$ minimizes

- ▶ $\|Ax - b\|$ over \mathbb{R}^n ,
- ▶ $\|x\|$ over $\operatorname{argmin} \|Ax - b\|$.

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Resolution of the first constraint:

$$\dot{T}_1 = J_1 \dot{\mathbf{q}} + \frac{\partial T_1}{\partial t} = -\lambda_1 T_1 \quad (1)$$

$$J_1 \dot{\mathbf{q}} = -\lambda_1 T_1 - \frac{\partial T_1}{\partial t} \quad (2)$$

$$\dot{\mathbf{q}}_1 \triangleq -J_1^+ (\lambda_1 T_1 + \frac{\partial T_1}{\partial t}) \quad (3)$$

Where J_1^+ is the (Moore Penrose) pseudo-inverse of J_1 .
 $\dot{\mathbf{q}}_1$ minimizes

- ▶ $\|J_1 \dot{\mathbf{q}} + \lambda_1 T_1 + \frac{\partial T_1}{\partial t}\| = \|\dot{T}_1 + \lambda_1 T_1\|$
- ▶ $\|\dot{\mathbf{q}}\|$ over $\operatorname{argmin} \|J_1 \dot{\mathbf{q}} + \lambda_1 T_1 + \frac{\partial T_1}{\partial t}\|$

Hence,

- ▶ if $\lambda_1 T_1 + \frac{\partial T_1}{\partial t}$ is in $\operatorname{Im}(J_1)$, (1) is satisfied

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Hierarchical task based control

In fact

$$\forall u \in \mathbb{R}^n, \quad J_1 (\dot{\mathbf{q}}_1 + (I_n - J_1^+ J_1) u) = J_1 \dot{\mathbf{q}}_1$$

therefore,

$$\dot{\mathbf{q}} = \dot{\mathbf{q}}_1 + (I_n - J_1^+ J_1) u$$

also minimizes $\|J_1 \dot{\mathbf{q}} + \lambda_1 T_1 + \frac{\partial T_1}{\partial t}\|$.

$P_1 = (I_n - J_1^+ J_1)$ is a projector on J_1 kernel:

$$J_1 P_1 = 0$$

$\forall u \in \mathbb{R}^n$, if $\dot{\mathbf{q}} = P_1 u$, then, $\dot{T}_1 = \frac{\partial T_1}{\partial t}$.

Hierarchical task based control

In fact

$$\forall u \in \mathbb{R}^n, \quad J_1 (\dot{\mathbf{q}}_1 + (I_n - J_1^+ J_1) u) = J_1 \dot{\mathbf{q}}_1$$

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Controlling the second task

We have

$$\dot{\mathbf{q}} = \dot{\mathbf{q}}_1 + P_1 u$$

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We want

$$\dot{T}_2 = -\lambda_2 T_2$$

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$$\dot{\mathbf{q}}_2 \triangleq \dot{\mathbf{q}}_1 + P_1 \mathbf{u}$$

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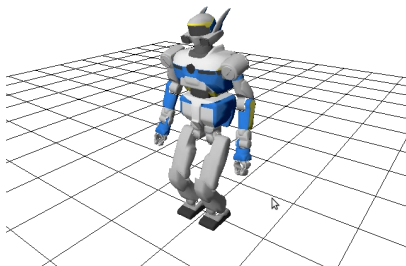
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Example



- ▶ T_1 : position of the feet + projection of center of mass,
- ▶ T_2 : position of the right wrist.

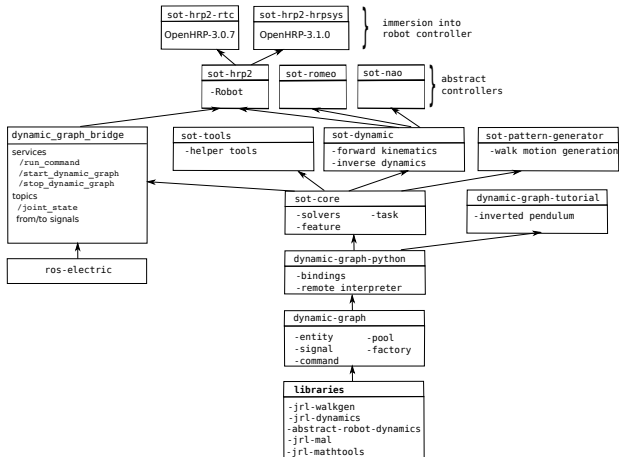
Outline

Introduction

Theoretical foundations

Software

Architecture overview



Libraries

- ▶ `jrl-mathtools`: implementation of small size matrices,
 - ▶ to be replaced by Eigen
- ▶ `jrl-mal`: abstract layer for matrices,
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- ▶ `abstract-robot-dynamics`: abstraction for humanoid robot description,
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- ▶ Entity
 - ▶ Signal: synchronous interface
 - ▶ Command: asynchronous interface
- ▶ Factory
 - ▶ builds a new entity of requested type,
 - ▶ new entity types can be dynamically added (advanced).
- ▶ Pool
 - ▶ stores all instances of entities,
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Synchronous interface storing a given data type

- ▶ output signals:
 - ▶ recomputed by a callback function, or
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Command

Asynchronous interface

- ▶ input in a fixed set of types,
- ▶ trigger an action,
- ▶ returns a result in the same set of types.

dynamic-graph-python

Python bindings to dynamic-graph

- ▶ module `dynamic_graph` linked to `libdynamic-graph.so`
 - ▶ class `Entity`
 - ▶ each C++ entity class declared in the factory generates a python class of the same name,
 - ▶ signals are instance members,
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 - ▶ method `help` lists commands
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dynamic-graph-tutorial

Simple use case for illustration

- ▶ Definition of 2 entity types
 - ▶ InvertedPendulum
 - ▶ input signal: force
 - ▶ output signal: state
 - ▶ FeedbackController
 - ▶ input signal: state
 - ▶ output signal: force

dynamic-graph-tutorial

```
>>> from dynamic_graph.tutorial import InvertedPendulum, FeedbackController
>>> a = InvertedPendulum ('IP')
>>> b = FeedbackController ('K')
>>> a.displaySignals ()
--- <IP> signal list:
|-- <Sig:InvertedPendulum(IP)::input(double)::force (Type Cst) AUTOPLUGGED
'-- <Sig:InvertedPendulum(IP)::output(vector)::state (Type Cst)
>>> a.help ()
Classical inverted pendulum dynamic model
```

List of commands:

```
-----
getCartMass:           Get cart mass
getPendulumLength:     Get pendulum length
getPendulumMass:       Get pendulum mass
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Integrate dynamics for time step provided as input

take one floating point number as input

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Class FeatureAbstract

- ▶ function of the robot and environment states
 - ▶ position of an end-effector,
 - ▶ position of a feature in an image (visual servoing)
- ▶ with values in a Lie group G ($SO(3)$, $SE(3)$, \mathbb{R}^n, \dots),
- ▶ with a mapping e from G into \mathbb{R}^m such that

$$e(0_G) = 0$$

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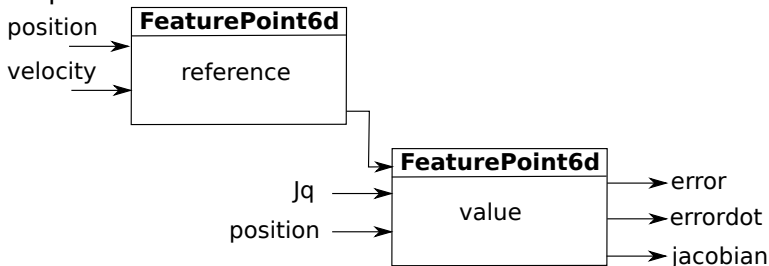
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Feature

When paired with a reference, features become *tasks*.

► Example

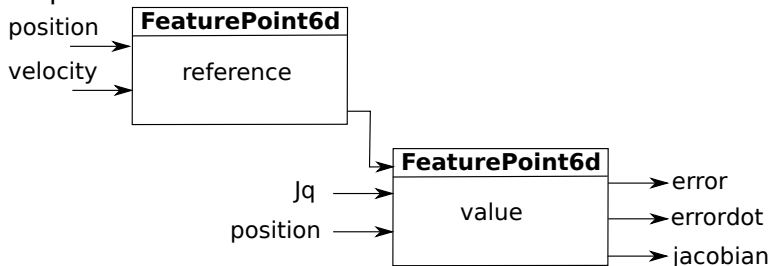


- $\text{error} = e(\text{value.position} \ominus \text{reference.position})$
- **error****dot**: derivative of error when `value.position` is constant.

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► Example

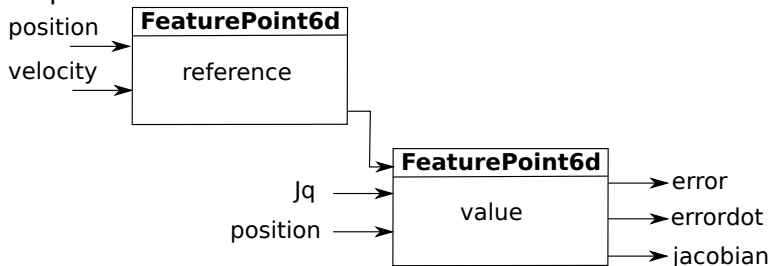


- $\text{error} = \mathbf{e}(\text{value.position} \ominus \text{reference.position})$
- **error****dot**: derivative of error when **value.position** is constant.

Feature

When paired with a reference, features become *tasks*.

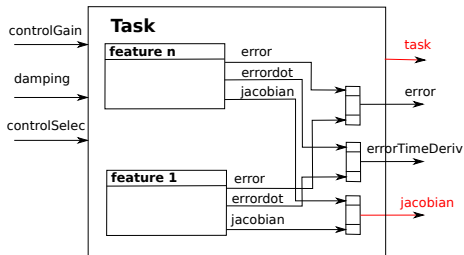
► Example



- $\text{error} = \mathbf{e}(\text{value.position} \ominus \text{reference.position})$
- **errordot**: derivative of error when `value.position` is constant.

Task

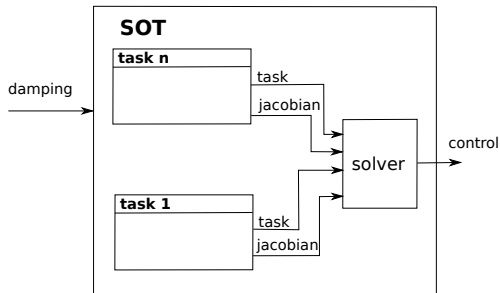
- ▶ Collection of features with a control gain,
- ▶ implements abstraction `TaskAbstract`



- ▶ `task = -controlGain.error`

Solver SOT

Hierarchical task solver



- computes robot joint velocity

sot-dynamic

`dynamic_graph.sot.dynamics.Dynamic` builds a kinematic chain from a file and

- ▶ computes forward kinematics
 - ▶ position and Jacobian of end effectors (wrists, ankles),
 - ▶ position of center of mass
- ▶ computes dynamics
 - ▶ inertia matrix.

Packages specific to robots

sot-hrp2

- ▶ defines a class `Robot` that provides
 - ▶ ready to use features for feet, hands, gaze and center of mass,
 - ▶ ready to use tasks for the same end effectors,
 - ▶ an entity `Dynamic`,
 - ▶ an entity `Device` (interface with the robot control system)

Utilities

- ▶ `dynamic_graph.writeGraph (filename)`: **writes the current graph in a file using graphviz dot format.**
- ▶ `dynamic_graph.sot.core.FeaturePosition` wraps two `FeaturePoint6d`: a value and a reference,
- ▶ `MetaTask6d`:
- ▶ `MetaTaskPosture`:
- ▶ `MetaTaskKine6d`:
- ▶ `MetaTaskKinePosture`:
- ▶ `MetaTaskCom`:

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Installation

Through `robotpkg`

- ```

▶ git clone http://trac.laas.fr/git/robots/robotpkg.git
 cd robotpkg
 ./bootstrap/bootstrap --prefix=<your_prefix>
 cd motion/sot-dynamic

 make install

```

Through github:

# Installation

## Through robotpkg

- ▶ 

```
git clone http://trac.laas.fr/git/robots/robotpkg.git
cd robotpkg
./bootstrap/bootstrap --prefix=<your_prefix>
cd motion/sot-dynamic

make install
```

## Through github:

- ▶ 

```
git clone git://github.com/jrl-umi3218/jrl-mal.git
git clone git://github.com/jrl-umi3218/jrl-mathtools.git
git clone git://github.com/laas/abstract-robot-dynamics.git
git clone git://github.com/jrl-umi3218/jrl-dynamics.git
git clone git://github.com/jrl-umi3218/dynamic-graph.git
git clone git://github.com/jrl-umi3218/dynamic-graph-python.git
git clone git://github.com/jrl-umi3218/sot-core.git
git clone git://github.com/laas/sot-tools.git

git clone git://github.com/jrl-umi3218/sot-dynamic.git
```
- ▶ for each package,  

```
mkdir package/build
cd package/build
cmake -DCMAKE_INSTALL_PREFIX=<your_prefix> ..

make install
```

# Installation

## Through robotpkg

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mkdir package/build
cd package/build
cmake -DCMAKE_INSTALL_PREFIX=<your-prefix> ..

make install
```

# Installation

## Through installation script

```
▶ git clone git://github.com/stack-of-tasks/install-sot.git
 cd install-sot/scripts

 ./install_sot.sh
```