The stack of tasks

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Outline

Introduction

Theoretical foundations

Software

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The stack of tasks provides a control framework for real-time redundant manipulator control

- implementation of a data-flow,
- control of the graph by python scripting,
- task-based hierarchical control,
- portable: tested on HRP-2, Nao, Romeo.

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Rigid body $\mathcal{B}$

- Configuration represented by an homogeneous matrix

$$M_{\mathcal{B}} = \left(\begin{array}{cc} R_{\mathcal{B}} & \mathbf{t}_{\mathcal{B}} \\ 0\ 0\ 0 & 1 \end{array} \right) \in SE(3)$$

$$R_{\mathcal{B}} \in SO(3) \Leftrightarrow R_{\mathcal{B}}^T R_{\mathcal{B}} = I_3$$

Point $\mathbf{x} \in \mathbb{R}^3$ in local frame of $\mathcal{B}$ is moved to $\mathbf{y} \in \mathbb{R}^3$ in global frame:

$$\left(\begin{array}{c} \mathbf{y} \\ 1 \end{array} \right) = M_{\mathcal{B}} \left(\begin{array}{c} \mathbf{x} \\ 1 \end{array} \right)$$

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Rigid body $\mathcal{B}$

- Velocity represented by $(\mathbf{v}_{\mathcal{B}}, \omega_{\mathcal{B}}) \in \mathbb{R}^6$ where

$$\dot{R}_{\mathcal{B}} = \hat{\omega}_{\mathcal{B}} R_{\mathcal{B}}$$

  and

$$\hat{\omega} = \begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix}$$

  is the matrix corresponding to the cross product operator

- Velocity of point P on $\mathcal{B}$

$$\mathbf{v}_p = \dot{\mathbf{t}}_{\mathcal{B}} + \omega_{\mathcal{B}} \times \vec{O_{\mathcal{B}}P}$$

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Configuration space

- Robot: set of rigid-bodies linked by joints $\mathcal{B}_0, \cdots \mathcal{B}_m$.
- Configuration: position in space of each body.

$$\mathbf{q} = (\mathbf{q}_{waist}, \theta_1, \cdots \theta_{n-6}) \in SE(3) \times \mathbb{R}^{n-6}$$
$$\mathbf{q}_{waist} = (x, y, z, roll, pitch, yaw)$$

- Position of $\mathcal{B}_i$ depends on $\mathbf{q}$:

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$$\begin{aligned} \dot{\mathbf{q}} &= (\dot{x}, \dot{y}, \dot{z}, \omega_x, \omega_y, \omega_z, \dot{\theta}_1, \cdots \dot{\theta}_{n-6}) \\ \omega &\in \mathbb{R}^3 \end{aligned}$$

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Task

- Definition: function of the
  - robot configuration,
  - time and
  - possibly external parameters

  that should converge to 0:

$$T \in C^{\infty}(\mathcal{C} \times \mathbb{R}, \mathbb{R}^m)$$

- Example: position tracking of an end-effector $\mathcal{B}_{ee}$
  - $M(\mathbf{q}) \in SE(3)$ position of the end-effector,
  - $M^*(t) \in SE(3)$ reference position

$$T(\mathbf{q}, t) = \begin{pmatrix} \mathbf{t}(M^{*-1}(t)M(\mathbf{q})) \\ u_{\theta}(R^{*-1}(t)R(\mathbf{q})) \end{pmatrix}$$

  where
  - $\mathbf{t}()$ is the translation part of an homogeneous matrix,
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Hierarchical task based control

Given

- a configuration $\mathbf{q}$,
- two tasks of decreasing priorities:
  - $T_1 \in C^\infty(\mathcal{C} \times \mathbb{R}, \mathbb{R}^{m_1})$,
  - $T_2 \in C^\infty(\mathcal{C} \times \mathbb{R}, \mathbb{R}^{m_2})$,

compute a control vector $\dot{\mathbf{q}}$

- that makes T_1 converge toward 0 and
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Jacobian:

- we denote
  - $J_i = \frac{\partial T_i}{\partial \mathbf{q}}$ for $i \in \{1, 2\}$
- then
  - $\forall \mathbf{q} \in \mathcal{C}, \forall t \in \mathbb{R}, \forall \dot{\mathbf{q}} \in \mathbb{R}^n,\ \dot{T}_i = J_i(\mathbf{q}, t)\dot{\mathbf{q}} + \frac{\partial T_i}{\partial t}(\mathbf{q}, t)$

We try to enforce

- $\dot{T}_1 = -\lambda_1 T_1 \quad \Rightarrow \quad T_1(t) = e^{-\lambda_1 t} T_1(0) \rightarrow 0$
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Moore Penrose pseudo-inverse

Given a matrix $A \in \mathbb{R}^{m \times n}$, the Moore Penrose pseudo inverse $A^+ \in \mathbb{R}^{n \times m}$ of A is the unique matrix satisfying:

$$\begin{aligned} AA^+A &= A \\ A^+AA^+ &= A^+ \\ (AA^+)^T &= AA^+ \\ (A^+A)^T &= A^+A \end{aligned}$$

Given a linear system:

$$Ax = b, \quad A \in \mathbb{R}^{m \times n}, \; x \in \mathbb{R}^n, \; b \in \mathbb{R}^m$$

$x = A^+b$ minimizes

- $\|Ax - b\|$ over $\mathbb{R}^n$,
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Resolution of the first constraint:

$$\dot{T}_1 = J_1\dot{\mathbf{q}} \quad + \quad \frac{\partial T_1}{\partial t} = -\lambda_1 T_1 \tag{1}$$

$$J_1\dot{\mathbf{q}} \quad = \quad -\lambda_1 T_1 - \frac{\partial T_1}{\partial t} \tag{2}$$

$$\dot{\mathbf{q}}_1 \quad \triangleq \quad -J_1^{+}(\lambda_1 T_1 + \frac{\partial T_1}{\partial t}) \tag{3}$$

Where J_1^{+} is the (Moore Penrose) pseudo-inverse of J_1.
$\dot{\mathbf{q}}_1$ minimizes

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- $\|\dot{\mathbf{q}}\|$ over argmin $\|J_1\dot{\mathbf{q}} + \lambda_1 T_1 + \frac{\partial T_1}{\partial t}\|$

Hence,

- if $\lambda_1 T_1 + \frac{\partial T_1}{\partial t}$ is in $Im(J_1)$, (1) is satisfied

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Hierarchical task based control

In fact

$$\forall u \in \mathbb{R}^n, \quad J_1 \left(\dot{\mathbf{q}}_1 + (I_n - J_1^+ J_1)u\right) = J_1 \dot{\mathbf{q}}_1$$

therefore,

$$\dot{\mathbf{q}} = \dot{\mathbf{q}}_1 + (I_n - J_1^+ J_1)u$$

also minimizes $\|J_1 \dot{\mathbf{q}} + \lambda_1 T_1 + \frac{\partial T_1}{\partial t}\|$.

$P_1 = (I_n - J_1^+ J_1)$ is a projector on J_1 kernel:
$J_1 P_1 = 0$
$\forall u \in \mathbb{R}^n$, if $\dot{\mathbf{q}} = P_1 u$, then, $\dot{T}_1 = \frac{\partial T_1}{\partial t}$.

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$$\forall u \in \mathbb{R}^n, \;\; J_1\left(\dot{\mathbf{q}}_1 + (I_n - J_1^{+}J_1)u\right) = J_1\dot{\mathbf{q}}_1$$

therefore,

$$\dot{\mathbf{q}} = \dot{\mathbf{q}}_1 + (I_n - J_1^{+}J_1)u$$

also minimizes $\|J_1\dot{\mathbf{q}} + \lambda_1 T_1 + \frac{\partial T_1}{\partial t}\|$.

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Controlling the second task

We have

$$\begin{aligned}\dot{\mathbf{q}} &= \dot{\mathbf{q}}_1 + P_1 u \\ \dot{T}_2 &= J_2\dot{\mathbf{q}} + \frac{\partial T_2}{\partial \mathbf{q}} \\ \dot{T}_2 &= J_2\dot{\mathbf{q}}_1 + \frac{\partial T_2}{\partial \mathbf{q}} + J_2 P_1 u\end{aligned}$$

We want

$$\dot{T}_2 = -\lambda_2 T_2$$

Thus

$$\begin{aligned}-\lambda_2 T_2 &= J_2\dot{\mathbf{q}}_1 + \frac{\partial T_2}{\partial \mathbf{q}} + J_2 P_1 u \\ J_2 P_1 u &= -\lambda_2 T_2 - J_2\dot{\mathbf{q}}_1 - \frac{\partial T_2}{\partial \mathbf{q}}\end{aligned}$$

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\dot{\mathbf{q}}_2 &\triangleq \dot{\mathbf{q}}_1 + P_1 u \\
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minimizes $\|\dot{T}_2 + \lambda_2 T_2\|$ over $\dot{\mathbf{q}}_1 + Ker\ J_1$.

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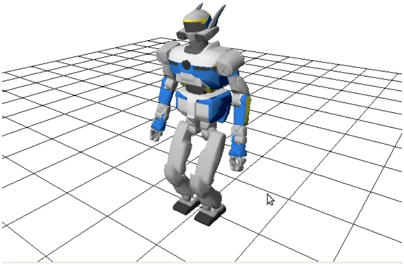


- T_1: position of the feet + projection of center of mass,
- T_2: position of the right wrist.

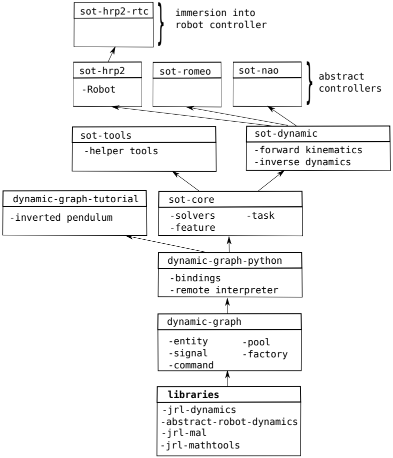
Outline

Introduction

Theoretical foundations

Software

Architeture overview



Libraries

- `jrl-mathtools`: implementation of small size matrices,
  - to be replaced by Eigen
- `jrl-mal`: abstract layer for matrices,
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- Entity
  - Signal: synchronous interface
  - Command: asynchronous interface
- Factory
  - builds a new entity of requested type,
  - new entity types can be dynamically added (advanced).
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  - stores all instances of entities,
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Command

Asynchronous interface

- input in a fixed set of types,
- trigger an action,
- returns a result in the same set of types.

dynamic-graph-python

Python bindings to dynamic-graph

- module dynamic_graph linked to libdynamic-graph.so
  - class Entity
    - each C++ entity class declared in the factory generates a python class of the same name,
    - signals are instance members,
    - commands are bound to instance methods
    - method help lists commands
    - method displaySignals displays signals
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    - property value to set and get signal value
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dynamic-graph-tutorial

Simple use case for illustration

- Definition of 2 entity types
  - `InvertedPendulum`
    - input signal: `force`
    - output signal: `state`
  - `FeedbackController`
    - input signal: `state`
    - output signal: `force`

dynamic-graph-tutorial

```
>>> from dynamic_graph.tutorial import InvertedPendulum, FeedbackController
>>> a = InvertedPendulum ('IP')
>>> b = FeedbackController ('K')
>>> a.displaySignals ()
--- <IP> signal list:
|-- <Sig:InvertedPendulum(IP)::input(double)::force (Type Cst) AUTOPLUGGED
`-- <Sig:InvertedPendulum(IP)::output(vector)::state (Type Cst)
>>> a.help ()
Classical inverted pendulum dynamic model

List of commands:
-----------------
  getCartMass:          Get cart mass
  getPendulumLength:    Get pendulum length
  getPendulumMass:      Get pendulum mass
  incr:                 Integrate dynamics for time step provided as input
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Package provides

- C++ code of classes `InvertedPendulum` and `FeedbackController`,
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sot-core

Class `FeatureAbstract`

- function of the robot and environment states
  - position of an end-effector,
  - position of a feature in an image (visual servoing)
- with values in a Lie group G ($SO(3)$, $SE(3)$, $\mathbb{R}^n$,...),
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$$e(0_G) = 0$$

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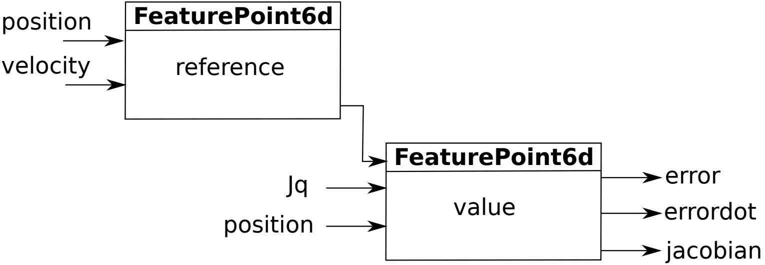
When paired with a reference, features become *tasks*.

- Example



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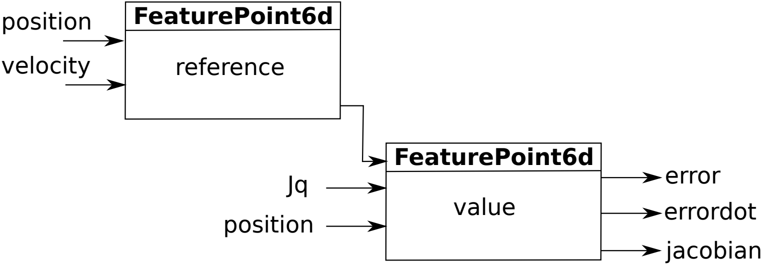

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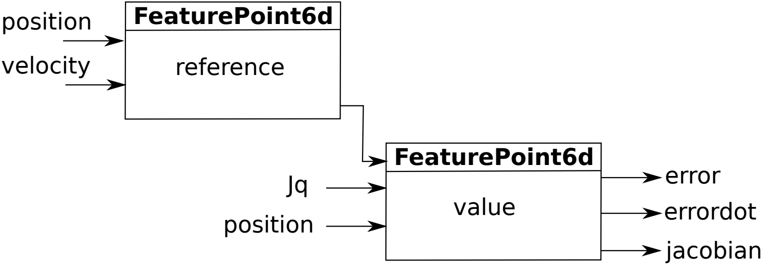

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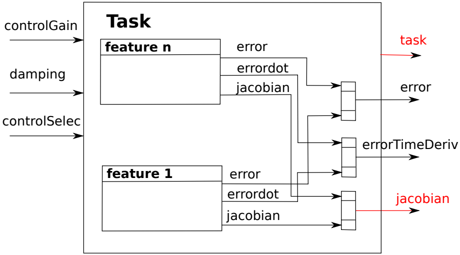
Task

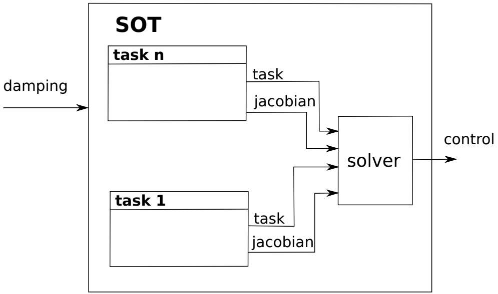
- Collection of features with a control gain,
- implements abstraction TaskAbstract



- $\mathtt{task} = -\mathtt{controlGain.error}$

Solver SOT

Hierarchical task solver



- computes robot joint velocity

sot-dynamic

`dynamic_graph.sot.dynamics.Dynamic` builds a kinematic chain from a file and

- computes forward kinematics
  - position and Jacobian of end effectors (wrists, ankles),
  - position of center of mass
- computes dynamics
  - inertia matrix.

Packages specific to robots

`sot-hrp2`

- defines a class `Robot` that provides
  - ready to use features for feet, hands, gaze and center of mass,
  - ready to use tasks for the same end effectors,
  - an entity `Dynamic`,
  - an entity `Device` (interface with the robot control system)

Utilities

- `dynamic_graph.writeGraph (filename)`: writes the current graph in a file using graphviz dot format.
- `dynamic_graph.sot.core.FeaturePosition` wraps two `FeaturePoint6d`: a value and a reference,
- `MetaTask6d`:
- `MetaTaskPosture`:
- `MetaTaskKine6d`:
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Installation

Through robotpkg

- ```
  git clone http://trac.laas.fr/git/robots/robotpkg.git
  cd robotpkg
  ./bootstrap/bootstrap --prefix=<your_prefix>
  cd motion/sot-dynamic

  make install
  ```

Through github:

- ```
  git clone git://github.com/jrl-umi3218/jrl-mal.git
  git clone git://github.com/jrl-umi3218/jrl-mathtools.git
  git clone git://github.com/laas/abstract-robot-dynamics.git
  git clone git://github.com/jrl-umi3218/jrl-dynamics.git
  git clone git://github.com/jrl-umi3218/dynamic-graph.git
  git clone git://github.com/jrl-umi3218/dynamic-graph-python.git
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- for each package,
  ```
  mkdir package/build
  cd package/build
  cmake -DCMAKE_INSTALL_PREFIX=<your_prefix> ..

  make install
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Installation

Through installation script

- ```
  git clone git://github.com/stack-of-tasks/install-sot.git
  cd install-sot/scripts

  ./install_sot.sh
  ```
  ```