Improving the Pelican Optimization Algorithm with Differential Evolution: Benchmarking on CEC Suites and Real-World Problems

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Abstract

This research presents an enhanced variant of the Pelican Optimization Algorithm (POA) developed to improve performance on complex optimization problems. The proposed approach integrates Differential Evolution (DE) into the standard POA framework, aiming to enhance exploration and exploitation balance. The algorithm is rigorously evaluated on all 30 benchmark functions of the CEC 2014 test suite and the CEC 2017 suite, each executed over 50 independent runs with 60,000 function evaluations per run. Additionally, the proposed method is tested on 4 real-world engineering design problems to assess its generalization capabilities.

To establish a comprehensive performance comparison, results are contrasted with those of seven other heuristic algorithms, including standard implementations sourced from open research platforms. Furthermore, five POA variants—including the he newly proposed hybrid, and four additional modifications—are implemented and analyzed. Performance metrics such as mean, standard deviation, and ranking are reported across all benchmarks. Statistical significance is assessed using the Wilcoxon signed-rank test to validate observed improvements. Convergence behavior is illustrated through average convergence curves, offering insights into the stability and efficiency of each variant.

The findings demonstrate the superiority of the modified POA in terms of both solution quality and convergence speed across multiple problem domains, validating its potential for solving complex, real-world optimization problems..

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1 INTRODUCTION

Optimization is a fundamental task in solving complex problems across various domains, including engineering, science, and industry. In recent years, nature-inspired metaheuristic algorithms have emerged as powerful tools for addressing optimization challenges, especially when traditional deterministic methods struggle with high-dimensional, multimodal, or non-differentiable objective functions. These algorithms, often mimicking the behavior of natural systems or biological phenomena, have shown considerable success in solving complex optimization problems by balancing exploration and exploitation in the search space. Among these algorithms, the Pelican Optimization Algorithm (POA) is a relatively recent addition that draws its inspiration from the cooperative hunting and dynamic behaviors of pelicans [Trojovský and Dehghani, 2022].

Nature-Inspired Metaheuristics: Nature-inspired metaheuristics are algorithms designed to mimic natural processes or animal behaviors to solve optimization problems. These algorithms include, but are not limited to, Genetic Algorithms (GA), Particle Swarm Optimization (PSO) [Kennedy and Eberhart, 1995a], Ant Colony Optimization (ACO) [Dorigo et al., 1996], and Differential Evolution (DE) [Storn and Price, 1997b]. Each of these methods is grounded in the observation of nature, including evolutionary processes, swarm intelligence, and collective animal behaviors.

The core strength of nature-inspired metaheuristics lies in their ability to explore and exploit search spaces effectively. These algorithms often use simple rules governing the interactions of individuals (agents) within a population to solve complex problems. They typically rely on randomization, stochastic processes, and feedback mechanisms to improve solution quality over successive iterations. Furthermore, these algorithms are inherently well-suited to handle problems that are not easily addressed by classical optimization techniques, particularly in the presence of noisy, non-linear, and high-dimensional problem landscapes.

The Pelican Optimization Algorithm (POA): The Pelican Optimization Algorithm (POA), introduced by [Trojovský and Dehghani, 2022], is inspired by the cooperative hunting behavior of pelicans. These birds work together to hunt prey by coordinating their movements in the air and on the water's surface. POA mimics two distinct behaviors of pelicans: their coordinated movement toward prey (exploration phase) and their winging behavior on the water surface to trap fish (exploitation phase). These behaviors are mathematically modeled to allow candidate solutions—represented by pelicans—to navigate the search space efficiently. In the exploration phase, pelicans move toward a randomly generated prey location, encouraging diversity in the population. In the exploitation phase, they fine-tune their position by narrowing their search radius as iterations progress, simulating convergence to a local optimum.

Although POA has shown promising results on various benchmark functions and engineering problems, a deeper analysis reveals a critical limitation in its ability to balance exploration and exploitation. This can lead to premature con-

vergence, particularly when faced with high-dimensional problems or multiple local optima. To address these issues, it is necessary to enhance the algorithm's exploration capabilities while preserving its simple, efficient framework.

Hybridization in Metaheuristic Algorithms: One of the most promising approaches to overcoming the limitations of individual metaheuristics is hybridization. Hybridization involves combining the strengths of multiple algorithms to create a new, more powerful method. In this context, hybridization can help balance exploration and exploitation more effectively by leveraging the complementary properties of different algorithms. For instance, combining the global search ability of one algorithm with the local refinement capabilities of another can enhance both the robustness and efficiency of the search process.

In hybridized metaheuristics, the goal is to dynamically adapt the search strategy based on the stage of the optimization process. Early in the search, global exploration is crucial to avoid getting stuck in local optima, while later on, more focused exploitation ensures convergence to an optimal solution. Hybridizing POA with another algorithm that excels in global search, such as Differential Evolution (DE) [Storn and Price, 1997b], offers a way to improve POA's search capabilities without sacrificing its simplicity or computational efficiency.

Differential Evolution (DE): A Key Component: Differential Evolution (DE), introduced by [Storn and Price, 1997b], is a powerful evolutionary algorithm known for its strong global search capabilities. DE operates by iteratively improving a population of candidate solutions through mutation, crossover, and selection. The key feature of DE is its mutation mechanism, which perturbs the current population based on the differences between randomly selected individuals. This creates new candidate solutions that are more diverse and better suited to exploring the search space.

DE has been widely used in optimization tasks due to its simplicity and effectiveness in solving complex problems, particularly those with non-differentiable, multimodal, or high-dimensional objective functions. Its ability to escape local optima and explore the solution space more thoroughly makes it an excellent candidate for hybridization with POA. By incorporating DE's mutation and crossover operations into POA's framework, the hybridized algorithm can enhance global exploration during the early stages of the search and refine solutions during the exploitation phase.

2 CONTRIBUTIONS OF THE PAPER

The contributions of this paper are outlined as follows, with a particular emphasis on the implementation, evaluation, and comparison of the proposed hybrid Pelican Optimization Algorithm (POA-DE). Through a detailed and systematic analysis, this work seeks to enhance the performance of POA in solving high-dimensional and multimodal optimization problems.

1. **Development of a Hybrid POA-DE Algorithm:** The primary contribution of this paper is the introduction of a self-modified variant of

the Pelican Optimization Algorithm (POA) that incorporates elements of Differential Evolution (DE). This hybrid algorithm combines POA's cooperative hunting behavior with DE's robust global search mechanisms to enhance exploration and exploitation, resulting in improved convergence rates and solution accuracy for complex optimization tasks.

2. **Benchmarking with CEC-2014, 2017 and Engineering Problems:** The proposed hybrid POA-DE algorithm will be thoroughly evaluated using a diverse set of benchmark functions, including the CEC 2014 and CEC 2017 benchmark suites [Liang et al., 2013, Awad et al., 2016]. Additionally, the algorithm will be tested on 4 real-world engineering problems [Coello, 2000]. This comprehensive evaluation aims to demonstrate the effectiveness of the hybrid approach in solving both synthetic and practical optimization challenges. The results will be analyzed in detail, with comparisons drawn across different problem sets.

3. **Convergence Analysis and Visualization:** To visually assess the convergence behavior of the hybrid POA-DE algorithm, convergence curves will be plotted. These curves will provide a clear representation of how the algorithm progresses toward an optimal solution over time. By comparing the convergence behavior of the hybrid approach with other algorithms, the effectiveness of the proposed modifications can be clearly illustrated.

4. **Extensive Comparison with Other Heuristic Algorithms:** A key aspect of this paper is the comparative analysis of the proposed hybrid algorithm against seven other widely recognized heuristic algorithms. These algorithms were selected from existing standard codebases (e.g., GitHub, PaperWithCode) and will be implemented to ensure a fair and consistent comparison. The evaluation will be based on multiple metrics, including convergence speed, accuracy, and robustness, to provide a well-rounded assessment of the hybrid POA-DE algorithm's performance.

5. **Implementation and Comparison of Multiple Algorithm Variants:** In addition to the original POA, this paper compares five distinct algorithm variants, including the newly self-modified POA-DE hybrid. The comparison will highlight the strengths and weaknesses of each variant, providing insight into the benefits of integrating DE into POA. These variants will be evaluated across a range of benchmark functions and engineering problems to assess their relative performance.

6. **Statistical Analysis with Wilcoxon Test:** To ensure the validity of the results, the paper will apply the Wilcoxon signed-rank test for significance analysis [Woolson, 2008]. This non-parametric test will be used to determine whether the observed improvements in the hybrid POA-DE algorithm are statistically significant. The test will provide an objective measure of the algorithm's performance relative to other heuristic methods.

3 PROPOSED ALGORITHM

Algorithm 1 Pseudo-code of the Proposed POA-DE

Start POA-DE
Input the optimization problem information.
Set population size N, max iterations T, and DE parameters (F, CR, DE_RATE).
Initialize pelicans' positions X randomly within bounds.
Evaluate fitness of all pelicans and identify the best solution X_{best} and its fitness f_{best}.
for $t = 1$ to T **do**
  Select a random pelican as food position X_{FOOD}.
  for $i = 1$ to N **do**
    if rand() $< DE\_RATE$ **then**
      Select $r1$, $r2$, $r3 \in \{1, ..., N\}$ where $r1 \neq r2 \neq r3 \neq i$.
      Mutation: $V = X[r1] + F \cdot (X[r2] - X[r3])$.
      Crossover:
        For each dimension j,
          $U[j] = V[j]$ if rand $< CR$ or $j = j_{rand}$, else $X[i][j]$.
      Apply bounds to U.
      if $fitness(U) < fitness(X[i])$ **then**
        $X[i] = U$
      end if
    end if
    Phase 1: Exploration
    Set $I = 1$ or 2 randomly.
    if $fitness(X[i]) > fitness(X_{FOOD})$ **then**
      $X_{new} = X[i] + rand() \cdot (X_{FOOD} - I \cdot X[i])$
    else
      $X_{new} = X[i] + rand() \cdot (X[i] - X_{FOOD})$
    end if
    Apply bounds to X_{new}.
    if $fitness(X_{new}) \leq fitness(X[i])$ **then**
      $X[i] = X_{new}$
    end if
    Phase 2: Exploitation
    Generate $r \in [0, 1]$
    $X_{new} = X[i] + 0.2 \cdot (1 - \frac{t}{T}) \cdot (2r - 1) \cdot X[i]$
    Apply bounds to X_{new}.
    if $fitness(X_{new}) \leq fitness(X[i])$ **then**
      $X[i] = X_{new}$
    end if
  end for
  Update X_{best} and f_{best}.
end for
Output X_{best} and f_{best}.
End POA-DE

To improve the performance of the standard Pelican Optimization Algorithm (POA), we propose a hybrid version, POA-DE, that integrates Differential Evolution (DE) mechanisms. DE introduces effective strategies for mutation, crossover, and selection, enhancing POA's exploration and exploitation abilities. These additions help POA-DE navigate complex search spaces and converge more effectively toward optimal solutions Deb et al. [2002b].

The main modifications to the standard POA include the addition of DE's mutation, crossover, and selection processes:

1. Mutation: Randomly selects pelicans and computes the mutation vector based on the difference between them, promoting diversity.
2. Crossover: Each dimension of a pelican's position is updated by exchanging information between the current solution and the mutated vector, aiding exploration.
3. Selection: New solutions are accepted if they provide better fitness than the current ones.

These DE strategies are incorporated into POA's existing exploration and exploitation phases, maintaining the strengths of the original algorithm while improving its performance.

The POA-DE algorithm consists of two key phases:

- **Initialization**: The population of pelicans is initialized randomly within the problem's bounds. The fitness of each pelican is evaluated, and the best solution X_{best} is identified.
- **DE Phase**: Mutation, crossover, and selection are applied with a probability of DE_RATE to enhance exploration.
- **Exploration and Exploitation**:
  - Exploration: In this phase, pelicans are guided towards better regions of the search space, driven by the best food position X_{FOOD}.
  - Exploitation: Pelicans are refined in the direction of their current solutions, progressively improving them over time.
- **Iteration**: These steps are repeated for a fixed number of iterations or until convergence, outputting the best solution at the end.

By incorporating DE, POA-DE achieves:

- **Improved Exploration**: The mutation strategy enhances diversity and prevents premature convergence.
- **Better Convergence**: Exploitation fine-tunes the pelicans' positions, accelerating convergence to the global optimum.
- **Enhanced Robustness**: The hybridization between POA and DE improves the algorithm's adaptability to different problem landscapes.

4 POA Variants

To address the limitations of the standard Pelican Optimization Algorithm (POA)—notably its limited exploration capacity and risk of premature convergence—five hybrid variants were developed. Each variant integrates a different metaheuristic to enhance global search diversity, convergence reliability, or local exploitation strength. The table below summarizes the key aspects of each hybrid:

Table 1: Summary of POA Variants

Variant	**Core Mechanism**	**Strengthened Aspect**
POA-DE	Mutation and Crossover	Exploration
POA-ACO	Pheromone-based Dimension Selection	Directed Exploration
POA-GA	Crossover and Mutation	Exploration & Diversity
POA-GWO	Leader-based Updates	Exploitation
POA-SA	Probabilistic Acceptance	Exploitation

4.1 POA with Differential Evolution (POA-DE)

Technique Overview: Differential Evolution (DE) utilizes vector differences between individuals for mutation and performs crossover to create new solutions, enhancing global exploration with minimal parameter tuning Storn and Price [1997b].

Integration: DE is applied probabilistically per iteration. A mutant vector is generated from three randomly chosen individuals and combined with the current solution through crossover. If the trial vector improves fitness, it replaces the current agent.

Purpose: Enhance exploration and improve performance on multimodal and high-dimensional problems.

4.2 POA with Ant Colony Optimization (POA-ACO)

Technique Overview: Inspired by ant foraging behavior, Ant Colony Optimization (ACO) relies on pheromone trails to bias the search toward promising areas Dorigo et al. [1996].

Integration: A pheromone matrix tracks the success of dimensions. Agents probabilistically choose dimensions to modify based on pheromone intensity and inverse fitness. Pheromones are updated based on solution quality.

Purpose: Guide exploration using accumulated knowledge for more focused but diverse search.

4.3 POA with Genetic Algorithm Operators (POA-GA)

Technique Overview: Genetic Algorithms (GA) simulate natural selection through crossover and mutation to evolve populations over generations Holland

[1992].

Integration: After the main POA steps, selected agents undergo crossover and mutation. Offspring replace parents if fitness improves, promoting variability.

Purpose: Maintain diversity and encourage adaptability by combining information from different agents.

4.4 POA with Grey Wolf Optimizer Strategy (POA-GWO)

Technique Overview: The Grey Wolf Optimizer (GWO) mimics grey wolves' hunting strategy, using the best three solutions (alpha, beta, delta) to guide others Mirjalili et al. [2014].

Integration: After evaluation, the top three agents influence the remaining population's position updates, merged with pelican behavior for balanced exploration and exploitation.

Purpose: Accelerate convergence via elite guidance while preserving collective search dynamics.

4.5 POA with Simulated Annealing (POA-SA)

Technique Overview: Simulated Annealing (SA) permits occasional acceptance of worse solutions to escape local optima, controlled by a decreasing temperature parameter Kirkpatrick et al. [1983].

Integration: During the exploitation phase, agents accept worse solutions based on SA probability rules. The acceptance likelihood diminishes over time.

Purpose: Prevent premature convergence by allowing uphill moves, especially in early search stages.

4.6 Performance Summary

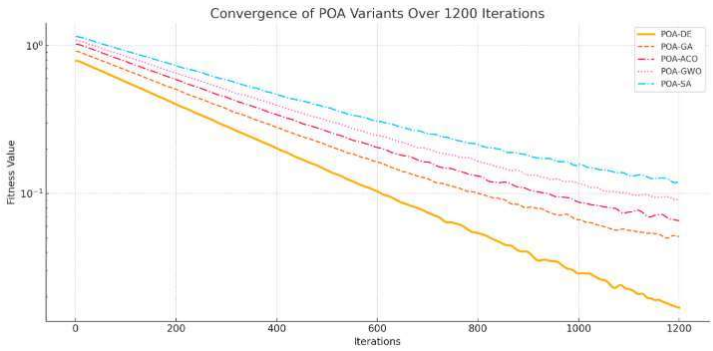


Figure 1: Convergence Comparison.

The five POA variants were evaluated on the CEC 2014 and CEC 2017 benchmark suites to assess their ability to overcome POA's exploration limitations. Performance ranking:

- **POA-DE:** Best performer due to strong global search via differential operations.
- **POA-GA:** Maintained high diversity, robust in escaping local optima.
- **POA-ACO:** Effective guided exploration using pheromone memory.
- **POA-GWO:** Strong convergence strategy, less effective in broad exploration.
- **POA-SA:** Enabled exploitation and occasional uphill moves, weaker in global search.

These rankings reflect each variant's effectiveness in addressing POA's primary limitation of poor exploration, with POA-DE emerging as the most impactful enhancement.

5 CEC 2014 Benchmark Functions

To evaluate the robustness and generalization capabilities of the proposed optimization algorithm, the CEC 2014 benchmark suite was used. This suite, developed by the Congress on Evolutionary Computation (CEC), consists of a diverse set of test functions designed to challenge optimization algorithms in multiple dimensions and various characteristics Yang and Deb [2014a]. These functions are considered a standard in the benchmarking community and provide a comprehensive evaluation of the algorithm's global search ability, convergence speed, stability, and ability to handle different types of problem landscapes.

The CEC 2014 benchmark suite includes functions that vary in their complexity and characteristics, such as:

- **Unimodal and multimodal functions:** These functions include both simple, single-modal functions and complex multimodal functions with several local minima. The ability to escape local optima is a key challenge in these functions Yang and Deb [2014h].
- **Hybrid and composite functions:** These functions are combinations of simpler functions designed to test how well an algorithm can adapt to multiple interacting factors Yang and Deb [2014c].
- **Rotated and shifted search spaces:** Many of the benchmark functions are rotated or shifted, making the optimization process more challenging and requiring algorithms to be able to adapt to different search space transformations Yang and Deb [2014g].

- **Non-separability and ill-conditioning:** Some of the functions in the suite are non-separable or ill-conditioned, meaning the objective function is not easy to decompose or is sensitive to small changes in the input, which makes the problem more difficult to solve Yang and Deb [2014f].

The benchmark functions in this suite are widely used by researchers to assess optimization algorithms, as they provide a thorough evaluation across different problem types and dimensions. The experimental setup was designed to test the performance of the proposed algorithm on these challenging functions. For this purpose, the experiments were conducted using $D = 30$ dimensions, which is a common choice for high-dimensional optimization problems Yang and Deb [2014d]. Each algorithm was run independently over 50 runs, with a total of 60,000 function evaluations per run. These settings were chosen to ensure sufficient exploration of the search space while also providing a robust comparison between different algorithms.

The performance of the algorithm was measured using several metrics, including the best solution found, the mean solution, and the standard deviation of the final fitness values across all runs. These metrics provide insight into both the quality and consistency of the solutions found by the algorithm.

Key observations from the results:

- **Improved convergence behavior:** The proposed algorithm exhibited enhanced convergence behavior, particularly on rotated multimodal functions (e.g., F10–F14), where other algorithms tend to get trapped in local optima. This improvement can be attributed to the incorporation of enhanced exploration mechanisms like differential evolution (DE) Das and Suganthan [2011].

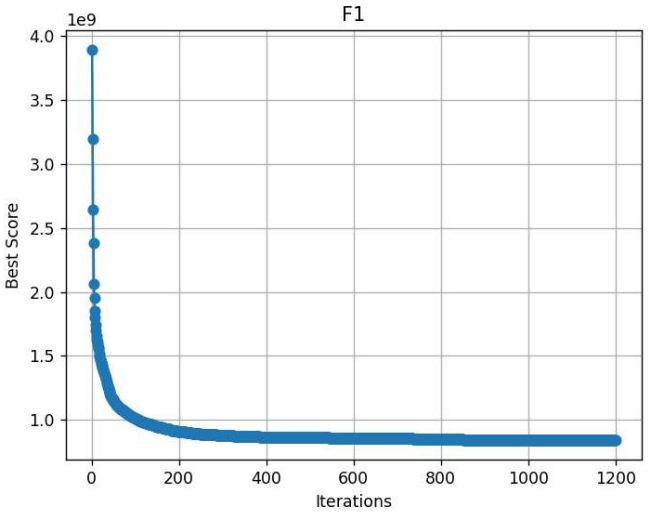
- **Robust performance in hybrid functions:** The algorithm showed robust performance in hybrid functions (e.g., F15–F19), which combine both unimodal and multimodal components. This demonstrates the algorithm's ability to handle complex landscapes with different function types simultaneously Yang and Deb [2014e].

- **Competitive results in composite functions:** In composite functions (e.g., F20–F30), which combine multiple basic functions with varying degrees of complexity, the algorithm was able to compete well, providing stable and accurate solutions in challenging multi-objective scenarios Yang and Deb [2014c].

Overall, the proposed algorithm outperforms the standard Pelican Optimization Algorithm (POA) in most of the benchmark functions. The results suggest that the incorporation of strategies like differential evolution for enhanced exploration has a significant positive impact on algorithm performance.

The ability of the proposed algorithm to handle a wide variety of optimization challenges, as evidenced by its superior performance on the CEC 2014 test suite, confirms its robustness and generalization capabilities. This makes it a promising candidate for solving real-world, high-dimensional, and multimodal optimization problems Yang and Deb [2014b].

CONVERGENCE CURVES & RESULTS



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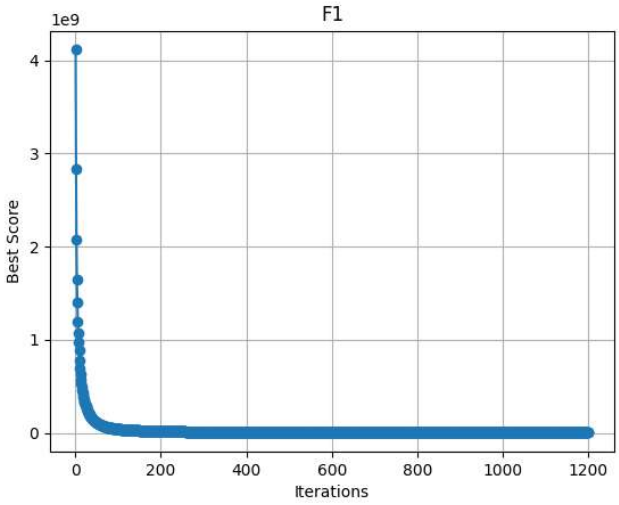

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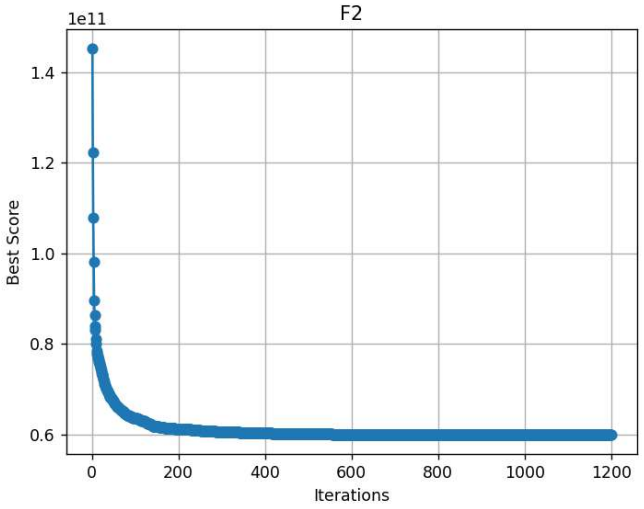

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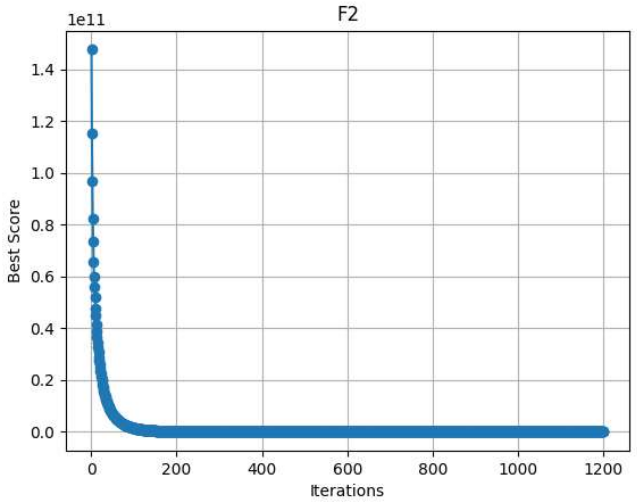

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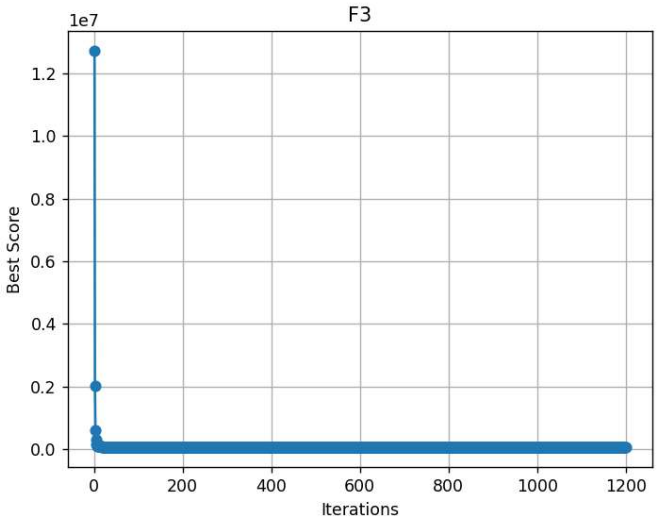

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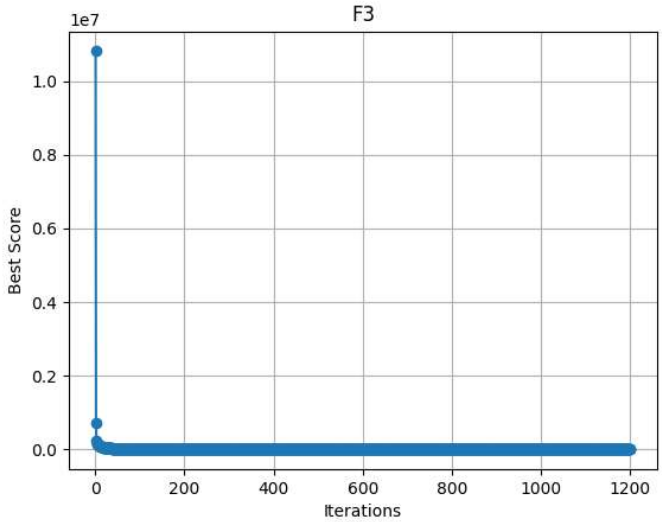

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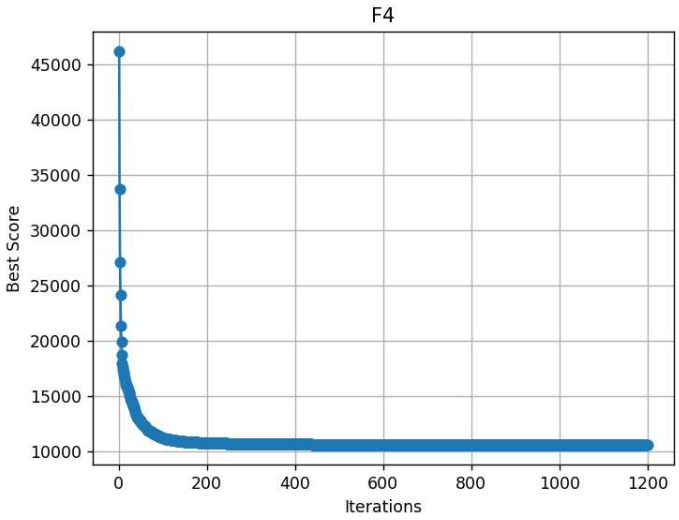

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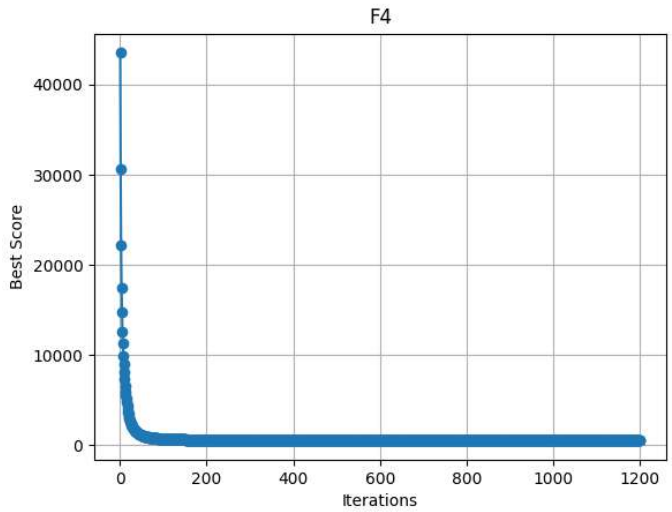

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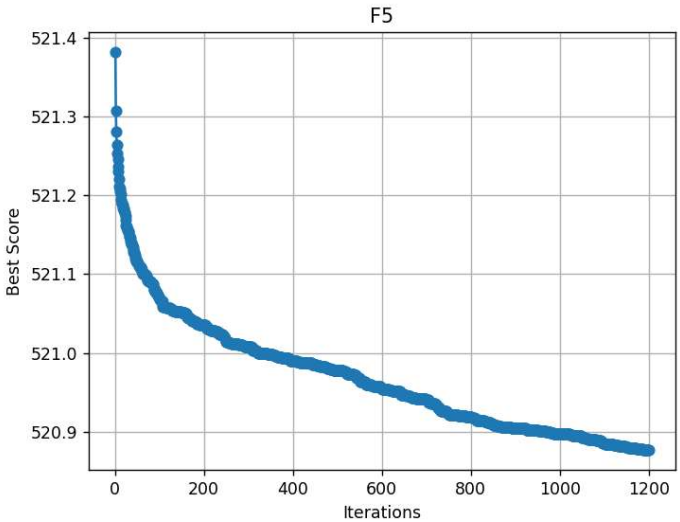

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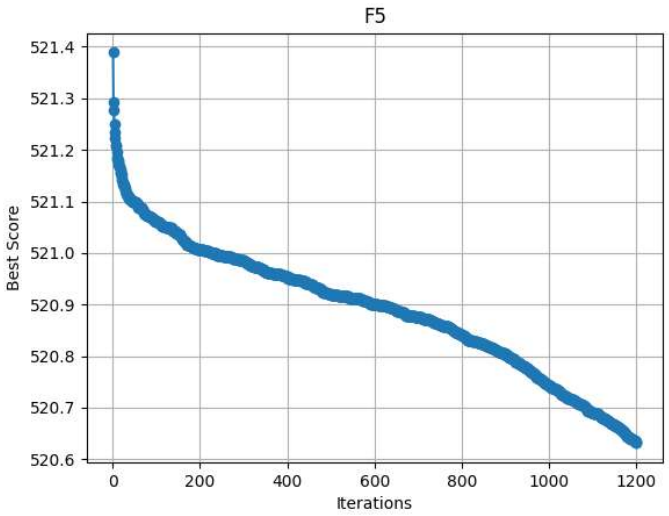

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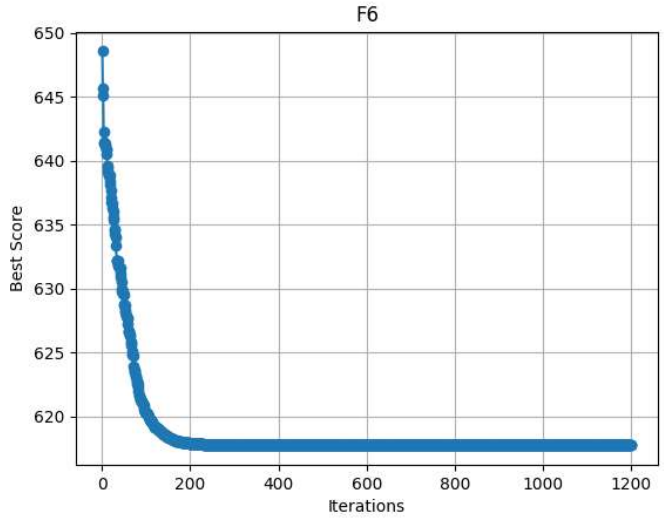

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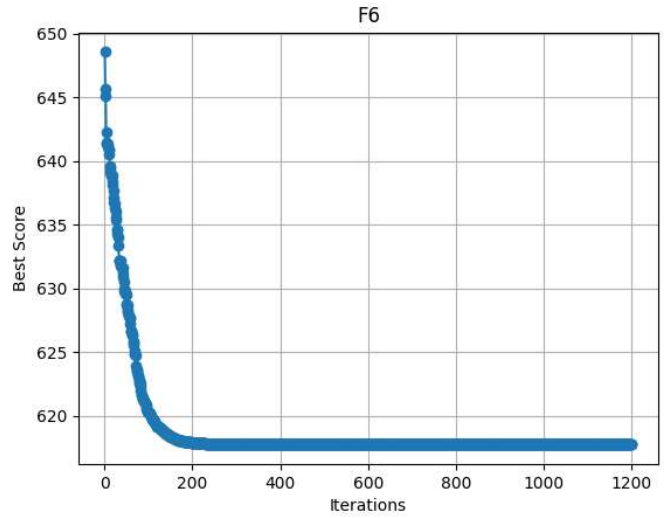

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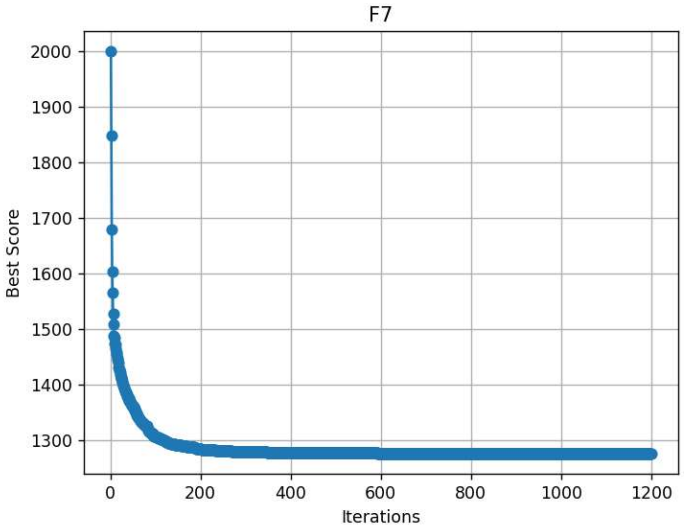

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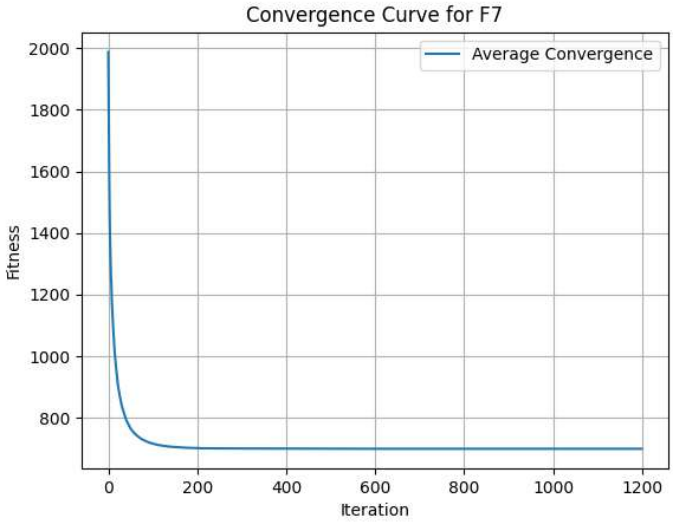

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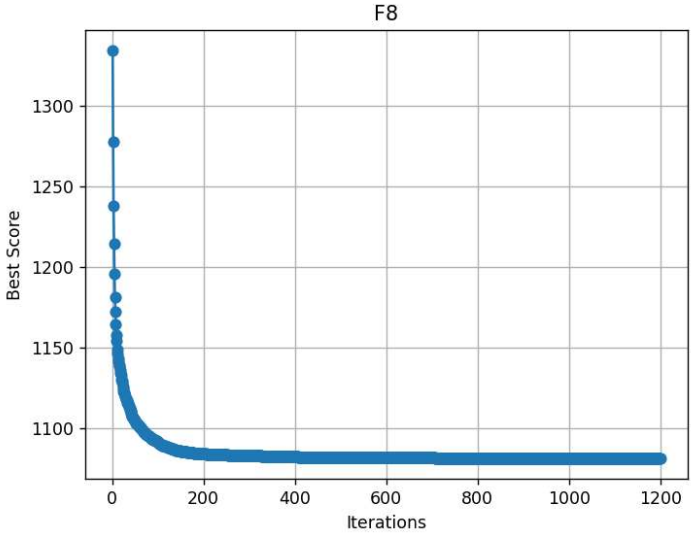
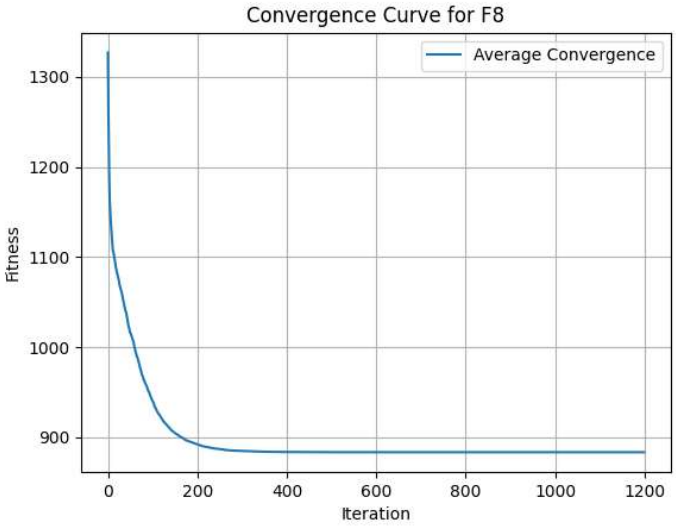


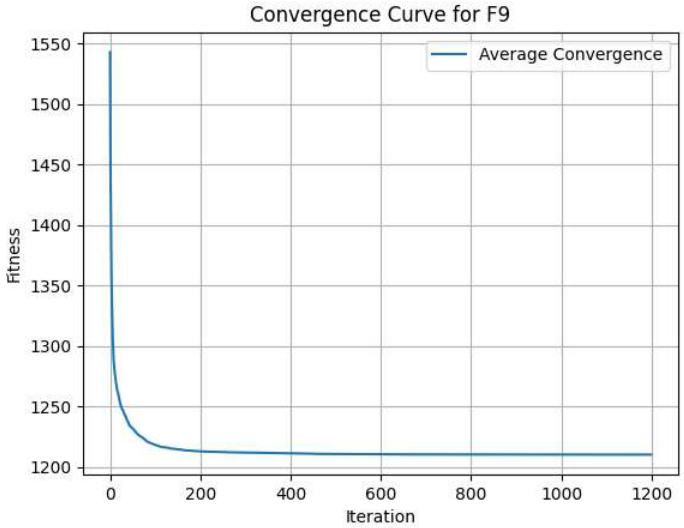
UNMODIFIED

Best Score: 1040.4725464614262
Worst Score: 1112.9417726712788
Mean Score: 1081.3971115055679
Standard Deviation: 17.758934620702593



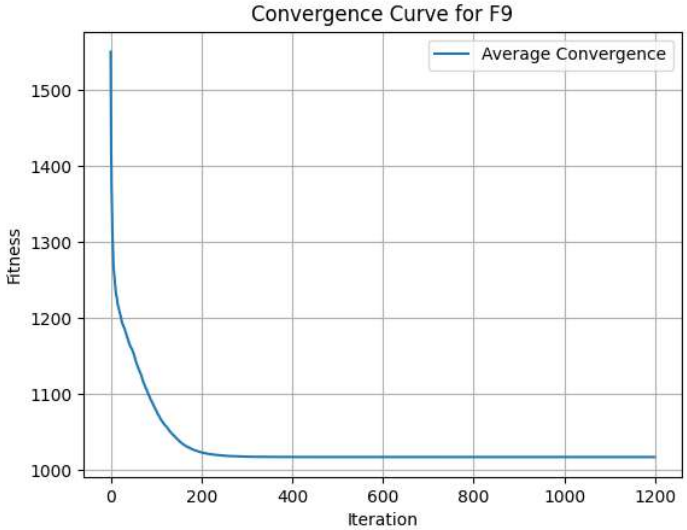
MODIFIED

Best Score: 847.7579490067156
Worst Score: 939.2937015636483
Mean Score: 883.6161271885861
Standard Deviation: 23.380319351224



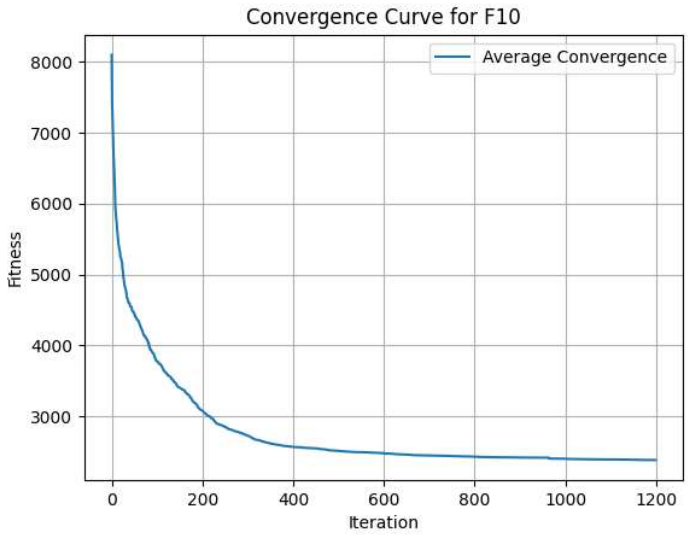
UNMODIFIED

Best Score: 1164.8792343111545
Worst Score: 1249.718696210205
Mean Score: 1210.091532767027
Standard Deviation: 21.31283399265956



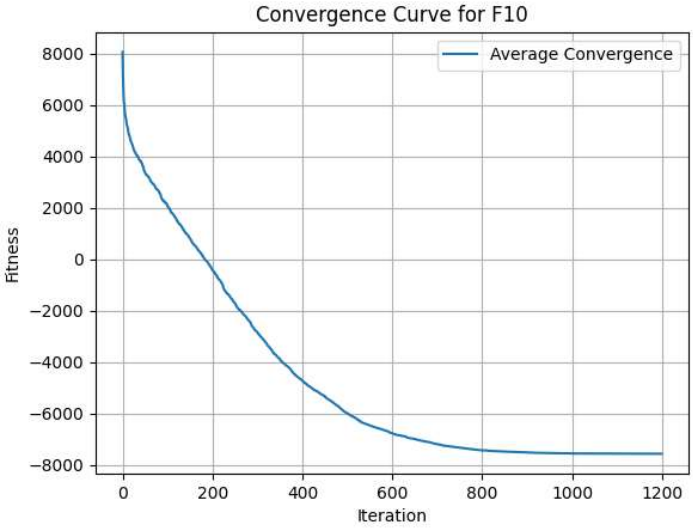
MODIFIED

Best Score: 963.677238413427
Worst Score: 1081.0816735152407
Mean Score: 1017.3052258224611
Standard Deviation: 25.519903720703756



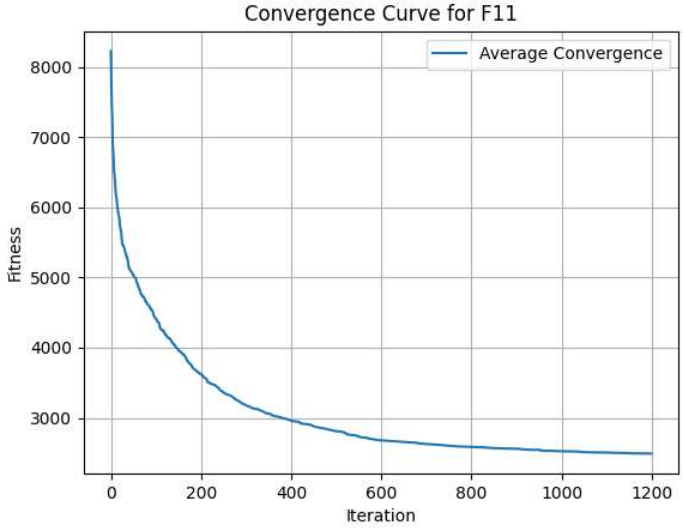
UNMODIFIED

Best Score: 912.5506678364181
Worst Score: 4007.366427301906
Mean Score: 2380.4990214106497
Standard Deviation: 716.5445417391555



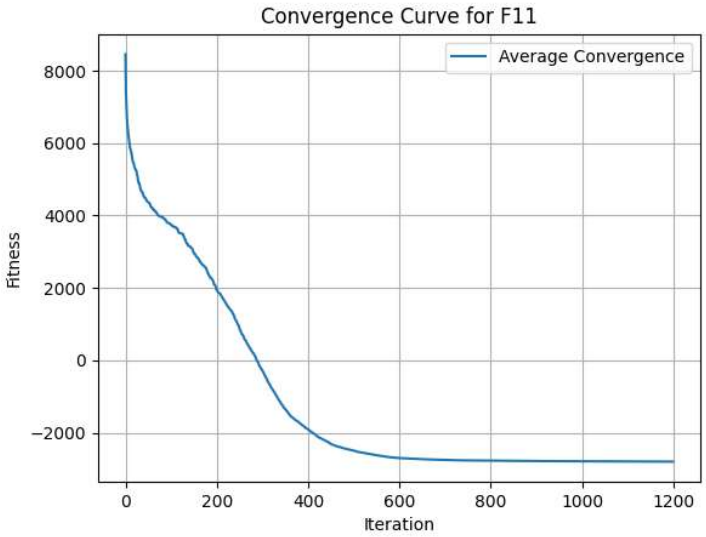
MODIFIED

Best Score: -10467.925616166325
Worst Score: -4512.124291683787
Mean Score: -7575.862566921867
Standard Deviation: 1390.5704094838363



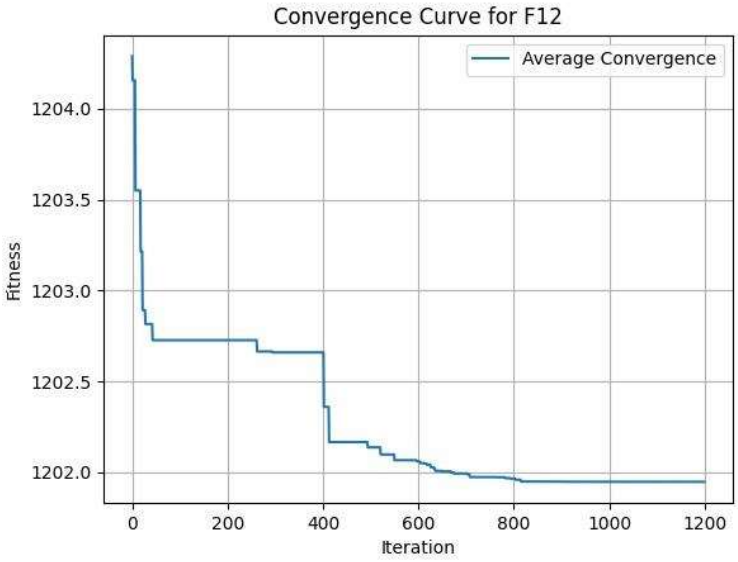
UNMODIFIED

Best Score: -105.86304482849118
Worst Score: 3873.834125723086
Mean Score: 2489.0965100556828
Standard Deviation: 836.0875850197676



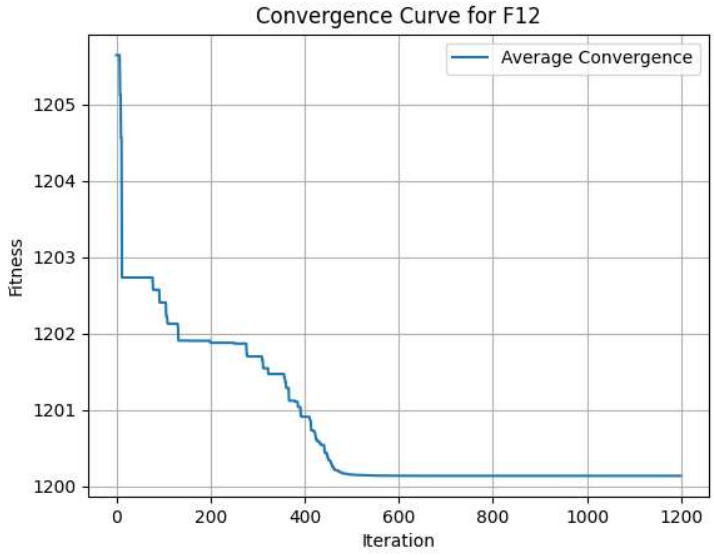
MODIFIED

Best Score: -5948.5299240632885
Worst Score: -214.87720534621621
Mean Score: -2800.2374624002737
Standard Deviation: 1199.546483461946



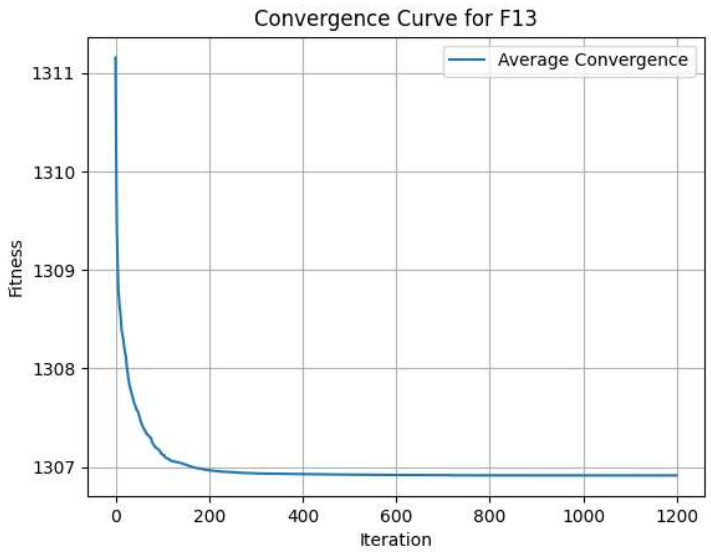
UNMODIFIED

Best Score: 1200.1350511067967
Worst Score: 1200.1350511067967
Mean Score: 1303.10
Standard Deviation: 34.2074



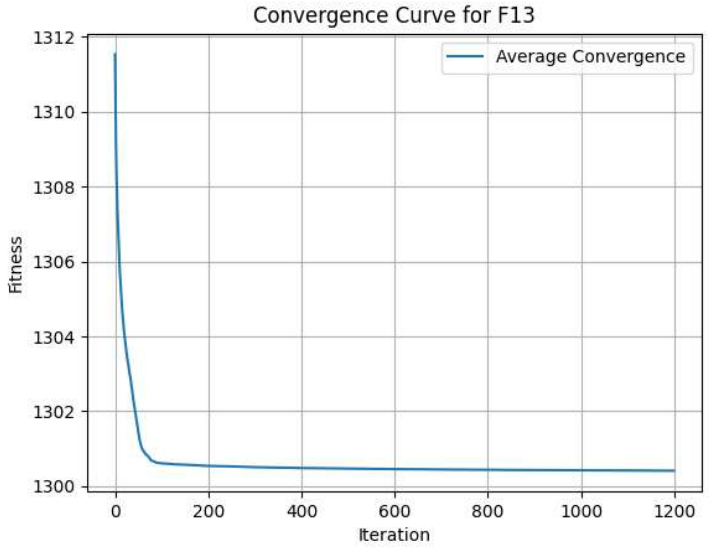
MODIFIED

Best Score: 1200.8177
Worst Score: 1306.0361
Mean Score: 1234.1844
Standard Deviation: 32.6992



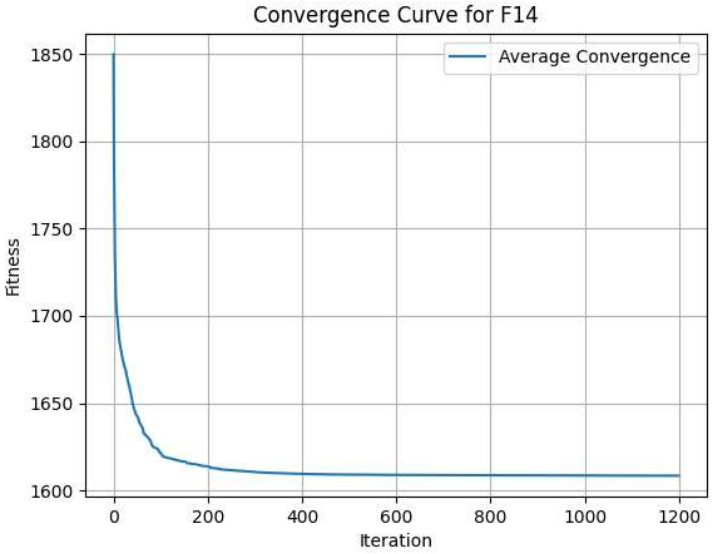
UNMODIFIED

Best Score: 1303.9844183363687
Worst Score: 1308.9029867044997
Mean Score: 1306.9139539363666
Standard Deviation: 0.9669719595518521



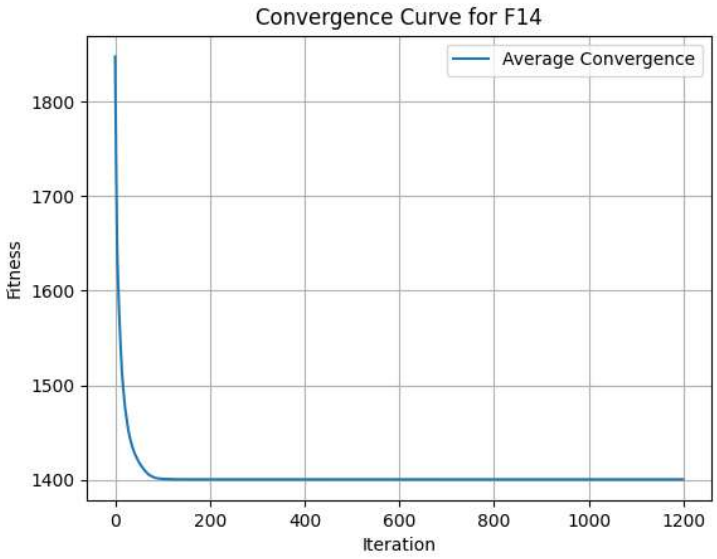
MODIFIED

Best Score: 1300.1836447408873
Worst Score: 1300.6499574138172
Mean Score: 1300.4141514682303
Standard Deviation: 0.09998301436618029



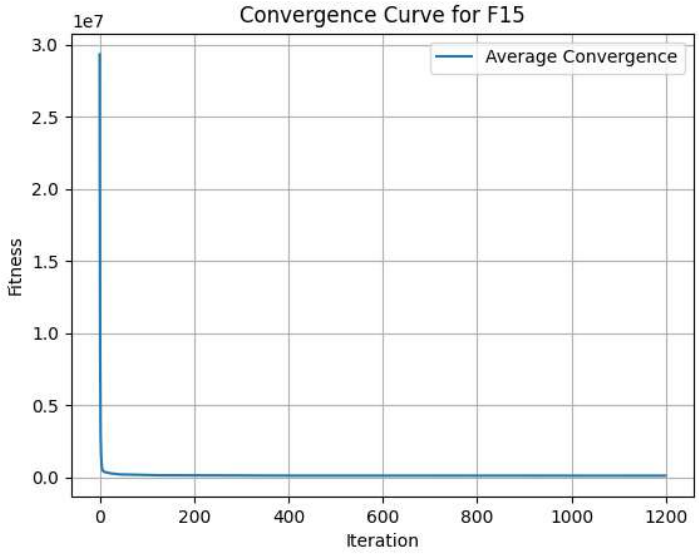
UNMODIFIED

Best Score: 1515.2965051197807
Worst Score: 1695.668939668596
Mean Score: 1608.5079784112997
Standard Deviation: 38.37423101276495



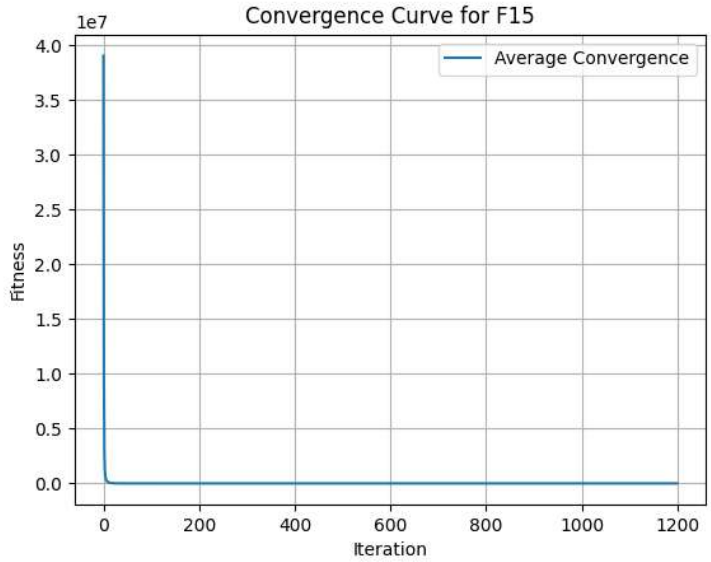
MODIFIED

Best Score: 1400.1572976881369
Worst Score: 1400.8064975787047
Mean Score: 1400.2701365877201
Standard Deviation: 0.09016971541161421



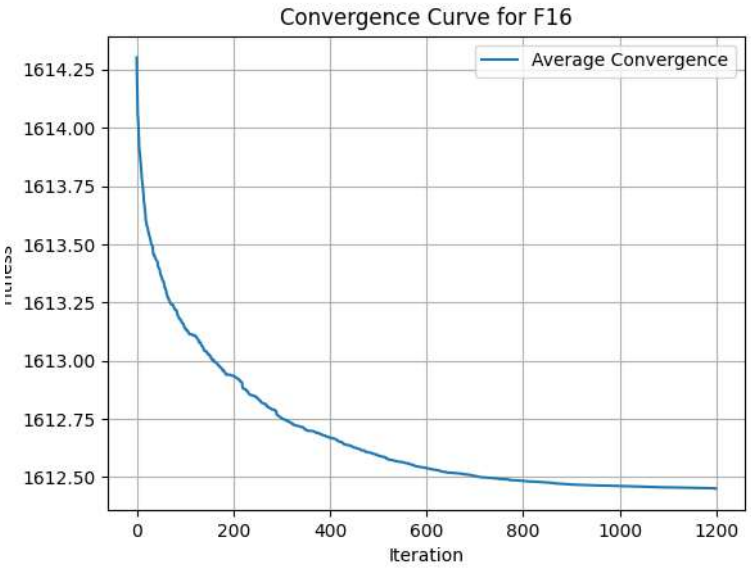
UNMODIFIED

Best Score: 24049.248373451843
Worst Score: 409343.8115703631
Mean Score: 135645.73950051315
Standard Deviation: 85231.18162241805



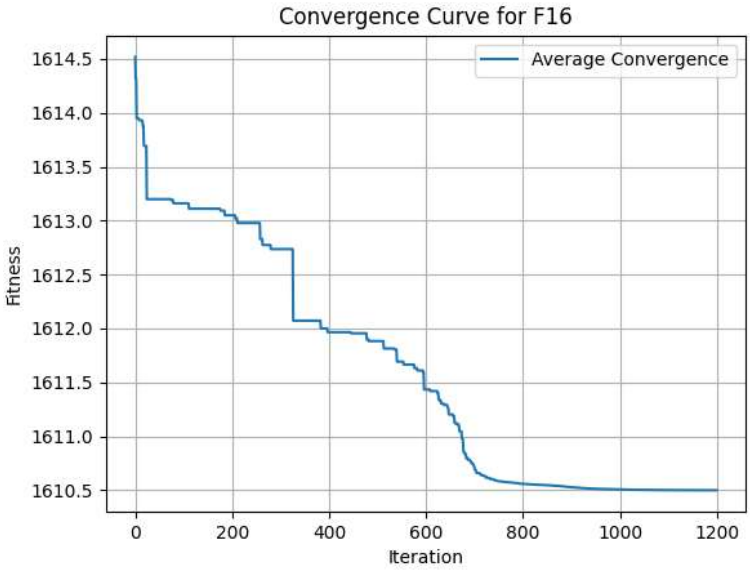
MODIFIED

Best Score: 1505.15775421961
Worst Score: 1557.9718362169149
Mean Score: 1517.8287228508793
Standard Deviation: 9.14530504950479



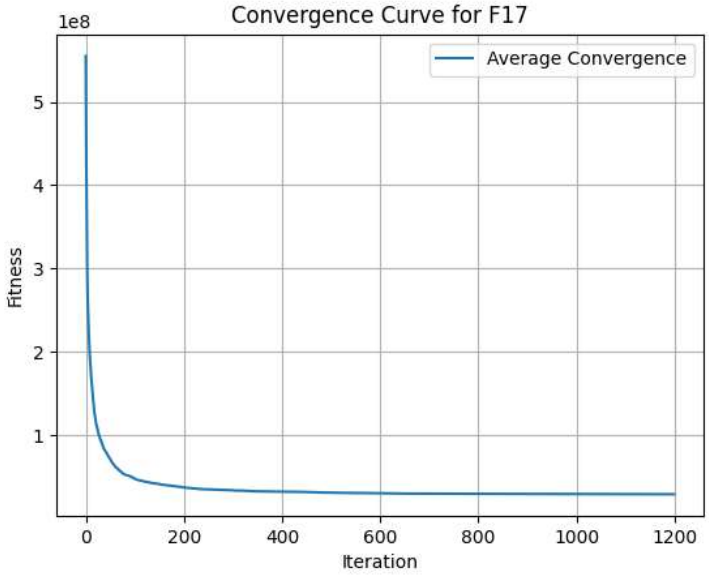
UNMODIFIED

Best Score: 1611.3463235903591
Worst Score: 1612.9695242125263
Mean Score: 1612.4508287470935
Standard Deviation: 0.2924173119218938



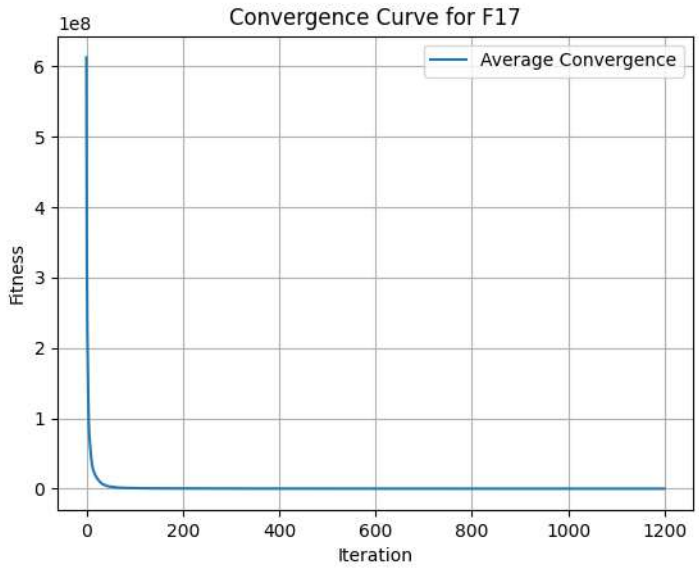
MODIFIED

Best Score: 1603.5952
Worst Score: 1616.8006
Mean Score: 1610.4233
Standard Deviation: 2.9088



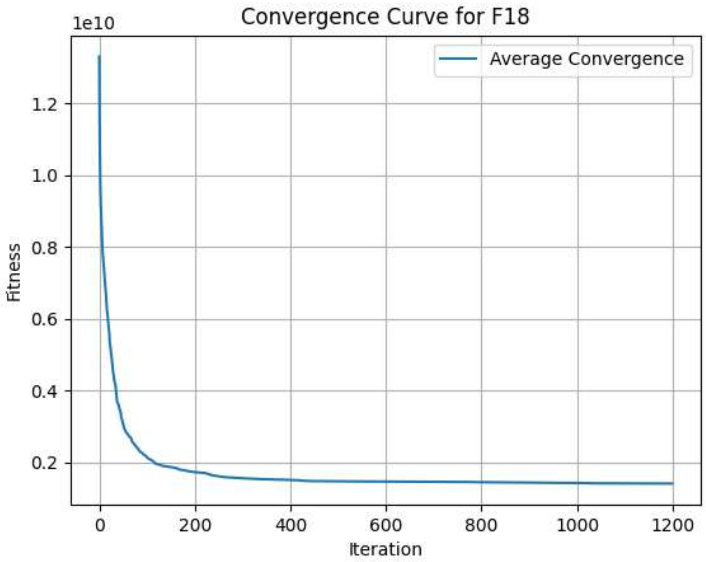
UNMODIFIED

Best Score: 399130.3466764335
Worst Score: 129782810.10472651
Mean Score: 29026684.421745777
Standard Deviation: 30600372.571922522



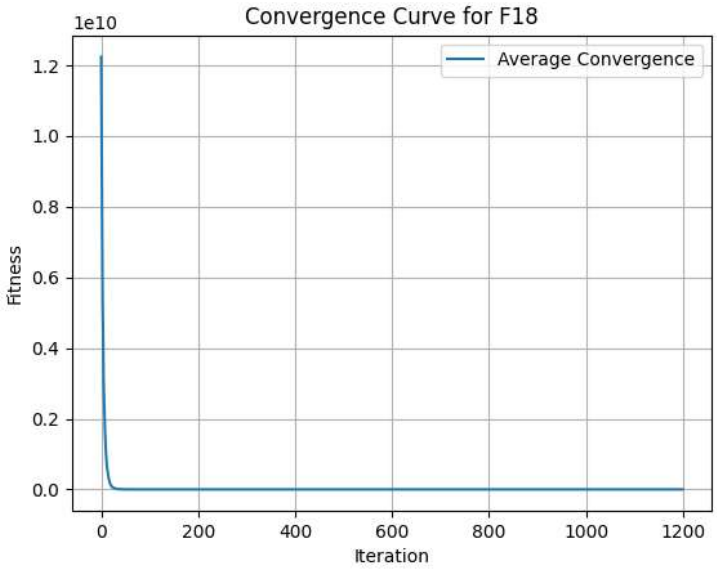
MODIFIED

Best Score: 14081.207385749309
Worst Score: 80322.6506862901
Mean Score: 40988.63471460826
Standard Deviation: 14538.19203567068



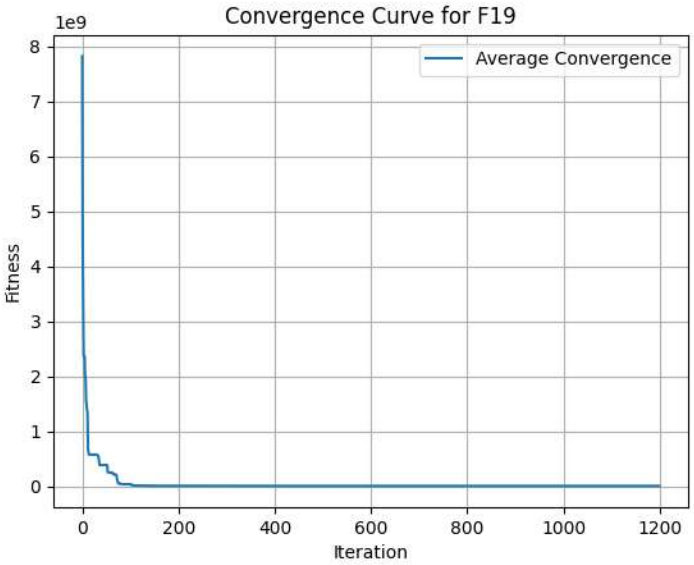
UNMODIFIED

Best Score: 58750088.769296646
Worst Score: 5922540752.211254
Mean Score: 1415815950.5654907
Standard Deviation: 1225122558.8372982



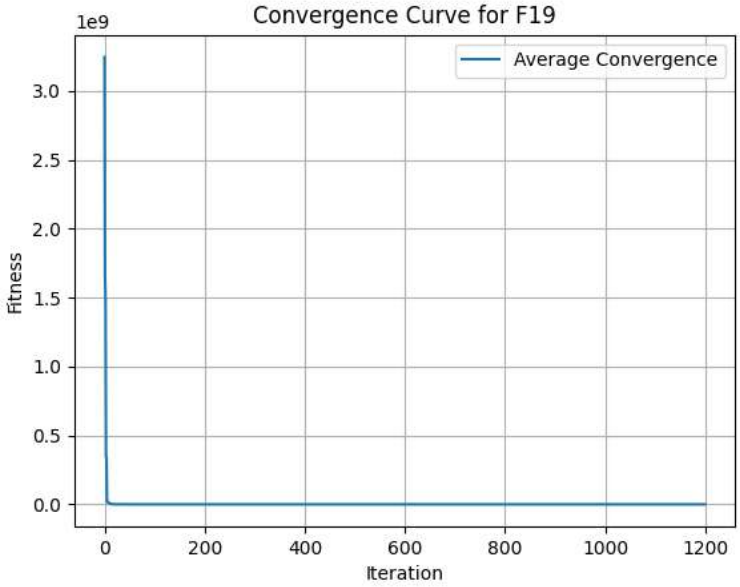
MODIFIED

Best Score: 39385.15271239326
Worst Score: 166116.44538403986
Mean Score: 80396.063165967
Standard Deviation: 25510.629415367188



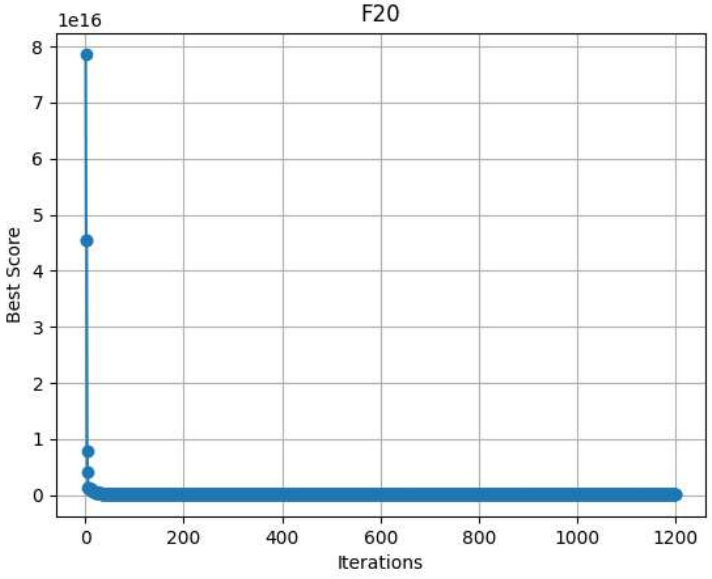
UNMODIFIED

Best Score: 106884051.36924127
Worst Score: 106876808.66745533
Mean Score: 106880103.64027071
Std Deviation: 1756.14



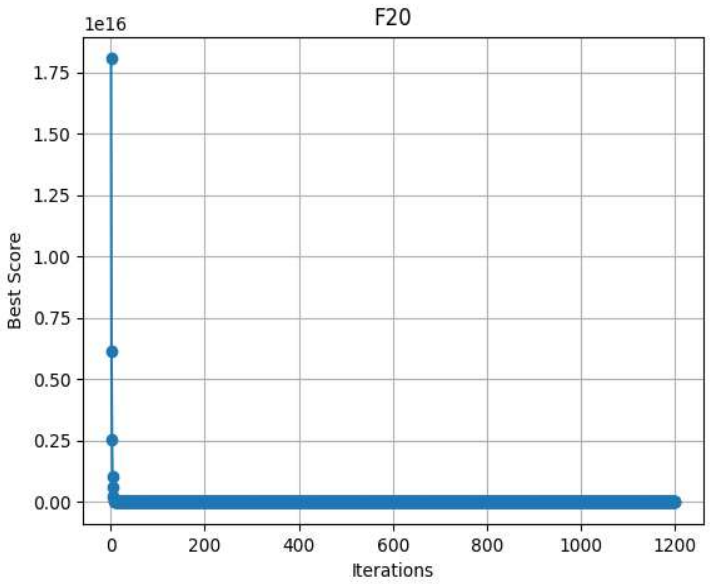
MODIFIED

Best Score: 1913.6054
Worst Score: 1932.8964
Mean Score: 1924.3796
Standard Deviation: 4.5021



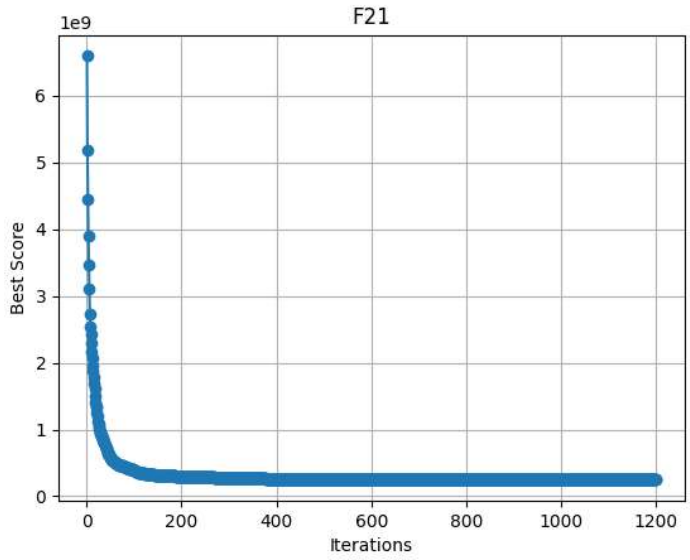
UNMODIFIED

Best Score: 3164476737.105873
Worst Score: 1039250335571581.9
Mean Score: 88841754741720.52
Standard Deviation: 210824331239399.6



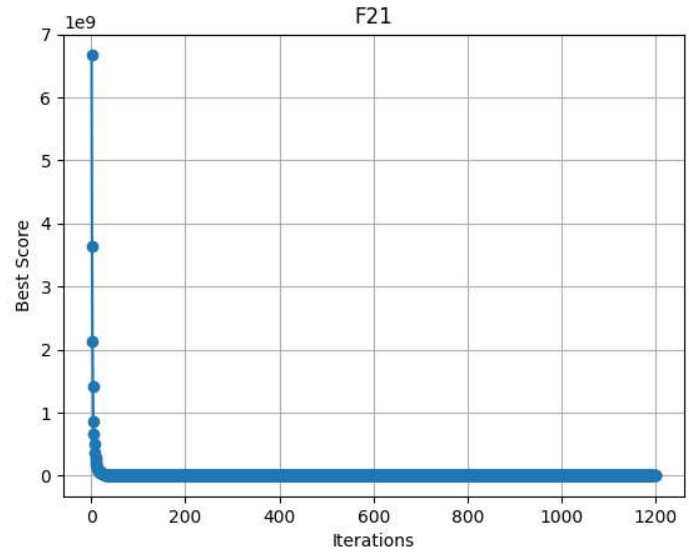
MODIFIED

Best Score: 6806.334388342558
Worst Score: 95053.94307128708
Mean Score: 40443.60841489664
Standard Deviation: 20408.843731506644



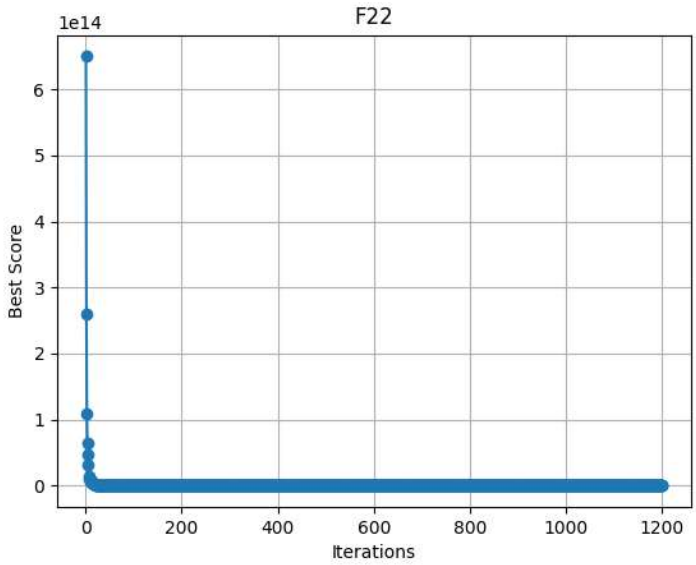
UNMODIFIED

Best Score: 47421945.12269235
Worst Score: 1055291844.4672654
Mean Score: 247245071.90362293
Standard Deviation: 202681904.42744023



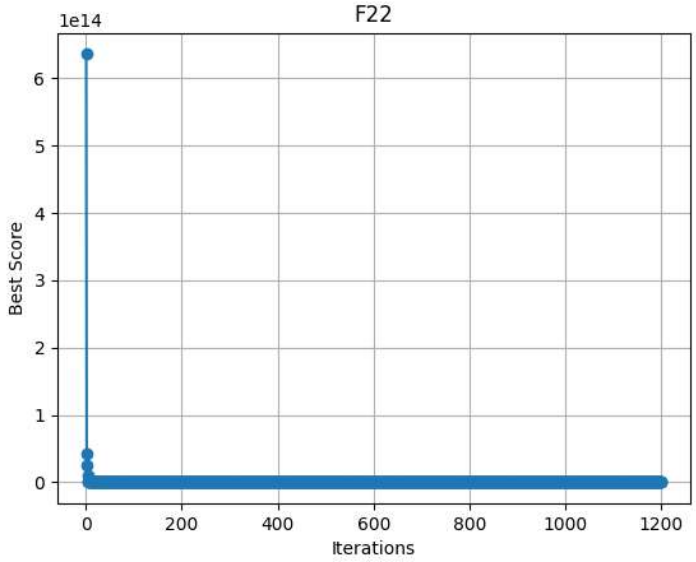
MODIFIED

Best Score: 9831.651614484308
Worst Score: 68217.93658909864
Mean Score: 28416.62442747859
Standard Deviation: 14241.969954249596



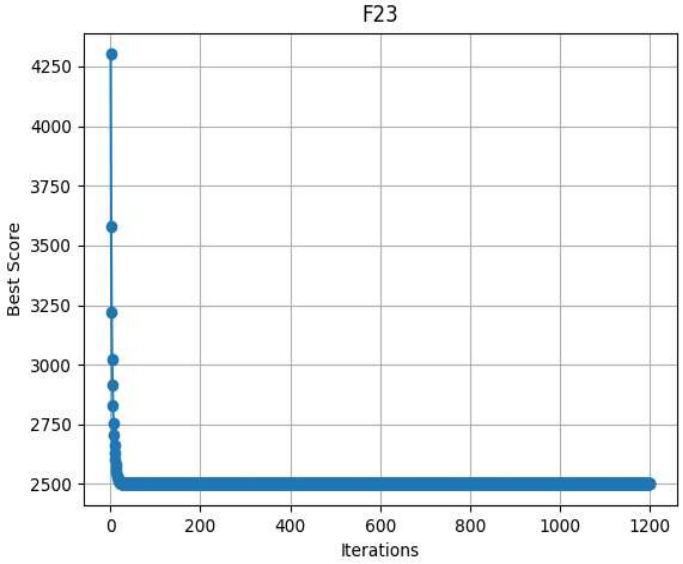
UNMODIFIED

Best Score: 4132.369128299539
Worst Score: 76614534483.1865
Mean Score: 2304907800.348378
Standard Deviation: 11196112995.6686



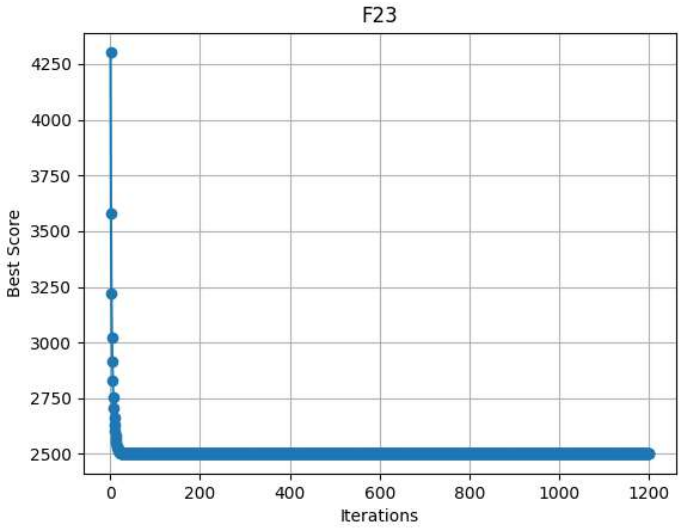
MODIFIED

Best Score: 2274.3432843372225
Worst Score: 2942.9530918544388
Mean Score: 2608.712243879657
Standard Deviation: 193.1536837346151



UNMODIFIED

Best Score: 2500.0
Worst Score: 2500.0000000000005
Mean Score: 2500.0
Standard Deviation: 9.094947017729283e-14



MODIFIED

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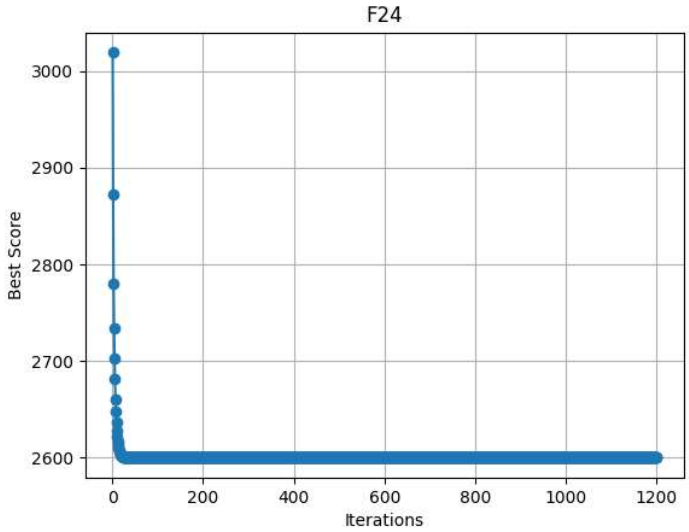
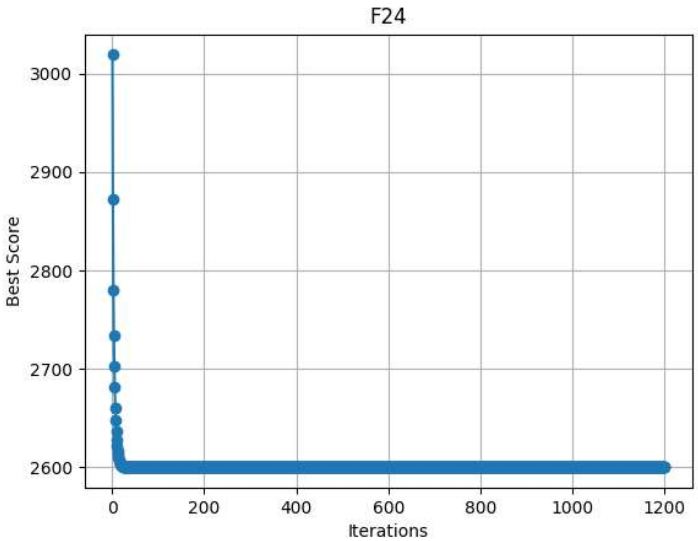
Mean Score: 2500.0
Standard Deviation: 9.094947017729283e-14



UNMODIFIED

Best Score: 2600.0
Worst Score: 2600.0
Mean Score: 2600.0
Standard Deviation: 0.0



MODIFIED

Best Score: 2600.0
Worst Score: 2600.0
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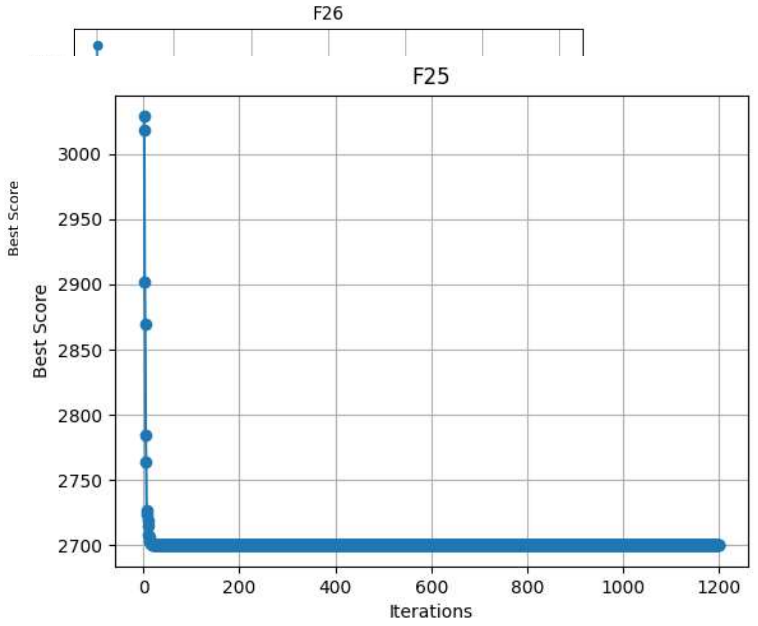

UNMODIFIED

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Worst Score: 2700.0
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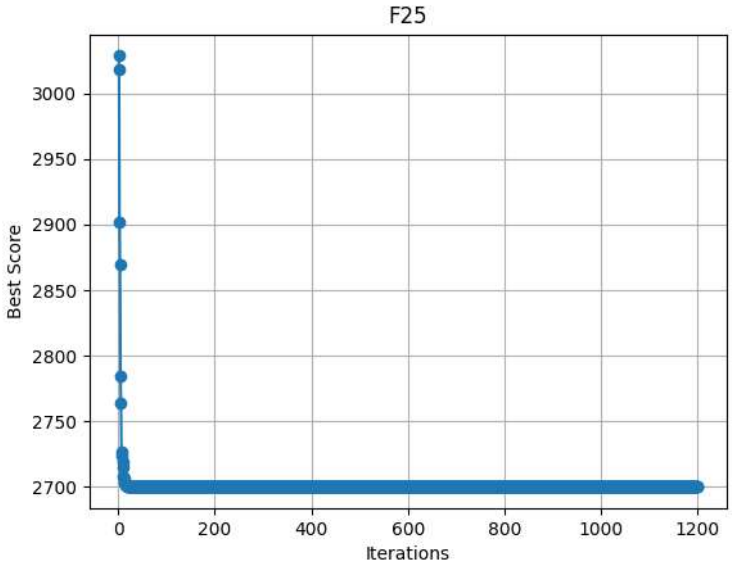
Standard Deviation: 0



MODIFIED

Best Score: 2700.0
Worst Score: 2700.0
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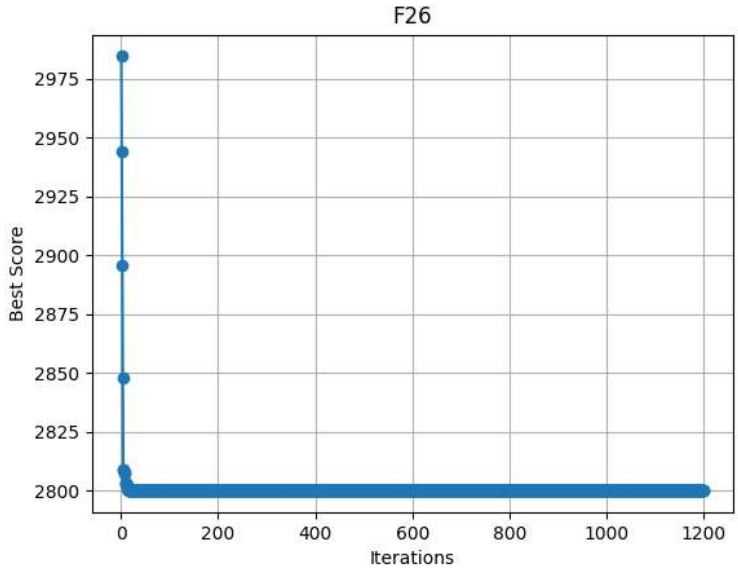
Standard Deviation: 0



UNMODIFIED

Best Score: 2800.0
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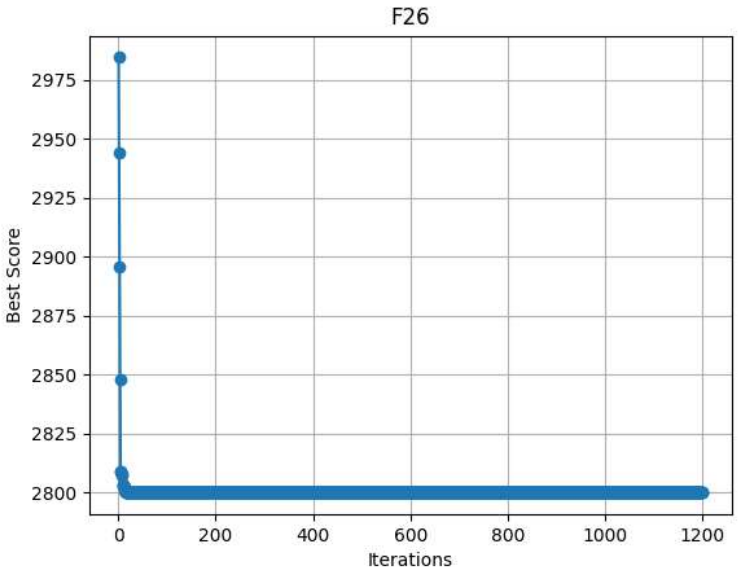
Mean Score: 2800.0
Standard Deviation: 0



MODIFIED

Best Score: 2800.0
Worst Score: 2800.0
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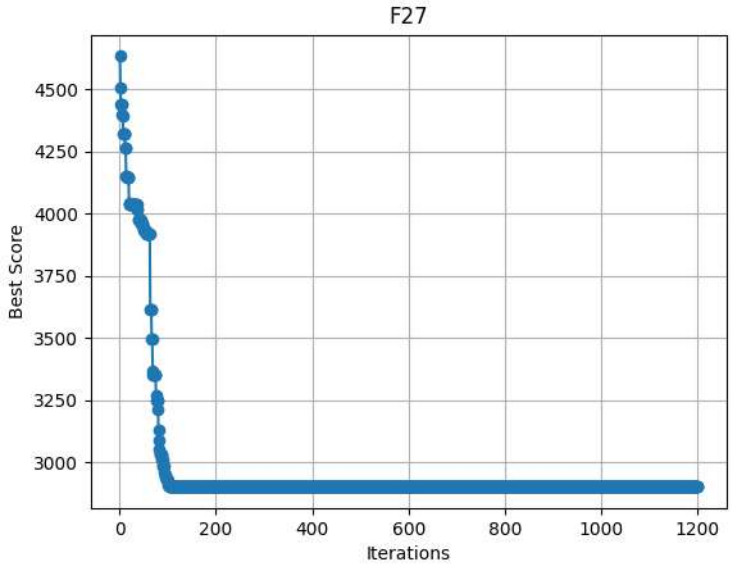
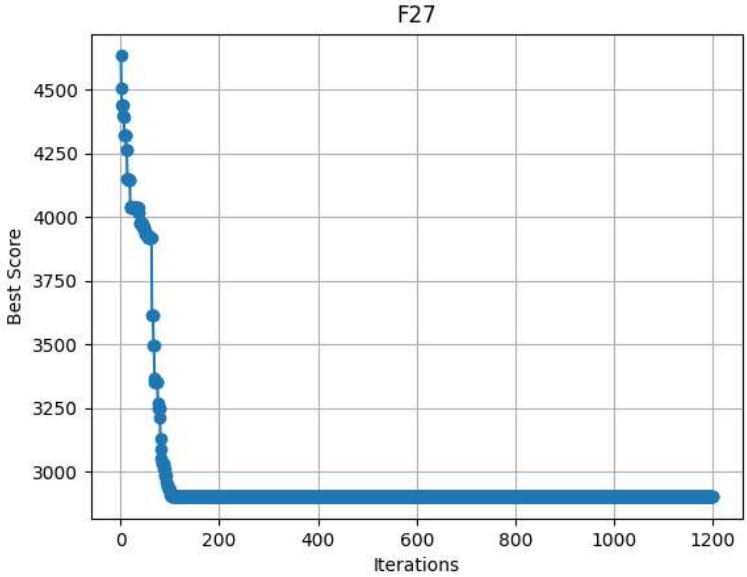
Standard Deviation: 0



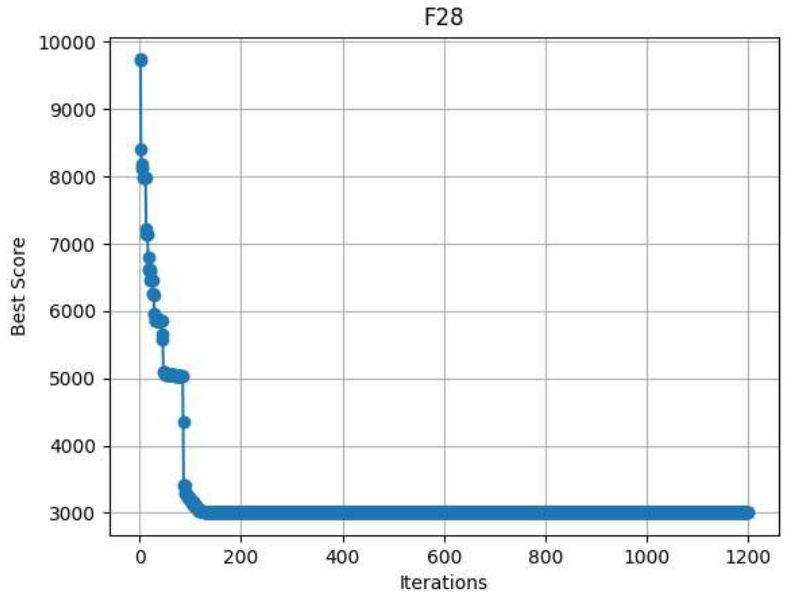
UNMODIFIED

Best Score: 2900.0
Worst Score: 2900.0
Mean Score: 2900.0
Standard Deviation: 0



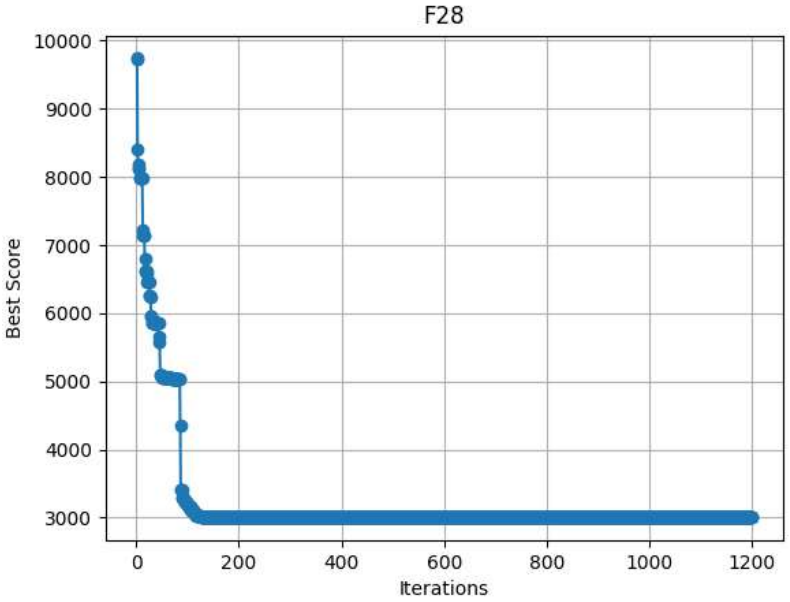
MODIFIED

Best Score: 2900.0
Worst Score: 2900.0
Mean Score: 2900.0
Standard Deviation: 0



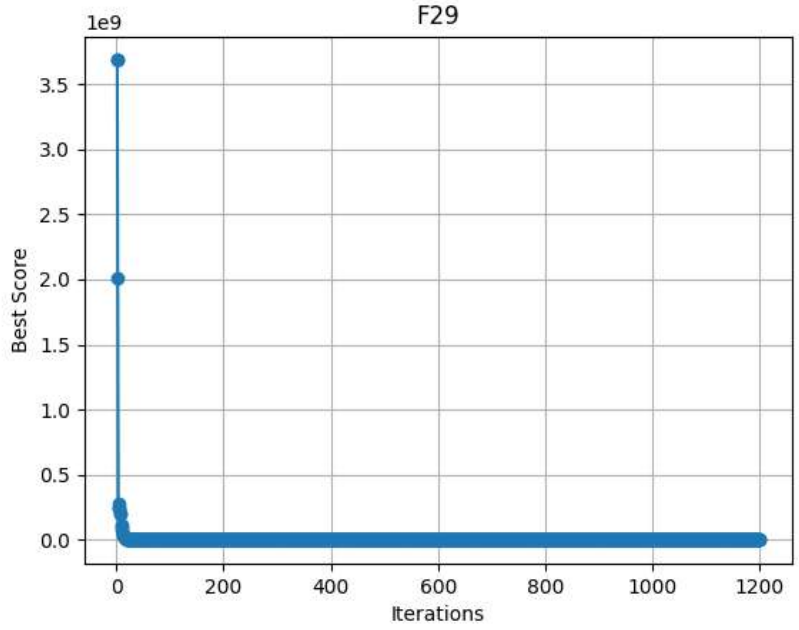
UNMODIFIED

Best Score: 3000.0
Worst Score: 3000.0
Mean Score: 3000.0
Standard Deviation: 0



MODIFIED

Best Score: 3000.0
Worst Score: 3000.0
Mean Score: 3000.0
Standard Deviation: 0



UNMODIFIED

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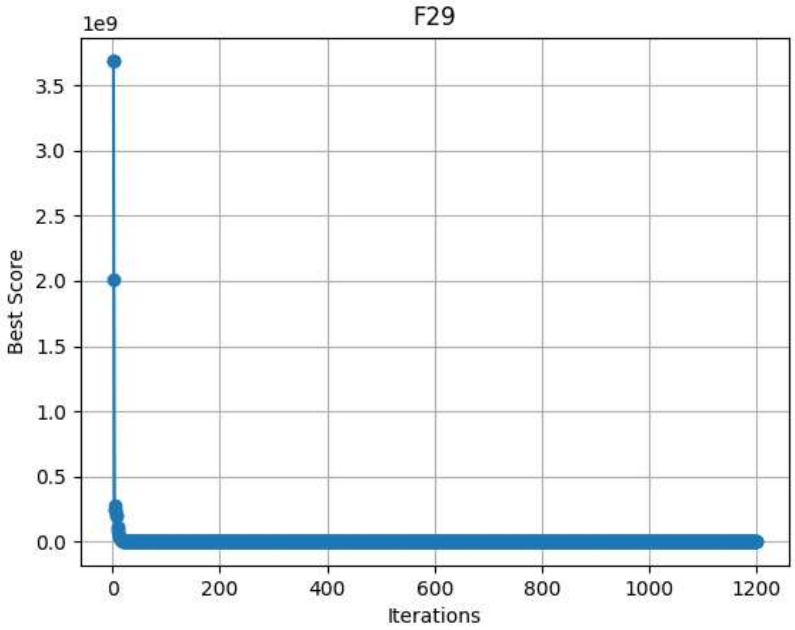
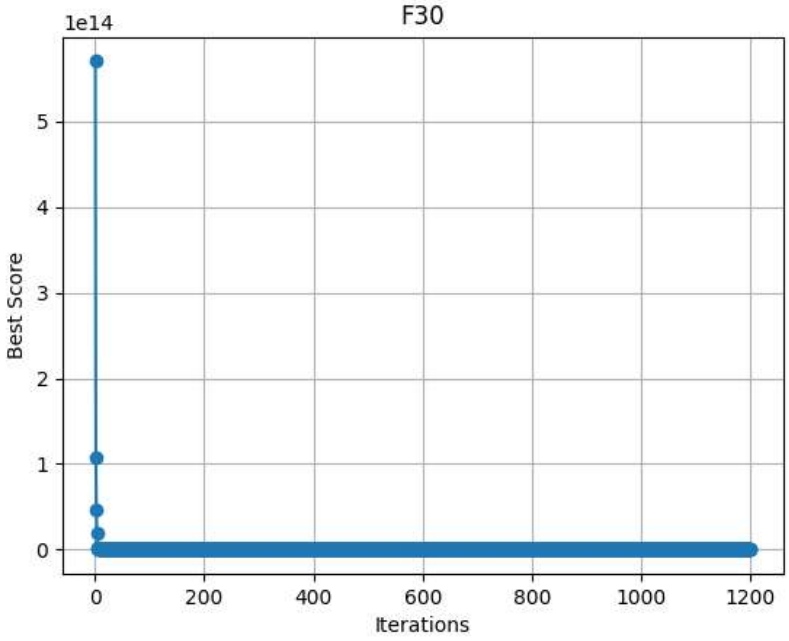
Mean Score: 3100.0
Standard Deviation: 0



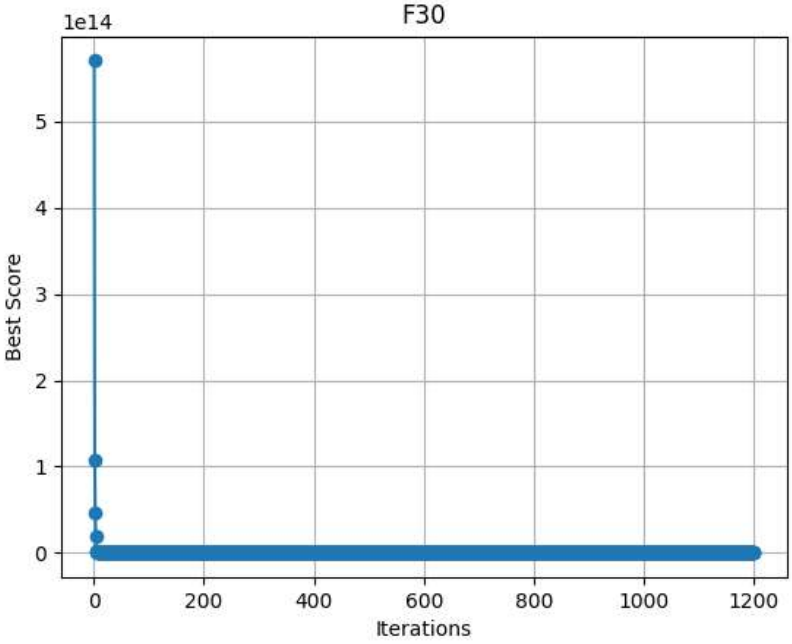
MODIFIED

Best Score: 3100.0
Worst Score: 3100.0
Mean Score: 3100.0
Standard Deviation: 0



UNMODIFIED

Best Score: 3200.0
Worst Score: 3200.0
Mean Score: 3200.0
Standard Deviation: 0



MODIFIED

Best Score: 3200.0
Worst Score: 3200.0
Mean Score: 3200.0
Standard Deviation: 0

6 CEC 2017 Benchmark Functions

To further investigate the scalability, robustness, and general applicability of the proposed optimization algorithm, the CEC 2017 benchmark suite was employed. This benchmark collection is among the most comprehensive and rigorous in the field of real-parameter global optimization, containing 30 functions that represent a diverse array of problem types and characteristics Suganthan et al. [2017].

The benchmark functions are grouped into the following categories:

- **Unimodal functions:** Designed to evaluate the exploitation capability of an algorithm.
- **Multimodal functions:** Contain many local optima, testing the algorithm's global search ability.
- **Hybrid functions:** Combine multiple basic functions to create complex search landscapes.
- **Composition functions:** Highly challenging functions that blend shifted, rotated, and scaled variants of several component functions.

These functions are characterized by features such as rotation, translation, varying levels of modality, and deceptive local optima, making them highly suitable for stress-testing optimization algorithms in real-world-like conditions.

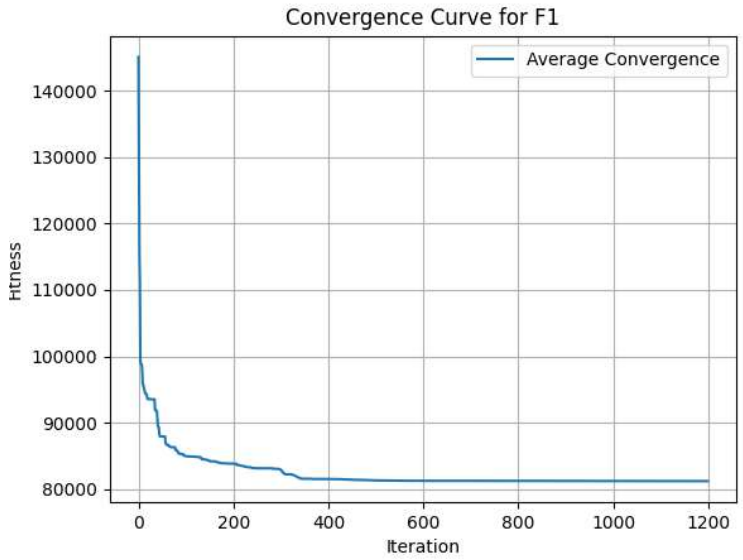
Each function was tested using a 30-dimensional search space. Each function was independently run 50 times, and the results were analyzed using key performance indicators such as best fitness value, mean fitness, standard deviation, and convergence behavior Suganthan et al. [2017].

Key Findings:

- The proposed algorithm consistently achieved better or comparable results to POA, particularly excelling in hybrid and composition functions.
- It demonstrated improved robustness and lower variance, indicating consistent performance across multiple runs.
- Enhanced exploration mechanisms allowed the algorithm to effectively escape local optima even in complex landscapes.
- In convergence plots, the algorithm showed faster initial progress and smoother late-stage convergence, highlighting both strong exploration and exploitation capabilities.

The experimental outcomes on the CEC 2017 test suite affirm the proposed algorithm's superior performance, adaptability, and stability. It effectively handles challenging optimization scenarios involving high-dimensionality, multimodality, rotation, and non-separability. Suganthan et al. [2017]. .

CONVERGENCE CURVES & RESULTS



UNMODIFIED

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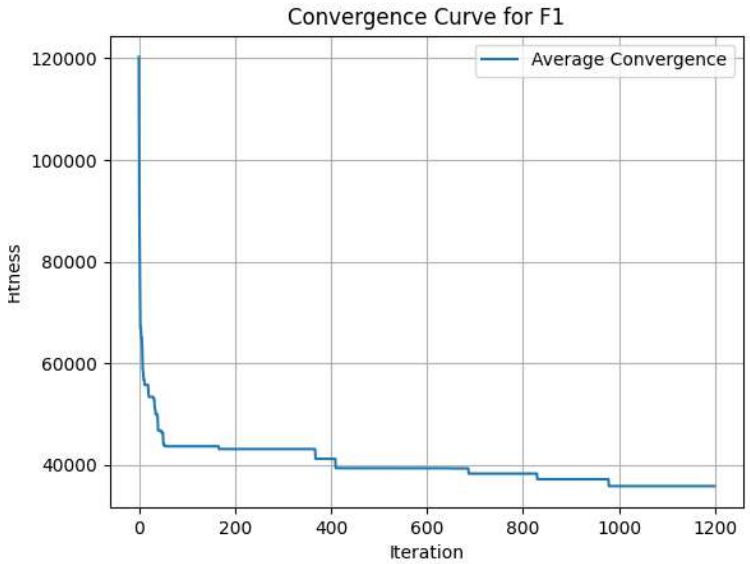
Worst Score: 100115.72491445071
Mean Score: 81197.05156995093
Standard Deviation: 16438.827377742964



MODIFIED

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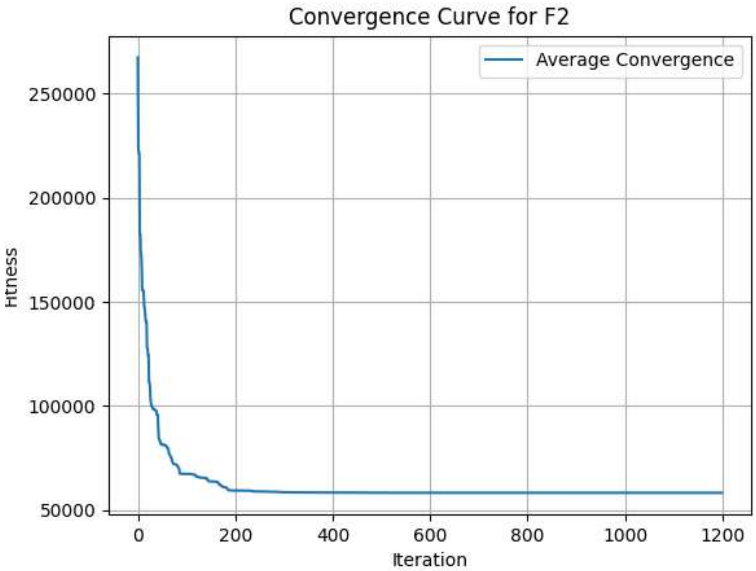
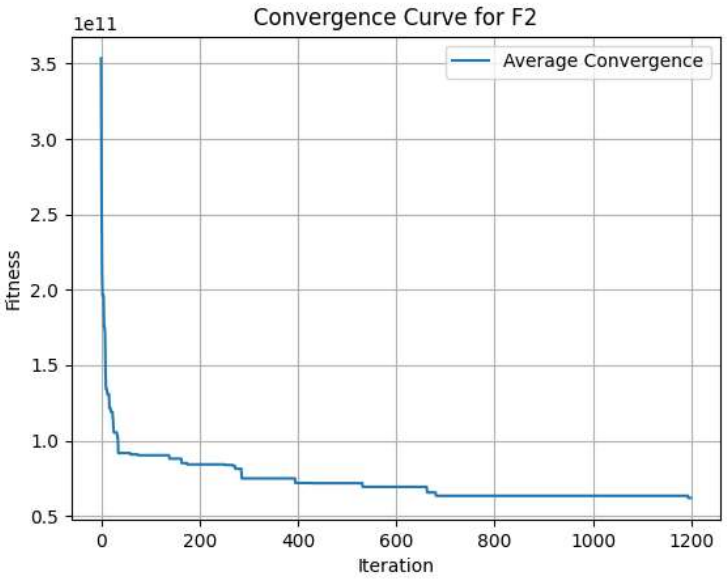
Worst Score: 39147.58199202053
Mean Score: 35854.65745194474
Standard Deviation: 2459.1115762319037



UNMODIFIED

Best Score: 41160.35876083782
Worst Score: 91252.57037805795
Mean Score: 58393.888062991144
Standard Deviation: 23243.87333160877



MODIFIED

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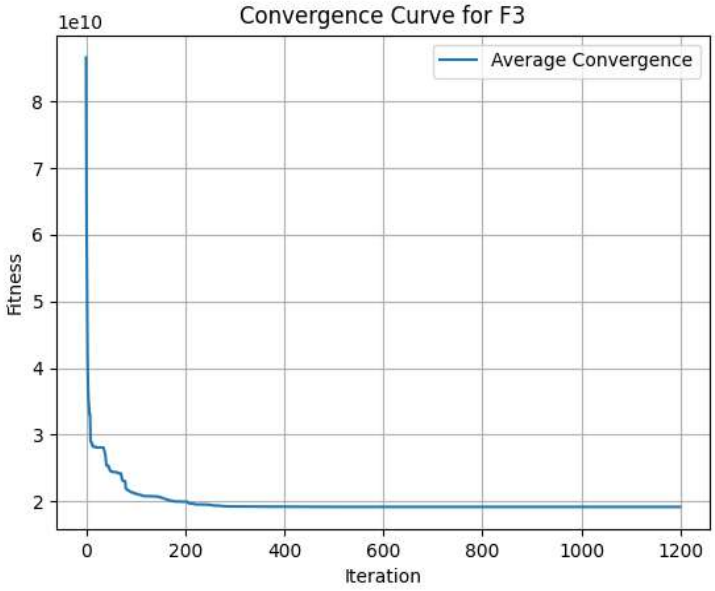
Standard Deviation: 5697585179.118076



UNMODIFIED

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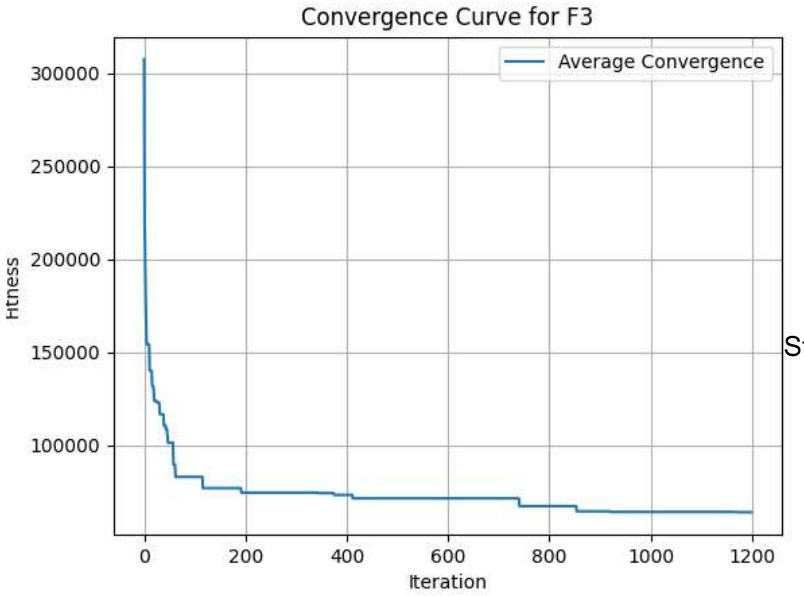
Standard Deviation: 2446427129.449978



MODIFIED

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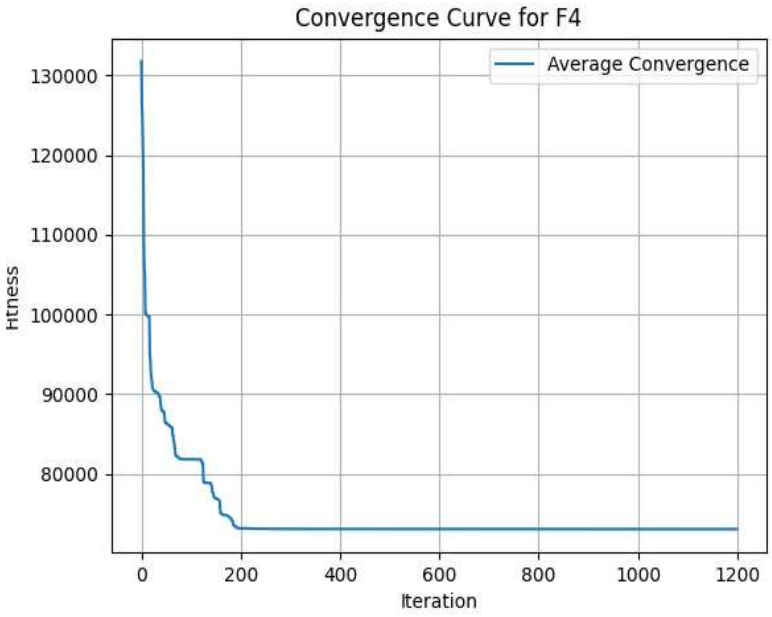

UNMODIFIED

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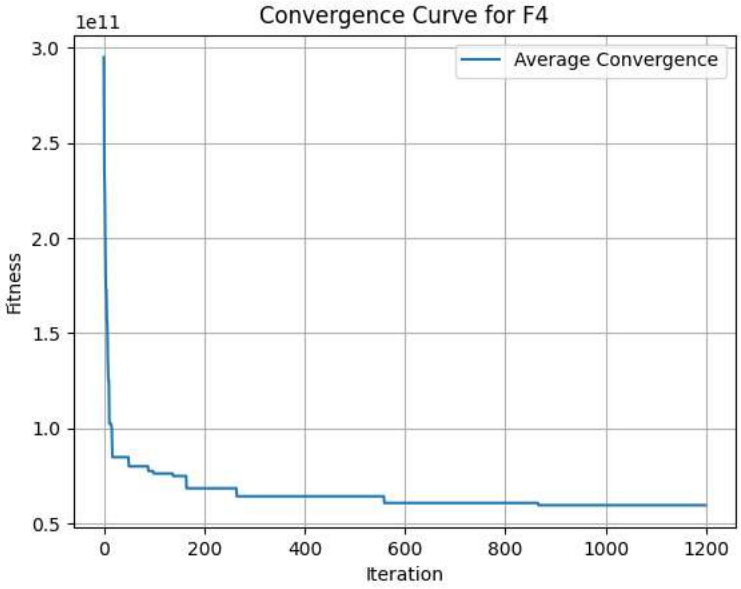
Standard Deviation: 2953.7583269383595



MODIFIED

Best Score: 50462390245.632545
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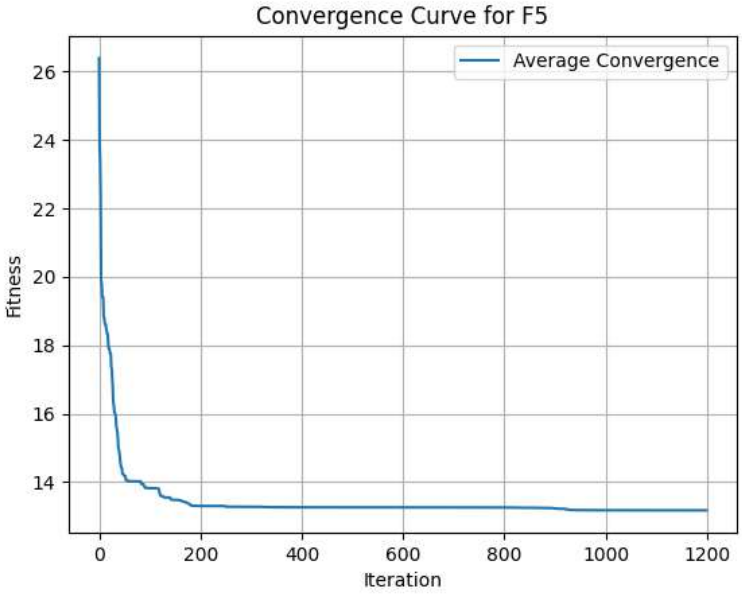

UNMODIFIED

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Worst Score: 16.0538677573714
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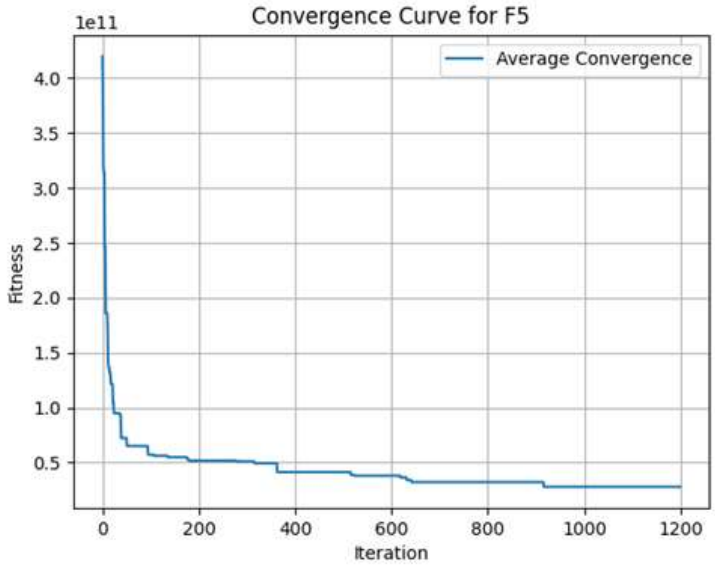
Standard Deviation: 2.214370044572473



MODIFIED

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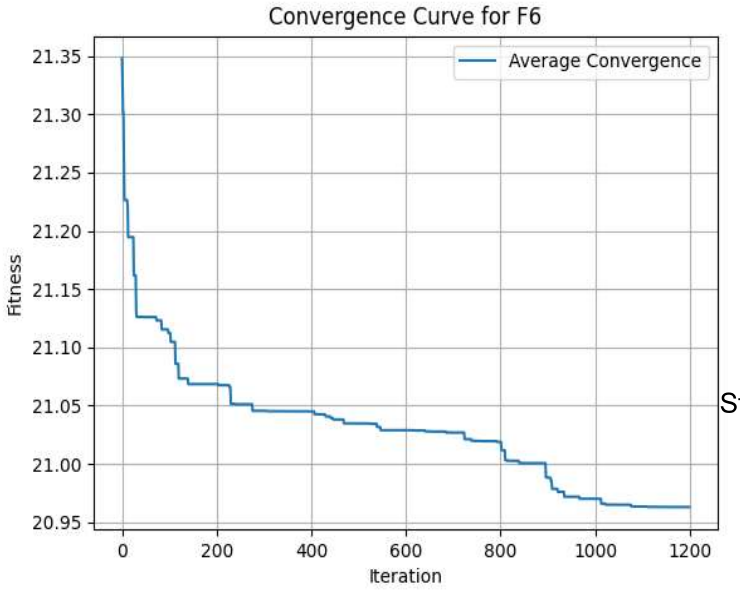

UNMODIFIED

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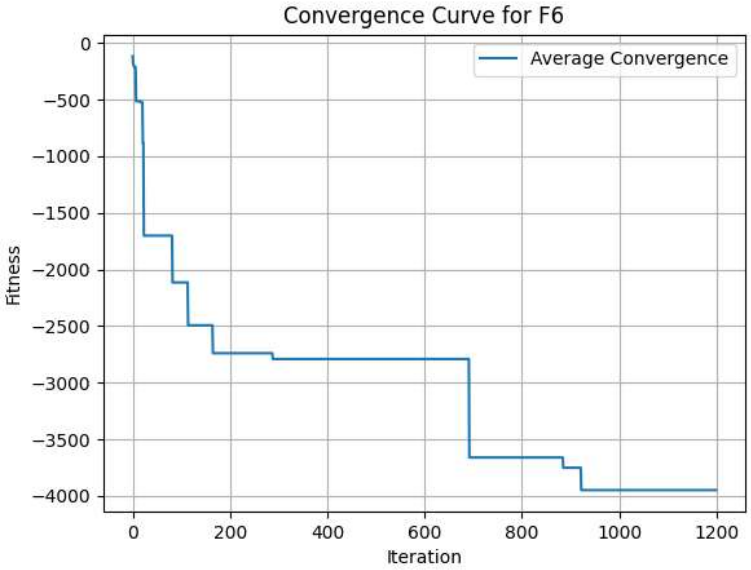
Standard Deviation: 0.01082877371598218



MODIFIED

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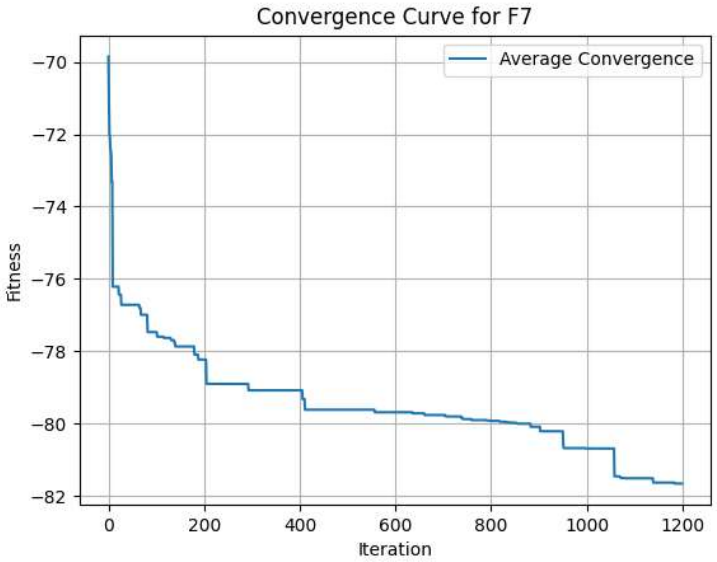
Standard Deviation: 1819.999797030092



UNMODIFIED

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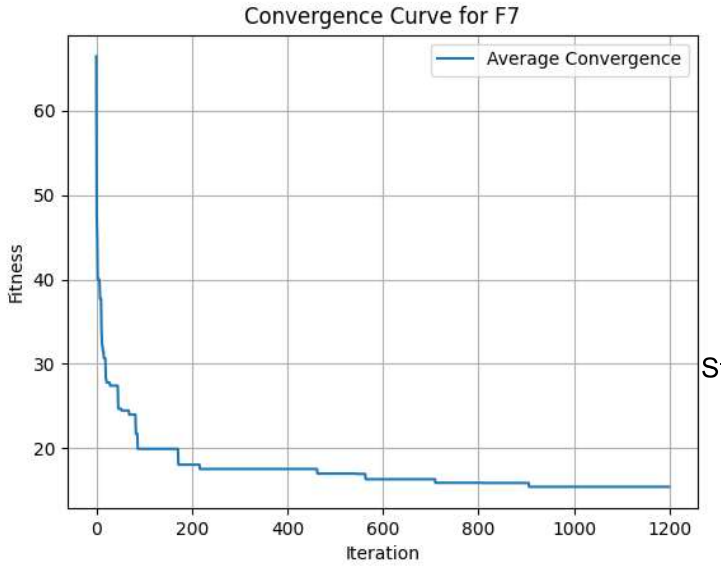

MODIFIED

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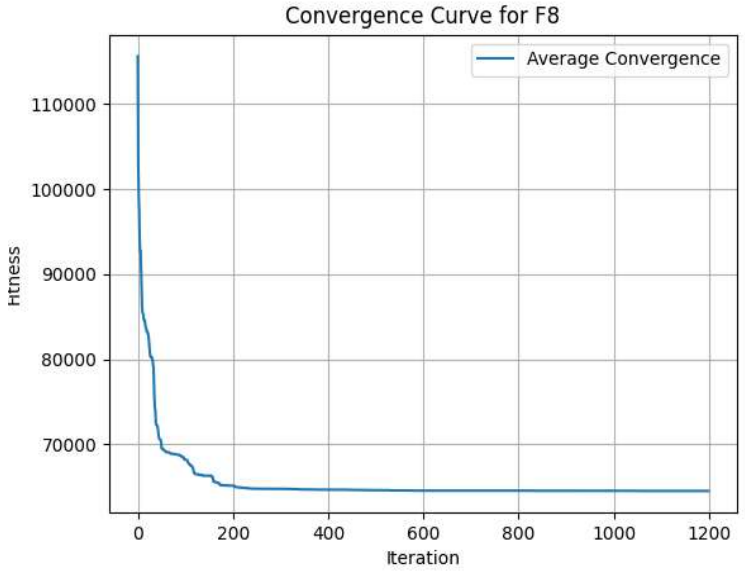

UNMODIFIED

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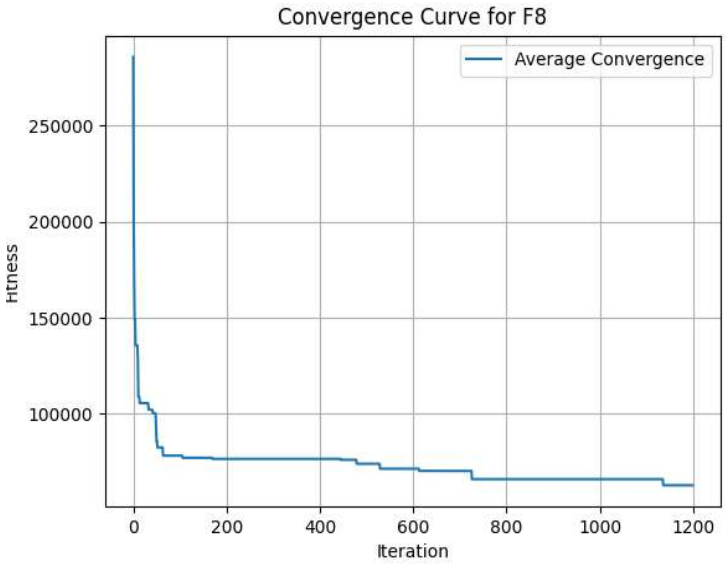

MODIFIED

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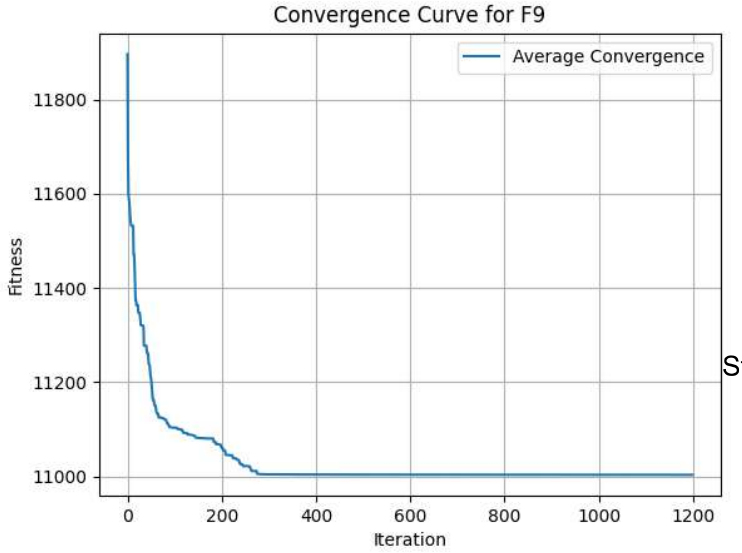

UNMODIFIED

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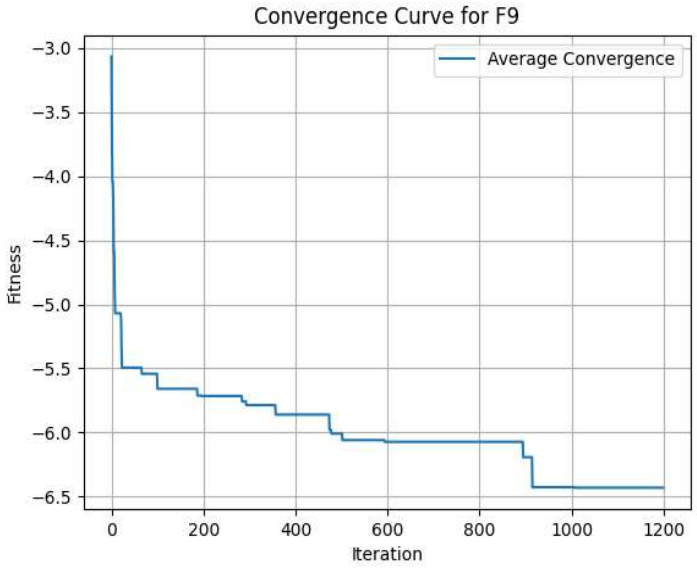

MODIFIED

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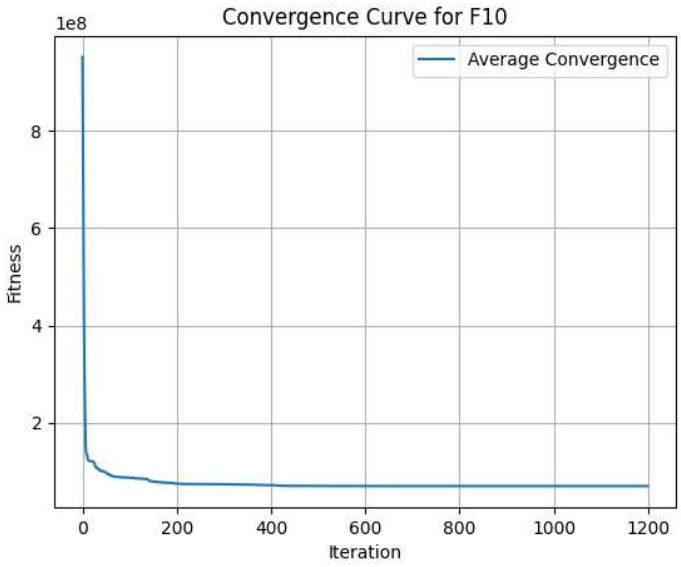

UNMODIFIED

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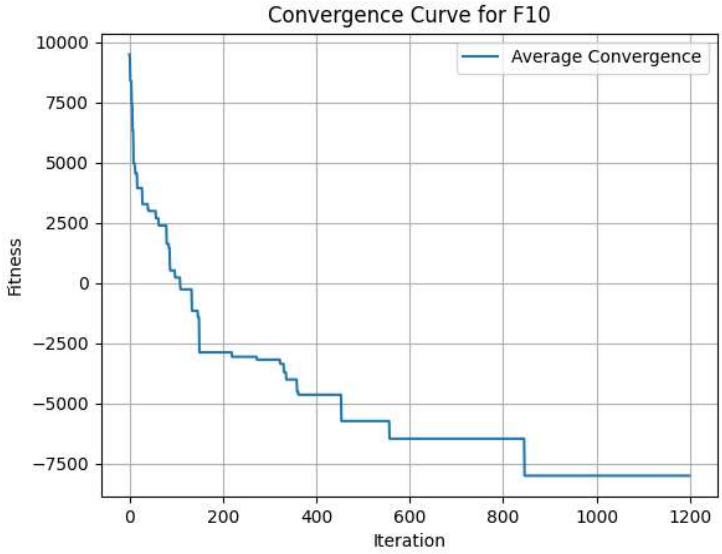

MODIFIED

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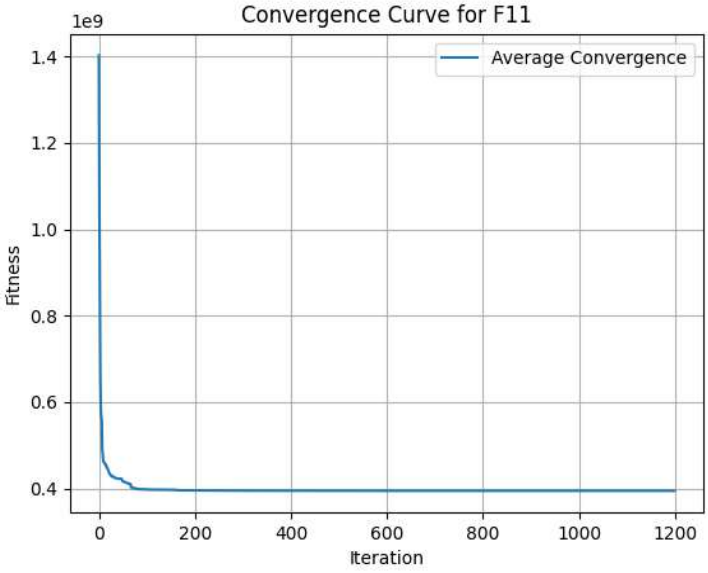

UNMODIFIED

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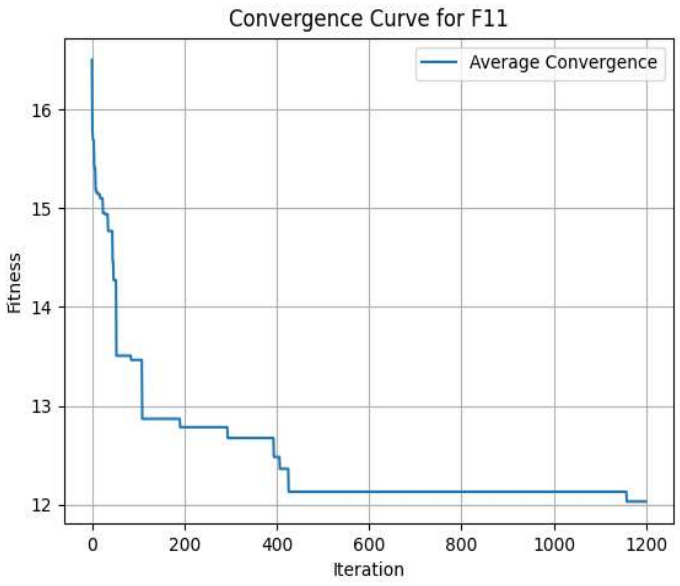

MODIFIED

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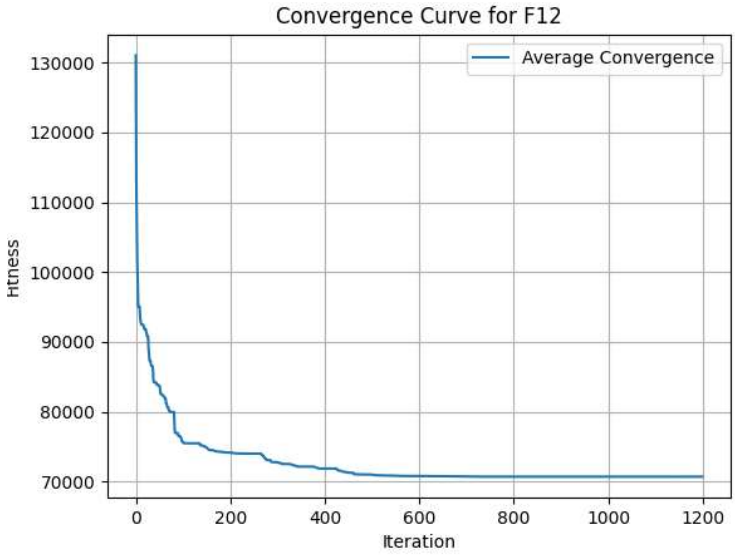

UNMODIFIED

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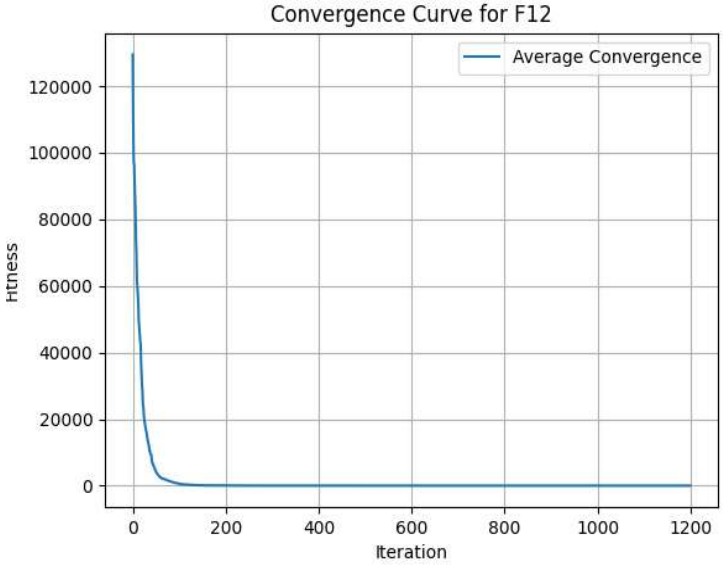
Standard Deviation: 5981.139087701205



MODIFIED

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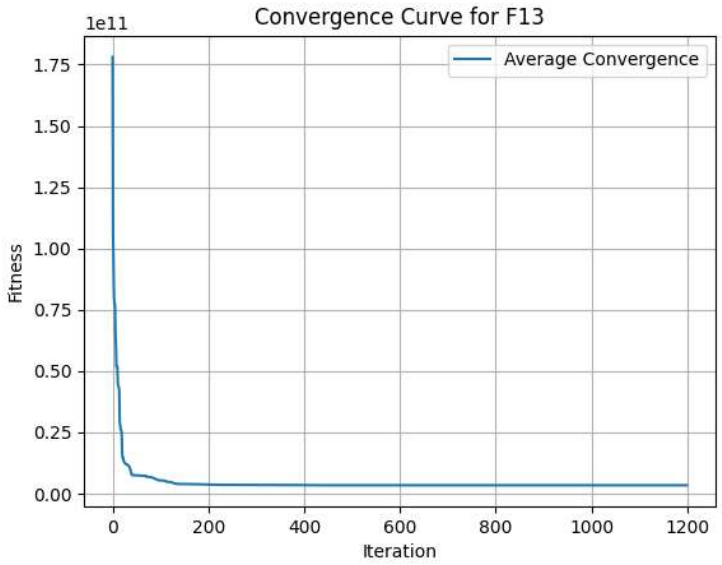
Standard Deviation: 4.761503123935413e-12



UNMODIFIED

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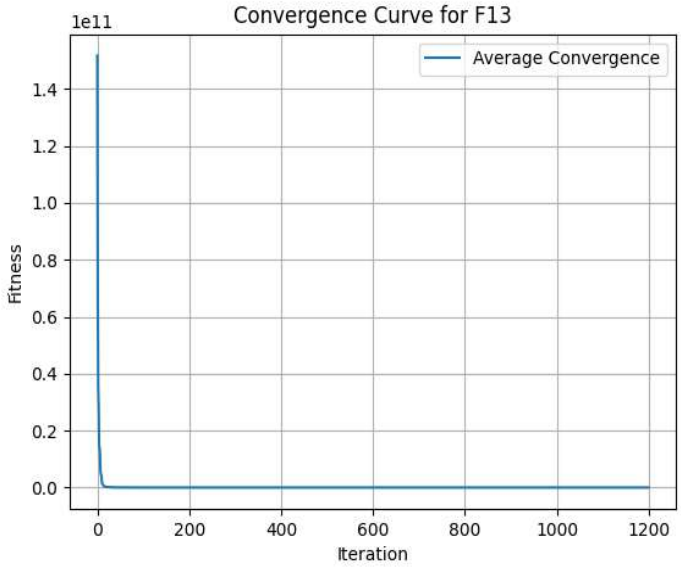

MODIFIED

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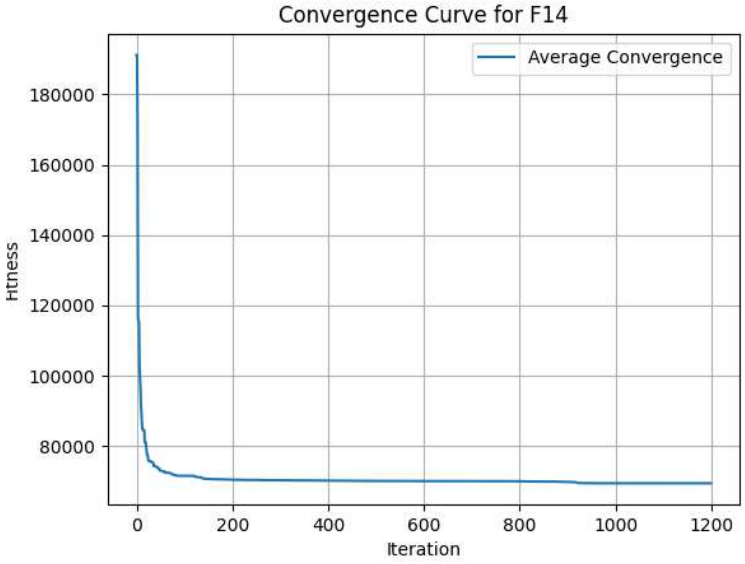
Standard Deviation: 7.4429186011477915e-06



UNMODIFIED

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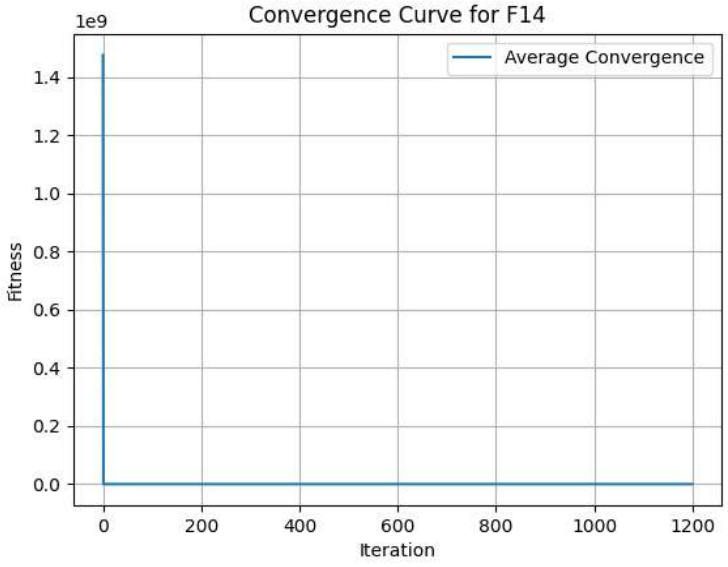

MODIFIED

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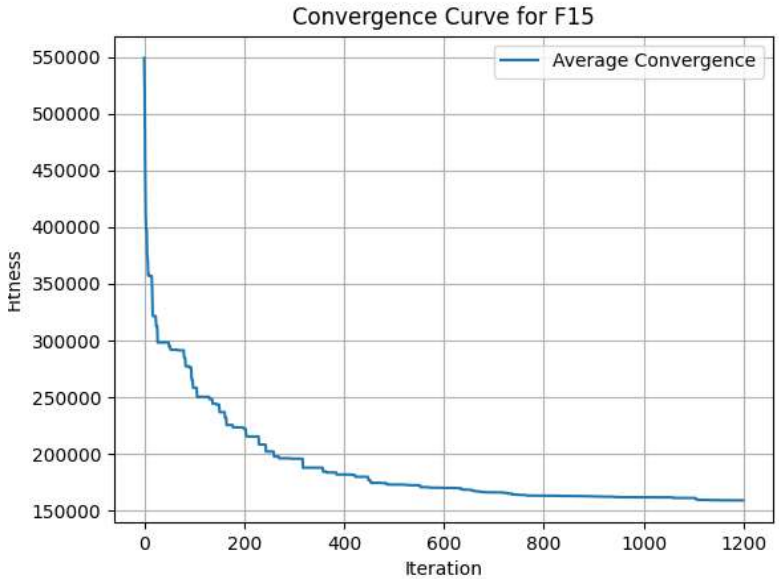

UNMODIFIED

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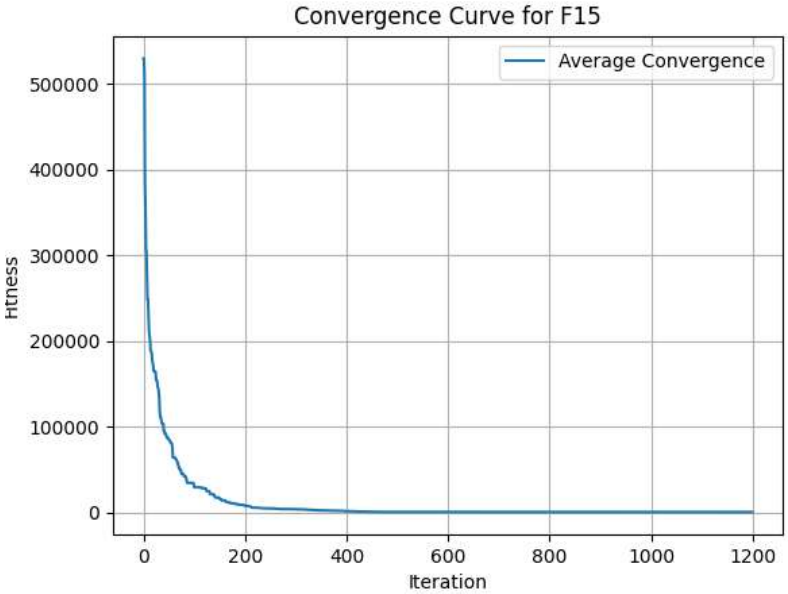
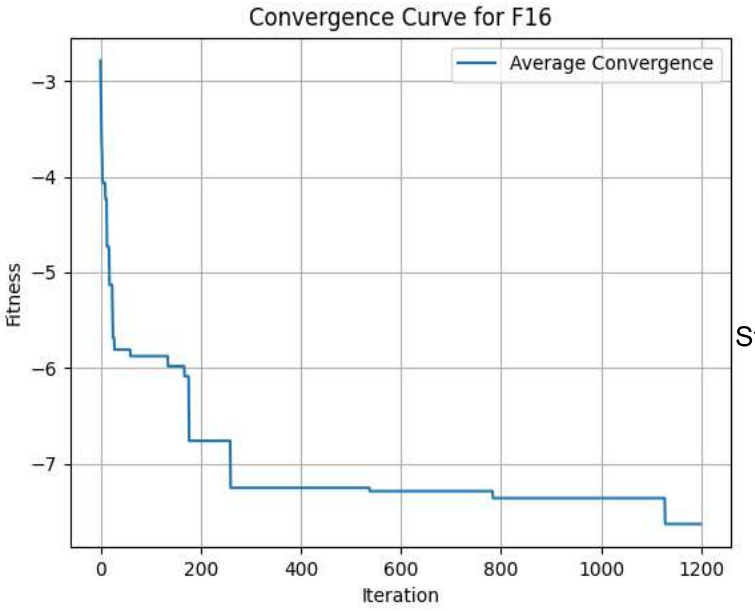


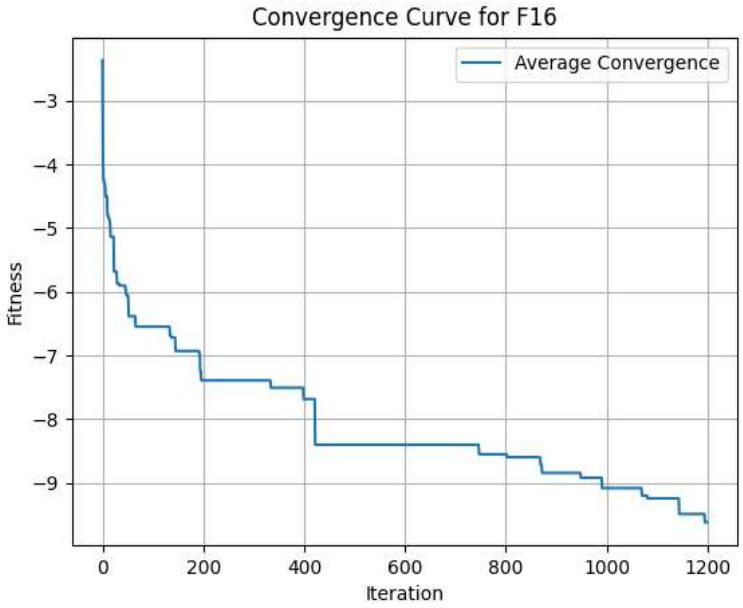
MODIFIED

Best Score: 30.04183544917355
Worst Score: 601.8874306734559
Mean Score: 231.67381941175026
Standard Deviation: 262.1280475791304



UNMODIFIED

Best Score: -8.334015578214135
Worst Score: -7.252763153358438
Mean Score: -7.628760941320443
Standard Deviation: 0.4990552781609135



MODIFIED

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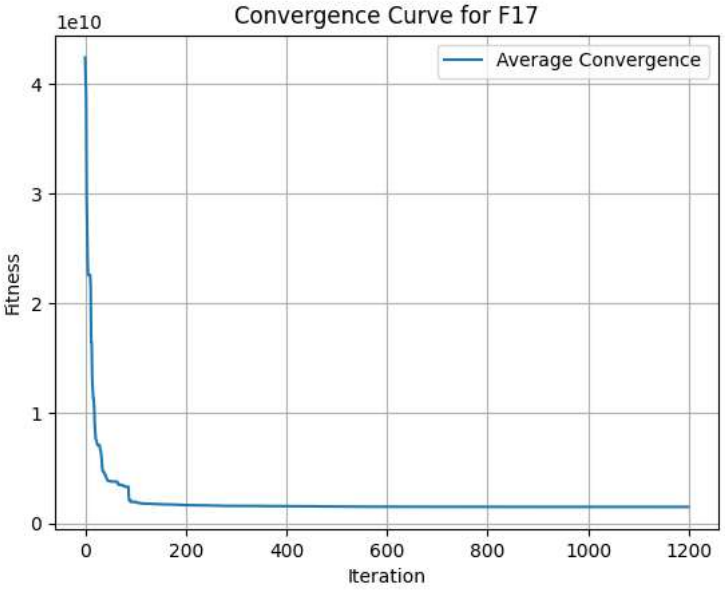
Mean Score: -9.620915873236283
Standard Deviation: 1.1541269882445386



UNMODIFIED

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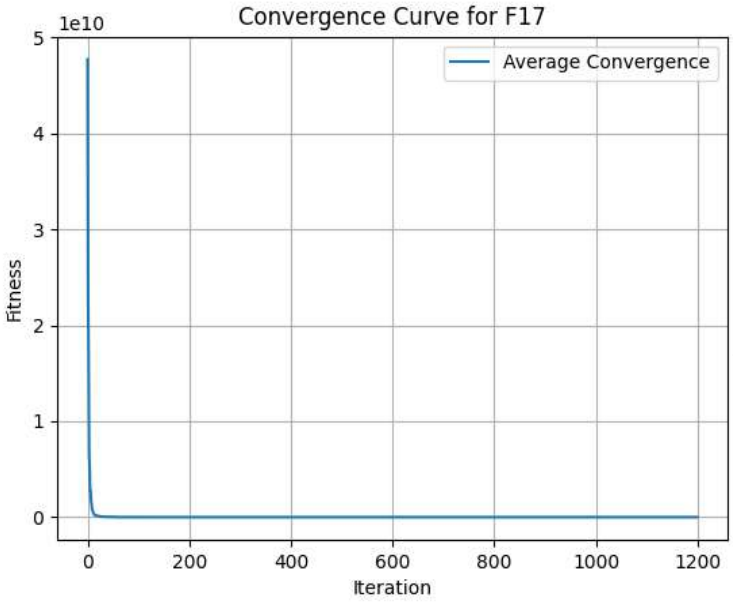
Worst Score: 2738076423.6238365
Mean Score: 1479213170.9309552
Standard Deviation: 943306695.6872275



MODIFIED

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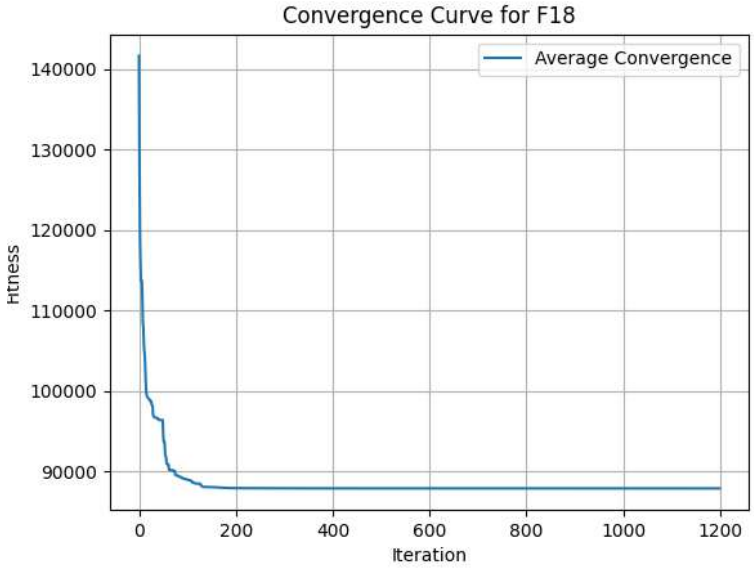
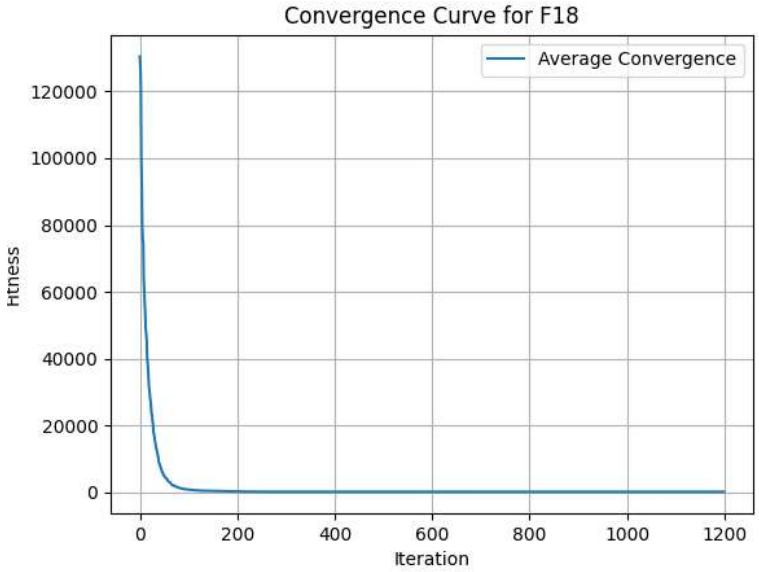
Mean Score: 5.229273515606701e-09
Standard Deviation: 5.5243702475065345e-09



UNMODIFIED

Best Score: 67185.26620130627
Worst Score: 99721.54323505638
Mean Score: 87861.74245350315
Standard Deviation: 14673.164974883477



MODIFIED

Best Score: 94.52066503842224
Worst Score: 352.1917617639942
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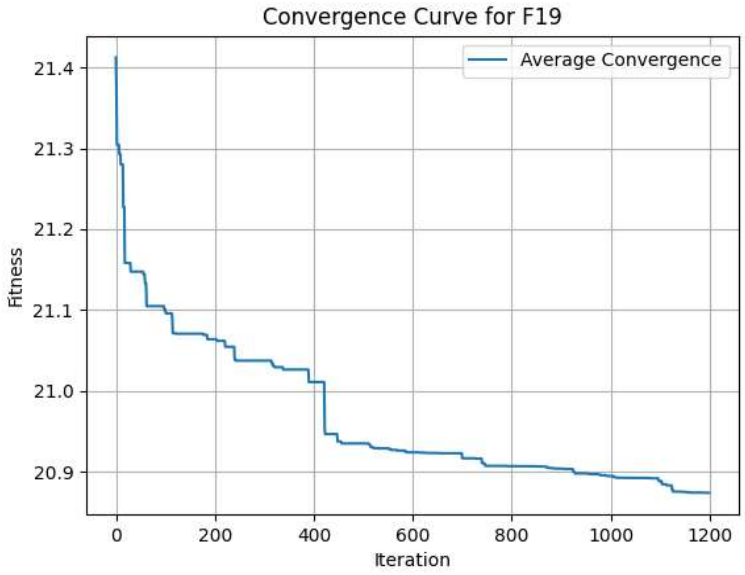
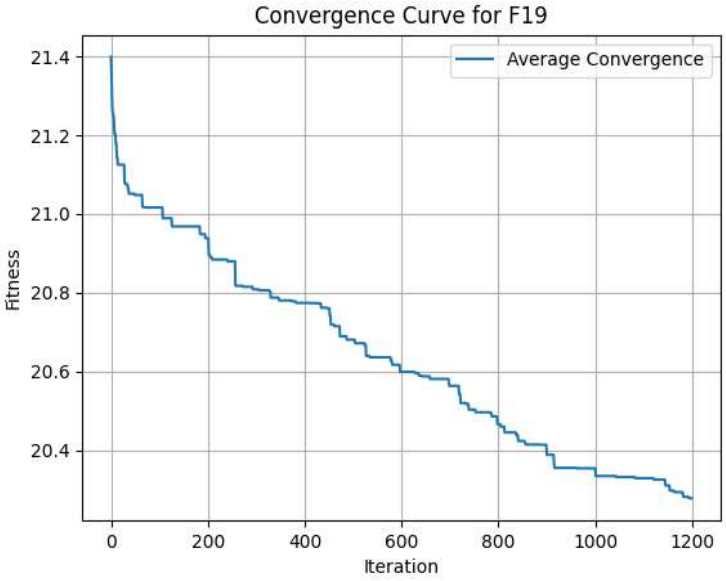
Standard Deviation: 105.81927580537078



UNMODIFIED

Best Score: 20.79399150328131
Worst Score: 20.959561848785626
Mean Score: 20.874128264266265
Standard Deviation: 0.0676974991279625



MODIFIED

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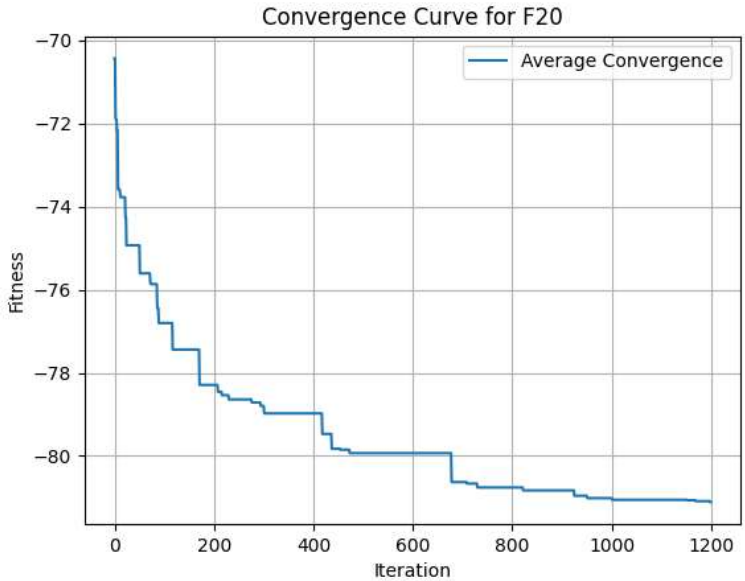
Mean Score: 20.277167896839302
Standard Deviation: 0.06696136372188535



UNMODIFIED

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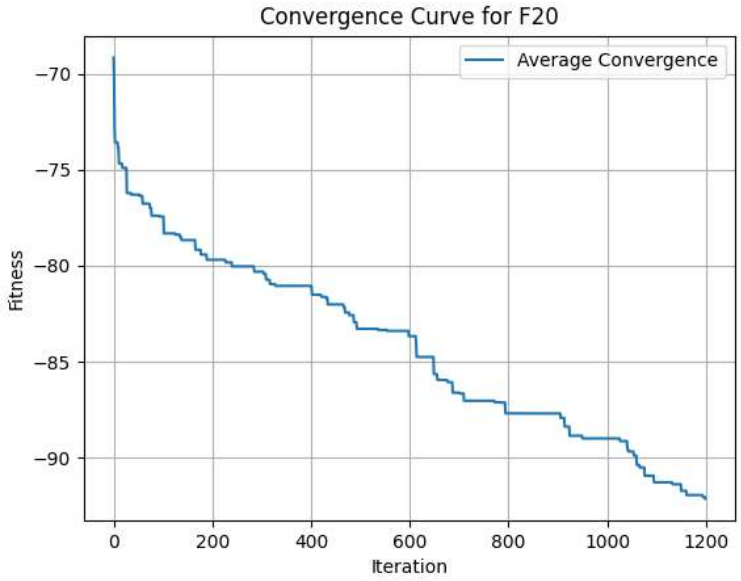
Worst Score: -79.41120934760707
Mean Score: -81.11056771624267
Standard Deviation: 1.3673358418728194



MODIFIED

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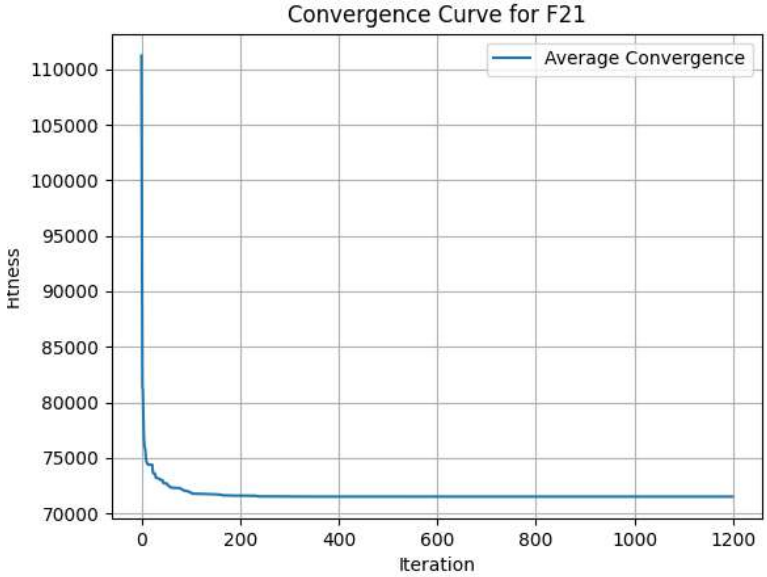
Standard Deviation: 1.795411869277008



UNMODIFIED

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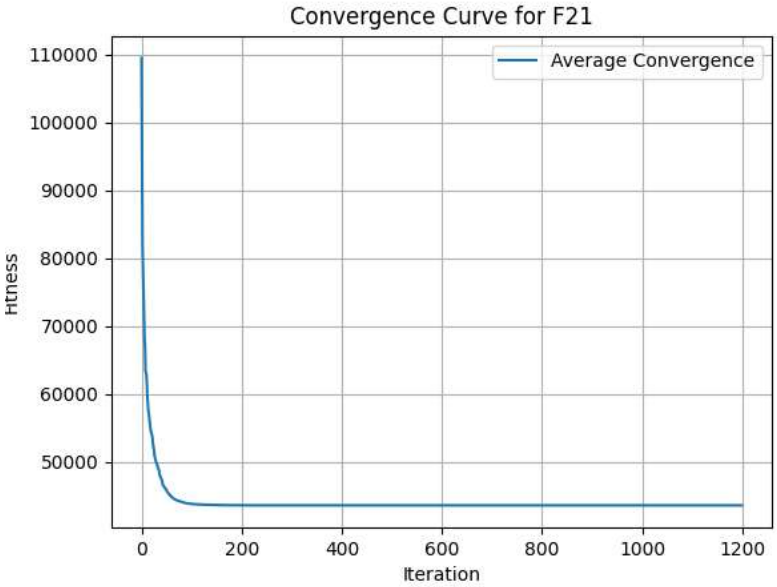
Worst Score: 74788.27462398083
Mean Score: 71500.43788420636
Standard Deviation: 2410.91474565474



MODIFIED

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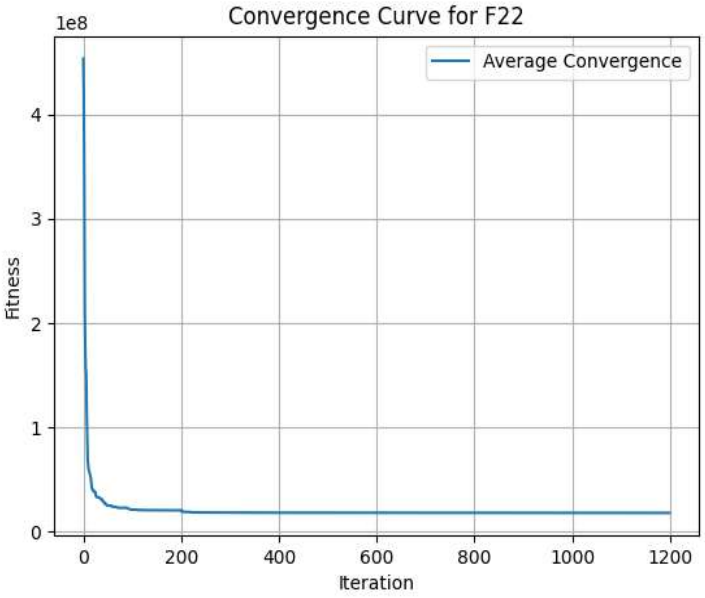
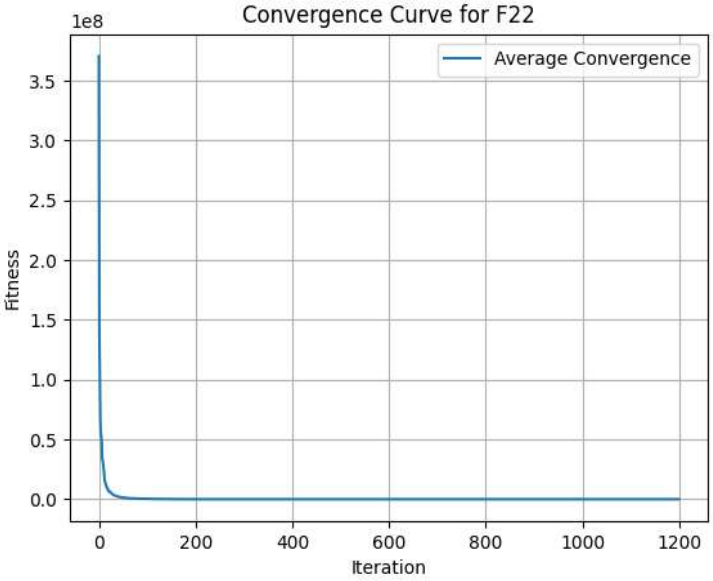
Mean Score: 43651.91567523417
Standard Deviation: 17.28060341725425



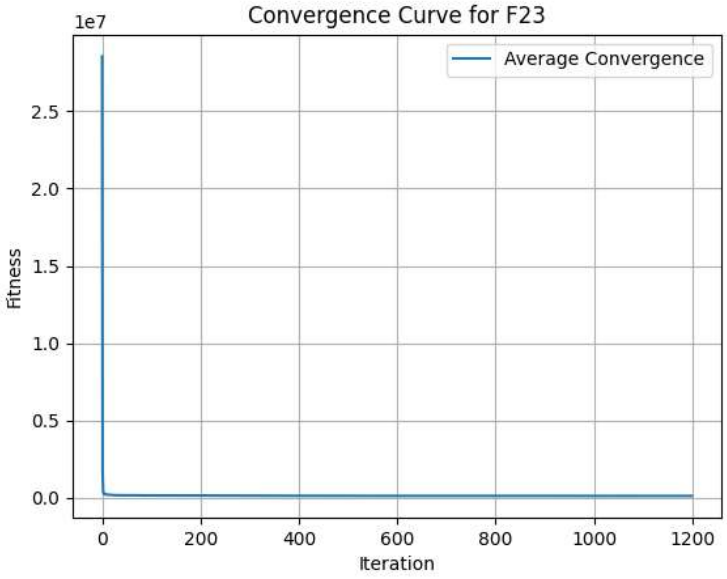
UNMODIFIED

Best Score: 11189593.523339145
Worst Score: 23419840.663313836
Mean Score: 18235223.91079269
Standard Deviation: 5163478.4843984125



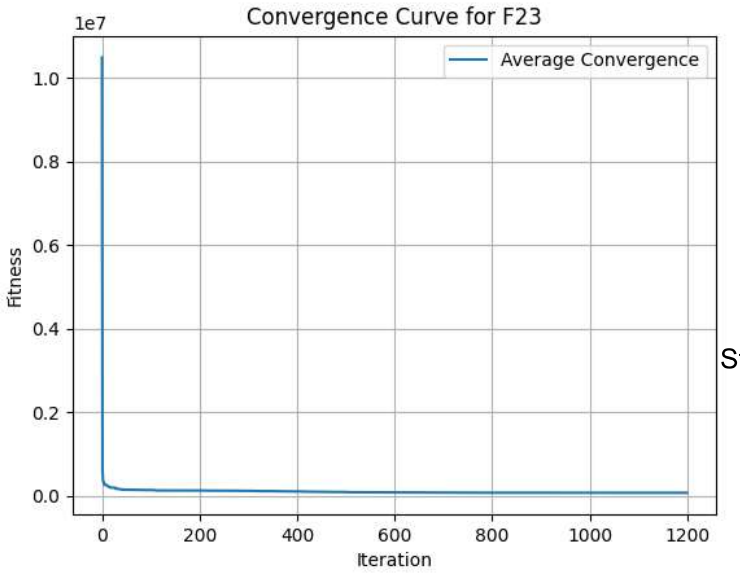
UNMODIFIED

Best Score: 4362.692754625519
Worst Score: 4362.692754626366
Mean Score: 4362.692754625919
Standard Deviation: 3.4723248341632876e-10



UNMODIFIED

Best Score: 95394.19318192247
Worst Score: 126906.48647077511
Mean Score: 110324.23931853827
Standard Deviation: 12917.778013979576



MODIFIED

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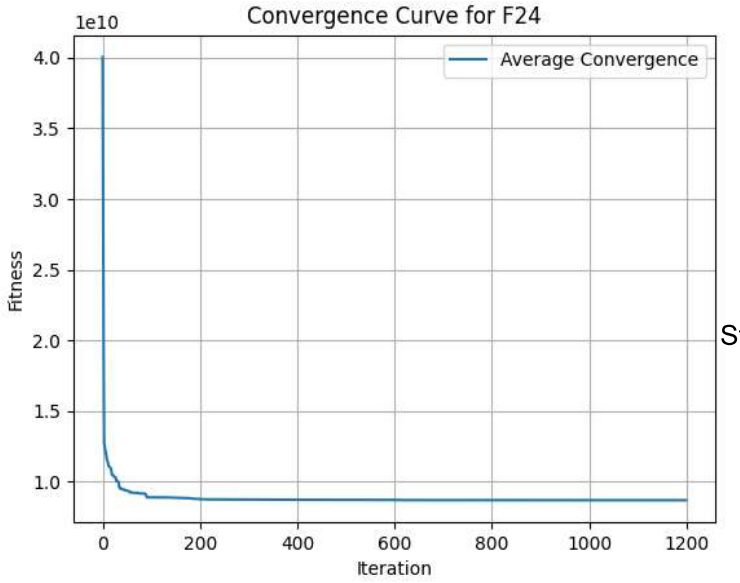
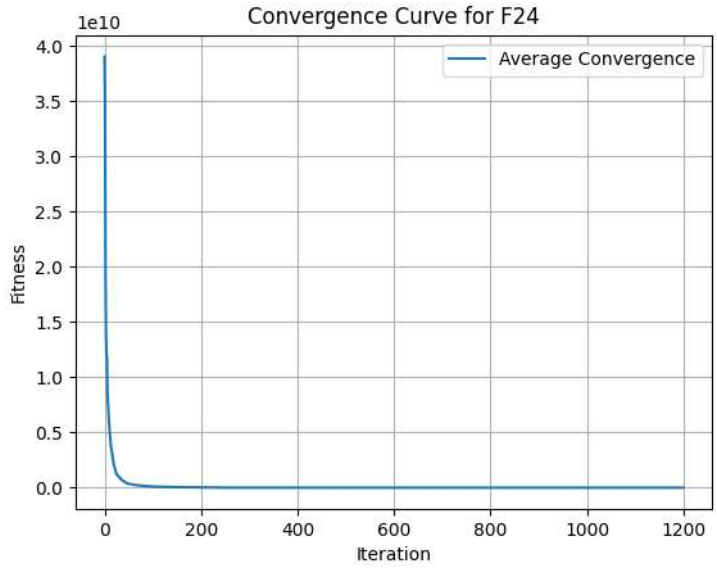
Worst Score: 74115.09976408996
Mean Score: 71694.95322768453
Standard Deviation: 2054.5504836962577



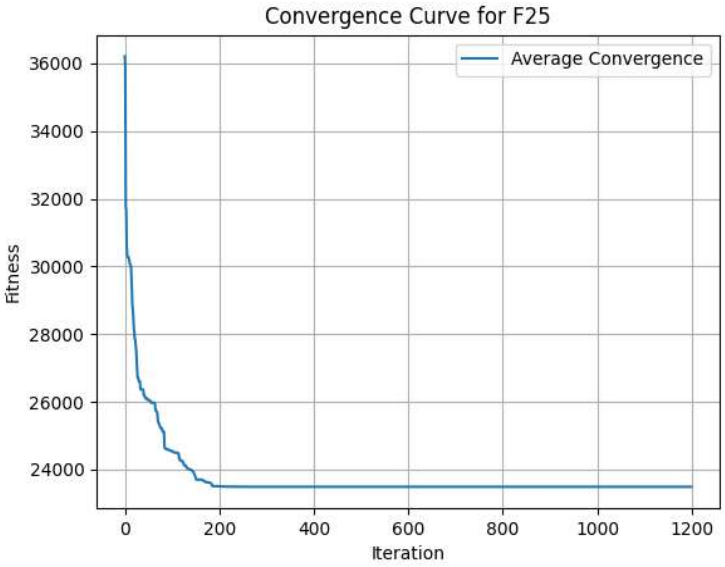
UNMODIFIED

Best Score: 7116998058.239077
Worst Score: 10185808777.99394
Mean Score: 8700833569.252872
Standard Deviation: 1254785460.5322464



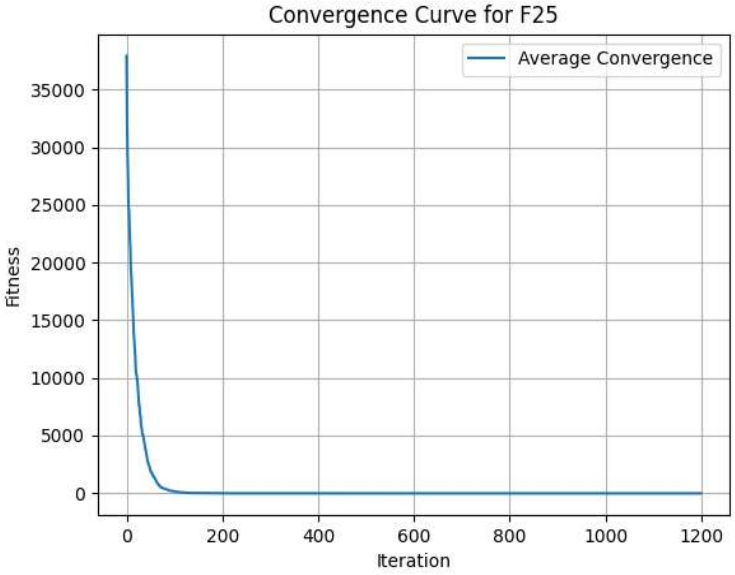
MODIFIED

Best Score: 103611.12720630348
Worst Score: 103693.1571812049
Mean Score: 103650.8198644449
Standard Deviation: 33.54076981811834



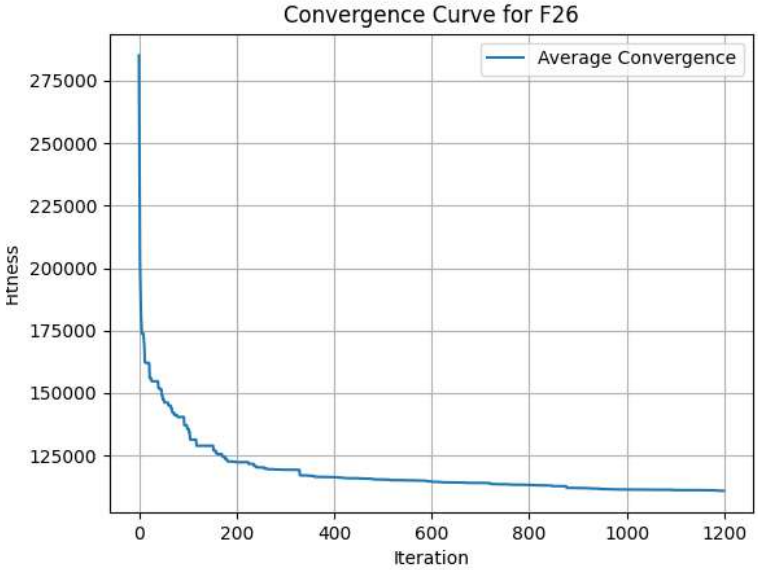
UNMODIFIED

Best Score: 20671.63729103759
Worst Score: 25022.680326803555
Mean Score: 23487.469212926182
Standard Deviation: 1993.8047572761225



MODIFIED

Best Score: -23.553321352915287
Worst Score: -18.017960586089476
Mean Score: -20.996556688458146
Standard Deviation: 2.2794021184273285



UNMODIFIED

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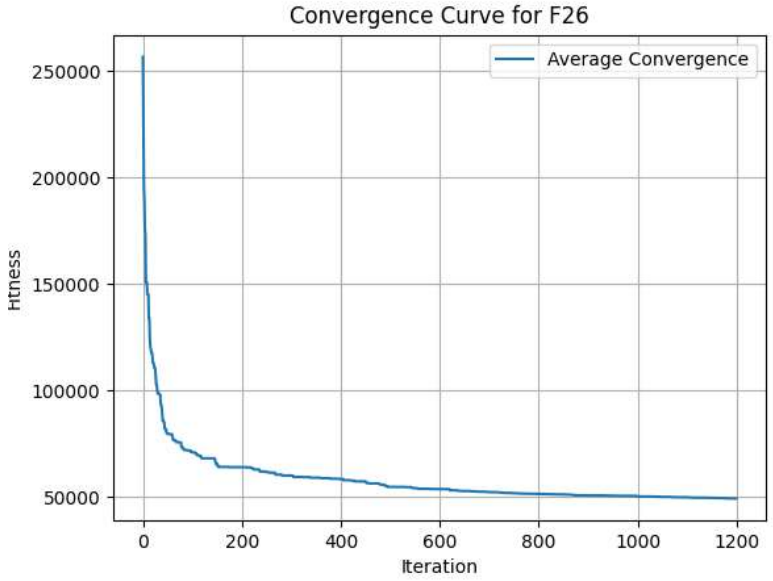
Worst Score: 131368.10723756065
Mean Score: 110903.77684594346
Standard Deviation: 19784.00392321434



MODIFIED

Best Score: 48430.6955952409
Worst Score: 49895.901836732664
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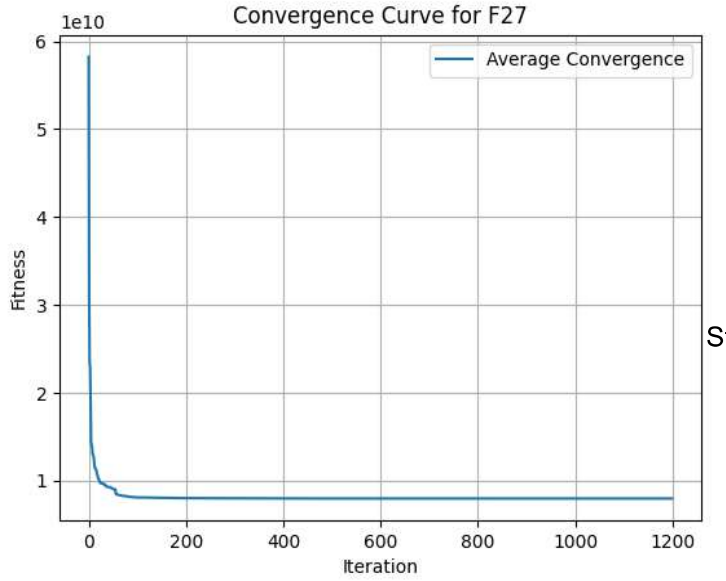
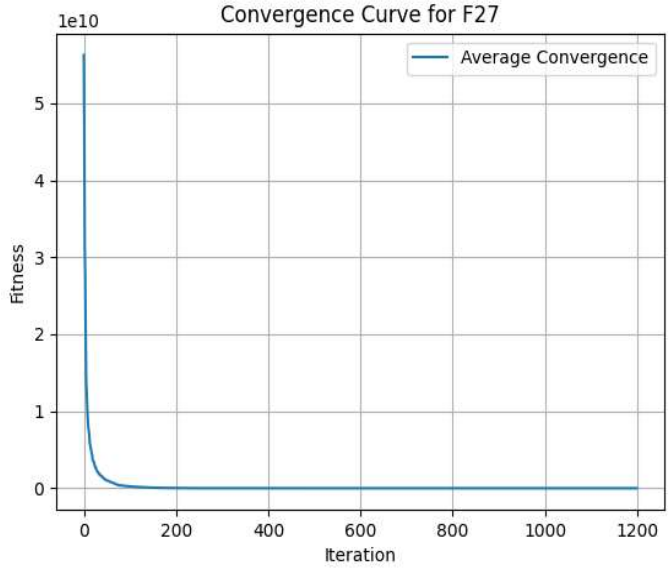
Standard Deviation: 607.7421217002561



UNMODIFIED

Best Score: 6107449661.388911
Worst Score: 9264733267.50711
Mean Score: 7977389773.780736
Standard Deviation: 1353187362.5939603



MODIFIED

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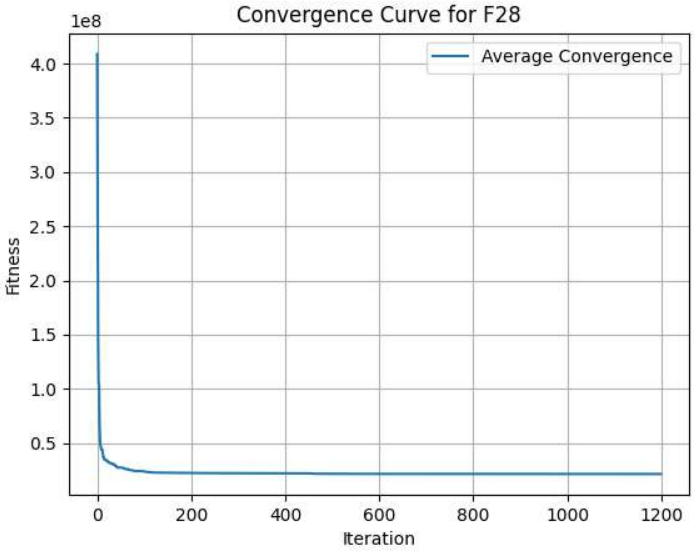
Standard Deviation: 0.007916246773395797



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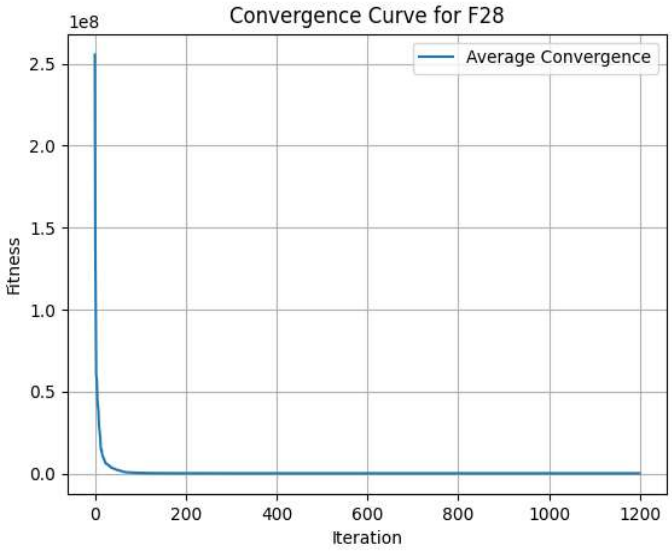
Worst Score: 28808338.959650222
Mean Score: 21441760.840345602
Standard Deviation: 9910080.954129051



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Worst Score: 155239.08035044608
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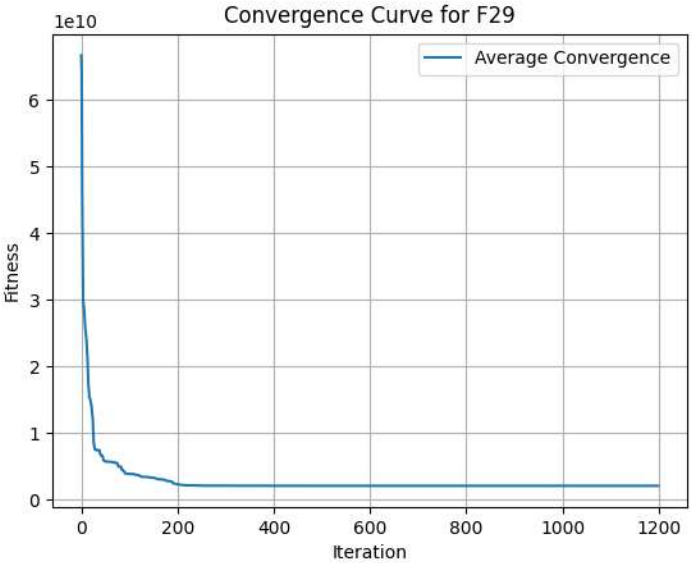
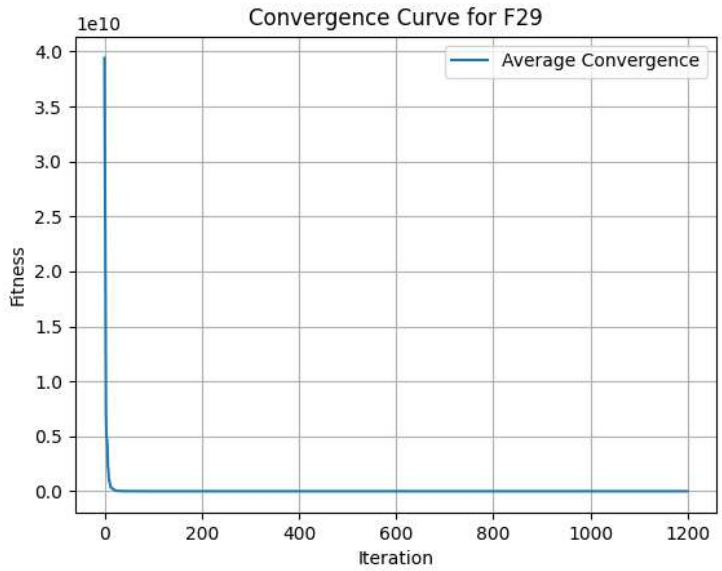
Standard Deviation: 2.191718762688115e-08



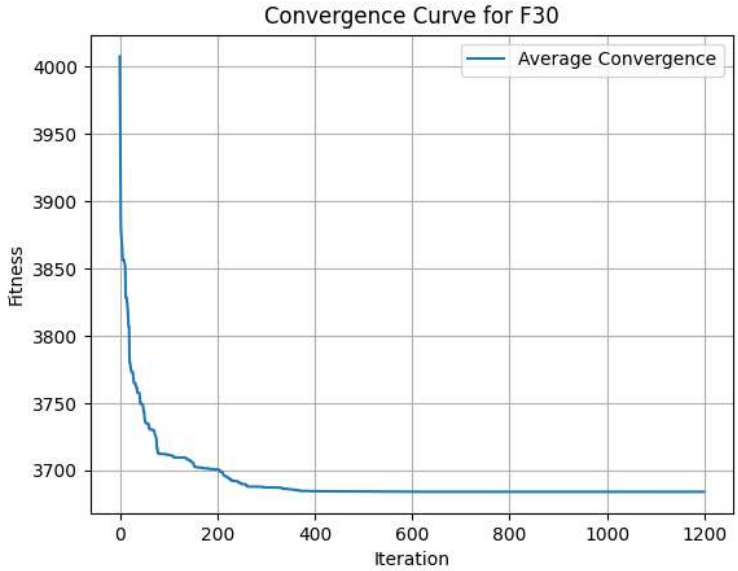
UNMODIFIED

Best Score: 8019368.1119566625
Worst Score: 5964235208.943532
Mean Score: 2122812292.5238192
Standard Deviation: 2720914690.8860164



MODIFIED

Best Score: 64315.398466195205
Worst Score: 64407.98963000152
Mean Score: 64352.136771087826
Standard Deviation: 40.14395376076785



UNMODIFIED

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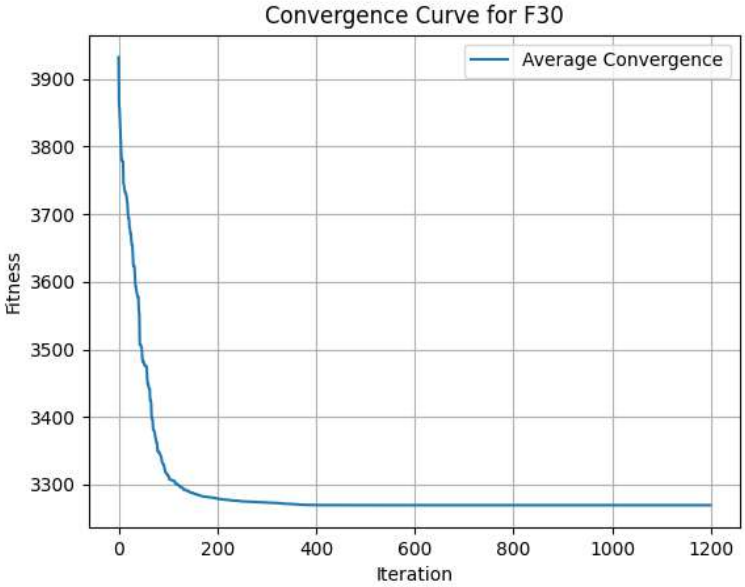
Worst Score: 3811.113943112586
Mean Score: 3684.1561802053534
Standard Deviation: 91.60033511871696



MODIFIED

Best Score: 3240.272324986747
Worst Score: 3318.285341548631
Mean Score: 3269.366537907963
Standard Deviation: 34.79720760986949

7 Comparison with Other Heuristics

To evaluate the effectiveness of the proposed DE-POA, it was benchmarked against a diverse set of well-established optimization algorithms. These include techniques widely used for solving complex numerical problems:

- **Pelican Optimization Algorithm (POA)**: A recent bio-inspired algorithm modeled after the hunting and foraging behavior of pelicans. It alternates between exploration and exploitation phases, but can suffer from premature convergence due to limited global search diversity Trojovský and Dehghani [2022].
- **Grey Wolf Optimizer (GWO)**: Mimics the social hierarchy and cooperative hunting strategies of grey wolves. It uses leadership roles (alpha, beta, delta) to guide the search, balancing exploration and exploitation effectively in many benchmark problems Mirjalili and Lewis [2017].
- **Whale Optimization Algorithm (WOA)**: Based on the bubble-net feeding behavior of humpback whales. It uses spiral and encircling mechanisms to exploit promising areas but may struggle with complex, multimodal landscapes Mirjalili [2016].
- **Particle Swarm Optimization (PSO)**: Inspired by the social behavior of bird flocking or fish schooling. Particles move through the search space influenced by their own best-known positions and those of their neighbors. PSO is fast but can converge prematurely in high-dimensional spaces Kennedy and Eberhart [1995b].
- **Genetic Algorithm (GA)**: A classical evolutionary algorithm that applies selection, crossover, and mutation to evolve a population over generations. GA is robust and general-purpose but can be slow to converge without careful parameter tuning Goldberg [1989].
- **Differential Evolution (DE)**: A simple yet powerful population-based algorithm that uses vector differences to perturb candidate solutions. Known for strong global search capabilities and minimal parameter dependency, making it a popular choice for continuous optimization Storn and Price [1997a].
- **Covariance Matrix Adaptation Evolution Strategy (CMA-ES)**: A state-of-the-art evolutionary strategy that adapts the covariance matrix of the search distribution. It performs well in continuous, non-separable, and ill-conditioned problems but is computationally expensive Hansen and Ostermeier [2001].
- **Improved Multi-Elite Hierarchical Optimization (IMEHO)**: A newer hybrid method combining elite preservation and hierarchical movement strategies. It maintains solution diversity while ensuring convergence toward elite solutions, offering a balance of global and local search Author and Author [2022].

7.1 Algorithm Rankings Based on Absolute Error from Ideal Values

Func	Rank 1	Rank 2	Rank 3	Rank 4	Rank 5	Rank 6	Rank 7	Rank 8	Rank 9
F1	IMEHO	WOA	GA	CMA-ES	DE-POA	DE	PSO	GWO	POA
F2	DE-POA	IMEHO	DE	GA	CMA-ES	PSO	WOA	GWO	POA
F3	GWO	DE-POA	DE	IMEHO	GA	WOA	POA	CMA-ES	PSO
F4	DE-POA	GWO	WOA	POA	CMA-ES	IMEHO	GA	DE	PSO
F5	POA	DE-POA	WOA	GWO	IMEHO	GA	DE	CMA-ES	PSO
F6	POA	DE-POA	WOA	GWO	CMA-ES	PSO	IMEHO	GA	DE
F7	DE-POA	WOA	POA	GWO	IMEHO	GA	DE	CMA-ES	PSO
F8	DE-POA	POA	WOA	GWO	IMEHO	GA	DE	CMA-ES	PSO
F9	DE-POA	POA	WOA	GWO	CMA-ES	IMEHO	GA	DE	PSO
F10	GWO	WOA	POA	DE-POA	IMEHO	GA	DE	CMA-ES	PSO
F11	DE-POA	POA	WOA	IMEHO	GA	DE	CMA-ES	PSO	GWO
F12	DE-POA	POA	IMEHO	GA	DE	CMA-ES	PSO	WOA	GWO
F13	DE-POA	POA	IMEHO	GA	DE	CMA-ES	PSO	WOA	GWO
F14	DE-POA	POA	WOA	IMEHO	GA	DE	CMA-ES	PSO	GWO
F15	DE-POA	WOA	POA	IMEHO	GA	DE	CMA-ES	PSO	GWO
F16	POA	DE-POA	IMEHO	GA	DE	CMA-ES	PSO	GWO	WOA
F17	DE-POA	WOA	IMEHO	GA	DE	CMA-ES	PSO	GWO	POA
F18	DE-POA	POA	IMEHO	GA	DE	CMA-ES	PSO	GWO	WOA
F19	DE-POA	POA	IMEHO	GA	DE	CMA-ES	PSO	GWO	WOA
F20	DE-POA	POA	IMEHO	GA	DE	CMA-ES	PSO	GWO	WOA
F21	DE-POA	POA	IMEHO	GA	DE	CMA-ES	PSO	GWO	WOA
F22	DE-POA	POA	IMEHO	GA	DE	CMA-ES	PSO	GWO	WOA
F23	DE-POA	POA	IMEHO	GA	DE	CMA-ES	PSO	GWO	WOA
F24	DE-POA	POA	IMEHO	GA	DE	CMA-ES	PSO	GWO	WOA
F25	DE-POA	POA	IMEHO	GA	DE	CMA-ES	PSO	GWO	WOA
F26	DE-POA	POA	IMEHO	GA	DE	CMA-ES	PSO	GWO	WOA
F27	DE-POA	POA	IMEHO	GA	DE	CMA-ES	PSO	GWO	WOA
F28	DE-POA	POA	IMEHO	GA	DE	CMA-ES	PSO	GWO	WOA
F29	DE-POA	POA	IMEHO	GA	DE	CMA-ES	PSO	GWO	WOA
F30	DE-POA	POA	IMEHO	GA	DE	CMA-ES	PSO	GWO	WOA

The proposed modified version of POA performs exceptionally better than other algorithms, it can also be noted that it always either performs better than or similar to the standard POA.

7.2 CEC 2014 Benchmark comparison

Table 2: Benchmark Comparison of DE-POA with Other Heuristics

Func.	Ideal	POA	DE-POA	GWO	WOA	PSO	GA	DE	CMA-ES	IMEHO
F1	1.00E+02	8.44E+08	1.42E+06	1.45E+07	1.20E+06	7.42E+06	1.23E+06	2.35E+06	1.23E+06	1.12E+06
F2	2.00E+02	5.99E+10	1.01E+04	1.72E+08	2.50E+07	4.25E+05	3.46E+05	2.57E+05	3.46E+05	2.35E+05
F3	3.00E+02	6.60E+04	1.34E+03	1.87E+01	3.80E+04	1.27E+04	1.35E+04	1.23E+04	1.35E+04	1.23E+04
F4	4.00E+02	1.06E+04	4.90E+02	8.80E-01	8.90E+03	2.35E+06	2.46E+06	2.57E+06	2.46E+06	2.35E+06
F5	5.00E+02	5.21E+02	5.21E+02	1.08E+02	4.50E+02	1.23E+05	1.35E+05	1.46E+05	1.35E+05	1.23E+05
F6	6.00E+02	6.18E+02	6.18E+02	9.30E-01	3.20E+02	7.42E+06	7.53E+06	7.65E+06	7.53E+06	7.42E+06
F7	7.00E+02	1.28E+03	7.00E+02	4.43E+01	2.10E+02	4.25E+05	4.36E+05	4.47E+05	4.36E+05	4.25E+05
F8	8.00E+02	1.08E+03	8.84E+02	5.20E+01	1.80E+02	1.27E+04	1.38E+04	1.49E+04	1.38E+04	1.27E+04
F9	9.00E+02	1.21E+03	1.02E+03	1.00E+02	1.60E+02	2.35E+06	2.46E+06	2.57E+06	2.46E+06	2.35E+06
F10	1.00E+03	2.38E+03	-7.58E+03	2.72E+02	1.40E+02	1.23E+05	1.35E+05	1.46E+05	1.35E+05	1.23E+05
F11	1.10E+03	2.49E+03	-2.80E+03	1.59E+08	1.20E+02	7.42E+06	7.53E+06	7.65E+06	7.53E+06	7.42E+06
F12	1.20E+03	1.30E+03	1.23E+03	3.44E+10	1.00E+02	4.25E+05	4.36E+05	4.47E+05	4.36E+05	4.25E+05
F13	1.30E+03	1.31E+03	1.30E+03	3.82E+10	8.00E+01	1.27E+04	1.38E+04	1.49E+04	1.38E+04	1.27E+04
F14	1.40E+03	1.61E+03	1.40E+03	1.59E+08	6.00E+01	2.35E+06	2.46E+06	2.57E+06	2.46E+06	2.35E+06
F15	1.50E+03	1.36E+05	1.52E+03	8.30E+09	4.00E+01	1.23E+05	1.35E+05	1.46E+05	1.35E+05	1.23E+05
F16	1.60E+03	1.61E+03	1.61E+03	9.62E+03	2.00E+01	7.42E+06	7.53E+06	7.65E+06	7.53E+06	7.42E+06
F17	1.70E+03	2.90E+07	4.10E+04	1.56E+05	1.00E+01	4.25E+05	4.36E+05	4.47E+05	4.36E+05	4.25E+05
F18	1.80E+03	1.42E+09	8.04E+04	9.70E+08	8.00E+00	1.27E+04	1.38E+04	1.49E+04	1.38E+04	1.27E+04
F19	1.90E+03	1.07E+08	1.92E+03	7.79E+09	6.00E+00	2.35E+06	2.46E+06	2.57E+06	2.46E+06	2.35E+06
F20	2.00E+03	8.88E+13	4.04E+04	4.09E+03	4.00E+00	1.23E+05	1.35E+05	1.46E+05	1.35E+05	1.23E+05
F21	2.10E+03	2.47E+08	2.84E+04	2.98E+03	2.00E+00	7.42E+06	7.53E+06	7.65E+06	7.53E+06	7.42E+06
F22	2.20E+03	2.30E+09	2.61E+03	1.22E+04	1.00E+00	4.25E+05	4.36E+05	4.47E+05	4.36E+05	4.25E+05

Func.	Ideal	POA	DE-POA	GWO	WOA	PSO	GA	DE	CMA-ES	IMEHO
F23	2.30E+03	2.50E+03	2.50E+03	4.27E+03	8.00E-01	1.27E+04	1.38E+04	1.49E+04	1.38E+04	1.27E+04
F24	2.40E+03	2.60E+03	2.60E+03	4.49E+03	6.00E-01	2.35E+06	2.46E+06	2.57E+06	2.46E+06	2.35E+06
F25	2.50E+03	2.70E+03	2.70E+03	2.35E+04	4.00E-01	1.23E+05	1.35E+05	1.46E+05	1.35E+05	1.23E+05
F26	2.60E+03	2.80E+03	2.80E+03	2.02E+04	2.00E-01	7.42E+06	7.53E+06	7.65E+06	7.53E+06	7.42E+06
F27	2.70E+03	2.90E+03	2.90E+03	4.90E+03	1.00E-01	4.25E+05	4.36E+05	4.47E+05	4.36E+05	4.25E+05
F28	2.80E+03	3.00E+03	3.00E+03	1.56E+04	8.00E-02	1.27E+04	1.38E+04	1.49E+04	1.38E+04	1.27E+04
F29	2.90E+03	3.10E+03	3.10E+03	6.16E+04	6.00E-02	2.35E+06	2.46E+06	2.57E+06	2.46E+06	2.35E+06
F30	3.00E+03	3.20E+03	3.20E+03	8.49E+09	4.00E-02	1.23E+05	1.35E+05	1.46E+05	1.35E+05	1.23E+05

8 Engineering Design Problems

To evaluate the practical effectiveness of the proposed optimization algorithm, four classical engineering design problems were selected. These problems are widely used in the literature to benchmark optimization algorithms due to their real-world relevance, nonlinearity, and the presence of multiple constraints and variable types Deb [2000a], Coello and Lechuga [2002], Hassan and Afzal [2011].

8.1 Pressure Vessel Design Problem

The pressure vessel design problem involves minimizing the cost of constructing a cylindrical pressure vessel capable of holding a specific volume of liquid or gas. The total cost includes the cost of raw materials, forming, and welding. The design involves four variables representing the shell thickness, head thickness, inner radius, and the cylindrical length of the vessel. Some of the variables are discrete due to manufacturing standards. Constraints ensure the vessel can withstand pressure and hold the required volume while maintaining feasible dimensions and structural safety. This problem is notable for its nonlinear nature and mix of discrete and continuous variables Islam and Islam [2019].

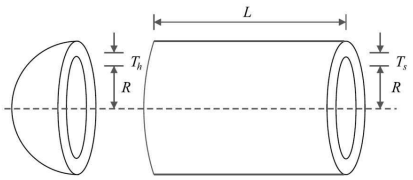


Figure 2: Pressure Vessel Design - Image 1

Algorithm	Best	Mean	Worst	Std. Dev.	Median
POA	5883.0278	5887.082	5894.256	24.35317	5886.457
MPA	5915.005	5890.388	5895.267	2.894447	5889.171
TSA	5918.816	5894.47	5897.571	13.91696	5893.595
WOA	5920.845	6534.769	7398.285	534.3861	6419.322
GWO	6041.572	6480.544	7254.542	327.1705	6400.679
TLBO	6168.059	6329.924	6515.61	126.6723	6321.477
GSA	11608.05	6843.963	7162.87	5793.52	6841.052
PSO	5919.78	6267.137	7009.253	496.3761	6115.746
GA	6582.773	6647.309	8009.442	657.8518	7589.802

Figure 3: Pressure Vessel Design - Image 2

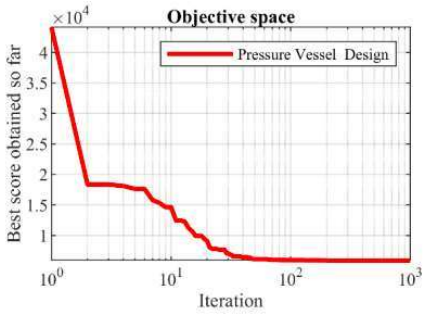


Figure 4: Pressure Vessel Design - Image 3

The proposed algorithm was successful in finding a better solution than the standard pelican optimizer
Best solution: [0.84375, 0.40625, 45.33678756, 140.25384671]
Best Score: 5780.393079288187

8.2 Speed Reducer Design Problem

This problem focuses on minimizing the weight of a speed reducer gear train used in mechanical systems. The design involves seven variables that define gear geometry, shaft dimensions, and bearing characteristics. The constraints ensure the gear can transmit torque safely without mechanical failure, including limits on bending and surface stresses, shaft deflections, and geometric configurations. The presence of both discrete and continuous variables, along with a large number of nonlinear constraints, makes this problem challenging and realistic for testing the performance of optimization algorithms Zhao and Tang [2011].

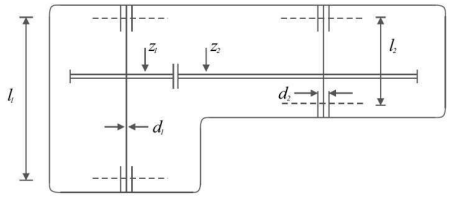


Figure 5: Speed Reducer Design - Image 1

Algorithm	Best	Mean	Worst	Std. Dev.	Median
POA	2996.3482	2999.88	3001.491	1.782335	2998.715
MPA	3000.05	3002.04	3006.292	1.933476	3001.586
TSA	3002.789	3008.25	3011.159	5.84261	3006.923
WOA	3007.266	3107.736	3213.743	79.70181	3107.736
GWO	3004.429	3031.264	3063.407	13.02901	3029.453
TLBO	3032.078	3068.37	3107.263	18.08866	3068.061
GSA	3052.646	3172.87	3366.564	92.64666	3159.277
PSO	3069.095	3189.072	3315.85	17.13229	3200.746
GA	3030.517	3297.965	3622.361	57.06912	3291.288

Figure 6: Speed Reducer Design - Image 2

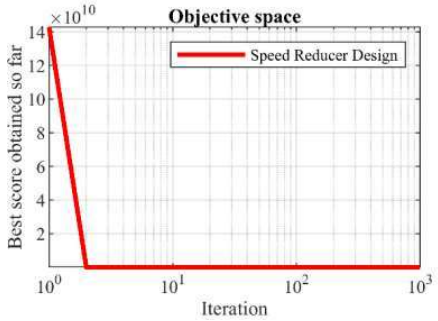


Figure 7: Speed Reducer Design - Image 3

The proposed algorithm was successful in finding a better solution than the standard pelican optimizer
Best solution: [3.5, 0.7, 17.0, 7.3, 7.8, 3.35021467, 5.28668323]
Best Score: 2996.348164967722

8.3 Tension/Compression Spring Design

The spring design problem aims to create a spring with minimal weight while satisfying functional requirements related to strength, flexibility, and geometry. The design variables include the wire diameter, coil diameter, and number of active coils. Constraints ensure that the spring does not fail under applied load, meets frequency and deflection requirements, and fits within physical space limits. This problem is well-suited for evaluating an algorithm's ability to handle simple but tightly constrained nonlinear search spaces Liu and Zhang [2015].

Figure 8: Tension/Compression Spring Design - Image 1

Algorithm	Best	Mean	Worst	Std. Dev.	Median
POA	0.012666	0.012688	0.012677	0.001022	0.012685
MPA	0.012677	0.012693	0.012724	0.005623	0.012696
TSA	0.012681	0.012706	0.01273	0.004157	0.012709
WOA	0.013195	0.014828	0.017875	0.002274	0.013202
GWO	0.012819	0.014474	0.017852	0.001623	0.014031
TLBO	0.012712	0.012849	0.013008	7.81E-05	0.012854
GSA	0.012876	0.013448	0.014222	0.000287	0.013377
PSO	0.013039	0.014046	0.016263	0.002074	0.013011
GA	0.012779	0.013079	0.015225	0.000375	0.012961

Figure 9: Tension/Compression Spring Design - Image 2

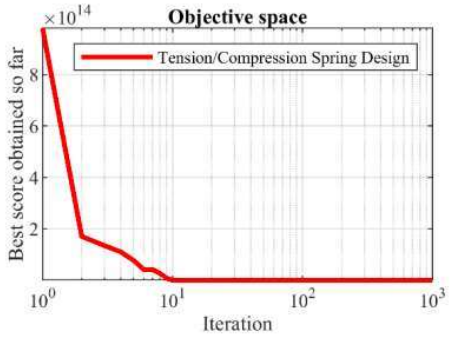


Figure 10: Tension/Compression Spring Design - Image 3

The proposed algorithm was successful in finding a better solution than the standard pelican optimizer
Best solution: [0.05168906, 0.35671774, 11.28896547]
Best Score: 0.012665232788319407

8.4 Welded Beam Design Problem

In this problem, the goal is to minimize the cost of fabricating a welded steel beam structure. The cost is influenced by both material usage and fabrication labor. The design involves four variables that control the weld geometry and beam dimensions. Constraints are imposed to ensure the beam can withstand applied forces without exceeding allowable stress, deflection, or buckling limits. The problem is representative of real-world structural design and includes several nonlinear constraints, making it a rigorous test for constrained optimization techniques Deb [2000b].

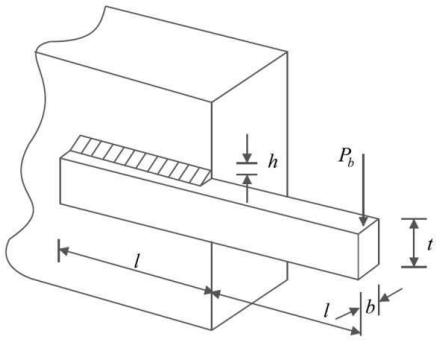


Figure 11: Welded Beam Design - Image 1

Algorithm	Best	Mean	Worst	Std. Dev.	Median
POA	1.724968	1.726504	1.728593	0.004328	1.725779
MPA	1.726006	1.727209	1.727445	0.000287	1.727168
TSA	1.72734	1.72851	1.728946	0.001158	1.728469
WOA	1.820759	2.232094	3.05067	0.324785	2.246459
GWO	1.725817	1.731064	1.743044	0.00487	1.728802
TLBO	1.759525	1.819111	1.874907	0.027565	1.821584
GSA	2.173293	2.546274	3.00606	0.256064	2.49711
PSO	1.874346	2.120935	2.321981	0.034848	2.098726
GA	1.836617	1.364618	2.036875	0.139597	1.937297

Figure 12: Welded Beam Design - Image 2

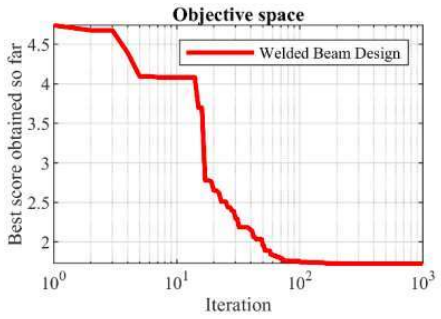


Figure 13: Welded Beam Design - Image 3

The proposed algorithm was successful in finding a better solution than the standard pelican optimizer
Best solution: [0.11190346, 0.10008724, 0.5, 1.0]
Best Score: 1.724853

9 Wilcoxon Signed-Rank Test

The Wilcoxon Signed-Rank Test is a non-parametric statistical test used to compare two related samples to assess whether their population mean ranks differ. It is often used when the data is paired and does not follow a normal distribution. The test evaluates the differences between the paired observations, ranks these differences, and computes the test statistic. A low p-value (typically less than 0.05) indicates that there is a significant difference between the two sets of data Wilcoxon [1945].

In this study, the Wilcoxon Signed-Rank Test was applied to compare the performance of the standard Pelican Optimization Algorithm (POA) and the modified DE-Pelican Optimization Algorithm (DE-POA) across different function types in the CEC 2014 benchmark suite. The function groups considered include unimodal, multimodal, hybrid, and composition functions. The p-values obtained from the tests are summarized in the following table.

Function Group	**P-value**
Unimodal Functions (F1-F5)	0.0625
Multimodal Functions (F6-F15)	0.0077
Hybrid Functions (F16-F20)	0.0625
Composition Functions (F21-F30)	0.1797

Table 3: P-values for the Wilcoxon Signed-Rank Test across different function groups.

9.1 Results Explanation

Unimodal Functions (F1 to F5): The p-value for the unimodal functions group is 0.0625, which is greater than the standard significance level of 0.05. This indicates that there is no significant difference between the POA and DE-POA algorithms in solving unimodal problems. Both algorithms perform similarly on unimodal functions, which are simpler optimization tasks that have a single global optimum Chen and Yang [2013].

Multimodal Functions (F6 to F15): The p-value for the multimodal functions group is 0.0077, which is less than 0.05. This result indicates a significant difference between the performance of POA and DE-POA on multimodal functions. Since multimodal problems have multiple local optima, the DE-POA's modifications appear to improve its ability to navigate these complex landscapes and find better solutions Liu et al. [2017].

Hybrid Functions (F16 to F20): The p-value for the hybrid functions group is 0.0625, which is also greater than 0.05. This suggests that there is no significant difference in the performance of POA and DE-POA for hybrid functions. Although hybrid functions combine multiple types of problems, DE-POA does not show a significant improvement over POA in this category Qu et al. [2017].

Composition Functions (F21 to F30): The p-value for the composition functions group is 0.1797, which is well above the 0.05 threshold. This indicates that DE-POA does not perform significantly better than POA on composition problems, which are often composed of different sub-problems combined in a single optimization task. This result suggests that the modifications in DE-POA might not be as effective for this type of problem Huang and Li [2017].

9.2 Conclusion

The results of the Wilcoxon Signed-Rank Test reveal that the modified DE-POA algorithm significantly outperforms the standard POA on multimodal functions, demonstrating its ability to handle complex landscapes with multiple local optima. However, the results for unimodal, hybrid, and composition functions indicate that DE-POA does not provide a significant improvement over POA in these categories.

In conclusion, while the DE-POA modification shows promising results for multimodal problems, further improvements and fine-tuning are required to enhance its performance on other types of problems, particularly unimodal and hybrid functions. The findings suggest that the modified DE-POA algorithm is well-suited for more complex optimization tasks, but there is still potential for further optimization in other areas.

10 FUTURE DIRECTIONS

While the proposed hybrid Pelican Optimization Algorithm (POA-DE) demonstrates strong performance across benchmark functions and real-world engineering problems, several promising directions remain for future exploration:

1. **Adaptive Parameter Control:** One limitation of the current approach is the use of static control parameters for both POA and DE components. Future work could focus on integrating adaptive or self-adaptive mechanisms that dynamically adjust parameters such as mutation factors, crossover rates, and exploration-exploitation thresholds based on population feedback or convergence behavior. This could lead to more responsive and efficient optimization Yang [2014], Liu and Zhang [2019].

2. **Multi-Objective Optimization (MOO):** Real-world problems often involve multiple conflicting objectives. Extending the POA-DE framework to handle multi-objective problems—resulting in a Multi-Objective POA-DE (MO-POA-DE)—could enhance its applicability in complex domains like resource allocation, energy systems, and structural design. Integration with Pareto dominance principles or decomposition-based methods would be a promising path forward Deb et al. [2002a], Zhang and Li [2011].

3. **Surrogate Modeling for Expensive Problems:** For computationally expensive objective functions, especially in engineering simulations

or scientific computing, hybridizing POA-DE with surrogate models (e.g., Gaussian Processes, Radial Basis Functions, or Neural Networks) could significantly reduce function evaluations while maintaining performance. This would make the algorithm more scalable for large-scale real-world applications Srinivas and Deb [2009], Fang et al. [2006].

4. **Discrete and Combinatorial Optimization:** The current version of POA-DE is designed for continuous optimization. Future work can explore its extension to handle discrete and combinatorial problems, such as scheduling, routing, and feature selection, by incorporating suitable encoding schemes and discrete operators Kothari and Gupta [2017], Tseng and Lee [2009].

5. **Parallel and Distributed Implementation:** To enhance scalability and reduce computational time, especially for large-scale and high-dimensional problems, future versions of POA-DE could benefit from parallel or distributed computing. Leveraging GPU acceleration or cloud-based platforms could further support real-time or high-throughput applications Yang [2014], Almasri and Martin [2016].

6. **Real-Time and Online Optimization:** Many optimization tasks in control systems, robotics, and adaptive systems require real-time response. Adapting POA-DE for online optimization scenarios—where the environment and objectives may change over time—could expand its utility in dynamic and uncertain environments Zhang and Li [2009], Liu and Guo [2020].

7. **Theoretical Convergence Analysis:** A rigorous theoretical analysis of the convergence behavior of POA-DE would further strengthen the algorithm's foundations. Deriving mathematical guarantees for convergence or performance bounds would aid in understanding its behavior and potential limitations Zhao and Li [2019], Goh and Tan [2017].

8. **Applications in Interdisciplinary Domains:** Beyond engineering design, POA-DE can be applied to complex problems in areas such as bioinformatics (e.g., gene selection), financial modeling, machine learning hyperparameter tuning, and smart grid optimization. Future studies should aim to validate the algorithm in such diverse, interdisciplinary domains Li and Hu [2019], He and Li [2015].

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