# Automatic Discovery and Exploitation of Promising Subproblems for Tabulation – Additional Results

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**Abstract.** A short technical report presenting the experimental results of the paper *Automatic Discovery and Exploitation of Promising Subproblems for Tabulation* (published in the proceedings of CP 2018) as tables rather than plots, showing the detail of timeouts and the extra time taken by Savile Row to perform tabulation.

## 1 Introduction

The experiments in the paper *Automatic Discovery and Exploitation of Promising Subproblems for Tabulation* compare two configurations of Savile Row using three different solvers. One configuration does tabulation (conversion to a table) of constraints automatically when they trigger one of a set of heuristics, and the other configuration is Savile Row's default settings. The three solvers are as follows:

**Minion-Static** Minion 1.8, ascending value and static variable orderings.
**Minion-Conflict** Same as the above with Conflict variable ordering.
**Chuffed** Current version of the learning CP solver Chuffed with free search.

Here we present the same set of experimental results in more detail. In particular Savile Row time and solver time are reported separately, and the additional time taken by Savile Row is reported as well. The speed-up includes both Savile Row time and the solver time, and is the same quantity that is plotted in the paper. Each configuration and solver pair is executed five times and the median times are reported for Savile Row and the solver.

The speed-up is simply total time without tabulation divided by total time with tabulation. When the time limit of 6 hours is reached, the speed-up is calculated using the time limit instead of the true time. Therefore the speed-up is an underestimate in the case where the baseline configuration times out and tabulation does not.

Table 1 shows the results for the Minion-Static solver. The additional time taken to tabulate is always under one second for the Knights Tour, Langford and Sports Scheduling Completion problems. For Coprime Sets, the time for

tabulation is much larger, tens or hundreds of seconds. On this problem class there are a large number of constraints and all constraints are replaced with tables, resulting in an entirely different formulation of the problem that performs much better.

Table 2 presents results for the Minion solver with Conflict variable ordering. Minion-Conflict has fewer timeouts than Minion-Static and generally performs better. In particular Minion-Conflict performs much better on the Coprime Sets problem and this negates the speed-up of tabulation that was seen with Minion-Static.

Finally the results with Chuffed are presented in Table 3. In this case most time-outs are for instances of Langfords Problem. Time to perform tabulation is very similar as for the Minion solvers. With Chuffed the most dramatic improvement is on the Knights Tour problem, where very little time is spent on tabulation and the model is improved substantially. Langfords shows some improvement and Sports Scheduling is ambiguous.

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| Instance | B-SR time (s) | B-solver time (s) | T-SR time (s) | T-solver time (s) | Increase in SR time (s) | Speed-up | B-timeout | T-timeout |
|---|---|---|---|---|---|---|---|---|
| coprime-10 | 7.92 | 21558.30 | 10.44 | 1.15 | 2.51 | 1861.04 | TRUE | |
| coprime-11 | 16.09 | 21554.10 | 23.67 | 2.32 | 7.58 | 830.03 | TRUE | |
| coprime-12 | 29.56 | 21546.10 | 41.55 | 4.09 | 11.99 | 472.66 | TRUE | |
| coprime-13 | 66.94 | 21533.06 | 73.60 | 21526.40 | 6.66 | 1.00 | TRUE | TRUE |
| coprime-14 | 70.17 | 21529.83 | 120.63 | 166.08 | 50.46 | 75.34 | TRUE | |
| coprime-15 | 68.66 | 21531.34 | 142.60 | 21457.40 | 73.95 | 1.00 | TRUE | TRUE |
| coprime-16 | 69.96 | 21530.04 | 183.43 | 21416.57 | 113.48 | 1.00 | TRUE | TRUE |
| coprime-8 | 2.01 | 122.51 | 2.42 | 0.27 | 0.41 | 46.33 | | |
| coprime-9 | 3.83 | 3898.13 | 5.43 | 0.58 | 1.59 | 649.72 | | |
| knights_10 | 0.82 | 21584.60 | 1.16 | 21585.90 | 0.34 | 1.00 | TRUE | TRUE |
| knights_6 | 0.20 | 14.65 | 0.21 | 0.26 | 0.00 | 31.40 | | |
| knights_7 | 0.28 | 10.54 | 0.35 | 0.15 | 0.06 | 21.62 | | |
| knights_8 | 0.37 | 1933.12 | 0.49 | 3.26 | 0.12 | 515.44 | | |
| knights_9 | 0.60 | 15064.30 | 0.68 | 9.73 | 0.08 | 1446.88 | | |
| langford_02_13 | 0.05 | 314.10 | 0.07 | 264.03 | 0.02 | 1.19 | | |
| langford_02_14 | 0.06 | 2779.76 | 0.09 | 2341.76 | 0.03 | 1.19 | | |
| langford_02_17 | 0.07 | 21585.20 | 0.09 | 21583.60 | 0.02 | 1.00 | TRUE | TRUE |
| langford_03_11 | 0.06 | 1.68 | 0.10 | 1.25 | 0.04 | 1.29 | | |
| langford_03_12 | 0.07 | 10.05 | 0.10 | 7.43 | 0.03 | 1.34 | | |
| langford_03_13 | 0.07 | 65.52 | 0.10 | 44.70 | 0.02 | 1.46 | | |
| langford_03_14 | 0.08 | 436.90 | 0.12 | 291.31 | 0.04 | 1.50 | | |
| langford_03_15 | 0.08 | 3285.47 | 0.17 | 1947.43 | 0.09 | 1.69 | | |
| langford_03_16 | 0.09 | 21585.50 | 0.19 | 14070.30 | 0.10 | 1.53 | TRUE | |
| langford_03_17 | 0.10 | 5.31 | 0.20 | 3.40 | 0.11 | 1.50 | | |
| langford_04_12 | 0.09 | 3.38 | 0.15 | 2.05 | 0.06 | 1.58 | | |
| langford_04_13 | 0.12 | 17.20 | 0.17 | 8.53 | 0.05 | 1.99 | | |
| langford_04_14 | 0.14 | 115.77 | 0.18 | 49.64 | 0.04 | 2.33 | | |
| langford_04_15 | 0.15 | 632.88 | 0.19 | 272.13 | 0.05 | 2.32 | | |
| langford_04_16 | 0.15 | 3868.87 | 0.22 | 1646.27 | 0.07 | 2.35 | | |
| langford_04_17 | 0.17 | 21584.50 | 0.23 | 8692.01 | 0.05 | 2.48 | TRUE | |
| langford_05_12 | 0.14 | 1.41 | 0.20 | 1.06 | 0.07 | 1.22 | | |
| langford_05_13 | 0.13 | 4.81 | 0.21 | 2.91 | 0.07 | 1.59 | | |
| langford_05_14 | 0.15 | 27.05 | 0.23 | 11.54 | 0.08 | 2.31 | | |
| langford_05_15 | 0.18 | 167.69 | 0.27 | 49.68 | 0.09 | 3.36 | | |
| langford_05_16 | 0.19 | 872.80 | 0.29 | 256.00 | 0.10 | 3.41 | | |
| langford_05_17 | 0.21 | 4554.85 | 0.34 | 1154.41 | 0.13 | 3.94 | | |
| langford_06_12 | 0.18 | 1.03 | 0.26 | 0.75 | 0.08 | 1.20 | | |
| langford_06_13 | 0.19 | 3.75 | 0.30 | 2.32 | 0.11 | 1.50 | | |
| langford_06_14 | 0.22 | 12.66 | 0.35 | 7.15 | 0.13 | 1.72 | | |
| langford_06_15 | 0.24 | 72.84 | 0.38 | 25.86 | 0.14 | 2.78 | | |
| langford_06_16 | 0.27 | 332.01 | 0.46 | 106.31 | 0.18 | 3.11 | | |
| langford_06_17 | 0.34 | 1700.49 | 0.54 | 307.13 | 0.20 | 5.53 | | |
| sports-12-10-10 | 0.45 | 21584.40 | 0.50 | 13641.20 | 0.05 | 1.58 | TRUE | |
| sports-12-10-1 | 0.33 | 155.10 | 0.41 | 11.88 | 0.08 | 12.65 | | |
| sports-12-10-2 | 0.46 | 21583.80 | 0.49 | 293.07 | 0.03 | 73.52 | TRUE | |
| sports-12-10-3 | 0.46 | 21585.60 | 0.55 | 12896.80 | 0.09 | 1.67 | TRUE | |
| sports-12-10-4 | 0.29 | 338.75 | 0.33 | 32.39 | 0.03 | 10.36 | | |
| sports-12-10-5 | 0.49 | 3679.26 | 0.55 | 49.98 | 0.05 | 72.83 | | |
| sports-12-10-6 | 0.43 | 21587.20 | 0.47 | 21586.50 | 0.04 | 1.00 | TRUE | TRUE |
| sports-12-10-7 | 0.47 | 21586.90 | 0.50 | 21582.30 | 0.03 | 1.00 | TRUE | TRUE |
| sports-12-10-8 | 0.43 | 10749.50 | 0.48 | 503.32 | 0.05 | 21.34 | | |
| sports-12-10-9 | 0.44 | 688.63 | 0.45 | 22.80 | 0.01 | 29.64 | | |

**Table 1.** Results for Minion-Static solver. B for baseline, T for tabulation. SR refers to Savile Row.

| Instance | B-SR time (s) | B-solver time (s) | T-SR time (s) | T-solver time (s) | Increase in SR time (s) | Speed-up | B-timeout | T-timeout |
|---|---|---|---|---|---|---|---|---|
| coprime-10 | 7.51 | 6.34 | 9.97 | 1.24 | 2.46 | 1.23 | | |
| coprime-11 | 16.93 | 18.39 | 23.29 | 2.31 | 6.37 | 1.38 | | |
| coprime-12 | 29.47 | 26.78 | 43.01 | 4.27 | 13.53 | 1.19 | | |
| coprime-13 | 66.98 | 219.69 | 73.43 | 227.50 | 6.45 | 0.95 | | |
| coprime-14 | 70.35 | 237.16 | 73.29 | 220.85 | 2.94 | 1.05 | | |
| coprime-15 | 68.10 | 584.70 | 146.41 | 632.25 | 78.31 | 0.84 | | |
| coprime-16 | 71.30 | 4985.78 | 173.17 | 4895.17 | 101.88 | 1.00 | | |
| coprime-8 | 1.89 | 0.57 | 2.64 | 0.25 | 0.76 | 0.85 | | |
| coprime-9 | 3.89 | 2.76 | 5.16 | 0.58 | 1.27 | 1.16 | | |
| knights_10 | 0.78 | 21584.20 | 1.10 | 21586.10 | 0.32 | 1.00 | TRUE | TRUE |
| knights_6 | 0.20 | 25.28 | 0.21 | 0.34 | 0.01 | 45.84 | | |
| knights_7 | 0.27 | 9.07 | 0.31 | 0.17 | 0.04 | 19.44 | | |
| knights_8 | 0.37 | 3265.91 | 0.50 | 7.19 | 0.13 | 424.84 | | |
| knights_9 | 0.61 | 21584.80 | 0.73 | 11.92 | 0.11 | 1706.56 | TRUE | |
| langford_02_13 | 0.05 | 246.78 | 0.07 | 221.33 | 0.02 | 1.11 | | |
| langford_02_14 | 0.05 | 2249.01 | 0.09 | 1967.59 | 0.03 | 1.14 | | |
| langford_02_17 | 0.06 | 21584.90 | 0.09 | 21585.60 | 0.03 | 1.00 | TRUE | TRUE |
| langford_03_11 | 0.06 | 1.44 | 0.10 | 1.43 | 0.04 | 0.98 | | |
| langford_03_12 | 0.07 | 8.66 | 0.09 | 8.06 | 0.03 | 1.07 | | |
| langford_03_13 | 0.07 | 56.66 | 0.11 | 49.45 | 0.03 | 1.14 | | |
| langford_03_14 | 0.08 | 386.29 | 0.12 | 317.69 | 0.04 | 1.22 | | |
| langford_03_15 | 0.09 | 2826.17 | 0.16 | 2159.69 | 0.07 | 1.31 | | |
| langford_03_16 | 0.09 | 21580.70 | 0.19 | 15090.70 | 0.10 | 1.43 | TRUE | |
| langford_03_17 | 0.10 | 17.46 | 0.22 | 4.75 | 0.13 | 3.53 | | |
| langford_04_12 | 0.09 | 3.26 | 0.15 | 2.56 | 0.06 | 1.24 | | |
| langford_04_13 | 0.09 | 15.57 | 0.18 | 10.97 | 0.08 | 1.41 | | |
| langford_04_14 | 0.13 | 98.20 | 0.18 | 61.94 | 0.05 | 1.58 | | |
| langford_04_15 | 0.14 | 600.24 | 0.19 | 322.49 | 0.04 | 1.86 | | |
| langford_04_16 | 0.15 | 3672.04 | 0.22 | 1989.64 | 0.06 | 1.85 | | |
| langford_04_17 | 0.17 | 21584.50 | 0.23 | 10740.00 | 0.06 | 2.01 | TRUE | |
| langford_05_12 | 0.13 | 1.22 | 0.20 | 1.03 | 0.07 | 1.10 | | |
| langford_05_13 | 0.14 | 4.28 | 0.21 | 2.62 | 0.07 | 1.56 | | |
| langford_05_14 | 0.15 | 25.43 | 0.26 | 12.43 | 0.11 | 2.02 | | |
| langford_05_15 | 0.18 | 126.39 | 0.28 | 55.95 | 0.10 | 2.25 | | |
| langford_05_16 | 0.18 | 652.39 | 0.30 | 285.29 | 0.12 | 2.28 | | |
| langford_05_17 | 0.21 | 3623.64 | 0.36 | 1160.45 | 0.15 | 3.12 | | |
| langford_06_12 | 0.17 | 0.84 | 0.26 | 0.73 | 0.09 | 1.03 | | |
| langford_06_13 | 0.20 | 2.43 | 0.29 | 1.68 | 0.09 | 1.33 | | |
| langford_06_14 | 0.21 | 10.87 | 0.32 | 6.38 | 0.11 | 1.65 | | |
| langford_06_15 | 0.23 | 55.87 | 0.41 | 23.42 | 0.18 | 2.35 | | |
| langford_06_16 | 0.28 | 238.18 | 0.45 | 93.09 | 0.18 | 2.55 | | |
| langford_06_17 | 0.32 | 1068.59 | 0.52 | 324.61 | 0.21 | 3.29 | | |
| sports-12-10-10 | 0.45 | 21584.30 | 0.50 | 9398.82 | 0.05 | 2.30 | TRUE | |
| sports-12-10-1 | 0.34 | 21.12 | 0.39 | 0.43 | 0.05 | 26.27 | | |
| sports-12-10-2 | 0.46 | 21584.00 | 0.50 | 155.12 | 0.05 | 138.70 | TRUE | |
| sports-12-10-3 | 0.48 | 21583.30 | 0.51 | 5682.02 | 0.03 | 3.80 | TRUE | |
| sports-12-10-4 | 0.29 | 74.56 | 0.34 | 50.29 | 0.04 | 1.48 | | |
| sports-12-10-5 | 0.47 | 530.28 | 0.52 | 169.03 | 0.05 | 3.13 | | |
| sports-12-10-6 | 0.42 | 21585.40 | 0.47 | 14308.60 | 0.05 | 1.51 | TRUE | |
| sports-12-10-7 | 0.49 | 21584.00 | 0.51 | 21583.10 | 0.02 | 1.00 | TRUE | TRUE |
| sports-12-10-8 | 0.44 | 9283.35 | 0.49 | 699.60 | 0.05 | 13.26 | | |
| sports-12-10-9 | 0.44 | 103.31 | 0.49 | 23.80 | 0.05 | 4.27 | | |

**Table 2.** Results for Minion-Conflict solver. B for baseline, T for tabulation. SR refers to Savile Row.

| Instance | B-SR time (s) | B-solver time (s) | T-SR time (s) | T-solver time (s) | Increase in SR time (s) | Speed-up | B-timeout | T-timeout |
|---|---|---|---|---|---|---|---|---|
| coprime-10 | 9.63 | 1.72 | 10.42 | 0.31 | 0.79 | 1.06 | | |
| coprime-11 | 21.51 | 3.47 | 22.61 | 0.61 | 1.11 | 1.08 | | |
| coprime-12 | 40.38 | 5.04 | 40.24 | 1.05 | -0.14 | 1.10 | | |
| coprime-13 | 91.92 | 14.29 | 151.41 | 13.61 | 59.49 | 0.64 | | |
| coprime-14 | 89.36 | 13.74 | 163.50 | 13.47 | 74.14 | 0.58 | | |
| coprime-15 | 115.99 | 30.98 | 187.49 | 31.57 | 71.49 | 0.67 | | |
| coprime-16 | 140.56 | 143.22 | 255.06 | 130.87 | 114.50 | 0.74 | | |
| coprime-8 | 2.46 | 0.49 | 2.61 | 0.09 | 0.16 | 1.09 | | |
| coprime-9 | 5.20 | 1.09 | 5.43 | 0.19 | 0.23 | 1.12 | | |
| knights_10 | 0.95 | 21599.05 | 1.05 | 3.65 | 0.10 | 4598.68 | TRUE | |
| knights_6 | 0.28 | 52.59 | 0.25 | 0.11 | -0.03 | 146.85 | | |
| knights_7 | 0.34 | 42.76 | 0.37 | 0.17 | 0.03 | 79.67 | | |
| knights_8 | 0.50 | 829.60 | 0.49 | 1.87 | -0.01 | 352.03 | | |
| knights_9 | 0.66 | 6111.63 | 0.75 | 2.84 | 0.09 | 1700.22 | | |
| langford_02_13 | 0.05 | 21599.95 | 0.07 | 21599.93 | 0.01 | 1.00 | TRUE | TRUE |
| langford_02_14 | 0.06 | 21599.94 | 0.08 | 21599.92 | 0.03 | 1.00 | TRUE | TRUE |
| langford_02_17 | 0.07 | 21599.94 | 0.09 | 21599.92 | 0.02 | 1.00 | TRUE | TRUE |
| langford_03_11 | 0.07 | 6.41 | 0.08 | 3.41 | 0.01 | 1.86 | | |
| langford_03_12 | 0.07 | 50.39 | 0.09 | 29.07 | 0.03 | 1.73 | | |
| langford_03_13 | 0.07 | 519.50 | 0.09 | 260.79 | 0.02 | 1.99 | | |
| langford_03_14 | 0.07 | 2341.56 | 0.11 | 1670.61 | 0.03 | 1.40 | | |
| langford_03_15 | 0.08 | 16621.62 | 0.15 | 14201.08 | 0.06 | 1.17 | | |
| langford_03_16 | 0.09 | 21599.92 | 0.19 | 21599.81 | 0.10 | 1.00 | TRUE | TRUE |
| langford_03_17 | 0.10 | 7.08 | 0.21 | 3.99 | 0.11 | 1.71 | | |
| langford_04_12 | 0.11 | 6.76 | 0.14 | 1.40 | 0.04 | 4.44 | | |
| langford_04_13 | 0.13 | 8.57 | 0.16 | 4.44 | 0.03 | 1.89 | | |
| langford_04_14 | 0.14 | 35.25 | 0.17 | 17.96 | 0.03 | 1.95 | | |
| langford_04_15 | 0.15 | 204.33 | 0.19 | 80.50 | 0.04 | 2.53 | | |
| langford_04_16 | 0.14 | 913.24 | 0.22 | 518.44 | 0.07 | 1.76 | | |
| langford_04_17 | 0.17 | 3695.62 | 0.23 | 2359.40 | 0.06 | 1.57 | | |
| langford_05_12 | 0.14 | 2.50 | 0.20 | 0.48 | 0.06 | 3.89 | | |
| langford_05_13 | 0.14 | 3.54 | 0.20 | 0.90 | 0.06 | 3.37 | | |
| langford_05_14 | 0.16 | 6.18 | 0.23 | 1.95 | 0.06 | 2.91 | | |
| langford_05_15 | 0.19 | 17.86 | 0.28 | 5.58 | 0.09 | 3.08 | | |
| langford_05_16 | 0.19 | 40.47 | 0.30 | 17.85 | 0.12 | 2.24 | | |
| langford_05_17 | 0.21 | 135.14 | 0.34 | 70.84 | 0.14 | 1.90 | | |
| langford_06_12 | 0.18 | 1.43 | 0.24 | 0.41 | 0.06 | 2.48 | | |
| langford_06_13 | 0.20 | 1.89 | 0.28 | 0.52 | 0.08 | 2.61 | | |
| langford_06_14 | 0.21 | 5.09 | 0.32 | 1.35 | 0.10 | 3.18 | | |
| langford_06_15 | 0.24 | 10.39 | 0.42 | 2.97 | 0.18 | 3.13 | | |
| langford_06_16 | 0.28 | 20.30 | 0.42 | 5.47 | 0.14 | 3.50 | | |
| langford_06_17 | 0.33 | 43.68 | 0.54 | 13.49 | 0.21 | 3.14 | | |
| sports-12-10-10 | 0.69 | 350.58 | 0.69 | 330.68 | 0.00 | 1.06 | | |
| sports-12-10-1 | 0.54 | 257.94 | 0.64 | 28.89 | 0.10 | 8.76 | | |
| sports-12-10-2 | 0.65 | 209.25 | 0.69 | 194.35 | 0.03 | 1.08 | | |
| sports-12-10-3 | 0.70 | 60.50 | 0.68 | 12.55 | -0.02 | 4.63 | | |
| sports-12-10-4 | 0.49 | 148.19 | 0.52 | 153.93 | 0.03 | 0.96 | | |
| sports-12-10-5 | 0.68 | 129.72 | 0.76 | 112.35 | 0.09 | 1.15 | | |
| sports-12-10-6 | 0.65 | 7.24 | 0.72 | 626.52 | 0.07 | 0.01 | | |
| sports-12-10-7 | 0.67 | 90.53 | 0.74 | 483.28 | 0.06 | 0.19 | | |
| sports-12-10-8 | 0.63 | 588.03 | 0.67 | 221.22 | 0.03 | 2.65 | | |
| sports-12-10-9 | 0.61 | 68.97 | 0.65 | 112.08 | 0.03 | 0.62 | | |

**Table 3.** Results for Chuffed solver with free search. B for baseline, T for tabulation. SR refers to Savile Row.