Ay 190 Assignment 14

Grid-Based Hydrodynamics

March 26, 2013

1 1D Planar Finite-Volume Hydrodynamics Code

The solution to the classic 1D shocktube problem was calculated in a number of ways and the results compared. A domain over [0,1] was initially set up with a fluid of density $\rho_L = 1.0$ and pressure $P_L = 1.0$ to the left of $x = 0.5$, and a second fluid of density $\rho_R = 0.1$ and pressure $P_R = 0.125$ to the right of $x = 0.5$. The system was evolved until $t = 0.2$.

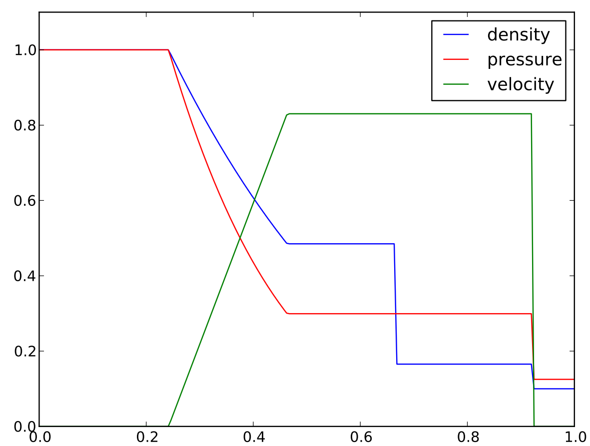

Figure 1: A solution to the shocktube problem initialied as described above, computed using Frank Timmes' exact Riemann solver.

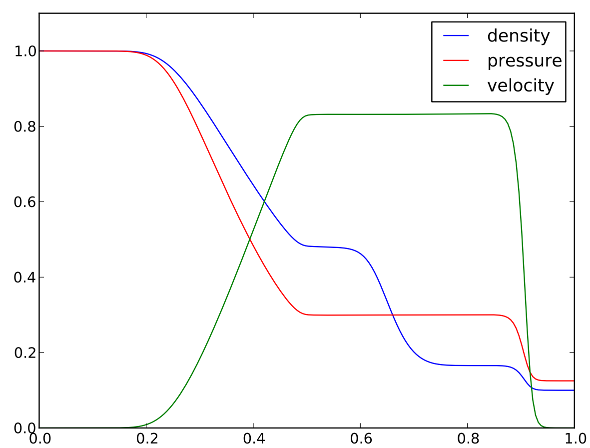

Figure 2: A solution for the same shocktube problem computed using a HLLE solver. The sharp transitions and discontinuities are smoothed out, likely due to the HLLE solver's approximation to the solution of the Riemann problem.

1.1 Comparisons of algorithm performance

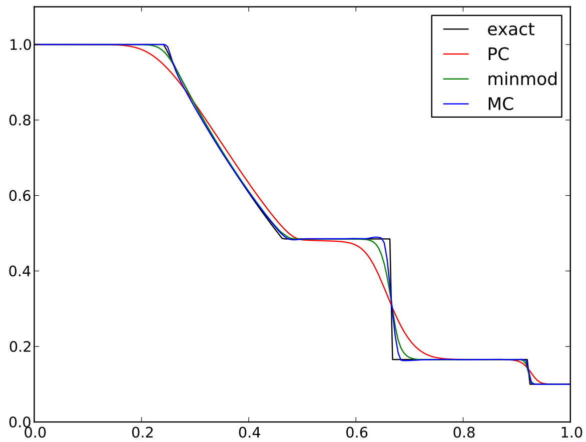

Figure 3: Comparison of density profiles produced using the HLLE approximate Riemann solver with piecewise constant, minmod, and monotonized central limiters for reconstruction compared with the exact Riemann solver. MC reproduces the transitions and discontinuities the best, though at the cost of introducing ripples in the solution, particularly at early times. At later times, the ripples exist near discontinuities. Minmod also develops ripples, but are suppressed earlier. PC reproduces the transitions and discontinuities the worst, but never develops ripples.

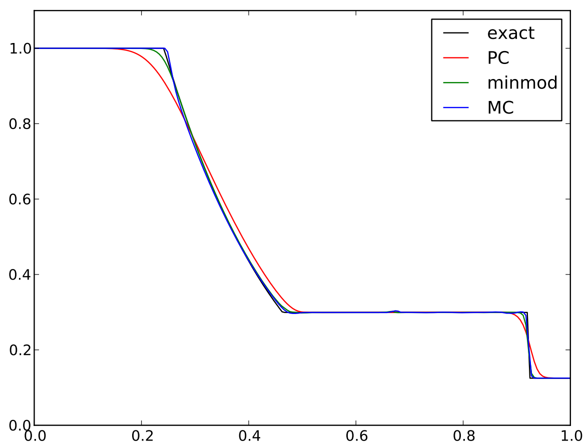

Figure 4: The same as Figure 3, but of pressures.

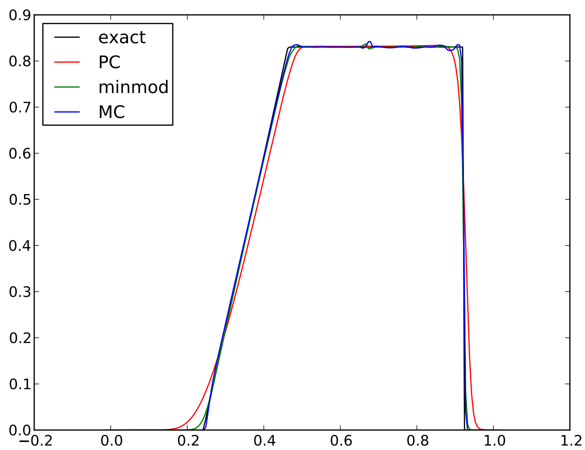

Figure 5: The same as Figure 3, but of velocities.

2 Simulating Stellar Collapse

The 1D planar finite-volume hydro code from above was adapted to spherical geometries and to compute the evolution of a star undergoing core collapse. The method was used to perform reconstruction. Unfortunately, the code broke (with $c_s^2 < 0$) at around 20 ms. The results until that point is shown below.

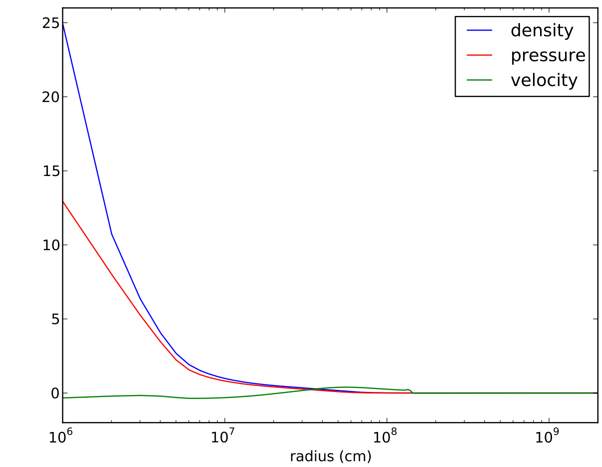

Figure 6: The state of the star at 20 ms into the collapse. Velocities are in units of 10^8 cm s^{-2}, and pressure and densities are in units of their maximum values at $t = 0$, which are $\rho_{0,max} = 10^{10}$ g cm^{-3} and $P_{0,max} = 3 \times 10^{27}$ g s^{-1} cm^{-1}.

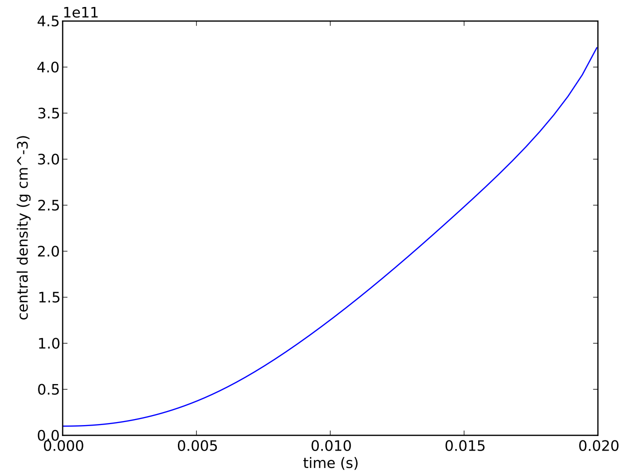

Figure 7: Central density over time until the code broke down, at which point $\rho_c = 4.2 \times 10^{11}$ g cm^{-3}.

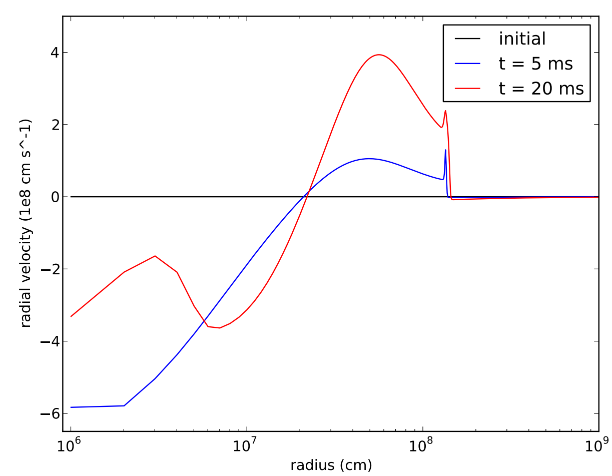

Figure 8: Radial velocities at $t = 0$ and 5 ms.

Appendices

The Python modules and scripts used in this assignment include

A 1D Finite Volume Planar Hydro Code: `hydro_planar.py`
B 1D Finite Volume Spherical Hydro Code: `hydro_spherical.py`
C Hybrid EOS for star undergoing core-collapse: `eos.py`

Frank Timmes' exact Riemann solver can be found at `cococubed.asu.edu/code_pages/exact_riemann.shtml`.