Ay190: Computational Astrophysics (Winter Term 2012)

Worksheet # 2 and Homework Assignment #2. All reading is due in class on Thursday, January 12. All homework is due on 2011/01/16 and must be submitted via version control before class (notify `cott@tapir.caltech.edu` about your submission.

Reading for Thursday, January 12

Read sections 4.5 to 5.2.2 in the lecture notes. An updated set of notes is available on the ay190 blog: http://ay190.blogspot.com.

In-Class Exercises

Document/version control your work in a sub-directory called "ws2_hw2" in your repository.

1. **Finite Difference Approximation and Convergence** (5 points)

   Discretize $f(x) = x^3 - 5x^2 + x$ on the interval $[-2, 6]$ and compute its first derivative with (i) forward differencing and (ii) central differencing. Demonstrate that (i) is first-order convergent and (ii) is second-order convergent by plotting the absolute error $f'(x; h_i) - f'(x)$ at resolutions h_1 and $h_2 = h_1/2$. At h_2 the absolute error should be reduced by the expected convergence factor.

2. **Interpolation: Cepheid Lightcurve** (5 points)

   Cepheids are stars that exhibit variations in their apparent magnitude due to radial pulsations driven by varying opacity (the κ-Mechanism). Astronomers have taken the following lightcurve data from a quickly varying (1-day period) cepheid:

   ```
   # time          apparent magnitude
   0.0             0.302
   0.2             0.185
   0.3             0.106
   0.4             0.093
   0.5             0.24
   0.6             0.579
   0.7             0.561
   0.8             0.468
   1.0             0.302
   ```

   (a) Construct (not by hand!) a single global Lagrange interpolation polynomial $p_8(x)$ of degree 8 that interpolates the lightcurve at its 9 known data points. Plot $p_8(x)$ and the data together.

   (b) Use piecewise linear and piecewise quadratic interpolation to interpolate the data. Plot the various results together in one figure.

Homework (due before class [via version control] on January 16, 2012)

1. Complete incomplete in-class exercises and document your work in a pdf document. Commit code and pdf document to your repository.

2. **Finite Difference Approximation and Convergence** (3 points)

   Derive a second-order central finite difference approximation for the second derivative of a function $f(x)$. Assume fixed step size h.

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3. **More Cepheid Lightcurve Interpolation** (7 points)

   (a) Re-do the interpolation by implementing piecewise cubic Hermite interpolation.

   (b) Re-do the interpolation with cubic spline interpolation. Do not implement it yourself, but learn to use the pylab library routines.

   Compare your new interpolation results with the old ones from the in-class exercise. All code/plots/documents to be commited to version control repository.