Hypothesis test

Answer sheet for exercise : The rainmakers

1. Name of the test.

One-sided test of the mean of a Gaussian sample with known variance

2. Describe with words
   (a) the population: annual rainfalls after insemination
   (b) the sample: 9 annual raintalls measured after insemination
   (c) the observed variable: the annual rainfall $X \sim \mathcal{N}(m, \sigma^2)$ $\sigma^2 = 100^2$
   (d) the observed parameter: m
3. Mathematical setting of the test. What are (in the context of the problem)
   (a) H_0: $[m = 600]$
   (b) H_1: $[m > 600]$
   (c) the significance level α: 0.05 (when not specified, we take this value)
   (d) the decision variable D:

   $$\bar{X}_9 = \frac{1}{9}(X_1 + \dots + X_9)$$

   (e) the distribution of D under assumption H_0:

   $$\bar{X}_9 \sim \mathcal{N}\left(600, \frac{100^2}{9}\right)$$

   (f) the observation of D on the sample, d: 610.2
   (g) the p-value or critical region expression :

   $$p = \mathbb{P}[\bar{X}_9 \geq 610.2 \mid m = 600] \ / \ k \text{ s.t. } \mathbb{P}[\bar{X}_9 \geq k \mid m = 600] = \alpha$$

   (h) the p-value or critical region $p = 0.38\,\%$ / $k \simeq 655$
4. Decision of the test.
   (a) Decision: We do not reject H_0
   (b) Reason for decision: $p >> \alpha$ / $d << k$
   (c) Conclusion: At a significance level of 5%, we have no sufficient evidence to reject H_0.

! p-value and critical value methods are two different but equivalent methods. It is good to know both but in real life one is enough!