❈ Appendix A: Classical confidence intervals (CI)

We assume that $(X_1, ..., X_n)$ is an n- sample of a random variable X. We recall

$$\overline{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i \quad \text{and} \quad S_n^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X}_n)^2$$

are estimators of the mean and the variance of X respectively, which are unbiased and convergent.The statsitic

$$T_n = \frac{1}{n}\sum_{i=1}^{n}(X_i - m)^2$$

is also an unbiased and convergent estimator of the variance when the mean m is known.

The observed values of this estimators are denoted $\overline{x}_n$, s_n and t_n.

In the following, we give the confidence intervals for different parameters in different cases at a confidence level $1-\alpha$. We recall the estimator of the parameter as well as its distribution under the given conditions. The value u_β denote the β-quantile of this very distribution.

A.1 CI for the mean and the variance of a Gaussian variable

We assume that $X \rightsquigarrow \mathcal{N}(m, \sigma^2)$.

CI for the mean with known variance

$$\frac{\overline{X}_n - m}{\sigma/\sqrt{n}} \rightsquigarrow \mathcal{N}(0,1)$$

Confidence intervals for m:

$$\left(-\infty, \overline{x}_n + \frac{\sigma}{\sqrt{n}}u_{1-\alpha}\right), \quad \left(\overline{x}_n - \frac{\sigma}{\sqrt{n}}u_{1-\alpha/2}, \overline{x}_n + \frac{\sigma}{\sqrt{n}}u_{1-\alpha/2}\right), \quad \left(\overline{x}_n - \frac{\sigma}{\sqrt{n}}u_{1-\alpha}, +\infty\right)$$

CI for the mean with unknown variance

$$\frac{\overline{X}_n - m}{S_n/\sqrt{n}} \rightsquigarrow \mathcal{T}_{n-1}$$

Confidence intervals for m:

$$\left(-\infty, \overline{x}_n + \frac{s_n}{\sqrt{n}}u_{1-\alpha}\right), \quad \left(\overline{x}_n - \frac{s_n}{\sqrt{n}}u_{1-\alpha/2}, \overline{x}_n + \frac{s_n}{\sqrt{n}}u_{1-\alpha/2}\right), \quad \left(\overline{x}_n - \frac{s_n}{\sqrt{n}}u_{1-\alpha}, +\infty\right)$$

CI interval for the variance with known mean

$$nT_n/\sigma^2 \rightsquigarrow \chi^2_n$$

Confidence intervals for σ^2:

$$\left(0, \frac{nt_n}{u_{\alpha/2}}\right), \quad \left(\frac{nt_n}{u_{1-\alpha/2}}, \frac{nt_n}{u_{\alpha/2}}\right), \quad \left(\frac{nt_n}{u_{1-\alpha/2}}, +\infty\right)$$

CI interval for the variance with unknown mean

$$(n-1)\frac{S_n^2}{\sigma^2} \rightsquigarrow \chi^2_{n-1}$$

Confidence intervals for σ^2:

$$\left(0, \frac{(n-1)s_n^2}{u_{\alpha/2}}\right), \quad \left(\frac{(n-1)s_n^2}{u_{1-\alpha/2}}, \frac{(n-1)s_n^2}{u_{\alpha/2}}\right), \quad \left(\frac{(n-1)s_n^2}{u_{1-\alpha/2}}, +\infty\right)$$

A.2 CI for a proportion

We assume that $X \rightsquigarrow \mathcal{B}(p)$ and that the sample is large.

$$\frac{\overline{X}_n - p}{\sqrt{\overline{X}_n(1-\overline{X}_n)}/\sqrt{n}} \rightsquigarrow \mathcal{N}(0,1) \text{ approximatively.}$$

Confidence intervals for p:

$$\left(-\infty, \overline{x}_n + \frac{u_{1-\alpha/2}}{\sqrt{n}}\sqrt{\overline{x}_n(1-\overline{x}_n)}\right), \quad \left(\overline{x}_n - \frac{u_{1-\alpha/2}}{\sqrt{n}}\sqrt{\overline{x}_n(1-\overline{x}_n)}, +\infty\right)$$

$$\left(\overline{x}_n - \frac{u_{1-\alpha/2}}{\sqrt{n}}\sqrt{\overline{x}_n(1-\overline{x}_n)}, \overline{x}_n + \frac{u_{1-\alpha/2}}{\sqrt{n}}\sqrt{\overline{x}_n(1-\overline{x}_n)}\right)$$

A.3 CI for the mean of a square integrable variable

We assume $\mathbb{E}[X] = m$, $\mathbb{V}\text{ar}[X] = \sigma^2$ and a sufficiently large sample. By Central Limit Theorem, the following intervals

$$\left(-\infty, \overline{x}_n + \frac{\sigma}{\sqrt{n}}u_{1-\alpha}\right), \quad \left(\overline{x}_n - \frac{\sigma}{\sqrt{n}}u_{1-\alpha/2}, \overline{x}_n + \frac{\sigma}{\sqrt{n}}u_{1-\alpha/2}\right), \quad \left(\overline{x}_n - \frac{\sigma}{\sqrt{n}}u_{1-\alpha}, +\infty\right)$$

are approximated confidence intervals for mean m with known variance σ^2 . In practice, if σ is unknown, we replace it with s_n.

There are no confidence interval for the variance in the non Gaussian case.