Method card 6

Course: statistics
Topic: Modeling with continuous distributions 2024-2025

For each distribution, you will find

- **some model example**
- **the PDF**
- **expectation and variance**

Uniform distribution $X \rightsquigarrow \mathcal{U}([a,b])$

Example of use: Drawing a piont uniformly on a bounded interval.
State space: $[a,b]$

PDF: $f(x) = \frac{1}{b-a}\mathbb{1}_{[a,b]}(x)$ for $x \in \mathbb{R}$.

Moments: $\mathbb{E}[X] = \frac{a+b}{2}$, $\mathbb{V}[X] = \frac{(b-a)^2}{12}$

Exponential distribution $X \rightsquigarrow \mathcal{E}(\lambda)$

Example of use: Waiting time between Poissionian arrivals.
State space: $\mathbb{R}_+^*$

PDF: $f(x) = \lambda e^{-\lambda x}\mathbb{1}_{\mathbb{R}_+}$ for $x \in \mathbb{R}$.

Moments: $\mathbb{E}[X] = \frac{1}{\lambda}$, $\mathbb{V}[X] = \frac{1}{\lambda^2}$

$$\boxed{\int_{\mathbb{R}_+} \lambda e^{-\lambda x} dx = 1, \quad \int_{\mathbb{R}_+} x\lambda e^{-\lambda x} dx = \frac{1}{\lambda}, \quad \int_{\mathbb{R}_+} x^2 \lambda e^{-\lambda x} dx = \frac{2}{\lambda^2}}$$

Gaussian distribution $X \rightsquigarrow \mathcal{N}(m, \sigma^2)$

Example of use: swiss knife for continuous symmetric unbouded with light tail or bounded, rounded shape distributions
State space: $\mathbb{R}$

PDF: $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}$ for $x \in \mathbb{R}$.

Moments: $\mathbb{E}[X] = m$, $\mathbb{V}[X] = \sigma^2$

$$\boxed{\int_{\mathbb{R}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2} dx = 1}$$

$$\boxed{\int_{\mathbb{R}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx = 1, \quad \int_{\mathbb{R}} \frac{x}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx = 0, \quad \int_{\mathbb{R}} \frac{x^2}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx = 1}$$

Triangular distribution of parameters (a,b,c), $a<c<b$

Example of use: Experiment whose values are taken in $[a,b]$ wit a maximum probability around $c \in (a,b)$.
State space: $\mathbb{R}$
PDF: For $x \in \mathbb{R}$,

$$f(x) = \begin{cases} 2\dfrac{x-a}{(b-a)(c-a)} \text{ if } x \in [a,c] \\ 2\dfrac{b-x}{(b-a)(b-c)} \text{ if } x \in [c,b] \\ 0 \text{ otherwise} \end{cases}$$

Moments: $\mathbb{E}\left[X\right] = \dfrac{a+b+c}{3}$, $\mathbb{V}\left[X\right] = \dfrac{a^2+b^2+c^2-ab-ac-bc}{18}$