IL NUOVO CIMENTO VOL. 65 A, N. 3 1 Ottobre 1981

A Unified Potential for Mesons and Baryons.

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(ricevuto il 30 Aprile 1981)

Summary. — We examine several phenomenological nonrelativistic $q\bar{q}$ potentials, with different radial dependences, that fit the low-lying charmonium spectra. With the familar Coulomb-plus-ramp–type dependence the same potential can reproduce the spectra of the light mesons using the Schrödinger equation. With half-strength and the addition of a small three-body term the ground states of the baryons are also obtained.

1. – Introduction.

In this paper, we shall use several phenomenological nonrelativistic $q\bar{q}$ potentials to generate the spectra of mesons and then calculate the ground-state masses of the baryons by assuming the qq interaction to be half as strong as the $q\bar{q}$ potential. The motivation of the work is threefold.

a) To test whether a $q\bar{q}$ potential whose parameters are chosen to fit the low-lying levels of charmonium is able to reproduce the spectra of the lighter mesons. This would be surprising, not because of the flavour independence of the $q\bar{q}$ interaction, but due to the apparent success of the nonrelativistic dynamics in light hadrons, where relativistic effects should be very large.

b) To test the sensitivity of the different radial forms of the $q\bar{q}$ interactions on the excited states of the lighter mesons.

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c) For the one-gluon exchange part, the qq potential in baryons is half as strong as the $q\bar{q}$ interaction in mesons. There is no real justification for applying this rule to the full phenomenological two-body interaction, except simplicity. The adequacy of this assumption is tested by calculating the ground-state baryon masses with a simple but well-tested method of solving the three-body problem.

We now go on to put the above points in perspective. From our calculations one cannot escape the conclusion that nonrelativistic dynamics appears to yield sensible results even for the lighter hadrons, so the relativistic effects are masked. This, of course, has been known for a long time through the successes in magnetic moments and mass formulae involving baryons [1,2]. More quantitative works include that of Graham and O'Donnell [3] on lighter mesons, and specially those of Isgur and Karl [4,5] on the baryonic states, all using nonrelativistic methods. We take a unified approach in the sense that we construct a $q\bar{q}$ potential to fit the levels of charmonium, and use it both for mesons and for baryons in fully dynamical nonrelativistic calculations. This point of view has been advocated by LIPKIN [6,7] and COHEN and LIPKIN [8]; indeed the latter have tried to find some justification for the masking of relativistic effects. LIPKIN [6], however, used only a specific radial form for the $q\bar{q}$ potential—the logarithmic form of Quigg and Rosner [9]—and did not examine the nodal excited states of the lighter mesons. We shall see that these are crucial in showing the inadequacy of the logarithmic form for the lighter mesons. Finally, we mention the works of Leal Ferreira *et al.* [10] and Stanley and Robson [11,12], who use relativistic dynamics and the one-gluon exchange potential model. LEAL FERREIRA *et al.* generate the relativistic wave functions by assuming that the quarks move in a one-body linear confinement potential and calculate the effect of the one-gluon exchange potential perturbatively. STANLEY and ROBSON [11,12] use the relativistic kinetic-energy operator and a more elaborate $q\bar{q}$ potential. For example, they add a two-gluon annihilation term in the

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(1) J. J. KOKKEDEE: *The Quark Model* (New York, N. Y., 1969).
(2) A. DE RÚJULA, H. GEORGI and S. L. GLASHOW: *Phys. Rev. D*, **12**, 147 (1975).
(3) R. H. GRAHAM and P. J. O'DONNELL: *Phys. Rev. D*, **19**, 284 (1979).
(4) N. ISGUR and G. KARL: *Phys. Rev. D*, **11**, 4187 (1978); **19**, 2653 (1979); **20**, 1191 (1979).
(5) L. COPLEY, N. ISGUR and G. KARL: *Phys. Rev. D*, **20**, 768 (1979).
(6) H. J. LIPKIN: *Phys. Lett. B*, **74**, 399 (1978).
(7) H. J. LIPKIN: in the *Proceedings of « Baryon 80 »*, edited by N. ISGUR (Toronto, 1981).
(8) I. COHEN and H. J. LIPKIN: *Phys. Lett. B*, **93**, 56 (1980).
(9) C. QUIGG and J. L. ROSNER: *Phys. Lett. B*, **71**, 153 (1977).
(10) P. LEAL FERREIRA, J. A. HELAYEL and N. ZAGURY: *Nuovo Cimento A*, **55**, 215 (1980).
(11) D. P. STANLEY and D. ROBSON: *Phys. Rev. D*, **21**, 3180 (1980).
(12) D. P. STANLEY and D. ROBSON: *Phys. Rev. Lett.*, **45**, 235 (1980).

potential for the isoscalar 0^- mesons like η and η_c, which in our simpler model we do not attempt to fit.

In sect. **2**, we describe the various $q\bar{q}$ potentials that we choose and the resulting meson spectra that we obtain by using them in the two-body Schrödinger equation. In sect. **3**, the method for solving the ground-state three-body problem is outlined and the corresponding results are presented. The success and the shortcomings of our model are discussed in the final section.

An appendix contains the main formulae that are used in the three-body problem.

2. – The $q\bar{q}$ potential and the meson spectra.

We use three different radial forms for the central part of the $q\bar{q}$ potential—all three forms have enjoyed some popularity in the literature. For convenience, we write

$$V_{q\bar{q}} = V^c_{q\bar{q}} + V^\sigma_{q\bar{q}}\,, \tag{1}$$

where $V^c_{q\bar{q}}$ is the central and $V^\sigma_{q\bar{q}}$ is the hyperfine spin-dependent potential. For simplicity, we do not consider spin-orbit and tensor potentials. The form of the spin-dependent potential is suggested by the one-gluon exchange mechanism and is taken throughout to be

$$V^\sigma_{q\bar{q}} = \frac{\hbar^2 \varkappa_\sigma}{m_i m_j c^2}\, 4\pi f(r)\, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j\,, \tag{2}$$

where m_i, m_j are the masses of the interacting q, $\bar{q}$, $\boldsymbol{\sigma}$ is the Pauli matrix, and the form factor is

$$f(r) = \exp[-r/r_0]/4\pi r_0^2 r\,. \tag{3}$$

Note that $f(r)$ reduces to $\delta(\boldsymbol{r})$ in the limit of $r_0 \to 0$, which is the form usually taken in perturbative calculations. Although the specific form of the form factor is not crucial, it is vital to use a finite-range function, otherwise there is a collapse for an attractive $\delta(\boldsymbol{r})$ in a dynamical calculation. The choice of the parameters $\varkappa_\sigma$ and r_0 will be discussed shortly. As mentioned earlier, three different choices for the central part $V^c_{q\bar{q}}$ are taken. The first is

$$V^c_{q\bar{q}}(\mathrm{I}) = \frac{\varkappa}{r} + \frac{r}{a^2} - \Delta\,, \tag{4}$$

whose form is suggested by QCD. This form has been used by EICHTEN *et al.* [13]

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[13] E. EICHTEN, K. GOTTFRIED, T. KINOSHITA, K. D. LANE and T. M. YAN: *Phys. Rev. D*, **21**, 203 (1980).

to make a detailed analysis of charmonium. The second form is due to MARTIN [16]:

$$V^{c}_{q\bar{q}}(\mathrm{II}) = -A + Br^{\beta}, \tag{5}$$

where again there are three adjustable parameters A, B and β which are determined through analysing charmonium and the heavier upsilon ($b\bar{b}$). The last form is due to QUIGG and ROSNER [9] and was used by LIPKIN [6] in his analysis:

$$V^{c}_{q\bar{q}}(\mathrm{III}) = \alpha \ln (\Lambda r) \tag{6}$$

with α and Λ the adjustable parameters. In all three cases, we add the same hyperfine term (2) and solve the Schrödinger equation. The parameters occurring in eqs. (2)-(6) are displayed in table I. A few comments about the choice of the parameters are in order. Consider, for example, the potential I given by eq. (4), together with the spin-dependent part (2). The latter has a small effect on the spectrum of charmonium because of the large mass m_c of the charmed quark. Thus the parameters $\varkappa$, a and Λ are essentially determined, for a choice of m_c, by fitting the $1S$, $1P$ and $2S$ states of charmonium. Fine tuning is necessary when $V^{\sigma}_{q\bar{q}}$ is switched on. Our choice for the numerical values of $\varkappa$, a and Λ are slightly different from those of Eichten *et al.* [13] because we take a larger m_c and also include the hyperfine term (2). The quark mass m_u ($= m_d$) is not a free parameter, but chosen from magnetic-moment considerations. For a given choice of the range parameter r_0 in (3), the strength $\varkappa_\sigma$ of the spin-dependent potential (2) is chosen to reproduce the π-ρ splitting. It is found that the best overall results are obtained when r_0 is in the vicinity of half a fm. Finally, the strange-quark mass m_s and the bottom-quark mass m_b are obtained by fitting the ground states of $\varphi(1020)$ and $\Upsilon(9434)$. When dealing with the other two radial forms (5) and (6) for the central $q\bar{q}$ potential, we found that we could retain the values of the masses m_u, m_s and m_c the same as in I, but m_b had to be changed somewhat. For simplicity, we also do not alter the parameters of the spin-dependent potential (2), although this means that the π-ρ splitting with potentials II and III is too small by about 20 MeV. Both the potentials II and III fail to fit the $4S$ level of charmonium because they rise too slowly [9,16], but otherwise the fit for the lower levels is satisfactory.

We examine now the excited states of the scalar and vector mesons as generated by the above potentials. In table II, the results for the potential $V^{c}_{q\bar{q}}(\mathrm{I}) + V^{\sigma}_{q\bar{q}}$, as defined by eqs. (2) and (4), are displayed. It shows that about thirty odd states may be identified in this model with experimental ones. Of course, in our model we cannot differentiate between states like $^3P_2$, $^3P_1$ and $^3P_0$.

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[16] A. MARTIN: CERN preprint TH-2980 (1980).

TABLE I. – *Parameters of the various* $q\bar{q}$ *potentials that are defined by eqs.* (1)-(6) *in the text.* For potentials II and III, the quark mass m_b need to fit the ground state of upsilon is $m_b = 5223$ MeV and 5235 MeV, respectively. The other quark masses remain the same in each case.

$V^{c}_{q\bar{q}}$ (I)	$V^{c}_{q\bar{q}}$ (II)	$V^{c}_{q\bar{q}}$ (III)	$V^{\sigma}_{q\bar{q}}$	Quark masses (MeV)
$\varkappa = 102.67$ MeV fm	$A = 8170$ MeV	$\alpha = 744$ MeV	$\varkappa_\sigma = 102.67$ MeV fm	$m_u = m_d = 337$
$a = 0.0326\,((\text{MeV})^{-1}\,\text{fm})^{\frac{1}{2}}$	$B = 8075$ MeV fm$^{-\beta}$	$\Lambda = 0.87$ fm^{-1}	$r_0^{-1} = 2.2$ fm^{-1}	$m_s = 600$, $m_c = 1870$
$\Delta = 913.5$ MeV	$\beta = 0.1$			$m_b = 5259$ (I)

TABLE II. – *The predicted meson spectrum in* MeV *calculated by solving the Schrödinger equation with potential* $V^{c}_{q\bar{q}}(\text{I}) + V^{\sigma}_{q\bar{q}}$.

		$S = 1$						
$q\bar{q}$		$1S$	$1P$	$2S$	$1D$	$3S$	$2D$	$4S$
$c\bar{c}$	theory	3 097	3 519	3 685	3 804	4 109	4 192	4 469
	experiment	3 097	3 521	3 685	3 768	4 030	4 159	4 415
$b\bar{b}$	theory	9 433	9 933	10 028	10 185	10 375	10 479	10 654

		1S	1P	2S	1D	1S	1P	2S	1D
$u\bar{u}$	theory	777	1254	1614	1678	136	1118	1311	1628
	experiment	ρ (776)	A_1(1240) [15]	ρ′(1600)	g(1700)	π(138)			A_3(1660)
			A_2(1317), D (1285) [15]						
$u\bar{s}$	theory	905	1375	1694	1774	520	1282	1501	1738
	experiment	K*(892)	Q_2(1400),	K*(1650)	K*(1780)	K(495)	Q_1(1280)		
			κ (1500), K*(1430)						
$s\bar{s}$	theory	1017	1468	1744	1834				
	experiment	φ(1020)	E(1425) [15], f′(1515)	φ′(1670) [15]					
$u\bar{c}$	theory	2020	2491	2767	2868	1886	2455	2697	2852
	experiment	D*(2010)				D(1870)			
$s\bar{c}$	theory	2101	2541	2768	2874	1996	2509	2715	2853
	experiment	F*(2140)				F(2030)			
$u\bar{b}$	theory	5350	5826	6086	6196	5301	5812	6061	6190
	experiment								

([14]) T. Böhringer, F. Costantini, J. Dobbins, P. Franzini, K. Han, S. W. Herb, D. M. Kaplan, L. M. Lederman, G. Mageras, D. Peterson, E. Rice, J. K. Yoh, G. Finocchiaro, J. Lee-Franzini, G. Giannini, R. D. Schamberger jr., M. Sivertz, L. J. Spencer and P. M. Tuts: *Phys. Rev. Lett.*, **44**, 1111 (1980).

([15]) L. Montanet: CERN preprint EP/80-163, review talk at the *International Conference on High-Energy Physics* (Madison, Wis., 1980).

Nevertheless, the agreement is impressive for this potential with the Coulomb-plus-ramp dependence. When, on the other hand, the radial forms given by (5) or (6) for $V^c_{q\bar{q}}$ are used to generate the excited states of the lighter mesons, these in general come out too low. This itself is not very surprising, because these two potentials were designed to reproduce the low-lying states of $c\bar{c}$ and were too shallow to reproduce even the $4S$ state of charmonium. For the lighter mesons like ρ and K*, this deficiency shows up even for the first excited S-state. For example, what should be ρ′(1600) comes at 1350 MeV with potential (5) and at 1289 MeV with potential (6). Discrepancies of about 150 and 200 MeV exist for K*(1650) with the same potentials. Although the potentials (5) and (6) fare poorly for the lightest mesons, they are adequate to reproduce the excited states of $s\bar{s}$—in this respect the Martin-type potential (5) does very well. This has been noted by RICHARD ([17]). If, however, we choose to ignore relativistic effects and insist on a universal potential for the mesons, then it appears that the radial form of Coulomb plus ramp, as given by eq. (4), is the most favoured potential. In the next section, we shall use this potential to calculate the ground-state masses of the baryons.

3. – The baryon masses.

If there is an overall colour-dependent factor of $\boldsymbol{F}_i \cdot \boldsymbol{F}_j$ in the two-body interaction, the qq potential V_{qq} in baryons is just $\frac{1}{2}V_{q\bar{q}}$, where $V_{q\bar{q}}$ is given by eq. (1). We can test this hypothesis with our phenomenological potentials. Apart from this overall factor of $\frac{1}{2}$, all parameters of the potential as well as the quark masses are kept unchanged. We use the Feshbach-Rubinow (FR) approximation ([18]) to obtain a variational solution of the three-body Schrödinger equation, appropriately generalized to handle unequal masses of the quarks and unequal force bonds ([19]). The method is explained in detail in the above references, but for the sake of completeness we outline the main steps of the method in the appendix. The method assumes that the ground-state three-body wave function is a function of a single variable $R = \frac{1}{2}(x + y + \eta z)$, where $x = |\boldsymbol{r}_2 - \boldsymbol{r}_3|$, and likewise for y and z. The variational parameter η takes account of the asymmetry in the force bonds and the masses. The three-body Schrödinger equation is then reduced to a single differential equation in the variable R, as shown in the appendix, with an effective mass and an effective potential that are dependent on the variational parameter η. The ground-state eigenvalue of the equation is minimized as a function of η. For sym-

([17]) J. M. RICHARD: CERN preprint TH-3006 (1980).

([18]) H. FESHBACH and S. I. RUBINOW: *Phys. Rev.*, **98**, 188 (1955).

([19]) L. ABOU-HADID and K. HIGGINS: *Proc. Phys. Soc.*, **79**, 34 (1962); R. K. BHADURI, Y. NOGAMI and W. VAN DIJK: *Nucl. Phys. B*, **1**, 269 (1967); **2**, 316(E) (1967).

metric systems like $\Delta(1232)$ and $\Omega(1672)$, the minimum is obtained for $\eta = 1$, but in general η may deviate considerably from unity, as shown in table III. The method is simple and accurate in the absence of tensor potentials. Its accuracy has been checked in atomic problems with Coulomb force ([20]), in nuclear problems with short-range force ([21]) and in an exactly solvable harmonic model ([22]) of three quarks. Recently, RICHARD ([17]) has compared the accuracy of the FR method used by us with the method of hyperspherical co-ordinates. If we use one of our earlier versions of Coulomb-plus-ramp interaction, the FR method gives a mass of 1699 MeV for Ω, while the hyperspherical method yields a value lower by 13 MeV. For the gentler Martin-type interaction, we have checked that the hyperspherical method is more accurate by about 6 MeV. The FR method, however, is very easy to implement even for unequal masses. It has the disadvantage, however, that the spectra of the excited states are not obtained, and we limit our investigation here to the ground state.

TABLE III. – *The ground states of* $S = \frac{1}{2}$ *and* $S = \frac{3}{2}$ *baryons as calculated in the* FR *approximation with the potential* $\frac{1}{2}(V^{c}_{q\bar{q}}(\mathrm{I}) + V^{\sigma}_{q\bar{q}})$. The optimized asymmetry parameter η and the calculated mass M (MeV) are shown in columns 3, 4 and 7, 8. The adiacent square-bracketed numbers are obtained by including a three-body term $C/(m_1 m_2 m_3)^{\frac{1}{3}}$.

qqq	$S = \frac{1}{2}$				$S = \frac{3}{2}$			
	Experiment	η	M		Experiment	η	M	
uud	$\mathcal{N}(939)$	0.74	1052	[939]	$\Delta(1232)$	1	1354	[1241]
uds	Λ (1115)	1.11	1196	[1111]				
uds	Σ (1193)	0.62	1281	[1198]	$\Sigma(1385)$	0.78	1477	[1394]
uss	Ξ (1318)	1.09	1387	[1324]	$\Xi(1533)$	1.32	1591	[1528]
sss					$\Omega(1672)$	1	1699	[1651]
udc ([23])	$\Lambda_c(2285)$	0.87	2334	[2286]				
udc ([23])	$\Sigma_c(2457)$	0.48	2511	[2463]		0.53	2585	[2537]
ssc		0.59	2717	[2693]		0.63	2776	[2751]
ccc						1	4813	[2804]

In table III, we display the results of our three-body calculation for the ground-state baryon masses, using potentials (4) and (2). As mentioned earlier, these potentials are multiplied by an overall factor of $\frac{1}{2}$ to represent the qq interaction in baryons. It is seen from table III that, although the splittings

([20]) R. K. BHADURI and Y. NOGAMI: *Phys. Rev. A*, **13**, 1986 (1976).
([21]) M. MCMILLAN: *Can. J. Phys.*, **43**, 463 (1965).
([22]) R. K. BHADURI, L. E. COHLER and Y. NOGAMI: *Phys. Rev. Lett.*, **44**, 1369 (1980).
([23]) P. MUSSET: CERN preprint EP/80-161, review talk at *Neutrino-80 Conference, 1980.*

$\mathcal{N}$-Δ, Λ-Σ, etc. come out about right, there is an underbinding of about 113 MeV for the nucleon and somewhat less for the heavier baryons. An error estimate of the FR approximation in the exactly solvable harmonic model indicated ([22]) an inaccuracy of about 20 MeV. For the more rapidly changing potential under consideration, this error may be somewhat more. Even then, there does appear to be some underbinding of the lighter baryons with the prescription that we have used. Very similar results for the baryon masses are obtained when potential (5) is used instead of the Coulomb-plus-ramp form, except that in this case the underbinding of the nucleon is considerably less—only 77 MeV. For this Martin-type potential, the calculated mass for $\Omega(1672)$ comes at 1684 MeV, which is remarkable. RICHARD has already noted this ([17]), but he did not consider the other baryons. Our results for the baryons are rather similar to those obtained by STANLEY and ROBSON ([12]), who also found underbinding in the lighter baryons. A variety of effects may contribute to this underbinding. Mass-dependent corrections may arise, for example, from the neglected momentum-dependent terms in the one-gluon exchange potential ([12]), or through some three-body force, amongst other possibilities. We get good agreement with experimental masses of baryons by adding an attractive three-body contribution of the form $C/(m_1 m_2 m_3)^{\frac{1}{3}}$, where m_i's are the masses of the three quarks. This form is quite *ad hoc* and C is chosen to adjust the mass of the nucleon to 939 MeV; $C/(m_u)^{\frac{2}{3}} = -113$ MeV for potential (4) and -77 MeV for (5). The baryon masses shifted in this way are shown in square brackets in table III. Only the results for the Coulomb-plus-ramp potential (4), which had the best meson spectra, are displayed. The quality of fit with the other radial forms is quite comparable, except that the underbinding is less. In all cases, we find the r.m.s. radius of the nucleon to be too small, about 0.5 fm. Since the FR method in its present form is not applicable for nonzero-angular-momentum states, we cannot examine the rich baryon spectra. In order to assert forcefully that the same potential is applicable to mesons as well as to baryons, one must examine also the baryon spectra. This is a non-trivial task and is being looked into.

4. – Discussion.

Motivated by the success of the nonrelativistic quark model, we set out to investigate whether a $q\bar{q}$ potential that fits charmonium can also describe the spectra of the lighter mesons. Relativistic effects should be very large, but we choose to ignore them, assuming tacitly that these may be incorporated in the effective masses of the quarks without altering the $q\bar{q}$ potential. In any case, one object of this work is only to examine, without theoretical justification, whether relativistic effects may be ignored. It appears that, if one takes a Coulomb-plus-ramp–type potential (4) together with a short-range spin-dependent term (2), it is possible to describe the main aspects of the meson spectra. Note, however, that one cannot reproduce the spin-splittings π-ρ *and* ψ-η_c with the same potential. We choose to ignore η_c because additional two-gluon contributions should be added ([12]) to describe it properly. Since the short-range part of the wave function should be sensitive to relativistic effects, which in turn determines the leptonic widths of the neutral vector mesons, these should be examined too. As emphasized by EICHTEN *et al.* ([13]), it is better to examine the ratios of the widths in the van Royen-Weisskopf formula to minimize theoretical uncertainties. For the charmonium $1S$ state, $\Gamma_{e^+e^-}(\psi)$ is experimentally known to be (4.8 ± 0.6) keV ([24]). With this input we obtain the leptonic widths of the other states from our nonrelativistic calculation. The numbers are in keV with the experimental numbers in brackets:

$$\psi(3685)\colon 2.2(2.1\pm0.3)\,, \qquad \Upsilon(9433)\colon 1.3(1.2\pm0.2)\,,$$

$$\Upsilon(9993)\colon 0.48(0.47\pm0.10)\,, \qquad \varphi(1020)\colon 1.3(1.31\pm0.15)\,,$$

$$\rho(776)\colon 3.3(6.54\pm0.9)\,.$$

The discrepancy is the largest for the lightest vector meson, but at least the order of magnitude is right.

One clue to the radial form of the confinement potential may be in the Regge trajectory. If one plots the angular momentum J *vs.* the experimental mass squared M^2 for $\rho(776)$, $A_3(1317)$ and $g(1700)$, which are known to have $J=1$, 2 and 3, respectively, one gets a linear Regge trajectory. A parallel straight line is obtained for K*(892), K*(1430) and K*(1780). For the potential (4) plus (2) that we are considering, the slopes of these lines are reproduced well with the $1S$, $1P$ and $1D$ energies, but states of higher L deviate significantly from the straight line, indicating that for the higher states the ramp confinement is too steep. With the Martin-type potential (4) or the logarithmic potential (6) with the parameters as fixed in table I the theoretical slopes are very different from the experimental one, as indeed is evident from our remarks earlier. In any case, the Regge trajectory shows that none of the potentials that we are considering here are realistic for high-angular-momentum states.

Another point that emerges from table I is that the strange-quark mass $m_s = 600$ MeV was needed to fit $\varphi(1020)$. Recall that m_u was fixed at 337 MeV

([24]) P. A. RAPIDIS, B. GOBBI, D. LÜKE, A. BARBARO-GALTIERI, J. M. DORFAN, R. ELY, G. J. FELDMAN, J. M. FELLER, A. FONG, G. HANSON, J. A. JAROS, B. P. KWAN, P. LECOMTE, A. M. LITKE, R. J. MADARAS, J. F. MARTIN, T. S. MAST, D. H. MILLER, S. I. PARKER, M. L. PERL, I. PERUZZI, M. PICCOLO, T. P. PUN, M. T. RONAN, R. R. ROSS, B. SADOULET, T. G. TRIPPE, V. VUILLEMIN and D. E. YOUNT: *Phys. Rev. Lett.*, **39**, 526 (1977).

from naive magnetic-moment consideration. In the same model, $\mu_\Lambda = -(m_u/3m_s)\mu_p$ and, to reproduce the experimental value of -0.61 nm, m_s should be 513 MeV. LIPKIN [6] has pointed out that the above value of $m_s(= 513 \text{ MeV})$ is compatible with the relation

$$M_\Lambda - M_{\mathcal{N}} = m_s - m_u\,, \tag{7}$$

which follows by noting that the spin-dependent term in the qq interaction does not contribute to the Λ-$\mathcal{N}$ mass difference and by further assuming that the nucleon and the lambda space wave functions are almost the same. The results of our three-body calculation show clearly that relationship (7) is not obeyed—the r.h.s. of the equation is 263 MeV, while the l.h.s. is 172 MeV or 144 MeV depending on whether three-body forces are included or not. This is because the asymmetry parameter η is very different for Λ and $\mathcal{N}$, indicating that the wave functions are very different for the two. To make the point forcefully, consider an exactly solvable harmonic model [25]

$$V_{ij} = \boldsymbol{F}_i\cdot\boldsymbol{F}_j\left[-\frac{\varkappa}{2}r_{ij}^2 - \frac{\lambda}{m_i m_j}\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j + C\right]. \tag{8}$$

Here $\boldsymbol{F}_i\cdot\boldsymbol{F}_j = -\frac{2}{3}$ for qq. The parameters are fixed by fitting the masses of $\mathcal{N}(939)$, $\mathcal{N}(1470)$, $\Delta(1232)$ and $\Lambda(1115)$ and insisting that $m_u = 336$ MeV. One then finds that $m_s = 574$ MeV, $\varkappa = 304.2 \text{ MeV fm}^{-2}$, $\lambda/m^2 = 73.2$ MeV and $C = 295$ MeV. It is easy to show that

$$M_\Lambda - M_{\mathcal{N}} = m_s - m_u + \frac{3}{2}\sqrt{\frac{2\varkappa}{m_u}}\left(\sqrt{\frac{2m_u + m_s}{3m_s}} - 1\right), \tag{9}$$

where the last term contributes about -60 MeV. With rapid spatial variations in the spin-dependent potential as in eqs. (2) and (3), the difference is even larger and m_s is in the vicinity of 600 MeV. This has the unpleasant consequence that the magnetic moment μ_Λ cannot be well reproduced in the naive model. This problem is generally overlooked in all nonrelativistic quark model calculations.

Despite these reservations, we may conclude by stating that the Coulomb-plus-ramp–type of interacion, together with short-range spin dependence, yields reasonable spectra for mesons and ground-state masses of baryons, light and heavy, if nonrelativistic dynamics is used.

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[25] D. A. LIBERMAN: *Phys. Rev. D*, **16**, 1542 (1977); and footnote [10] in ref. [22]. The baryon and vector-meson masses are well reproduced.

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This work was supported by the Natural Sciences and Engineering Research Council of Canada. This paper is dedicated to Professor Hajime NARUMI on the occasion of his sixtieth birthday.

APPENDIX

There are two methods that have been extensively used in recent years for solving three-body problems, the Faddeev equation and the hyperspherical expansion ([17,26]). The former as such is not suitable for the three-quark problem because there is no quark-quark scattering. The hyperspherical expansion method is certainly a powerful one, but it requires considerable computation. To our knowledge no hyperspherical harmonic calculation has been done for asymmetric three-quark systems such as the octet baryons. The Feshbach-Rubinow method which we have used in this work is much simpler and, as was discussed in sect. **3**, is reasonably accurate. Since the FR method does not seem to be well known, we summarize the main formulae so that the interested reader can reproduce our results easily.

Let us begin with the octet baryons whose wave functions are more complicated than the decuplet ones. For the spin and flavour part of the wave function we take the following:

$$\text{(A.1)}\qquad \begin{cases} p = \chi_s uud\,, & \Lambda = \chi_a \dfrac{1}{\sqrt{2}}(ud - du)s\,, \\ \Xi^0 = \chi_s ssu\,, & \Sigma^0 = \chi_s \dfrac{1}{\sqrt{2}}(ud + du)s\,, \end{cases}$$

where χ_s and χ_a are the spin functions defined by

$$\text{(A.2)}\qquad \chi_s = \frac{1}{\sqrt{6}}\left(|\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle - 2|\uparrow\uparrow\downarrow\rangle\right),$$

$$\text{(A.3)}\qquad \chi_a = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\rangle\right).$$

The indices 1, 2, 3 for the quarks are arranged such that $m_1 = m_2 (= m)$. With the above wave functions, the spin factors $\langle \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \rangle$ in V^{σ}_{qq} turn out to be

$$\text{(A.4)}\qquad \langle \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \rangle = \begin{cases} 1 & \text{for } \mathcal{N}, \Sigma, \Xi\,, \\ -3 & \text{for } \Lambda\,, \end{cases}$$

$$\text{(A.5)}\qquad \langle \boldsymbol{\sigma}_2 \cdot \boldsymbol{\sigma}_3 \rangle = \langle \boldsymbol{\sigma}_3 \cdot \boldsymbol{\sigma}_1 \rangle = \begin{cases} -2 & \text{for } \mathcal{N}, \Sigma, \Xi\,. \\ 0 & \text{for } \Lambda\,. \end{cases}$$

([26]) F. S. MORAES, H. T. COELHO and R. CHANDA: *Lett. Nuovo Cimento*, **26**, 466 (1979).

As was mentioned in sect. **3**, the spacial part of the wave function is assumed to be a function $\Phi(R)$ of a single variable $R = \frac{1}{2}(x + y + \eta z)$, where η takes care of the asymmetry due to the unequal masses and bonds. The expectation value of the three-body Hamiltonian can be reduced to ([19])

$$\langle\Phi|H|\Phi\rangle = \int_0^\infty \left\{\frac{\hbar^2}{4m_R}\left(\frac{\mathrm{d}\Phi}{\mathrm{d}R}\right)^2 + \sum_{i>j} W_{ij}(R)\,\Phi^2\right\} R^5\,\mathrm{d}R\,, \tag{A.6}$$

where

$$\frac{1}{m_R} = \left(\frac{1+\eta^2}{m} + \frac{1}{m_3}\right)2\xi + \left(\frac{2}{m} + \frac{1}{m_3}\right)\eta\zeta\,, \tag{A.7}$$

$$\xi = \frac{4(8+5\eta+\eta^2)}{15(1+\eta)^5}\,, \qquad \zeta = \frac{8(5+\eta)}{15(1+\eta)^5}\,. \tag{A.8}$$

The « effective potential » $W_{ij}(R)$ will be given later. We also obtain $\langle\Phi|\Phi\rangle = 2\xi\int_0^\infty \Phi^2 R^5\,\mathrm{d}R$. The wave function Φ is determined from $\delta\langle\Phi|H-E|\Phi\rangle = 0$, which leads to

$$-\frac{\hbar^2}{m_R}\left(\frac{\mathrm{d}^2}{\mathrm{d}R^2} - \frac{15}{4R^2}\right)u + 4\sum_{i>j} W_{ij}(R)\,u = 8\xi E u\,, \tag{A.9}$$

where $u(R) = R^{\frac{5}{2}}\Phi(R)$.

If we denote the potential in H by

$$V = V_{12}(z) + V_{23}(x) + V_{31}(y)\,, \tag{A.10}$$

we find $W_{ij}(R)$ to be

$$W_{12}(R) = \frac{1}{R^5}\int_0^{\gamma R} \mathrm{d}z\, V_{12}(z)\, z^2\left(2R^2 - 2Rz - \frac{1-3\eta^2}{6}z^2\right), \tag{A.11}$$

$$W_{23}(R) = W_{31}(R) = \frac{\gamma^3}{R^5}\left[\int_0^{\gamma R} \mathrm{d}x\, V_{23}(x)\, x^2\left(2R^2 - 2Rx + \frac{3-\eta}{6}x^2\right) - \right.$$
$$\left. - \frac{2\eta}{3(1-\eta)^3}\int_R^{\gamma R} \mathrm{d}x\, V_{23}(x)\, x(R-x)^2\{8R - (5+3\eta^2)x\}\right], \tag{A.12}$$

where $\gamma = 2/(1+\eta)$.

The potentials that we have considered in this work consist of terms of the form of r^n, $\ln r$ and $\exp[-r/r_0]/(r/r_0)$. Apart from the multiplying constant factors, $W_{ij}(R)$'s arising from these terms are given as follows.

For r^n

$$W_{12}=\frac{(n+4)(n+6)+3(n+5)\eta+3\eta^2}{3(n+3)(n+4)(n+5)}\gamma^{n+5}R^n\,, \tag{A.13}$$

$$W_{23}=\frac{8\{(n+2)(1-\eta)(\eta+\gamma^{n+3})+2\eta\gamma(1-\gamma^{n+2})\}}{(1-\eta)^3(n+2)(n+3)(n+4)(n+5)}\gamma^2R^n\,. \tag{A.14}$$

Note that, because of the scaling nature of r^n, the resulting W's are also of the form of R^n. W_{ij} for $\ln r^n$ is equal to $[\mathrm{d}(W_{ij}\text{ for }r^n)/\mathrm{d}n]_{n=0}$. This is because $\ln r=(\mathrm{d}r^n/\mathrm{d}n)_{n=0}$. For $\exp[-(r/r_0)]/(r/r_0)$

$$W_{12}=\frac{2}{\gamma^3\varrho^5}\left\{\exp[-\gamma\varrho]\left[-\frac{1}{6}\gamma^5\varrho^3+\frac{1}{2}\eta\gamma^4\varrho^2+(2-\eta^2\gamma^2)\gamma\varrho-\right.\right.$$
$$\left.\left.-2(1-\gamma)-\eta^2\gamma^2\right]+\gamma^2\varrho^2-2\eta\gamma^2\varrho+2(1-\gamma)+\eta^2\gamma^2\right\}, \tag{A.15}$$

$$W_{23}=\frac{2\gamma^2}{\varrho^5}\left\{(1-\gamma)(2\exp[-\varrho]-\varrho^2+2\varrho-2)+\right. \tag{A.16}$$
$$+(\varrho-1)(\exp[-\varrho]+\varrho-1)+\frac{1}{1-\gamma}\varrho(\varrho-1)\exp[-\gamma\varrho]-$$
$$\left.-\frac{1}{(1-\gamma)^2}[(2\varrho-1)\exp[-\gamma\varrho]+\exp[-\varrho]]+\frac{2}{(1-\gamma)^3}(\exp[-\gamma\varrho]-\exp[-\varrho])\right\},$$

where $\varrho=R/r_0$. The asymmetry parameter η is determined by minimizing E.

For the decuplet baryons, the spin functions are all taken to be $|\uparrow\uparrow\uparrow\rangle$. The spacial part of the wave function is symmetric and $\eta=1$, and hence $W_{12}=$ $=W_{23}=W_{31}$.

● RIASSUNTO (*)

Si esaminano alcuni potenziali non relativistici $q\bar{q}$, con diverse dipendenze radiali, che approssimano gli spettri bassi del charmonio. Con la familiare dipendenza di tipo Coulomb più rampa, lo stesso potenziale può riprodurre gli spettri dei mesoni leggeri mediante l'equazione di Schrödinger. Con metà della forza e l'aggiunta di un piccolo termine a tre corpi, si ottengono anche gli stati fondamentali dei barioni.

(*) *Traduzione a cura della Redazione.*

Единый потенциал для мезонов и барионов.

Резюме (*). — Мы исследуем несколько феноменологических нерелятивистских $q\bar{q}$ потенциалов с различными радиальными зависимостями, которые соответствуют низко-лежащим спектрам чармония. В случае обычного Кулона плюс зависимость типа наклонной плоскости, тот же потенциал может воспроизвести спектры легких мезонов, используя уравнение Шредингера. Используя вдвое меньший потенциал и добавляя малый трех-частичный член, получаются основные состояния барионов.

(*) *Переведено редакцией.*