PID Parameter Tables for the Control of standard Systems optimized with PSO

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***Abstract*—Optimized parameters for PID controllers are of significant value in current and future control engineering. In this publication, the particle swarm optimization (PSO) method is applied to minimize the integral of time weighted absolute error (ITAE) and integral of absolute error (IAE). For a large number of standard transfer functions, pre- calculated tables were generated that provide the optimized controller parameters. This publication provides tables for the control of time-delayed systems and overshooting second-order systems. Since additional parameters such as time constants, system damping, or controller output limits are also taken into account, these tables can be used to control a wide range of systems. Furthermore the correct application of the tables is demonstrated in an example.**

Keywords—PID Control, Particle Swarm Optimization, PSO, PTn Systems, Damped Systems, ITAE Criterion, IAE Criterion

I. INTRODUCTION

PID controllers are still state-of-the-art also in the most modern technical systems. In the past, the first optimizations were parameter tables that achieved stable closed-loop behavior, but they subsequently required further tuning to achieve a good transient response. [1-2]. However, in recent times, more and more optimization methods and algorithms from the fields of computer science and artificial intelligence have been applied. In particular, the particle swarm optimization PSO is often used as a solution approach for parameter optimization in control theory. Some very well-converging algorithms have been used for control with H∞ [3-4]. For the broad field of mechatronics, this was also used for the PID control of electric motors [5-7]. But also PID controllers for non-linear systems have been optimized with PSO [8]. Furthermore PSO was also used to optimize PID temperature controllers [9]. However, many methods related to this methods have also been used to optimize PID controllers [10-18].

In this publication, PSO is used to calculate parameter tables for the control of a large number of time-delayed systems (PTn) and weakly damped second-order systems. Chapter 2 presents the fundamentals and identification of the systems. Chapter 3 presents the application of the PSO algorithm to the calculation of the parameter tables. Chapter 4 contains the calculated parameter tables, which are the core of the publication. Chapter 5 shows an application of the parameter tables to a concrete example, and chapter 6 contains the discussion and outlook.

II. IDENTIFICATION OF THE SYSTEMS

The block diagram with the system and controller structure is shown in Fig. 1. The PID control parameters are also visible here. Later, the generally applicable parameter tables for Kp, Ti, and Td are calculated using PSO. The later discussed transfer functions of the systems to be controlled are shown on the right: PTn systems on the one hand and weakly damped second-order systems on the other hand. Subsequently, the PID parameters of these systems are optimized for step responses.

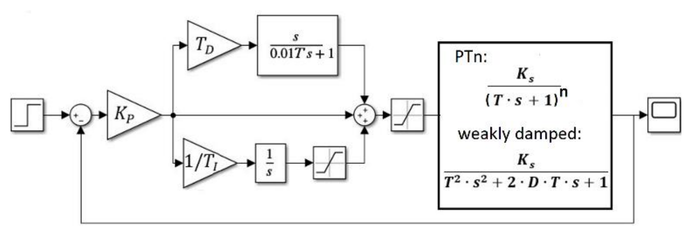

Fig. 1. PID controlled PTn and weakly damped 2nd order, Blockdiagram.

A. *Identification of the PTn System*

Time-delayed systems are very often controlled with PID controller structures. Therefore, they will be identified below. There are several ways to do this, one of which is the Schwarze method, which is described in many textbooks on control engineering [19]. Using this method, the measured step response is identified with a PTn system.

$$G(s) = K_s \cdot \frac{1}{(s\cdot T+1)^n} \tag{1}$$

Ks is the static gain. Using the measurement, the parameters t10, t50, and t90 can first be determined according to Fig. 2. First, an auxiliary parameter μ is calculated. From this, the order n and further parameters, α10, α50, and α90, can then be determined using Table 1. Finally, the new equivalent time constant T can be calculated.

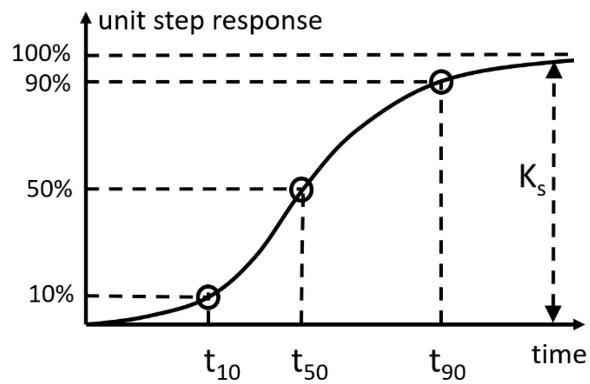

Fig. 2. System identification of a PTn System.

$$\mu = \frac{t_{10}}{t_{90}} \tag{2}$$

TABLE I. TABLE TYPE STYLES

μ	identification of order n and α- parameters			
	Order n	*α 10*	*α 50*	*α 90*
0.137	2	1.880	0.596	0.257
0.207	3	0.907	0.374	0.188
0.261	4	0.573	0.272	0.150
0.304	5	0.411	0.214	0.125
0.340	6	0.317	0.176	0.108

$$T = \frac{1}{3} \cdot (\propto_{10}\, t_{10} + \propto_{50}\, t_{50} + \propto_{90}\, t_{90}) \tag{3}$$

B. Identification of the weakly damped overshoot system

A second example of a common system that can be controlled with PID controllers, are weakly damped second-order systems. These can also be quite easily identified from step responses. The transfer function is generally calculated according to (4).

$$G(s) = \frac{K_S}{T^2 \cdot s^2 + 2 \cdot D \cdot T \cdot s + 1} \tag{4}$$

Again, Ks is the static gain. The attenuation D and the time constant T are also required for identification.

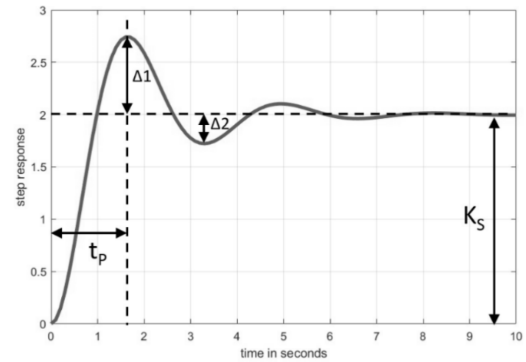

Fig. 3. Step response of a weakly damped 2nd order system.

The two values tp and the overshoot Δ1 are found from the step response. Δ1 is relevant in relation to the static gain Ks. From the graph in Fig. 3, one can find: Ks = 2 and Δ1/Ks = 0.36. Using this, the attenuation D can be found using (5), [19].

$$D = -\frac{\ln(\Delta 1/Ks)}{\sqrt{\pi^2 + (\ln(\Delta 1/Ks))^2}} \tag{5}$$

The resulting attenuation D is 0.31. Next, the time constant T must be identified. This is calculated using (6). The value tp is read off in Fig. 2 as 1.6s. Thus, T is calculated to be approximately 0.5s.

$$T = \frac{t_p}{\pi} \tag{6}$$

III. PARTICLE SWARM OPTIMIZATION FOR THE CALCULATION OF PID PARAMETERS

The core of this publication is the provision of precalculated tables for optimal PID control of standard transfer functions (1) and (4) according to a quality criterion. Therefore, integral of time weighted absolute error criterion (ITAE) and the integral of absolute error criterion (IAE) are applied here. These are criteria based on a unit-step response in the time domain.

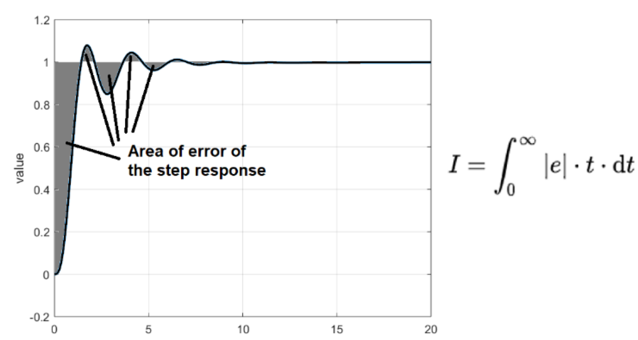

Fig. 4. Step response of a weakly damped 2nd order system.

ITAE and IAE are fundamentally similar; the only difference is the time weighting 't' in the integral, which is missing in IAE.

Thus, in ITAE, the error areas that occur later have a greater influence on the criterion. Both criteria are established standards for evaluating controlled systems in the time domain.

The tables to be calculated should be generally valid. Therefore, the parameters T, Ks, the order n, and the attenuation D must also be part of them. In principle, one could simply calculate the criteria in the entire parameter space with Kp, Ti, Td, and also various controller output limitations. The parameters of the smallest ITAE or IAE are then assumed to be optimal. This leads to a complexity of O(n^4). In principle, it's possible to calculate this, but depending on the desired accuracy, it requires a lot of computing power. For a specific application, it is important to perform the calculations as quickly as possible and on local computers. Therefore, particle swarm optimization (PSO) is used here. [20-22]. This is a stochastic method that does not calculate the entire parameter space, but rather uses a scattering of particles to find the optimal parameters. In this example, a particle has the parameters Kp, Ti, and Td. This can thus be represented three-dimensionally, as shown in Fig. 5.

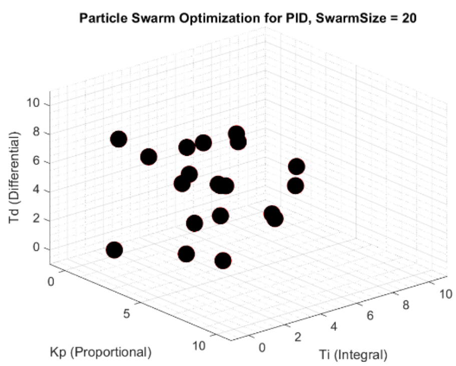

Fig. 5. PSO Particles, representing PID parameters, random distribution

Each particle moves from iteration step to iteration step with a velocity vector, which is composed of the current velocity, the spatial distance to the smallest ITAE or IAE criterion that the particle itself has ever reached (personal best) and the spatial distance to the smallest criterion that the best of all particles has ever reached (global best).

$$
\begin{aligned}
& v_i(t+1) = w \cdot v_i(t) && \text{actual velocity} \\
& +c_1 \cdot r_1 \cdot \left(P_{best,i}(t) - P_i(t)\right) && \text{cognitive learning factor} \\
& +c_2 \cdot r_2 \cdot \left(G_{best}(t) - P_i(t)\right) && \text{social learning factor}
\end{aligned} \tag{7}
$$

Thus, all particles jointly search for the best solution and also strive for a common best point. There are also weighting parameters, namely constants w, c1, c2 (example 0.5, 2, 2), as well as random values r1, r2, [0…1]. It turns out that this method for calculating the parameters for PID controllers converges very quickly and is also relatively robust against parameter changes. Fig. 6 shows the situation of a parameter calculation after 28 or 75 iterations.

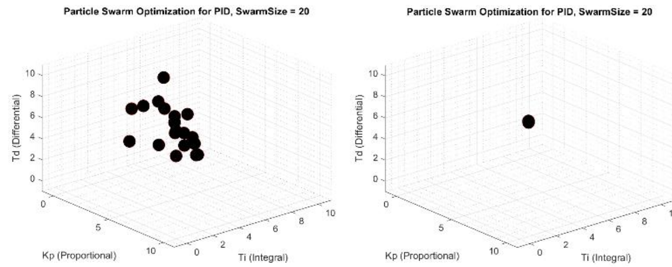

Fig. 6. PSO Particles, representing PID parameters after 28 and 75 iterations.

In its actual implementation, PSO also has also some disadvantages. For example, it doesn't always find the global optimum, but often only a local one. Furthermore, it doesn't always converge at the same speed. However, it is quite possible to improve this with suitable methods by performing multiple calculations with randomized start particles and then taking only the smallest results. In practice, it has been shown that PSO finds the optimal parameters significantly faster than other calculations across the entire parameter space. This makes the calculations much more efficient and feasible on local desktop computers or laptops.

IV. The Pre-calcuated PID Parameter Tables for the Control of Standard Systems

This chapter summarizes the results. The parameters were calculated using the PSO method, but a cross-comparison was also performed with the application of other methods [17]. Large parameter values are limited here by 10. Since the controller outputs of real control loops are always limited, the parameters were calculated for typical control loop limits of +/-2, +/-3, +/-5, and +/-10. The definition of the controller output limitation is described in the tables.

A. *Parameter tables for control of time delayed systems*

The following tables II and III contain the precalculated parameters for IAE and ITAE criterions for the time-delayed systems, PTn (1). As the static gain, Ks, and the time constant T are part of the table values, it can be used to find the optimal PID parameters for the control of a large number of those standard systems.

TABLE II. **IAE Criterion**

System order n	**(control signal limitation – control signal before the step) divided by (control signal for stationary end value – control signal before step)**			
	+/- 2	+/- 3	+/- 5	+/- 10
1	Kp·Ks=10 Ti = 3.1·T Td = 0	Kp·Ks=10 Ti = 2·T Td = 0	Kp·Ks= 10 Ti = 1.3·T Td = 0	Kp·Ks = 10 Ti = 1·T Td = 0
2	Kp·Ks = 10 Ti = 9.6·T Td = 0.3·T	Kp·Ks = 10 Ti = 7.3·T Td = 0.3·T	Kp·Ks = 10 Ti = 5.6·T Td = 0.3·T	Kp·Ks = 10 Ti = 3.7·T Td = 0.2·T
3	Kp·Ks= 5.4 Ti = 9.4·T Td = 0.7·T	Kp·Ks= 7 Ti = 10·T Td = 0.7·T	Kp·Ks= 8.4 Ti = 9.8·T Td = 0.7·T	Kp·Ks = 10 Ti = 9.7·T Td = 0.7·T

System order n	(control signal limitation – control signal before the step) divided by (control signal for stationary end value – control signal before step)			
	+/- 2	+/- 3	+/- 5	+/- 10
4	Kp·Ks = 2 Ti = 5.2·T Td = 1.1·T	Kp·Ks = 2.9 Ti = 6.5·T Td = 1.2·T	Kp·Ks= 3.3 Ti = 7.1·T Td = 1.3·T	Kp·Ks= 3.3 Ti = 6.9·T Td = 1.3·T
5	Kp·Ks= 1.7 Ti = 5.8·T Td = 1.6·T	Kp·Ks =1.8 Ti = 5.9·T Td = 1.6·T	Kp·Ks= 1.8 Ti = 5.8·T Td = 1.6·T	Kp·Ks= 1.7 Ti = 5.5·T Td = 1.6·T
6	Kp·Ks= 1.3 Ti = 5.9·T Td = 1.9·T	Kp·Ks = 1.3 Ti = 5.8·T Td = 1.9·T	Kp·Ks= 1.3 Ti = 5.8·T Td = 1.9·T	Kp·Ks= 1.3 Ti = 5.6·T Td = 1.9·T

TABLE III. **ITAE CRITERION**

System order n	(control signal limitation – control signal before the step) divided by (control signal for stationary end value – control signal before step)			
	+/- 2	+/- 3	+/- 5	+/- 10
1	Kp·Ks=9.3 Ti = 2.9·T Td = 0	Kp·Ks=9.5 Ti = 1.9·T Td = 0	Kp·Ks= 9.1 Ti = 1.2·T Td = 0	Kp·Ks = 10 Ti = 1·T Td = 0
2	Kp·Ks = 10 Ti = 9.6·T Td = 0.3·T	Kp·Ks = 10 Ti = 7.3·T Td = 0.3·T	Kp·Ks= 9.6 Ti = 5.4·T Td = 0.3·T	Kp·Ks = 9.8 Ti = 4.7·T Td = 0.3·T
3	Kp·Ks= 5.4 Ti = 9.4·T Td = 0.7·T	Kp·Ks= 7 Ti = 10·T Td = 0.7·T	Kp·Ks= 8.2 Ti = 9.6·T Td = 0.7·T	Kp·Ks = 10 Ti = 9.7·T Td = 0.7·T
4	Kp·Ks= 1.9 Ti = 5·T Td = 1.1·T	Kp·Ks = 2.4 Ti = 5.9·T Td = 1.2·T	Kp·Ks= 2.3 Ti = 5.7·T Td = 1.2·T	Kp·Ks= 2.1 Ti = 5·T Td = 1.1·T
5	Kp·Ks= 1.4 Ti = 5.3·T Td = 1.4·T	Kp·Ks =1.4 Ti = 5.2·T Td = 1.4·T	Kp·Ks= 1.4 Ti = 5.2·T Td = 1.4·T	Kp·Ks= 1.4 Ti = 5·T Td = 1.4·T
6	Kp·Ks= 1.1 Ti = 5.5·T Td = 1.7·T	Kp·Ks = 1.1 Ti = 5.5·T Td = 1.7·T	Kp·Ks= 1.1 Ti = 5.4·T Td = 1.7·T	Kp·Ks= 1.1 Ti = 5.3·T Td = 1.7·T

B. Parameter tables for control of overshooting systems

The next table shows the controller parameters precalculated with PSO for an overshooting 2nd order system (4). The controller output limitations were also taken into account in the table.

TABLE IV. **ITAE CRITERION, OVERSHOOTING 2ND ORDER**

System damping D	(control signal limitation – control signal before the step) divided by (control signal for stationary end value – control signal before step)			
	+/- 2	+/- 3	+/- 5	+/- 10
1.0 Δ1/Ks = 0	Kp·Ks=10 Ti = 9.6·T Td = 0.3·T	Kp·Ks=10 Ti = 7.3·T Td = 0.3·T	Kp·Ks= 9.6 Ti = 5.4·T Td = 0.3·T	Kp·Ks = 9.8 Ti = 4.7·T Td = 0.3·T
0.7 Δ1/Ks = 0.05	Kp·Ks = 10 Ti = 8.6·T Td =0.35·T	Kp·Ks = 10 Ti = 6.8·T Td =0.35·T	Kp·Ks= 10 Ti = 5.4·T Td =0.35·T	Kp·Ks = 9.9 Ti = 4.6·T Td =0.35·T
0.6 Δ1/Ks = 0.10	Kp·Ks= 9.8 Ti = 8.3·T Td = 0.4·T	Kp·Ks= 10 Ti = 6.9·T Td = 0.4·T	Kp·Ks= 10 Ti = 5.2·T Td =0.35·T	Kp·Ks = 9.9 Ti = 4.9·T Td = 0.4·T
0.5 Δ1/Ks = 0.16	Kp·Ks= 9.9 Ti = 8.1·T Td = 0.4·T	Kp·Ks = 9.8 Ti = 6.5·T Td = 0.4·T	Kp·Ks= 9.8 Ti = 5.3·T Td = 0.4·T	Kp·Ks= 9.9 Ti = 4.7·T Td = 0.4·T
0.4 Δ1/Ks = 0.25	Kp·Ks= 9.7 Ti = 7.6·T Td = 0.4·T	Kp·Ks =10 Ti = 6.4·T Td = 0.4·T	Kp·Ks= 10 Ti = 5.2·T Td = 0.4·T	Kp·Ks= 9.9 Ti = 4.5·T Td = 0.4·T
0.3 Δ1/Ks = 0.37	Kp·Ks= 9.4 Ti = 7.3·T Td =0.45·T	Kp·Ks = 9.7 Ti = 6.3·T Td =0.45·T	Kp·Ks= 9.9 Ti = 5.4·T Td =0.45·T	Kp·Ks= 9.9 Ti = 4.8·T Td = 0.45·T
0.2 Δ1/Ks = 0.53	Kp·Ks= 9.7 Ti = 7.3·T Td =0.45·T	Kp·Ks = 9.9 Ti = 6.2·T Td =0.45·T	Kp·Ks= 9.9 Ti = 5.2·T Td =0.45·T	Kp·Ks= 9.9 Ti = 4.6·T Td =0.45·T
0.1 Δ1/Ks = 0.73	Kp·Ks= 9.9 Ti = 7.5·T Td = 0.5·T	Kp·Ks = 9.8 Ti = 6.3·T Td = 0.5·T	Kp·Ks = 10 Ti = 5.5·T Td = 0.5·T	Kp·Ks = 9.9 Ti = 4.9·T Td = 0.5·T
0 Δ1/Ks = 1	Kp·Ks= 10 Ti = 7.3·T Td = 0.5·T	Kp·Ks = 10 Ti = 6.2·T Td = 0.5·T	Kp·Ks= 10 Ti = 5.3·T Td = 0.5·T	Kp·Ks= 9.9 Ti = 4.7·T Td = 0.5·T

V. AN EXAMPLE FOR USING THE TABLE

These pre-calculated tables can be used to control the standard transfer functions, wich are identified after chapter 2. As an example, the step response of an overshooting system is measured according to Fig. 7.

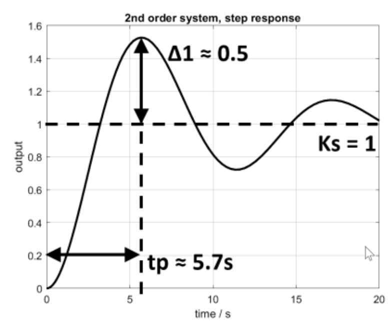

Fig. 7. Step response of a 2nd order system.

The identification of the parameters results in tp = 5.7s, Δ1 = about 0.5 and Ks = 1. Using (5) and (6), this results in D = 0.22 and T = 1.81 for the identified parameters of the transfer function in (4). This system will now to be controlled with pre-calculated PID- parameters of the minimum quality criterion ITAE according to the block diagram in Fig. 8.

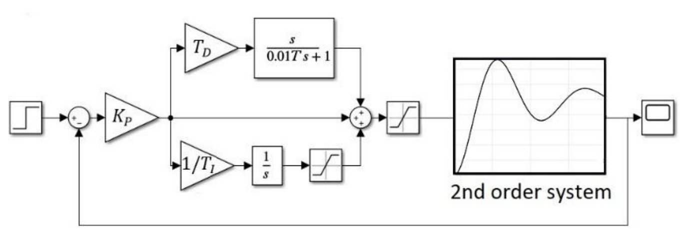

Fig. 8. Closed loop block diagram of the controlled 2nd order system.

Assuming that the controller output is limited to +/- 10 and one typically wants to implement steps of 1, this results in an output limitation of the controller of +/- 10. Taken from table (IV), this results with an attenuation D = 0.2: Kp·Ks = 9.9; Ti = 4.6·T; Td = 0.45·T.

Thus, Kp = 9.9; Ti = 8.33; and Td = 0.81. The corresponding graph for the step response of the closed-loop system realized with these parameters is shown in Fig. 9, also compared to the open-loop system.

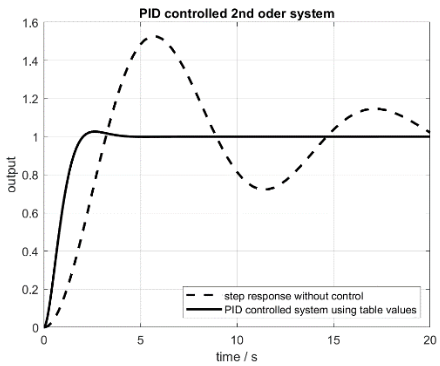

Fig. 9. Step responses of open loop and closed loop system.

VI. Discussion and further Improvements

The example in chapter 5 impressively demonstrates what an optimally tuned PID controller can achieve. Although the tables presented here are only suitable for controlling standard systems such as PTn and overshooting second-order systems, since parameters such as Ks, D, and T are also taken into account in the tables, they can be used to control a very large number of real systems without further calculations. The tables also contain the controller output limits, which are important in practice. The values provided have been calculated to approximately two significant digits. For the attenuation of the 2nd-order systems, the table values are in increments of 0.1. This initially seems rather coarse. However, many of the author's tests with real systems show that in most cases, the identification is not more accurate in any case. Thus, parameters accurate to more significant digits would only result in an apparent accuracy.

The PSO method is also fundamentally a very good combination in conjunction with the finding of parameters for PID controllers. Compared to other methods [17], the table values could be determined relatively quickly and with reasonable computational effort. On the one hand, the tables presented here cover a very wide range of possible controlled systems. On the other hand, it is desirable to be able to find suitable parameters for general systems with minimal computational effort. The idea here would be automated identification, even of unstable systems. For such systems, suitable controller parameters could also be found on local computers using the PSO algorithm. The next steps would be an algorithm that can automate this process as much as possible for any measured step response or even for unstable systems. It would be great, if this publication could make a valuable contribution to current and future control theory.

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