Numerical Simulation of Compressible Flows with Immersed Boundaries Using Discontinuous Galerkin Methods

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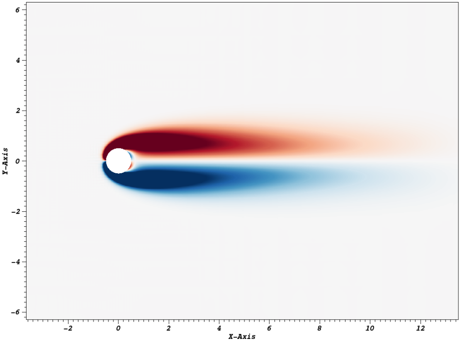

Outline

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Introduction

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Flow Properties

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- Compressible flow
- Ideal gas
  - Heat capacity ratio $\gamma = \dfrac{c_p}{c_v} = 1.4$
- Newtonian fluid
  - Stress tensor $\tau_{ij} = \mu \left[\left(\dfrac{\partial u_i}{\partial x_j} + \dfrac{\partial u_j}{\partial x_i} \right) - \dfrac{2}{3} \dfrac{\partial u_k}{\partial x_k} \delta_{ij} \right]$
- Mach number $\mathrm{Ma} = \dfrac{v_\infty}{a_\infty} = 0.2$
- Reynolds number $\mathrm{Re} = \dfrac{\rho_\infty V_\infty L}{\mu_\infty} \propto \dfrac{\text{inertia forces}}{\text{viscous forces}}$
- Prandtl number $\mathrm{Pr} = \dfrac{\mu_\infty c_p}{k_\infty} \propto \dfrac{\text{viscous diffusion rate}}{\text{thermal diffusion rate}}$

Non-dimensional 2D Compressible Navier-Stokes Equations

- 2D
- Non-dimensional conserved flow variables: density ρ, momentum ρu, ρv, energy ρE

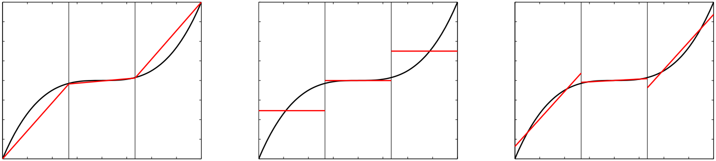
$$\boxed{\frac{\partial U}{\partial t}} + \boxed{\left(\frac{\partial F_c^x(U)}{\partial x} + \frac{\partial F_c^y(U)}{\partial y}\right)} - \boxed{\left(\frac{\partial F_v^x(U,\nabla U)}{\partial x} + \frac{\partial F_v^y(U,\nabla U)}{\partial y}\right)} = 0$$

$$U = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho E \end{pmatrix} \quad F_c^x = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho u v \\ u(\rho E + p) \end{pmatrix} \quad F_c^y = \begin{pmatrix} \rho v \\ \rho u v \\ \rho v^2 + p \\ v(\rho E + p) \end{pmatrix}$$

- Temporal derivative
- Convective fluxes
- Viscous fluxes

$$F_v^x = \frac{1}{\mathrm{Re}} \begin{pmatrix} 0 \\ \tau_{xx} \\ \tau_{xy} \\ \tau_{xx} u + \tau_{xy} v + \frac{\gamma}{\mathrm{Pr}(\gamma - 1)} \kappa \frac{\partial T}{\partial x} \end{pmatrix} \quad F_v^y = \frac{1}{\mathrm{Re}} \begin{pmatrix} 0 \\ \tau_{xy} \\ \tau_{yy} \\ \tau_{xy} u + \tau_{yy} v + \frac{\gamma}{\mathrm{Pr}(\gamma - 1)} \kappa \frac{\partial T}{\partial y} \end{pmatrix}$$

The Discontinuous Galerkin Space Discretisation



(a) First order FEM (b) Zeroth order DG (FVM) (c) First order DG

Abbildung: Comparison of FEM, FVM and DG

DG space discretisation Vorgehen, Bildchen, fluxes

The Immersed Boundary Method

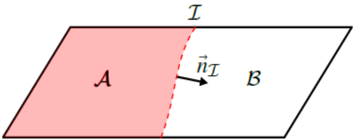
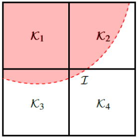


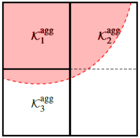
Abbildung: Cut cell with physical (red) and void region (white) [4]

regions mit Bild, Aufteilung Integrale mass matrix rk time discretisation formel cell agglomeration

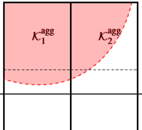
Cell Agglomeration



(a) Initial mesh partitioning



(b) Cell agglomeration with small agglomeration threshold



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Problem Specification

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- Supersonic inlet
- Adiabatic slip wall
- Flow properties:
  - Symmetrical flow
  - Isentropic inviscid flow with $\frac{p}{\rho^\gamma} = \text{const}$
    ➔ Entropy $s = 0$

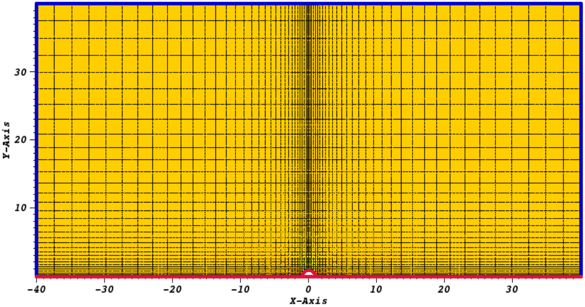

Abbildung: Mesh with 64 Cells per Direction

- Domain:
  - $0 \leq y \leq 40$
  - $-40 \leq x \leq 40$
  - Cylinder with radius $r = 1$ at $(0, 0)$
    ➔ Level set $\varphi = x^2 + y^2 - 1$ set as adiabatic slip wall

Robustness Study – Preparation

shift, degree 1 bis 3, agglo 0.5, 64 mal 64 cells Parameter, was wird getan

Robustness Study – Evaluation

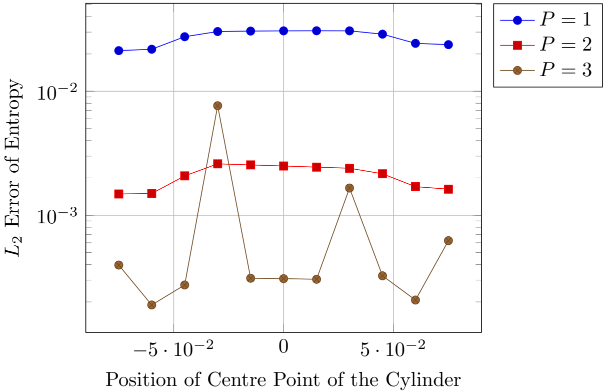

Abbildung: Convergence Plot

Ergebnisse, Plot, komischer punkt wird angeschaut

Comvergence Study – Preparation

Parameter, was wird getan

Convergence Study – Evaluation

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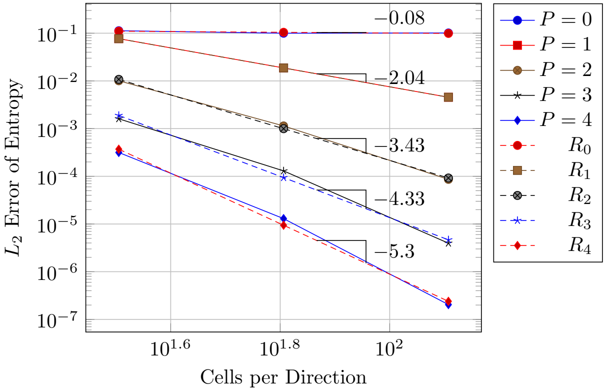

Abbildung: Convergence Plot

Ergebnisse, Plot

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Theory – Differentiation into Flow Regimes

- $40 - 50 < \mathrm{Re} < 190$: laminar vortex shedding,
- $190 < \mathrm{Re} < 260$: 3D wake-transition regime,

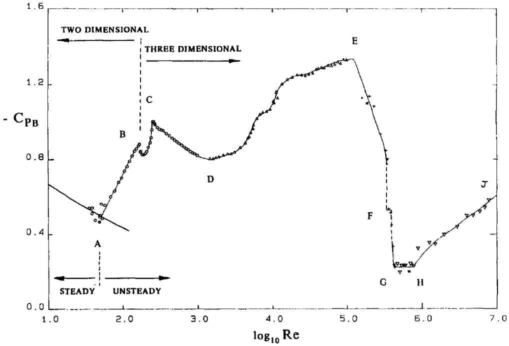


Abbildung: Base Suction Coefficient over Reynolds Numbers [4]

Theory – Laminar Steady Regime

laminar steady regime Bild

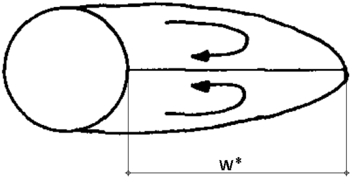


Abbildung: Recirculation Area [4]

Theory – Laminar Vortex Shedding

Bild

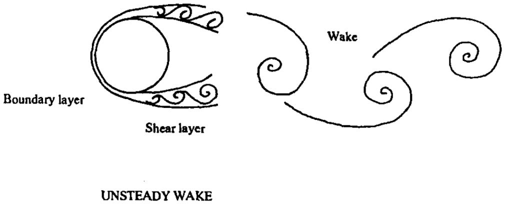


Abbildung: Kármán Vortex Street [4]

Karman vortex street frequency /strouhal

Simulation Properties

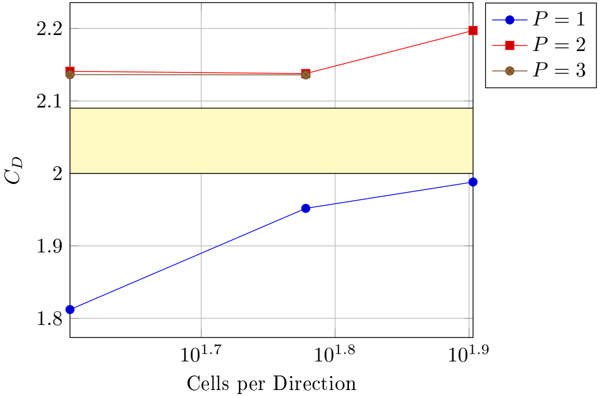
simulation parameter gitter cD, CL, W*, St

Simulation at Re = 20 I

Re = 20	Source	2D/3D	W^*	C_D
Numerical – Incompressible	Dennis and Chang	2D	0.94	2.05
	Fornberg	2D	0.91	2.00
	Linnick and Fasel	2D	0.93	2.06
Experimental	Coutanceau and Bouard	-	0.93	-
	Tritton	-	-	2.09
Numerical – Compressible	Brehm, Hader and Fasel (Ma = 0.1)	3D	0.96	2.02
	Ayers	2D	0.975	2.06
	Present Results:	**2D**	**0.928**	**2.136**

Simulation at Re = 20 II

- hier
- kommt
- beschreibung hin

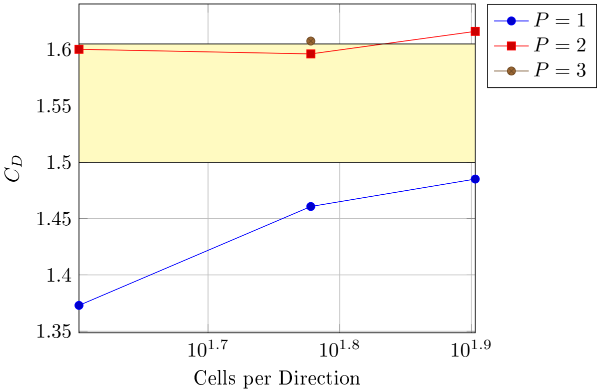


Simulation at Re = 40 I

Re = 40	Source	2D/3D	W^*	C_D
Numerical – Incompressible	Dennis and Chang	2D	2.35	1.52
	Fornberg	2D	2.24	1.50
	Linnick and Fasel	2D	2.28	1.54
Experimental	Coutanceau and Bouard	-	2.13	-
	Tritton	-	-	1.59
Numerical – Compressible	Brehm, Hader and Fasel (Ma = 0.1)	3D	2.26	1.51
	Ayers	2D	2.250	1.605
	Present Results:	**2D**	**2.201**	**1.608**

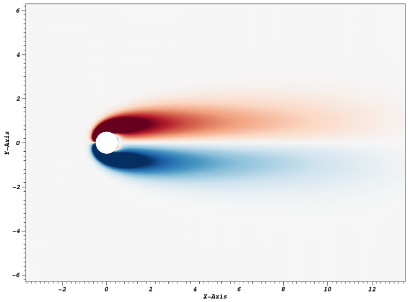
Simulation at Re = 40 II

- hier
- kommt
- beschreibung hin

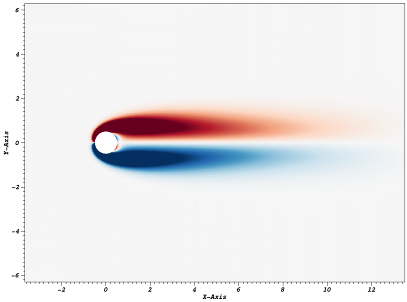


Comparison of Re = 20 and Re = 40

	W^*	C_D
Re = 20	0.928	2.136
Re = 40	2.201	1.608



(a) Re = 20



(b) Re = 40

Simulation at Re = 100 I

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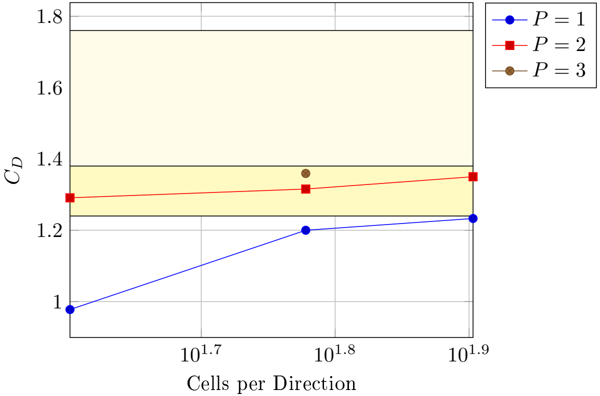
Re = 100	Source	2D/3D	St	C_D	C_L
Numerical – Incompressible	Gresho, Chan, Lee, et al.	2D	0.18	1.76	-
	Linnick and Fasel (λ = 0.056)	2D	0.169	1.38 ± 0.010	± 0.337
	Linnick ad Fasel (λ = 0.023)	2D	0.1696	1.34 ± 0.009	± 0.333
	Persillon and Braza	2D	0.165	1.253	-
	Saiki and Biringen	2D	0.171	1.26	-
	Persillon and Braza	3D	0.164	1.240	-
	Liu, Zheng and Sung	3D	0.165	1.35 ± 0.012	± 0.339
Experimental	Berger and Wille	-	$0.16 - 0.17$	-	-
	Clift, Grace and Weber	-	-	1.24	-
	Williamson	-	0.164	-	-
Numerical – Compressible	Brehm, Hader and Fasel (Ma = 0.1)	3D	0.165	1.32 ± 0.01	± 0.32
	Ayers	2D	0.167	1.371 ± 0.011	± 0.333
	Present Results:	**2D**	**0.1669**	$\mathbf{1.3593 \pm 0.00805}$	$\mathbf{\pm 0.3291}$

Simulation at Re = 100 II

- hier
- kommt
- beschreibung hin



Simulation at Re = 200 I

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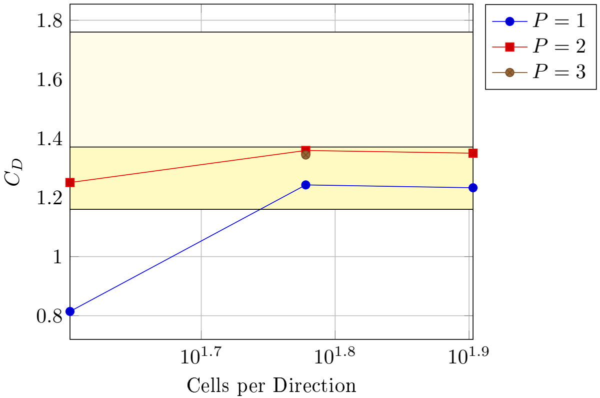
Re = 200	Source	2D/3D	St	C_D	C_L
Numerical – Incompressible	Belov, Martinelli and Jameson	2D	0.193	1.19 ± 0.042	± 0.64
	Gresho, Chan, Lee et al.	2D	0.21	1.76	-
	Linnick and Fasel (λ = 0.056)	2D	0.199	1.37 ± 0.046	± 0.70
	Linnick and Fasel (λ = 0.023)	2D	0.197	1.34 ± 0.044	± 0.69
	Miyake, Sakamoto, Tokunaga et al.	2D	0.196	1.34 ± 0.043	± 0.67
	Persillon and Braza	2D	0.198	1.321	-
	Saiki and Biringen	2D	0.197	1.18	-
	Persillon and Braza	3D	0.181	1.306	-
	Liu, Zheng and Sung	3D	0.192	1.31 ± 0.049	± 0.69
Experimental	Berger and Wille	-	$0.18 - 0.19$	-	-
	Clift, Grace and Weber	-	-	1.16	-
	Williamson	-	0.181	-	-
Numerical – Compressible	Brehm, Hader and Fasel (Ma = 0.1)	3D	0.192	1.3 ± 0.04	± 0.66
	Ayers	2D	0.201	1.371 ± 0.011	± 0.70
	Present Results:	**2D**	**0.2002**	$\mathbf{1.344 \pm 0.0462}$	$\mathbf{\pm 0.6887}$

Simulation at Re = 200 II

- hier
- kommt
- beschreibung hin



Simulation at Re = 200 III

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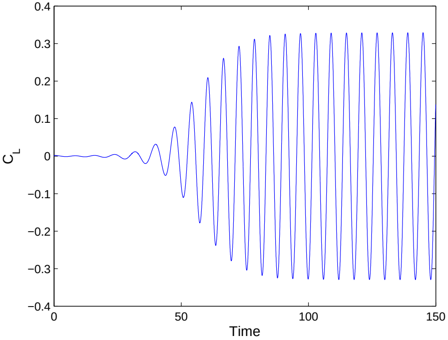
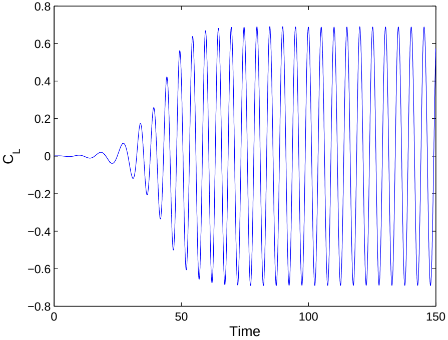
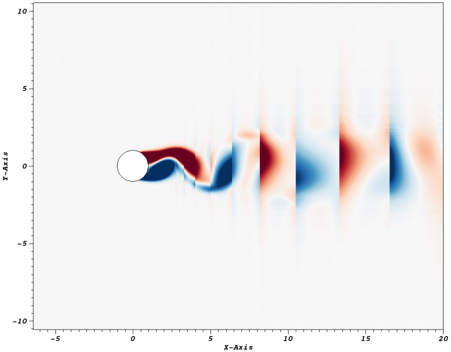
re 200 tabelle, plot, lift over time, vorticity

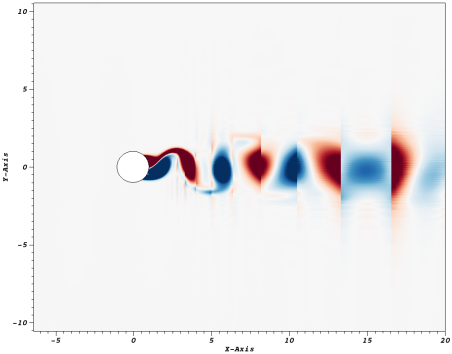
Comparison of Re = 100 and Re = 200 I





Comparison of Re = 100 and Re = 200 II





	St	C_D	C_L
Re = 100	0.1669	1.359 ± 0.00805	± 0.3291
Re = 200	0.2002	1.344 ± 0.0462	± 0.6887

Summary

conclusion

Outlook

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future works

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The End

ende, fragen

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[3] [Müller, 2016] B. Müller, S. Krämer-Eis, F. Kummer et al.
A high-order Discontinuous Galerkin method for compressible flows with immersed boundaries
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alle tabellen und graphen die man brauchen könnte in anhang