Numerical Simulation of Compressible Flows with Immersed Boundaries Using Discontinuous Galerkin Methods

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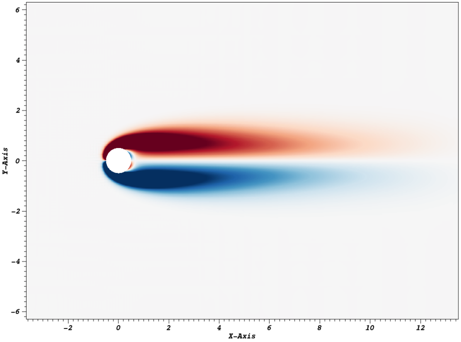

Outline

Introduction

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Flow Properties

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- Compressible flow
- Ideal gas
  - Heat capacity ratio $\gamma = \dfrac{c_p}{c_v} = 1.4$
- Newtonian fluid
  - Stress tensor $\tau_{ij} = \mu \left[\left(\dfrac{\partial u_i}{\partial x_j} + \dfrac{\partial u_j}{\partial x_i} \right) - \dfrac{2}{3} \dfrac{\partial u_k}{\partial x_k} \delta_{ij} \right]$
- Mach number $\mathrm{Ma} = \dfrac{v_\infty}{a_\infty} = 0.2$
- Reynolds number $\mathrm{Re} = \dfrac{\rho_\infty V_\infty L}{\mu_\infty} \propto \dfrac{\text{inertia forces}}{\text{viscous forces}}$
- Prandtl number $\mathrm{Pr} = \dfrac{\mu_\infty c_p}{k_\infty} \propto \dfrac{\text{viscous diffusion rate}}{\text{thermal diffusion rate}}$

Non-dimensional 2D Compressible Navier-Stokes Equations

- 2D
- Non-dimensional conserved flow variables: density ρ, momentum ρu, ρv, energy ρE

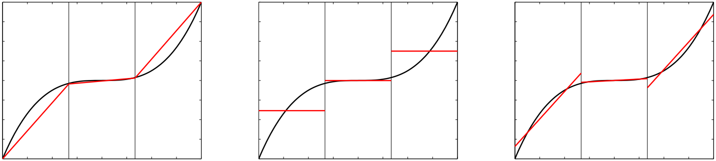
$$\boxed{\frac{\partial U}{\partial t}} + \boxed{\left(\frac{\partial F_c^x(U)}{\partial x} + \frac{\partial F_c^y(U)}{\partial y}\right)} - \boxed{\left(\frac{\partial F_v^x(U,\nabla U)}{\partial x} + \frac{\partial F_v^y(U,\nabla U)}{\partial y}\right)} = 0$$

$$U = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho E \end{pmatrix} \quad F_c^x = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho u v \\ u(\rho E + p) \end{pmatrix} \quad F_c^y = \begin{pmatrix} \rho v \\ \rho u v \\ \rho v^2 + p \\ v(\rho E + p) \end{pmatrix}$$

- Temporal derivative
- Convective fluxes
- Viscous fluxes

$$F_v^x = \frac{1}{\mathrm{Re}} \begin{pmatrix} 0 \\ \tau_{xx} \\ \tau_{xy} \\ \tau_{xx} u + \tau_{xy} v + \frac{\gamma}{\mathrm{Pr}(\gamma - 1)} \kappa \frac{\partial T}{\partial x} \end{pmatrix} \quad F_v^y = \frac{1}{\mathrm{Re}} \begin{pmatrix} 0 \\ \tau_{xy} \\ \tau_{yy} \\ \tau_{xy} u + \tau_{yy} v + \frac{\gamma}{\mathrm{Pr}(\gamma - 1)} \kappa \frac{\partial T}{\partial y} \end{pmatrix}$$

The Discontinuous Galerkin Space Discretisation



(a) First order FEM (b) Zeroth order DG (FVM) (c) First order DG

Figure: Comparison of FEM, FVM and DG

DG space discretisation Vorgehen, Bildchen, fluxes

The Immersed Boundary Method

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- Division into
  - the physical region: $\mathcal{A} = \{\vec{x} \in \Omega_h : \varphi(\vec{x}) > 0\}$,
  - the void region: $\mathcal{B} = \{\vec{x} \in \Omega_h : \varphi(\vec{x}) < 0\}$,
  - the immersed boundary: $\mathcal{I} = \{\vec{x} \in \Omega_h : \varphi(\vec{x}) = 0\}$

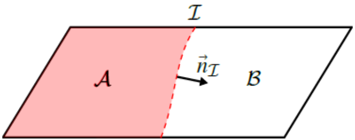

Figure: Cut cell with physical (red) and void region (white) [3]

- Restrict problem domain to physical region:

$$\int_{\mathcal{A}_i} \frac{\partial c_i}{\partial t} \Phi_{i,j} \, dV + \sum_{e=1}^{E_i} \int_{\mathcal{E}_{i,e}^{\mathcal{A}}} f\left(c^-, c^+, \mathbf{n}\right) \Phi_{i,j} \, dA + \int_{\mathcal{I}_i} f\left(c^-, c^+, \mathbf{n}_\mathcal{I}\right) \Phi_{i,j} \, dA - \int_{\mathcal{A}_i} \mathbf{f}\left(c_i\right) \cdot \nabla \Phi_{i,j} \, dV = 0$$

- Solution via explicit Euler time discretisation
  - Time step size depends on cell with smallest volume
    ➔ Cell agglomeration

Cell Agglomeration

- Agglomeration threshold $0 \leq \alpha \leq 1$
- Cells $\mathcal{K}_s^{\text{src}}$ with $\text{frac}(\mathcal{A}_i) = \frac{\text{meas}(\mathcal{A}_i)}{\text{meas}(\mathcal{K}_i)} \leq \alpha$ get agglomerated to neighbouring cell $\mathcal{K}_s^{\text{tar}}$
- Neighbouring cells are weakly coupled via fluxes
  ➔ basis $\vec{\Phi}_i$ can be extended from the target cell into the source cell

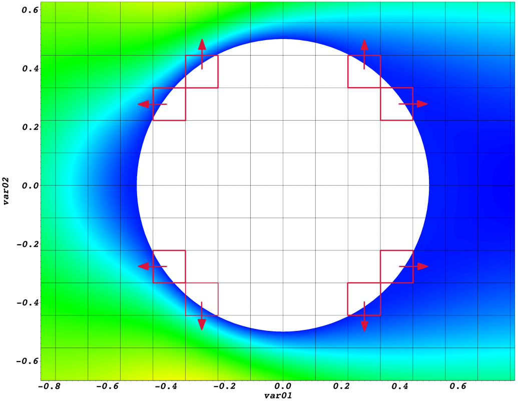

Figure: Cell agglomeration for α = 0.3

Problem Specification

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- Supersonic inlet
- Adiabatic slip wall
- Flow properties:
  - Symmetrical flow
  - Isentropic inviscid flow with $\frac{p}{\rho^{\gamma}} = \text{const}$
    ➔ Entropy $s = 0$

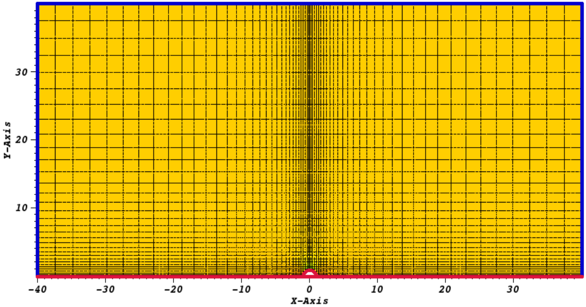

Figure: Mesh with 64 Cells per Direction

- Domain:
  - $0 \leq y \leq 40$
  - $-40 \leq x \leq 40$
  - Cylinder with radius $r = 1$ at $(0, 0)$
    ➔ Level set $\varphi = x^2 + y^2 - 1$ set as adiabatic slip wall

Robustness Study – Preparation

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- Variation of polynomial degree $1 \leq P \leq 3$
- Variation of position of the cylinder's centre point from -0.075 to $+0.075$ with step size 0.015
  ➔ Level set $\varphi = (x - \text{shift})^2 + y^2 - 1$
- Mesh as shown before (64 cells per direction)
- Constant agglomeration threshold $\alpha = 0.5$
  ➔ several different cell agglomerations occur

Aim: Proving the robustness of the solver

Robustness Study – Evaluation

- Constant error of entropy for degrees 1 and 2
  ➔ Only slight influence by cell agglomeration
- Large error differences for degree 3
  - For some cases huge effort in getting calculations to work (early breakup)
  - Discordant cases should be examined more closely

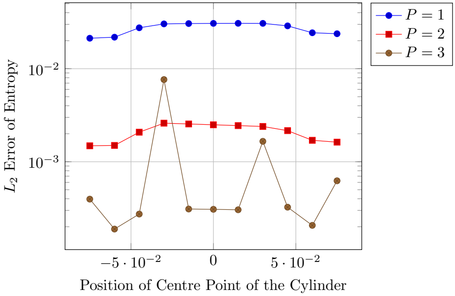

Figure: Convergence Plot

➔ Sufficiently robust for lower degrees, higher degree calculations strongly influenced by cell agglomeration

Comvergence Study – Preparation

- Variation of polynomial degree $0 \leq P \leq 4$
- Variation of mesh size: 32, 64 and 128 cells per direction
- Constant level set $\varphi = x^2 + y^2 - 1$
- Constant agglomeration threshold $\alpha = 0.3$

Aim: Proving convergence rate of $\mathcal{O}(h^{P+1})$

Convergence Study – Evaluation

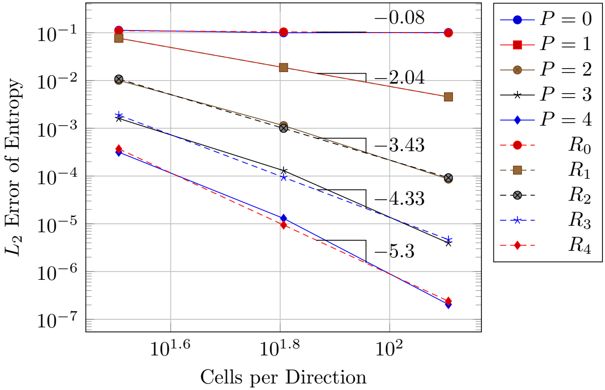

Figure: Convergence Plot

Ergebnisse, Plot

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Theory – Viscid Flow Around a Cylinder

- Re $\leq 40-50$: laminar steady regime
- $40-50 \leq$ Re ≤ 190: laminar vortex shedding (Kármán vortex street)
- $190 \leq$ Re: increasing 3D effects
- Characteristic values:
  - Coefficient of drag C_D
  - Coefficient of lift C_L
  - Wake separation length W^*
  - Strouhal number (frequency of vortex shedding)

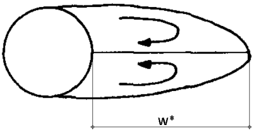

Figure: Laminar steady regime [4]

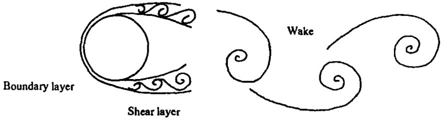

Figure: Kármán Vortex Street [4]

Simulation Properties

- Domain:
  - $-20 \leq x \leq 20$
  - $-20 \leq y \leq 20$
  - Cylinder with radius $r = 0.5$ at $(0, 0)$
    ➔ Level set $\varphi = x^2 + y^2 - 0.25$ set as isothermal wall
- Variation of mesh size: 40, 60 and 80 cells per direction
- Variation of polynomial degree $1 \leq P \leq 3$
- Constant agglomeration threshold $\alpha = 0.3$

➤ Supersonic inlet

➤ Isothermal wall

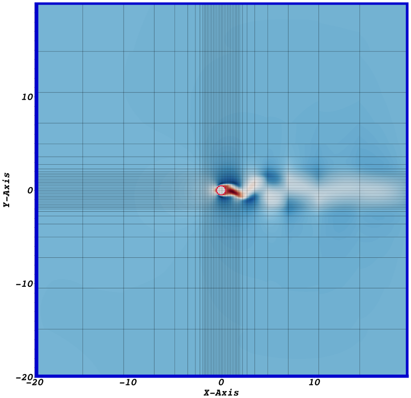

Figure: Coarsest mesh with 40×40 cells

Compared Values

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- Steady flow simulations for Re = 20 and Re = 40:
  - Coefficient of drag C_D
  - Wake separation length W^*

    ➔ found from evaluating x at $y = 0$ where x-velocity $u = 0$
- Unsteady flow simulations for Re = 100 and Re = 200:
  - Coefficient of drag C_D
  - Coefficient of lift C_L
  - St found from evaluating frequency of C_L

$$C_D = \frac{d}{q_\infty L_\infty}$$

$$C_L = \frac{l}{q_\infty L_\infty}$$

$$\mathrm{St} = \frac{f L_\infty}{V_\infty}$$

d : drag force

l : lift force

q_∞ : dynamic pressure

L_∞ : cylinder diameter

V_∞ : flow velocity

Simulation at Re = 20 I

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Re = 20	Source	2D/3D	W^*	C_D
Numerical – Incompressible	Dennis and Chang	2D	0.94	2.05
	Fornberg	2D	0.91	2.00
	Linnick and Fasel	2D	0.93	2.06
Experimental	Coutanceau and Bouard	-	0.93	-
	Tritton	-	-	2.09
Numerical – Compressible	Brehm, Hader and Fasel (Ma = 0.1)	3D	0.96	2.02
	Ayers	2D	0.975	2.06
	Present Results:	**2D**	**0.928**	**2.136**

Simulation at Re = 20 II

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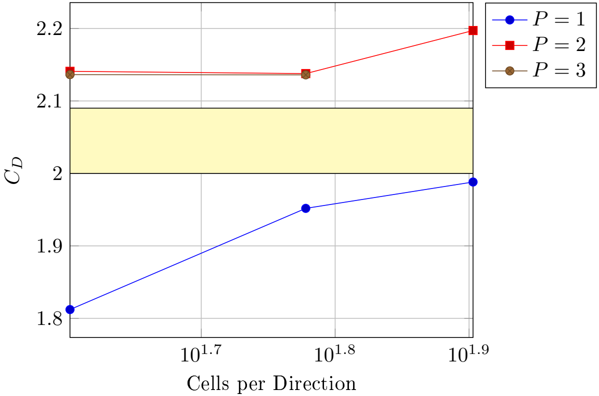
- beschreibung hin



Simulation at Re = 40 I

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Re = 40	Source	2D/3D	W^*	C_D
Numerical – Incompressible	Dennis and Chang	2D	2.35	1.52
	Fornberg	2D	2.24	1.50
	Linnick and Fasel	2D	2.28	1.54
Experimental	Coutanceau and Bouard	-	2.13	-
	Tritton	-	-	1.59
Numerical – Compressible	Brehm, Hader and Fasel (Ma = 0.1)	3D	2.26	1.51
	Ayers	2D	2.250	1.605
	Present Results:	**2D**	**2.201**	**1.608**

Simulation at Re = 40 II

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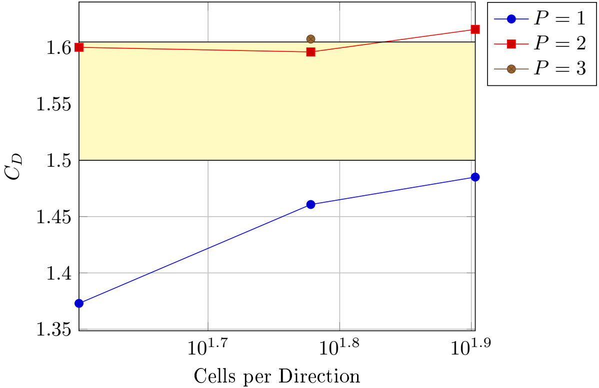
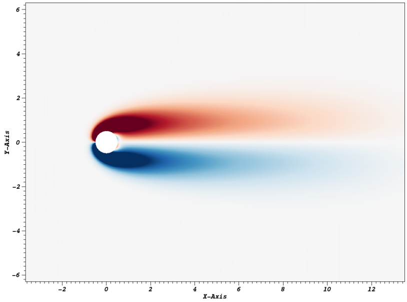
- beschreibung hin

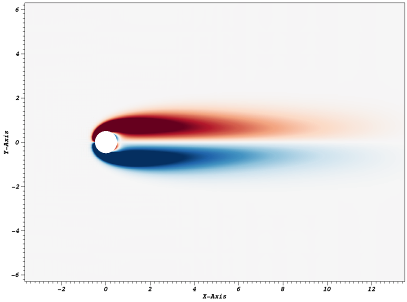


Comparison of Re = 20 and Re = 40

	W^*	C_D
Re = 20	0.928	2.136
Re = 40	2.201	1.608



(a) Re = 20



(b) Re = 40

Simulation at Re = 100 I

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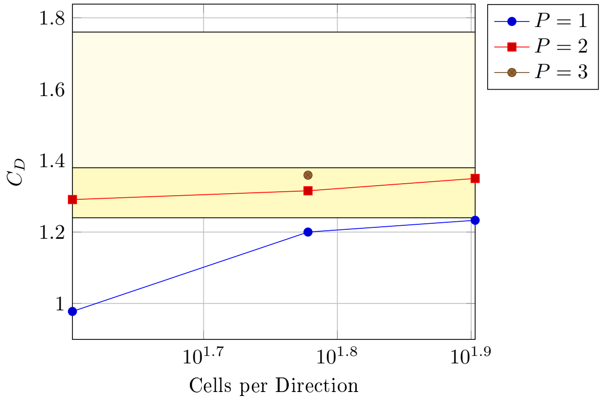
Re = 100	Source	2D/3D	St	C_D	C_L
Numerical – Incompressible	Gresho, Chan, Lee, et al.	2D	0.18	1.76	-
	Linnick and Fasel (λ = 0.056)	2D	0.169	1.38 ± 0.010	± 0.337
	Linnick ad Fasel (λ = 0.023)	2D	0.1696	1.34 ± 0.009	± 0.333
	Persillon and Braza	2D	0.165	1.253	-
	Saiki and Biringen	2D	0.171	1.26	-
	Persillon and Braza	3D	0.164	1.240	-
	Liu, Zheng and Sung	3D	0.165	1.35 ± 0.012	± 0.339
Experimental	Berger and Wille	-	$0.16 - 0.17$	-	-
	Clift, Grace and Weber	-	-	1.24	-
	Williamson	-	0.164	-	-
Numerical – Compressible	Brehm, Hader and Fasel (Ma = 0.1)	3D	0.165	1.32 ± 0.01	± 0.32
	Ayers	2D	0.167	1.371 ± 0.011	± 0.333
	Present Results:	**2D**	**0.1669**	$\mathbf{1.3593 \pm 0.00805}$	$\mathbf{\pm 0.3291}$

Simulation at Re = 100 II

- hier
- kommt
- beschreibung hin



Simulation at Re = 200 I

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Re = 200	Source	2D/3D	St	C_D	C_L
Numerical – Incompressible	Belov, Martinelli and Jameson	2D	0.193	1.19 ± 0.042	± 0.64
	Gresho, Chan, Lee et al.	2D	0.21	1.76	-
	Linnick and Fasel (λ = 0.056)	2D	0.199	1.37 ± 0.046	± 0.70
	Linnick and Fasel (λ = 0.023)	2D	0.197	1.34 ± 0.044	± 0.69
	Miyake, Sakamoto, Tokunaga et al.	2D	0.196	1.34 ± 0.043	± 0.67
	Persillon and Braza	2D	0.198	1.321	-
	Saiki and Biringen	2D	0.197	1.18	-
	Persillon and Braza	3D	0.181	1.306	-
	Liu, Zheng and Sung	3D	0.192	1.31 ± 0.049	± 0.69
Experimental	Berger and Wille	-	$0.18 - 0.19$	-	-
	Clift, Grace and Weber	-	-	1.16	-
	Williamson	-	0.181	-	-
Numerical – Compressible	Brehm, Hader and Fasel (Ma = 0.1)	3D	0.192	1.3 ± 0.04	± 0.66
	Ayers	2D	0.201	1.371 ± 0.011	± 0.70
	Present Results:	**2D**	**0.2002**	$\mathbf{1.344 \pm 0.0462}$	$\mathbf{\pm 0.6887}$

Simulation at Re = 200 II

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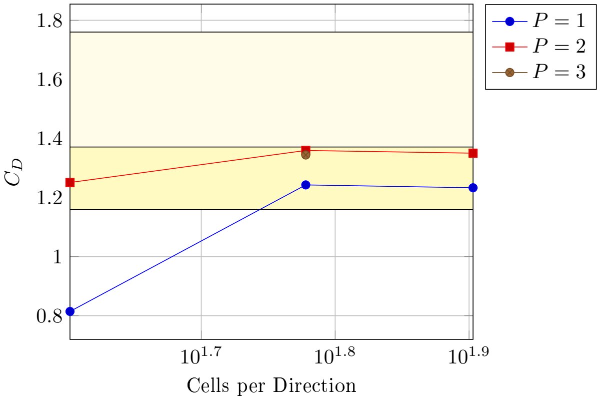

Simulation at Re = 200 III

re 200 tabelle, plot, lift over time, vorticity

Comparison of Re = 100 and Re = 200 I

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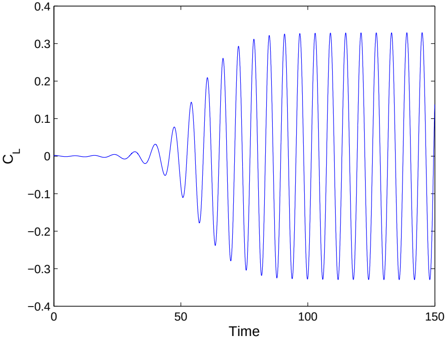

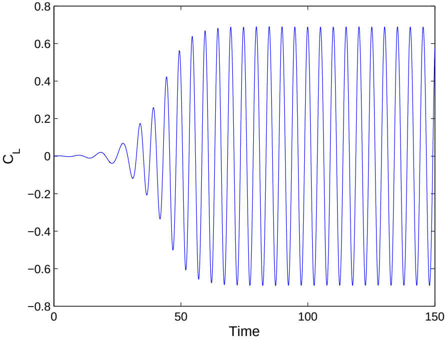

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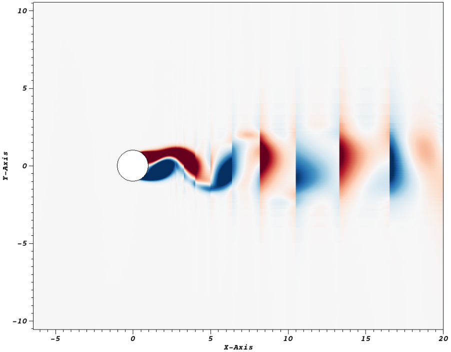
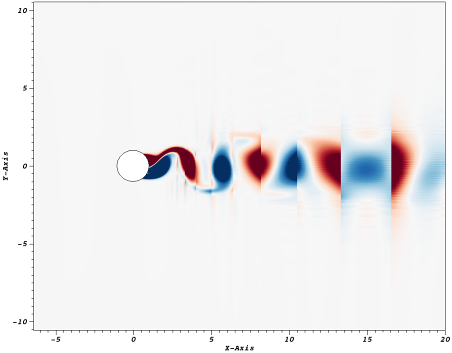
Comparison of Re = 100 and Re = 200 II





	St	C_D	C_L
Re = 100	0.1669	1.359 ± 0.00805	± 0.3291
Re = 200	0.2002	1.344 ± 0.0462	± 0.6887

Summary

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conclusion

Outlook

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future works

The End

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