CSC421 A3

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Q1.

Q1. (p|q|¬r) & ((¬r|q|p) -> ((r|q) & ¬q & ¬p))

Show unsatisfiable

1. (I)

(p|q|¬r) & (¬(¬r|q|p) | ((r|q) & ¬q & ¬p))

(Negation, distributive)

(p|q|¬r) & ((r & ¬q & ¬p) | ((r|q) & ¬q & ¬p))

(p|q|¬r) & ((r & ¬q & ¬p) | (r|q)) & ((r & ¬q & ¬p) | ¬q) & ((r & ¬q & ¬p) | ¬p)

① ② ③ ④

(Distributive, Operators)

① (p|q|¬r) &

② ((r|r|q) & (¬q|r|q) & (¬p|r|q)) &

③ ((r|¬q) & (¬q|¬q) & (¬p|¬q)) &

④ ((r|¬p) & (¬q|¬p) & (¬p|¬p))

Distributive, remove brackets

(p|q|¬r) & (r|r|q) & (¬q|r|q) & (¬p|r|q) & (r|¬q) & (¬q|¬q) & (¬p|¬q) & (r|¬p) & (¬q|¬p) & (¬p|¬p)

Clausal form

Derive empty clause using resolution

1.	$(p \mid q \mid \neg r)$	Premise
2.	$(r \mid r \mid q)$	"
3.	$(\neg q \mid r \mid q)$	"
4.	$(\neg p \mid r \mid q)$	"
5.	$(r \mid \neg q)$	"
6.	$(\neg q \mid \neg q)$	"
7.	$(\neg p \mid \neg q)$	"
8.	$(r \mid \neg p)$	"
9.	$(\neg q \mid \neg p)$	"
10.	$(\neg p \mid \neg p)$	"
11.	$\{p\}$	1, 5
12.	$\{\neg r, \neg q\}$	2, Negated conclusion, Idempotent law
13.	$\{\neg p\}$	4, 12
14.	$\{\}$	11, 13

Q2.

a) **First order logic:**

all x all y (Horse(x) & Dog(y) => Faster(x, y)).
exists y (Greyhound(y) & all z (Rabbit(z) => Faster(y, z))).
all y (Greyhound(y) => Dog(y)).
all x all y all z (Faster(x, y) & Faster(y, z) => Faster(x, z)).

Conclusion
all x all y (Horse(x) & Rabbit(y) -> Faster(x, y)).

b) **First order logic:**

All x (Hummingbird(x) => bird(x))
All x (Hummingbird(x) => richlyColored(x))
All x (livesOnHoney(x) => ~largeBird(x))
All x (richlyColored(x) =>livesOnHoney(x))

Conclusion:

All x (Hummingbird(x) => richlyColored(x)) AND
All x (richlyColored(x) =>livesOnHoney(x)) AND
All x (livesOnHoney(x) => ~largeBird(x))
By law of syllogism
All x (Hummingbird(x) => ~largeBird(x))

c) **First order logic:**

All x (myGardner(x) => worth_listening_military(x))
All x (remember_waterloo(x) => old(x))
All x (worth_listening_military(x) => remember_waterloo(x))

Conclusion:

All x (myGardner(x) => worth_listening_military(x)) AND
All x (worth_listening_military(x) => remember_waterloo(x)) AND
All x (remember_waterloo(x) => old(x))
By law of syllogism
All x (myGardner(x) => old(x))

Q3.

$$P(p_{13} \mid b_{12}, b_{21}):$$

$$= \alpha \sum_{p_{22}} \sum_{p_{31}} P_r(b_{12} \mid P_{22}, P_{13})\, P_r(b_{21} \mid P_{22}, P_{31})\, P_r(P_{13})\, P_r(P_{22})\, P_r(P_{31})$$

$$\begin{aligned}
= \alpha [\, & P_r(b_{12} \mid P_{13}, P_{22}) \cdot P(b_{21} \mid P_{31}, P_{22}) \cdot P(P_{13}) \cdot P(P_{22}) \cdot P(P_{31}) + \\
& P(b_{12} \mid P_{13}, \neg P_{22}) \cdot P(b_{21} \mid P_{31}, \neg P_{22}) \cdot P(P_{13}) \cdot P(\neg P_{22}) \cdot P(P_{31}) + \\
& P(b_{12} \mid P_{13}, P_{22}) \cdot P(b_{21} \mid \neg P_{31}, P_{22}) \cdot P(P_{13}) \cdot P(P_{22}) \cdot P(\neg P_{31}) + \\
& P(b_{12} \mid P_{13}, \neg P_{22}) \cdot P(b_{21} \mid, \neg P_{31}, \neg P_{22}) \cdot P(P_{13}) \cdot P(\neg P_{22}) \cdot P(\neg P_{31})\,]
\end{aligned}$$

$$\begin{aligned}
= \alpha [\, & 1 \cdot 1 \cdot 0.01 \cdot 0.01 \cdot 0.01 + \\
& 1 \cdot 1 \cdot 0.01 \cdot 0.99 \cdot 0.01 + \\
& 1 \cdot 1 \cdot 0.01 \cdot 0.01 \cdot 0.99 + \\
& 0 \\
]
\end{aligned}$$

$$\begin{aligned}
= \alpha [\, & 0.000001 + \\
& 0.000099 + \\
& 0.000099 + \\
& 0 \\
]
\end{aligned}$$

$$= 0.000199$$

$$P(P_{22} \mid b_{12}, b_{21}):$$

$$= \alpha \sum_{P_{13}} \sum_{P_{31}} P(b_{12} \mid P_{13}, P_{22})\, P(b_{21} \mid P_{22}, P_{31}) \cdot P(P_{13}) \cdot P(P_{22}) \cdot P(P_{31})$$

$$\begin{aligned}
= \alpha \big[& P(b_{12} \mid P_{13}, P_{22})\, P(b_{21} \mid P_{22}, P_{31}) \cdot P(P_{13}) \cdot P(P_{22}) \cdot P(P_{31}) + \\
& P(b_{12} \mid \neg P_{13}, P_{22}) \cdot P(b_{21} \mid P_{22}, P_{31}) \cdot P(\neg P_{13}) \cdot P(P_{22}) \cdot P(P_{31}) + \\
& P(b_{12} \mid P_{13}, P_{22}) \cdot P \cdot (b_{21} \mid P_{22}, \neg P_{31}) \cdot P(P_{13}) \cdot P(P_{22}) \cdot P(\neg P_{31}) + \\
& P(b_{12} \mid \neg P_{13}, P_{22}) \cdot P(b_{21} \mid P_{22}, \neg P_{31}) \cdot P(\neg P_{13}) \cdot P(P_{22}) \cdot P(\neg P_{31}) \\
\big] &
\end{aligned}$$

$$\begin{aligned}
= \alpha \big[& 1 \cdot 1 \cdot 0.01 \cdot 0.01 \cdot 0.01 + \\
& 1 \cdot 1 \cdot 0.99 \cdot 0.01 \cdot 0.01 + \\
& 1 \cdot 1 \cdot 0.01 \cdot 0.01 \cdot 0.99 + \\
& 1 \cdot 1 \cdot 0.99 \cdot 0.01 \cdot 0.99 \\
\big] &
\end{aligned}$$

$$\begin{aligned}
= \alpha \big[& 0.000001 + \\
& 0.000099 + \\
& 0.000099 + \\
& 0.009801 \\
\big] &
\end{aligned}$$

$$= \alpha\ 0.01$$

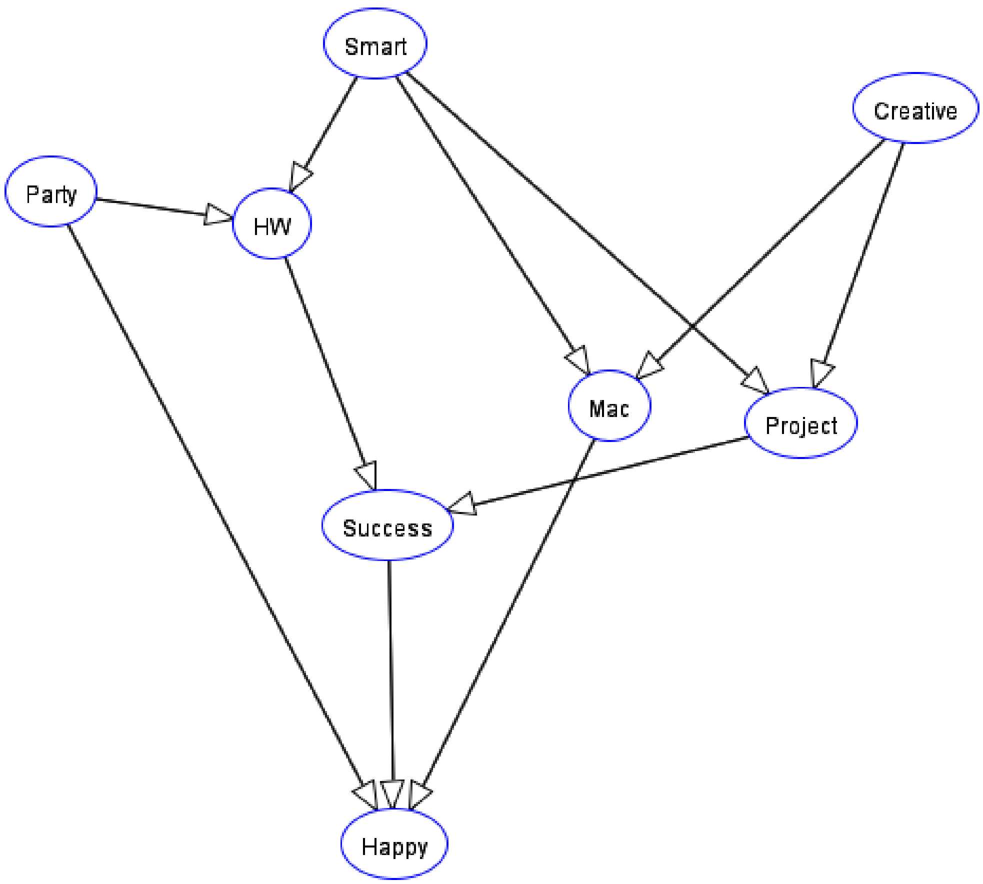
The probability of P(p13 | b12,b21) ends up to be quite low (a * 0.000199) whereas P(p22 | b12,b21) ends up quite higher (a * 0.01).

With a logical agent, all squares look the same, Therefore, the logical agent would choose either one with equal chance(⅓) of being chosen. This means the agent will die a third of the time.

With a probabilistic agent, it will never choose to go [2,2], it will always choose [1,3] or [3,1] based on probability.

Q4.

i) Draw the bayesian network



ii) Estimate the probabilities of the conditional probability tables using the data provided

P(HW | party, smart)

Party	Smart	cnt1	cnt2	p(hw |party, smart)
0	0	610	186	0.304918
0	1	1379	1239	0.898477
1	0	866	81	0.093533
1	1	2145	1722	0.802797

P(Mac | smart, creative)

Creative	Smart	cnt1	cnt2	P(Mac | creative, smart)
0	0	436	53	0.12156
0	1	1067	441	0.413308
1	0	1040	933	0.897115
1	1	2457	1685	0.685796

P(Project | smart, creative)

Creative	Smart	cnt1	cnt2	P(Project | creative, smart)
0	0	436	46	0.105505
0	1	1067	847	0.793814
1	0	1040	419	0.402885
1	1	2457	2224	0.905169

P (Success | HW, project)

HW	Project	cnt1	cnt2	p(Success | HW, project)
0	0	906	45	0.049669
0	1	866	179	0.206697
1	0	558	171	0.306452
1	1	2670	2394	0.896629

P(Happy | party, success, mac)

Party	Success	Mac	cnt1	cnt2	P(Happy | party, success, mac)
0	0	0	309	29	0.093851
0	0	1	483	99	0.204969
0	1	0	440	135	0.306818
0	1	1	757	271	0.357992
1	0	0	507	213	0.420118
1	0	1	912	449	0.492325
1	1	0	632	456	0.721519
1	1	1	960	921	0.959375

iii)

note* wouldn't let me enter past 5 decimal places for probability tables. IE answers might be slightly off.

3.

$p(\text{Happy} \mid \text{party, smart}, \neg \text{creative})$

$= a \sum_{Hw} \sum_{mac} \sum_{success} \sum_{project} p(Hw \mid p, s)\, p(Mac \mid s, \neg c)\, p(Proj \mid s, \neg c)$
$P(Success \mid Hw, project) \cdot P(Happy \mid p, Success, proj)\, P(p) \cdot P(s) \cdot P(\neg C)$

$= a \cdot (\ldots)$

$p(\neg \text{Happy} \mid p, s, \neg c)$

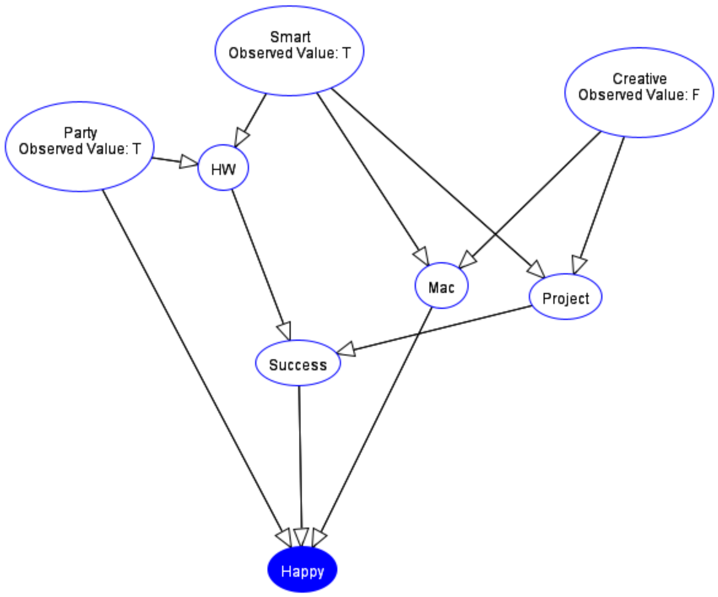
$= a \sum_{Hw} \sum_{mac} \sum_{success} \sum_{project} p(Hw \mid p, s)\, p(Mac \mid s, \neg c)\, p(Proj \mid s, \neg c)$
$P(success \mid Hw, proj) \cdot P(Happy \mid p, success, proj) \cdot P(p) \cdot P(s) \cdot P(\neg C)$

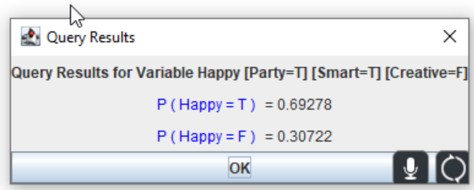
$= a \cdot (\ldots)$

$alpha = 1 / (P(h \mid p, s, \neg c) + P(\neg h \mid p, s, \neg c))$

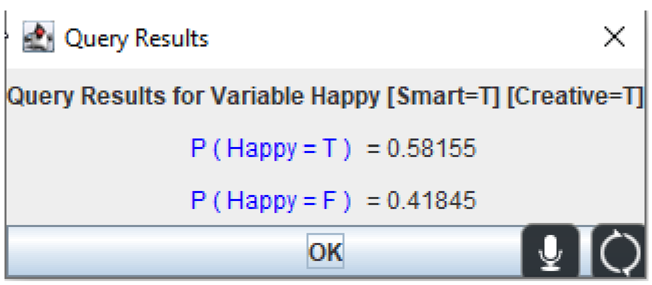
Using Aitool

$P(Happy \mid p, s, \neg c) = 0.69278$

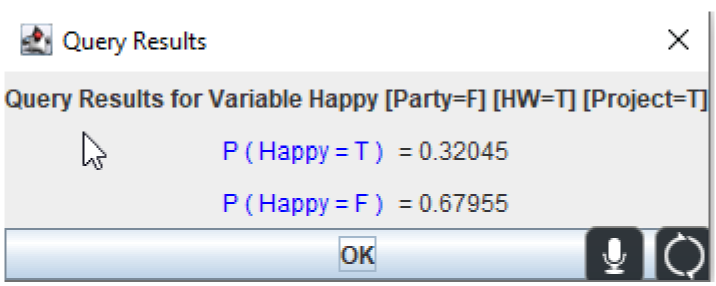




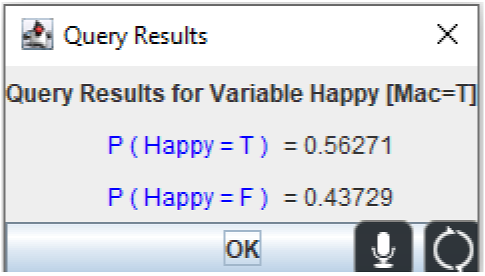
iv) Smart and Creative



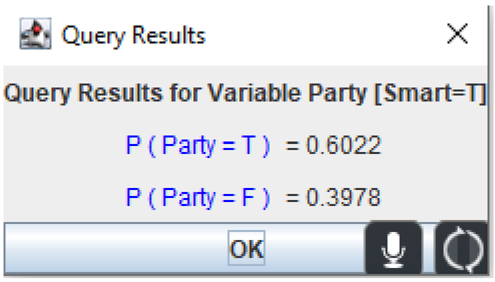
v) Doesn’t party, does his homework and project



vi) Own’s a mac



vii) Wicked smart



viii)