

# DiffTrace: Efficient Whole-Program Trace Analysis and Diffing

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**Abstract—** Abstract to be written

**Index Terms—** diffing, tracing, debugging

## I. INTRODUCTION

### BEGIN Ganesh

Debugging high performance computing code remains a challenge at all levels of scale. Conventional HPC debuggers [?], [?], [?] excel at many tasks such as examining the execution state of a complex simulation at a detailed level and allowing the developer to re-execute the program close to the point of failure. However, they do not provide a good understanding of why a program version that worked earlier failed upon upgrade or feature addition. Innovative solutions are needed to highlight the salient differences between two executions in a manner that makes debugging easier as well as more systematic. A recent study conducted under the auspices of the DOE [?] provides a comprehensive survey of existing debugging tools. It classifies them under *four* software organizations (serial, multithreaded, multi-process, and hybrid), *six* method types (formal methods, static analysis, dynamic analysis, nondeterminism control, anomaly detection, and parallel debugging), and lists a total of 30 specific tools. Despite this abundance of tools and approaches, many significant problems remain to be solved before debugging *can be approached by the HPC community as a collaborative activity* so that HPC developers can share their solutions and extend a common framework **Need more build-up; will do later.**

In this paper, we provide our fundamentally fresh look at debugging. We point out three significant problems that we have addressed in our work, and provide our preliminary solutions backed up by case studies. While our work has not (yet) addressed the situations in which millions of threads and thousands of processes run for days and produce an error, we strongly believe that we can get there only through a series of *rigorous* approaches that overcome key limitations found in conventional debugging approaches in a step-by-step manner, accompanied by careful measurements of the merits of the new approach. The main contribution of this paper is the first such critical measurements of our proposed approach.

a) *Problem-1: Need to Generalize Approaches for Outlier Detection.*: Almost all debugging approaches seek to find outliers (“unexpected executions”) amongst thousands of

running processes and threads. The approach taken by most existing tools is to look for symptoms in a specific bug-class that they cover. Unfortunately, this approach calls for a programmer having a good guess of what the underlying problem might be, and to then pick the right set of tools to deploy. If the guess is wrong, the programmer has no choice but to refine their guess and look for bugs in another class, re-executing the application and hoping for better luck with another tool. This iterative loop of re-execution followed by applying a best-guess tool for the suspected bug class can potentially consume large amounts of execution cycles and also waste an expert developer’s time.

*Solution to Problem-1: Whole Program Tracing for Debugging:* In this setting, our first contribution is a debugging approach in which the application is not merely run with a single symptom-specific tool attached as described earlier. Instead, we collect *whole program traces* of function calls and returns, using a PIN-based function call tracing facility called ParLoT that we have developed and previously reported [?]. We store these traces for potential examination by multiple tools and approaches. The advantage of whole program binary tracing supported in ParLoT is that we can collect function calls at *any desired level of abstraction*. For instance, if the programmer wants to cover activities at the MPI level, the OpenMP level and perhaps even lower levels (e.g., the MPI library or the OpenMP runtime), they can do so using ParLoT.

Clearly, the more APIs at which function calls are recorded, the more burdensome trace collection becomes. However, the advantage is that correspondingly more tools can then be applied to the collected traces. There is always a sweet-spot in this trade-off space, depending on the particular debugging situation. However, our fundamental insight is that *given the inevitability of heterogeneous programming* (the use of multiple concurrency models), it is important to be collecting traces from a few related APIs at a time, so that one can study bugs in one of the concurrency models or a bug resulting from a bad cross-model interaction from a *single run of the program*.

In our research, we have thus far demonstrated the advantage of ParLoT with respect to collecting both MPI and OpenMP traces from a *single run of a hybrid MPI/OpenMP program*. We demonstrate that from this single type of traces, it is possible to pick out MPI-level bugs or OpenMP-level bugs.

While we have not covered all these combinations in our work so far, the main contribution claimed is the ability to cover multiple APIs while debugging, and without re-executions or guess-work.

While this approach to whole-program tracing may sound extremely computation intensive, we employ novel on-the-fly compression techniques within ParLoT. In our previous study [?], we report compression efficiencies exceeding 16,000. This allows us to bring out the function call traces without significantly burdening the memory subsystem or I/O networks in the HPC cluster.

*b) Problem-2: Need to Generalize Approaches for Outlier Detection::* Given that outlier detection is central to debugging, it is important to be employing efficient representations of the traces collected from threads and processes so that one can compute *distances* between these traces more systematically, without involving human reasoning in the loop. The representation must also be versatile enough to be able to “Diff” the traces<sup>1</sup> with respect to *an extensible number of vantage points*. These vantage points could be diffing with respect to process level activities, diffing with respect to thread-level activities, a combination thereof, or even finite sequences of process/thread calls (say, to locate *changes* in caller/callee relationships).

*Solution to Problem-2: Use of Concept Lattices in Debugging:* In DiffTrace, we employ *concept lattices* to amalgamate the collected traces. Concept lattices have previously been employed in HPC to perform structural clustering of process behaviors [?] to present performance data more meaningfully to users. The authors of that paper employ the notion of *Jaccard distances* to cluster performance results that are closely related to process structures (determined based on caller/callee relationships).

In DiffTrace, we employ incremental algorithms for building and maintaining concept lattices from the ParLoT-collected traces. In addition to Jaccard distances, in our work we also perform hierarchical clustering of traces and provide a tunable threshold for outlier detection. We believe that these uses of concept lattices and more refinement approaches for outlier detection are new in HPC debugging.

*c) Problem-3: Loop Detection::* Most programs spend most of their time in loops. Therefore it is important to employ state-of-the-art algorithms for loop extraction from execution traces. It is also important to be able to diff two executions with respect to changes in their looping behaviors.

*Solution to Problem-3: Rigorous Approaches to Loop Analysis:* In DiffTrace, we employ the notion of NLRs (what does it stand for?) for extracting loops. Each repetitive loop structure is given an identifier, and nested loops are expressed as repetitions of this identifier exponentiated (as with regular expressions). This approach to summarizing loops can help manifest bugs where the program does not hang or crash, but nevertheless run differently in a manner that informs the developer engaged in debugging.

<sup>1</sup>Hence the name of our tool, **DiffTrace**.

To summarize, the key contributions of this paper are the following [[fix the section numbers later]]:

- A method to organize function call traces collected from processes and threads into concept lattices, and a method to detect loops from dynamic traces (Section ??).
- Details of the algorithms employed in DiffTrace (Section ??).
- Experimental studies on a heterogeneous program called Iterated Local Champion Search (ILCS, Section ??).
- Strengths and limitations of DiffTrace, plans for future work (Section ??).

## END Ganesh

[[Ganesh and Saeed have written some text before for the intro which is available in v0/intro.tex (also available but commented in current file). Current version is based on our discussion on May 8th]]

- Importance of whole program diffing : understand changes, debug (DOE REPORT [1])
- Efficient tracing supports selective monitoring at multiple levels
  - Bugs not there at a predictable API level
  - Prior work (ParLoT) supports whole program tr.
- Dissimilarity is important to know: bugs, changes during porting,...
- Key enablers of meaningful diffing:
  - Formal concepts (novel contrib to debugging)
  - Loop detection (loop diffing can help)
- Importance, given the growing heterogeneity

**\*\* TODO: Highlights of results obtained as a result of the above thinking should be here. This typically comes before ROADMAP of paper.**

In summary, this paper makes the following main contributions:

- A tunable tracing and trace-analysis tool-chain for HPC application program understanding and debugging
- A variation of the NLR algorithm to compress traces in lossless fashion for easier analysis and detecting (broken) loop structures
- An FCA-based clustering approach to efficiently classify traces with similar behavior
- A tunable ranking mechanism to highlight suspicious trace instances for deeper study
- A visualization framework that reflects the points of differences or divergence in a pair of sequences.

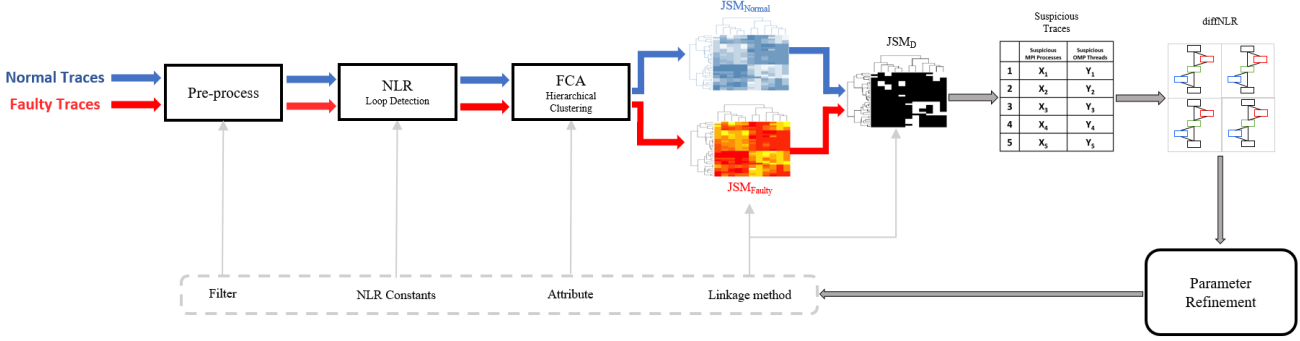
The rest of the paper is as follows:

- Sec 2: Background
- Sec 3: Components
- Sec 4: Case Study: ILCS
- Sec 5: Related Work
- Sec 6: Concluding Remarks

## II. DIFFTRACE OVERVIEW

The general idea is to utilize ParLoT [2] traces for studying HPC application behaviors towards fault detection and

Figure 1. DiffTrace Overview



localization. ParLOT collects whole-program function calls and returns via dynamic binary instrumentation [3], and incrementally compresses them on-the-fly. ParLOT instruments and encodes functions within the binary to unique integers at two major levels: *main image* only contains functions from the source-code and included APIs and *all images* that cover system and library functions as well. Upon termination of the application, ParLOT flushes out per-thread trace files each contains compressed sequence of executed function IDs. The compression mechanism of ParLOT significantly reduces the time, memory and disk overhead, leaves the majority of the system bandwidth for the application.

With the mindset of “pay a little upfront to dramatically reduce the number of overall debug iterations”, ParLOT well overcomes the challenge of whole-program *trace collection* and leaves the *trace analysis* for offline post-mortem analysis. However, raw ParLOT traces are just compressed sequences of executed function IDs per thread. Also, enabling ParLOT on top of large-scale HPC applications execution would result in thousands of often-long traces. The collected traces require to get prepared, grouped and transformed to meaningful representations that reveal facts about the dynamic behavior of the program.

In this paper, we introduce DiffTrace, a tool-chain (figure 1) that provides infrastructures for *iterative and configurable trace search space reduction towards localizing the fault* in a set of faulty traces. Considering a “successful” termination of the application as *normal behavior*, DiffTrace iteratively compares the faulty traces against normal traces to detect any *abnormal behavior*. Each abnormal behavior is a potential fault cause or manifestation. However, faults in HPC applications may occur or influence the program behavior at different locations and granularities, due to the often high and hybrid level of parallelism. Also, typical HPC applications spend most of their execution time in a main loop until convergence or over time-steps. A fault may get triggered at some point at the code and reflects itself right away, or after some progress from triggered point. Given a set of normal traces and faulty traces, DiffTrace feeds both sets into a sequence of data transformation and classification. Then suggests a few traces for deeper study (e.g., comparing actual traces from a specific vantage point) by *measuring and locating points of differences*.

Figure 2. Simplified MPI implementation of Odd/Even Sort

	Main Function	oddEvenSort()
1	int main(){	oddEvenSort(rank, cp){
2	int rank, cp;	...
3	MPI_Init();	for (int i=0; i < cp; i++)
4	MPI_Comm_rank(..., &rank);	{
5	MPI_Comm_size(..., &cp);	int ptr = findPtr(i, rank);
6	// Initialize data to sort	...
7	int *data[data_size];	if (rank % 2 == 0) {
8	...	MPI_Send(..., ptr, ...);
9	oddEvenSort(rank, cp);	MPI_Recv(..., ptr, ...);
10	...	} else {
11	MPI_Finalize();	MPI_Recv(..., ptr, ...);
12	}	MPI_Send(..., ptr, ...);
13		}
14		...
15		}
16		}

Since each DiffTrace components have multiple parameters and constants, evidences about what went wrong that made the program fails should get obtained *iteratively*. In each iteration, by picking semantically well-tuned parameters, DiffTrace can put a flash-light on a specific aspect of the program dynamic behavior.

The work-flow of DiffTrace starts with *pre-processing* traces. ParLOT traces are highly compressed and often contain functions that might not be interesting from a certain point of analysis. DiffTrace decompresses and prunes traces, providing “finer” traces for later phases. Loops in source codes reflect themselves as a sequence of *repetitive patterns*, resulting in often-long but redundant trace entries. A “nested loop recognition” mechanism then mines loops from traces as “a *measure of progress*” per thread, and also a loss-less abstraction to ease the rest of trace analysis. The control flow of events in parallel architectures often follow a *regular pattern* such as SPMD, odd-even and master/slave. This characteristic often makes all traces of a single execution tend to fall into just a few “conceptually equivalent classes”. **definition of “fault” maybe? a software bug, node failure, a library version update or porting to a new system that cause failure to a working software** To study the impact of a fault on a normal execution, DiffTrace classify collected traces and measures the *similarity* between the equivalent classes of faulty execution against the corresponding fault-free execution. The basic idea is to find out which traces (and consequently processes/threads) are falling into a different class (i.e., cluster) when the fault is introduced.

Table I  
PRE-DEFINED FILTERS

Category	Sub-Category	Description
Primary	Returns	Filter out all returns
	PLT	Filter out the ".plt" function calls for external functions/procedures that their address needs to be resolved dynamically from Procedure Linkage Table (PLT)
MPI	MPI All	Only keep functions that start with "MPI_"
	MPI Collectives	Only keep MPI collective calls (MPI_Barrier, MPI_Allreduce, etc)
	MPI Send/Recv	Only keep MPI_Send, MPI_Isend, MPI_Recv, MPI_Irecv and MPI_Wait
	MPI Internal Library	Keep all inner MPI library calls
OMP	OMP All	Only keep OMP calls (starting with GOMP_)
	OMP Critical	Only keep OMP_CRITICAL_START and OMP_CRITICAL_END
	OMP Mutex	Only keep OMP_Mutex calls
System	Memory	Keep any memory related functions (memcpy, memchk, alloc, malloc, etc)
	Network	Keep any network related functions (network, tcp, sched, etc)
	Poll	Keep any poll related functions (poll, yield, sched, etc)
	String	Keep any string related functions (strlen, strcpy, etc)
Advanced	Custom	Any regular expression can be captured
	Everything	Does not filter anything

Based on the observed similarity of two clusterings, DiffTrace suggests top suspicious traces that are suffering the most from the fault for deeper analysis.

The rest of this section illustrates DiffTrace components on sets of ParLOT traces collected from MPI odd/even sort example (figure 2). Odd/Even sort is a variant of the bubble-sort operates in two alternate phases: *Phase-even* where even processes exchange (compare and swap) values with right neighbors and *Phase-odd* where odd processes exchange values with right neighbors. The for loop in line 4 of `oddEvenSort()` iterates over phases of the algorithm and based on the phase, the appropriate partner for each rank is getting discovered by the function `findPtr()` (line 6). The odd/even ranks then exchange their chunks of data (lines 9-13) and a set of sort, merge and copy operations would be performed on received data by each rank (which are replaced by `...` in line 15 for simplicity).

According to MPI Standard, `MPI_Send` is a *blocking send* used the *standard* communication mode in which MPI may buffer outgoing messages and the send call may complete before a matching receive is invoked. On the other hand, buffer space may be unavailable, or MPI may choose not to buffer outgoing messages, for performance reasons. In this case, the send call will not complete until a matching receive has been posted, and the data has been moved to the receiver. So based on the MPI implementation, a swap of order in line 11-12 of figure 7 code might end up causing a deadlock. This section ends with showing that DiffTrace can extract evidences from traces to justify and locate the cause of such deadlocks and other faults. Later in the paper, we evaluate DiffTrace on a real world hybrid MPI+OpenMP example.

#### A. Pre-processing

Using ParLOT decoder, each trace needs to get decompressed for further analysis. Supporting the iterative approach of DiffTrace, corresponding function names of each function ID in traces are checked with some pre-defined (table I) or custom regular expressions (i.e., *filters*). In case of a match, DiffTrace would keep that function ID for later phases, otherwise that function ID would not be stored in the current iteration of DiffTrace.

Table II  
THE GENERATED TRACES FOR ODD/EVEN EXECUTION WITH FOUR PROCESSES

$T_0$	$T_1$	$T_2$	$T_3$
...	...	...	...
main	main	main	main
MPI_Init	MPI_Init	MPI_Init	MPI_Init
MPI_Comm_Rank	MPI_Comm_Rank	MPI_Comm_Rank	MPI_Comm_Rank
MPI_Comm_Size	MPI_Comm_Size	MPI_Comm_Size	MPI_Comm_Size
...	...	...	...
oddEvenSort	oddEvenSort	oddEvenSort	oddEvenSort
...	...	...	...
findPtr	findPtr	findPtr	findPtr
MPI_Send	MPI_Recv	MPI_Send	MPI_Recv
MPI_Recv	MPI_Send	MPI_Recv	MPI_Send
...	...	...	...
findPtr	findPtr	findPtr	findPtr
MPI_Send	MPI_Recv	MPI_Send	MPI_Recv
MPI_Recv	MPI_Send	MPI_Recv	MPI_Send
...	...	...	...
MPI_Finalize	MPI_Finalize	MPI_Finalize	MPI_Finalize

Figure 3. Trace NLRs

$T_0$	$T_1$	$T_2$	$T_3$
MPI_Init()	MPI_Init()	MPI_Init()	MPI_Init()
MPI_Comm_Rank()	MPI_Comm_Rank()	MPI_Comm_Rank()	MPI_Comm_Rank()
MPI_Comm_Size()	MPI_Comm_Size()	MPI_Comm_Size()	MPI_Comm_Size()
$L0 \wedge 2$	$L1 \wedge 4$	$L0 \wedge 4$	$L1 \wedge 2$
MPI_Finalize()	MPI_Finalize()	MPI_Finalize()	MPI_Finalize()

Table II shows pre-processed traces ( $T_i$ ) of odd/even sort execution with 4 processes.  $T_i$  is the trace that stores function calls of process  $i$ .

#### B. Nested Loop Representation

HPC applications often spend most of their times in *loops*. Function calls within each loop body reflect themselves as *repetitive patterns* in ParLOT traces. Inspired by ideas from detection of repetitive patterns in strings [48] and other data structures [47], we have adapted the Nested Loop Recognition (NLR) algorithm from Ketterlin et al [6] to detect repetitive patterns in ParLOT traces (more details in section III-A). Detecting such patterns “measures progress” per trace and reflects the delayed or unfinished/broken loops that potentially caused by a fault.

For example, the loop in line 3 of `oddEvenSort()` (figure 2) iterates 4 times in execution with 4 processes. Thus each  $T_i$  contains 4 consecutive occurrence of either `MPI_Send()` – `MPI_Recv()` (even  $T$ s) or `MPI_Recv()` – `MPI_Send()` (odd  $T$ s). By keeping only MPI functions and converting each  $T_i$  to its equivalent NLR (Nested Loop Representation), table II can be reduced to figure 3. **L0** (pair of `MPI_Send` - `MPI_Recv`) and **L1** (pair of `MPI_Recv` - `MPI_Send`) are two main body loops that have been detected among pre-processed filters. Note that, since first and last processes have only one-way communication with their neighbors,  $T_0$  and  $T_3$  iterates over the loop half of other processes.

connect this section to next

#### C. Hierarchical Clustering via FCA

Underlying parallel software architecture of HPC applications often follow a *regular pattern* with respect to sequences of executed functions. Considering this characteristic, per-thread individual function call traces can be classified into

Figure 4. Formal Concept Definition

For subsets  $A \subseteq G$  of objects and subsets  $B \subseteq M$  of attributes, one defines two derivation operators as follows:  
 $A' = \{m \in M \mid (g,m) \in I \text{ for all } g \in A\}$ , and dually  
 $B' = \{g \in G \mid (g,m) \in I \text{ for all } m \in B\}$ .  
 Applying either derivation operator and then the other constitutes two closure operators:  
 $A \mapsto A'' = (A')'$  for  $A \subseteq G$  (extent closure), and  
 $B \mapsto B'' = (B')'$  for  $B \subseteq M$  (intent closure).  
**Definition of a formal concept:** a pair  $(A,B)$  is a formal concept of a context  $(G, M, I)$  provided that:  
 $A \subseteq G$ ,  $B \subseteq M$ ,  $A' = B$ , and  $B' = A$ .  
 Equivalently and more intuitively,  $(A,B)$  is a **formal concept** precisely when:  
 - every object in  $A$  has every attribute in  $B$ ,  
 - for every object in  $G$  that is not in  $A$ , there is some attribute in  $B$  that the object does not have,  
 - for every attribute in  $M$  that is not in  $B$ , there is some object in  $A$  that does not have that attribute.

a few equivalent groups based on the conceptual structure of trace contents (i.e., sequence of functions). Such classification would distinguish naturally different threads (e.g., MPI processes from OpenMP threads in hybrid MPI+OpenMP applications), reduce the search space into just a few representative classes of traces and ease the process of detecting outlier traces. In addition, the set of per-thread traces should be studied as “a whole” since there is a strong conceptual and causal relation among threads/processes. In order to integrate all collected traces into a *single model of execution* and forming “equivalent classes of traces”, we have adapted the idea of *Structural Clustering* [39] by applying *formal concept analysis* (FCA)[7] techniques to ParLOT traces.

A *formal context* is a triple  $K = (G, M, I)$ , where  $G$  is a set of **objects**,  $M$  is a set of **attributes**, and  $I \subseteq G \times M$  is an incidence relation that expresses *which objects have which attributes*. Table II-C) shows the formal context of odd/even sort pre-processed traces.

In this context, attributes are trace entries (function calls or detected loop bodies) without involving their frequency (e.g., loop counts). However, any set of attributes can be extracted from traces (table YY). **Definition of formal concept (needed?) figure 4 :**

A *concept lattice* can be derived from a *formal context* by specifying *formal concepts* (figure 4) and a *partial order* on them. Concept lattices are represented as a directed acyclic graph where concepts are nodes and the order on them determines the edges. Figure 5 shows the concept lattice of the formal context in The formal context in table II-C shows that all traces has the functions `MPI_Init()`, `MPI_Comm_size()`, `MPI_Comm_rank()` and `MPI_Finalize()`. Even traces have the loop `L0` and odd traces have the loop `L1`. The resulting concept lattice from this context is shown in figure 5 and reads as:

- The top node indicates that all traces share `MPI_Init()`, `MPI_Comm_size()`, `MPI_Comm_rank()` and `MPI_Finalize()`.
- The bottom node signifies that none of the traces share

Table III  
FORMAL CONTEXT OF ODD/EVEN SORT EXAMPLE

	MPI_Init()	MPI_Comm_Size()	MPI_Comm_Rank()	L0	L1	MPI_Finalize()
Trace 0	x	x	x	x		x
Trace 1	x	x	x		x	x
Trace 2	x	x	x	x		x
Trace 3	x	x	x		x	x

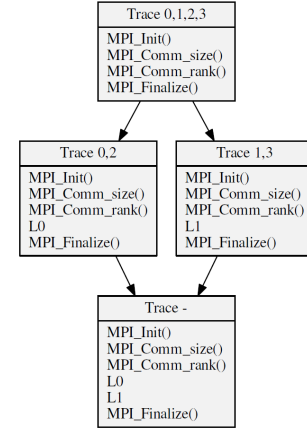


Figure 5. Sample Concept Lattice from Obj-Atr Context in table II-C

all attributes.

- The middle nodes show that  $T_0$  and  $T_2$  are different from  $T_1$  and  $T_3$

We have used concept lattices to obtain pair-wise *similarity scores* among traces. The similarity metric that we have used is *Jaccard Index*, also known as *Intersection over Union*. For any pair of  $(T_i, T_j)$ , number of attributes in the Lowest Common Ancestor (LCA) node of  $T_i$  and  $T_j$  is the number of attributes that  $T_i$  and  $T_j$  have in common (intersection). The sum of the number of attributes of nodes in the path from each  $T_i$  and  $T_j$  to their LCA is the union. Figure 6 shows the heatmap of full pair-wise Jaccard Similarity Matrix (JSM) obtained from the concept lattice in figure 5. DiffTrace uses obtained JSMs as *linkage* function to form equivalent classes of traces by hierarchical clustering. In the next phase of DiffTrace, we show how the differences of two hierarchical clustering from two executions (faulty vs. normal) can reveal which traces has been changed the most by the fault.

#### D. Detecting Suspicious Traces via DiffJSM

Up to this point, DiffTrace can narrow down the search space from numerous long traces to their equivalent JSM (i.e., clusters). As the original intention of DiffTrace implies, we are interested in detecting what has changed the most when a fault is introduced with respect to the “natural asymmetry” of the application. In other words, DiffTrace abstracts function call traces into JSMs which are reflections of asymmetry among traces. Now the hierarchical clustering based on the *DiffJSM*, the subtraction of the faulty JSM from its corresponding normal JSM, would put the trace(s) that changed the most in a separate cluster from the others (i.e., outlier). The obtained outlier traces are candidates of the potential cause of the change in the program behavior, thus a potential fault root



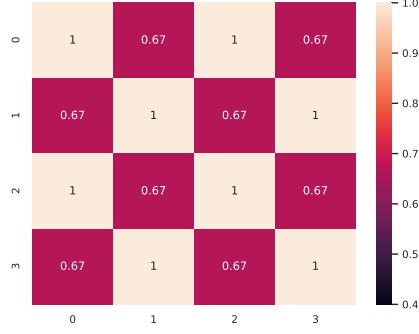


Figure 6. Pair-wise Jaccard Similarity Matrix (JSM) of MPI processes in Sample code

cause or fault manifestation. However, a single iteration of DiffTrace (with a single set of parameters shown as dotted box in figure 1) might not reflect such asymmetries in the normal and faulty traces. Also different set of parameters might produce inaccurate suggestions (false positives). To improve the accuracy of suggested suspicious traces (ranking table), we have sorted the suggestion table based on the *B-score* similarity metric of two hierarchical clustering [13]. (more in section 3.B-score).

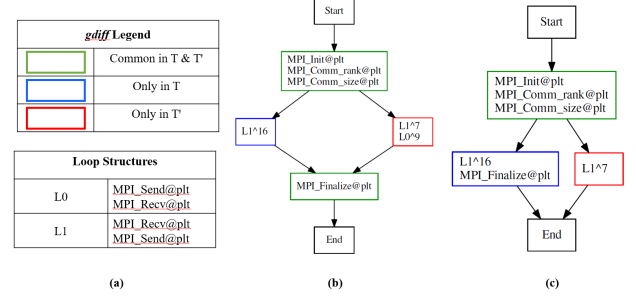
To evaluate the effectiveness of DiffJSM idea on suggesting suspicious traces, we have planted two artificial bugs (*swapBug* and *dlBug*) in the code in figure 2 and launched the code with 16 processes. *swapBug* swaps the order of MPI\_Send and MPI\_Recv in rank 5 after 7th iteration (of the loop in line 3 of oddEvenSort) simulating a potential deadlock and *dlBug* simulates an actual deadlock (e.g., infinite loop) in the same location (rank 5 after 7th iteration). Upon collection of ParLOT traces from execution above buggy versions, DiffTrace first decompresses traces and filters out all non-MPI functions. Then two major loops are detected, **L0** and **L1** (figure 8-(a)) that are supposed to occur 16 times in even and odd traces, respectively (except for first and last trace which is 8 times). After constructing concept lattices and their corresponding JSMs,  $T_7 = 5$  has been suggested as the most probable trace that got affected by the artificial bugs.

1) *introducing diffNLR*: To show the actual points of differences in suspicious traces with their corresponding normal trace, DiffTrace visualizes the common and different blocks of a pair of pre-processed trace via *diffNLR*, a graphical visualization of *diff* algorithm [12]. *diff* takes two sequences  $S_A$  and  $S_B$  and computes the minimal *edit* to convert  $S_A$  to  $S_B$ . This algorithm has been used in GNU *diff* to compare two text files and in git for efficiently keeping track of file changes. Since ParLOT preserves the order of function calls in the binary, each per thread trace  $T_i$  is totally ordered, thus *diff* can reflect the differences of a pair of  $T$ s. *diffNLR* aligns common and different blocks of a pair of sequences (e.g., traces) horizontally and vertically, making it easier for analyst to see the differences of a pair of sequences in a glance.

Figure 7. A line change in oddEvenSort (left) that might cause a deadlock in oddEvenSort\_DL (right)

	oddEvenSort()	oddEvenSort_DL()
1	<code>oddEvenSort(rank, cp){</code>	<code>oddEvenSort_DL(rank, cp){</code>
2	<code>... for (int i=0; i &lt; cp; i++)</code>	<code>... for (int i=0; i &lt; cp; i++)</code>
3	<code>{</code>	<code>{</code>
4	<code>int ptr = findPtr(i, rank);</code>	<code>int ptr = findPtr(i, rank);</code>
5	<code>... if (rank % 2 == 0) {</code>	<code>... if (rank % 2 == 0) {</code>
6	<code>... MPI_Send(..., ptr, ...);</code>	<code>... MPI_Send(..., ptr, ...);</code>
7	<code>... MPI_Recv(..., ptr, ...);</code>	<code>... MPI_Recv(..., ptr, ...);</code>
8	<code>... } else {</code>	<code>... } else {</code>
9	<code>... MPI_Recv(..., ptr, ...);</code>	<code>... MPI_Send(..., ptr, ...);</code>
10	<code>... MPI_Send(..., ptr, ...);</code>	<code>... MPI_Recv(..., ptr, ...);</code>
11	<code>... }</code>	<code>... }</code>
12	<code>... }</code>	<code>... }</code>
13	<code>... }</code>	<code>... }</code>
14	<code>... }</code>	<code>... }</code>
15	<code>... }</code>	<code>... }</code>
16	<code>}</code>	<code>}</code>

Figure 8. (a) The legend of *gdifff* and the list of loop structures (b) *gdifff*(5) of *swapBug* (c) *gdifff*(5) of *dlBug*



For simplicity, our implementation of *gdifff* only takes one argument  $x$  as *the suspicious trace*

$$gdifff(x) \equiv gdifff(T_x, T'_x)$$

where  $T_x$  is the trace of thread/process  $x$  of a normal/successful execution and  $T'_x$  is the corresponding trace of faulty execution.

Figure 8-(b) shows the *diffNLR*(5) of *swapBug* where  $T_5$  iterates over the loop [MPI\_Recv - MPI\_Send] for 16 times ( $L1^{16}$ ) after the MPI initialization while the order swap has well reflected in  $T'_5$  ( $L1^7 - L0^9$ ). Both processes seem to be terminated fine by executing MPI\_Finalize(). However, *diffNLR*(5) of *dlBug* (figure 8-(c)) shows that while  $T_5$  have executed MPI\_Finalize and terminated well,  $T'_5$  got stuck after executing L1 seven times and have never reached MPI\_Finalize.

This example shows that our approach can locate the impacted part of each execution by a fault. Having a pre-understanding of *how the application should behave normally* would reduce the number of iterations by picking the right set of parameters on each pass.

### III. ALGORITHMS UNDERLYING DIFFTRACE

#### A. NLR

To recognize loop structures in traces, we have adopted ideas from Kobayashi [5] paper where he defines loops in a sequence of instructions as “a string of instruction executions in which a particular sequence of distinct instructions (called the *cycle* of the loop) is successively repeated”. This idea have been later expanded by Ketterline et al in [6] where they

have introduced Nested Loop Recognition (NLR) algorithm for compressing data access addresses and predicting next accessing addresses. NLR is a memory-bounded algorithm that start reading from the beginning of the sequence

store them in the stack

upon each push to stack

checks for top 3 equal size sub-sequence for isomorphism (equal length and equal corresponding elements)

checks if top  $n$  elements of the stack matches with any previous detected loops. if yes, increment the loop count and pop  $n$  elements from the stack

the above procedure would be repeated any time a change happens in the stack.

There is a pre-defined size for Max Stack. If stack reaches that point, a fixed number of elements would be popped from the bottom of the stack to free the space for rest of elements.

figure ?? shows the final product

complexity is  $\Theta(K^2N)$  where  $K$  is a fixed priori and  $N$  is the size of the input.

### B. Concept Lattice Construction

- 1 paragraph background on FCA
- 1-2 paragraphs on advantages of FCA and what we would gain from FCA? Answer: Full pair-wise Jaccard Similarity Matrix (JSM)
- 1 paragraph how JSMs are going to help us (referring to the major figure at the beginning of this section)
  - Some background about Jaccard Similarity Score
  - How to obtain full pair-wise Jaccard Similarity Matrix (JSM) from a concept lattice (e.g., LCA approach)
- 1-2 paragraphs on CL generation (related work and our approach)
  - Batch vs. Incremental [11]
  - Complexity:  $O(2^{2K}||E||)$  where  $K$  is an upper bound for number of attributes (e.g., distinct function calls in the whole execution) and  $||E||$  is the number of objects (e.g., number of PTs).
- 1-2 paragraphs (+ 1-2 figures) explaining the FCA ideas on odd/even sort example.

### C. Hierarchical Clustering, Construction and Comparison

JSMs and Linkage functions from Scipy B-score to compare clusterings

Table IV  
ILCSTSP.MC1-MC-6-N1.M.8.AUTO

Filter	Attributes	Link Method	Thresh	B-score	Top Procs (JSMD)	TOP Threads(JSMD)
01.mem.ompall.cust.0K10	sing.actual	average	4	0.308594		6.2 , 6.4 , 5.2 , 5.4 , 7.3 ,
11.mem.ompall.cust.0K10	sing.actual	average	4	0.308594		6.2 , 6.4 , 5.2 , 5.4 , 7.3 ,
01.mem.ompmutex.cust.0K10	sing.actual	weighted	4	0.321883		6.2 , 3.4 , 5.3 , 4.2 , 7.4 ,
11.mem.ompmutex.cust.0K10	sing.actual	weighted	4	0.321883		6.2 , 3.4 , 5.3 , 4.2 , 7.4 ,
11.plt.mem.mpi.ompertit.cust.0K10	doub.actual	weighted	4	0.324427	3 ,	0.2 , 1.1 , 3.1 , 4.3 , 4.4 , 5.2 ,
01.plt.mem.mpi.ompertit.cust.0K10	doub.actual	weighted	4	0.324427	3 ,	0.2 , 1.1 , 3.1 , 4.3 , 4.4 , 5.2 ,
01.mem.ompall.cust.0K10	doub.actual	weighted	4	0.33266	3 ,	6.4 , 7.1 , 3.4 , 4.3 , 4.4 , 5.2 ,
11.mem.ompall.cust.0K10	doub.actual	weighted	4	0.33266	3 ,	6.4 , 7.1 , 3.4 , 4.3 , 4.4 , 5.2 ,
01.mem.ompertit.cust.0K10	sing.actual	weighted	4	0.354396	6 ,	7.2 , 7.4 , 3.4 , 4.2 , 4.3 , 4.4 ,
11.mem.ompertit.cust.0K10	sing.actual	weighted	4	0.354396	6 ,	7.2 , 7.4 , 3.4 , 4.2 , 4.3 , 4.4 ,
11.plt.mem.mpi.ompertit.cust.0K10	doub.actual	average	4	0.381646		3.3 , 2.4 ,

Table V  
ILCSTSP.MC1-MC-2-2.M.8.AUTO

Filter	Attributes	Link Method	Thresh	B-score	Top Procs (JSMD)	TOP Threads(JSMD)
01.mem.ompall.cust.0K10	doub.actual	weighted	4	0.309391	3 ,	6.2 , 6.4 , 7.1 , 7.4 , 1.3 , 3.1 ,
11.mem.ompall.cust.0K10	doub.actual	weighted	4	0.309391	3 ,	6.2 , 6.4 , 7.1 , 7.4 , 1.3 , 3.1 ,
11.plt.mem.cust.0K10	doub.actual	weighted	4	0.318003	7 , 4 ,	1.3 , 2.2 , 3.3 , 3.4 , 4.2 , 4.3 ,
01.plt.mem.cust.0K10	doub.actual	weighted	4	0.318003	7 , 4 ,	1.3 , 2.2 , 3.3 , 3.4 , 4.2 , 4.3 ,
01.mem.ompall.cust.0K10	doub.actual	average	4	0.34462	7 , 3 ,	7.1 , 1.3 , 1.4 , 2.2 , 2.3 , 3.1 ,
11.mem.ompall.cust.0K10	doub.actual	average	4	0.34462	7 , 3 ,	7.1 , 1.3 , 1.4 , 2.2 , 2.3 , 3.1 ,
11.plt.mem.mpi.ompertit.cust.0K10	doub.actual	average	4	0.350702	7 , 3 ,	6.2 , 6.3 , 7.2 , 2.4 , 3.3 , 4.2 ,
01.plt.mem.mpi.ompertit.cust.0K10	doub.actual	average	4	0.350702	7 , 3 ,	6.2 , 6.3 , 7.2 , 2.4 , 3.3 , 4.2 ,
11.plt.mem.cust.0K10	doub.actual	weighted	3	0.357334	7 ,	2.2 , 3.3 , 3.4 , 4.3 , 4.4 ,
01.plt.mem.cust.0K10	doub.actual	weighted	3	0.357334	7 ,	2.2 , 3.3 , 3.4 , 4.3 , 4.4 ,
01.mem.ompall.cust.0K10	doub.actual	weighted	3	0.380481	3 ,	7.1 , 1.3 , 3.1 , 4.3 , 4.4 ,

## IV. CASE STUDY: ILCS

Table VIII  
ILCSTSP.BC1-WS-3-NN.M.8.AUTO

Filter	Attributes	Link Method	Thresh	B-score	Top Procs (JSMD)	TOP Threads(JSMD)
11.mpi.cust.0K10	sing.actual	weighted	4	0.385229	6 ,	6.2 , 6.3 , 6.4 , 7.2 , 7.3 , 7.4 ,
11.mpi.all.cust.0K10	sing.actual	weighted	4	0.385229	6 ,	6.2 , 6.3 , 6.4 , 7.2 , 7.3 , 7.4 ,
01.mpi.all.cust.0K10	sing.actual	weighted	4	0.385229	6 ,	6.2 , 6.3 , 6.4 , 7.2 , 7.3 , 7.4 ,
01.mpi.col.cust.0K10	sing.actual	weighted	4	0.385229	6 ,	6.2 , 6.3 , 6.4 , 7.2 , 7.3 , 7.4 ,
11.mpi.col.cust.0K10	sing.actual	weighted	4	0.385229	6 ,	6.2 , 6.3 , 6.4 , 7.2 , 7.3 , 7.4 ,
01.mpi.cust.0K10	sing.actual	weighted	4	0.385229	6 ,	6.2 , 6.3 , 6.4 , 7.2 , 7.3 , 7.4 ,
01.plt.cust.0K10	sing.actual	weighted	4	0.448188	7 , 3 ,	6.2 , 6.4 , 7.1 , 7.4 , 3.3 , 3.4 ,
11.plt.cust.0K10	sing.actual	weighted	4	0.448188	7 , 3 ,	6.2 , 6.4 , 7.1 , 7.4 , 3.3 , 3.4 ,
11.mpi.cust.0K10	sing.actual	weighted	3	0.465043	6 ,	6.2 , 6.3 , 6.4 , 7.2 , 7.3 , 7.4 ,
11.mpi.all.cust.0K10	sing.actual	weighted	3	0.465043	6 ,	6.2 , 6.3 , 6.4 , 7.2 , 7.3 , 7.4 ,
01.mpi.all.cust.0K10	sing.actual	weighted	3	0.465043	6 ,	6.2 , 6.3 , 6.4 , 7.2 , 7.3 , 7.4 ,

Table VI  
ILCSTSP.AR2-WO-2-NN.M.8.AUTO

Filter	Attributes	Link Method	Thresh	B-score	Top Procs (JSMD)	TOP Threads(JSMD)
01.plt.cust.0K10	doub.actual	average	4	0.392446	6 ,	2.3 , 2.4 , 4.2 , 4.3 , 4.4 ,
11.plt.cust.0K10	doub.actual	average	4	0.392446	6 ,	2.3 , 2.4 , 4.2 , 4.3 , 4.4 ,
01.plt.cust.0K10	sing.log10	centroid	4	0.945946	0 ,	6.2 , 0.4 , 1.1 , 0.1 , 2.3 , 2.4 ,
01.plt.cust.0K10	sing.log10	median	4	0.945946	0 ,	6.2 , 0.4 , 1.1 , 0.1 , 2.3 , 2.4 ,
11.plt.cust.0K10	sing.log10	centroid	4	0.945946	0 ,	6.2 , 0.4 , 1.1 , 0.1 , 2.3 , 2.4 ,
11.plt.cust.0K10	sing.log10	median	4	0.945946	0 ,	6.2 , 0.4 , 1.1 , 0.1 , 2.3 , 2.4 ,
01.plt.cust.0K10	sing.log10	median	3	0.947368	0 ,	6.2 , 0.4 , 1.1 , 0.1 , 2.3 , 2.4 ,
11.plt.cust.0K10	sing.log10	median	3	0.947368	0 ,	6.2 , 0.4 , 1.1 , 0.1 , 2.3 , 2.4 ,
01.plt.cust.0K10	sing.log10	complete	3	1	0 ,	0.1 , 7.1 , 1.1 , 3.1 , 3.3 ,
01.plt.cust.0K10	sing.log10	complete	4	1	0 ,	0.1 , 7.1 , 1.1 , 3.1 , 3.3 ,
01.plt.cust.0K10	sing.log10	average	4	1	0 ,	0.1 , 7.1 , 1.1 , 3.1 , 3.3 ,

Table VII  
ILCSTSP.BC2-WR-3-NN.M.8.AUTO

Filter	Attributes	Link Method	Thresh	B-score	Top Procs (JSMD)	TOP Threads(JSMD)
01.plt.cust.0K10	doub.actual	centroid	4	0.512309	2 ,	6.4 , 7.3 , 1.2 , 1.3 , 2.1 , 2.2 ,
11.plt.cust.0K10	doub.actual	centroid	4	0.512309	2 ,	6.4 , 7.3 , 1.2 , 1.3 , 2.1 , 2.2 ,
01.plt.cust.0K10	sing.actual	average	4	0.513221	7 ,	3.4 , 5.2 , 4.2 , 4.3 ,
01.plt.cust.0K10	sing.actual	average	4	0.513221	7 ,	3.4 , 5.2 , 4.2 , 4.3 ,
11.mpi.cust.0K10	sing.actual	median	4	0.544807	0 ,	0.1 , 6.4 , 7.3 , 7.4 , 1.3 , 0.2 ,
11.mpi.all.cust.0K10	sing.actual	median	4	0.544807	0 ,	0.1 , 6.4 , 7.3 , 7.4 , 1.3 , 0.2 ,
01.mpi.all.cust.0K10	sing.actual	median	4	0.544807	0 ,	0.1 , 6.4 , 7.3 , 7.4 , 1.3 , 0.2 ,
11.mpi.col.cust.0K10	sing.actual	median	4	0.544807	0 ,	0.1 , 6.4 , 7.3 , 7.4 , 1.3 , 0.2 ,
01.mpi.col.cust.0K10	sing.actual	median	4	0.544807	0 ,	0.1 , 6.4 , 7.3 , 7.4 , 1.3 , 0.2 ,
01.mpi.cust.0K10	sing.actual	median	4	0.544807	0 ,	0.1 , 6.4 , 7.3 , 7.4 , 1.3 , 0.2 ,
01.plt.cust.0K10	doub.actual	centroid	3	0.639268	2 ,	6.4 , 7.3 , 1.2 , 1.3 , 2.1 , 2.2 ,



## V. CASE STUDY: ILCS

### A. Experimental Methodology

So far, we are able to collect whole-program execution traces, preprocess them (decompress, filter, detect loops, extract attributes) and inject each *PT* to concept lattice data structure. Concept lattices help us having a single model for the execution of HPC application with thousands of processes/threads. Concept lattices also classify *PTs* based on their Jaccard distance. Full pair-wise Jaccard distance matrix can be extracted from the concept lattice in linear time and reduces the search space from thousands of *PTs* to just a few equivalent classes of *PTs*. Studying JSM by itself helps the user to understand the program behavior as a whole, and how each process/thread behaving. However, comparing the JSM of the bug-free version of the application versus the buggy version would reveal insights about how the bug impacted the behavior of the application. In particular, we are interested to see how the bug changes the formation of equivalent classes of *PTs*. Inspired by a method for comparing two different clustering [13], we count the number of objects (*PTs*) in each cluster and see which *PT(s)* fall into different clusters once the bug is introduced. A set of candidate *PTs* then would be reported to the user for more in-depth study. Here is where we take advantage of diffNLR to see how does the bug changes the control flow of a candidate *PT* comparing to its corresponding *PT* of native run.

Table 9 shows different parameters that we can pre-process *PTs* with. Each combination of these parameters would result in a different concept lattice, thus different JSM and different clusterings. A table similar to X is created for each injected bug. Each row of the table is showing the set of parameters used to create JSMs. Then by calculating  $|JSM(buggy) - JSM(bugfree)|$  we are interested to see which *PT* changes the most after the bug injected and falls into a single cluster. The object(s) in the cluster with the fewest members (below a threshold) are potential candidates of *threads that are manifesting the bug* and the diff(buggy,bug-free) is in our interest to see how does the bug changes its control flow.

### B. Case Study: ILCS-TSP

Here is the ILCS framework pseudo-code. User needs to write `CPU_Init()`, `CPU_Exec()` and `CPU_Output()`.

```
int main(argc, argv){
    MPI_Init();
    MPI_Comm_size()
    MPI_Comm_rank(my_rank)
    //Figuring local number of CPUs
    MPI_Reduce() // Figuring global number of CPUs
    CPU_Init();
    //For storing local champion results
    champ[CPUs] = malloc();
    MPI_Barrier();
    #pragma omp parallel num_threads(CPUs+1)
```

```
{
    rank = omp_get_thread_num()
    if (rank == 0){ //communication thread
        do{
            //Find and report the thread with
            //local champion, global champion
            MPI_AllReduce();
            //Find and report the process with
            //global champion
            MPI_AllReduce();
            //The process with the global champion
            //copy its results to bcast_buffer
            if (my_rank == global_champion){
                #pragma omp cirtical
                memcpy(bcast_buffer, local_champ)
            }
            //Broadcast the champion
            MPI_Bcast(bcast_buffer)
        } while (no_change_threshold);
        cont=0 // signal worker threads to stop
    } else{ // worker threads
        while(cont){
            //Calculate Seed
            local_result = CPU_Exec()
            if (local_result < champ[rank]){
                #pragma omp cirtical
                memcpy(champ[rank], local_result)
            }
        }
    }
    //Find and report the thread with
    //local champion, global champion
    MPI_AllReduce();
    //Find and report the process with
    //global champion
    MPI_AllReduce();
    // The process with the global champion
    // copy its results to bcast_buffer
    if (my_rank == global_champion){
        #pragma omp cirtical
        memcpy(bcast_buffer, local_champ)
    }
    //Broadcast the champion
    MPI_Bcast(bcast_buffer)
    if (my_rank==0){
        CPU_Output(champ)
    }
    MPI_Finalize()
}
/* User code for TSP problem */
CPU_Init(){
    // Read In data from cities
    // Calculate distances
```

Filters										CL Attributes	Clustering	
Prime		General		MPI		OMP		Other				
Filter	Description	Filter	Description	Filter	Description	Filter	Description	Filter	Description			
ret	Filter Returns	@plt	...@plt	mpi	MPI_...	ompcrit	OMP critical	Custom	Defining specific regex to filter	Objects: Traces Attributes: set of <atr:freq>		single
.plt	Filter .plt	mem	Memory related malloc memcpy etc	mpiall	..MPI... MPID... PMPI...	ompmu tex	OMP mutex	incEveryth ing	Include whatever is not in the Filters	Single: set of single trace entries atr: sing	No Frequency: only presence of attribute entries matters freq:-	complete
		net	Network related	mpicol	MPI collectives	omppall	OMP all functions			Double: set of 2-consecutive entries atr: doub	Log10: log(freq) of each entry matters (for large frequency numbers freq: log10(#atr)	average
		poll	Poll Related poll, yield	mpisr	MPI send/recv						Actual: actual frequency of each entry matters freq: #atr	weighted
		str	String related strcpy strcmp etc									centroid
												median
												ward

Figure 9. Filters, Attributes and other Parameters used to pre-process ParLOT Traces (PTs)

```

// Return data structure to store champion
}

CPU_Exec(){
// Find local champions (TSP tours)
}

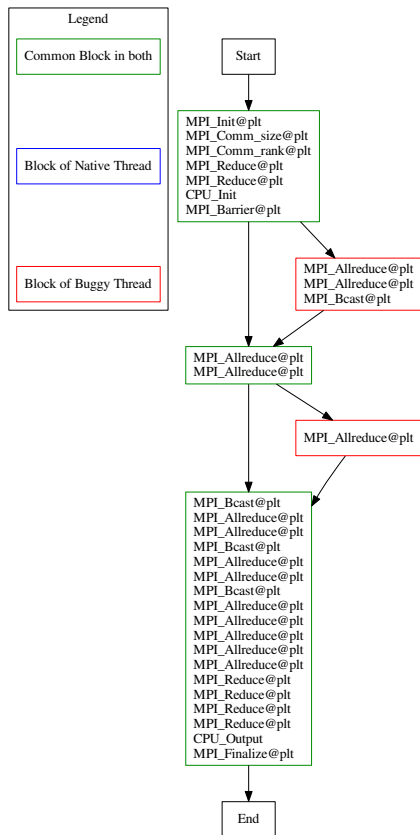
CPU_Output(){
// Output champion
}

```

Table ?? describes the bug that I injected to ILCS-TSP

Table IX  
INJECTED BUGS TO ILCS-TSP

ID	Level	Bugs	Description
1	MPI	allRed1wrgOp-1-all-x	Different operation (MPI_MAX) in only one process (buggyProc = 2) for MPI_ALLREDUCE() in Line 21
2		allRed1wrgSize-1-all-x	Wrong size in only one process (buggyProc = 2) for MPI_ALLREDUCE() in Line 21
3		allRed1wrgSize-all-all-x	Wrong Size in all processes for MPI_ALLREDUCE() in Line 21
4		allRed2wrgOp-1-all-x	Different operation (MPI_MAX) in only one (buggyProc) for first MPI_ALLREDUCE() – L277:ilcsTSP.c
5		allRed2wrgSize-1-all-x	Wrong size in only one (buggyProc) for first MPI_ALLREDUCE() – L277:ilcsTSP.c
6		allRed2wrgSize-all-all-x	Wrong Size in all processes for second MPI_ALLREDUCE() – L277:ilcsTSP.c
7		bcastWrgSize-1-all-x	Wrong Size in only one (buggyProc) of MPI_Bcast() – L290:ilcsTSP.c
8		bcastWrgSize-all-all-x	Wrong Size n all processes for MPI_Bcast() – L240:ilcsTSP.c
9	OMP	misCrit-1-1-x	Missing Critical Section in buggyProc and buggyThread – L170:ilcsTSP.c
10		misCrit-all-1-x	Missing Critical Section in buggyThread and all procoesses – L170:ilcsTSP.c
11		misCrit-1-all-x	Missing Critical Section in buggyProc and all threads – L170:ilcsTSP.c
12		misCrit-all-all-x	Missing Critical Section in all procs and threads – L170:ilcsTSP.c
13		misCrit2-1-1-x	Missing Critical Section in buggyProc and buggyThread – L230:ilcsTSP.c
14		misCrit2-all-1-x	Missing Critical Section in buggyThread – L230:ilcsTSP.c
15		misCrit2-1-all-x	Missing Critical Section in buggyProc and all threads – L230:ilcsTSP.c
16		misCrit2-all-all-x	Missing Critical Section in all procs and threads – L230:ilcsTSP.c
17		misCrit3-1-all-x	Missing Critical Section in buggyProc and all threads – L280:ilcsTSP.c
18		misCrit3-all-all-x	Missing Critical Section in all procs and threads – L280:ilcsTSP.c
19	General	infLoop-1-1-1	Injected an infinite loop after CPU_EXEC() in buggyProc,buggyThread & buggyIter L164:ilcsTSP.c

Figure 10. Bug1: diffNLR  $PT_{0,0}$  - buggy vs. native

1) *Bug1: Wrong Operation in MPI AllReduce()*: We have injected a bug (row 1 table IX) where `MPI_Allreduce()` had been invoked with a wrong operation in one of the processes ( $P_2$ ) (`MPI_MAX` instead of `MPI_MIN`).

**::What is the runtime reaction to this bug::** Program terminated well without any error, crash, hang or throwing any exception. But the results might be corrupted. This might be a silent bug that diffTrace could reveal

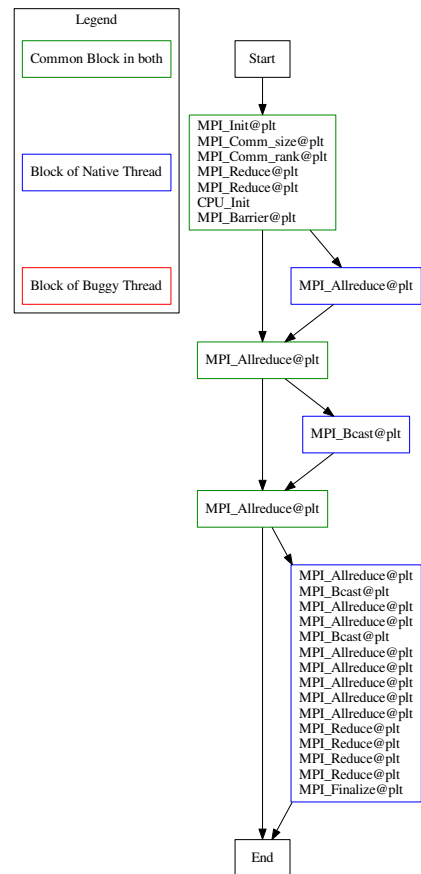
The last row of table X is telling us that among all combinations of parameters (filters, attributes, etc.) PT 0 (ParLOT trace that belongs to thread 0 of process 0) got impacted the most after we inject the bug.

The target `MPI_Allreduce()` that we injected the bug to, finds the rank (i.e., process) that has the “champion” result among all of ranks using `MPI_MIN` operator. Then that champion rank copies its “champion results” to a global data-structure and broadcast the “champion results” to all other ranks for the next time step. However, since we changed the `MPI_MIN` to `MPI_MAX` in only one of the ranks, the true champion rank would get lost, instead a false champion rank (which turned to be rank 0 or  $PT_{0,0}$ ) would broadcast its results as champion in the first time-step, causing a potential wrong answer.

Table X  
BUG 1: WRONG MPI OPERATION IN ALLREDUCE() CANDIDATE TABLE

Filter	Attribute	K: # of diff Clusters	# Objects in each Cluster (CL i)	Candidate PT Outliers
11.mpi.cust.0K10	sing.orig	2	CL 0:34	-
			CL 1:6	-
			CL 0:34	-
11.mpi.cust.0K10	sing.orig	3	CL 1:5	{2 3 4 27 29 }
			CL 2:1	{22 }
			CL 0:3	{24 38 39 }
11.mpi.cust.0K10	sing.orig	4	CL 1:31	-
			CL 2:5	{2 3 4 27 29 }
			CL 3:1	{22 }
11.mpi.cust.0K10	sing.log10	2	CL 0:34	-
			CL 1:6	-
			CL 0:34	-
11.mpi.cust.0K10	sing.log10	3	CL 1:5	{2 3 4 27 29 }
			CL 2:1	{22 }
			CL 0:3	{24 38 39 }
11.mpi.cust.0K10	sing.log10	4	CL 1:31	-
			CL 2:5	{2 3 4 27 29 }
			CL 3:1	{22 }
11.mpi.cust.0K10	sing.actual	2	CL 0:34	-
			CL 1:6	-
			CL 0:34	-
11.mpi.cust.0K10	sing.actual	3	CL 1:5	{2 3 4 27 29 }
			CL 2:1	{22 }
			CL 0:3	{24 38 39 }
11.mpi.cust.0K10	sing.actual	4	CL 1:31	-
			CL 2:5	{2 3 4 27 29 }
			CL 3:1	{22 }
11.mpi.cust.0K10	doub.orig	2	CL 0:39	-
			<b>CL 1:1</b>	<b>{0}</b>
11.mpi.cust.0K10	doub.orig	3	CL 0:32	-
			CL 1:7	-
			<b>CL 2:1</b>	<b>{0}</b>
11.mpi.cust.0K10	doub.orig	4	CL 0:32	-
			CL 1:7	-
			<b>CL 2:1</b>	<b>{0}</b>
11.mpi.cust.0K10	doub.log10	2	CL 0:39	-
			<b>CL 1:1</b>	<b>{0}</b>
11.mpi.cust.0K10	doub.log10	3	CL 0:32	-
			CL 1:7	-
			<b>CL 2:1</b>	<b>{0}</b>
11.mpi.cust.0K10	doub.log10	4	CL 0:32	-
			CL 1:7	-
			<b>CL 2:1</b>	<b>{0}</b>
11.mpi.cust.0K10	doub.actual	2	CL 0:39	-
			<b>CL 1:1</b>	<b>{0}</b>
11.mpi.cust.0K10	doub.actual	3	CL 0:32	-
			CL 1:7	-
			<b>CL 2:1</b>	<b>{0}</b>
11.mpi.cust.0K10	doub.actual	4	CL 0:32	-
			CL 1:7	-
			<b>CL 2:1</b>	<b>{0}</b>
		<b>TOP Suspicious Traces to check</b>	<b>1-0 2-2 3-3</b>	-





3) *Bug3: Wrong Size in MPI AllReduce() (all processes)*: We have injected a bug (row 3 table IX) where `MPI_Allreduce()` had been invoked with a wrong size.

**::MORE EXPLANATIONS ABOUT WHAT THE BUG IS::**

Similar to table X, the same ranking system tells us to check  $PT_{1,0}$  and  $PT_{3,0}$ . Note that the bug injected to only one process ( $P_3$ ) to have the wrong size.)

## EXPLANATIONS OF OBSERVATIONS:



4) *Bug4: Wrong Op in MPI AllReduce(): no effect!, program terminates fine:* maybe all images show some reflection

5) *Bug5: Wrong Size in next MPI AllReduce()(one process)::no effect, program terminates fine:* maybe all images show some reflection

6) *Bug6: Wrong Size in next MPI AllReduce()(all processes)::no effect, program terminates fine:* maybe all images show some reflection

7) *Bug7: Wrong Size in MPI Bcast()(one process)::* maybe all images show some reflection

8) *Bug8: Wrong Size in next MPI Bcast()(all processes)::no effect, program terminates fine:* maybe all images show some reflection

9) *3: Missing Critical Section one thread in on process:* I planted the bug (missing critical section) in process 2

## VI. RELATED WORK

### A. Program Understanding

- Score-P [14]
- TAU [15]
- ScalaTrace: Scalable compression and replay of communication traces for HPC [16]
- Barrier Matching for Programs with Textually unaligned barriers [17]
- Pivot Tracing: Dynamic causal monitoring for distributed systems - Johnathan mace [18]
- Automated Charecterization of parallel application communication patterns [19]
- Problem Diagnosis in Large Scale Computing environments [20]
- Probablistic diagnosis of performance faults in large-scale parallel applications [21]
- detecting patterns in MPI communication traces - robert preissl [22]
- D4: Fast concurrency debugging with parallel differntial analysis - bozhen liu [23]
- Marmot: An MPI analysis and checking tool - bettina krammer [24]
- MPI-checker - Static Analysis for MPI - Alexandrer droste [25]
- STAT: stack trace analysis for large scale debugging - Dorian Arnold [26]
- DMTracker: Finding bugs in large-scale parallel programs by detecting anomaly in data movements [27]
- SyncChecker: Detecting synchronization errors between MPI applications and libraries - [28]
- Model Based fault localization in large-scale computing systems - Naoya Maruyama [29]
- Synoptic: Studying logged behavior with inferred models - ivan beschastnikh [30]
- Mining temporal invariants from partially ordered logs - ivan beschastnikh [31]
- Scalable Temporal Order Analysis for Large Scale Debugging - Dong Ahn [32]
- Inferring and asserting distributed system invariants - ivan beschastnikh - stewart grant [33]

- PRODOMETER: Accurate application progress analysis for large-scale parallel debugging - subatra mitra [34]
- Automaded : Automata-based debugging for dissimilar parallel tasks - greg [35]
- Automaded : large scale debugging of parallel tasks with Automaded - ignacio [36]
- Inferring models of concurrent systems from logs of their behavior with CSight - ivan [37]

### B. Trace Analysis

- Trace File Comparison with a hierarchical Sequence Alignment algorithm [38]
- structural clustering : matthias weber [39]
- building a better backtrace: techniques for postmortem program analysis - ben liblit [40]
- automatically charecterizing large scale program behavior - timothy sherwood [41]

### C. Visualizations

- Combing the communication hairball: Visualizing large-scale parallel execution traces using logical time - katherine isaacs [42]
- recovering logical structure from charm++ event traces [43]
- ShiViz - Debugging distributed systems - [44]

### D. Concept Lattice and LCA

- Vijay Garg - Applications of lattice theory in distributed systems
- Dmitry Ignatov [?] - Concept Lattice Applications in Information Retrieval
- [7] [11] [45] [8] [12]

### E. Repetitive Patterns

- [46] [47] [48] [49] [50]

### F. STAT

Parallel debugger STAT[26]

- STAT gathers stack traces from all processes
- Merge them into prefix tree
- Groups processes that exhibit similar behavior into equivalent classes
- A single representative of each equivalence can then be examined with a full-featured debugger like TotalView or DDT

What STAT does not have?

- FP debugging
- Portability (too many dependencies)
- Domain-specific
- Loop structures and detection

45	(7)11.mem.ompcrit.cust.0K10	sing.actual	1:(3_0,4_0):0.75	1:(2_1,3_1):0.00	1:(3_2,6_2):0.57	1:(1_3,2_3):0.89
			2:(2_0,3_0):0.75	2:(0_1,4_1):0.00	2:(3_2,5_2):0.57	2:(0_3,7_3):0.89
			3:(1_0,6_0):0.75	3:(0_1,5_1):0.00	3:(2_2,4_2):0.50	3:(4_3,6_3):0.50
46	(7)11.mem.ompcrit.cust.0K10	sing.log10	1:(3_0,4_0):0.75	1:(2_1,3_1):0.00	1:(0_2,2_2):0.33	1:(1_3,2_3):0.33
			2:(2_0,3_0):0.75	2:(0_1,4_1):0.00	2:(3_2,7_2):0.33	2:(6_3,7_3):0.33
			3:(1_0,6_0):0.75	3:(0_1,5_1):0.00	3:(3_2,6_2):0.33	3:(1_3,4_3):0.20
47	(7)11.mem.ompcrit.cust.0K10	sing.orig	1:(3_0,4_0):0.75	1:(2_1,3_1):0.00	1:(0_2,2_2):0.33	1:(1_3,2_3):0.33
			2:(2_0,3_0):0.75	2:(0_1,4_1):0.00	2:(3_2,7_2):0.33	2:(6_3,7_3):0.33
			3:(1_0,6_0):0.75	3:(0_1,5_1):0.00	3:(3_2,6_2):0.33	3:(1_3,4_3):0.20

Figure 14. Part of ranking table for MisCrit 1-1

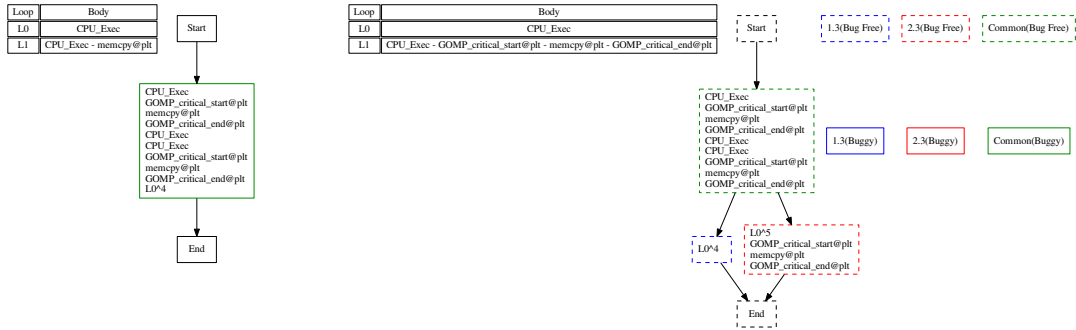


Figure 15. diffNLR of process 1 thread 3 and process 2 thread 3 buggy(missing critical section) vs. bug-free

## VII. CONCLUDING REMARKS

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## APPENDIX