DiffTrace: Efficient Whole-Program Trace Analysis and Diffing

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Abstract— Abstract to be written Index Terms—diffing, tracing, debugging

I. Introduction

BEGIN Ganesh

Debugging high performance computing code remains a challenge at all levels of scale. Conventional HPC debuggers [?], [?], [?] excel at many tasks such as examining the execution state of a complex simulation at a detailed level and allowing the developer to re-execute the program close to the point of failure. However, they do not provide a good understanding of why a program version that worked earlier failed upon upgrade or feature addition. Innovative solutions are needed to highlight the salient differences between two executions in a manner that makes debugging easier as well as more systematic. A recent study conducted under the auspices of the DOE [?] provides a comprehensive survey of existing debugging tools. It classifies them under four software organizations (serial, multithreaded, multi-process, and hybrid), six method types (formal methods, static analysis, dynamic analysis, nondeterminism control, anomaly detection, and parallel debugging), and lists a total of 30 specific tools. Despite this abundance of tools and approaches, many significant problems remain to be solved before debugging can be approached by the HPC community as a collaborative activity so that HPC developers can share their solutions and extend a common framework Need more build-up; will do later.

In this paper, we provide our fundamentally fresh look at debugging. We point out three significant problems that we have addressed in our work, and provide our preliminary solutions backed up by case studies. While our work has not (yet) addressed the situations in which millions of threads and thousands of processes run for days and produce an error, we strongly believe that we can get there only through a series of *rigorous* approaches that overcome key limitations found in conventional debugging approaches in a step-by-step manner, accompanied by careful measurements of the merits of the new approach. The main contribution of this paper is the first such critical measurements of our proposed approach.

a) Problem-1: Need to Generalize Approaches for Outlier Detection:: Almost all debugging approaches seek to find outliers ("unexpected executions") amongst thousands of

running processes and threads. The approach taken by most existing tools is to look for symptoms in a specific bugclass that they cover. Unfortunately, this approach calls for a programmer having a good guess of what the underlying problem might be, and to then pick the right set of tools to deploy. If the guess is wrong, the programmer has no choice but to refine their guess and look for bugs in another class, re-executing the application and hoping for better luck with another tool. This iterative loop of re-execution followed by applying a best-guess tool for the suspected bug class can potentially consume large amounts of execution cycles and also waste an expert developer's time.

Solution to Problem-1: Whole Program Tracing for Debugging: In this setting, our first contribution is a debugging approach in which the application is not merely run with a single symptom-specific tool attached as described earlier. Instead, we collect whole program traces of function calls and returns, using a PIN-based function call tracing facility called ParLoT that we have developed and previously reported [?]. We store these traces for potential examination by multiple tools and approaches. The advantage of whole program binary tracing supported in ParLoT is that we can collect function calls at any desired level of abstraction. For instance, if the programmer wants to cover activities at the MPI level, the OpenMP level and perhaps even lower levels (e.g., the MPI library or the OpenMP runtime), they can do so using ParLoT.

Clearly, the more APIs at which function calls are recorded, the more burdensome trace collection becomes. However, the advantage is that correspondingly more tools can then be applied to the collected traces. There is always a sweet-spot in this trade-off space, depending on the particular debugging situation. However, our fundamental insight is that *given the inevitability of heterogeneous programming* (the use of multiple concurrency models), it is important to be collecting traces from a few related APIs at a time, so that one can study bugs in one of the concurrency models or a bug resulting from a bad cross-model interaction from a *single run of the program*.

In our research, we have thus far demonstrated the advantage of ParLoT with respect to collecting both MPI and OpenMP traces from a *single run of a hybrid MPI/OpenMP program*. We demonstrate that from this single type of traces, it is possible to pick out MPI-level bugs or OpenMP-level bugs.

While we have not covered all these combinations in our work so far, the main contribution claimed is the ability to cover multiple APIs while debugging, and without re-executions or guess-work.

While this approach to whole-program tracing may sound extremely computation intensive, we employ novel on-the-fly compression techniques within ParLoT. In our previous study [?], we report compression efficiencies exceeding 16,000. This allows us to bring out the function call traces without significantly burdening the memory subsystem or I/O networks in the HPC cluster.

b) Problem-2: Need to Generalize Approaches for Outlier Detection:: Given that outlier detection is central to debugging, it is important to be employing efficient representations of the traces collected from threads and processes so that one can compute distances between these traces more systematically, without involving human reasoning in the loop. The representation must also be versatile enough to be able to "Diff" the traces¹ with respect to an extensible number of vantage points. These vantage points could be diffing with respect to process level activities, diffing with respect to thread-level activities, a combination thereof, or even finite sequences of process/thread calls (say, to locate changes in caller/callee relationships).

Solution to Problem-2: Use of Concept Lattices in Debugging: In DiffTrace, we employ concept lattices to amalgamate the collected traces. Concept lattices have previously been employed in HPC to perform structural clustering of process behaviors [?] to present performance data more meaningfully to users. The authors of that paper employ the notion of Jaccard distances to cluster performance results that are closely related to process structures (determined based on caller/callee relationships).

In DiffTrace, we employ incremental algorithms for building and maintaining concept lattices from the ParLoT-collected traces. In addition to Jaccard distances, in our work we also perform hierarchical clustering of traces and provide a tunable threshold for outlier detection. We believe that these uses of concept lattices and more refinement approaches for outlier detection are new in HPC debugging.

c) Problem-3: Loop Detection:: Most programs spend most of their time in loops. Therefore it is important to employ state-of-the-art algorithms for loop extraction from execution traces. It is also important to be able to diff two executions with respect to changes in their looping behaviors.

Solution to Problem-3: Rigorous Approaches to Loop Analysis: In DiffTrace, we employ the notion of NLRs (what does it stand for?) for extracting loops. Each repetitive loop structure is given an identifier, and nested loops are expressed as repetitions of this identifier exponentiated (as with regular expressions). This approach to summarizing loops can help manifest bugs where the program does not hang or crash, but nevertheless run differently in a manner that informs the developer engaged in debugging.

To summarize, the key contributions of this paper are the following [[fix the section numbers later]]:

- A method to organize function call traces collected from processes and threads into concept lattices, and a method to detect loops from dynamic traces (Section ??).
- Details of the algorithms employed in DiffTrace (Section ??).
- Experimental studies on a heterogeneous program called Iterated Local Champion Search (ILCS, Section ??).
- Strengths and limitations of DiffTrace, plans for future work (Section ??).

END Ganesh

[[Ganesh and Saeed have written some text before for the intro which is available in v0/intro.tex (also available but commented in current file). Current version is based on our discussion on May 8th]]

- Importance of whole program diffing : understand changes, debug (DOE REPORT [1])
- Efficient tracing supports selective monitoring at multiple levels
 - Bugs not there at a predictable API level
 - Prior work (ParLoT) supports whole program tr.
- Dissimilarity is important to know: bugs, changes during porting,...
- Key enablers of meaningful diffing:
 - Formal concepts (novel contrib to debugging)
 - Loop detection (loop diffing can help)
- · Importance, given the growing heterogeneity

** TODO: Highlights of results obtained as a result of the above thinking should be here. This typically comes before ROADMAP of paper.

In summary, this paper makes the following main contribu-

- A tunable tracing and trace-analysis tool-chain for HPC application program understanding and debugging
- A variation of the NLR algorithm to compress traces in lossless fashion for easier analysis and detecting (broken) loop structures
- An FCA-based clustering approach to efficiently classify traces with similar behavior
- A tunable ranking mechanism to highlight suspicious trace instances for deeper study
- A visualization framework that reflects the points of differences or divergence in a pair of sequences.

The rest of the paper is as follows:

Sec 2: Background

- Sec 3: Components

- Sec 4: Case Study: ILCS

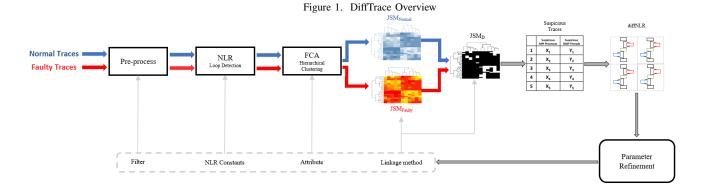
Sec 5: Related Work

Sec 6: Concluding Remarks

II. DIFFTRACE OVERVIEW

DiffTrace employs ParLOT's [2] whole-program functioncall and return traces for fault detection and localization.

¹Hence the name of our tool, **DiffTrace**.



ParLOT is based on the dynamic binary instrumentation toolkit Pin [3] and incrementally compresses the generated traces on-the-fly. It can capture functions at two levels: the main image, which does not include library code, and all *images*, which includes all application code. As the application runs, ParLOT generates per-thread trace files that contain the compressed sequence of the IDs of the executed functions. The compression mechanism is light-weight yet effective, thus not only reducing the required bandwidth and storage but also the runtime relative to not compressing the traces. As a result, ParLOT can capture whole-program traces at low overhead while leaving the majority of the disk bandwidth to the application. Using whole-program traces substantially reduces the number of overall debug iterations because it allows us to repeatedly analyze the traces offline with different filters. Despite these benefits, raw ParLOT traces are just compressed sequences of executed function IDs. Moreover, using ParLOT on a large-scale HPC application tends to yield thousands of long traces. These traces need to be filtered, grouped, and transformed into a meaningful representation that reveals important facts about the dynamic behavior of the program and hides uninteresting details. This is what DiffTrace does.

The DiffTrace tool-chain (Figure 1) provides the sophisticated infrastructure needed for iterative and configurable trace search space reduction for localizing errors in a set of traces. Considering a "successful" termination of the application as *normal behavior*, DiffTrace iteratively compares the traces from faulty runs against those from normal runs to progressively zero in on any abnormal behavior. Each abnormal behavior is considered a potential fault cause or manifestation. However, faults may occur or influence the program behavior at different locations and granularities. A fault may get triggered at some point in the code and be immediately visible, or it may only manifest itself some time later. DiffTrace feeds the normal and faulty traces through a sequence of data transformation and classification steps. It then suggests a few traces for a more detailed study (e.g., comparing the traces from a different vantage point) based on measuring and locating points of differences. Since each DiffTrace step can be parameterized in multiple ways, additional evidence about what went wrong may be obtained

Figure 2. Simplified MPI implementation of Odd/Even Sort

	Main Function	oddEvenSort()
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15	<pre>int main() { int rank,cp; MPI_Init() MPI Comm rank(, &rank); MPI Comm size(, &cp); // initialize data to sort int *data[data_size]; oddEvenSort(rank, cp); MPI_Finalize(); }</pre>	<pre>oddEvenSort(rank, cp){ for (int i=0; i < cp; i++) { int ptr = findPtr(i, rank); if (rank % 2 == 0) { MPI Send(, ptr,); MPI Recv(, ptr,); else { MPI_Recv(, ptr,); MPI_Send(, ptr,); } }</pre>

iteratively. In each iteration, DiffTrace puts the spotlight on a separate aspect of the program's dynamic behavior.

DiffTrace starts by *pre-processing* the traces. ParLOT traces are highly compressed and typically contain functions that are not of interest to the current analysis. Thus, DiffTrace first decompresses and prunes the traces. Loops appear as repetitive patterns, resulting in often long but redundant trace information. A "nested loop recognition" mechanism then extracts the loops from the traces. The resulting information not only serves as a lossless abstraction to ease the rest of the trace analysis but also as a per-thread "measure of progress". The control flow in parallel programs often follows a specific pattern such as SPMD, odd-even, or master/slave [?]. As a consequence, the traces of a single execution tend to fall into just a few "equivalence classes". definition of "fault" maybe? a software bug, node failure, a library version update or porting to a new system that causes failure to a working software To study the impact of a fault on the execution, DiffTrace classifies the collected traces and measures the similarity between the equivalence classes from the faulty and the faultfree execution. The basic idea is to find out which traces (and, consequently, processes/threads) are falling into a different class (i.e., cluster) when the fault is introduced. Based on the observed similarity among the two clusterings, DiffTrace suggests the topmost suspicious traces, i.e., the traces that suffer the most from the fault, for deeper analysis.

The rest of this section illustrates the DiffTrace components on sets of ParLOT traces from the aforementioned MPI

Table I PRE-DEFINED FILTERS

Category	Sub-Category	Description
Primary	Returns	Filter out all returns
riillary		Filter out the ".plt" function calls for external functions/procedures that
	PLT	their address needs to be resolved dynamically from Procedure Linkage
		Table (PLT)
	MPI All	Only keep functions that start with "MPI_"
MPI	MPI Collectives	Only keep MPI collective calls (MPI_Barrier, MPI_Allreduce, etc)
MITI	MPI Send/Recv	Only keep MPI_Send, MPI_Isend, MPI_Recv, MPI_Irecv and MPI_Wait
	MPI Internal Library	Keep all inner MPI library calls
	OMP All	Only keep OMP calls (starting with GOMP_)
OMP	OMP Critical	Only keep OMP_CRITICAL_START and OMP_CRITICAL_END
	OMP Mutex	Only keep OMP_Mutex calls
	Memory	Keep any memory related functions (memcpy, memchk, alloc, malloc, etc)
System	Network	Keep any network related functions (network, tcp, sched, etc)
System	Poll	Keep any poll related functions (poll, yield, sched, etc)
	String	Keep any string related functions (strlen, strcpy, etc)
Advanced	Custom	Any regular expression can be captured
Advanced	Everything	Does not filter anything

odd/even sort example (Figure 2). Odd/even sort is a parallel version of bubble sort and operates in two alternating phases: in the *even phase*, the even processes exchange (conditionally swap) values with their right neighbors, and in the *odd phase*, the odd processes exchange values with their right neighbors. The for loop in line 4 of oddEvenSort () iterates over the phases of the algorithm. Based on the phase, the appropriate partner for each rank is computed by the function findPtr() (line 6). The odd/even ranks then exchange their chunks of data (lines 9-13) and sort, merge, and copy operations are performed on the received data (denoted by . . . in line 15 for simplicity).

According to the MPI standard, MPI_Send may block, meaning MPI may buffer the outgoing message, in which case the send call completes before a matching receive is invoked, or it may choose not to buffer the message. In the latter case, the send call will not complete until a matching receive has been posted and the data has been transferred to the receiver. So, based on the dynamic behavior, swapping the statements on lines 11 and 12 of Figure ?? might end up causing a deadlock. DiffTrace can extract evidence from the traces to locate the cause of such deadlocks and other faults.

A. Pre-processing

Using ParLOT's decoder, each trace is first decompressed. Next, the desired functions are extracted based on predefined (Table I) or custom regular expressions (i.e., *filters*) and kept for later phases. All other trace information is discarded.

Table II shows the pre-processed traces (T_i) of odd/even sort with four processes. T_i is the trace that stores the function calls of process i.

B. Nested Loop Representation

Most programs, and HPC applications in particular, spend most of their runtime in *loops*. Function calls within a loop body yield *repetitive patterns* in ParLOT traces. Inspired by ideas for the detection of repetitive patterns in strings [51] and other data structures [50], we have adapted the Nested Loop Recognition (NLR) algorithm by Ketterlin et al. [13] to detect repetitive patterns in ParLOT traces (cf. Section III-A). Detecting such patterns can be used to measure the progress of each thread, revealing unfinished or broken loops that may be the consequence of a fault.

Table II
THE GENERATED TRACES FOR ODD/EVEN EXECUTION WITH FOUR

T_0	T_1	T_2	T_3
main	main	main	main
MPI_Init	MPI_Init	MPI_Init	MPI_Init
MPI_Comm_Rank	MPI_Comm_Rank	MPI_Comm_Rank	MPI_Comm_Rank
MPI_Comm_Size	MPI_Comm_Size	MPI_Comm_Size	MPI_Comm_Size
oddEvenSort	oddEvenSort	oddEvenSort	oddEvenSort
findPtr	findPtr	findPtr	findPtr
MPI_Send	MPI_Recv	MPI_Send	MPI_Recv
MPI_Recv	MPI_Send	MPI_Recv	MPI_Send
findPtr	findPtr	findPtr	findPtr
MPI_Send	MPI_Recv	MPI_Send	MPI_Recv
MPI_Recv	MPI_Send	MPI_Recv	MPI_Send
MPI_Finalize	MPI_Finalize	MPI_Finalize	MPI_Finalize

Table III NLR OF TRACES

T_0	T_1	T_2	T_3
MPI_Init	MPI_Init	MPI_Init	MPI_Init
MPI_Comm_Rank	MPI_Comm_Rank	MPI_Comm_Rank	MPI_Comm_Rank
MPI_Comm_Size	MPI_Comm_Size	MPI_Comm_Size	MPI_Comm_Size
L0 ^ 2	L1 ^ 4	L0 ^ 4	L1 ^ 2
MPI_Finalize	MPI_Finalize	MPI_Finalize	MPI_Finalize

For example, the loop in line 3 of oddEvenSort () (Figure 2) iterates four times when run with four processes. Thus each T_i contains four occurrences of either [MPI_Send-MPI_Recv] (even i) or [MPI_Recv-MPI_Send] (odd i). By keeping only MPI functions and converting each T_i into its equivalent NLR (Nested Loop Representation), Table II can be reduced to Table III where ${\bf L0}$ and ${\bf L1}$ represent the loop body [MPI_Send-MPI_Recv] and [MPI_Recv-MPI_Send], respectively. The integer after the ^ symbol in NLR represents the loop iteration count. Note that, since the first and last processes only have one-way communication with their neighbors, T_0 and T_3 perform only half as many iterations.

connect this section to next

C. Hierarchical Clustering via FCA

HPC applications often follow a regular pattern of executed functions. Due to this characteristic, per-thread function-call traces can generally be classified into a few equivalence groups based on their content. Such a classification would distinguish structurally different threads from each other (e.g., MPI processes from OpenMP threads in hybrid MPI+OpenMP applications) and reduce the search space into just a few representative classes of traces. In addition, the set of per-thread traces should be studied as "a whole" since there is a strong conceptual and causal relation among threads/processes. To integrate the collected traces into a single model of execution and forming "equivalence classes of traces", we have adapted the idea of Structural Clustering [42] by applying formal concept analysis (FCA) [14] techniques to our traces.

A formal context is a triple K = (G, M, I), where G is a set of **objects**, M is a set of **attributes**, and $I \subseteq G \times M$ is an incidence relation that expresses which objects have

Figure 3. Formal Concept Definition

For subsets $A \subseteq G$ of objects and subsets $B \subseteq M$ of attributes, one defines two derivation operators as follows:

 $A' = \{m \in M \mid (g,m) \in I \text{ for all } g \in A\}, \text{ and dually }$

 $B' = \{g \in G \mid (g,m) \in I \text{ for all } m \in B\}.$

Applying either derivation operator and then the other constitutes two closure operators:

 $A \mapsto A'' = (A')'$ for $A \subseteq G$ (extent closure), and $B \mapsto B'' = (B')'$ for $B \subseteq M$ (intent closure).

Definition of a formal concept: a pair (A,B) is a formal concept of a context (G, M, I) provided that:

 $A \subseteq G$, $B \subseteq M$, A' = B, and B' = A.

Equivalently and more intuitively, (A,B) is a **formal concept** precisely when:

- every object in A has every attribute in B,
- for every object in G that is not in A, there is some attribute in B that the object does not have,
- for every attribute in M that is not in B, there is some object in A that does not have that attribute.

Table IV FORMAL CONTEXT OF ODD/EVEN SORT EXAMPLE

	MPI_Init()	MPI_Comm_Size()	MPI_Comm_Rank()	L0	L1	MPI_Finalize()
Trace 0	×	×	×	×		×
Trace 1	×	×	×		×	×
Trace 2	×	×	×	×		×
Trace 3	×	×	×		×	×

which attributes. Table IV shows the formal context of the preprocessed odd/even sort traces.

In this context, attributes are trace entries (function calls or detected loop bodies) without their frequency (e.g., iteration counts). Table (the table that shows attributes in the next section) shows the attributes that we have extracted from the traces. The context shows that all traces include the functions MPI_Init(), MPI_Comm_size(), MPI_Comm_rank() and MPI_Finalize(). The even traces contain the loop *L0* and the odd traces the loop *L1*. Definition of formal concept (needed?) figure 3:

A concept lattice can be derived from a formal context by specifying formal concepts (Figure 3) and a partial order on them. Concept lattices are represented as a directed acyclic graph where concepts are nodes and the order on them determines the edges. Figure 4 shows the concept lattice derived from the formal context in Table IV and is interpreted as follows:

- The top node indicates that all traces share MPI_Init(), MPI_Comm_size(), MPI_Comm_rank() and MPI_Finalize().
- The bottom node signifies that none of the traces share all attributes.
- The middle nodes show that T_0 and T_2 are different from T_1 and T_3 .

Once the redundant labels are removed from the lattice, each object (trace) and attribute appears in the lattice exactly once. Consequently, the nodes of the lattice form the desired grouping, since it is guaranteed that each trace belongs to exactly one group. However, the concept lattice itself does not provide similarity values for the distinct groups of traces.

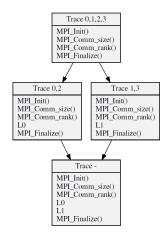


Figure 4. Sample Concept Lattice from Obj-Atr Context in Table IV

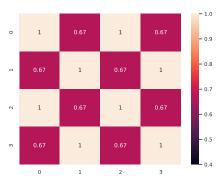


Figure 5. Pairwise Jaccard Similarity Matrix (JSM) of MPI processes in sample code

The Jaccard Index, also known as Intersection over Union, measures the distance between sets A and B in terms of the ratio of the intersection size of A and B over the size of their union. The complete pairwise Jaccard Similarity Matrix (JSM) can easily be computed from the concept lattice.

For any pair of (T_i,T_j) , the number of attributes in the Lowest Common Ancestor (LCA) node of T_i and T_j in the lattice is the number of attributes that T_i and T_j have in common (intersection). The sum of the number of attributes of the nodes on the path from each T_i and T_j to their LCA is the union. This property is one of the motivations for using concept lattices as classifier since it makes computing the union and intersection easy and fast.

Some algorithms for extracting concepts from contexts and constructing the concept lattice require the whole context to be present in the memory. For large-scale executions with thousands of threads, this is not feasible. Through an incremental concept-lattice-construction approach, DiffTrace extracts attributes (table that shows attributes) from NLR traces and injects them into a concept lattice one trace at a time (cf. Section III-B). Figure 5 shows the heatmap (explain this) of the JSM obtained from the concept lattice in Figure 4. DiffTrace uses the JSM to form equivalence classes of traces

by hierarchical clustering. Next, we show how the differences between two hierarchical clusterings from two executions (faulty vs. normal) reveal which traces have been affected the most by the fault.

D. Detecting Suspicious Traces via DiffJSM

So far, we have explained how DiffTrace can narrow down the search space from numerous long traces to just a few equivalent JSMs (i.e., clusters). $JSM_{normal}[i][j]$ ($JSM_{faulty}[i][j]$) shows the Jaccard similarity score of T_i and T_j from the normal trace (T_i' and T_j').

However, we are interested in detecting what changed the most due to the fault. The hierarchical clustering based on *DiffJSM*, followed by the subtraction of the faulty JSM from its corresponding normal JSM, typically puts the trace(s) that changed the most in a separate, often singleton, cluster and thus detects the outlier.

$$DiffJSM = |JSM_{faulty} - JSM_{normal}|$$

The resulting outlier traces are candidates for the potential cause of the change in the program behavior and thus a potential fault root cause or fault manifestation. However, a single iteration of DiffTrace (with a single set of parameters shown as a dashed box in Figure 1) may not detect the problem. To improve the accuracy, we have sorted the suggestion table based on the *B-score* similarity metric of two hierarchical clusterings [20] (cf. Section III-C).

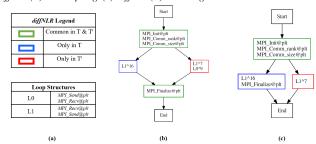
To evaluate the effectiveness of DiffJSM, we planted two artificial bugs (swapBug and dlBug) in the code from Figure 2 and ran it with 16 processes. swapBug swaps the order of MPI_Send and MPI_Recv in rank 5 after the seventh iteration of the loop in line 3 of oddEvenSort, simulating a potential deadlock. dlBug simulates an actual deadlock in the same location (rank 5 after the seventh iteration). Upon collection of ParLOT traces from the execution of the buggy code versions, DiffTrace first decompresses them and filters out all non-MPI functions. Then two major loops are detected, L0 and L1 (Figure 6-(a)), that are supposed to loop 16 times in the even and odd traces, respectively (except for the first and last traces, which loop just eight times).

After constructing concept lattices and their corresponding JSMs, trace 5 appears as the trace that got affected the most by the bugs because row 5 (showing the similarity score of T_5 relative to all other traces) $(JSM_{normal}[5][i]$ for $i \in [0,16)$) changed the most after the bug was introduced. * The differences between the suggested suspicious trace (T_s) and its corresponding normal trace (T_s) is visualized by diffNLR.

1) diffNLR: To highlight the differences in an easy-tounderstand manner, DiffTrace visually separates the common and different blocks of a pair of pre-processed traces via diffNLR, a graphical visualization of the diff algorithm [19].

diff takes two sequences S_A and S_B and computes the minimal edit to convert S_A to S_B . This algorithm is used in the GNU diff utility to compare two text files and in git for efficiently keeping track of file changes. Since ParLOT preserves the order of function calls, each trace T_i is totally ordered. Thus diff can expose the differences of a pair of

Figure 6. (a) The legend of diffNLR and the list of loop structures (b) diffNLR(5) of swapBug (c) diffNLR(5) of dlBug



Ts. diffNLR aligns common and different blocks of a pair of sequences (e.g., traces) horizontally and vertically, making it easier for the analyst to see the differences at a glance. For simplicity, our implementation of gdiff only takes one argument x that denotes the suspicious trace.

 $\operatorname{diffNLR}(x) \equiv \operatorname{diffNKR}(T_x, T_x')$ where T_x is the trace of thread/process x of a normal execution and T_x' is the corresponding trace of the faulty execution.

Figure 6-(b) shows the diffNLR(5) of swapBug where T_5 iterates over the loop [MPI_Recv - MPI_Send] 16 times (L1^16) after the MPI initialization while the order swap is well reflected in T_5' (L1^7 - L0^9). Both processes seem to terminate fine by executing MPI_Finalize(). However, diffNLR(5) of dlBug (Figure 6-(c)) shows that, while T_5 executed MPI_Finalize, T_5' got stuck after executing L1 seven times and never reached MPI_Finalize.

This example illustrates how our approach can locate the part of each execution that was impacted by a fault. Having an understanding of *how the application should behave normally* can reduce the number of iterations by picking the right set of parameters sooner.

III. ALGORITHMS UNDERLYING DIFFTRACE

A. NLR

Nested Loop Recognition (NLR) algorithm [13] is originally designed for prediction and compression of data access traces (memory addresses) through detecting the "linear progression function" in the sequence of addresses. Two main operations of NLR algorithm are 1) recognizing the start of a loop and forming its initial syntactic structure and 2) recognizing if what follows a loop is just another iteration and extend the upper bound. These operations are performed incrementally and recursively to recognize loops at different depths (linear function of loop index i and depth d) until the input sequence is completely consumed and no more loop is detected.

Previous to NLR, Kobayashi [12] proposed a similar bottom-up strategy to build loop nests from a trace (sequence of string instructions) where each recognized loop is replaced with a new symbol, which is remembered so that identical loops at different locations can be identified. This process is restarted once the whole trace has been analyzed for depth-2 loops, and so on until some replacement is performed.

Inspired by Kobayashi approach, we have re-implemented the NLR algorithm (DiffTrace-NLR) for detecting repetitive patterns in a sequence of function calls (trace) and summarizing the trace to NLR representation. DiffTrace-NLR works by incrementally pushing trace entries (function IDs) into a stack of *elements* (i.e., function IDs and already detected loop structures). Whenever an element is pushed to the stack, the upper elements of the stack are recursively examined for potential loop detection or loop extensions (Procedure 1).

```
Reduce (S):
   for i:1....3K do
      b = i/3
      if Top 3 b-long elements of S are isomorphic then
         pop i elements from S
         LB = S[b:1] \ LC = 3
         LS = (LB, LC)
         push LS to S
         add LB to Loop Table
         Reduce (S)
      end
      if S[i] is a loop (LS) and S[i-1:1]
      isomorphic to its loop bodyLB then
         LC = LC + 1
         pop i-1 elements from S
         Reduce (S)
      end
```

Procedure 1: Reduce Procedure Adapted From the NLR algorithm

Each loop structure LS is a tuple of (loop body LB, loop count LC) where LB is a sequence of elements and LC is an integer showing the frequency of consecutive occurrence of LB. To avoid coincidental regularity, the algorithm needs at least three consecutive terms to form a LS. The Reduce procedure checks to see if any three consecutive sub-sequences of top of the stack are isomorphic. In our context, subsequences are isomorphic if they have equal lengths and all correspoding elements of each subsequence are identical. Then the three subsequences are popped from the stack and the LS=(LB,3) is pushed to stack (LB is either of subsequences). We store all distinct LBs into a hash-table, assigning each a unique ID which can be applied as heuristic to detect loops in other traces and more readable representation (e.g., LB_i^*LC). The maximum length of subsequences to examin is decided by a fixed priori K. The complexity of NLR algorithm is $\Theta(K^2N)$ where N is the size of the input.

B. Concept Lattice Construction

- 1 paragraph background on FCA
- 1-2 paragraphs on advantages of FCA and what we would gain from FCA? Answer: Full pair-wise Jaccard Similarity Matrix (JSM)
- 1 paragraph how JSMs are going to help us (referring to the major figure at the beginning of this section)

- Some background about Jaccard Similarity Score
- How to obtain full pair-wise Jaccard Similarity Matrix (JSM) from a concept lattice (e.g., LCA approach)
- 1-2 paragraphs on CL generation (related work and our approach)
 - Batch vs. Incremental [18]
 - Complexity: $O(2^{2K}||E||)$ where K is an upper bound for number of attributes (e.g., distinct function calls in the whole execution) and ||E|| is the number of objects (e.g., number of PTs).
- 1-2 paragraphs (+ 1-2 figures) explaining the FCA ideas on odd/even sort example.

There are algorithms that can extract concepts and their partial order from a formal context as a batch or incrementally. For large scale executions

Other kinds of similarity indexes can be extracted from concept lattices [17], in addition to many applications of concept lattices properties have been exploited in many computer science fields have been e different similarity metrics can be derived from the

C. Hierarchical Clustering, Construction and Comparison

JSMs and Linkage functions from Scipy B-score to compare clusterings

Table V ILCSTSP.MC1-MC-6-N1.M.8.AUTO

Filter	Attributes	Link Method	Thresh	B-score	Top Procs (JSMD)	TOP Threads(JSMD)
01.mem.ompall.cust.0K10	sing.actual	average	4	0.308594		6.2 , 6.4 , 5.2 , 5.4 , 7.3 ,
11.mem.ompall.cust.0K10	sing.actual	average	4	0.308594		6.2 , 6.4 , 5.2 , 5.4 , 7.3 ,
01.mem.ompmutex.cust.0K10	sing.actual	weighted	4	0.321883		6.2 , 3.4 , 5.3 , 4.2 , 7.4 ,
11.mem.ompmutex.cust.0K10	sing.actual	weighted	4	0.321883		6.2 , 3.4 , 5.3 , 4.2 , 7.4 ,
11.plt.mem.mpi.ompcrit.cust.0K10	doub.actual	weighted	4	0.324427	3,	0.2 , 1.1 , 3.1 , 4.3 , 4.4 , 5.2 ,
01.plt.mem.mpi.ompcrit.cust.0K10	doub.actual	weighted	4	0.324427	3,	0.2 , 1.1 , 3.1 , 4.3 , 4.4 , 5.2 ,
01.mem.ompall.cust.0K10	doub.actual	weighted	4	0.33266	3,	6.4 , 7.1 , 3.4 , 4.3 , 4.4 , 5.2 ,
11.mem.ompall.cust.0K10	doub.actual	weighted	4	0.33266	3,	6.4 , 7.1 , 3.4 , 4.3 , 4.4 , 5.2 ,
01.mem.ompcrit.cust.0K10	sing.actual	weighted	4	0.354396	6,	7.2 , 7.4 , 3.4 , 4.2 , 4.3 , 4.4 ,
11.mem.ompcrit.cust.0K10	sing.actual	weighted	4	0.354396	6,	7.2 , 7.4 , 3.4 , 4.2 , 4.3 , 4.4 ,
11.plt.mem.mpi.ompcrit.cust.0K10	doub.actual	average	4	0.381646		3.3 . 2.4 .

Table VI ILCSTSP.MC1-MC-2-2.M.8.AUTO

Filter	Attributes	Link Method	Thresh	B-score	Top Procs (JSMD)	TOP Threads(JSMD)
01.mem.ompall.cust.0K10	doub.actual	weighted	4	0.309391	3,	6.2 , 6.4 , 7.1 , 7.4 , 1.3 , 3.1 ,
11.mem.ompall.cust.0K10	doub.actual	weighted	4	0.309391	3,	6.2 , 6.4 , 7.1 , 7.4 , 1.3 , 3.1 ,
11.plt.mem.cust.0K10	doub.actual	weighted	4	0.318003	7,4,	1.3 , 2.2 , 3.3 , 3.4 , 4.2 , 4.3 ,
01.plt.mem.cust.0K10	doub.actual	weighted	4	0.318003	7,4,	1.3 , 2.2 , 3.3 , 3.4 , 4.2 , 4.3 ,
01.mem.ompall.cust.0K10	doub.actual	average	4	0.34462	7,3,	7.1 , 1.3 , 1.4 , 2.2 , 2.3 , 3.1 ,
11.mem.ompall.cust.0K10	doub.actual	average	4	0.34462	7,3,	7.1 , 1.3 , 1.4 , 2.2 , 2.3 , 3.1 ,
11.plt.mem.mpi.ompcrit.cust.0K10	doub.actual	average	4	0.350702	7,3,	6.2 , 6.3 , 7.2 , 2.4 , 3.3 , 4.2 ,
01.plt.mem.mpi.ompcrit.cust.0K10	doub.actual	average	4	0.350702	7,3,	6.2 , 6.3 , 7.2 , 2.4 , 3.3 , 4.2 ,
11.plt.mem.cust.0K10	doub.actual	weighted	3	0.357334	7,	2.2 , 3.3 , 3.4 , 4.3 , 4.4 ,
01.plt.mem.cust.0K10	doub.actual	weighted	3	0.357334	7,	2.2 , 3.3 , 3.4 , 4.3 , 4.4 ,
01.mem.ompall.cust.0K10	doub.actual	weighted	3	0.380481	3,	7.1 , 1.3 , 3.1 , 4.3 , 4.4 ,

IV. CASE STUDY: ILCS

Table IX ILCSTSP.BC1-WS-3-NN.M.8.AUTO

Filter	Attributes	Link Method	Thresh	B-score	Top Procs (JSMD)	TOP Threads(JSMD)
11.mpi.cust.0K10	sing.actual	weighted	4	0.385229	6,	6.2 , 6.3 , 6.4 , 7.2 , 7.3 , 7.4 ,
11.mpiall.cust.0K10	sing.actual	weighted	4	0.385229	6,	6.2 , 6.3 , 6.4 , 7.2 , 7.3 , 7.4 ,
01.mpiall.cust.0K10	sing.actual	weighted	4	0.385229	6,	6.2 , 6.3 , 6.4 , 7.2 , 7.3 , 7.4 ,
01.mpicol.cust.0K10	sing.actual	weighted	4	0.385229	6,	6.2 , 6.3 , 6.4 , 7.2 , 7.3 , 7.4 ,
11.mpicol.cust.0K10	sing.actual	weighted	4	0.385229	6.	6.2 , 6.3 , 6.4 , 7.2 , 7.3 , 7.4 ,
01.mpi.cust.0K10	sing.actual	weighted	4	0.385229	6,	6.2 , 6.3 , 6.4 , 7.2 , 7.3 , 7.4 ,
01.plt.cust.0K10	sing.actual	weighted	4	0.448188	7,3,	6.2 , 6.4 , 7.1 , 7.4 , 3.3 , 3.4 ,
11.plt.cust.0K10	sing.actual	weighted	4	0.448188	7,3,	6.2 , 6.4 , 7.1 , 7.4 , 3.3 , 3.4 ,
11.mpi.cust.0K10	sing.actual	weighted	3	0.465043	6,	6.2 , 6.3 , 6.4 , 7.2 , 7.3 , 7.4 ,
11.mpiall.cust.0K10	sing.actual	weighted	3	0.465043	6,	6.2 , 6.3 , 6.4 , 7.2 , 7.3 , 7.4 ,
01.mpiall.cust.0K10	sing.actual	weighted	3	0.465043	6.	6.2 . 6.3 . 6.4 . 7.2 . 7.3 . 7.4 .

Table VII ILCSTSP.AR2-WO-2-NN.M.8.AUTO

Filter	Attributes	Link Method	Thresh	B-score	Top Procs (JSMD)	TOP Threads(JSMD)
01.plt.cust.0K10	doub.actual	average	4	0.392446	6,	2.3 , 2.4 , 4.2 , 4.3 , 4.4 ,
11.plt.cust.0K10	doub.actual	average	4	0.392446	6,	2.3 , 2.4 , 4.2 , 4.3 , 4.4 ,
01.plt.cust.0K10	sing.log10	centroid	4	0.945946	0,	6.2, 0.4, 1.1, 0.1, 2.3, 2.4,
01.plt.cust.0K10	sing.log10	median	4	0.945946	0,	6.2, 0.4, 1.1, 0.1, 2.3, 2.4,
11.plt.cust.0K10	sing.log10	centroid	4	0.945946	0,	6.2, 0.4, 1.1, 0.1, 2.3, 2.4,
11.plt.cust.0K10	sing.log10	median	4	0.945946	0,	6.2, 0.4, 1.1, 0.1, 2.3, 2.4,
01.plt.cust.0K10	sing.log10	median	3	0.947368	0 ,	6.2 , 0.4 , 1.1 , 0.1 , 2.3 , 2.4 ,
11.plt.cust.0K10	sing.log10	median	3	0.947368	0,	6.2, 0.4, 1.1, 0.1, 2.3, 2.4,
01.plt.cust.0K10	sing.log10	complete	3	1	0,	0.1 , 7.1 , 1.1 , 3.1 , 3.3 ,
01.plt.cust.0K10	sing.log10	complete	4	1	0,	0.1 , 7.1 , 1.1 , 3.1 , 3.3 ,
01.plt.cust.0K10	sing.log10	average	4	1	0.	0.1 . 7.1 . 1.1 . 3.1 . 3.3 .

Table VIII ILCSTSP.BC2-WR-3-NN.M.8.AUTO

Filter	Attributes	Link Method	Thresh	B-score	Top Procs (JSMD)	TOP Threads(JSMD)
01.plt.cust.0K10	doub.actual	centroid	4	0.512309	2,	6.4 , 7.3 , 1.2 , 1.3 , 2.1 , 2.2 ,
11.plt.cust.0K10	doub.actual	centroid	4	0.512309	2,	6.4 , 7.3 , 1.2 , 1.3 , 2.1 , 2.2 ,
01.plt.cust.0K10	sing.actual	average	4	0.513221	7,	3.4 , 5.2 , 4.2 , 4.3 ,
11.plt.cust.0K10	sing.actual	average	4	0.513221	7,	3.4 , 5.2 , 4.2 , 4.3 ,
11.mpi.cust.0K10	sing.actual	median	4	0.544807	0,	0.1, 6.4, 7.3, 7.4, 1.3, 0.2,
11.mpiall.cust.0K10	sing.actual	median	4	0.544807	0,	0.1, 6.4, 7.3, 7.4, 1.3, 0.2,
01.mpiall.cust.0K10	sing.actual	median	4	0.544807	0,	0.1, 6.4, 7.3, 7.4, 1.3, 0.2,
01.mpicol.cust.0K10	sing.actual	median	4	0.544807	0,	0.1, 6.4, 7.3, 7.4, 1.3, 0.2,
11.mpicol.cust.0K10	sing.actual	median	4	0.544807	0,	0.1 , 6.4 , 7.3 , 7.4 , 1.3 , 0.2 ,
01.mpi.cust.0K10	sing.actual	median	4	0.544807	0,	0.1, 6.4, 7.3, 7.4, 1.3, 0.2,
01 plt cust 0K10	doub actual	centroid	3	0.639268	2	64 73 12 13 21 22

V. CASE STUDY: ILCS

{

A. Experimental Methodology

So far, we are able to collect whole-program execution traces, preprocess them (decompress, filter, detect loops, extract attributes) and inject each PT to concept lattice data structure. Concept lattices help us having a single model for the execution of HPC application with thousands of processes/threads. Concept lattices also classify PTs based on their Jaccard distance. Full pair-wise Jaccard distance matrix can be extracted from the concept lattice in linear time and reduces the search space from thousands of PTs to just a few equivalent classes of PTs. Studying JSM by itself helps the user to understand the program behavior as a whole, and how each process/thread behaving. However, comparing the JSM of the bug-free version of the application versus the buggy version would reveal insights about how the bug impacted the behavior of the application. In particular, we are interested to see how the bug changes the formation of equivalent classes of PTs. Inspired by a method for comparing two different clustering [20], we count the number of objects (PTs) in each cluster and see which PT(s) fall into different clusters once the bug is introduced. A set of candidate PTs then would be reported to the user for more in-depth study. Here is where we take advantage of diffNLR to see how does the bug changes the control flow of a candidate PT comparing to its corresponding PT of native run.

Table 7 shows different parameters that we can pre-process PTs with. Each combination of these parameters would result in a different concept lattice, thus different JSM and different clusterings. A table similar to XI is created for each injected bug. Each row of the table is showing the set of parameters used to create JSMs. Then by calculating |JSM(buggy) - JSM(bugfree)| we are interested to see which PT changes the most after the bug injected and falls into a single cluster. The object(s) in the cluster with the fewest members (below a threshold) are potential candidates of threads that are manifesting the bug and the diff(buggy,bugfree) is in our interest to see how does the bug changes its control flow.

B. Case Study: ILCS-TSP

Here is the ILCS framework pseudo-code. User needs to write CPU_Init(), CPU_Exec() and CPU_Output().

```
int main(argc, argv){
   MPI_Init();
   MPI_Comm_size()
   MPI_Comm_rank(my_rank)
   // Figuring local number of CPUs
   MPI_Reduce() // Figuring global number of CPUs
   CPU_Init();
   // For storing local champion results
   champ[CPUs] = malloc();
   MPI_Barrier();
   #pragma omp parallel num_threads(CPUs+1)
   //
```

```
rank = omp_get_thread_num()
  if (rank == 0) { //communication thread
   do{
    //Find and report the thread with
    //local champion, global champion
    MPI AllReduce();
    //Find and report the process with
    //global champion
    MPI AllReduce();
    //The process with the global champion
    //copy its results to bcast_buffer
    if (my_rank == global_champion){
     #pragma omp cirtical
     memcpy(bcast_buffer,local_champ)
    //Broadcast the champion
    MPI_Bcast(bcast_buffer)
   } while (no_change_threshold);
   cont=0 // signal worker threads to stop
  } else { // worker threads
   while (cont) {
    // Calculate Seed
    local_result = CPU_Exec()
    if (local result < champ[rank]){</pre>
     #pragma omp cirtical
     memcpy(champ[rank],local_result)
 //Find and report the thread with
 //local champion, global champion
 MPI_AllReduce();
 //Find and report the process with
 //global champion
 MPI_AllReduce();
 // The process with the global champion
 // copy its results to bcast_buffer
 if (my_rank == global_champion){
 #pragma omp cirtical
 memcpy (bcast buffer, local champ)
 //Broadcast the champion
 MPI_Bcast(bcast_buffer)
 if (my_rank==0)
  CPU_Output (champ)
 MPI_Finalize()
/* User code for TSP problem */
CPU Init(){
 // Read In data from cities
 // Calculate distances
```

	Filters											
Pı	Prime		eneral		MPI		OMP		Other	CL Attributes		Clustering
Filter	Descriptio n	Filter	Description	Filter	Descriptio n	Filter	Description	Filter	Description	OL AMBUICS		
ret	Filter Returns	@plt	@plt	mpi	MPI	ompcrit	OMP critical	Custom	Defining specific regex to filter		ects: Traces set of <atr:freq></atr:freq>	single
.plt	Filter .plts	mem	Memory related malloc memcpy etc	mpiall	MPI MPID PMPI	ompmu tex	OMP mutex	incEveryt hing	Include whatever is not in the Filters	Single: set of single trace entries atr: sing	No Frequency: only presence of attribute entries matters freq:-	complete
		net	Network related	mpicol	MPI collectives	ompall	OMP all functions			Double: set of 2-consecutive entries atr: doub	Log10: log(freq) of each entry matters (for large frequency numbers freq: log10(#atr)	average
		poll	Poll Related poll, yield	mpisr	MPI send/recv						Actual: actual frequency of each entry matters freq: #atr	weighted
		str	String related stcpy strcmp etc									centroid
												median
												ward

Figure 7. Filters, Attributes and other Parameters used to pre-process ParLOT Traces (PTs)

```
// Return data structure to store champion
}
CPU_Exec(){
   // Find local champions (TSP tours)
}
CPU_Output(){
   // Output champion
}
```

Table ?? describes the bug that I injected to ILCS-TSP

Table X
INJECTED BUGS TO ILCS-TSP

ID	T1	D	Description
ID	Level	Bugs	Description
1		allRed1wrgOp-1-all-x	Different operation (MPI_MAX) in only one process (buggyProc = 2) for MPI_ALLREDUCE() in Line 21
2		allRed1wrgSize-1-all-x	Wrong size in only one process (buggyProc = 2) for MPI_ALLREDUCE() in Line 21
3		allRed1wrgSize-all-all-x	Wrong Size in all processes for MPI_ALLREDUCE() in Line 21
4	MPI	allRed2wrgOp-1-all-x	Different operation (MPI_MAX) in only one (buggyProc) for first MPI_ALLREDUCE() – L277:ilcsTSP.c
5	IVIFI	allRed2wrgSize-1-all-x	Wrong size in only one (buggyProc) for first MPI_ALLREDUCE() – L277:ilcsTSP.c
6		allRed2wrgSize-all-all-x	Wrong Size in all processes for second MPI_ALLREDUCE() – L277:ilcsTSP.c
7		bcastWrgSize-1-all-x	Wrong Size in only one (buggyProc) of MPI_Bcast() – L290:ilcsTSP.c
8		bcastWrgSize-all-all-x	Wrong Size n all processes for MPI_Bcast() – L240:ilcsTSP.c
9		misCrit-1-1-x	Missing Critical Section in buggyProc and buggyThread – L170:ilcsTSP.c
10		misCrit-all-1-x	Missing Critical Section in buggyThread and all processes – L170:ilcsTSP.c
11		misCrit-1-all-x	Missing Critical Section in buggyProc and all threads – L170:ilcsTSP.c
12		misCrit-all-all-x	Missing Critical Section in all procs and threads – L170:ilcsTSP.c
13	OMP	misCrit2-1-1-x	Missing Critical Section in buggyProc and buggyThread – L230:ilcsTSP.c
14	OMF	misCrit2-all-1-x	Missing Critical Section in buggyThread – L230:ilcsTSP.c
15		misCrit2-1-all-x	Missing Critical Section in buggyProc and all threads – L230:ilcsTSP.c
16		misCrit2-all-all-x	Missing Critical Section in all procs and threads – L230:ilcsTSP.c
17		misCrit3-1-all-x	Missing Critical Section in buggyProc and all threads – L280:ilcsTSP.c
18		misCrit3-all-all-x	Missing Critical Section in all procs and threads – L280:ilcsTSP.c
19	General	infLoop-1-1-1	Injected an infinite loop after CPU_EXEC() in buggyProc,buggyThread & buggyIter L164:ilcsTSP.c

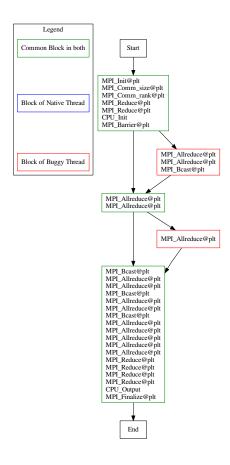


Figure 8. Bug1: diffNLR $PT_{0,0}$ - buggy vs. native

1) Bug1: Wrong Operation in MPI AllReduce(): We have injected a bug (row 1 table X) where MPI_Allreduce() had been invoked with a wrong operation in one of the processes (P_2)(MPI_MAX instead of MPI_MIN).

::What is the runtime reaction to this bug:: Program terminated well without any error, crash, hang or throwing any exception. But the results might be corrupted. This might be a silent bug that diffTrace could reveal

The last row of table XI is telling us that among all combinations of parameters (filters, attributes, etc.) PT 0 (ParLOT trace that belongs to thread 0 of process 0 got impacted the most after we inject the bug.

The target MPI_Allreduce() that we injected the bug to, finds the rank (i.e., process) that has the "champion" result among all of ranks using MPI_MIN operator. Then that champion rank copies its "champion results" to a global data-structure and broadcast the "champion results" to all other ranks for the next time step. However, since we changed the MPI_MIN to MPI_MAX in only one of the ranks, the true champion rank would get lost, instead a false champion rank (which turned to be rank 0 or $PT_{0,0}$) would broadcast its results as champion in the first time-step, causing a potential wrong answer.

Table XI
BUG 1: WRONG MPI OPERATION IN ALLREDUCE() CANDIDATE TABLE

Filter	Attribute	K: # of	# Objects in each	Candidate PT	
1 11101		diff Clusters	Cluster (CL i)	Outliers	
11.mpi.cust.0K10	sing.orig	2	CL 0:34	-	
1			CL 1:6	-	
11			CL 0:34	- (2.2.4.27.20.)	
11.mpi.cust.0K10	sing.orig	3	CL 1:5 CL 2:1	{2 3 4 27 29 }	
				{22 }	
	sing.orig	4	CL 0:3 CL 1:31	{24 38 39 }	
11.mpi.cust.0K10			CL 1:31 CL 2:5	{2 3 4 27 29 }	
			CL 2.3 CL 3:1	{22}	
			CL 0:34	122 5	
11.mpi.cust.0K10	sing.log10	2	CL 0.54	-	
	sing.log10	3	CL 0:34	_	
11.mpi.cust.0K10			CL 1:5	{2 3 4 27 29 }	
l			CL 2:1	{22}	
			CL 0:3	{24 38 39 }	
11 OV 10	-:110	4	CL 1:31	-	
11.mpi.cust.0K10	sing.log10	4	CL 2:5	{2 3 4 27 29 }	
			CL 3:1	{22 }	
11.mpi.cust.0K10	sing.actual	2	CL 0:34	-	
11.mpi.cust.oK10	Sing.actual	2	CL 1:6	-	
			CL 0:34	-	
11.mpi.cust.0K10	sing.actual	3	CL 1:5	{2 3 4 27 29 }	
			CL 2:1	{22 }	
	sing.actual	4	CL 0:3	{24 38 39 }	
11.mpi.cust.0K10			CL 1:31	-	
T T T T T T T T T T T T T T T T T T T			CL 2:5	{2 3 4 27 29 }	
			CL 3:1	{22 }	
11.mpi.cust.0K10	doub.orig	2	CL 0:39 CL 1:1	- (0)	
			CL 1:1 CL 0:32	{0}	
11.mpi.cust.0K10	doub.orig	3	CL 0:32 CL 1:7	-	
11.iiipi.cust.0K10			CL 1.7 CL 2:1	{0}	
	doub.orig	4	CL 0:32	\ U \\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\	
11.mpi.cust.0K10			CL 0.32	-	
11.mpi.cust.orc10			CL 2:1	{0}	
		_	CL 0:39	-	
11.mpi.cust.0K10	doub.log10	2	CL 1:1	{0}	
			CL 0:32	-	
11.mpi.cust.0K10	doub.log10	3	CL 1:7	-	
			CL 2:1	{0}	
			CL 0:32	-	
11.mpi.cust.0K10	doub.log10	4	CL 1:7	-	
			CL 2:1	{0 }	
11.mpi.cust.0K10	doub.actual	2	CL 0:39	-	
11.mpi.cust.orc10	doub.actuar		CL 1:1	{0 }	
	doub.actual	3	CL 0:32	-	
11.mpi.cust.0K10			CL 1:7	-	
			CL 2:1	{0}	
11	doub.actual	4	CL 0:32	-	
11.mpi.cust.0K10			CL 1:7	- (0)	
			CL 2:1	{0 }	
		TOP Suspicious	1-0 2-2		
		Traces to check	3-3	-	
		<u> </u>	J-3		

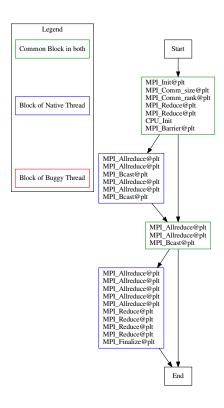


Figure 9. Bug2: diffNLR $PT_{3,0}$ - buggy vs. native

2) Bug2: Wrong Size in MPI AllReduce() (one process): We have injected a bug (row 2 table X) where MPI_Allreduce() had been invoked with a wrong size. ::MORE EXPLANATIONS ABOUT WHAT THE BUG IS::

Similar to table XI, the same ranking system tells us to check $PT_{1,0}$ and $PT_{3,0}$. Note that the bug injected to only one process (P_3) to have the wrong size.)

::EXPLANATIONS OF OBSERVATIONS::

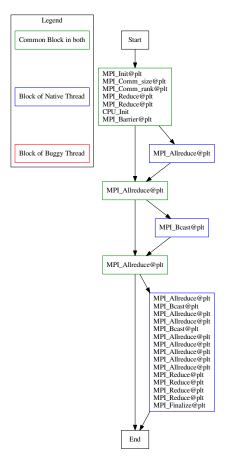


Figure 10. Bug2: diffNLR $PT_{1,0}$ - buggy vs. native



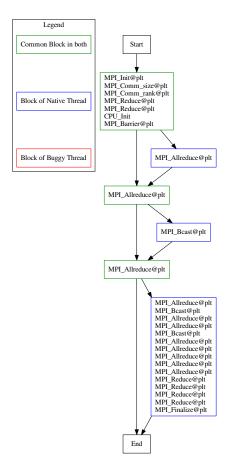
Figure 11. Bug2: diffNLR $PT_{3,0}$ - buggy vs. native

3) Bug3: Wrong Size in MPI AllReduce() (all processes): We have injected a bug (row 3 table X) where MPI_Allreduce() had been invoked with a wrong size. ::MORE EXPLANATIONS ABOUT WHAT THE BUG IS::

::What is the runtime reaction to this bug:: on node 3 (rank 3 in comm 0): Fatal error in PMPI_Bcast: Invalid root

Similar to table XI, the same ranking system tells us to check $PT_{1,0}$ and $PT_{3,0}$. Note that the bug injected to only one process (P_3) to have the wrong size.)

::EXPLANATIONS OF OBSERVATIONS::



- 4) Bug4: Wrong Op in MPI AllReduce(): no effect!,program terminates fine: maybe all images show some reflection
- 5) Bug5: Wrong Size in next MPI AllReduce()(one process)::no effect, program terminates fine: maybe all images show some reflection
- 6) Bug6: Wrong Size in next MPI AllReduce()(all processes)::no effect,program terminates fine: maybe all images show some reflection
- 7) Bug7: Wrong Size in MPI Bcast()(one process)::: maybe all images show some reflection
- 8) Bug8: Wrong Size in next MPI Bcast()(all processes)::no effect,program terminates fine: maybe all images show some reflection
- 9) 3: Missing Critical Section one thread in on process: I planted the bug (missing critical section) in process 2

VI. RELATED WORK

A. Program Understanding

- Score-P [6]
- TAU [5]
- ScalaTrace: Scalable compression and replay of communication traces for HPC [21]
- Barrier Matching for Programs with Textually unaligned barriers [22]
- Pivot Tracing: Dynamic causal monitoring for distributed systems - Johnathan mace [23]
- Automated Charecterization of parallel application communication patterns [24]
- Problem Diagnosis in Large Scale Computing environments [25]
- Probablistic diagnosis of performance faults in large-scale parallel applications [26]
- detecting patterns in MPI communication traces robert preissl [27]
- D4: Fast concurrency debugging with parallel differntial analysis bozhen liu [28]
- Marmot: An MPI analysis and checking tool bettina krammer [29]
- MPI-checker Static Analysis for MPI Alexandrer droste [30]
- STAT: stack trace analysis for large scale debugging Dorian Arnold [7]
- DMTracker: Finding bugs in large-scale parallel programs by detecting anomaly in data movements [31]
- SyncChecker: Detecting synchronization errors between MPI applications and libraries - [32]
- Model Based fault localization in large-scale computing systems - Naoya Maruyama [33]
- Synoptic: Studying logged behavior with inferred models
 ivan beschastnikh [34]
- Mining temporal invariants from partially ordered logs ivan beschastnikh [35]
- Scalable Temporal Order Analysis for Large Scale Debugging - Dong Ahn [36]
- Inferring and asserting distributed system invariants ivan beschastnikh - stewart grant [37]

- PRODOMETER: Accurate application progress analysis for large-scale parallel debugging - subatra mitra [38]
- Automaded: Automata-based debugging for dissimilar parallel tasks - greg [39]
- Automaded : large scale debugging of parallel tasks with Automaded - ignacio [8]
- Inferring models of concurrent systems from logs of their behavior with CSight ivan [40]

B. Trace Analysis

- Trace File Comparison with a hierarchical Sequence Alignment algorithm [41]
- structural clustering : matthias weber [42]
- building a better backtrace: techniques for postmortem program analysis - ben liblit [43]
- automatically charecterizing large scale program behavior
 timothy sherwood [44]

C. Visualizations

- Combing the communication hairball: Visualizing largescale parallel execution traces using logical time - katherine e isaacs [45]
- recovering logical structure from charm++ event traces
 [46]
- ShiViz Debugging distributed systems [47]

D. Concept Lattice and LCA

- Vijay Garg Applications of lattice theory in distributed systems
- Dimitry Ignatov [?] Concept Lattice Applications in Information Retrieval
- [14] [18] [48] [15] [19]

E. Repetitive Patterns

• [49] [50] [51] [52] [53]

F. STAT

Parallel debugger STAT[7]

- STAT gathers stack traces from all processes
- Merge them into prefix tree
- Groups processes that exhibit similar behavior into equivalent classes
- A single representative of each equivalence can then be examined with a full-featured debugger like TotalView or DDT

What STAT does not have?

- FP debugging
- Portability (too many dependencies)
- Domain-specific
- Loop structures and detection

45	(7)11.mem.ompcrit.cust.0K10	sing.actual	1:(3_0,4_0):0.75	1:(2_1,3_1):0.00	1:(3_2,6_2):0.57	1:(1_3,2_3):0.89
			2:(2_0,3_0):0.75	2:(0_1,4_1):0.00	2:(3_2,5_2):0.57	2:(0_3,/_3):0.09
			3:(1_0,6_0):0.75	3:(0_1,5_1):0.00	3:(2_2,4_2):0.50	3:(4_3,6_3):0.50
46	(7)11.mem.ompcrit.cust.0K10	sing.log10	1:(3_0,4_0):0.75	1:(2_1,3_1):0.00	1:(0_2,2_2):0.33	1:(1_3,2_3):0.33
			2:(2_0,3_0):0.75	2:(0_1,4_1):0.00	2:(3_2,7_2):0.33	2:(6_3,7_3):0.33
			3:(1_0,6_0):0.75	3:(0_1,5_1):0.00	3:(3_2,6_2):0.33	3:(1_3,4_3):0.20
47	(7)11.mem.ompcrit.cust.0K10	sing.orig	1:(3_0,4_0):0.75	1:(2_1,3_1):0.00	1:(0_2,2_2):0.33	1:(1_3,2_3):0.33
			2:(2_0,3_0):0.75	2:(0_1,4_1):0.00	2:(3_2,7_2):0.33	2:(6_3,7_3):0.33
			3:(1_0,6_0):0.75	3:(0_1,5_1):0.00	3:(3_2,6_2):0.33	3:(1_3,4_3):0.20

Figure 12. Part of ranking table for MisCrit 1-1

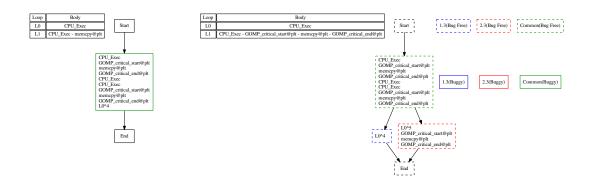


Figure 13. diffNLR of process 1 thread 3 and process 2 thread 3 buggy(missing critical section) vs. bug-free

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