INTEGRAL ESTIMATION IN QUANTUM PHYSICS

by

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A dissertation submitted to the faculty of
The University of Utah
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

Department of Mathematics

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The University of Utah Graduate School

STATEMENT OF DISSERTATION APPROVAL

The dissertation of   **Jane Doe**

has been approved by the following supervisory committee members:

Cornelius Lánczos ,	Chair(s)	**17 Feb 2016**
		Date Approved
Hans Bethe ,	Member	**17 Feb 2016**
		Date Approved
Niels Bohr ,	Member	**17 Feb 2016**
		Date Approved
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		Date Approved
Paul A. M. Dirac ,	Member	**17 Feb 2016**
		Date Approved

by **Petrus Marcus Aurelius Featherstone-Hough** , Chair/Dean of

the Department/College/School of **Mathematics**

and by **Alice B. Toklas** , Dean of The Graduate School.

ABSTRACT

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

For my parents, Alice and Bob.

CONTENTS

LIST OF FIGURES

LIST OF TABLES

NOTATION AND SYMBOLS

α	fine-structure (dimensionless) constant, approximately 1/137
α	radiation of doubly-ionized helium ions, He++
β	radiation of electrons
γ	radiation of very high frequency, beyond that of X rays
γ	Euler's constant, approximately $0.577\,215\ldots$
δ	stepsize in numerical integration
$\delta(x)$	Dirac's famous function
ϵ	a tiny number, usually in the context of a limit to zero
$\zeta(x)$	the famous Riemann zeta function
$\ldots$	$\ldots$
$\psi(x)$	logarithmic derivative of the gamma function
ω	frequency

CHAPTER 1

THE FIRST

This is a chapter. Remember that there should *always* be at least of few lines of prose after each sectional heading: failure to do so is a disservice to your readers, and also produces incorrect vertical spacing.

1.1 The first section

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

In **Figure 1.1** on the following page, we have a picture, and the LaTeX markup to include it looks like this:

```
\begin{figure}[t]
    \centerline{\includegraphics{fig1}}
    \caption{The first figure.}%
    \figlabel{fig1}
\end{figure}
```

We intentionally omitted an extension on the filename, so that this document can be processed with `latex` to get an output `.dvi` file, or with `pdflatex` to get an output `.pdf` file. The first case uses the file `fig1.eps`, and the second uses `fig1.pdf`. The `distill` or `ps2pdf` commands can be used to convert from *Encapulated PostScript* files to *Portable Document Format* files.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Figure 1.1. The first figure.

1.1.1   The first subsection

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

1.1.2   The second subsection

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

1.1.3   The third subsection

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

1.1.3.1   The first subsubsection

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

1.1.3.2   The second subsubsection

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

1.1.3.2.1   The first numbered paragraph   Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah

blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

1.1.3.2.2 The second numbered paragraph Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

1.2 The second section

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

In **Figure 1.2**, we have another picture.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

This is Figure 2

Figure 1.2. The second figure.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

In **Table 1.1**, we show the 24-character lowercase Greek alphabet.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Table 1.1. Lowercase Greek letters.

α	alpha
β	beta
γ	gamma
δ	delta
ϵ, ε	epsilon
ζ	zeta
η	eta
θ, ϑ	theta
ι	iota
κ	kappa
λ	lambda
μ	mu
ν	nu
ξ	xi
o	omicron
π	pi
ρ	rho
σ, ς	sigma
τ	tau
υ	upsilon
ϕ, φ	phi
χ	chi
ψ	psi
ω	omega

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

1.3   The third section

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

In **Table 1.2** on the next page, we show the 24-character uppercase Greek alphabet, 13 of which are identical with Latin letters, because the Romans borrowed several letters from the earlier Greek alphabet. However, the letter sounds do not always carry over: notice in particular the different names of the letter shapes **H** and **P**. In Modern Greek, β is pronounced *veeta*; the letter pair $\mu\pi$ is used to get a *bee* sound;

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Table 1.2. Uppercase Greek letters. Notice that several have the same letter shapes as Latin letters, and for those, TEX does not define macro names. For convenience, we supply our own definitions of these macros: `\Alpha`, `\Beta`, `\Epsilon`, `\Zeta`, `\Eta`, `\Iota`, `\Kappa`, `\Mu`, `\Nu`, `\Omicron`, `\Rho`, `\Tau`, and `\Chi`.

A	Alpha
B	Beta
Γ	Gamma
Δ	Delta
E	Epsilon
Z	Zeta
H	Eta
Θ	Theta
I	Iota
K	Kappa
Λ	Lambda
M	Mu
N	Nu
Ξ	Xi
O	Omicron
Π	Pi
P	Rho
Σ	Sigma
T	Tau
Υ	Upsilon
Φ	Phi
X	Chi
Ψ	Psi
Ω	Omega

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

1.4   Free software packages

The Free Software Foundation offers almost 300 software packages, most easily portable to many different operating systems and CPU platforms. They include at least these:

a2ps, acct, acm, adns, alive, anubis, apl, archimedes, aris, aspell, auctex, autoconf-archive, autoconf, autogen, automake, avl, ballandpaddle, barcode, bash, bayonne, bc, binutils, bison, bool, bpel2owfn, c-graph, ccaudio, ccd2cue, ccrtp,

ccscript, cfengine, cflow, cgicc, chess, cim, classpath, classpathx, clisp, combine, commoncpp, complexity, config, coreutils, cpio, cppi, cssc, cursynth, dap, datamash, ddd, ddrescue, dejagnu, denemo, dico, diction, diffutils, dionysus, direvent, dismal, dominion, easejs, ed, edma, electric, emacs, emms, enscript, fdisk, ferret, findutils, fisicalab, flex, fontutils, freedink, freefont, freeipmi, gama, garpd, gawk, gcal, gcc, gcide, gcl, gcompris, gdb, gdbm, gengen, gengetopt, gettext, gforth, ggradebook, ghostscript, gift, gleem, glibc, global, glpk, gmp, gnash, gnats, gnatsweb, gnu-c-manual, gnu-crypto, gnu-pw-mgr, gnubatch, gnubik, gnucap, gnucobol, gnudos, gnue, gnugo, gnuit, gnujump, gnukart, gnumach, gnun, gnunet, gnupod, gnuprologjava, gnuradio, gnurobots, gnuschool, gnushogi, gnusound, gnuspeech, gnuspool, gnustep, gnutls, gnutrition, gnuzilla, goptical, gperf, gprolog, greg, grep, groff, grub, gsasl, gsegrafix, gsl, gslip, gsrc, gss, gtypist, guile-gnome, guile-gtk, guile-ncurses, guile-opengl, guile-rpc, guile-sdl, guile, gv, gvpe, gxmessage, gzip, halifax, health, hello, help2man, hp2xx, httptunnel, hurd, hyperbole, idutils, ignuit, indent, inetutils, intlfonts, jacal, jel, jwhois, kawa, less, libcdio, libextractor, libffcall, libiconv, libidn, libmatheval, libmicrohttpd, librejs, libsigsegv, libtasn1, libtool, libunistring, libxmi, lightning, lilypond, liquidwar6, lsh, m4, macchanger, mailman, mailutils, make, marst, maverik, mc, mcron, mcsim, mdk, metahtml, mifluz, mig, miscfiles, mit-scheme, moe, motti, mpc, mpfr, mpria, mtools, myserver, nano, ncurses, nettle, non-gnu, ocrad, octave, oleo, orgadoc, osip, paperclips, parallel, parted, patch, pem, pexec, phantom, pies, plotutils, proxyknife, pspp, psychosynth, pth, pyconfigure, radius, rcs, readline, recutils, reftex, remotecontrol, rottlog, rpge, rush, sather, sauce, savannah, scm, screen, sed, serveez, sharutils, shishi, shmm, shtool, sipwitch, slib, smalltalk, solfege, spacechart, spell, sqltutor, src-highlite, stow, superopt, swbis, tar, termcap, termutils, teseq, teximpatient, texinfo, thales, time, tramp, trueprint, unifont, units, unrtf, userv, uucp, vc-dwim, vcdimager, vera, wb, wdiff, websocket4j, wget, which, windows, xaos, xboard, xhippo, xlogmaster, xnee, xorriso, and zile.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

In **Figure 1.3**, we have yet another picture.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

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Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

This is Figure 3

Figure 1.3. The third figure. This one has both short and long captions. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

1.5   Resizing figures

In **Figure 1.4** through **Figure 1.8** on the following page, we show how graphics files can be rescaled to convenient sizes, with input like this:

```
\begin{figure}[p]
    \centerline{\includegraphics[scale = 0.5]{fig1}}
    \caption{The fourth figure (at 50\% scale).}%
    \figlabel{fig4}
\end{figure}

\begin{figure}[p]
    \centerline{\includegraphics[scale = 0.75]{fig1}}
    \caption{The fifth figure (at 75\% scale).}%
    \figlabel{fig5}
\end{figure}
```

This is Figure 1

Figure 1.4. The fourth figure (at 50% scale).

This is Figure 1

Figure 1.5. The fifth figure (at 75% scale).

This is Figure 1

Figure 1.6. The sixth figure (at native size).

This is Figure 1

Figure 1.7. The seventh figure (at 125% scale).

This is Figure 1

Figure 1.8. The eighth figure (at 175% scale).

```
\begin{figure}[p]
    \centerline{\includegraphics{fig1}}
    \caption{The sixth figure (at native size).}%
    \figlabel{fig6}
\end{figure}

\begin{figure}[p]
    \centerline{\includegraphics[scale = 1.25]{fig1}}
    \caption{The seventh figure (at 125\% scale).}%
    \figlabel{fig7}
\end{figure}

\begin{figure}[p]
    \centerline{\includegraphics[scale = 1.75]{fig1}}
    \caption{The eighth figure (at 175\% scale).}%
    \figlabel{fig8}
\end{figure}
```

You can include multiple images, each with its own caption inside a single *unbreakable* `figure` environment, like this example shown in **Figure 1.9** and **Figure 1.10** on the next page, although you might want to adjust interfigure vertical space with a \vspace{} command:

```
\begin{figure}[t]
    \centerline{\includegraphics[scale = 0.5]{fig1}}
    \caption{The fourth figure (at 50\% scale).}%
    \figlabel{fig9}
    \vspace{3ex}
    \centerline{\includegraphics[scale = 0.75]{fig1}}
    \caption{The fifth figure (at 75\% scale).}%
    \figlabel{fig10}
\end{figure}
```

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

This is Figure 1

Figure 1.9. The ninth figure (at 50% scale), boxed with the tenth figure.

This is Figure 1

Figure 1.10. The tenth figure (at 75% scale), boxed with the ninth figure.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

As a final example in this chapter, **Figure 1.11** on the following page shows how you can use LaTeX `picture` mode for annotating and positioning graphics images prepared outside LaTeX. The input that produced that figure looks like this:

```
\begin{figure}[t]
    %% The original image is 216bp wide by 72bp high, but we
    %% rescale it to 150 picture units divided by \unitlength:
    %% 150 / 0.75 = 112.5 mm
    \newcommand {\myfig} {\includegraphics[width = 112.5mm]{fig1}}

    \begin{center}
        %% The \unitlength is chosen to make the complete picture fit
        %% within the page margins

        \setlength{\unitlength}{0.75mm}

        %%%     insert (width,height)(lower-left-x,lower-left-y)
        \begin{picture}(170,70)(10,10)
            %% Place the included image FIRST!
            \put(10,10) {\myfig}

            %% Everything that follows OVERLAYS the original image!

            \graphpaper[10](0,0)(170,70)

            %% Mark the image center and corners by centered bullets
            \newcommand {\thedot} {\makebox (0,0) {$\bullet$}}
            \put( 85, 35) {\thedot}
            \put( 10, 10) {\thedot}
```

Figure 1.11. Using LaTeX `picture` mode for figure labeling and positioning.

```
        \put( 10, 60) {\thedot}
        \put(160, 10) {\thedot}
        \put(160, 60) {\thedot}

        \put( 10, 10) {\makebox (0,0) [r] {lower-left}}
        \put(160, 10) {\makebox (0,0) [l] {lower-right}}
        \put( 10, 60) {\makebox (0,0) [r] {upper-left}}
        \put(160, 60) {\makebox (0,0) [l] {upper-right}}
      \end{picture}
    \end{center}

    \vspace{2\baselineskip}

    \caption[Using \LaTeX{} \texttt{picture} mode]
            {Using \LaTeX{} \texttt{picture} mode for figure labeling
             and positioning.}
    \figlabel{picture-mode}
\end{figure}
```

1.6   Summary and conclusions

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah

blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

   Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

CHAPTER 2

THE SECOND

This is a chapter.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

CHAPTER 3

THE THIRD

This is a chapter.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

CHAPTER 4

THE FOURTH

This is a chapter.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

4.1   More on the topic

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

4.2   Even more on the topic

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah

blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

4.3   Summary and conclusions

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

APPENDIX A

THE FIRST

This is an appendix. Notice that the LaTeX markup for an appendix is, surprisingly, \chapter. The \appendix command does not produce a heading; instead, it just changes the numbering style from numeric to alphabetic, and it changes the heading prefix from **CHAPTER** to **APPENDIX**.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

APPENDIX B

THE SECOND

This is an appendix.

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

APPENDIX C

THE THIRD

This is an appendix.

There are several books [12, 19–21, 23–25, 27–30] listed in our bibliography.

We also reference several journal articles [1, 2, 4, 8–10, 13–18, 22, 31, 32] and three famous doctoral theses of later winners [3, 6, 7] of the Nobel Prize in Physics (1922, 1933, and 1921):

Notice that, even though those citations appeared in LaTeX \cite{...} commands with their BibTeX citation labels in reverse alphabetical order, thanks to the citesort package, their reference-list numbers have been sorted in numerically ascending order, and then range-reduced.

Mention should also be made of a famous Dutch computer scientist's first publication [5].

Font metrics are an important, albeit low-level, aspect of typesetting. See the *Adobe Systems* manual about that company's procedures [26].

The bibliography at the end of this thesis contains several examples of documents with non-English titles, and their BibTeX entries provide title translations following the practice recommended by the American Mathematical Society and SIAM. Here is a sample entry that shows how to do so:

```
@PhdThesis{Einstein:1905:NBM,
  author =       "Albert Einstein",
  title =        "{Eine Neue Bestimmung der Molek{\"u}ldimensionen}.
                 ({German}) [{A} new determination of molecular
                 dimensions]",
  type =         "Inaugural dissertation",
  school =       "Bern Wyss.",
  address =      "Bern, Switzerland",
  year =         "1905",
  bibdate =      "Fri Dec 17 10:46:57 2004",
```

```
  bibsource =    "http://www.math.utah.edu/pub/tex/bib/einstein.bib",
  note =          "Published in \cite{Einstein:1906:NBM}.",
  acknowledgement = ack-nhfb,
  language =      "German",
  advisor =       "Alfred Kleiner (24 April 1849--3 July 1916)",
  URL =           "http://en.wikipedia.org/wiki/Alfred_Kleiner",
  remark =        "Received August 19, 1905 and published February 8,
                  1906.",
  Schilpp-number = "6",
}
```

The `note` field in that entry refers to another bibliography entry that need not have been directly cited in the document text. Such cross-references are common in BIBTEX files, especially for journal articles where there may be later comments and corrigenda that should be mentioned. Embedded `\cite{}` commands ensure that those possibly-important other entries are always included in the reference list when the entry is cited. The last bibliography entry [32] in this thesis has a long `note` field that tells more about what some may view as the most important paper in mathematics in the last century.

When entries cite other entries that cite other entries that cite other entries that ..., multiple passes of LATEX and BIBTEX are needed to ensure consistency. That is another reason why document compilation should be guided by a `Makefile` or a batch script, rather than expecting the user to remember just how many passes are needed.

BIBTEX entries are *extensible*, in that arbitrary key/value pairs may be present that are not necessarily recognized by any bibliography style files. The `advisor`, `acknowledgement`, `bibdate`, `bibsource`, `language`, `remark`, and `Schilpp-number` fields are examples, and may be used by other software that processes BIBTEX entries, or by humans who read the entries. `DOI` and `URL` fields are currently recognized by only a few styles, but that situation will likely change as publishers demand that such important information be included in reference lists.

In BIBTEX `title` fields, braces protect words, such as proper nouns and acronyms, that cannot be downcased if the selected bibliography style would otherwise do so. In German, all nouns are capitalized, and the simple way to ensure their protection is to brace the entire German text in the title, as we did in the entry above.

The world's first significant computer program may have been that written in 1842

by Lady Augusta Ada Lovelace (1815–1852) for the computation of Bernoulli numbers [16,18]. She was the assistant to Charles Babbage (1791–1871), and they are the world's first computer programmers. The programming language *Ada* is named after her, and is defined in the ANSI/MIL-STD-1815A Standard; its number commemorates the year of her birth.

We do not discuss mathematical *transforms* in this dissertation, but you can find that phrase in the index (except that this sample thesis doesn't have one!)

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

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