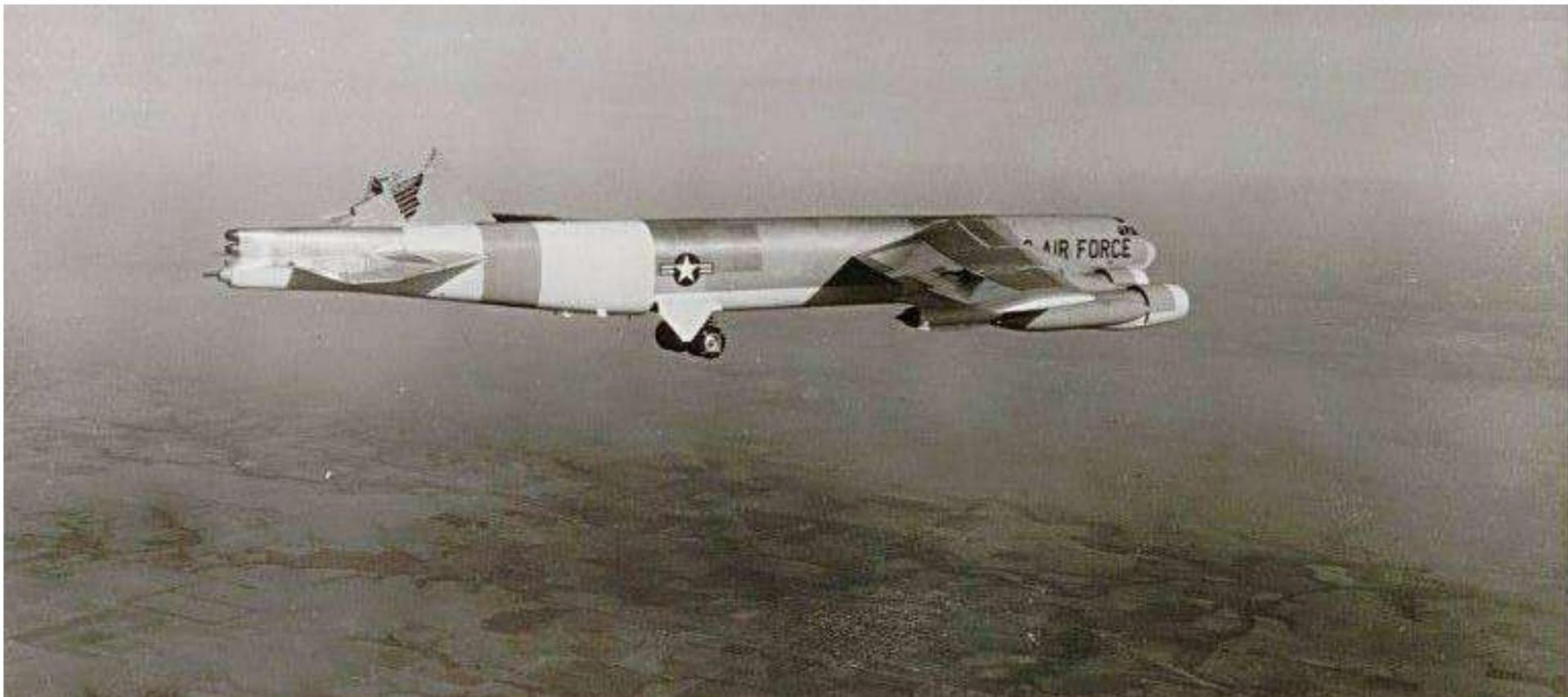


Welcome to the course

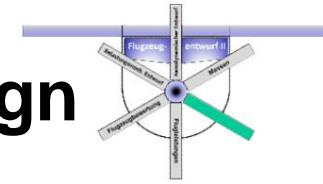
# Aircraft design II



**Andreas Bardenhagen  
Andreas Gobbin**

# F Fundamentals of flight mechanical design

## Overview



**F.1 Longitudinal movement**

**F.2 Lateral movement**

**F.3 Static longitudinal stability and controllability**

**F.3.1 Lift, neutral position and pitch moment of the aircraft without horizontal tail**

**F.3.2 Lift and pitch moment of the isolated horizontal tail unit**

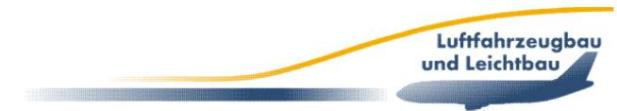
**F.3.3 Lift and pitching moment of the aircraft without rudder deflection**

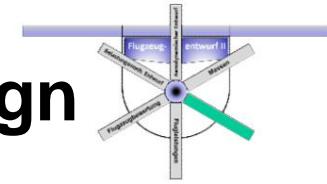
**F.3.4 Lift and pitch moment of the aircraft at elevator deflection**

**F.3.5 Static longitudinal stability with fixed rudder**

**F.3.6 Controllability at the forward centre of gravity**

**F.3.7 Controllability on curved flight path**





# F Fundamentals of flight mechanical design

## Overview

### F.4 Horizontal tail unit design

#### F.4.1 Horizontal tail unit dimensioning

##### F.4.1.1 Horizontal tail pitch angle

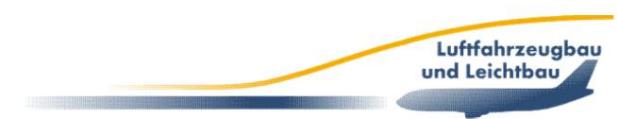
##### F.4.1.2 Tail lever arm

#### F.4.2 Horizontal tail configurations

#### F.4.3 Dynamic stability

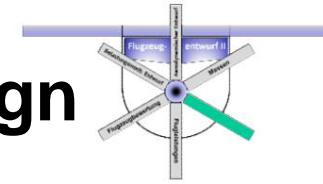
#### F.4.4 General design criteria

### F.5 Vertical stabilizer design



# F Fundamentals of flight mechanical design

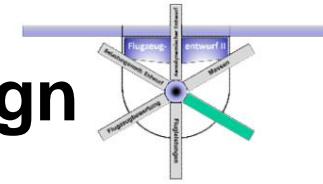
## Overview



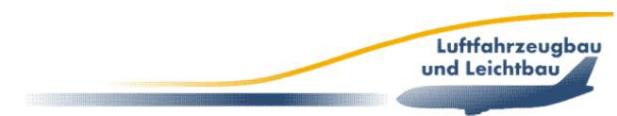
- The previous considerations referred to the movements of the aircraft's center of gravity, which are determined by the forces acting on the aircraft. • However, this assumes that – at any time, ie at any angle of attack, the aircraft by operating the control elements (elevator, rudder, ailerons or spoilers) the force and moment equilibrium can be maintained and controlled over the intended angle of attack range.
  - the flight characteristics are such that the aircraft is sufficiently stable and adequately controllable in all occurring and permissible flight attitudes and flight maneuvers.

# F Fundamentals of flight mechanical design

## Overview

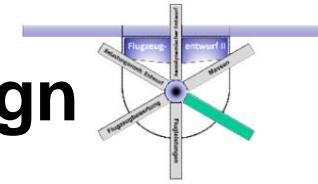


- To achieve this, certain requirements must be met with regard to the dimensions of the tail units, control elements and the center of gravity positions.
- The flight mechanical stability of an aircraft is defined as its ability to return to its initial position without any influence from the pilot after its stationary starting position has been disturbed by external influences (e.g. gusts, control impulses, short-term changes in the center of gravity).
- The controllability of an aircraft is the ability to maintain a state of equilibrium using the controls, to move from one state of equilibrium to another and to make accelerated changes to the flight path (e.g. interception)



# F Fundamentals of flight mechanical design

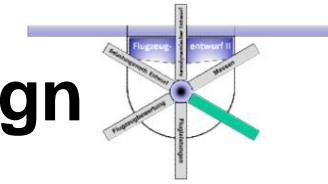
## Overview



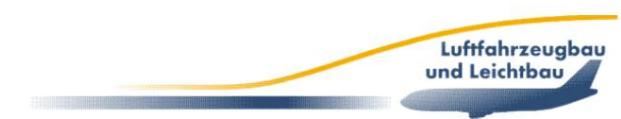
- The aircraft must be designed in such a way that every permissible speed can be maintained without the use of excessive forces.
- The stability must not be so great that controllability is impaired. Otherwise, for example, it may no longer be possible to control high angles of attack if the aircraft is too top-heavy (large distance between the center of gravity and the neutral point).
- The design regulations require that the aircraft be manoeuvrable around all axes (longitudinal, transverse and directional) during takeoff, climb, cruise and descent as well as landing, at any possible center of gravity position.

# F Fundamentals of flight mechanical design

## Overview

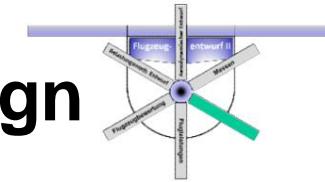


- The same applies to stability, whereby the above-mentioned Both static and dynamic stability must be demonstrated under certain conditions and different flap and landing gear positions (retracted, extended and during flight) must also be taken into account.



# F Fundamentals of flight mechanical design

## Overview

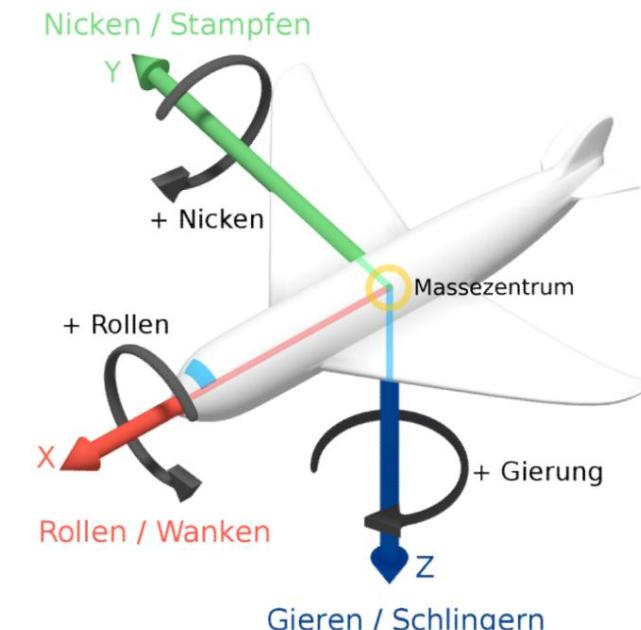


- In the general stability test,  
a rigid aircraft with fixed rudders 6 degrees of freedom.
- With freely movable rudders, the degrees of freedom  
the rudder angle.

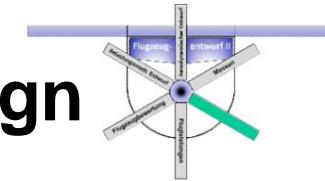
**Axis Translation**                                   **rotation**

x	$v_x$ – longitudinal speed $\dot{\gamma}_x$ - rolling
y	$v_y$ – lateral velocity $\dot{\gamma}_y$ – pitch
e	$v_z$ – normal speed $\dot{\gamma}_z$ - yaw

- It turns out that in most studies the 6 degrees of freedom can be broken down into 3 (for longitudinal movement) and 2 degrees of freedom (for roll and yaw movement).



Source: <https://de.wikipedia.org/wiki/Gierachse>



# F Fundamentals of flight mechanical design

## 1 Longitudinal movement

Here, the wing chord is parallel to the fuselage longitudinal axis and the center of gravity moves horizontally and vertically with the 3 Degrees of freedom  $v_x$ ,  $v_z$  and  $\dot{y}_y$  and the resulting maximum flight speed

$$v \ddot{y} v \ddot{y} \sqrt{\frac{2}{x^2} + \frac{2}{e^2}},$$

Angle of attack

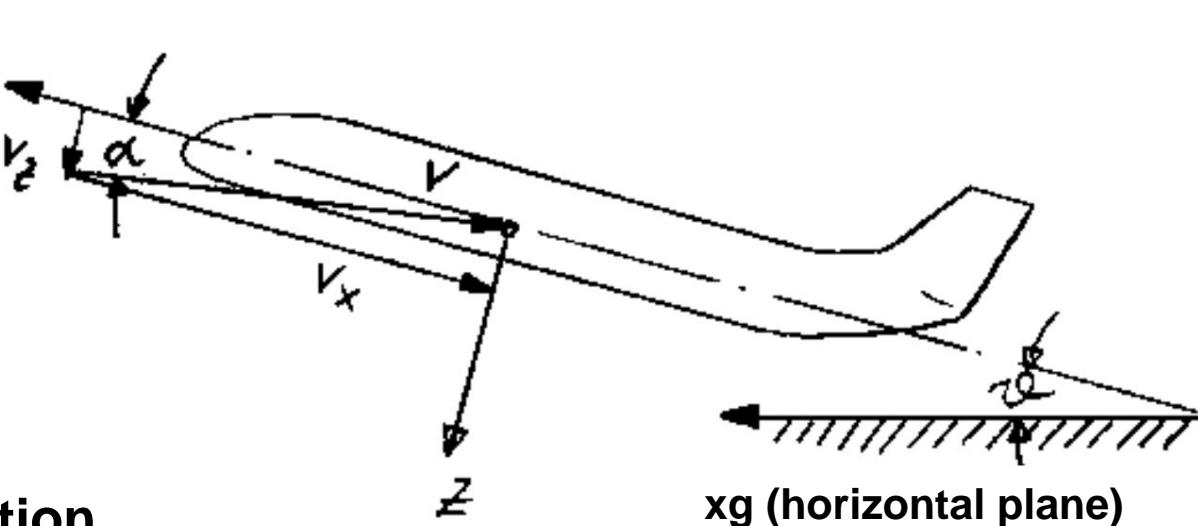
$$\ddot{y} \ddot{y} = \arctan \frac{\ddot{y} v_e \ddot{y}}{\ddot{y} v_x \ddot{y}},$$

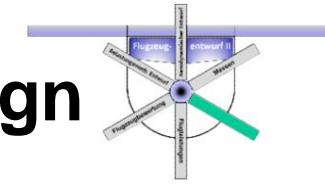
and the  
longitudinal inclination

$$\ddot{y} \ddot{y} \ddot{y} \ddot{y} \ddot{y} \ddot{y} \ddot{y}$$

$$\ddot{y} \quad y \quad \text{engl}$$

t

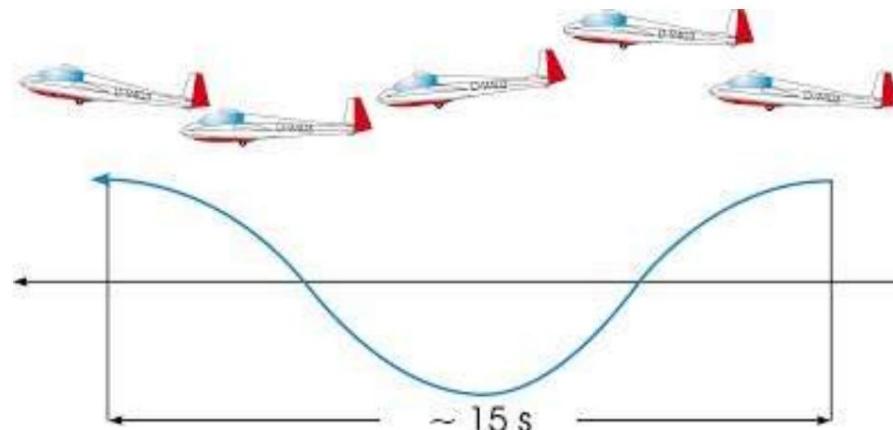




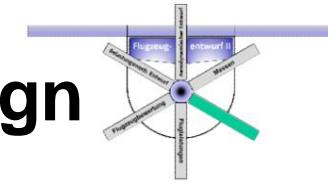
# F Fundamentals of flight mechanical design

## 1 Longitudinal movement

- **Longitudinal stability & controllability (Longitudinal stability & control)** is described by the degrees of freedom  $v$ ,  $\dot{y}$  and  $\ddot{y}$ .
- If the aircraft experiences a disturbance of one of the degrees of freedom in a stationary flight condition, two different dynamic longitudinal movements occur:
  1. A slow  $v$ ,  $\dot{y}$  - oscillation with a long duration ( $T \ddot{y} 30$  to 60 seconds), where  $\ddot{y}$  remains constant (phugoid).



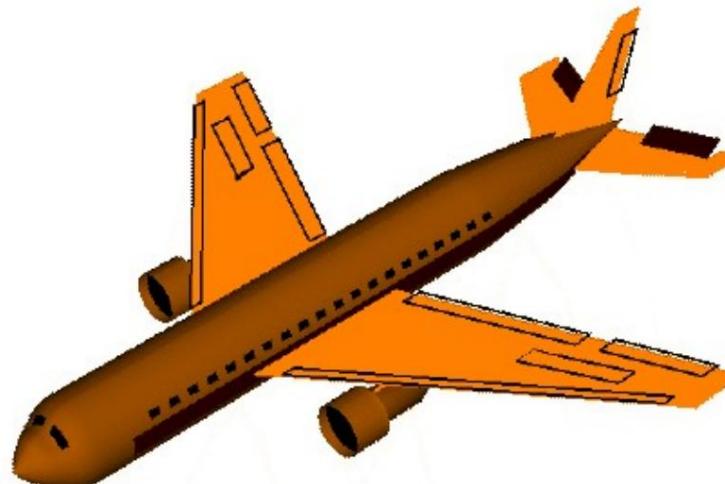
Source: <https://sites.google.com/site/nanarderies/phagoide>



# F Fundamentals of flight mechanical design

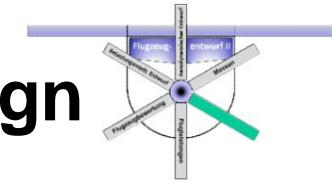
## 1 Longitudinal movement

**2. A fast  $\ddot{\gamma}$  oscillation with a short duration ( $T \approx 3$  to 5 seconds), where  $v$  and  $\dot{\gamma}$  remain constant.**



Source: NASA - Glenn Research Center, NASA, Public Domain, <https://commons.wikimedia.org/w/index.php?curid=566665>

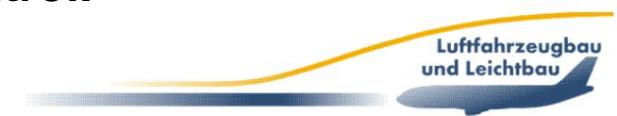
- For small aircraft and gliders, these times are significantly shorter due to their lower moments of inertia (e.g. 12 - 15 seconds or 1 to 2 seconds).

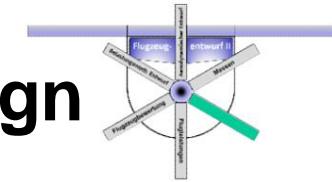


# F Fundamentals of flight mechanical design

## 1 Longitudinal movement

- For the fast  $\ddot{\gamma}$ -oscillation, very good damping is required, ie static and dynamic stability (FAR Part 25: “All short-period oscillations occurring in the range between the stall speed and the maximum permissible speed must be strongly damped with loose and fixed main controls”).
- With the slow  $\ddot{\gamma}$  oscillation, an increase in oscillation is not dangerous, as it can be easily detected and stopped with a counter-reaction that requires only little skill from the pilot.
- “Loose rudder”: Rudder angle adjusts itself according to Pressure distribution (only in small aircraft, since rudders in commercial aircraft are usually controlled hydraulically).
- “Fixed rudder”: Rudder angle is determined by control.



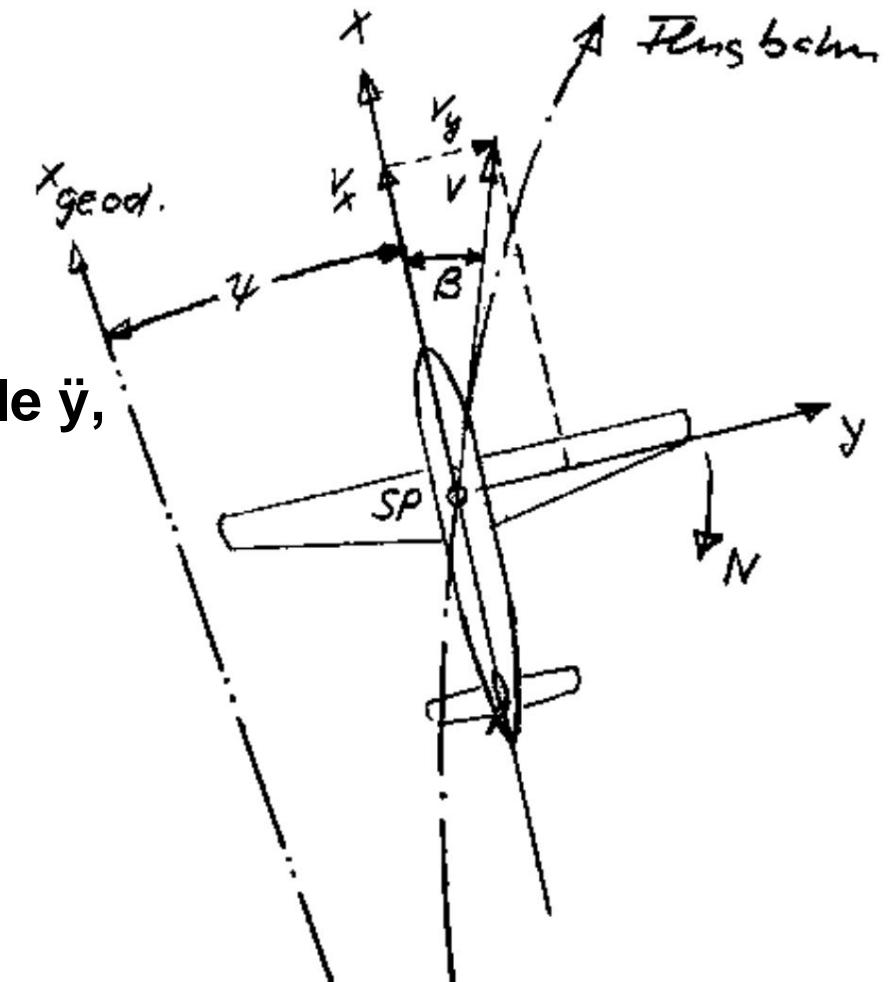


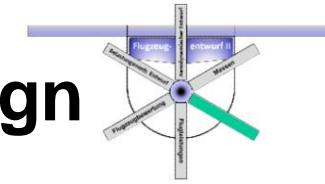
# F Fundamentals of flight mechanical design

## 2 Side movement

This movement with 3 degrees of freedom contains the components

- Roll  $\ddot{\gamma}_x$  (rotate around the Longitudinal axis, not visible here) with the roll angle  $\dot{\gamma}$ ,
- Yaw  $\ddot{\gamma}_z$  with the Yaw angle  $\dot{\gamma}$  and
- Slide  $\ddot{v}_y$  with the sliding angle  $\dot{\gamma}$ .





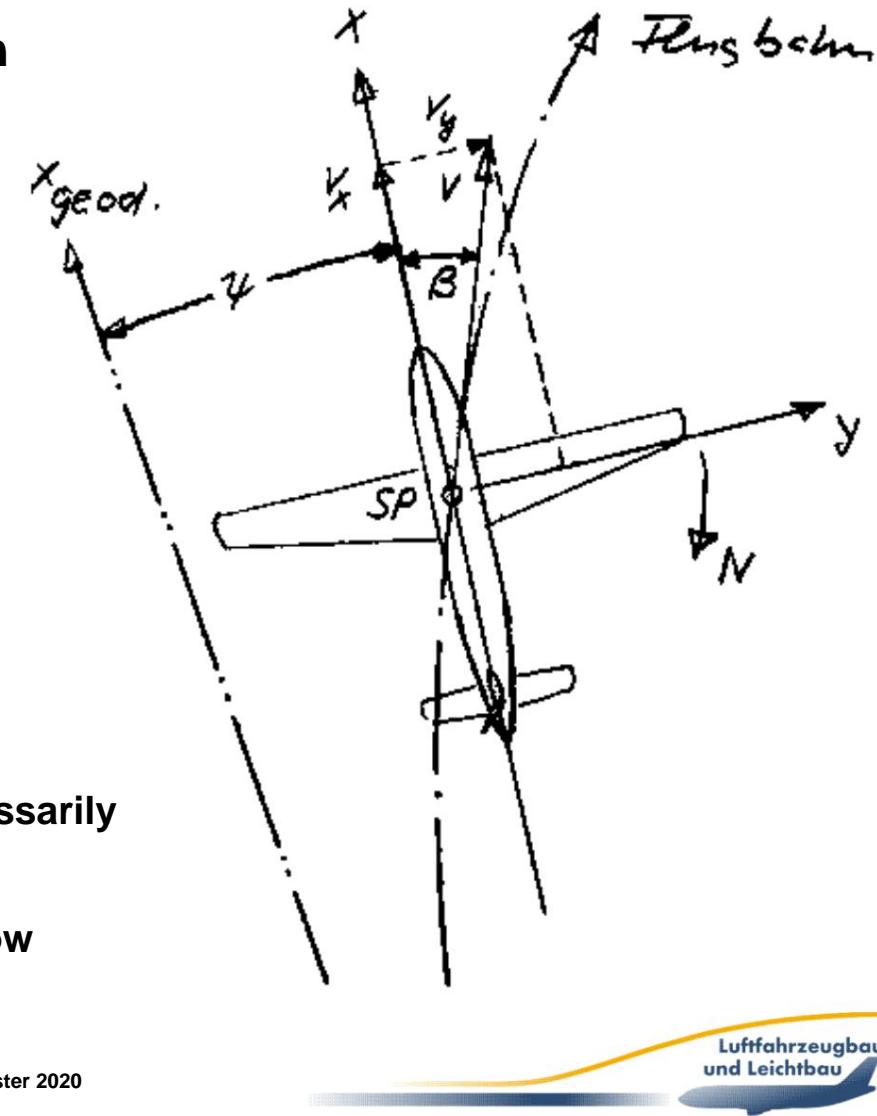
# F Fundamentals of flight mechanical design

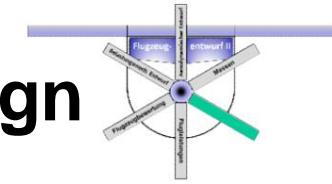
## 2 Side movement

- The sliding angle  $\ddot{\gamma}$  is the angle between the flow direction (tangential to the flight path) and the aircraft's longitudinal axis.

- The yaw angle  $\dot{\gamma}$  is the angular deviation of the aircraft's longitudinal axis from a defined azimuth direction in the earth-fixed coordinate system.
- In a  $360^\circ$  turn, the

Aircraft “yaws” by  $360^\circ$ , but does not necessarily have to “push” during the maneuver, ie have experienced a drag-increasing sideflow ( $\ddot{\gamma}$ ).

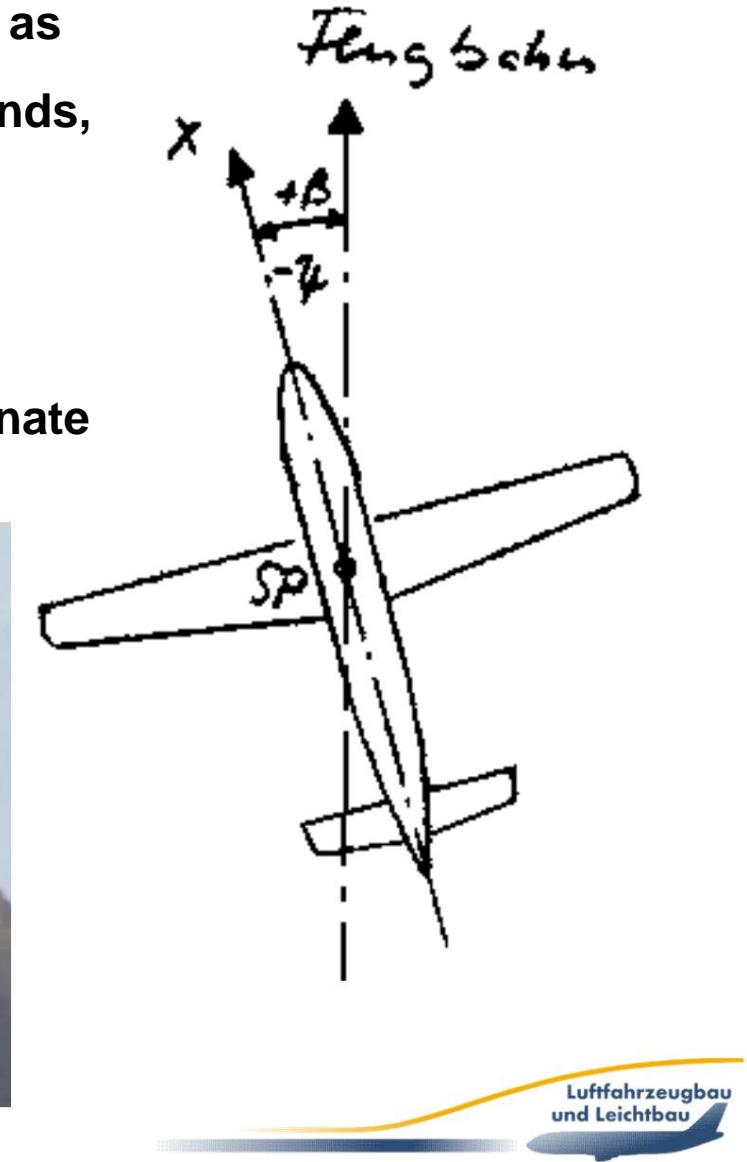


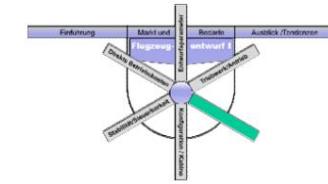


# F Fundamentals of flight mechanical design

## 2 Side movement

In practice, the yaw angle can be experienced as a “windward angle” during landing in crosswinds, since there is a visible difference between the flight direction in the aircraft-fixed coordinate system (heading) and the flight direction (flight path) in the earth-fixed coordinate system (track).





# F Flight mechanical design - basics

## 2 Side movement

- Investigation of lateral stability and controllability  
deals with the degrees of freedom  $\ddot{\gamma}$ ,  $\dot{\gamma}$  and  $\ddot{\gamma}$  and the  
Forms of movement after a disturbance of the stationary  
Flight status:

**1. Rolling movement**



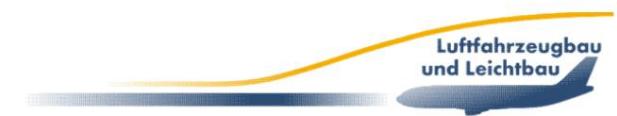
**2. Roll-yaw oscillation  
(Dutch roll)**



**3. Spiral fall**

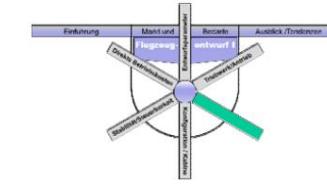
Source: Picascho - Own work,  
Public  
Domain, <https://commons.wikimedia.org/w/index.php?curid=3373136>

- When considering lateral movement, it is useful to distinguish  
between directional and transverse stability and controllability.

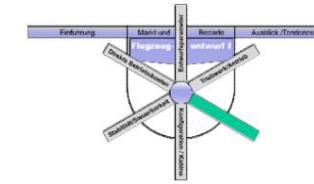


# F Flight mechanical design - basics

## 2 Side movement



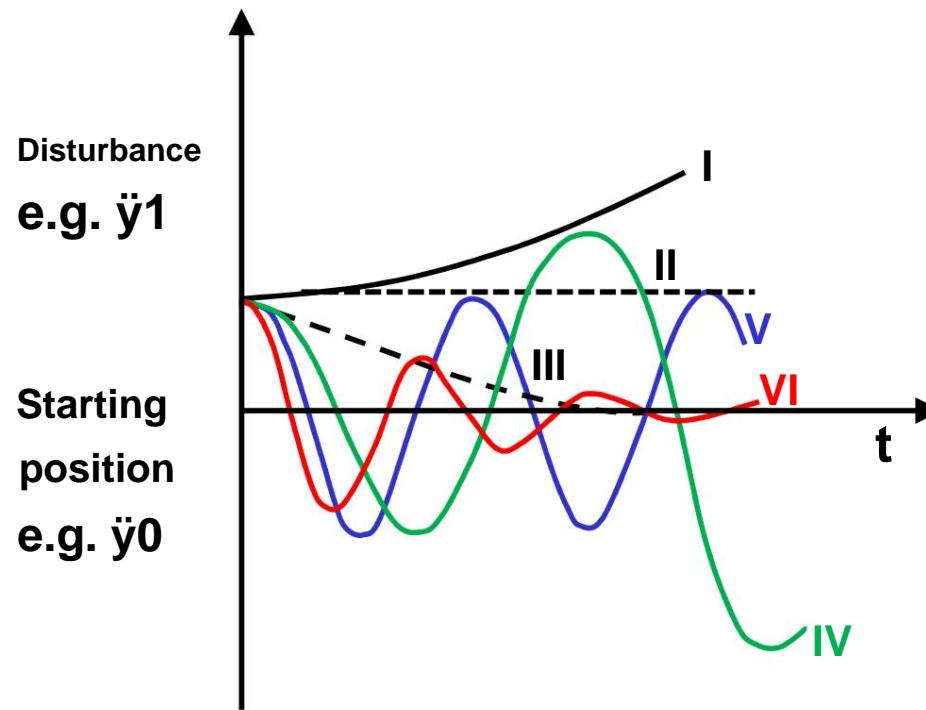
- Directional stability is the aircraft's attempt to independently end a slip flight condition with a loose rudder. • In principle, the vertical stabilizer (fin) acts like a wind vane.
- Lateral stability refers to the ability of a hanging wing to lift itself in side-slip flight with a loose rudder.
- Here too, all short-period vibrations must be strongly damped, i.e. dynamic directional and transverse stability must be ensured.
  - However, this stability is no longer present at larger angles of inclination (> V-shape of the wing) (natural spiral dive tendency).



F

# Flight mechanical design - basics

## Basic forms of stability behaviour



I	Statically and dynamically unstable	Aperiodic
II	Static and dynamic indifferent	Aperiodic
III	Static and dynamic (extremely) stable	Aperiodic
IV	Statically stable, dynamically unstable	Periodically
V	Statically stable, dynamically indifferent	Periodically
VI	Statically and dynamically stable	Periodically

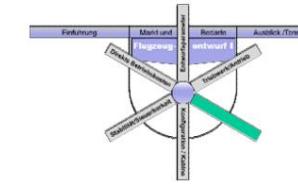
A distinction is made

- **Statically stable behaviour**, whereby the aircraft shows an initial tendency to return to the starting position after a disturbance of balance (curves III, IV, V, VI).

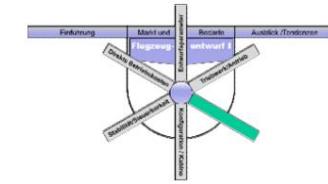
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# Flight mechanical design - basics

## Basic forms of stability behaviour



- **Dynamically stable behavior**, where the aircraft returns to equilibrium after a disturbance in the form of periodic movements (curve VI) or aperiodic movements (curve III).
- Due to unfavourable damping properties, a rocking oscillation movement can occur, which ultimately can no longer be controlled by a full rudder deflection.
- Despite existing static stability, the angle of attack increasingly moves away from the equilibrium state and becomes so large that the wing flow breaks off and dangerous flight conditions (e.g. dynamic tipping, slipping over the tail unit, spinning due to coupling of the pitching movement with movement around the vertical and longitudinal axes) are initiated.

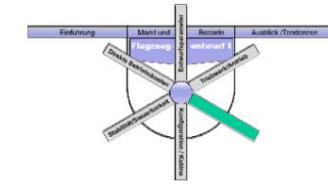


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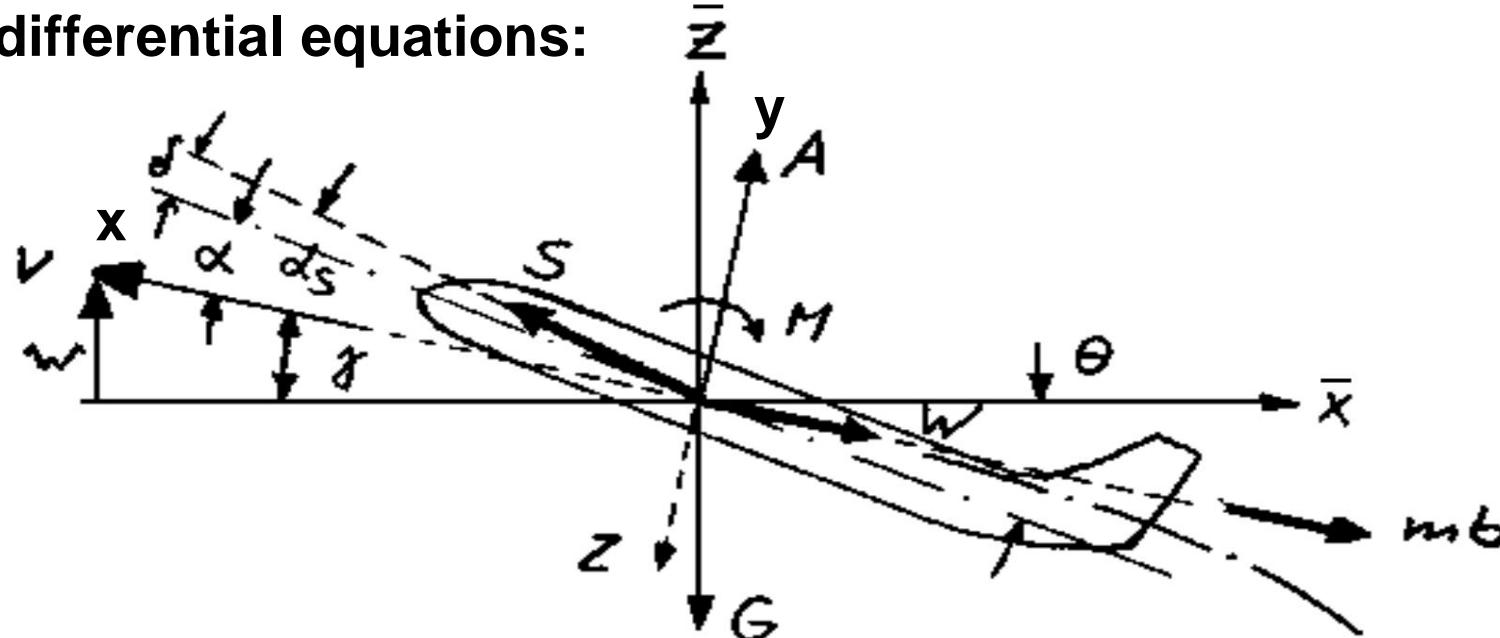
# Flight mechanical design - basics

## Basic forms of stability behaviour

- The condition for static stability is a necessary but not a sufficient condition for dynamic stability.
- In addition, sufficient damping of the vibrations is required, which can be achieved, for example, by large tail fins.

**F****Flight mechanical design - basics****3 Static longitudinal stability and controllability**

- The general longitudinal movement with fixed rudders and the Degrees of freedom  $v$ ,  $\ddot{y}$  and  $\dot{y}$  leads to the 3 differential equations:



Forces in the direction of travel:

$$\ddot{y} = K_0 m v W G \sin S \cos \theta$$

$$\ddot{y} \ddot{y} \ddot{y} \ddot{y}$$

s

Forces perpendicular to the path direction:  $K_0 m v A G \cos S \sin \theta$

$$\ddot{y} \ddot{y} \ddot{y} \ddot{y} \ddot{y} \ddot{y} \ddot{y} \ddot{y}$$

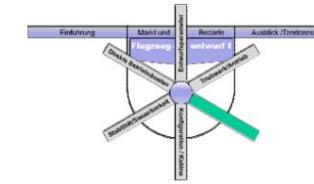
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Moments around y axis:

$$\ddot{y} = M_0 I_M \ddot{y} \ddot{y} \ddot{y} \ddot{y} \ddot{y}$$

# F Flight mechanical design - basics

## 3 Static longitudinal stability and controllability



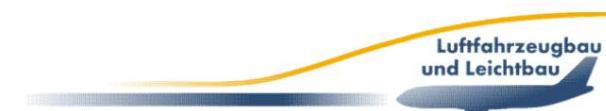
- By further processing these differential equations  
(Introduction of relative acceleration, linearization, Introduction of dimensionless quantities) can be reached with the Solution approach eyt to a characteristic equation 4-th Grades

$$AB\ddot{y}^4 + CD\dot{y}^3 + E\ddot{y}^2 + F\dot{y} + G = 0$$

- This allows the time-variant movement of the aircraft to be determined and thus statements to be made about its dynamic behavior.
- Dynamic stability is present when all Coefficients and the discriminant

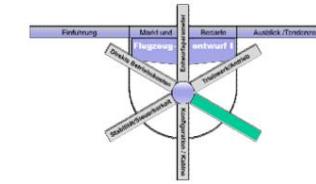
$$BCDAD^2B\ddot{y}^4 + C^2\dot{y}^2 + D^2 = 0$$

The  $\ddot{y}$ -oscillation (phugoid) is then damped.

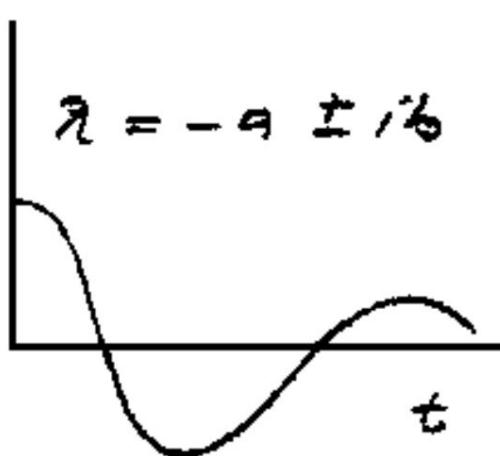


# F Flight mechanical design - basics

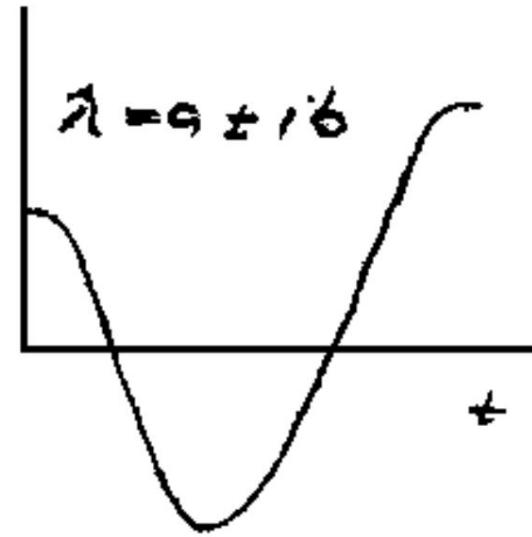
## 3 Static longitudinal stability and controllability



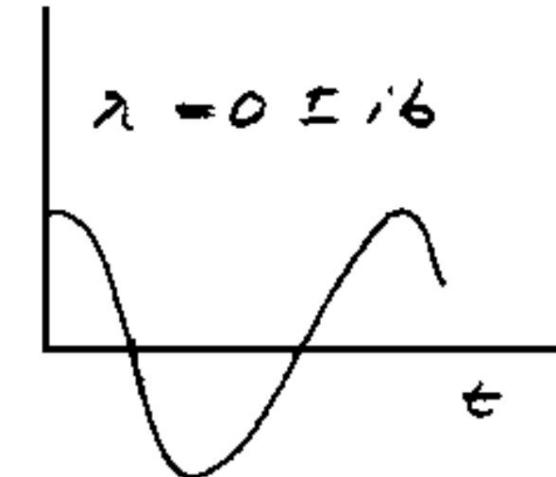
Graphically, the solutions can look like this:



steamed



undamped

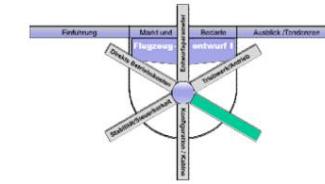


indifferent

Since the condition  $E > 0$  can also be obtained by cancelling all terms that contain a derivative with respect to time,  $E > 0$  is also called the condition of static longitudinal stability.

# F Flight mechanical design - basics

## 3 Static longitudinal stability and controllability

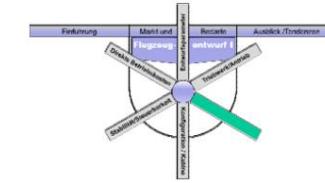


- If all  $\ddot{y}$  are real numbers, the motion is aperiodic, convergent if the sign is negative, divergent if the sign is positive
- If  $\ddot{y}$  forms a complex pair of numbers, the movement is oscillatory, positively damped if the real part is negative, and negatively damped if the real part is positive.
- If all coefficients of the characteristic equation are positive, no positive real roots, ie no pure divergence, can occur.
- A negative coefficient results in increasing Oscillation or pure divergence.
- If the discriminant  $BC \cdot D - A \cdot D^2 - B^2 \cdot E > 0$ , the real part of a complex pair of numbers cannot be positive, and no negatively damped oscillation can occur.



# F Flight mechanical design - basics

## 3 Static longitudinal stability and controllability

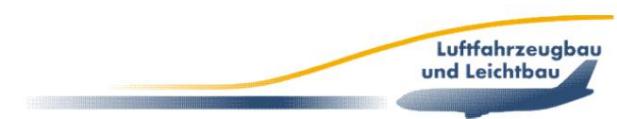


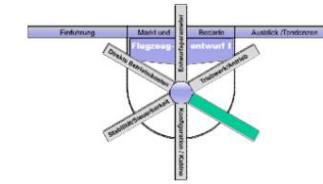
- If the discriminant = 0, neutrally damped oscillation occurs.
- $E > 0$  is the necessary condition for static stability. • It can be shown that this is equivalent to the statement  $c_m \ddot{y} < 0$  and consequently the static stability also applies

$$\frac{dcM}{d\ddot{y}} \ddot{y} < 0$$

- Since  $c_A$  and  $\ddot{y}$  are linearly coupled, the static stability is also

$$\frac{dc_M}{dc_A} \ddot{y} < 0 .$$

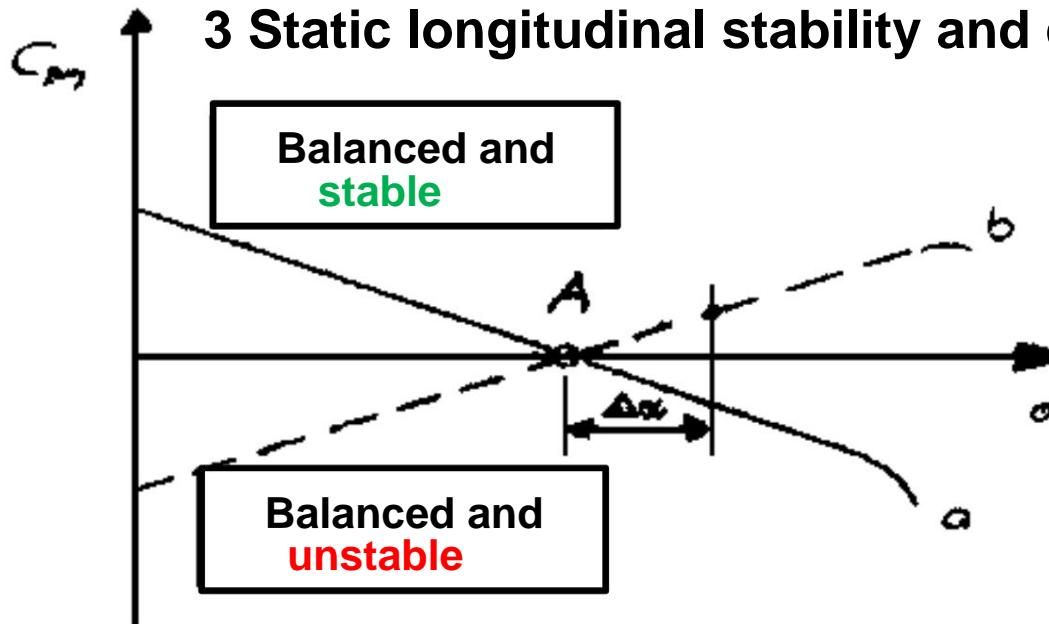




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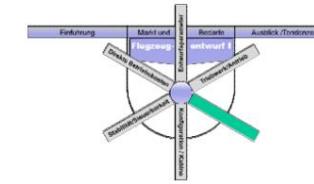
# Flight mechanical design - basics

## 3 Static longitudinal stability and controllability



- Static stability always occurs when, starting from a stationary initial state, a restoring moment occurs after a disturbance of the longitudinal movement.
- Curve a with  $dcm/d\ddot{\gamma} < 0$  is therefore stable, because a gust would also generate a top-heavy, back-rotating moment with the increase in angle of attack.

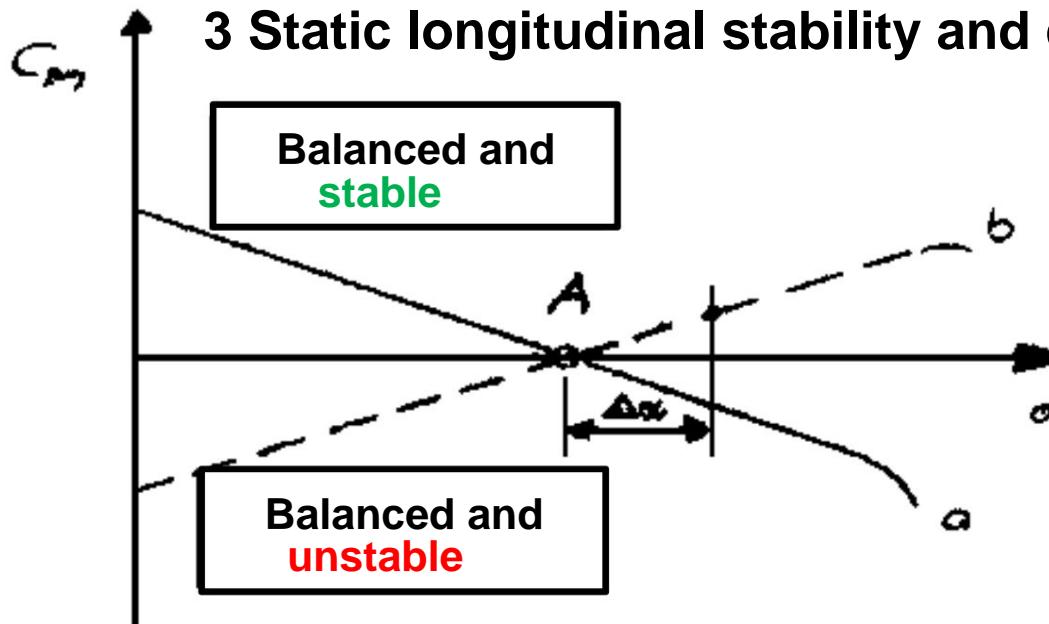




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# Flight mechanical design - basics

## 3 Static longitudinal stability and controllability



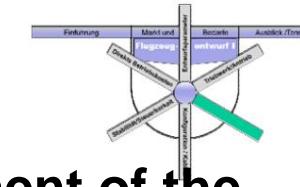
- This is not the case for curve b ( $dcm/d\ddot{\gamma} > 0$ )  $\ddot{\gamma}$  unstable. • A stationary, unaccelerated flight with a fixed Rudder (deflection is constant) occurs when the pitching moment disappears (point A). Here, therefore,  $cM = 0$  must apply. This is the condition for the trimmed flight state!
- Therefore  $cM_0 > 0$  must also apply!



**F**

# Flight mechanical design - basics

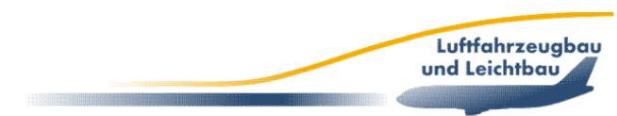
## 3.1 Lift, neutral point position and pitching moment of the Aircraft without horizontal tail

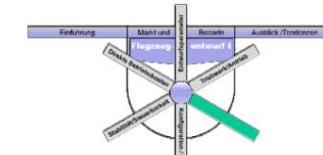


- For the sake of simplicity, it is assumed below that only the wing, fuselage, engine nacelles and horizontal tail assembly are responsible for the magnitude of the lift and the pitching moments.
- The high position of the centre of gravity and an eccentric thrust application point are neglected.
- In relation to the zero lift direction, the following applies to the lift of the wing

$$A c_{pq} \ddot{y} F = AF \quad \text{with} \quad c \ddot{y} c \ddot{y} \ddot{y} \ddot{y} = \frac{AF}{AF} = \frac{F}{F}$$

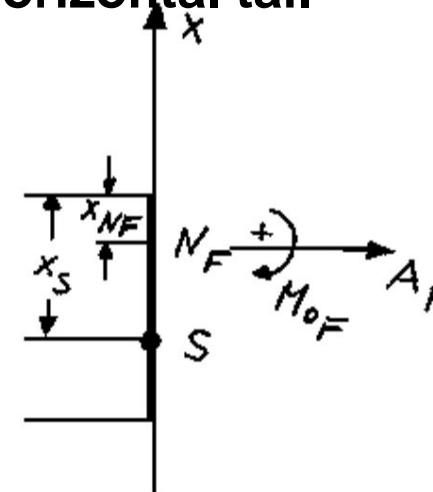
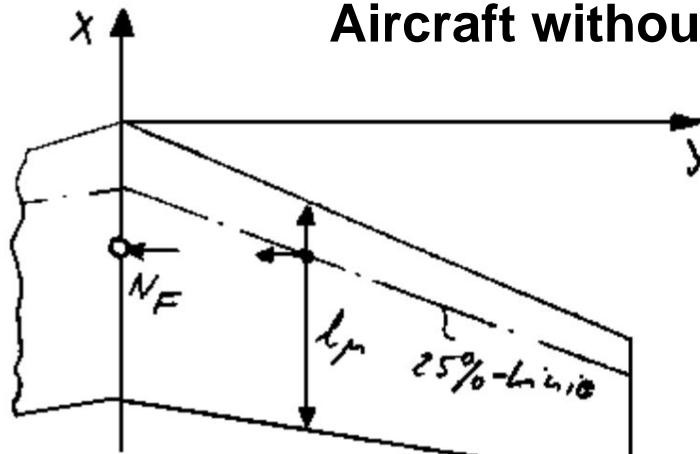
where  $\ddot{y}F$  refers to the zero lift direction.



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# Flight mechanical design - basics

## 3.1 Lift, neutral point position and pitching moment of the Aircraft without horizontal tail



**NF** neutral point  
wing

**M0F** zero moment  
wing

**l̄y** Reference wing  
depth

The pitch moment of the wing around the center of gravity can be written as

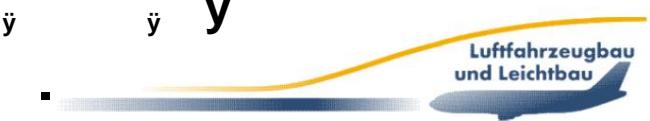
$$M_{M0F} = c_M \cdot I_y \cdot (\bar{y}_F - \bar{y}_S) + c_{M0F} \cdot I_y \cdot (\bar{y}_F - \bar{y}_N)$$

with the coefficients

$$c_M = \frac{\bar{y}_F - \bar{y}_S}{I_y} \quad c_{M0F} = \frac{\bar{y}_F - \bar{y}_N}{I_y}$$

and after forming

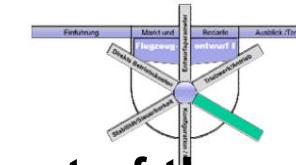
$$c_M = \frac{\bar{y}_F - \bar{y}_S}{I_y} \quad c_{M0F} = \frac{\bar{y}_F - \bar{y}_N}{I_y}$$



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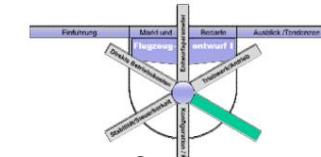
# Flight mechanical design - basics

## 3.1 Lift, neutral point position and pitching moment of the Aircraft without horizontal tail



- For isolated fuselages and nacelles, the same relationships can be established as for the wing, but the mutual influence must be taken into account when combining with the wing.
- As a first approximation, it can be assumed that the loss of lift in the area of the wing covered by the fuselage or, if applicable, the nacelles is compensated by the lift of these components themselves.
- Furthermore, the influence of the displacement effect of the fuselage or nacelles on the wing angle of attack is negligible.
- However, fuselage and nacelles generally shift the neutral point forward, which leads to an increase in the zero moment coefficient.



**F****Flight mechanical design - basics****3.1 Lift, neutral point position and pitching moment of the Aircraft without horizontal tail**

- For the wing-fuselage-nacelle combination, the index  $oH$  (ie without horizontal tailplane):
- You can write:

$$A c_{\theta H} \quad \text{with} \quad c_{\theta H} \quad \text{Oh} \quad A o H \quad F$$

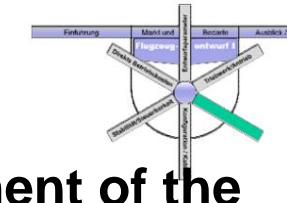
and

$$M_{oH} \quad \text{with} \quad C_{M_{oH}} \quad \frac{\ddot{y}_s}{I_s} \quad \frac{x_{n_{oH}}}{I_s} \quad \ddot{y}$$

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# Flight mechanical design - basics

## 3.1 Lift, neutral point position and pitching moment of the Aircraft without horizontal tail

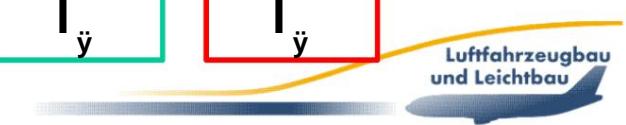


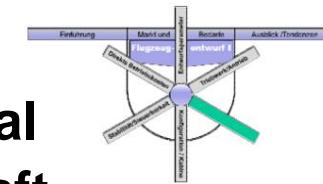
- The zero moment coefficient of the wing-fuselage-nacelle combination is mainly composed of the wing and fuselage components:

$$\begin{matrix} c \ddot{y} & c \ddot{y} & c \\ \text{MoH} & & \end{matrix} \quad \text{M0F} \quad \boxed{\text{M0R}}$$

- The fuselage coefficient is determined empirically. For high-wing configurations, a positive addition to the moment coefficient of 0.004 is included, and for low-wing configurations, a reduction of the same amount is included.
- The neutral point position of the wing-fuselage-nacelle combination is determined from the wing and corresponding Surcharges also of empirical origin for the two  $\ddot{y}$  other components

$$\frac{x_{\text{NoH}}}{I_{\ddot{y}}} \quad \frac{x_{\text{NF}}}{I_{\ddot{y}}} \quad \frac{x_{\text{NO}}}{I_{\ddot{y}}} \quad \frac{\ddot{y} x_{\text{NG}}}{I_{\ddot{y}}}$$

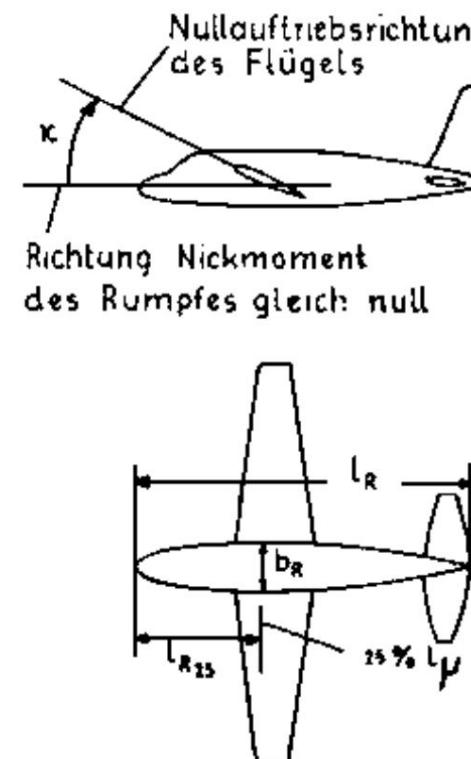
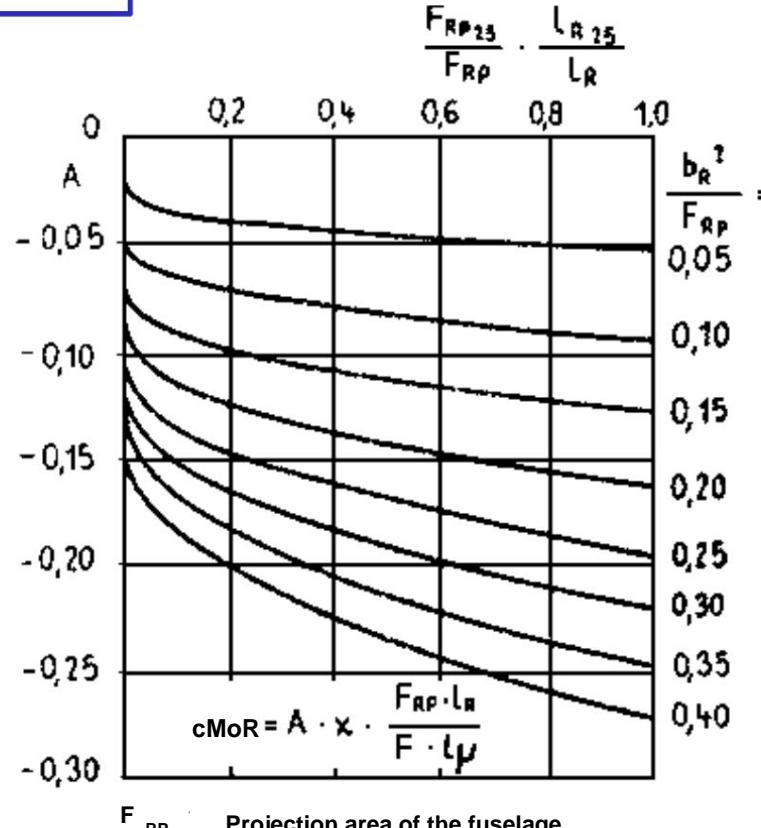


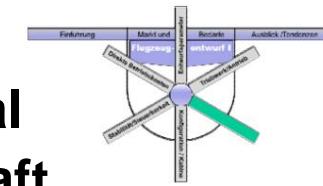


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## Flight mechanical design - Basics 3.1 Lift, neutral point position and pitching moment of the aircraft without horizontal tail unit

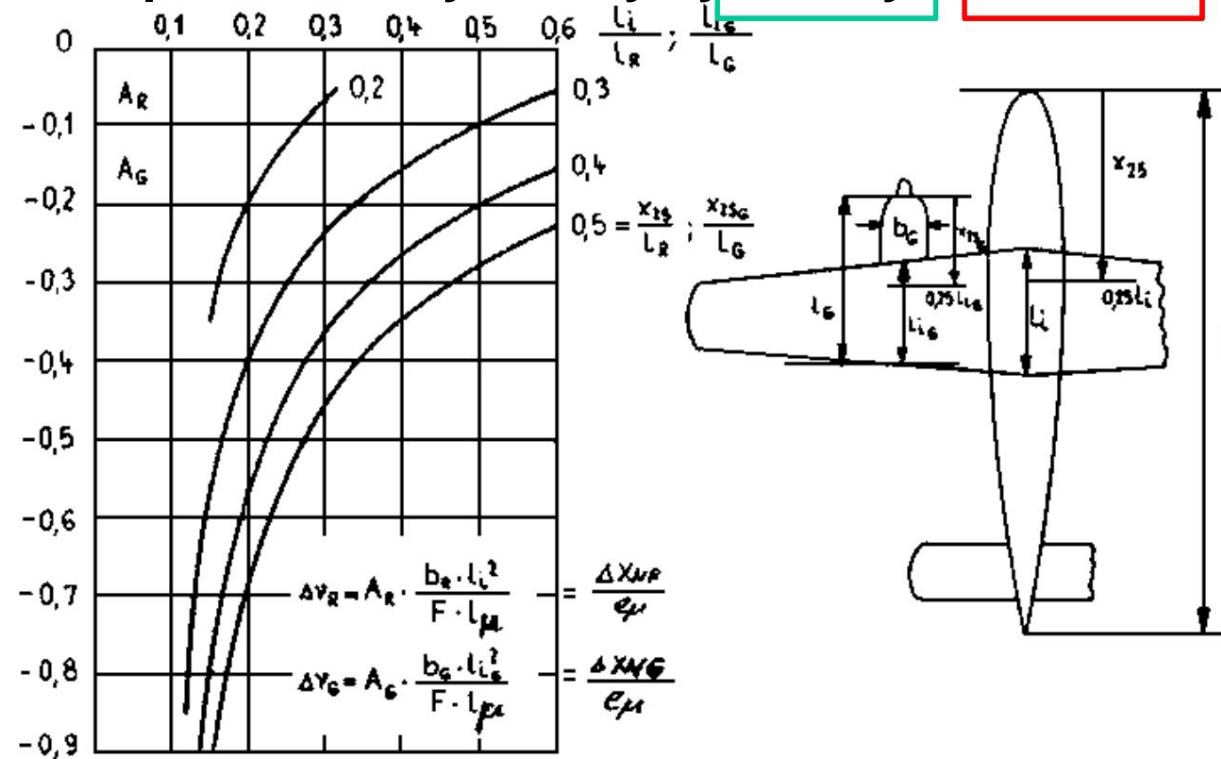
### cMoR from RAS data sheet



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## Flight mechanics design - Basics 3.1 Lift, neutral point position and pitching moment of the aircraft without horizontal tail unit

**Neutral point shifts  $\ddot{y}_xNR / \ddot{y}_x , \ddot{y}_xNG / \ddot{y}_x$**



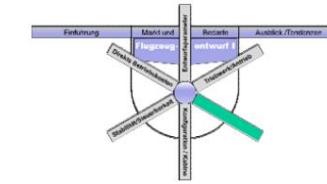
- The readings should be increased by 5% for low-wing aircraft and decreased by 5% for high-wing aircraft.

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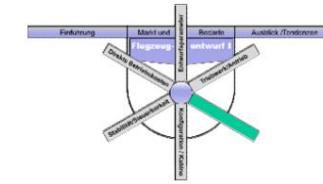
# Flight mechanical design - basics

## 3.2 Lift and pitching moment of the isolated

### Horizontal tail assembly

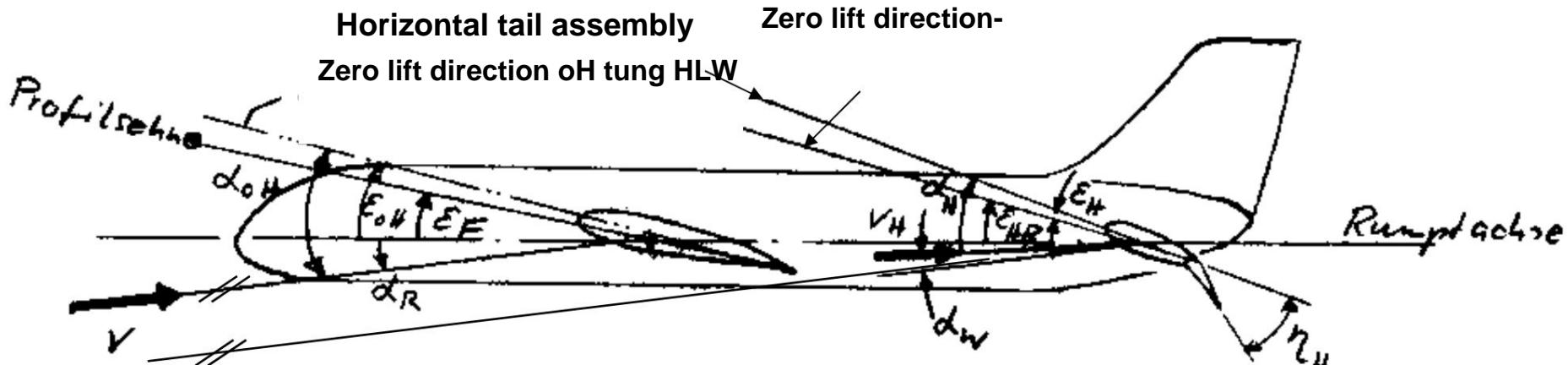


- The forces and moments of an isolated horizontal tail unit can be represented in a similar manner as for the isolated wing.
- However, large interactions must be taken into account when the horizontal tail assembly is installed on the aircraft.
- The horizontal tail assembly is located in the downwash field of the aircraft.
- the flow direction is different from that of the wing,
- it is influenced by the fuselage and • it may have a different dynamic pressure than the wing (slip stream in propeller aircraft, thrust dependent!).



# F Flight mechanical design - basics

## 3.2 Lift and pitching moment of the isolated



$\ddot{\gamma}_{oH}$  Angle of attack of the zero lift direction of the wing-fuselage-nacelle combination

$\ddot{\gamma}_R$  Angle of attack of the fuselage axis

$\ddot{\gamma}_{oH}$  Setting angle of the zero lift direction of the wing-fuselage-nacelle combination to the fuselage axis

Setting angle of the reference wing profile chord to the fuselage axis

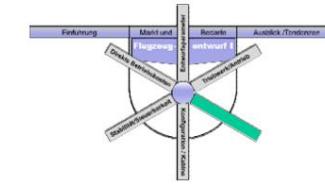
$\ddot{\gamma}_F \ddot{\gamma}_H$  Elevator deflection

$\ddot{\gamma}_{HR}$  Setting angle of the zero lift direction of the horizontal tail unit ( $\ddot{\gamma}_H = 0!$ ) to the fuselage axis

$\ddot{\gamma}_H$  Pitch angle difference between horizontal tail and wing

$\ddot{\gamma}_w$  Downwind angle



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# Flight mechanical design - basics

## 3.2 Lift and pitching moment of the isolated

### Horizontal tail assembly

This gives the angle of attack of the horizontal tail unit without

**Elevator deflection:**

$$\frac{c_A}{c_H} \frac{H}{y_H} = \frac{c_A}{c_H} \frac{H}{y_{OH}}$$

The following applies to buoyancy:

$$\frac{c_A}{c_H} \frac{H}{y_H} = \frac{c_A}{c_H} \frac{H}{y_{OH}} + \frac{c_A}{c_H} \frac{H}{y_{HA}}$$

and with

$$\frac{c_A}{c_H} \frac{H}{y_H} = \frac{c_A}{c_H} \frac{H}{y_{OH}} + \frac{c_A}{c_H} \frac{H}{y_{HA}} + \frac{c_A}{c_H} \frac{H}{y_{W}}$$

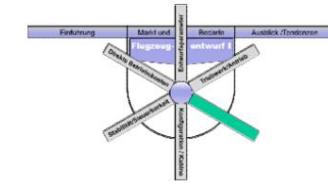
and a linear dependence of the downwind angle of

**Wing angle of attack**

$$\frac{c_A}{c_H} \frac{H}{y_H} = \frac{c_A}{c_H} \frac{H}{y_{OH}} + \frac{c_A}{c_H} \frac{H}{y_{W}}$$

one can write for the horizontal tail lift coefficient:

$$c_{UH} = c_A \frac{H}{y_H} = c_A \frac{H}{y_{OH}} + c_A \frac{H}{y_W}$$

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# Flight mechanical design - basics

## 3.2 Lift and pitching moment of the isolated

### Horizontal tail assembly

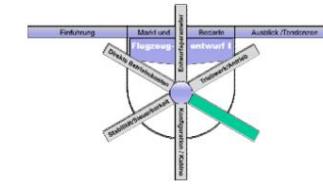
- In this case, the lift increase of the horizontal tail unit may need to be the hull influence must be taken into account.
- This can be done by multiplying the lift gradient with a tail unit efficiency  $\gamma_H$  :

$$1 \leq \gamma_H \leq 0.87 \quad \frac{F_R}{F_H}^2$$

FR stands for the tail area covered by the fuselage

- This allows the buoyancy gradient to be corrected:

$$c_{A_H} = \frac{\gamma c_{A_H}}{\gamma_H}$$



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# Flight mechanical design - basics

## 3.2 Lift and pitching moment of the isolated

### Horizontal tail assembly

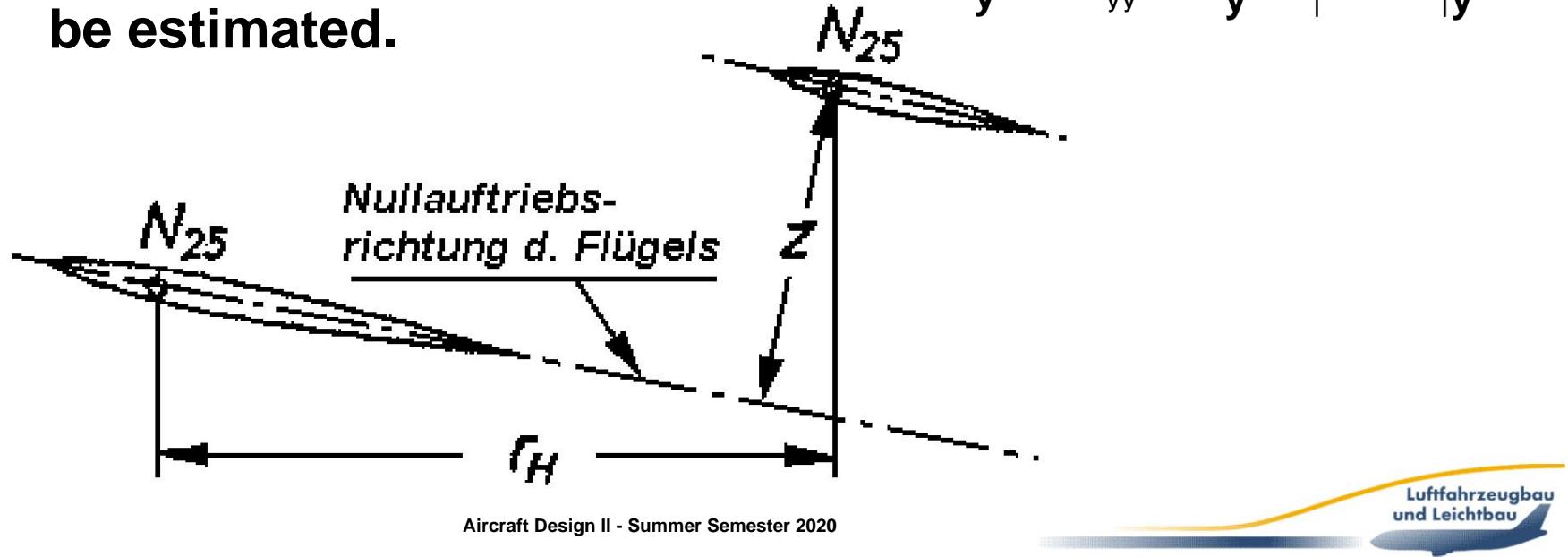
$\ddot{y} \ddot{y} W$

- The downdraft gradient depends on the distance of the  $\ddot{y} \ddot{y}$

**Tail unit from the wing in horizontal ( $r_H$ ) and vertical direction ( $z_H$ ) and can be determined using the approximation function**

$$\frac{\ddot{y} \ddot{y}_W}{\ddot{y} \ddot{y}} = \frac{0.557 c^{\ddot{y}}}{\ddot{y} \ddot{y} \ddot{y} \ddot{y} r_H^{\ddot{y} 0.25} \ddot{y} \ddot{y} 1} \left| \frac{2 z_H}{b} \right|^{\ddot{y}}$$

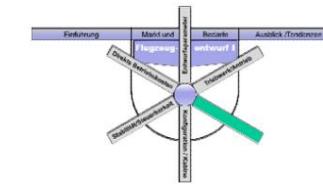
be estimated.



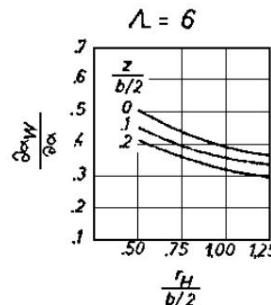
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# Flight mechanical design - basics

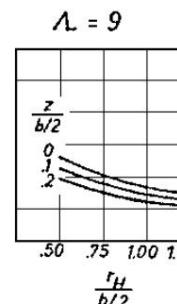
## 3.2 Lift and pitching moment of the isolated

**Horizontal tail assembly**

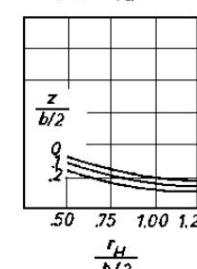
$$l_t/l_a = 1$$



$$\Lambda_L = 9$$

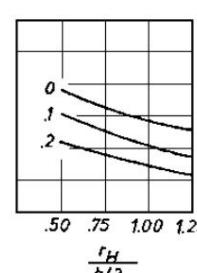
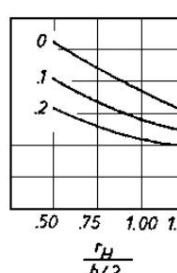
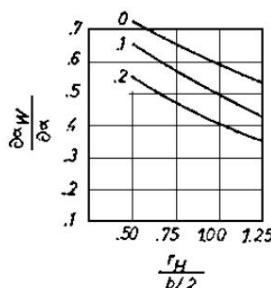
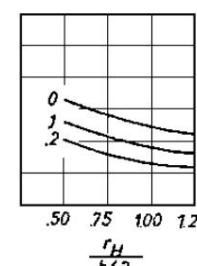
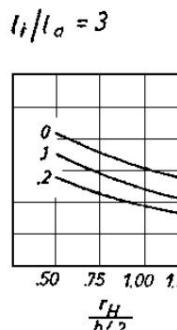
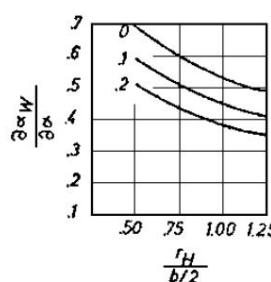


$$\Lambda_L = 12$$

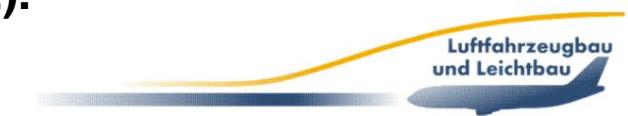


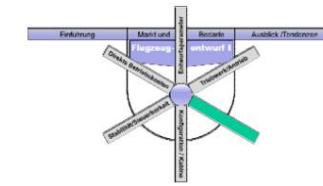
- The adjacent

Table presents potential-theoretically determined results for the Downwind angle on the Base low vertical Wing-tail distances and taking into account wing aspect ratio  $\bar{y}$ , reciprocal taper  $l_t/l_a$  and tail lever arms



$r_H / (b/2)$ .

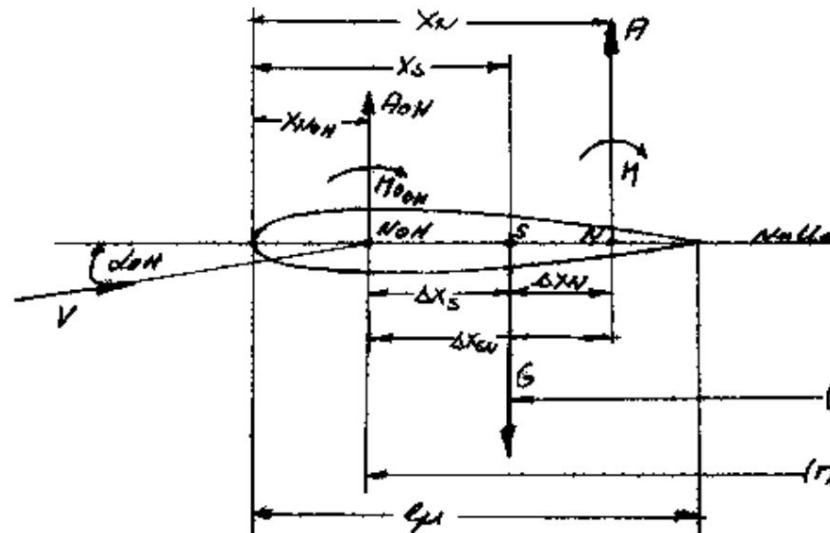




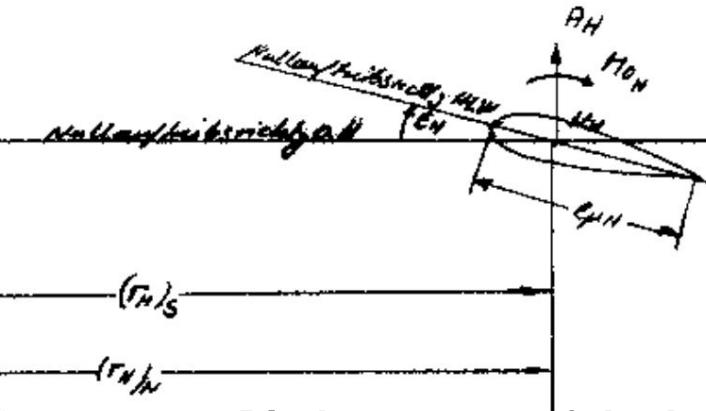
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# Flight mechanical design - basics

## 3.2 Lift and pitching moment of the isolated



Horizontal tail assembly



Pitch moment of the horizontal tail  
unit around the center

$$M_H \ddot{y}_H A_H \ddot{y}_H r_H \ddot{y}_H$$

of gravity  $q_H F_I$

$$c_{M_H} = \frac{A_H \ddot{y}_H r_H \ddot{y}_H}{I_H} \quad c_{A_H} = \frac{F_H q_H}{I_H}$$

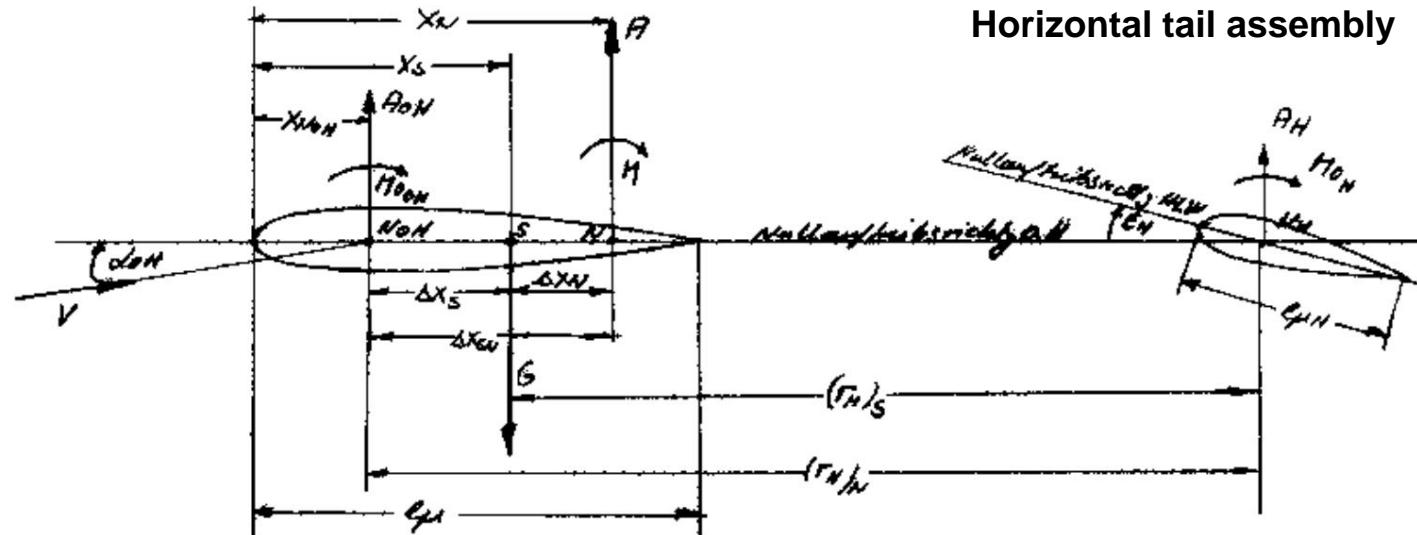
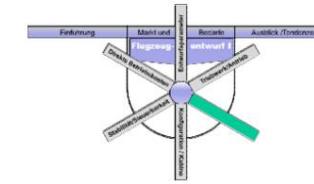
$$c_{M_H} = \frac{F_H q_H}{I_H} \quad c_{A_H} = \frac{F_H q_H}{I_H}$$

$$c_{M_{0H}} = \frac{F_H q_H}{I_H} \quad c_{A_H} = \frac{\ddot{y}_{HS} \ddot{y}_H}{I_H}$$

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# Flight mechanical design - basics

## 3.2 Lift and pitching moment of the isolated

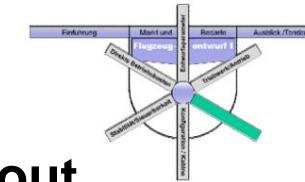


- The zero moment coefficient of the horizontal tail unit can usually be neglected, but in special configurations with very large tail unit volume (e.g. tandem wings) this component must be taken into account.
- However, in order to maintain general validity, it will be taken into account in the following considerations.

**F**

# Flight mechanical design - basics

## 3.3 Lift and pitching moment of the aircraft without Rudder deflection



- The total lift of the aircraft is the sum of the lift produced by the wing-fuselage combination and that of the horizontal tail unit. This means

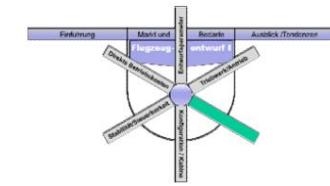
$$A \ddot{y} A_{\text{OH}} \ddot{y} A_{\text{H}}$$

- With coefficients this results in

$$A \ddot{y} \ddot{y} \ddot{y} c_{\text{ccc}} \quad U_{\text{H}} \frac{F_{\text{HH}, q}}{F_q}$$

and with the lift gradients or angles of attack

$$\begin{aligned} \ddot{y} \ddot{y} \ddot{y} & c_{\text{y}} \\ A A_{\text{OH}} & \alpha_{\text{OH}} \end{aligned} \quad \begin{aligned} \ddot{y} \ddot{y} \ddot{y} c_{\text{y}} & \\ \alpha_{\text{H}} & \alpha_{\text{H}} \end{aligned} \quad \begin{aligned} \ddot{y} & \\ \alpha_{\text{H}} & \alpha_{\text{H}} \end{aligned} \quad \begin{aligned} \ddot{y} & \\ \alpha_{\text{H}} & \alpha_{\text{H}} \end{aligned} \quad \begin{aligned} \ddot{y} & \\ \alpha_{\text{H}} & \alpha_{\text{H}} \end{aligned} \quad \begin{aligned} \ddot{y} & \\ \alpha_{\text{H}} & \alpha_{\text{H}} \end{aligned} \quad \begin{aligned} F_{\text{HH}, q} & \\ F_q & \end{aligned}$$



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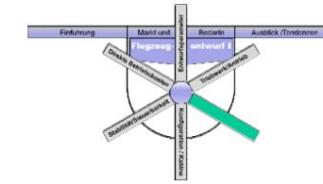
## Flight mechanics design - Basics 3.3 Lift and pitching moment of the aircraft without rudder deflection

- Solved for the angle of attack and the setting angle component results in  $\dot{\gamma}$

$$\frac{cc_{AA} \dot{\gamma}_{oH}}{1 + \frac{\dot{\gamma} c_A}{c_A_{oH}}} = \frac{F_{HH} q}{F q} \quad \frac{\ddot{\gamma} c_A}{\ddot{\gamma} c_A_{oH}} = \frac{W}{F q} \cdot \frac{F_{HH} q}{F q} = \frac{W}{q}$$

- The derivative with respect to the angle of attack  $\dot{\gamma} = \dot{\gamma}_{oH}$  leads to Aircraft lift increase

$$\frac{\ddot{\gamma} c_A}{\ddot{\gamma} c_A_{oH}} = \frac{1 + \frac{\dot{\gamma} c_A}{c_A_{oH}}}{\frac{F_{HH} q}{F q}} = \frac{1}{1 - \frac{F_{HH} q}{F q}}$$



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## Flight mechanics design - Basics 3.3 Lift and pitching moment of the aircraft without rudder deflection

Zeroing results in the zero angle of attack of the aircraft, which here is related to the zero lift direction of the wing. After

$$0c^y_{\text{A}_{\text{oH}}} \quad \frac{c^y_{\text{A}_H}}{c^y_{\text{A}_{\text{oH}}}} \cdot \frac{F_{\text{HH}}}{F} \cdot \frac{q}{q} = 1$$

$$\frac{\ddot{y}y_w}{\ddot{y}y_{\text{Oh}}} = \frac{\ddot{y}y_{\text{A}_H}}{\ddot{y}y_H}$$

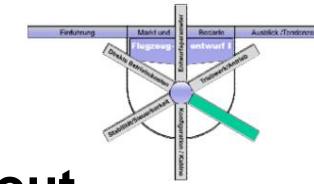
solving, we get

$$\frac{c^y_{\text{A}_H} \cdot \frac{F_{\text{HH}}}{F} \cdot \frac{q}{q}}{c^y_{\text{A}_{\text{oH}}} \cdot 1 + \frac{c^y_{\text{A}_H}}{c^y_{\text{A}_{\text{oH}}}} \cdot \frac{F_{\text{HH}}}{F} \cdot \frac{q}{q}}$$

and with the known denominator (lift increase of the aircraft)

$$\frac{\ddot{y}c_{\text{A}_H}}{c_A} \cdot \frac{F_{\text{HH}}}{F} \cdot \frac{q}{q} = \ddot{y}y_H$$



**F**

# Flight mechanical design - basics

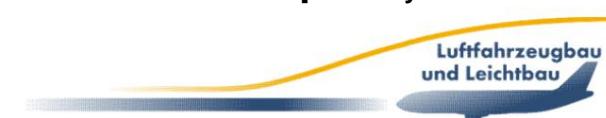
## 3.3 Lift and pitching moment of the aircraft without Rudder deflection

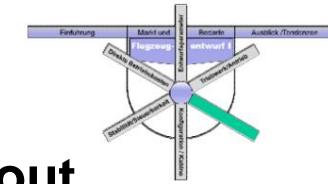
- The presence of a horizontal tail unit therefore leads to an increased lift gradient for the aircraft.
- The tail setting influences the zero lift direction.
- This changes the resolved polar in position (with a generally negative horizontal tail angle to the right (or aerodynamically arching) and direction (steeper climb)).
- For the moment related to the center of gravity we obtain

$$\text{so } M \ddot{y} = M \ddot{y} A_{0oH} \ddot{y} x \ddot{y} x_{0H} \ddot{y} A_s \ddot{y} r \ddot{y}_{NoH} \ddot{y} H H S \ddot{y} - \ddot{y} M oH$$

and with  $\ddot{y}xS = (xS - xNoH)$  and expressed in coefficients:

$$C_{M\ddot{y}0oH} = \frac{\ddot{y} c}{A_{0H}} \frac{x_s}{I_{\ddot{y}}} \cdot C_{A_H} \frac{F_{HHH} S q}{F q} \cdot \frac{\ddot{y} \ddot{y}}{\ddot{y} c} \frac{M_{0H}}{I_{\ddot{y}}} \cdot \frac{F_{HH}}{F} \frac{q}{q} \cdot \frac{I_{\ddot{y}H}}{I_{\ddot{y}}}$$



**F**

# Flight mechanical design - basics

## 3.3 Lift and pitching moment of the aircraft without Rudder deflection

- With  $c_{AoH} \frac{F_{HH}}{F}, \frac{q}{q}$

and  $\ddot{y}_{MM} \ddot{y}_{M0H} \ddot{y}_{xS}$

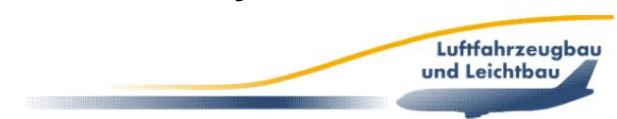
you get

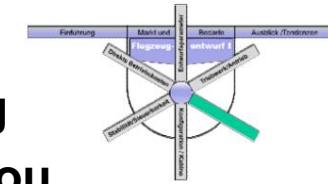
$$c_{c_{MM}} \frac{\ddot{y}}{\ddot{y}} \frac{\ddot{y}}{\ddot{y}} c_{A,A} \frac{F_{HH}}{F} \frac{q}{q} \frac{\ddot{y}}{\ddot{y}} \frac{\ddot{y} x_s}{I_{\ddot{y}}} \frac{\ddot{y} x_s}{I_{\ddot{y}}} c_{A_H} \frac{F_{HH}}{F} \frac{q}{q} \frac{\ddot{y}}{\ddot{y}} \frac{\ddot{y} x_s}{I_{\ddot{y}}} \frac{\ddot{y} x_s}{I_{\ddot{y}}}$$

$$\ddot{y} c_{M_{0H}} \frac{F_{HH}}{F} \frac{q}{q} \frac{I_{\ddot{y}_H}}{I_{\ddot{y}}}$$

or.

$$c_{c_{MM}} \frac{\ddot{y}}{\ddot{y}} c_A \frac{\ddot{y} x_{SHH}}{I_{\ddot{y}}} \frac{F}{F} \frac{q}{q} \frac{\ddot{y}}{\ddot{y}} c_{A_H} \frac{\ddot{y} x_{SHH}}{I_{\ddot{y}}} \frac{\ddot{y}}{\ddot{y}} c_{M_{0H}} \frac{I_{\ddot{y}_H}}{I_{\ddot{y}}} \frac{\ddot{y}}{\ddot{y}}$$



**F**

## Flight mechanics design - Basics 3.3 Lift and pitching moment of the aircraft without rudder deflection • If you set the lift coefficient of the horizontal tailplane

**coefficient of the horizontal tailplane**

$$c_{A_H} = c_{A_H} \cdot \frac{C_L}{C_L} = 1 \quad \frac{C_L}{C_L} = \frac{C_L}{C_L} = 1 \quad \frac{C_L}{C_L} = \frac{C_L}{C_L}$$

**and additionally the lift increase of the aircraft**

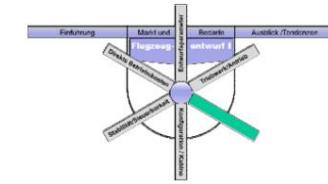
$$c_A = c_A \cdot \frac{C_L}{C_L} = 1 \quad \frac{C_L}{C_L} = \frac{C_L}{C_L} = 1 \quad \frac{C_L}{C_L} = \frac{C_L}{C_L}$$

$$c_{A_{OH}} = c_{A_{OH}} \cdot \frac{F_{q_{OH}}}{F_{q_{OH}}} = 1 \quad \frac{F_{q_{OH}}}{F_{q_{OH}}} = \frac{F_{q_{OH}}}{F_{q_{OH}}} = 1 \quad \frac{F_{q_{OH}}}{F_{q_{OH}}} = \frac{F_{q_{OH}}}{F_{q_{OH}}}$$

**The aircraft moment coefficient is:**

$$MMA_{00H} = \frac{I_x q_{SHH} F_{q_{OH}}}{I_y} = \frac{\frac{C_L}{C_L} \cdot \frac{C_L}{C_L} \cdot \frac{C_L}{C_L}}{I_y} = \frac{\frac{C_L}{C_L} \cdot \frac{C_L}{C_L} \cdot \frac{C_L}{C_L}}{I_y}$$

Luftfahrtbau  
und Leichtbau



F

## Flight mechanics design - Basics 3.3 Lift and pitching moment of the aircraft without rudder deflection • This

**expression can be converted into the previously presented**

**Straight line equation**

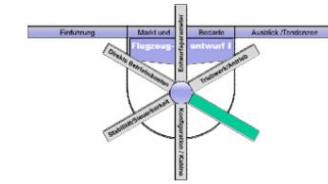
$$\frac{\dot{y}_c}{M} = \frac{c}{A}$$

and it results

$$C_M = C_{M_{0,0H}} + \frac{q_{HH}}{q} \frac{F}{I_y} + \frac{r_{HN}}{I_y} + \frac{I_H}{I_y} \frac{\dot{y}_H}{\dot{y}}$$

The first two summands correspond to the zero moment coefficient and the expression in brackets to the moment increase.

$$C_A = \frac{F c_{HA1}}{F C_{A_{0H}}} \frac{\dot{y}_H}{\dot{y}_1} + \frac{\dot{y}_W}{\dot{y}}$$

**F**

## Flight mechanics design - Basics 3.3 Lift and pitching moment of the aircraft without rudder deflection • The factor in front of

**cA** is the moment increase known as the stability measure **s** or the negative slope of the straight line  $cM = f(cA)$ .

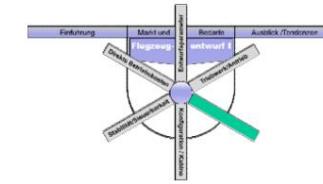
$$\begin{array}{c}
 \frac{\ddot{y} c_{A_H}}{F} \quad \ddot{y} \quad 1 \quad \frac{\ddot{y} \ddot{y}_w \ddot{y}}{\ddot{y} \ddot{y}} \\
 \hline
 s \quad \frac{\ddot{y} c_M}{\ddot{y} c_A} \quad \frac{\ddot{y} x q_{SHHHN} \ddot{y} \ddot{y} q}{I} \quad \frac{F}{F} \quad \frac{\ddot{y} r \ddot{y}}{I} \quad \frac{\ddot{y} c_{A_{oH}}}{F c_{HA1}} \quad \ddot{y} \ddot{y} \quad \frac{\ddot{y} \ddot{y}_w \ddot{y}}{\ddot{y} \ddot{y}} \\
 \hline
 \ddot{y} c_A \quad I \quad F \quad I \quad \ddot{y} \ddot{y}_H \quad \ddot{y} \ddot{y} \quad \ddot{y} \ddot{y} \quad \ddot{y} \ddot{y}_w \ddot{y} \\
 \hline
 F \quad c_{A_{oH}} \quad 1 \quad \ddot{y} \quad \ddot{y} \ddot{y} \quad \ddot{y} \ddot{y} \quad \ddot{y} \ddot{y}_w \ddot{y}
 \end{array}$$

• The stability is therefore strongly dependent on the center of gravity. •

**Definition:** horizontal tail volume (ratio) **VH**, tail factor

$$V_H = \frac{F r_{HHN}}{F} \frac{\ddot{y} \ddot{y}}{I} \quad 0.5 \dots 1.0$$





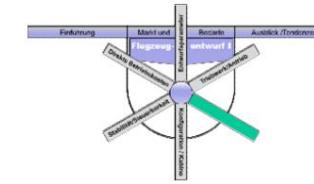
F

## Flight mechanics design - Basics 3.3 Lift and pitching moment of the aircraft without rudder deflection

- At the centre of gravity with the stability factor  $s = 0$ , the Airplane has an indifferent equilibrium.
- The center of gravity is then exactly at the aircraft neutral point.
- Setting the equation to zero gives:

$$\begin{array}{c}
 s = 0 \\
 ! \\
 \ddot{y} rx \quad \text{SHHHHHHQ} \\
 \hline
 I_{\dot{y}}
 \end{array}
 \quad
 \begin{array}{c}
 F \\
 \ddot{y} \quad \ddot{y} \\
 \hline
 F \quad I_{\dot{y}}
 \end{array}
 \quad
 \begin{array}{c}
 \ddot{y} \quad \ddot{y} \\
 \hline
 \ddot{y} \quad \ddot{y} \quad W \quad \ddot{y} \\
 \hline
 c A_H \quad \ddot{y} \quad \ddot{y} \quad \ddot{y} \quad \ddot{y} \\
 \hline
 c A_{Oh} \quad \ddot{y} \quad \ddot{y} \quad \ddot{y} \quad \ddot{y} \\
 \hline
 \ddot{y} \\
 \hline
 F \quad c A_{H1} \quad \ddot{y} \quad \ddot{y} \quad H \quad \ddot{y} \quad \ddot{y} \quad W \quad \ddot{y} \\
 \hline
 F \quad c A_{Oh} \quad 1 \quad \ddot{y} \quad \ddot{y} \quad \ddot{y} \quad \ddot{y} \quad \ddot{y} \quad \ddot{y}
 \end{array}$$





## F

## Flight mechanics design - Basics 3.3 Lift and pitching moment of the aircraft without rudder deflection

- Use of  $\frac{\ddot{y}}{x_s I} = \frac{x_{N\ddot{y}}}{I_{\ddot{y}}} = \frac{x_{NoH}}{I_{\ddot{y}}}$  and

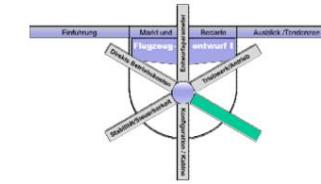
- After adjusting, you get the neutral point position of the aircraft:

$$\begin{array}{c}
 \ddot{y}_{HNHA}, \frac{F}{q}, \frac{c^{\ddot{y}}_H}{c_{A_{oH}}}, \ddot{y}_1, \frac{\ddot{y}\ddot{y}_w}{\ddot{y}\ddot{y}} \\
 \hline
 \frac{x_N}{I_{\ddot{y}}}, \frac{x_{NoH}}{I_{\ddot{y}}} \quad \frac{F}{q}, \frac{c^{\ddot{y}}_{A_{oH}}}{c_{A_{oH}}}, \ddot{y}\ddot{y}, \ddot{y}\ddot{y} \\
 \hline
 F c_{A1} \frac{\ddot{y}\ddot{y}_H}{c_{A_{oH}}}, \ddot{y}_1, \frac{\ddot{y}\ddot{y}_w}{\ddot{y}\ddot{y}}
 \end{array}$$

- This is located between the wing and the altitude  
Tail neutral point according to the tail area, the dynamic pressure ratio, the tail lever arm and the quotient of the lift gradients, i.e. the geometric shape of the wing and tail.

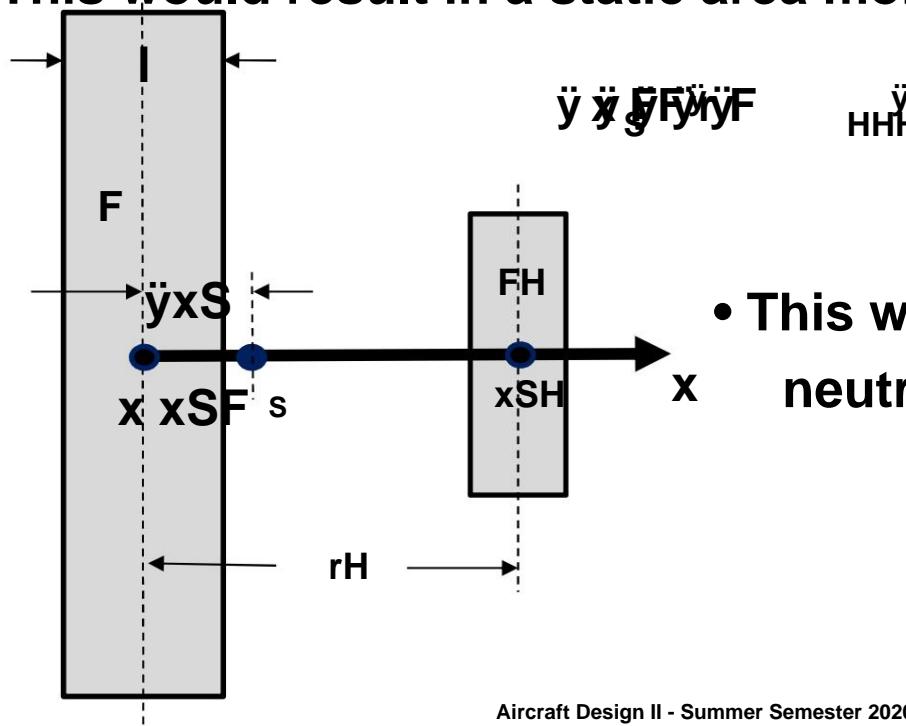
**F**

## Flight mechanics design - Basics 3.3 Lift and pitching moment of the aircraft without rudder deflection • A



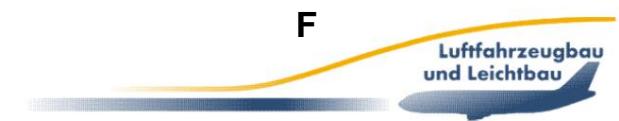
simplified estimation of the neutral point position based on a center of gravity calculation is possible in the first approach, but would not take the aerodynamic influences ( $cA'$ ,  $\dot{y}_w$ ,  $q$ ) into account.

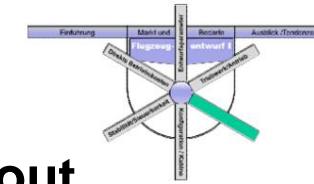
- This would result in a static area moment of:



- This would correspond to the neutral point position

$$\frac{x_N}{I_y} = \frac{x_{NoH}}{I_y} = \frac{I_y}{F_1 y_H}$$

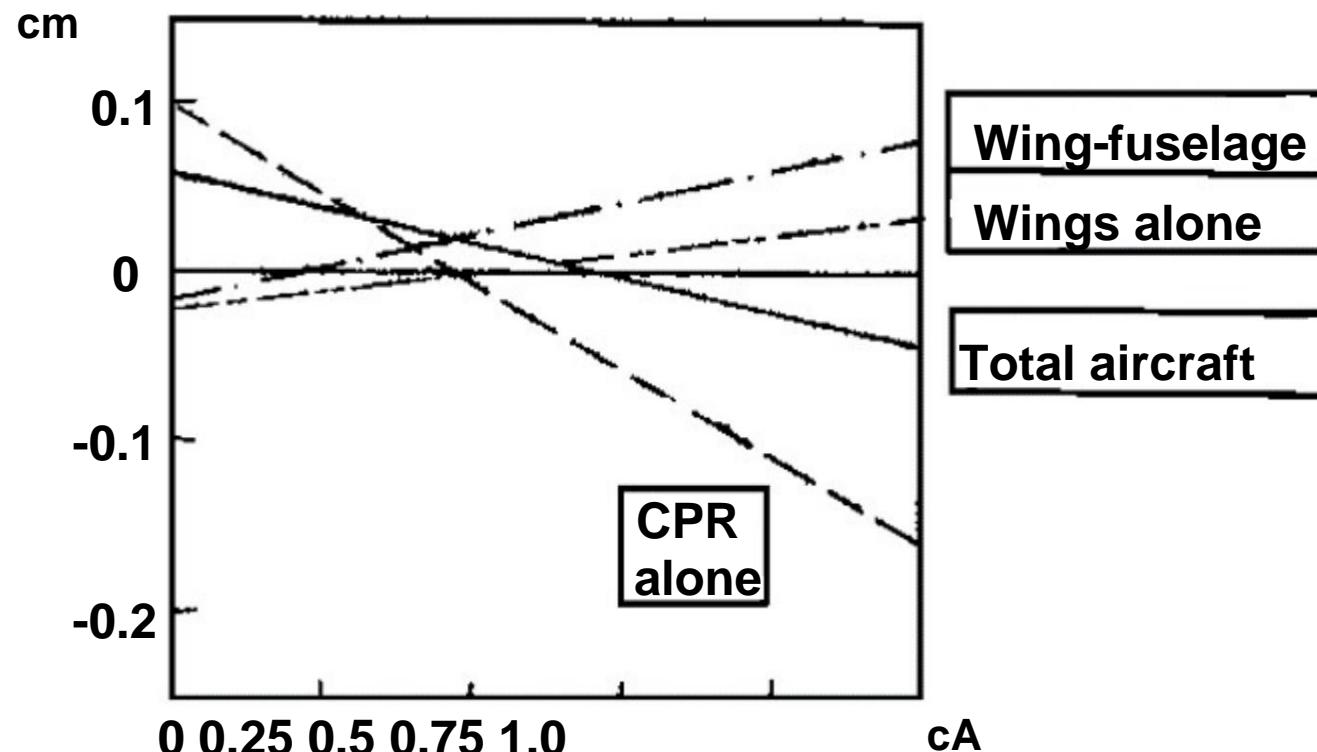




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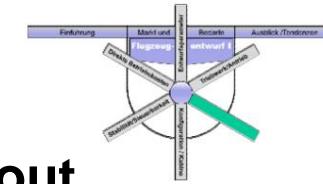
# Flight mechanical design - basics

## 3.3 Lift and pitching moment of the aircraft without Rudder deflection



- The destabilizing parts of the wing and fuselage must be overcompensated by the stabilizing effect of the horizontal tail unit in order to obtain a statically stable aircraft configuration.

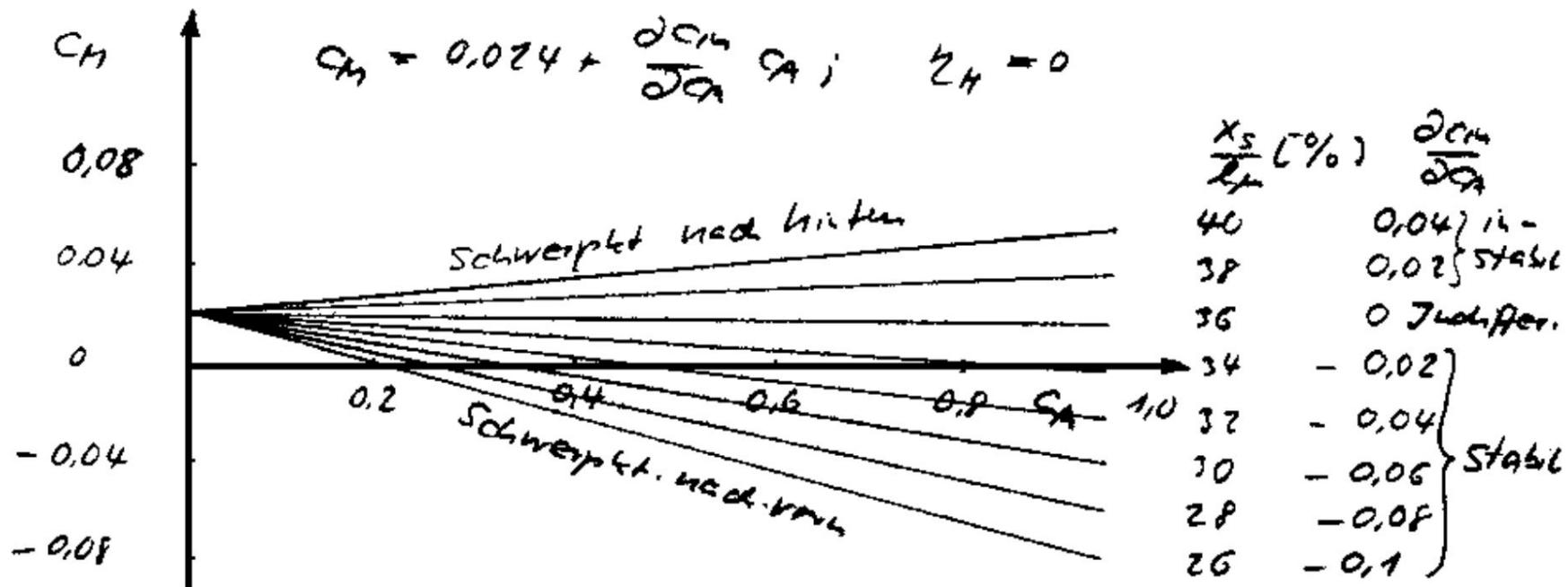




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# Flight mechanical design - basics

## 3.3 Lift and pitching moment of the aircraft without Rudder deflection

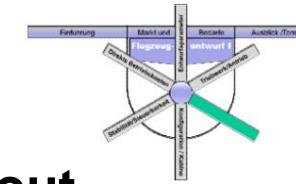


- An aircraft can only fly stationary with a lift coefficient at which the moment coefficient disappears. • All other flight speeds and thus lift coefficients can only be achieved with constant rudder deflection by changing the center of gravity.

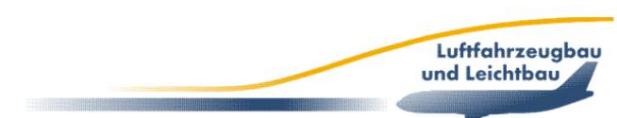
**F**

# Flight mechanical design - basics

## 3.3 Lift and pitching moment of the aircraft without Rudder deflection



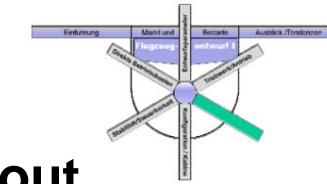
- It can be seen that in the case shown above, at  $dcM / dcA$  between -0.02 and -0.1 moment equilibrium for the  $cA$  range from 0.2 to 1.0.
- With a fixed tail unit ( $\ddot{y}H = \text{const.}$ ) only one selected trimmed flight condition can be achieved with a center of gravity position.
- The further back the center of gravity is, the slower the aircraft becomes, because the equilibrium position is reached with a higher lift coefficient.
- In principle, the flight control of an aircraft can be achieved by shifting the center of gravity.
- In the example shown, the entire speed range between  $xS / l_m = 26\%$  and  $34\%$  can be covered.



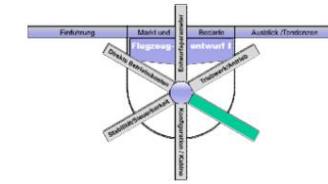
# F

## Flight mechanical design - basics

### 3.3 Lift and pitching moment of the aircraft without Rudder deflection



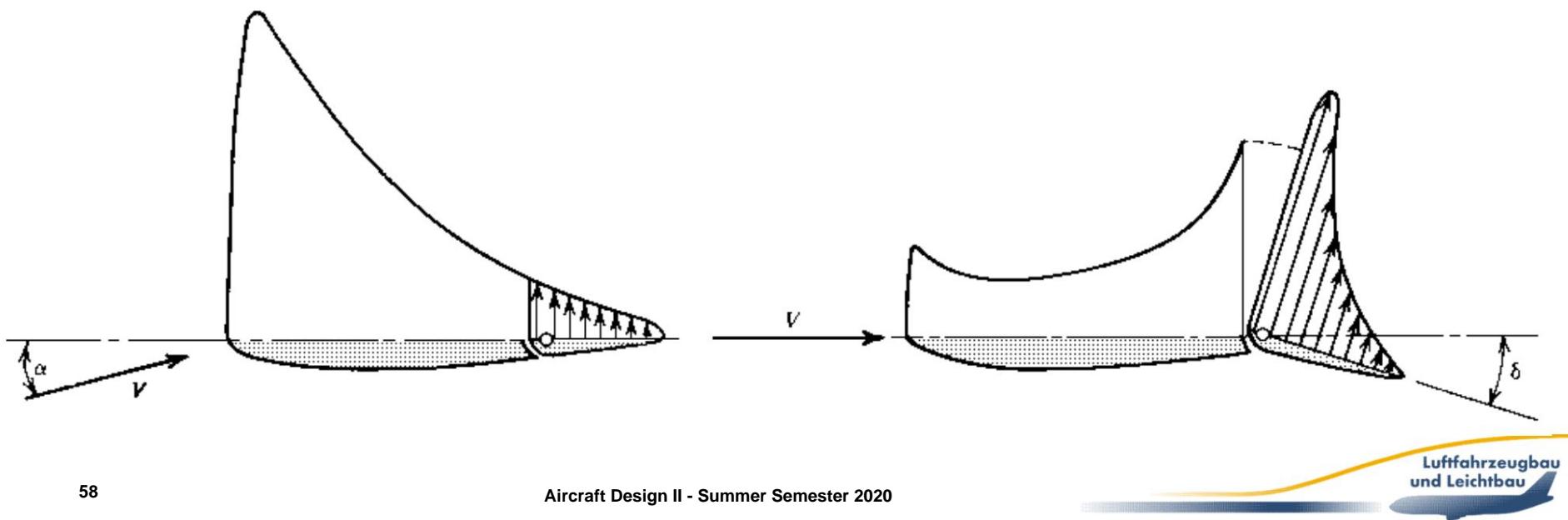
- This control principle was already used by Lilienthal and is still used today in hang gliders and kites applied.
- It is controlled by moving the pilot's body relative to the neutral point of the wing.
- The disadvantage here is the change in the degree of stability during the control processes.

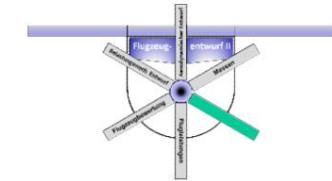
**F**

# Flight mechanical design - basics

## 3.4 Lift and pitching moment of the aircraft at Elevator deflection

- As a result of an elevator deflection, the lift coefficient of the horizontal stabilizer changes.
- If the lift coefficient remains constant, the angle of attack must change with rudder deflection.
- This also changes the overall pressure distribution and therefore the moment.





## F

# Flight mechanical design - basics

## 3.4 Lift and pitching moment of the aircraft at Elevator deflection

- In addition to a change in lift, a rudder deflection causes a rearward shift in the centre of lift, which represents the increased top-heavy (negative) moment.
- The lift coefficient with rudder deflection can be calculated using linear approaches for these influences.

$$c_{A_H} = \frac{\ddot{y} c_{A_H}}{\ddot{y} \ddot{y}_H}$$

$$\ddot{y} c_{A_H} = \ddot{y} \ddot{y}_H \ddot{y}$$

$$\frac{\ddot{y} c_{A_H}}{\ddot{y} \ddot{y}_H} \ddot{y} \ddot{y}_H \ddot{y}$$

$$\frac{\ddot{y} c_{A_H}}{\ddot{y} \ddot{y}_H} \ddot{y} \ddot{y}_H \ddot{y}$$

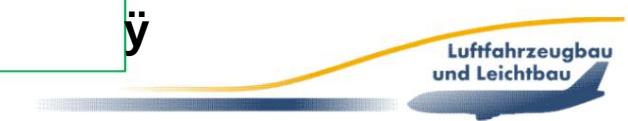
- A positive elevator angle means a downward deflection, ie an increase in lift is achieved.
- The overall horizontal tail lift coefficient is then:

$$c_{A_H} = c_{A_H} + c_{A_H} \frac{\ddot{y}}{\ddot{y}} \frac{\ddot{y} \ddot{y} \ddot{y}_{OH}}{\ddot{y}}$$

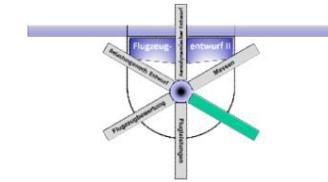
$$c_{A_H} = c_{A_H} \frac{\ddot{y}}{\ddot{y}} \frac{\ddot{y} \ddot{y} \ddot{y}_{OH}}{\ddot{y}}$$

$$\frac{\ddot{y} \ddot{y}_W}{\ddot{y} \ddot{y}_H} \ddot{y} \ddot{y} \ddot{y}_H \ddot{y}$$

$$\frac{\ddot{y} \ddot{y}_H}{\ddot{y} \ddot{y}_H} \ddot{y} \ddot{y}_H \ddot{y}$$



## F Flight mechanics design - Basics 3.4 Lift and pitching moment of the aircraft at Elevator deflection



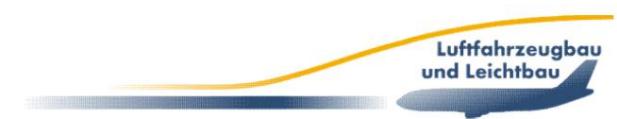
• The two partial differentials

$$\frac{c_{UH}}{\ddot{y}y_H} \quad \text{and} \quad \frac{\ddot{y}y_H}{\ddot{y}y_H}$$

are referred to as rudder effectiveness.

- The expression on the right describes exactly the Change in the zero angle of attack due to rudder deflection (change in camber).
- A fairly good approximation for the rudder effectiveness of the camber flap commonly used for control purposes is given by the following polynomial:

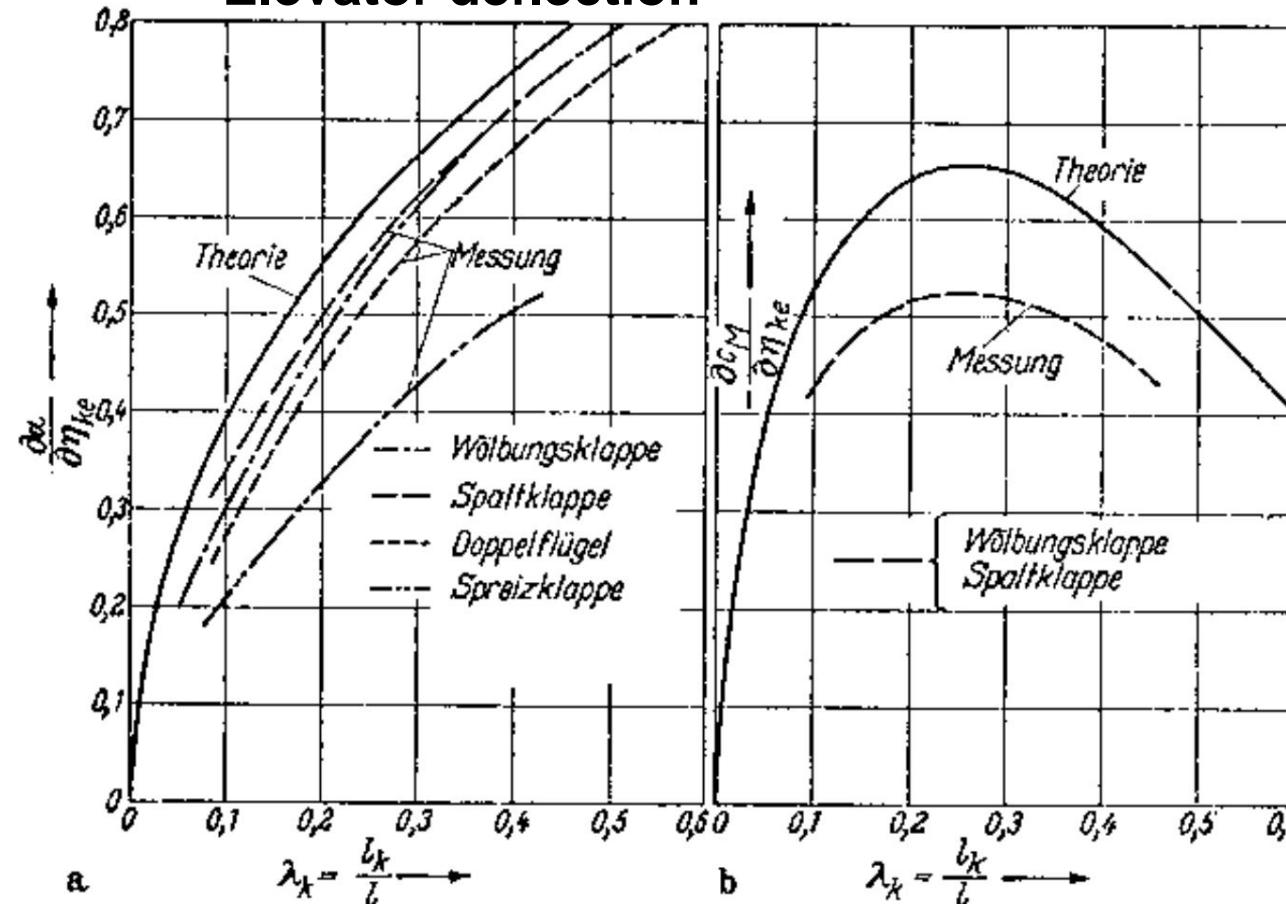
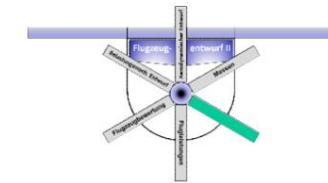
$$\frac{\ddot{y}y}{\ddot{y}y} = 2,9 \cdot \frac{\dot{y}^1 - k}{\dot{y}^1} \cdot \frac{\ddot{y}^2 + 2,7 \dot{y}^1 \ddot{y}^1 - k \dot{y}^1}{\dot{y}^1 \ddot{y}^1}^2$$



## F

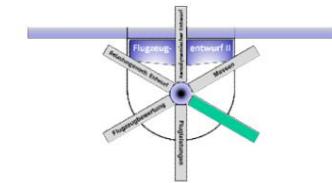
# Flight mechanical design - basics

## 3.4 Lift and pitching moment of the aircraft at Elevator deflection



Klappenwirksamkeit verschiedener Anordnungen; Theorie und Messung.

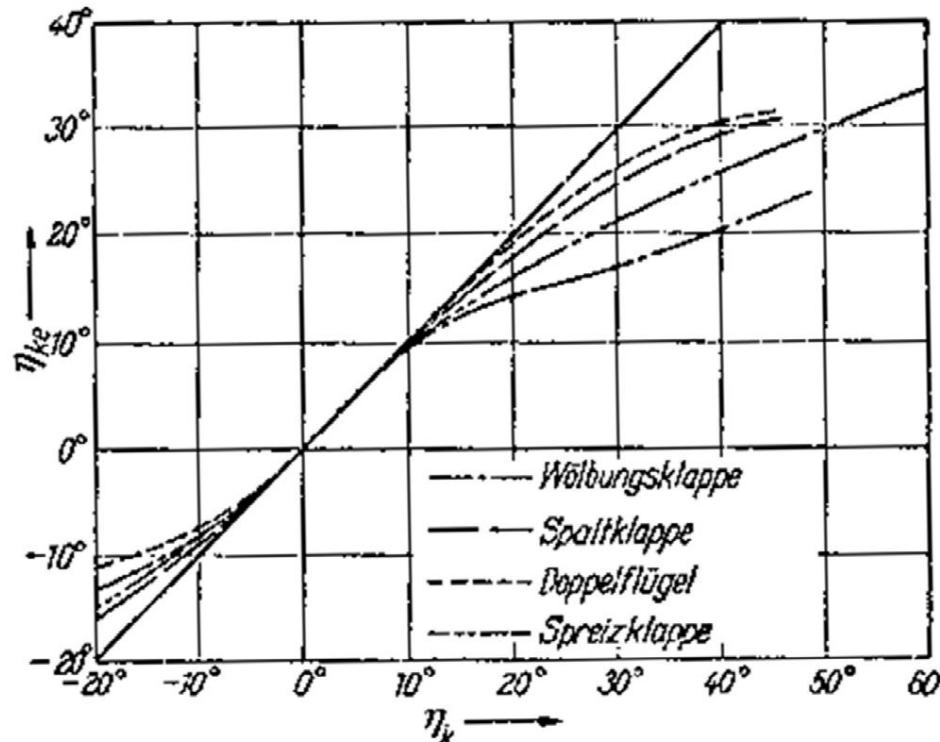
- a) Anstellwinkeländerung infolge Klappenausschlag  $\frac{\partial \alpha}{\partial \eta_{ke}}$  in Abhängigkeit vom Klappentiefenverhältnis  $\lambda_k$ ; b) Momentenänderung infolge Klappenausschlag  $\frac{\partial c_M}{\partial \eta_{ke}}$  in Abhängigkeit vom Klappentiefenverhältnis  $\lambda_k$ .

**F**

# Flight mechanical design - basics

## 3.4 Lift and pitching moment of the aircraft at Elevator deflection

- The linearity between rudder angle and change in lift coefficient assumed in the above relationships does not apply to the entire deflection range, as the following graphic shows:



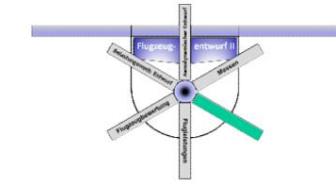
Zuordnung des effektiven Klappenausschlages  $\eta_{ke}$  zum geometrischen Klappenrusschlag  $\eta_k$  für verschiedene Klappenanordnungen.



## F

# Flight mechanical design - basics

## 3.4 Lift and pitching moment of the aircraft at Elevator deflection

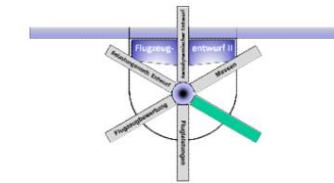


- An approximation is also given for the effective rudder angle, which was determined for cambered flaps:

$$\delta_r \approx 0.3 \sin \alpha_{\text{eff}}$$

## F

## Flight mechanics design - Basics 3.4 Lift and pitching moment of the aircraft at Elevator deflection

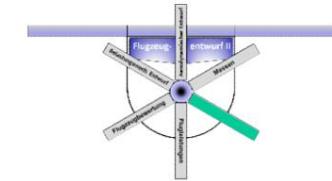


- Finally, this extension can also be used to Lift of the entire aircraft as a function of Rudder deflection can be formulated:

$$C_{M_H} = \frac{F \cdot c_q}{q} \cdot C_{A_{OH}} + \frac{\ddot{y}_H}{I} \cdot C_{A_H}$$

- For the pitch moment coefficient of the horizontal tail unit without Rudder deflection was:

$$C_{M_{OH}} = \frac{F}{q} \cdot C_{A_{OH}} + \frac{\ddot{y}_H}{I} \cdot C_{A_H}$$

**F**

## Flight mechanics design - Basics 3.4 Lift and pitching moment of the aircraft at Elevator deflection

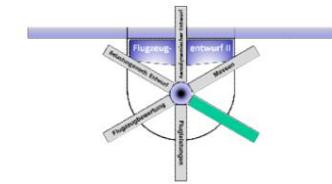
- The zero moment of the horizontal tail assembly is now increased by linear differential approach for the rudder deflection:  $\ddot{y}_c$

$$c_{M_{0H}} \frac{\ddot{y}}{\ddot{y}_c} c_{M_{0FH}} \frac{\ddot{y}}{\ddot{y}_c} \frac{M_{0FH}}{\ddot{y}\ddot{y}_H} \ddot{y}\ddot{y}_H$$

- The index FH stands for “fixed height control”. • This gives the moment coefficient of the

Total horizontal tail assembly:

$$c_{M_H} \frac{F_{HH}}{F} \frac{q}{q} = c_{M_{0FH}} \frac{\ddot{y}}{\ddot{y}_H} \frac{\ddot{y}_c}{\ddot{y}_H} \frac{M_{0FH}}{\ddot{y}\ddot{y}_H} \frac{\ddot{y}}{\ddot{y}_H} \frac{I}{I} \frac{\ddot{y}\ddot{y}_{HS}}{\ddot{y}\ddot{y}_H} c_{A_H} \frac{\ddot{y}}{\ddot{y}_H} \frac{\ddot{y}\ddot{y}_{oH}}{\ddot{y}\ddot{y}_H} \ddot{y}_1 \frac{\ddot{y}\ddot{y}_W}{\ddot{y}\ddot{y}} \frac{\ddot{y}\ddot{y}_H}{\ddot{y}\ddot{y}_H} \frac{\ddot{y}\ddot{y}_H}{\ddot{y}\ddot{y}_H} \frac{\ddot{y}\ddot{y}_H}{\ddot{y}\ddot{y}_H}$$

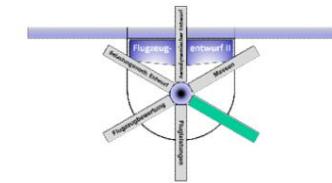


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## Flight mechanics design - Basics 3.4 Lift and pitching moment of the aircraft at Elevator deflection

- Now the rudder effect is also calculated for the aircraft moment introduced.
- The moment equation without rudder deflection was:

$$\begin{aligned}
 & \text{cc}_{MM} \frac{\ddot{y}}{y} c_{A_{oH}} = \frac{\ddot{y} x_{SHH} F}{I} \frac{q}{q}, \quad \left\{ \begin{array}{l} \ddot{y} \ddot{v} \\ \ddot{c}_{A_H} \end{array} \right. , \quad \left. \begin{array}{l} \ddot{c}_A \frac{\ddot{y}}{y} \frac{\ddot{y} \ddot{W}}{y} \\ \ddot{c}_{A_{oH}} \frac{F}{q} \frac{\ddot{y} \ddot{W}}{y} \end{array} \right. , \quad \left. \begin{array}{l} \ddot{y} \ddot{y} \ddot{c}_{HM} \\ \ddot{y} \ddot{y} \end{array} \right. , \quad \left. \begin{array}{l} I \\ I \end{array} \right. \\
 & \text{cc}_{MM} \frac{\ddot{y}}{y} c_{A_{oH}} = \frac{\ddot{y} x_{SHH} F}{I} \frac{q}{q}, \quad \left. \begin{array}{l} \ddot{y} \ddot{v} \\ \ddot{c}_{A_H} \end{array} \right. , \quad \left. \begin{array}{l} \ddot{c}_A \frac{\ddot{y}}{y} \frac{\ddot{y} \ddot{W}}{y} \\ \ddot{c}_{A_{oH}} \frac{F}{q} \frac{\ddot{y} \ddot{W}}{y} \end{array} \right. , \quad \left. \begin{array}{l} \ddot{y} \ddot{y} \ddot{c}_{HM} \\ \ddot{y} \ddot{y} \end{array} \right. , \quad \left. \begin{array}{l} I \\ I \end{array} \right. 
 \end{aligned}$$



## F

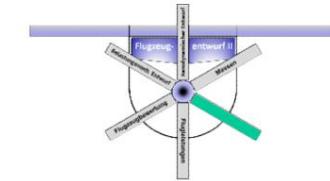
## Flight mechanics design - Basics 3.4 Lift and pitching moment of the aircraft at Elevator deflection

- Both the angle of attack term  $\dot{\gamma}_H$  (brackets) and the zero moment coefficient now have a rudder effect:

$$\begin{aligned}
 & \frac{\ddot{\gamma}_x}{I} = \frac{S_{HH} \dot{\gamma}_c}{F} = \frac{q}{q}, \\
 & \frac{\ddot{\gamma}_{HN}}{I} = c_{A_H} \dot{\gamma}, \\
 & \frac{\ddot{\gamma}_{\xi 1}}{F} = \frac{c_{A_{OH}}}{q} \dot{\gamma} + c_{A_{OH}} \frac{\ddot{\gamma}_H}{q}, \\
 & \text{Red circles highlight } \ddot{\gamma}_H \text{ and } c_{A_{OH}} \text{ terms.}
 \end{aligned}$$

# F Flight mechanical design - basics

## 3.4 Lift and pitching moment of the aircraft at Elevator deflection



- Now, again, a transformation into the now bilinear form of the moment equation can be carried out:

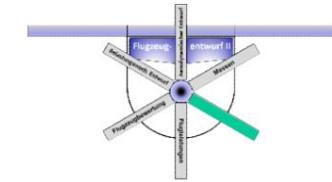
$$c_{MM} \ddot{y} = \frac{\dot{y} c_M}{\dot{y} c_A} \ddot{y} \phi_A + \frac{\dot{y} c_M}{\dot{y} y_H} \ddot{y} \ddot{y}_H$$

- The derivations lead back to the derivatives, whereby the Stability measure  $s$  does not change due to the elevator angle and by

$$\ddot{y} s = \frac{\dot{y} \dot{y} c_M}{\dot{y} c_A} \ddot{y} + \frac{s}{I_y} \ddot{y} r_{NH} \ddot{y} + \frac{F_q H}{I_y F c} \ddot{y} + \frac{\frac{C_{A_H}}{C_{A_{OH}}} \ddot{y} \ddot{y}_1 \ddot{y} \ddot{y}_H \ddot{y} w}{\frac{1}{F c} \frac{HA}{A_{OH}} \ddot{y} \ddot{y}_1 \ddot{y} \ddot{y}_H \ddot{y} w}$$

described.



**F**

## Flight mechanics design - Basics 3.4 Lift and pitching moment of the aircraft at Elevator deflection

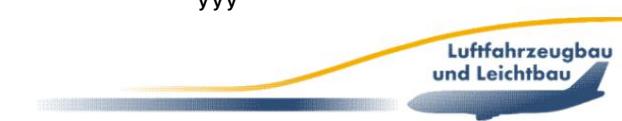
- For the rudder effectiveness of the aircraft, the Form the derivative  $\frac{\partial c_M}{\partial \delta_H}$  from the moment equation:

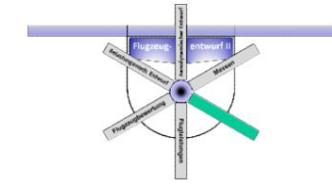
$$c_{M,0,H} = \frac{\frac{\partial c_A}{\partial \delta_H} \cdot \frac{\partial F_{H,H}}{\partial q} \cdot \frac{q}{I}}{\frac{\partial c_A}{\partial \delta_H} + \frac{\partial c_{A,0,H}}{\partial q} \cdot \frac{F_{H,H}}{I}}$$

$$\frac{\partial c_{M,0,H}}{\partial \delta_H} = \frac{\frac{\partial c_A}{\partial \delta_H} \cdot \frac{\partial F_{H,H}}{\partial q} \cdot \frac{q}{I}}{\frac{\partial c_A}{\partial \delta_H} + \frac{\partial c_{A,0,H}}{\partial q} \cdot \frac{F_{H,H}}{I}}$$

- You get the relationship

$$\frac{\partial c_M}{\partial \delta_H} = \frac{F_{H,H} \cdot q}{I} \cdot \frac{1}{\frac{\partial c_A}{\partial \delta_H} + \frac{\partial c_{A,0,H}}{\partial q} \cdot \frac{F_{H,H}}{I}}$$



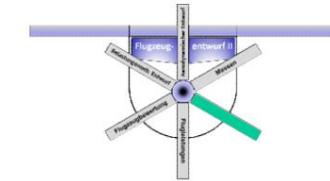
**F****Flight mechanical design - basics****3.4 Lift and pitching moment of the aircraft at  
Elevator deflection**

- In summary, the moment equation for the Aircraft with elevator deflection  $\ddot{\gamma}$  as follows:**

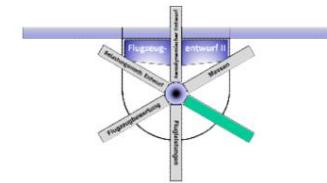
$$\begin{aligned}
 c_M &= c_{M_0} + \frac{q F_r \ddot{\gamma} - q F_l \ddot{\gamma}_N}{I_H} \\
 &\quad + \frac{\frac{c_A H}{I_H} \ddot{\gamma} \ddot{\gamma}_N}{I_H} \\
 &\quad + \frac{\frac{F_c A}{I_H} \ddot{\gamma} \ddot{\gamma}_N}{I_H} \\
 &\quad + \frac{\frac{c_A H}{I_H} \ddot{\gamma} \ddot{\gamma}_N}{I_H} \\
 &\quad + \frac{\frac{F_c A}{I_H} \ddot{\gamma} \ddot{\gamma}_N}{I_H} \\
 &\quad + \frac{\frac{c_A H}{I_H} \ddot{\gamma} \ddot{\gamma}_N}{I_H} \\
 &\quad + \frac{\frac{F_c A}{I_H} \ddot{\gamma} \ddot{\gamma}_N}{I_H}
 \end{aligned}$$

# F Flight mechanical design - basics

## 3.4 Lift and pitching moment of the aircraft at Elevator deflection

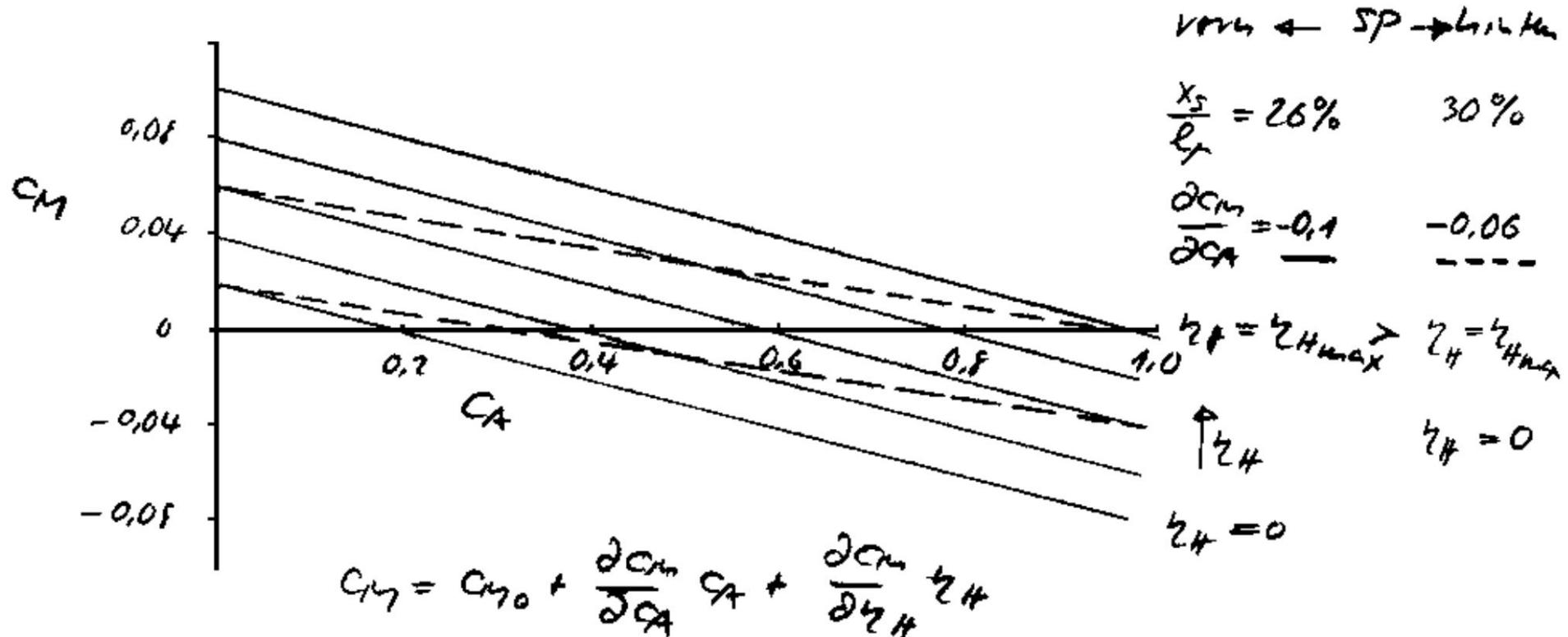


- In aerodynamic control by means of rudder deflection the stability measure  $s$  remains constant.
- There is a parallel shift of the  $cM-cA$  curves with the rudder deflection, which allows the trimming of the entire  $cA$  range for a given center of gravity position.

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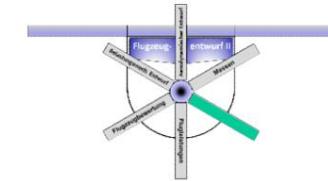
# Flight mechanical design - basics

## 3.4 Lift and pitching moment of the aircraft at Elevator deflection

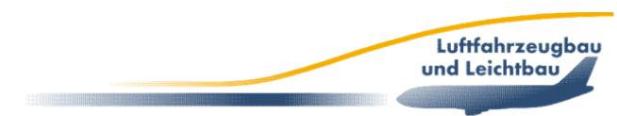


# F Flight mechanical design - basics

## 3.4 Lift and pitching moment of the aircraft at Elevator deflection

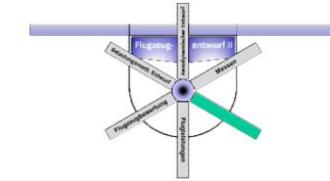


- The zero moment coefficient is changed by the rudder deflection.
  - The influence of the center of gravity on the static longitudinal stability. • In order to be able to fly with a certain lift coefficient, a much larger elevator deflection is required with the center of gravity at the front than with the center of gravity at the rear.
  - This means that the stability at the forward center of gravity must not be too great, otherwise the elevator deflection may not be sufficient to achieve moment equilibrium at the maximum lift coefficient. •
- The main task of flight mechanics design is to match the effectiveness of the tail unit to the center of gravity area.**

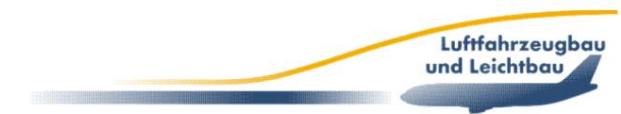


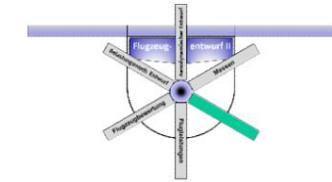
# F Flight mechanical design - basics

## 3.4 Lift and pitching moment of the aircraft at Elevator deflection

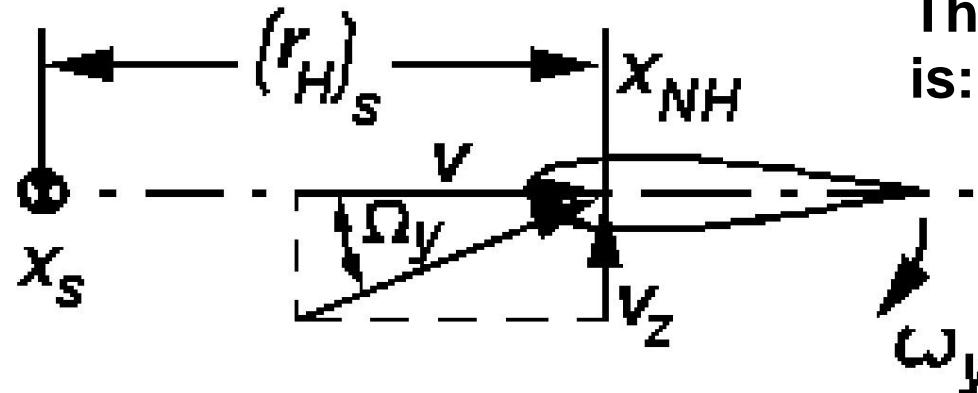


- One of the criteria for dimensioning the tail unit must therefore be that the entire flight range from high speed (small  $c_A$ ) to minimum speed ( $c_{A\max}$ ) can be trimmed with an appropriate rudder deflection.
- The rudder must be dimensioned in such a way that the wing can be brought into the position of maximum angle with the rudder deflection during landing.
- To design an aircraft with regard to its flight mechanics The mechanical properties not only have to ensure stability and trimming, but it must also be possible to generate a positive, i.e. tail-heavy pitch acceleration in every flight condition and configuration (cruise, take-off, landing). This is particularly necessary for recovering from high-speed flight (stepping flight).



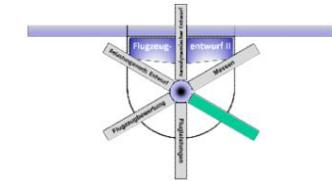
**F****Flight mechanical design - basics****3.4 Lift and pitching moment of the aircraft at  
Elevator deflection**

- The ability to catch, i.e. the ability to make positive longitudinal pitch changes, must be ensured for all flight speeds.
- Rotational movements of the aircraft around the lateral axis have a dampening effect on the pitching moment.
- If the aircraft rotates around the lateral axis at a rotational speed  $\ddot{\gamma}_y$ , this induces a vertical movement in the tail unit, which leads to a change in the angle of attack.



The vertical speed  
is:

$$v_{e\dot{y}} = \dot{y} \cdot r_y$$



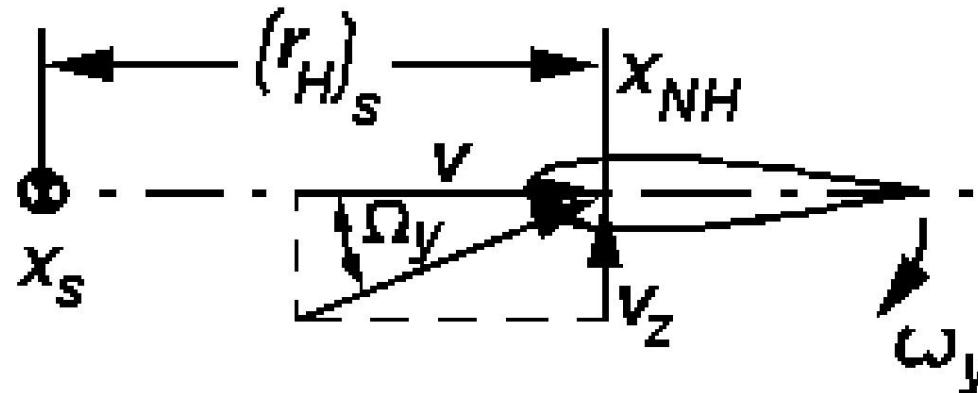
# F Flight mechanical design - basics

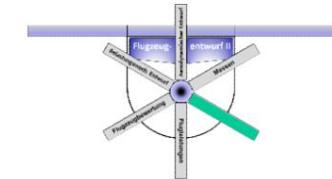
## 3.4 Lift and pitching moment of the aircraft at Elevator deflection

- The angle of attack change due to this vertical speed for small angles is

$$\dot{\gamma} = \frac{\tan \alpha}{V} = \frac{V_e}{V}$$

- $\dot{\gamma}$  is also called dimensionless tilt angle velocity designated.



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# Flight mechanical design - basics

## 3.4 Lift and pitching moment of the aircraft at Elevator deflection

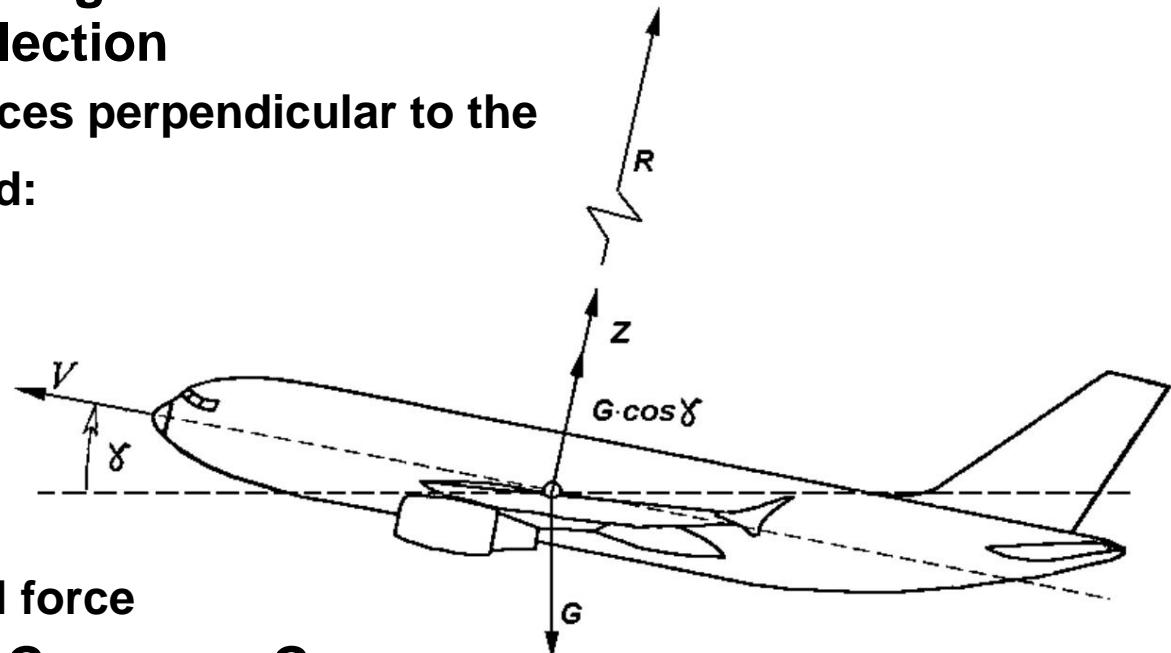
- From the equilibrium of forces perpendicular to the

The flight path is obtained:

$$A \ddot{y} G \ddot{y} \cos \ddot{y} \ddot{y} Z$$

and with

$$\ddot{y} \ddot{y} \ddot{y} \ddot{y} \frac{V}{R}$$

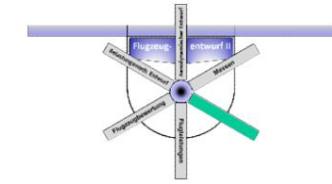


follows for the centrifugal force

$$Z \ddot{y} m \ddot{y} b \ddot{y} \ddot{y} \ddot{y} \ddot{y} \frac{G}{R} \frac{v^2}{R} \frac{G}{G} v \ddot{y} \ddot{y} \ddot{y} \ddot{y} \ddot{y} \ddot{y} \frac{G}{G}$$

- For recovery from a dive ( $\ddot{y} = 90^\circ$ ) with maximum boost

$$ZA_c \text{ Max } \ddot{y} \text{ Amax } \frac{\ddot{y}}{2} v^2 F \text{ applies } \ddot{y} \ddot{y} c \text{ Amax } \frac{\ddot{y}}{2} \frac{vg}{GF}$$

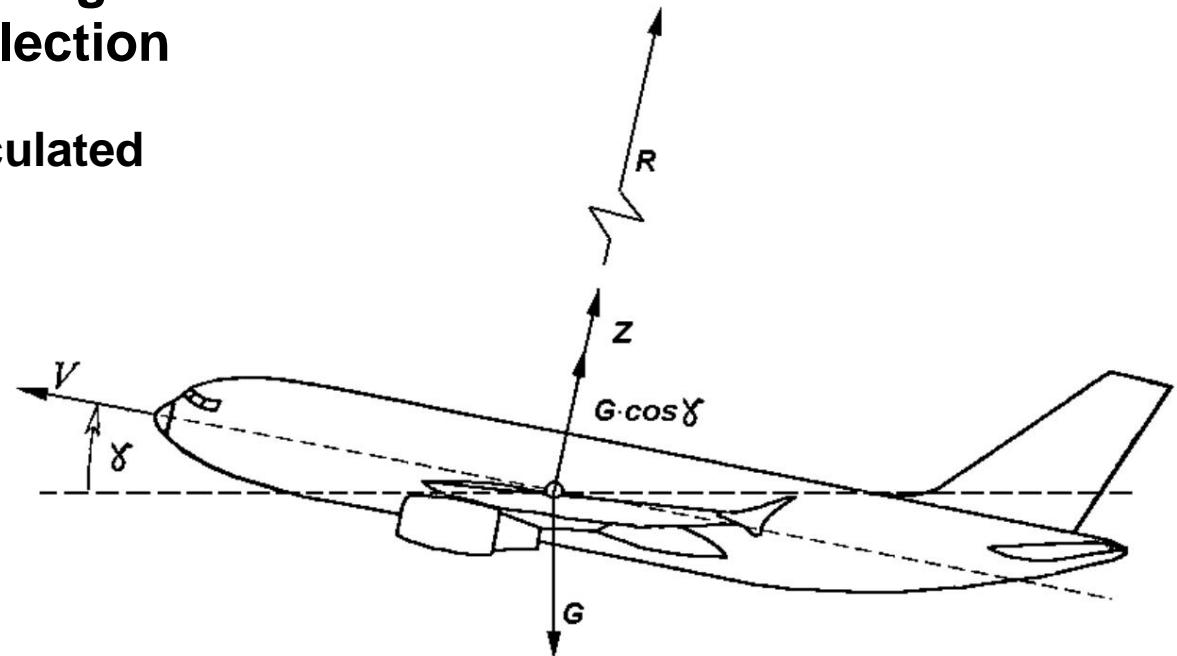
**F**

# Flight mechanical design - basics

## 3.4 Lift and pitching moment of the aircraft at Elevator deflection

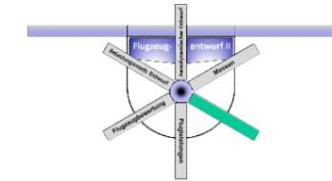
- The tilt angle speed is calculated as

**Help of the tail lever arms**



$$\frac{\dot{\gamma}}{\dot{y}} = \frac{v_{z_{\max}}}{v} = \frac{\ddot{y}_H \ddot{y}_S}{v} = \frac{c_A \dot{\gamma}_{\max}}{2} = \frac{\dot{y}}{\dot{y}_H \dot{y}_S} = \frac{G}{GF}$$

$$c_A \dot{\gamma}_{\max} = \frac{\dot{y}}{2} = \dot{y}_r \dot{y}_H \dot{y}_S = \dot{y} \dot{y}_x s = \frac{G}{GF}$$



# F Flight mechanical design - basics

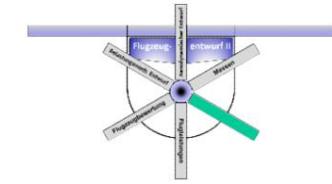
## 3.4 Lift and pitching moment of the aircraft at Elevator deflection

- Using this angle  $\dot{\gamma}_y$ , the moment equation must now be expanded with elevator deflection.

$$\begin{aligned}
 & \frac{c c_{M M} \dot{\gamma}_{0 H}}{F q} = \frac{\dot{\gamma} \dot{\gamma} r_{H N}}{I \dot{\gamma}}, \quad \ddot{\gamma} \ddot{\gamma} \ddot{\gamma} \ddot{\gamma} \ddot{\gamma} \ddot{\gamma} c \boxed{\ddot{\gamma}} \ddot{\gamma} c_{0 H}, \quad \frac{I}{I} \frac{\dot{\gamma}_H}{\dot{\gamma}} \dot{\gamma} \\
 & \ddot{\gamma} \ddot{\gamma} c_A \frac{\dot{\gamma} x_s}{I \dot{\gamma}} = \frac{\dot{\gamma} \dot{\gamma} r_{N H q}}{I F q F c}, \quad \frac{H H}{H A 1 \dot{\gamma} \dot{\gamma}_H} \frac{\ddot{\gamma} \ddot{\gamma} \ddot{\gamma} \ddot{\gamma} \ddot{\gamma} \ddot{\gamma} w}{F c A_{0 H} \dot{\gamma} \dot{\gamma} \dot{\gamma} \dot{\gamma} \dot{\gamma} \dot{\gamma} w} \\
 & \ddot{\gamma}_H \frac{F q_H}{F q l \ddot{\gamma}} = \frac{\dot{\gamma} I \dot{\gamma}_H}{\dot{\gamma} \dot{\gamma}_H} \frac{\ddot{\gamma} C_{M 0}}{I \dot{\gamma}} \frac{r_{H N}}{\dot{\gamma}_H}, \quad \ddot{\gamma} \ddot{\gamma} \ddot{\gamma} \ddot{\gamma} \ddot{\gamma} \ddot{\gamma} c_{A_H} \ddot{\gamma} \ddot{\gamma} \ddot{\gamma} \ddot{\gamma} \ddot{\gamma} \ddot{\gamma} H
 \end{aligned}$$

# F Flight mechanical design - basics

## 3.4 Lift and pitching moment of the aircraft at Elevator deflection

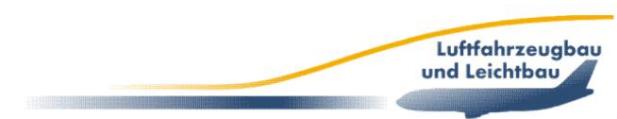


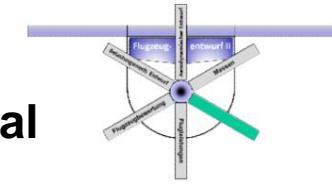
- The general moment equation has the following Look:

$$c_{M,yy} = \frac{\dot{y}c_M}{\dot{y}c_A} \dot{y}\theta_A + \frac{\dot{y}c_M}{\dot{y}y_H} \ddot{y}y_H + \frac{\dot{y}c_M}{\dot{y}y_y} \ddot{y}y_y$$

- The change in the moment coefficient is compared with the Tilt angle velocity obtained by differentiation with respect to  $\dot{y}y$  :

$$\frac{\dot{y}c_M}{\dot{y}y_y} = \frac{\ddot{y}_H \ddot{V}_H}{I_y} = \frac{F}{F} \cdot \frac{q}{q} \cdot c_{UH}^{\dot{y}}$$



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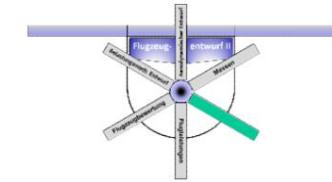
## Flight mechanics design - Basics 3.5 Static longitudinal stability with fixed rudder • The stability measure is unchanged

$$\frac{s \ddot{y} \ddot{y}}{\frac{\ddot{y} c_M}{\ddot{y} c_A} \ddot{y} \ddot{y}} = \frac{\ddot{y} x_s}{\ddot{y} \ddot{y} I \ddot{y}} + \frac{x_{N_{oH}}}{I \ddot{y}} \ddot{y} \frac{\ddot{y} \ddot{V}_{HMH}}{I \ddot{y}}, \frac{F}{F}, \frac{q}{q}, \frac{\frac{c_{A_H}}{c_{A_{oH}}} \ddot{y}}{\frac{F c_{HA1}}{F} \ddot{y} \ddot{y}_H} \frac{\ddot{y} \ddot{y}_1}{\ddot{y} \ddot{y}} \frac{\ddot{y} \ddot{y}_W \ddot{y}}{\ddot{y} \ddot{y}}$$

- At the center of gravity position for which the stability measure disappears, the aircraft exhibits an indifferent equilibrium, since the center of gravity then lies exactly at the aircraft ne
- Setting the equation to zero leads to

$$\frac{0 \ddot{y} \ddot{y}}{\frac{\ddot{y} x_N}{I \ddot{y}} \ddot{y}} = \frac{x_{N_{oH}}}{I \ddot{y}} \ddot{y} \frac{\ddot{y} \ddot{V}_{HMH}}{I \ddot{y}}, \frac{F}{F}, \frac{q}{q}, \frac{\frac{c_{A_H}}{c_{A_{oH}}} \ddot{y}}{\frac{F c_{HA1}}{F} \ddot{y} \ddot{y}_H} \frac{\ddot{y} \ddot{y}_1}{\ddot{y} \ddot{y}} \frac{\ddot{y} \ddot{y}_W \ddot{y}}{\ddot{y} \ddot{y}}$$



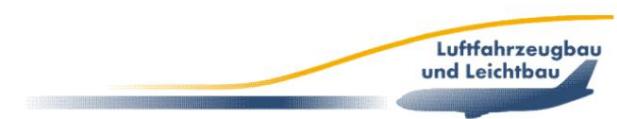
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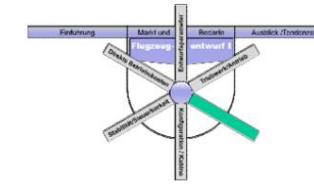
## Flight mechanical design - Basics 3.5 Static longitudinal stability with fixed rudder

- This gives the neutral point position of the aircraft:

$$\begin{array}{c}
 \ddot{y} \ddot{v} \\
 \text{HAA} F \\
 \hline
 \frac{x_N}{\dot{y}} \quad \frac{x_{N_{oH}}}{\dot{y}} \quad \ddot{y} \quad F \quad q \quad c_{A_{oH}}^{\dot{y}} \quad \ddot{y}_1 \quad \ddot{y} \ddot{y} \quad w \quad \ddot{y} \\
 \frac{I}{\dot{y}} \quad \frac{I}{\dot{y}} \quad \frac{F c_{A_1}}{\dot{y}} \quad \ddot{y} \ddot{y}_H \quad \ddot{y}_1 \quad \ddot{y} \ddot{y} \quad w \quad \ddot{y} \\
 \frac{F}{\dot{y}} \quad \frac{c_{A_{oH}}}{\dot{y}} \quad \ddot{y} \quad \ddot{y} \ddot{y} \quad \ddot{y} \ddot{y}
 \end{array}$$

- The neutral point is between the wing and horizontal tail neutral point as per
  - the tail area ratio, – the dynamic pressure ratio, – the tail lever arm and – the quotient of the lift gradients.

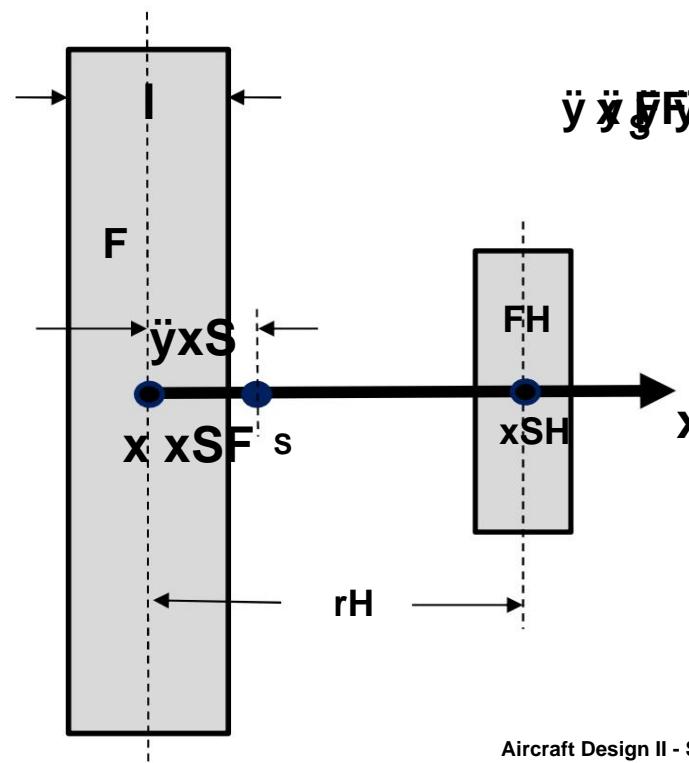




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## Flight mechanical design - Basics 3.5 Static longitudinal stability with fixed rudder

- A simplified estimation of the neutral point position based on a center of gravity calculation is possible in the first approach, but would not take into account the aerodynamic influences ( $cA'$ ,  $\ddot{y}_w$ ,  $q$ ).
- This would result in a static area moment of:



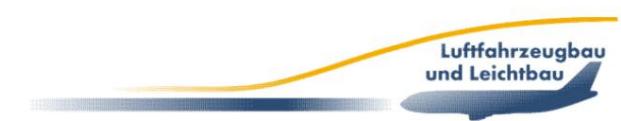
- This would correspond to the neutral point position

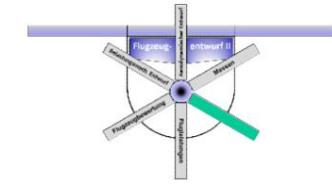
$$\frac{\ddot{y} \times s}{I} = \frac{\ddot{x} \times s}{I} \quad \text{or} \quad \frac{\ddot{y}}{I} = \frac{\ddot{x}}{I}$$

$$\frac{\ddot{y}}{I} = \frac{\ddot{x}}{I} \quad \text{or} \quad \ddot{y} = \ddot{x}$$

$$\frac{\ddot{y}}{r_H} = \frac{\ddot{x}}{x_N} \quad \text{or} \quad \ddot{y} = \frac{x_N}{r_H} \ddot{x}$$

$$\frac{\ddot{y}}{r_H} = \frac{\ddot{x}}{x_{NoH}} \quad \text{or} \quad \ddot{y} = \frac{x_{NoH}}{r_H} \ddot{x}$$





## F Flight mechanics design - Basics 3.5 Static longitudinal stability with fixed rudder • With the aircraft neutral point, a second Germanation of the stability measure is possible.

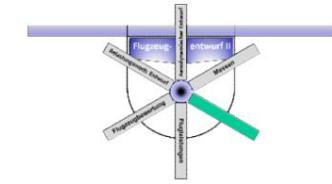
- If you solve the above equation for the wing neutral point, you get

$$\frac{x_{N_{oH}}}{I_y} \cdot \frac{x_N}{I_y} \cdot \frac{\ddot{y} \ddot{v}_{HNHH}}{F} \cdot \frac{F}{q} \cdot \frac{c_{A_H}^{\dot{y}} \ddot{y}_1 \ddot{y} \ddot{y}_w \ddot{y}}{c_{A_{oH}}^{\dot{y}} \ddot{y} \ddot{y} \ddot{y} \ddot{y}} = \frac{F c_{HA1}^{\dot{y}} \ddot{y} \ddot{y}_H \ddot{y}_1 \ddot{y} \ddot{y}_w \ddot{y}}{F c_{A_{oH}}^{\dot{y}} \ddot{y} \ddot{y} \ddot{y} \ddot{y}}$$

- If the above expression is converted into the following, known equation the stability measure  $s$  is used  $\ddot{y} \ddot{y}$

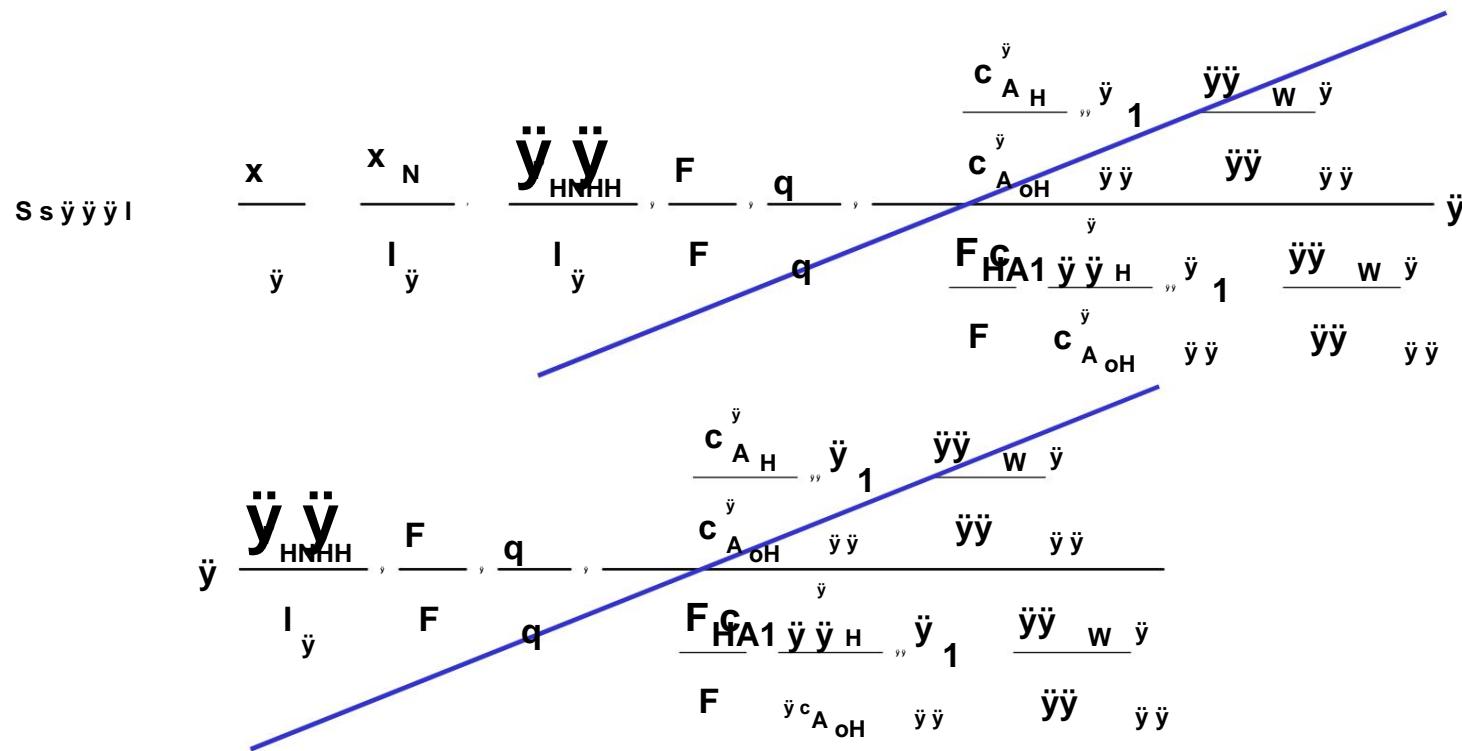
$$s \ddot{y} \ddot{y} \cdot \frac{\ddot{y} c_M}{\ddot{y} c_A} s \ddot{y} \ddot{y} \ddot{y} + \frac{x_{N_{oH}}}{I_y} \cdot \frac{r_{HNHH}}{I_y} \cdot \frac{F}{q} \cdot \frac{c_{A_H}^{\dot{y}} \ddot{y}_1 \ddot{y} \ddot{y}_w \ddot{y}}{c_{A_{oH}}^{\dot{y}} \ddot{y} \ddot{y} \ddot{y}} \text{ for } \ddot{y} \ddot{y} \ddot{y}$$

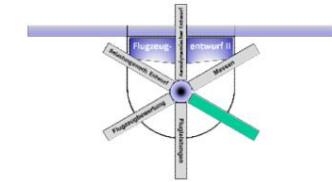




F

## Flight mechanical design - Basics 3.5 Static longitudinal stability with fixed rudder



**F****Flight mechanical design - basics****3.5 Static longitudinal stability with fixed rudder**

- Then the stability measure can also be written as:

$$s \ddot{y} = \frac{\ddot{y} X_N}{I_{\dot{y}}} - \frac{x_{NS}}{I_{\dot{y}}}$$

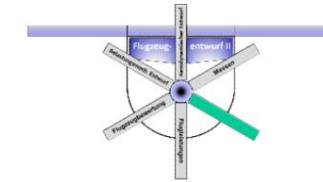
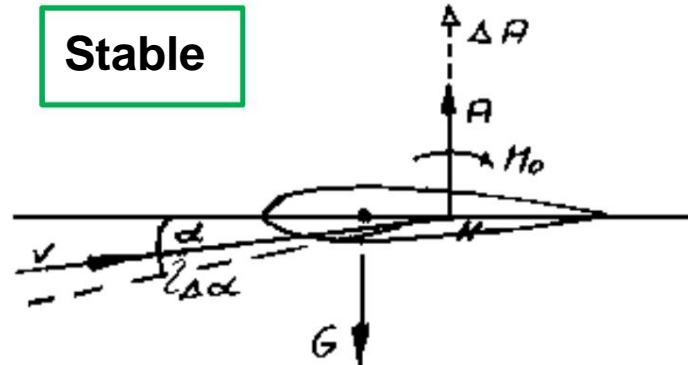
- For stability, it is important to note that the following applies to the centre of gravity:

$$\frac{x_S}{I_{\dot{y}}} \ddot{y} = \frac{X_N}{I_{\dot{y}}}$$

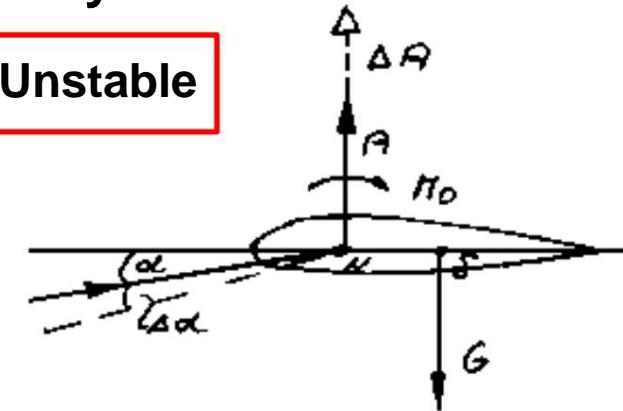
**F**

# Flight mechanical design - basics

## 3.5 Static longitudinal stability with fixed rudder

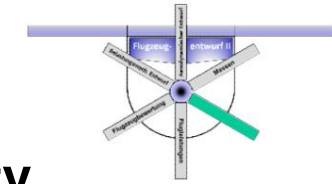
**Stable**

→ reverse rotation moment

**Unstable**

→ turning moment

- For a stable aircraft, the center of gravity must always be in front of the overall neutral point!
- The further forward the center of gravity, the more longitudinally stable the Airplane.
- The maximum rear center of gravity position is critical for longitudinal stability.

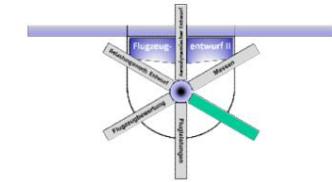
**F**

# Flight mechanical design - basics

## 3.6 Controllability at the forward center of gravity

- As explained, the stability must not be too great, otherwise controllability will be impaired.
- This is especially true when controlling high angles of attack using the elevator at forward centre of gravity positions.
- The moment equation with elevator deflection was:

$$C_{MM} \ddot{y} = 0 \quad \frac{\dot{y} C_M}{\dot{y} C_A} \ddot{y} \theta_A = \frac{\dot{y} C_M}{\dot{y} \dot{y}_H} \ddot{y} \ddot{y}_H$$



## F

## Flight mechanical design - Basics 3.6

### Controllability at the forward center of gravity

- Objective: The maximum possible centre of gravity should be achieved at the Lift coefficient can be controlled at full negative rudder deflection
- Then the stability measure in the equilibrium state

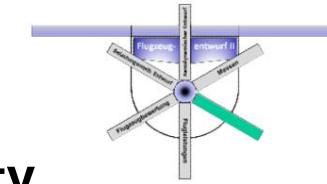
$$0 \leq \frac{\dot{c}_M}{c_A} \leq c_{A_{\max}} \quad \text{and} \quad \frac{\dot{c}_M}{c_A} = \frac{\dot{c}_M}{H}$$

only have the following size:

$$s \leq \frac{x_{NS}}{I} \quad \frac{\dot{c}_M}{c_A} \leq \frac{C_{M0}}{C_{A_{\max}}} \quad \frac{\dot{c}_M}{c_A} \leq \frac{\dot{c}_M}{H} = \frac{H_{\max}}{C_{A_{\max}}}$$

# F Flight mechanical design - basics

## 3.6 Controllability at the forward center of gravity



- If the equation for **rudder efficiency** and **Zero moment coefficient** used here follows:

$$\frac{s}{c_{A_{\text{Max}} q l}} \cdot \frac{\dot{y}_H_{\text{Max}}}{F q_H} = \frac{\dot{y}_H I_{\dot{y}_H}}{\dot{y}_H H} + \frac{\dot{y}_C_{M0}}{I_{FH}} + \frac{\dot{y}_r_{HN}}{I_{HN}} \cdot C_{A_H}^{\dot{y}_H} + \frac{\dot{y}_y_H}{I_{y_H}} \cdot \dot{y}$$
  

$$\frac{1}{c_{A_{\text{Max}}}} \cdot \frac{\dot{y}}{C_{M0_{OH}}} = \frac{F q_H}{F q} \cdot \frac{\dot{y}_y_r_{HN}}{I_{y}} + u_H \cdot \frac{\dot{y}_y_y_{cc}}{I_{y}} + \frac{I_{y_H}}{I_{y}} \cdot M_{0_{FH}}$$

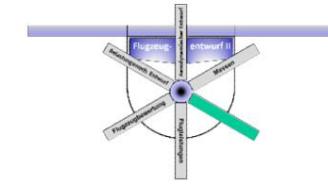
- or the permissible front center of gravity position for trimability:

$$\frac{x_S}{I_{y_s}} \leq \frac{x_N}{I_{y}}$$

## F

# Flight mechanical design - basics

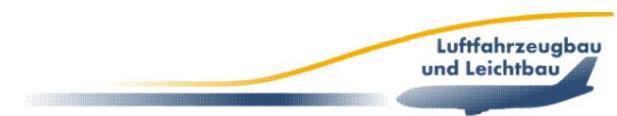
## 3.7 Controllability on curved flight paths

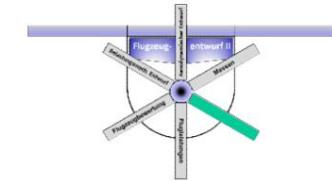


- Not only the ability to trim all operating speeds is an applicable controllability criterion
- It must also be ensured that a positive change in runway inclination can be achieved at all speeds using the elevator.
- This interceptability requirement is critical at the forward center of gravity position.
- When recovering from a dive, the aircraft experiences a Tilt angle velocity around the transverse axis, the

a dampening moment  $\ddot{y}$

$$\frac{\ddot{y} c_M}{y} \text{ causes. 0}$$





## F

# Flight mechanical design - basics

## 3.7 Controllability on curved flight paths

- To achieve this maneuver with an elevator deflection  $H_{\max} \ddot{\gamma}$

to achieve the maximum lift coefficient,  $c$

$A_{\max}$

the stability measure in the equilibrium state

$$C_M = \frac{\ddot{\gamma} C_M}{\ddot{\gamma} C_A} = M_0 = \frac{\ddot{\gamma} C_M}{\ddot{\gamma} C_A} = \frac{\ddot{\gamma} C_M}{\ddot{\gamma} H} = \frac{\ddot{\gamma} C_M}{\ddot{\gamma} y} = \frac{\ddot{\gamma} C_M}{\ddot{\gamma} y}$$

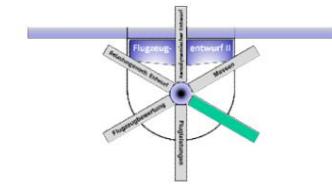
only by size

$$s = \frac{\ddot{\gamma} c_M}{\ddot{\gamma} c_A} = \frac{c_{M0}}{c_{A_{\max}}} = \frac{\ddot{\gamma} c_M}{\ddot{\gamma} y_H} = \frac{\ddot{\gamma} H_{\max}}{c_{\max}} = \frac{\ddot{\gamma} \ddot{\gamma}_M}{\ddot{\gamma} y_A} = \frac{\ddot{\gamma} \ddot{\gamma}_M}{c_{\max}}$$

•

Also already known:

$$\frac{\ddot{\gamma} c_M}{\ddot{\gamma} y} = \frac{\ddot{\gamma} \ddot{\gamma}_H}{I_y F_q} = \frac{F_H}{c_{A_H}} = c_{A_H}$$

**F**

# Flight mechanical design - basics

## 3.7 Controllability on curved flight paths

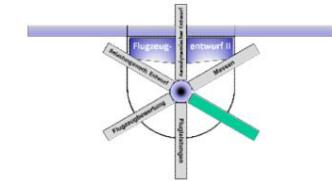
- With the moment coefficient for pitching motion

$$\begin{aligned}
 & s \quad \ddot{y} \quad \frac{\ddot{y} c_{MNS}}{I_y} \quad \ddot{y} \\
 & \ddot{y} C_A \quad I_y \\
 & \boxed{\ddot{y} H_{\max} \quad F q_H \quad \ddot{y} I \quad \ddot{y}_H \quad \ddot{y} C_M \quad 0_{FH} \quad \ddot{y} \ddot{y}_H \quad C_{A_H} \quad \ddot{y} \ddot{y}_H \quad \ddot{y} H \quad \ddot{y} \\
 & C_{Agl} \quad \ddot{y} \ddot{y} \quad \ddot{y} \ddot{y}_H \quad I_y \quad \ddot{y} \ddot{y}_H \quad \ddot{y} \ddot{y}} \\
 & \boxed{\ddot{y} \quad 1 \quad \ddot{y} \quad C_M \quad 0_{oH} \quad F q_H \quad \ddot{y} \ddot{y}_r \quad r_H \quad \ddot{y} \quad U_H \quad \ddot{y} \ddot{y} \ddot{y} \quad cc \quad I_{\ddot{y}_H} \quad \ddot{y} \ddot{y} \quad M \quad 0_{FH} \quad \ddot{y} \ddot{y}} \\
 & \boxed{\ddot{y} \quad y \quad H \quad H \quad M \quad y \quad F \quad q \quad \ddot{y} \quad C_{A_H} \quad I_F \quad q} \quad \text{you get the}
 \end{aligned}$$

$x_S$  permissible front center of gravity  $\ddot{y} \ddot{y} s$

$$\frac{x_N}{I_y} = \frac{I_{\ddot{y}_H}}{I_y}$$





# F Flight mechanical design - basics

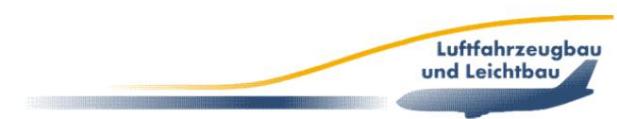
## 3.7 Controllability on curved flight paths

- Somewhat simplified, this results in

$$\begin{aligned}
 & \frac{\ddot{y}_{c_{xx}}}{M_{NS} \ddot{y}} = \frac{\ddot{y}_c}{I_{\ddot{y}}} \\
 & \frac{\ddot{y}_{H_{Max}}}{q \ddot{y}_{Max}} = \frac{\ddot{y}_F q_H \ddot{y}_c F}{I_{\ddot{y}}} , \quad \frac{\ddot{y}_{r_H} \ddot{y}_N}{\ddot{y}_c A_H} , \quad \frac{\ddot{y}_{\ddot{y}}_{HHH} \ddot{y}_F q}{I_{\ddot{y}_H} F q_l} , \quad \frac{\ddot{y}_c_{M_{0FH}} \ddot{y}}{\ddot{y}_{\ddot{y}_H} \ddot{y}_y} \\
 & \frac{1}{c_{A_{Max}}} , \quad c_{M_{0oH}} , \quad \frac{\ddot{y}_{\ddot{y}_H F_H}}{I_F q} , \quad \frac{\ddot{y}_{\ddot{y}_H Y_I}}{A_H Y_I} , \quad \frac{I_{\ddot{y}_H}}{\ddot{y}_y} c_{M_{0FH}} , \quad \frac{\ddot{y}}{\ddot{y}_y}
 \end{aligned}$$

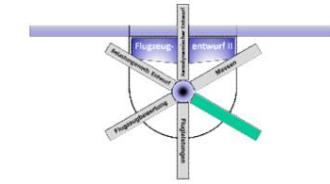
and you get the permissible front  
Center of gravity

$$\frac{x_s}{I_{\ddot{y}}} \ddot{y} \ddot{y}_s = \frac{x_N}{I_{\ddot{y}}}$$



# F Flight mechanical design - basics

## 4.1 Horizontal tail unit dimensioning

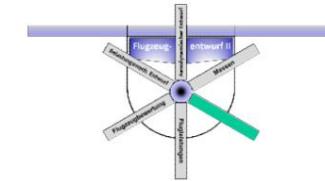


- The main design parameters for the tail unit are
  - the area ratio,
  - the tail unit layout (lift gradient),
  - the rudder geometry (effectiveness) and
  - the maximum rudder angle.
- From these parameters, the two quantities necessary to ensure the flight mechanical requirements can be determined:
  - Horizontal tail angle and
  - required tail lever arm
- However, any other approach to linking of the mentioned sizes possible.

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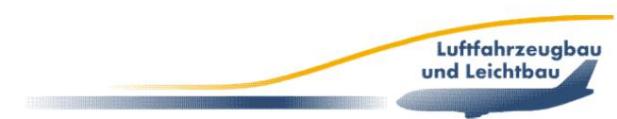
# Flight mechanical design - basics

## 4.1.1 Horizontal tail pitch angle



- **Assumption:** Under steady cruise conditions and at middle center of gravity position, the moment equilibrium not with the help of the elevator, but with the tail unit adjustment (no trim).
- Then the moment equation of the aircraft can be calculated without Elevator deflection can be used to determine the horizontal tail angle.
- This is set to zero and after the setting angle dissolved.
- The lift coefficient used here is that of cruise flight at medium weight:

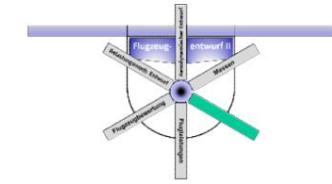
$$c_{AR} \frac{G}{q F}$$



## F

# Flight mechanical design - basics

## 4.1.1 Horizontal tail pitch angle



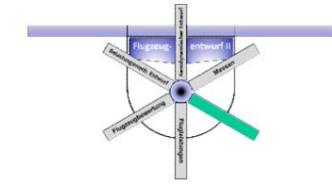
$$\begin{aligned}
 c_M &= \frac{c_{M_{0_{OH}}}}{c_{A_H}} = \frac{\frac{q F_r}{I} \ddot{y}_H}{\frac{I}{I} \ddot{y}} = \frac{\ddot{y}_{cc}}{\ddot{y}} \\
 &\quad \text{where } c_{M_{0_{OH}}} = \frac{\ddot{y}_{xq}}{q F_l} = \frac{\ddot{y}_N}{\frac{c_{A_H}}{c_{A_{OH}}} \ddot{y}_1 \ddot{y}_W} \\
 &\quad \text{and } c_{A_H} = \frac{F_{c_{HA1}}}{F_c} \ddot{y}_H = \frac{\ddot{y}_1 \ddot{y}_W}{\ddot{y}_Y}
 \end{aligned}$$

- Resolution according to the setting angle for  $cM = 0$  and CAR brings

$$\begin{aligned}
 c_{M_{0_{OH}}} &= \frac{q F_H}{q F_l} = \frac{I}{I} \ddot{y}_H = \frac{\ddot{y}_{cc}}{\ddot{y}} \\
 &\quad \text{where } q F_l = \frac{q F_{HA}}{M_{A_{OH}}} = \frac{\ddot{y}_N}{\ddot{y}} \text{ and } I = \frac{s_m}{I} \\
 &\quad \text{and } c_{A_H} = \frac{\ddot{y}_{c_{A_H}}}{\ddot{y}_{A_{OH}}} \ddot{y}_W = \frac{\ddot{y}_1 \ddot{y}_W}{\ddot{y}_Y} \\
 &\quad \text{and } q F_{HA} = \frac{F_{c_{HA1}}}{F_c} \ddot{y}_H = \frac{\ddot{y}_1 \ddot{y}_W}{\ddot{y}_Y}
 \end{aligned}$$

$$\ddot{y}_H = \frac{\ddot{y}_{c_{A_H}}}{I F q} = \frac{\ddot{y}_{c_{A_H}}}{c_{A_H}}$$

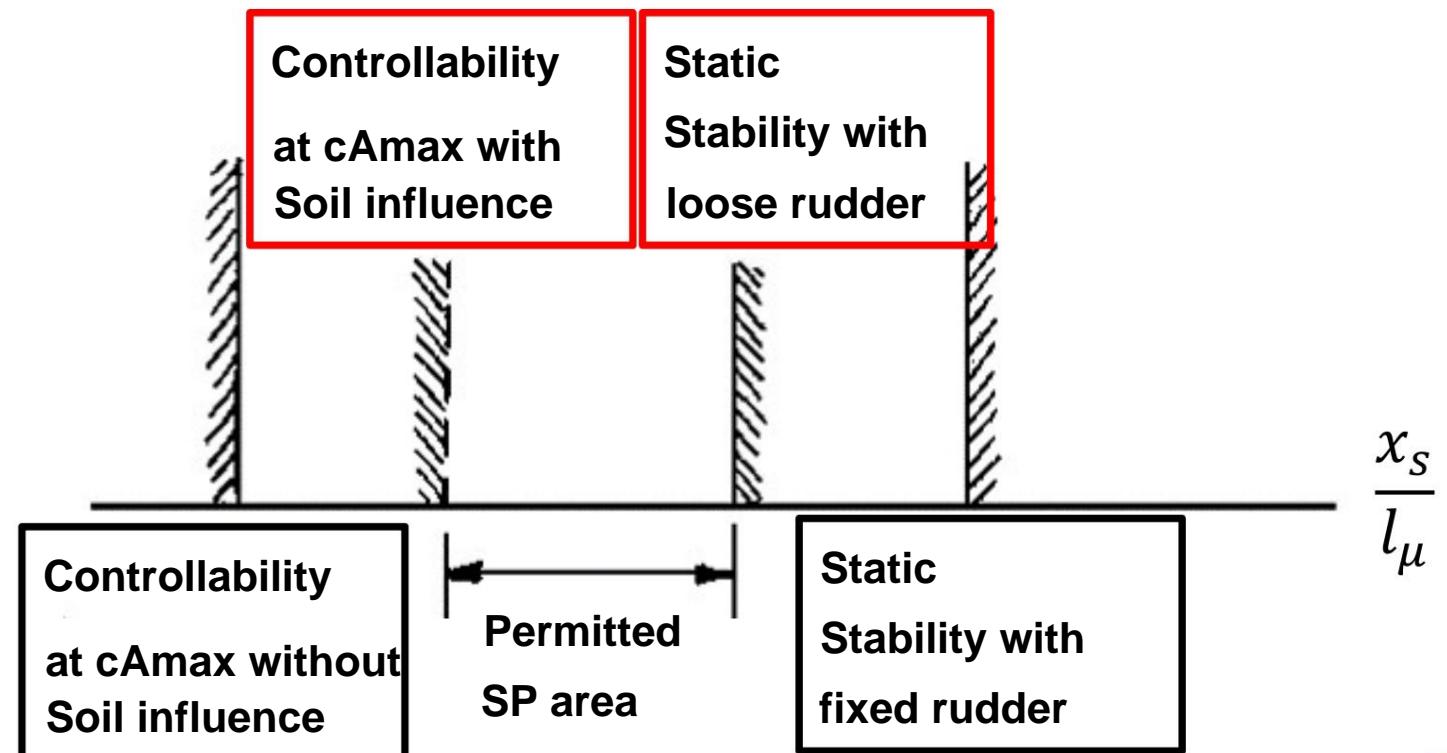
- This usually results in negative values  $\ddot{y}_H$ .



# F Flight mechanical design - basics

## 4.1.2 Tail lever arm

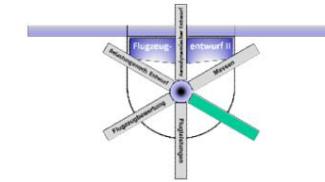
- The permissible centre of gravity range can be determined from the requirements described.
- **Two effects** must be taken into account, which cause constriction:



# F

# Flight mechanical design - basics

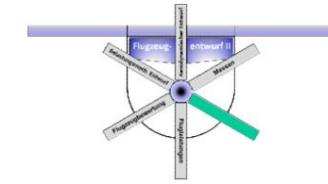
## 4.1.2 Tail lever arm



- Static stability with loose rudder means the  
**Stability of an aircraft whose elevator can move freely around the axis of rotation and thus adjust itself to the air forces.**
- Stability is lower with a free rudder than with a fixed one.
- Since the horizontal tail unit is partially unable to generate lift when the rudder is free, depending on the rudder configuration, control unit design and flow conditions, the analysis of this case is very complex and is therefore omitted here.

# F Flight mechanical design - basics

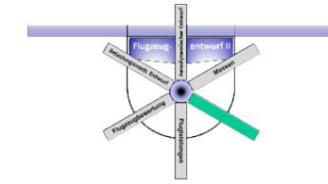
## 4.1.2 Tail lever arm



- The proximity to the ground influences the flow behind the wing during take-off and landing, so that
  - $\ddot{y}W$  is reduced and • the lift gradient of the wing  $iA$  is increased to a greater extent than that of the tail unit.
- The influences of the reduced gradient quotient and the reduced downwash angle on the position of the aircraft neutral point partially compensate each other.
- However, the starting configuration is usually the decisive criterion for dimensioning the maximum rudder deflection.
- The “minimum control speed air” is adjusted if necessary defined by this.

# F Flight mechanical design - basics

## 4.1.2 Tail lever arm



- The relationships derived above for the three-dimensional cases

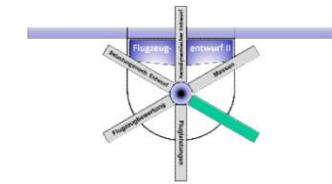
I. Static longitudinal stability at rearmost centre of gravity

II. Controllability at the front center of gravity

III. Interceptability at the forward center of gravity

can be calculated with more or less algebraic effort  
after the tail unit lever arms.

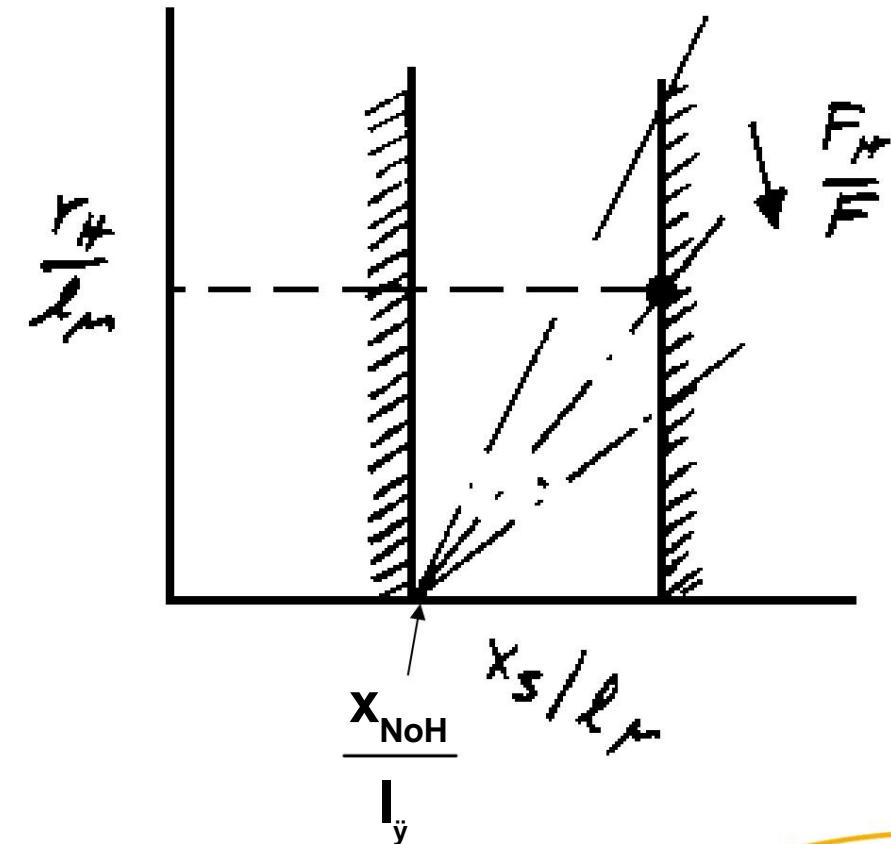
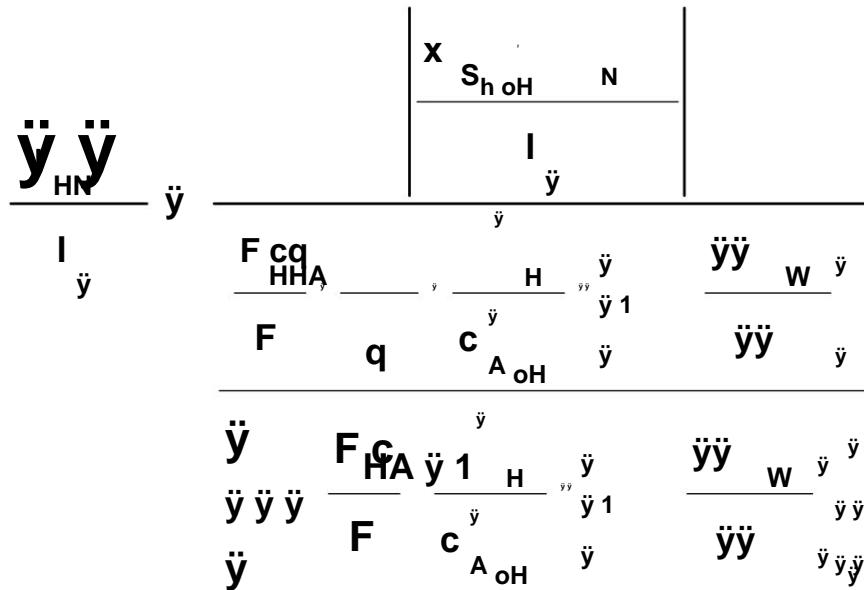
- This results in dimensioning guidelines for the tail area, the lever arm of the horizontal tail, the maximum rudder deflection and, indirectly via the rudder efficiency, the rudder depth.

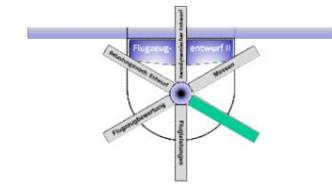
**F**

# Flight mechanical design - basics

## 4.1.2 Tail lever arm

### I. Stability at rearmost centre of gravity

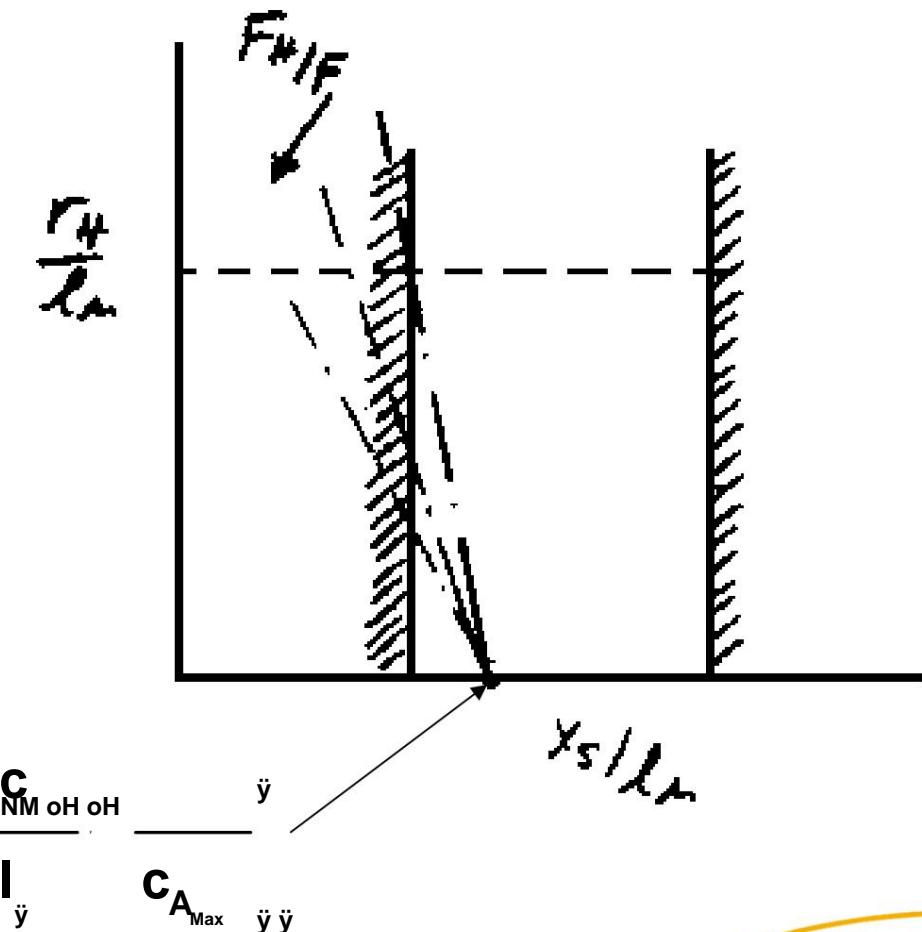


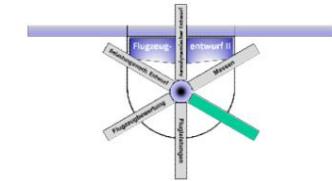


# F Flight mechanical design - basics

## 4.1.2 Tail lever arm

### II. Controllability at the front center of gravity

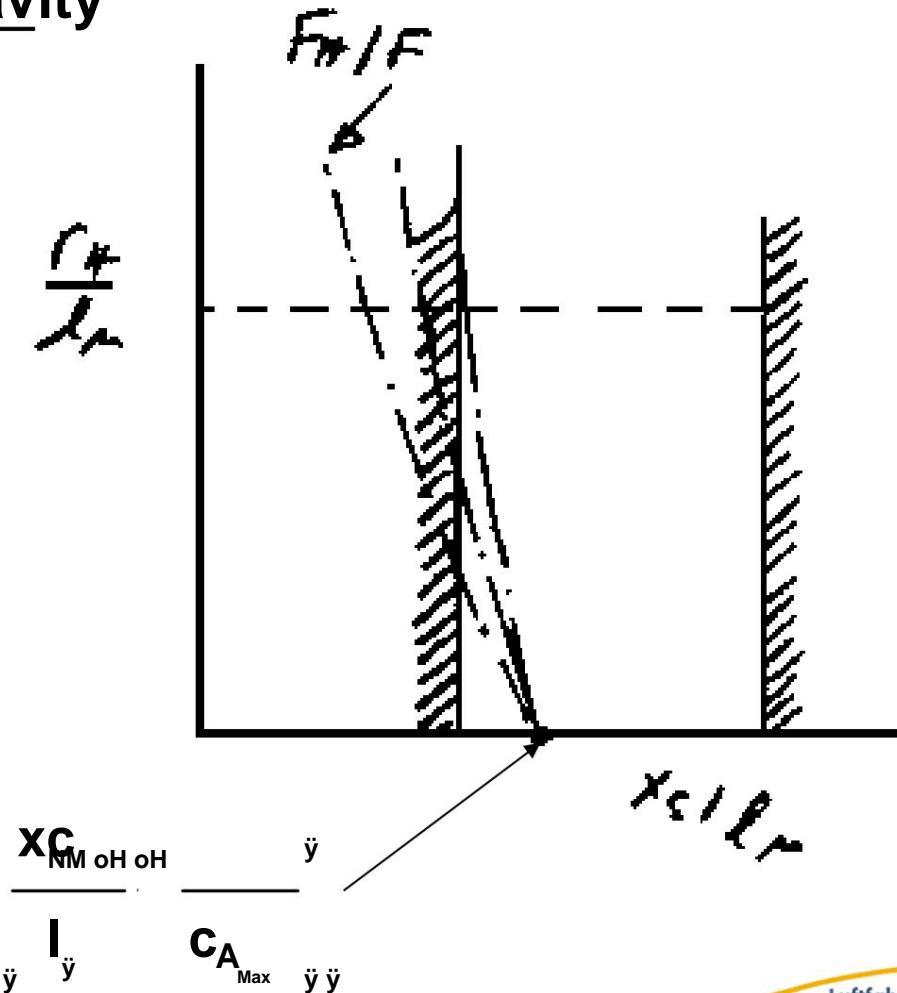


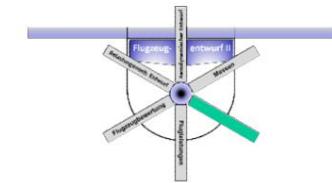


# F Flight mechanical design - basics

## 4.1.2 Tail lever arm

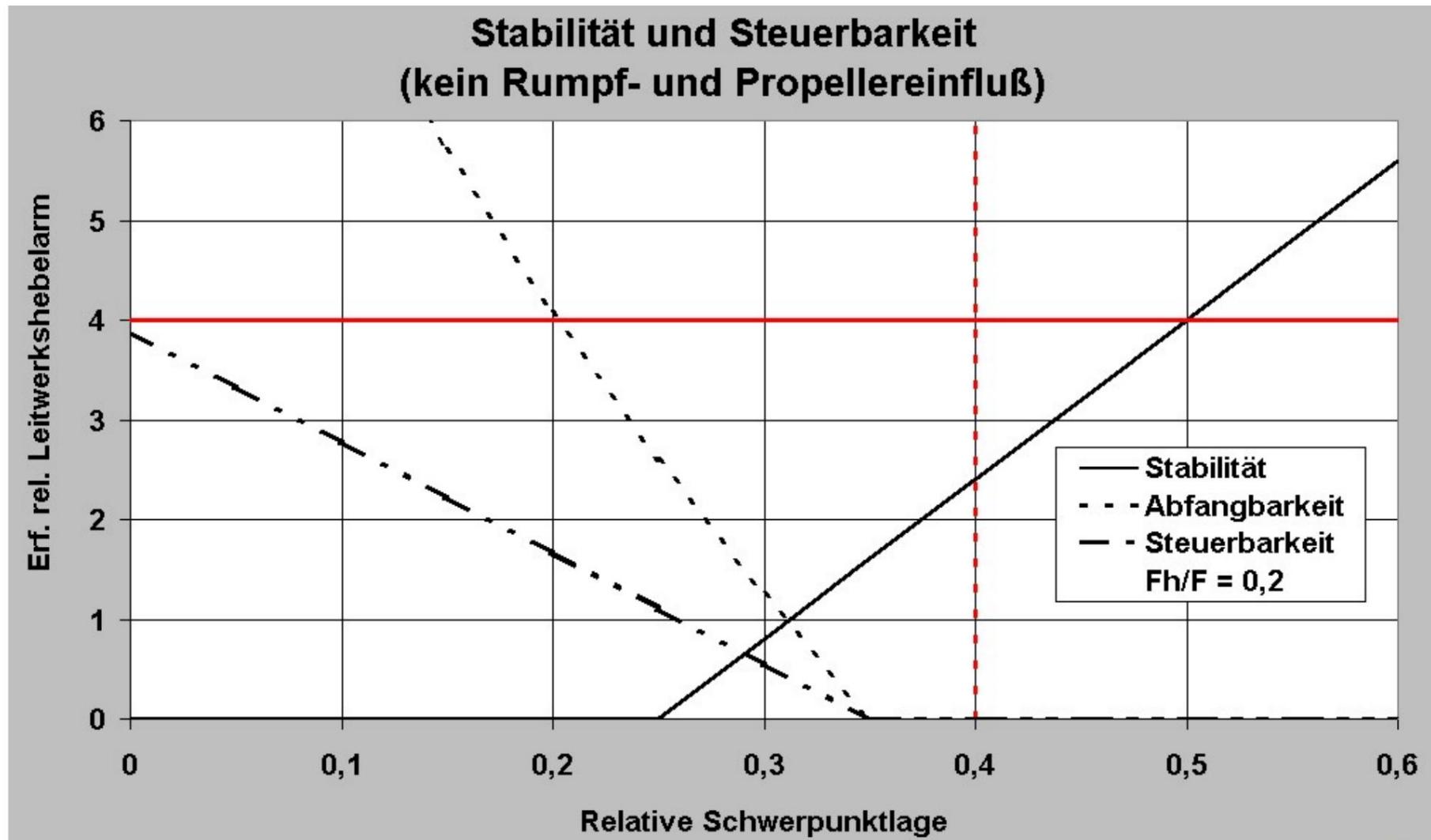
### III. Controllability with curved flight path (interceptability) and forward center of gravity





# F Flight mechanical design - basics

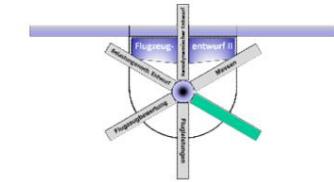
## 4.1.2 Tail lever arm



# F

# Flight mechanical design - basics

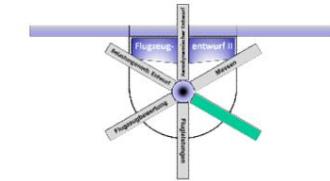
## 4.2 Horizontal tail configurations



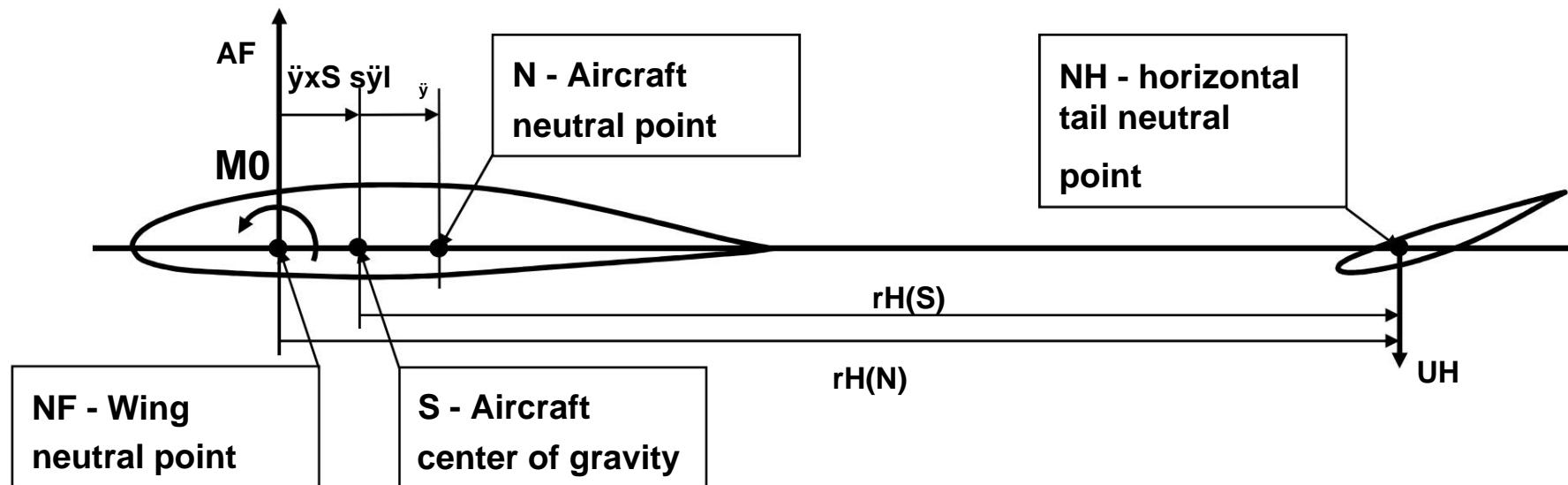
- The installation of horizontal tailplanes on the aircraft is done for two reasons:
  1. Generation of positive zero moments. This means that the wing may be additionally loaded by the tail unit downforce and the trim resistance increases.
  2. Relocation of the overall neutral point to increase stability.

# F Flight mechanical design - basics

## 4.2 Horizontal tail configurations



- Kite configuration (standard) in travel case:

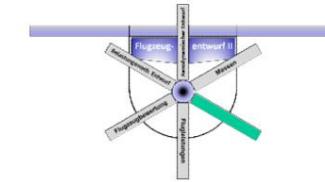


- Since for wings with positive camber the zero moment coefficient value  $< 0$  (top-heavy), the tail unit must generate downforce if a moment equilibrium is to be achieved.
- It follows that either the horizontal tail angle and/or the horizontal tail camber must be negative in the trimmed flight condition.

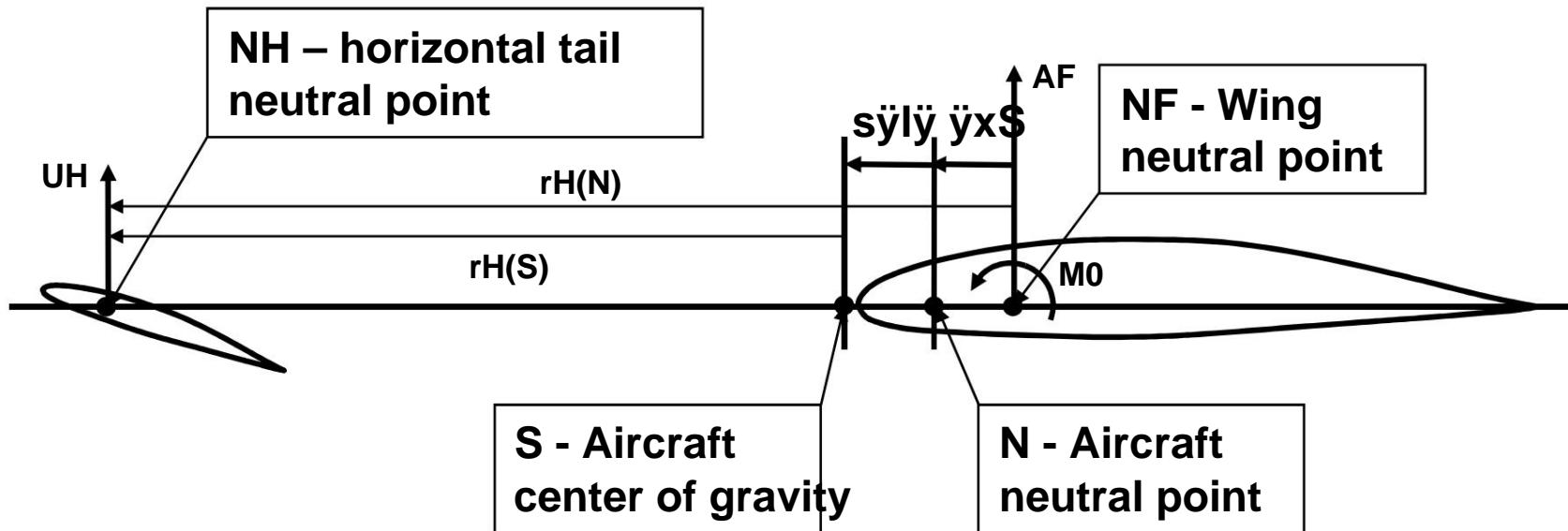
## F

# Flight mechanical design - basics

## 4.2 Horizontal tail configurations



- The canard configuration has the tail unit in front of the wing:

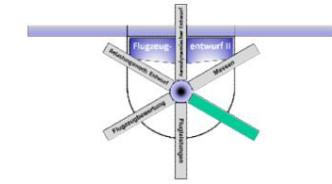


- For wings with positive camber ( $0 < C_{M_{0oh}} < 0$ ) the altitude  $\ddot{y}$  tail lift  $> 0$  and the angle of attack in cruise flight is positive.
- The advantage of this configuration is the generation of a positive zero moment while simultaneously increasing the total lift or reducing the load on the wing.

# F

# Flight mechanical design - basics

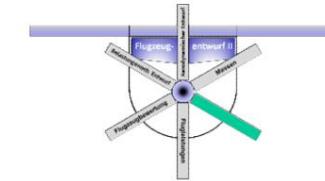
## 4.2 Horizontal tail configurations



- A special case between the duck and the dragon configuration  
    This configuration represents the tandem configuration.
- The wing and tail surfaces are of the same size.
- With identical wings, the lift is distributed between both  
    lift generators exclusively in accordance with the resulting  
    downwash angle (superposition of upwash and downwash!).

# F Flight mechanical design - basics

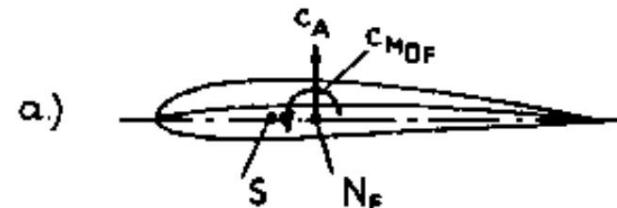
## 4.2 Horizontal tail configurations



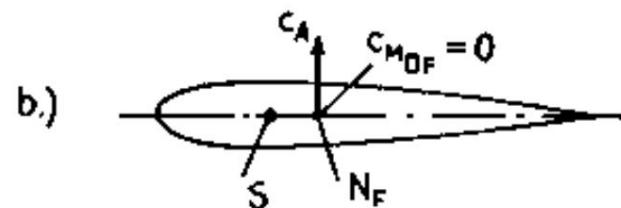
- The flying wing aircraft represents another extreme configuration

represents.

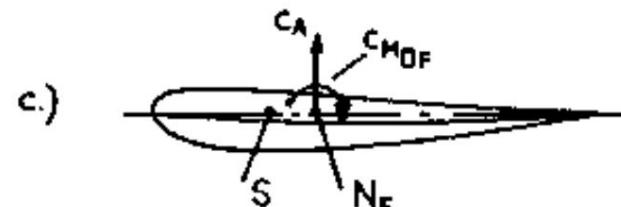
- The horizontal tail assembly is completely missing. With a straight wing  
The wing curvature alone determines whether moment equilibrium can be achieved.



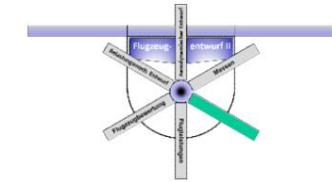
kein Momentengleichgewicht für positive  $c_A$



Momentengleichgewicht bei  $c_A = 0$



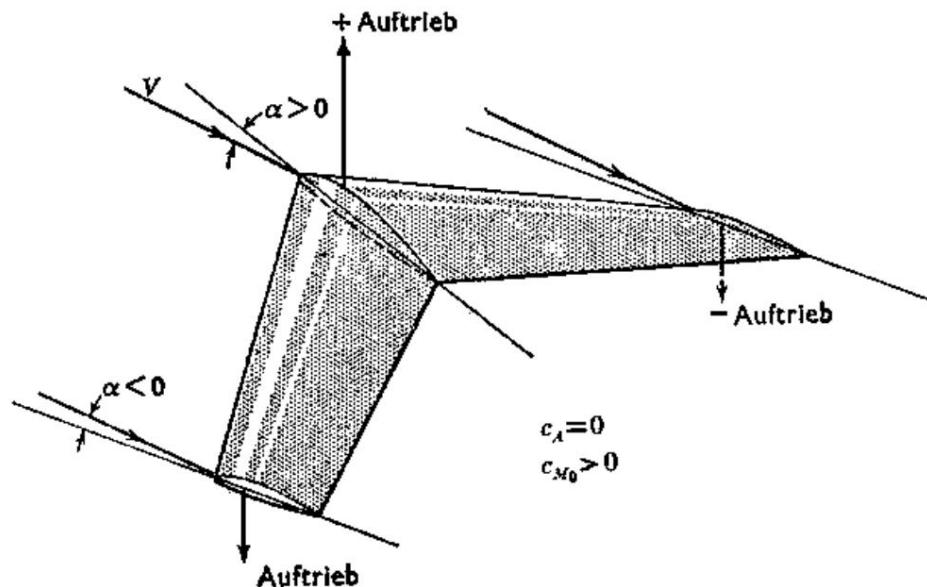
Momentengleichgewicht möglich bei positiven  $c_A$

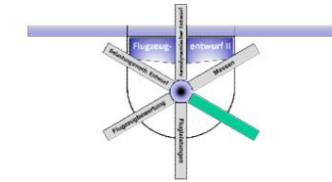
**F**

# Flight mechanical design - basics

## 4.2 Horizontal tail configurations

- For straight wing aircraft, only the negative profile camber (case c) the conditions for static longitudinal stability
- However, a positive zero moment would also be possible with symmetrical and positively curved wings if the wing
  - positively swept and
  - would be strongly geometrically twisted.

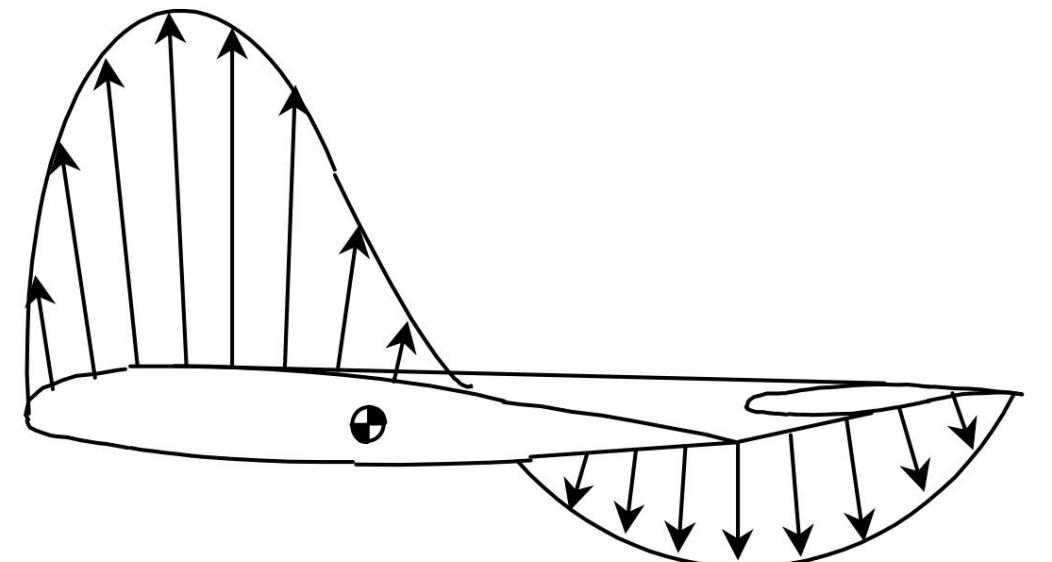




# F Flight mechanical design - basics

## 4.2 Horizontal tail configurations

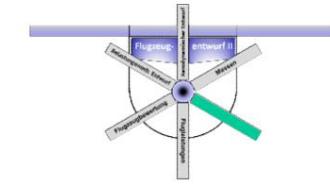
- Due to the twisting, the arrow wing must  
The lift center shifted outwards is clearly shifted towards  
the center of the wing and a bell-shaped lift  
distribution is created so that a downward force can act  
on a sufficient lever arm against the top-heavy  
wing moment (Gebr. Horten principle).



**F**

# Flight mechanical design - basics

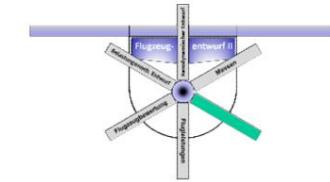
## 4.2 Horizontal tail configurations



- The advantage of the flying wing configuration is the small surface area, the low weight due to the lack of bending stress and the structural simplicity.
- Disadvantages are
  - poor dynamic stability characteristics (angle of attack oscillation, low inertia around the transverse axis),
  - small permissible centre of gravity area,
  - poor coating properties and
  - very large trim resistance (here as induced Drag) due to bell-shaped lift distribution and small lever arms.

# F Flight mechanical design - basics

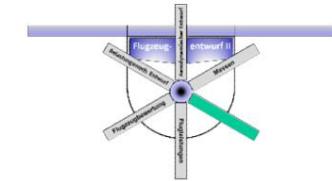
## 4.2 Horizontal tail configurations



- A complex, differentiated control of the Trailing edge flaps to control such aircraft.
- An example of a flying wing configuration is the SB13 glider of the Akaflieg Braunschweig.
- Lateral stability is ensured by oversized winglets at the tips of the positively swept, high aspect ratio wing.



Aircraft Design II - Summer Semester 2020

**F**

# Flight mechanical design - basics

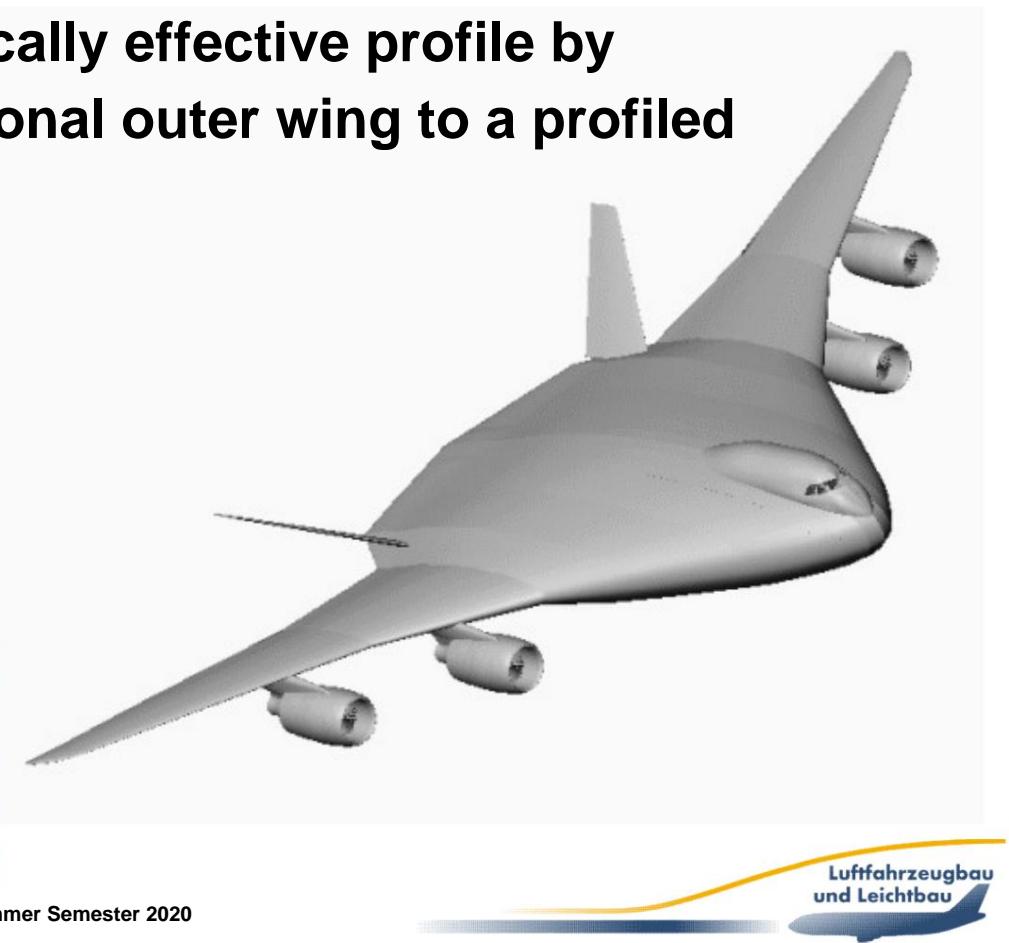
## 4.2 Horizontal tail configurations

- A variant of the flying wing that is being followed with great interest by the industry is the “Blended Wing Body” (BWB)
- The BWB solves the problem of accommodating a payload in an aerodynamically effective profile by attaching a more conventional outer wing to a profiled fuselage body.



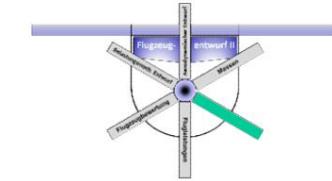
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Aircraft Design II - Summer Semester 2020

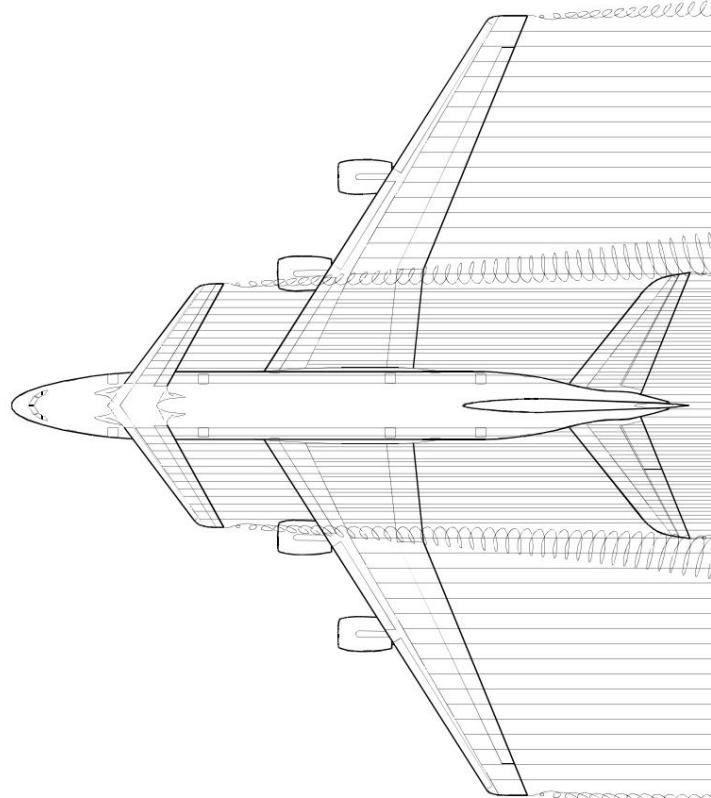
Luftfahrzeubau  
und Leichtbau

# F Flight mechanical design - basics

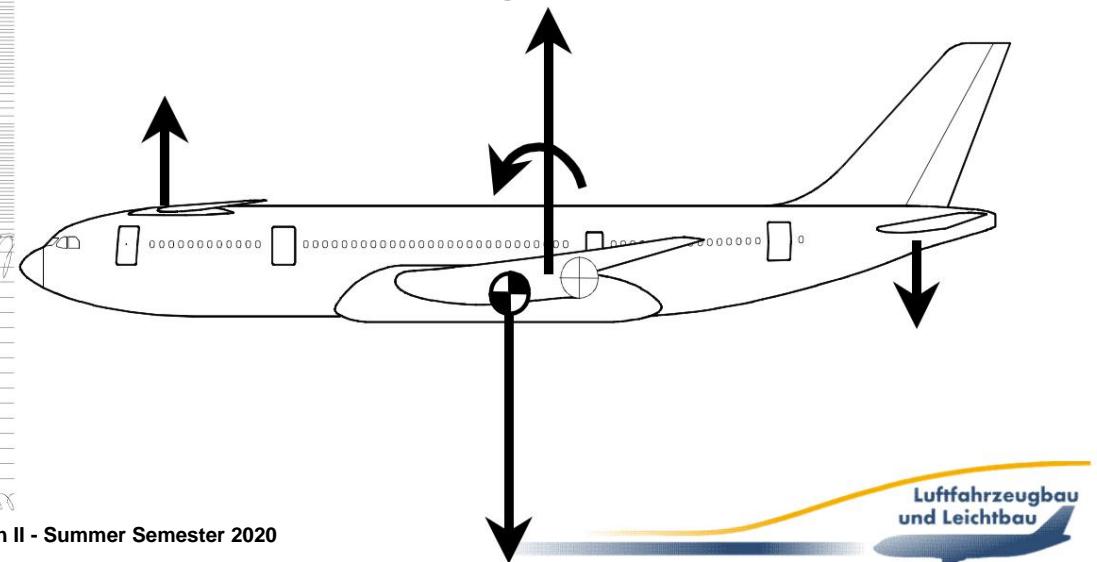
## 4.2 Horizontal tail configurations



- Another variant of a tail unit arrangement is the three-surface configuration.
- In addition to the conventional kite arrangement, another surface arranged at the bow of the fuselage acts like an additional canard.

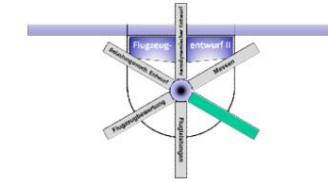


- The lift of this surface reduces the downforce on the rear tail unit and relieves the load on the fuselage structure.

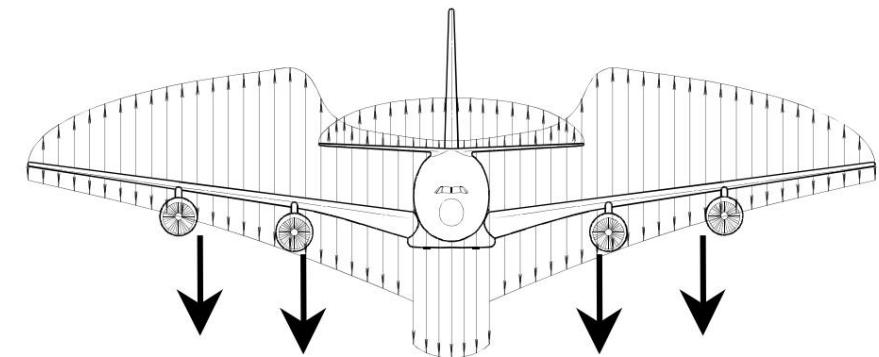
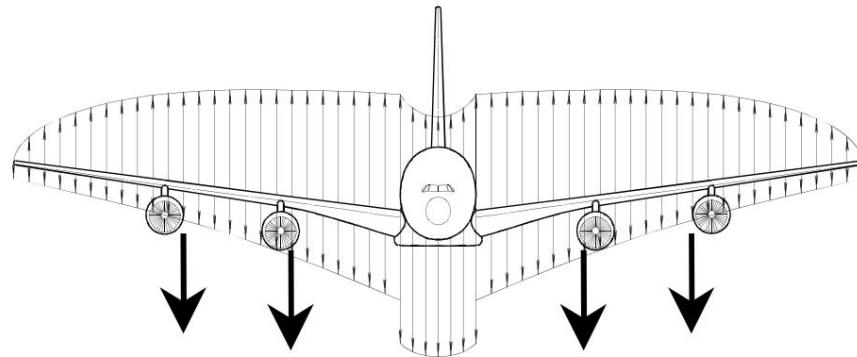


# F Flight mechanical design - basics

## 4.2 Horizontal tail configurations



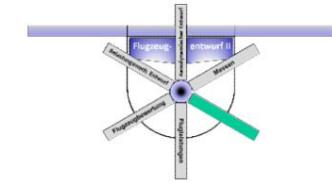
- The following sketch shows the kite configuration (left) and the Tri-Wing (right) shows the distribution of lift and weight forces and is intended to particularly illustrate the difference in wing lift distribution.



- The additional lift produced by the canard wing causes through its downwash field, the angle of attack on the wing is reduced to such an extent that the gain is lost there again. • The additional weight of the additional control surface and its additional interference with the fuselage increases the drag.

# F Flight mechanical design - basics

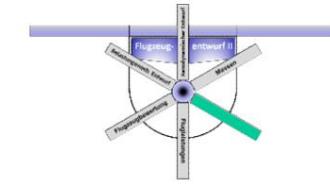
## 4.3 Dynamic stability



- Dynamic stability means that the horizontal tail assembly must not only generate a reverse rotational moment when the equilibrium is disturbed (static stability), but it must also dampen this movement well. • This requirement is met particularly well when the distance between the tail assembly neutral point and the center of gravity is large.  $\ddot{y}_{HN} / \dot{y}$
- The size  $\frac{\ddot{y}_{HN}}{\dot{y}} \cdot \frac{\dot{y}^2}{F_H}$  is an important criterion for dynamic Stability.
- In the preliminary draft, it is initially calculated using statistical comparisons should be taken into account.
- The verification of dynamic stability is discussed in  
In this context, courses on flight mechanics are omitted.

# F Flight mechanical design - basics

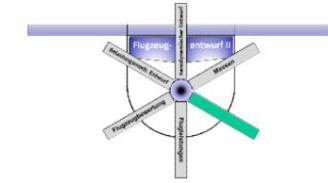
## 4.4 General design criteria



- The taper of the horizontal tail unit is not as important as the wing taper. • For reasons of strength, it should not be chosen too large. • Values of around 0.4 represent a good compromise between optimal aerodynamic and structural quality.
- The extension of the horizontal tail unit influences its effect clearly and should not be too small.
- Here too, the component weight is counteracted and the aero-Elastic problems are less with small stretches.
- In addition, the maximum angle of attack of the tail unit increases with lower aspect ratio.
- This means that the wing reaches its maximum lift in front of the horizontal stabilizer. The aircraft still remains controllable.

# F Flight mechanical design - basics

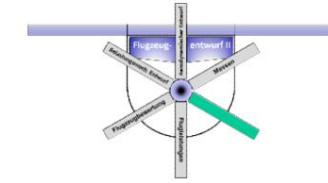
## 4.4 General design criteria



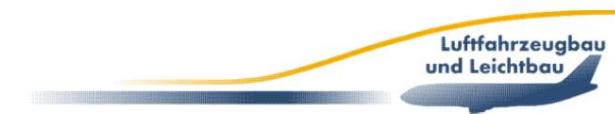
- This good-natured behavior is further supported by the downwash angle, which always ensures a lower angle of attack on the tail unit than on the wing. • The aspect ratio of the horizontal tail unit is therefore a value that can only be optimized with the help of detailed investigations.
- However, the horizontal tail assembly should always have a higher critical Mach number than the wing.
- This is achieved with smaller aspect ratios, larger sweeps and smaller profile thicknesses than with the wing.
- Typical horizontal tail plane extensions are between 3 and 4 for conventional aircraft configurations.

# F Flight mechanical design - basics

## 4.4 General design criteria

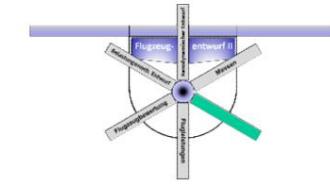


- Profiling is carried out according to the following criteria:
  - Minimum drag in cruise flight, ie a negative Camber that is adjusted to the cruise lift coefficient.
  - Maximum lift with rudder deflection greater than that for The value required to control an intercepting arc with maximum wing lift is the value required, ie the profile thickness is chosen to be exactly as large as necessary.
  - The rudder depth should be just as large as necessary to ensure the ability to catch the rudder at a given maximum rudder deflection and as small as possible to keep the rudder forces and hinge moments controllable.
  - The maximum rudder deflection in the effective deflection range, i.e. in the linear range of  $\ddot{y}_y/\dot{y}_y$ .



# F Flight mechanical design - basics

## 4.4 General design criteria

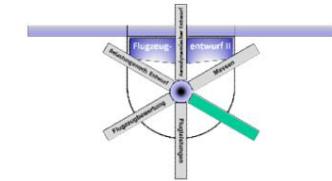


- The V-shape (dehedral upwards, unihedral downwards) is chosen with small angles to avoid thrust jet interference and is therefore of interest in connection with the engine arrangement. •

Particularly with propeller aircraft, care must be taken to that the tail unit is either completely in the propeller wake or completely outside it.

# F Flight mechanical design - basics

## 4.4 General design criteria

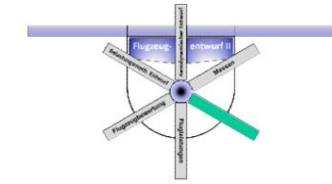


- For production cost reasons, in practice
 

Particularly in small aircraft, symmetrical profiles and pendulum tail units are common.
- In pendulum tail units, the rudder function is realized by changing the setting angle of the rudderless fin. • For these, the rudder effectiveness  $\ddot{\gamma}_H / \ddot{\gamma}_H$  takes the value 1.
- The variable setting of the horizontal tail is usually also used to trim the flight speed (THS – Trimmable Horizontal Stabilizer). • The adjustment is made so that the rudder always remains in the neutral position and thus does not cause an increase in drag.
- The rudder is only used during flight maneuvers.

# F Flight mechanical design - basics

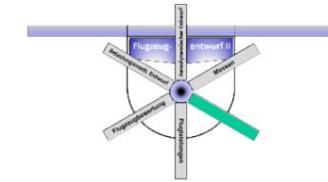
## 4.4 General design criteria



- There are also concepts where the horizontal tail unit is like the wing is firmly mounted on the fuselage. The setting angle is adjusted to the cruising condition with a medium center of gravity. • Both trimming and maneuver control are carried out with the rudder, which must be aerodynamically very effective (profile drag through flap deflection, slats may be required) and designed with a relatively large rudder depth.
- The aerodynamic disadvantage in off-design operating conditions is, in particular in very large aircraft, more than compensated for by the weight savings in the statically indeterminate clamping of the tail unit.

# F Flight mechanical design - basics

## 4.4 General design criteria



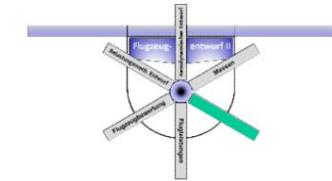
- Various aerodynamic trimming options are available.  
The basic principle is that a small auxiliary rudder attached to the trailing edge of the rudder either increases or decreases the rudder hinge moment.
- Either the auxiliary rudder, with its low lift force acting on the large lever arm, increases the control force when deflected in the same direction in order to give the pilot of a small aircraft greater feedback of the control forces
- or the auxiliary rudder provides for an opposite direction of stroke for its reduction, whereby the trim alone determines the stationary rudder setting.

**Power amplification**



**Trim**





# F Flight mechanical design - basics

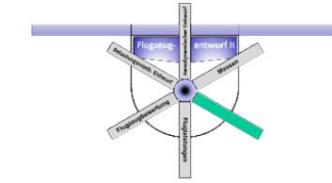
## 4.4 General design criteria

- Problems with excessive control forces, especially in smaller aircraft with direct force coupling between the control and the rudder surface, can be compensated with the help of compensating surfaces.
  - Part of the Rudder surface in the outer area is guided far in front of the axis of rotation in order to create an opposing hinge moment during a deflection with simultaneous To achieve an increase in buoyancy.
- MS528**

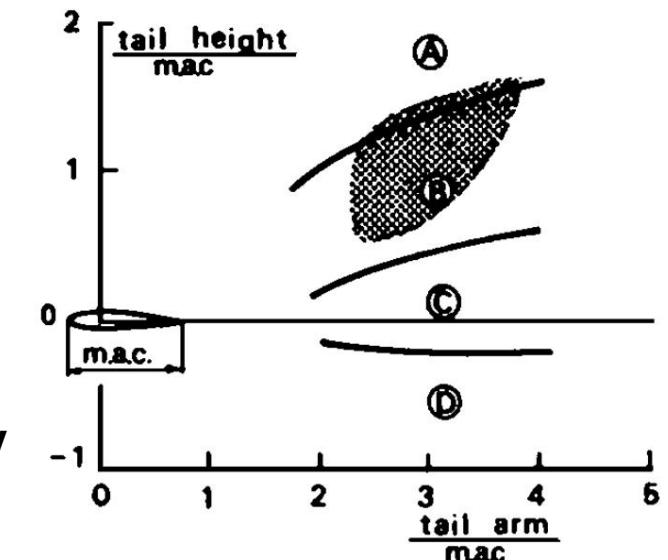


# F Flight mechanical design - basics

## 4.4 General design criteria

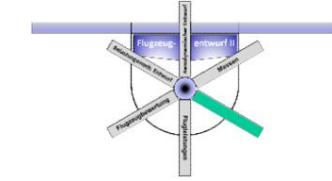


- There are many options for the installation location of the horizontal tail unit. possibilities.
- The installation position must be taken into account with regard to the expected position of a vortex band which leaves the wing to the rear and upwards in the stalled flight condition.
- This vortex band should run either above or below the horizontal tail unit and should not hit it in order to ensure controllability even in this condition.
  - The relative position of the vertical stabilizer is also crucial, as its effectiveness is significantly influenced by this.

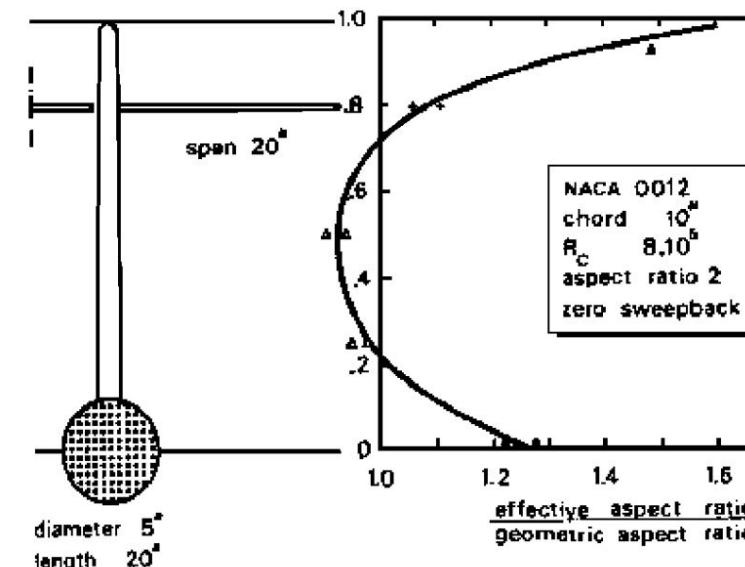


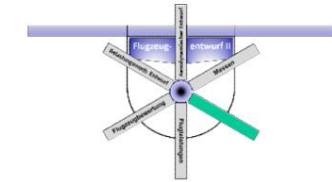
# F Flight mechanical design - basics

## 4.4 General design criteria



- In a T-arrangement, the horizontal tail unit acts like a Edge disc of the finite vertical stabilizer.
- Its effective aspect ratio is thereby increased while maintaining the same geometric aspect ratio, which results in an increase in its lift characteristics and a reduction in the induced drag in the sliding state.
- In the same, but not as The conventional fuselage arrangement of the horizontal stabilizer also works effectively.

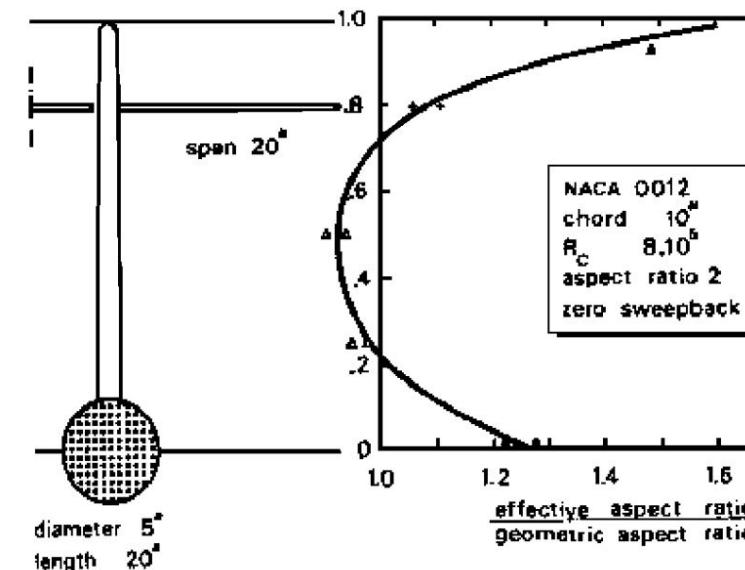


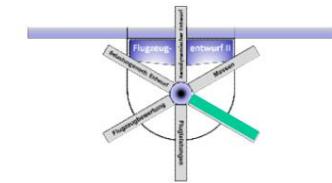
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# Flight mechanical design - basics

## 4.4 General design criteria

- On the other hand, the cross arrangement actually results in a reduction in the effective aspect ratio of the vertical stabilizer.
- The effect of the vertical stabilizer on the horizontal stabilizer is not significant.





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# Flight mechanical design - basics

## 4.4 General design criteria



VFW614



HFB320



ATR75

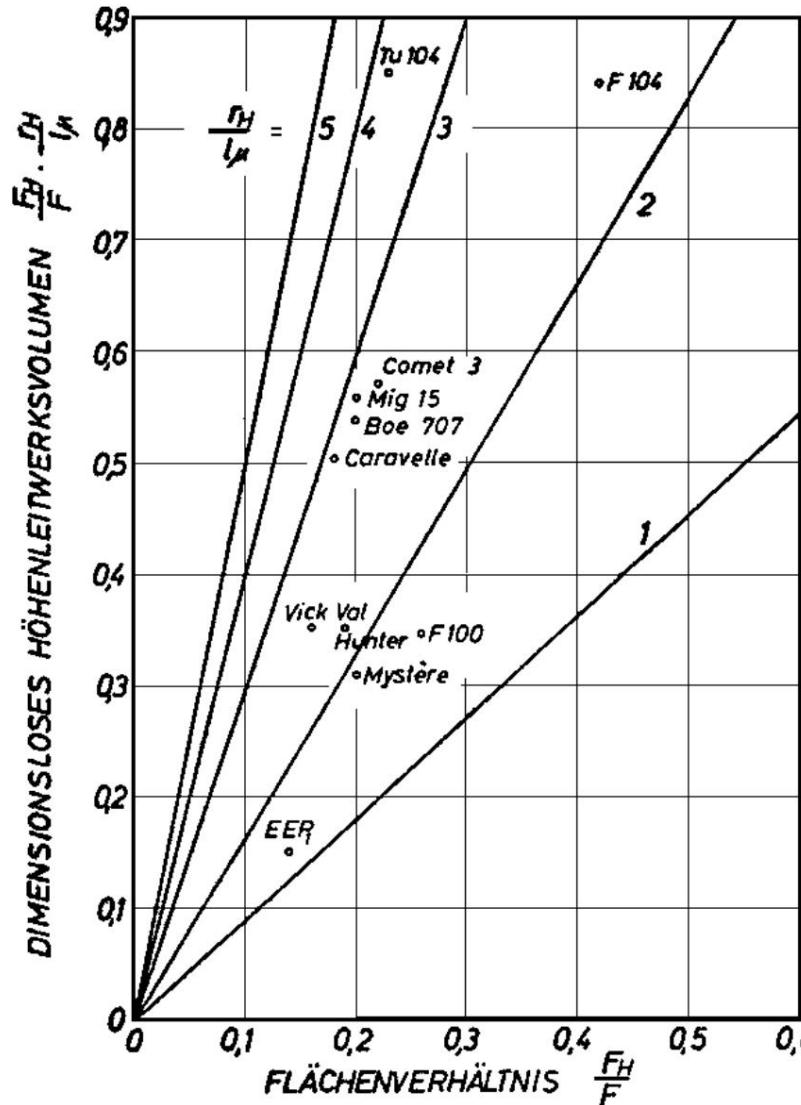
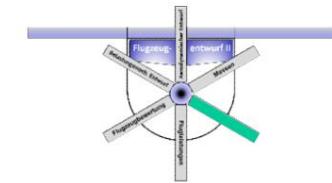


Dornier Seastar

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# Flight mechanical design - basics

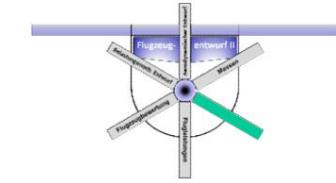
## 4.4 General design criteria



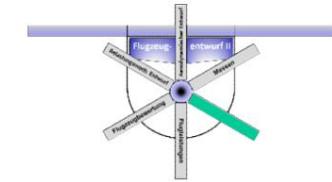
- Pre-dimensioning of the horizontal tail unit can be done by statistical evaluation of the tail unit volume.

# F Flight mechanical design - basics

## 5 Vertical stabilizer design



- The tasks of the vertical stabilizer are
  - the creation of sufficient stability to vertical axis,
  - the damping of yaw oscillations (Dutch roll),
  - moment compensation for asymmetric loads (engine failure, drag asymmetry),
  - the generation of lateral forces and moments for Control of the aircraft's attitude, – generating a lateral force to end the spin and
  - enabling a slip-free landing at crosswind.



# F Flight mechanical design - basics

## 5 Vertical stabilizer design

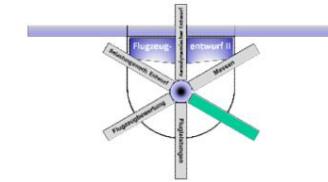
- Lateral stability for aircraft with a conventional vertical stabilizer arrangement behind the wing is made possible by the wind vane phenomenon. • If the aircraft is brought into a sideslip condition by a side gust, a lateral angle of attack is created on the vertical stabilizer, which is far from the center of gravity, which generates a reverse lateral force.
- The static and dynamic stability is also with relative FS S small tail volumes are easily ensured.

$$F \propto I_y$$

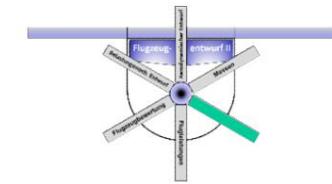
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# Flight mechanical design - basics

## 5 Vertical stabilizer design



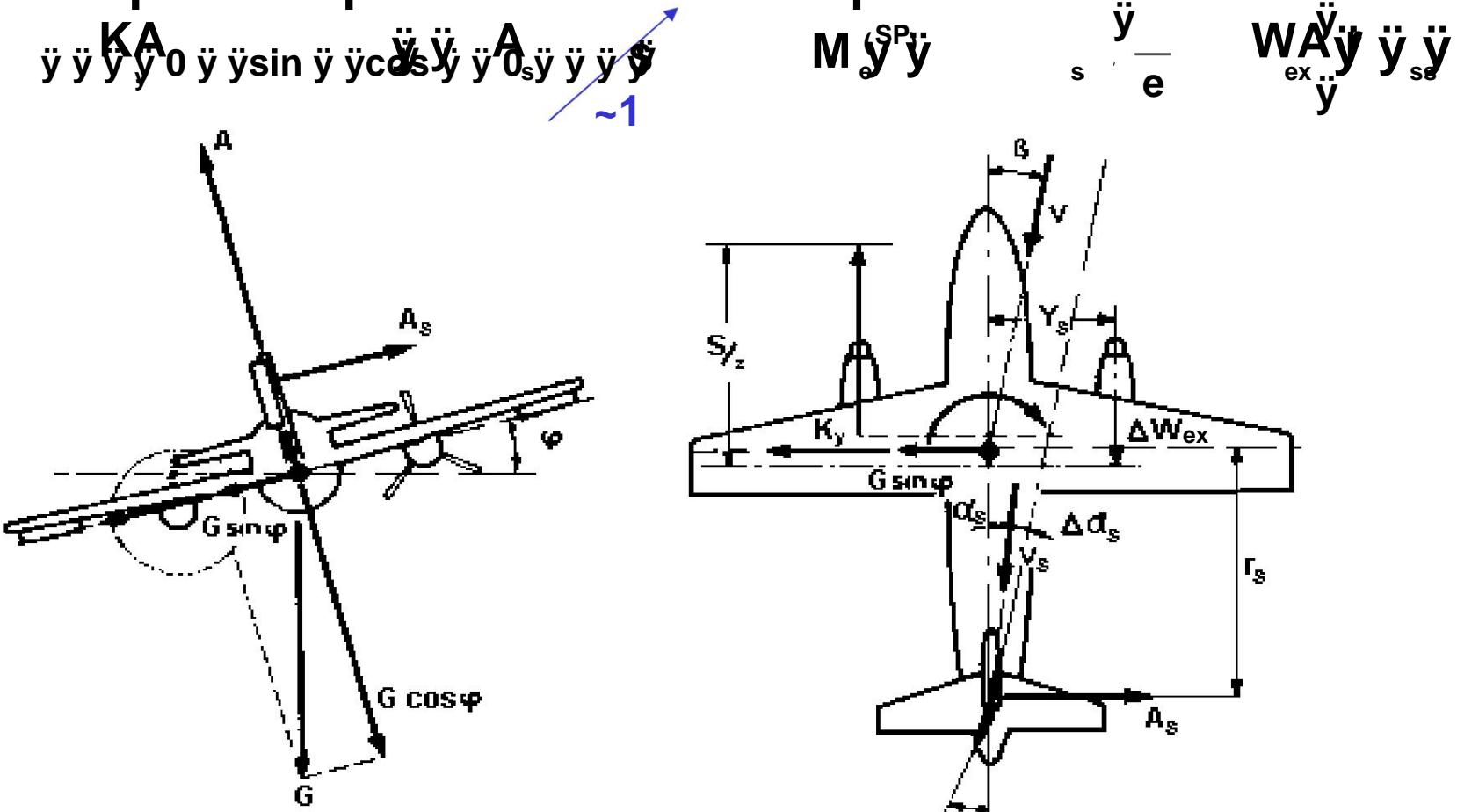
- However, the situation is different when it comes to controllability. • Sufficient lateral forces must be produced to prevent uncontrollable rotation around the vertical axis in the critical case of failure of the critical engine, which generates the maximum moment around the vertical axis.
- Compensation of the asymmetric thrust is dimensioned for the tail area, the maximum rudder deflection and the rudder depth with a tail lever arm usually predetermined by the horizontal tail arrangement.



# F Flight mechanical design - basics

## 5 Vertical stabilizer design

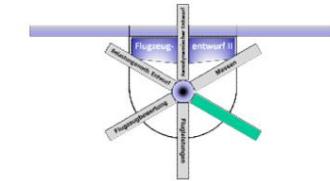
- The basis for considering lateral stability is again the simplified equilibrium relationships:



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# Flight mechanical design - basics

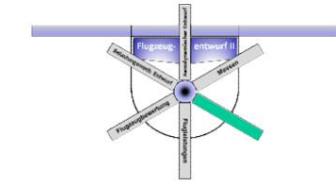
## 5 Vertical stabilizer design



- The value  $S/z$  describes the asymmetrical part of the thrust which results from the total installed thrust divided by the number of engines  $z$ .
- $\ddot{y}_{Wex}$  describes the resistance increase of the idle Engine.
- Overall, and in the event of engine failure, the decisive factor is the sum of the drag increases resulting from the engine failure and the resulting asymmetric flight condition.

# F Flight mechanical design - basics

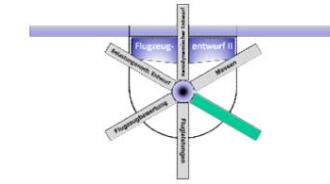
## 5 Vertical stabilizer design



- The shares are detailed
  1. the resistance of the stationary engine (wind milling drag),
  2. the induced vertical tail drag,
  3. the increase in profile and induced drag due to aileron and rudder deflection,
  4. Increase in drag due to side blowing of the fuselage and nacelles and
  5. the resistance change of the sliding wing and of the horizontal tail assembly (sweep effect).
- In the latter case, the reversed Effects of both wing halves cancel each other out at sweep angles that are greater than the sideslip angle.

# F Flight mechanical design - basics

## 5 Vertical stabilizer design

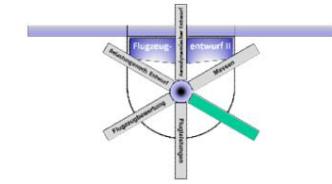


- An eccentric thrust drop, which generates a side force on the vertical stabilizer due to the sideslip angle, can be compensated in flight by a laterally directed component of lift.
- To determine the hanging angle  $\beta$ , set the angle vertical stabilizer force resolved relationship for the force balance in the moment equation and initially obtains

$$0 = \frac{\dot{y}_S}{e} - W_a \frac{\dot{y}}{\dot{y}} A_s \sin \beta \dot{y} \ddot{y} - \dot{y} \dot{y} r_s$$

and with coefficients and the dimensionless quantities for the Tail lever arm, eccentricity and thrust

$$0 = \frac{\dot{y}_S}{b} - \frac{\dot{y}_{years}}{2} \frac{b}{e} \frac{\dot{y} c_A}{G} \frac{\dot{y}}{\dot{y}} - c_{w_{ex}} \frac{\dot{y}}{\dot{y}} \frac{\dot{y} c_A \sin \beta}{\dot{y}} \frac{\dot{y} \dot{y}}{\dot{y}} \frac{\dot{y} r_s}{I} \frac{\dot{y}}{\dot{y}}$$



# F Flight mechanical design - basics

## 5 Vertical stabilizer design •

Solving for the hang angle gives  $\gamma = \frac{c_A s}{e G} \frac{w_{ex}}{\dot{y}}$

$$\tan \gamma = \frac{\frac{c_A s}{e G} w_{ex}}{\dot{y}} = \frac{c_A s}{e G} \frac{w_{ex}}{\dot{y}}$$

$$\gamma = \arctan \left( \frac{c_A s}{e G} \frac{w_{ex}}{\dot{y}} \right)$$

If the aircraft rotates at this angle against the thrust eccentricity with the wing hanging, the equation for the horizontal force equilibrium is:

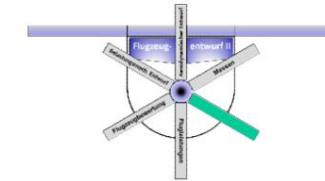
$$c_{A_s} c_A \sin \gamma = F$$

• This

shows that slow flight can become critical with regard to exceeding the maximum lift coefficient on the vertical stabilizer (vMCA!).

# F Flight mechanical design - basics

## 5 Vertical stabilizer design



- In the case that the thrust eccentricity must be compensated by a rudder deflection alone, as is the case, for example, during landing immediately before touching down without thrust, only the moment equilibrium with  $\ddot{y} = 0$  is used and the result is

$$\ddot{y}M \ddot{y}_e^{(SP)} 0 \ddot{y} \ddot{y} \ddot{y} \ddot{y}_e \ddot{y} y_A r W y_{ex} s$$

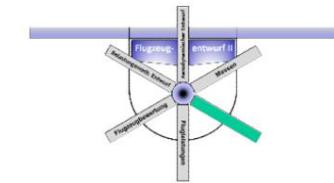
and with coefficients follows

$$0 \ddot{y} \frac{s}{G} \frac{1}{e} \frac{2}{b} \frac{ss}{s}, \frac{b}{2} \frac{c_q F}{c_q' F}$$

$$A_s \frac{\ddot{y}_{ss}}{I_{\ddot{y}}} \ddot{y} l \ddot{y} \ddot{y} \ddot{y} c_q F_w_{ex} \frac{2}{b} \frac{ss}{s}, \frac{b}{2}$$

# F Flight mechanical design - basics

## 5 Vertical stabilizer design



- With the lift coefficient of the vertical stabilizer  $c_{A_s}$  receives one

$$\frac{S}{G} \cdot \frac{1}{e} \cdot \frac{2 \text{ years}_s}{b} \cdot \frac{b}{2} \cdot c_{A_s} F_c = \frac{\ddot{y}_{ss}}{I} \cdot \ddot{y}_{ly} \cdot \ddot{y}_{ly} c_{W_{ex}} \cdot \frac{2 \text{ years}_s}{b} \cdot \frac{b}{2}$$

- After forming, the sliding angle is

$$\frac{2 \text{ years}_s}{b} \cdot \frac{b}{2 \text{ litres}_y} \cdot \frac{c_{A_s}}{c_{W_{ex}}} = \frac{\ddot{y}_{ss}}{\ddot{y}_{ly}} \cdot \frac{\ddot{y}_{ly}}{\ddot{y}_{ly}} \cdot \frac{c_{A_s}}{\ddot{y}_{ly}} = \frac{c_{A_s}}{\ddot{y}_{ly}}$$

- The expression  $2\ddot{y}_l \frac{b}{\ddot{y}}$  can be sufficiently accurately determined with half the wing aspect ratio since  $\ddot{y}_l \approx l_m$ .

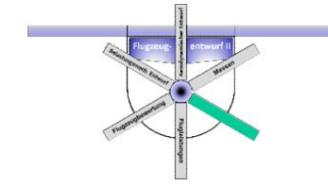
Wing aspect ratio can be equated since  $\ddot{y}_l \approx l_m$ .



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# Flight mechanical design - basics

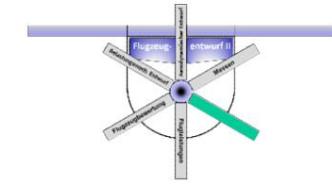
## 5 Vertical stabilizer design



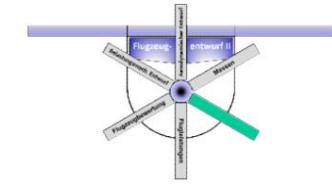
- This function can now be used to check whether the rudder angle required to fully compensate for the thrust eccentricity and the corresponding maximum side force coefficient can also be achieved in landing configuration at minimum speed with  $cA_{max}$ , or how large the vMCA (minimum control speed air) is.
- To determine the aerodynamic coefficients, the wing with camber flap at maximum lift is to be used.

# F Flight mechanical design - basics

## 5 Vertical stabilizer design

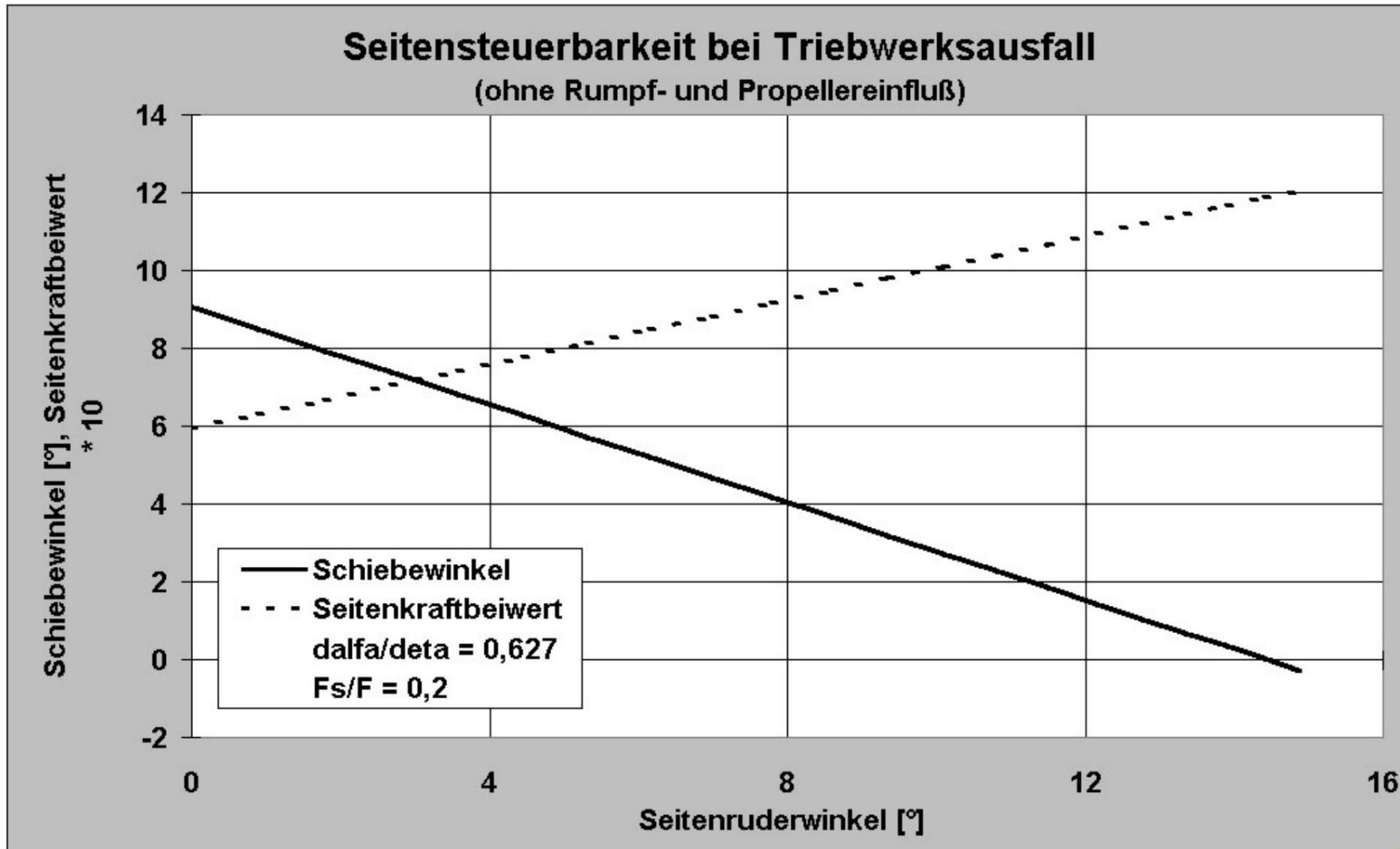


- In the above relationship it is assumed that the sliding angle  $\ddot{\gamma}$  also corresponds to the angle of attack of the vertical stabilizer.
- This is not always the case. Depending on the configuration, the flow direction is influenced locally, similar to the downwash field of the wing, by the fuselage or, even more clearly in propeller aircraft, by the propeller wake. • Particular attention must be paid to the spin in the wake area, which can induce large lateral velocity components on the vertical stabilizer.
- In this case, a change in local back pressure is also to be expected.



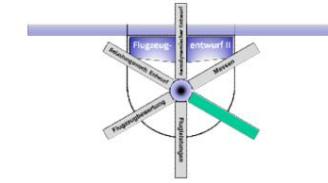
# F Flight mechanical design - basics

## 5 Vertical stabilizer design



# F Flight mechanical design - basics

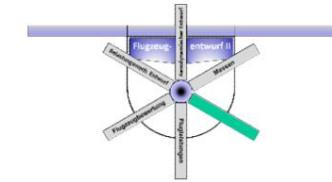
## 5 Vertical stabilizer design



- Another important design criterion for the side guide Factory configuration is the control work of a crosswind during landing.
- To achieve a yaw angle that allows a maximum assumed crosswind component  $v_w = v \cdot \sin(\ddot{\gamma})$ , the maximum rudder deflection is required:

$$\frac{\arcsine \frac{\ddot{\gamma} v_w}{v}}{\frac{\ddot{\gamma} v_{\min}}{\ddot{\gamma}_s}}$$

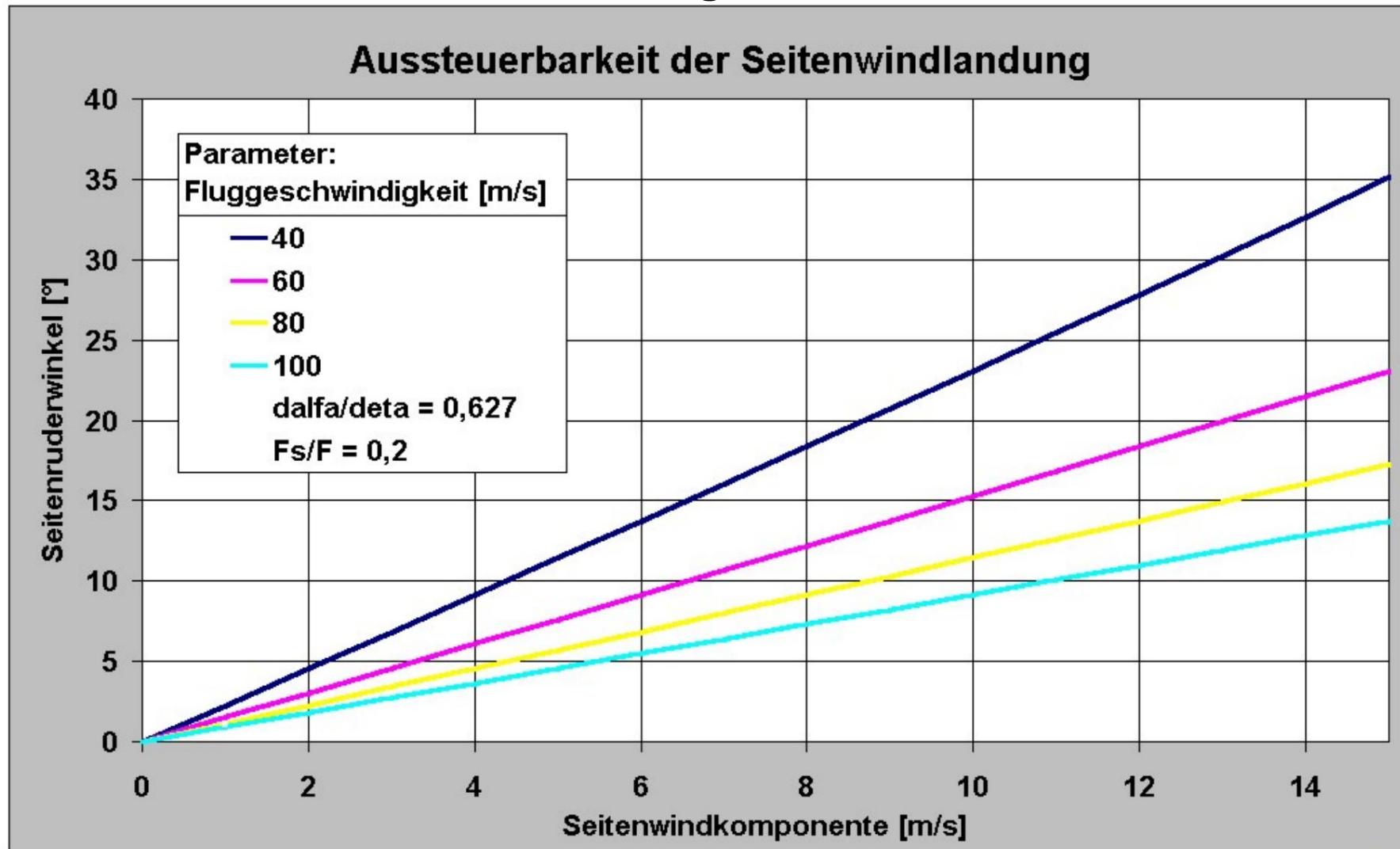
- Vertical stabilizers should be designed for crosswind components of at least 15 m/s at the minimum flight speed.
- Here too, the landing configuration is critical.



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# Flight mechanical design - basics

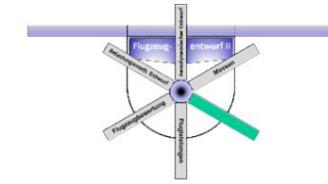
## 5 Vertical stabilizer design



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# **Flight mechanical design - basics**

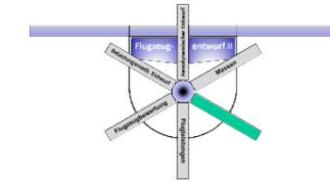
## **5 Vertical stabilizer design**



- The same considerations apply to the geometric design of the vertical stabilizer as for the horizontal stabilizer, but here you are usually dealing with a symmetrical profile that has no zero angle of attack and moment and is aligned in the direction of the fuselage's longitudinal axis with minimal drag. • A vertical stabilizer trim is necessary for asymmetrical flow conditions.  
tions in the wake of a propeller.
- A so-called “bail edge” may be helpful, particularly in smaller GA (General Aviation) aircraft. This is a laterally bent sheet of metal on the trailing edge of the rudder and is dimensioned for cruise flight.
- Since the lateral force characteristics depend strongly on the relative installation location of the horizontal tail assembly, the above remarks on the effective vertical tail assembly stretch must be observed.

## F Flight mechanical design - basics

### 5 Vertical stabilizer design



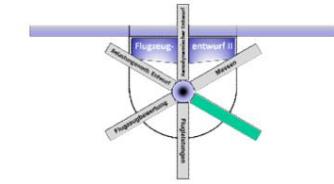
- One measure to improve insufficient spin behavior is to increase the depth of the tail unit in the fuselage area, by installing a “dorsal fin”, which is often found in practice. • The Saab

340 has a pronounced “dorsal fin”.



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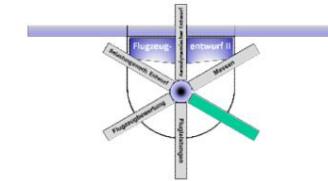
## 5 Vertical stabilizer design



- To avoid large vibration amplitudes, large aircraft are constantly adjusting their yaw angles with the help of a regulator, the “yaw damper”.
- This also prevents the occurrence of sideslip angles, which could lead to flow separation at the vertical stabilizer.

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## 5 Vertical stabilizer design



- For special designs such as the transport aircraft “Beluga”, which has problems with the vertical stabilizer effectiveness due to the large-volume fuselage body and also with the wind vane stability due to the extraordinary fuselage height in the front area of the aircraft, additional vertical fins at the ends of the horizontal stabilizer can also provide a remedy.

