Welcome to the course

Aircraft design II

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4 Range and flight time

• The actual transport work takes place during cruise flight. • This

is the part of a transport mission in which the aircraft is flown over a longer period of time under optimal, stationary or quasi-stationary operating conditions. • Depending on the application, there are

different

Species for optimal conditions.

• It can be flown in three ways:

1. constant flight altitude, constant angle of attack (or lift coefficient) and variable speed,
2. constant speed, constant angle of attack and variable altitude and
3. constant altitude, constant speed and variable angle of attack

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4 Range and flight time

- The different cruise flight strategies result in different flight programs with different optimal parameters.

- While flight case 2 (H = var.) yields the most economical results nis, in practice the aviation operations tend to proceed according to strategy 3 (v = const., H = const.).

- Today's air traffic control systems are based on the separation Organizing traffic by means of altitude staggering with a fixed flight route.

- The constant, undefined change of altitude levels as in the continuous "cruise climb" (case 2) is difficult to control in this air traffic control system.

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4 Range and flight time

- **It is preferable to fly in a mixed mode:**
  - **The initial cruise altitude, at which a residual climb capability of 300 ft/min must be ensured, is initially kept constant until the reduced flight weight provides the climb performance potential to reach the next permitted altitude level (4000 ft higher with 300 ft/min residual climb capability).**
  - **Then the altitude is increased to the next height with the best range.**
- **This step-shaped flight path approximates the optimal flight path of flight case 2. • Flight case 1 (a**

= const.) is used less frequently in practice, since the pilot and autopilot are less able to control the angle of attack than the flight speed (case 3).

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4.1 Basic equation of range flight

- The distance covered in a time interval with a certain speed is generally given in a time interval $\mathrm{dR} = \mathrm{v}\,\mathrm{dt}$. With it

$$R = \int_0^{t_R} v\, dt$$

- Due to the fuel consumed, the aircraft loses weight

dG b Sÿtÿ g dt S

- This gives the time flown per unit of fuel

engl ÿÿ $\frac{dG}{b\, S\, g}$

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4.1 Basic equation of range flight

- The distance travelled per unit of fuel can be expressed over the weight interval (fuel mileage)

$$dR = -\frac{v}{b_S \cdot S \cdot g} \cdot dG$$

- The initial weight for a cross-country flight is

$$G_a = G_A + G_{Ksteig} + G_{Kstart}$$

- This makes the range equation

$$R = -\int_{G_a}^{G_e} \frac{v}{b_S \cdot S \cdot g} \cdot dG$$

- With the horizontal force equilibrium e = S/G and after Swapping the integration limits follows

$$R = \int_{G_e}^{G_a} \frac{v}{b_S \cdot G \cdot \varepsilon \cdot g} \cdot dG$$

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4.1 Basic equation of range flight • Using

the vertical force balance as a condition for the speed and the definition of e gives: V

$$R = -\int_{G_a}^{G_e} \frac{\sqrt{\frac{2G}{\rho} \cdot \frac{1}{F} \cdot \frac{1}{c_A}}}{b \cdot G \cdot g} \cdot \frac{c_A}{c_W} \cdot dG \quad \text{or.} \quad R = -\int_{G_e}^{G_a} \frac{\sqrt{\frac{2}{\rho} \cdot \frac{1}{F}}}{b \cdot g \cdot G^{\frac{1}{2}}} \cdot \frac{\sqrt{c_A}}{c_W} \cdot dG$$

- For different requirements regarding flight altitude, lift coefficient and speed, different solutions result from the integration.

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4.1.1 Range at constant altitude and angle of attack

- At constant flight altitude and constant angle of attack (c_A = const.) – i.e. case 1 – the thrust is proportional to the weight and the speed is proportional to the square root of the weight.

- Thrust and speed increase continuously during flight. dig. Assuming that the specific consumption remains constant under these conditions, the following results:

$$R = \int_{G_e}^{G_a} \frac{\sqrt{\frac{2}{ÿ}\frac{1}{F}}}{b g} \cdot \frac{\sqrt{c_A}}{c_{\text{Shared flat}}} \cdot \frac{1}{\sqrt{G}}\, dG = \frac{2}{b g} \cdot \sqrt{\frac{2G_a}{ÿ\, F}} \cdot \frac{\sqrt{c_A}}{c_W} \left(1 - \sqrt{\frac{G_e}{G_a}}\right)$$

- This means that at a constant altitude and pitch, the maximum range is achieved when flying at a speed that corresponds to the maximum value of $\sqrt{c_A}/c_W$

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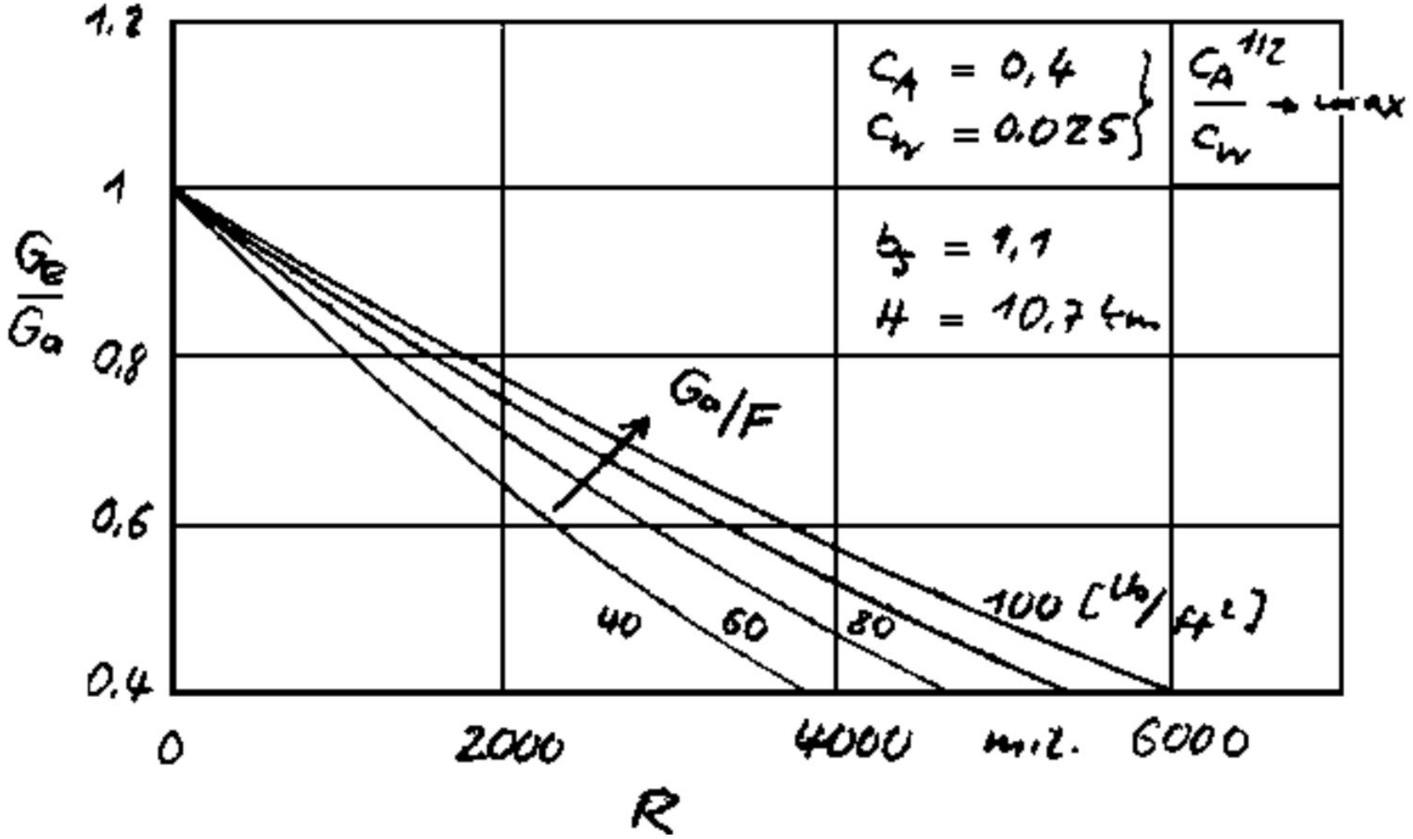
4.1.1 Range at constant altitude and angle of attack

• Furthermore, the range depends on the flight altitude, $\sqrt{1/ÿ}$ the engine quality with 1/bs and the wing loading Ga F as well as the weight ratio with $\sqrt{\ }\ /$ $\sqrt{Ge/Ga}$.



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4.1.2 Range with constant speed and angle of attack

- If the cruising speed and lift coefficient remain constant during range flight – i.e. case 2 – thrust and density are proportional to weight. However, the ratio of weight to density remains constant:

$$v \;\text{ÿ}\; \sqrt{\frac{2G}{\text{ÿ}_a} \text{ ÿ } \frac{a}{F} \text{ ÿ } \frac{1}{c_{Aa}}} \quad \text{with the density} \quad \text{ÿ ÿ ÿ ÿ}_a \frac{G}{G_a}$$

- This means that the aircraft climbs continuously at a constant speed and constant attitude (cruise climb) and the range equation has the following structure:

$$R \;\text{ÿ}\; \sqrt{\frac{2G}{\text{ÿ}_a} \text{ ÿ } \frac{a}{F} \text{ ÿ } \frac{1}{c_{ASa}}} \text{ ÿ } \frac{1}{b \text{ ÿe ÿ } G} \text{ ÿ ÿ}_{G_e}^{G_a} \frac{dG}{G}$$

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4.1.2 Range with constant speed and angle of attack

- This solution is known as Breguet's range formula:

$$R = \sqrt{\frac{2G_a}{\rho_a} \cdot \frac{1}{F} \cdot \frac{1}{c_{A_a}}} \cdot \frac{1}{b_S g \varepsilon} \cdot \ln\left(\frac{G_a}{G_e}\right)$$

- or.

$$R = \frac{v}{b_S \cdot \varepsilon \cdot g} \cdot \ln\left(\frac{G_a}{G_e}\right)$$

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4.1.3 Range at constant speed and altitude • Flight at constant altitude and constant speed

corresponds to

case 3. • Here the lift coefficient is proportional to the weight. The thrust is proportional to the weight multiplied by the ratio

c_W / c_A . It is therefore

$$c_A \ddot{y}\ddot{y} c_{A.a} \frac{G}{G_a}$$

• and with the quadratic polar approximation

$$c_W \ddot{y} c_{W0} \ddot{y} \ddot{y}\ddot{y}\ddot{y}\ddot{y} \frac{2 c_{A\,a}}{\ddot{y}\ddot{y}\ddot{y}\, e} \ddot{y} \left(\frac{G}{G_a} \right)^2$$

• The range equation is therefore

$$R \ddot{y} \frac{v}{b g_S} \ddot{y} \int_{G_e}^{G_a} \frac{1}{G \ddot{y} e}\, dG$$

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4.1.3 Range at constant speed and altitude

- **You get the solution ÿ**

$$R \text{ ÿ } \frac{v}{b g_S \text{ ÿ ÿ } \sqrt{\dfrac{c_{W0}}{\text{ÿÿÿÿ } e}}} \text{ ÿ } \arctan \frac{\dfrac{G_a \text{ ÿ ÿ ÿ } G_e}{q F} \text{ ÿ } \sqrt{\dfrac{1}{c_{W_0} \text{ ÿÿÿÿÿ } e}} \text{ ÿÿ}}{\text{ÿ ÿ } \dfrac{G_a \text{ ÿ ÿ1 ÿ } G_e}{q^2 F^2} \text{ ÿ } \dfrac{1}{c_{W_0} \text{ ÿÿÿÿÿ } e}} \text{ ÿÿÿÿÿ}$$

- **In this case the aircraft flies with a variable pitch and therefore with variable drag and variable glide ratio.**

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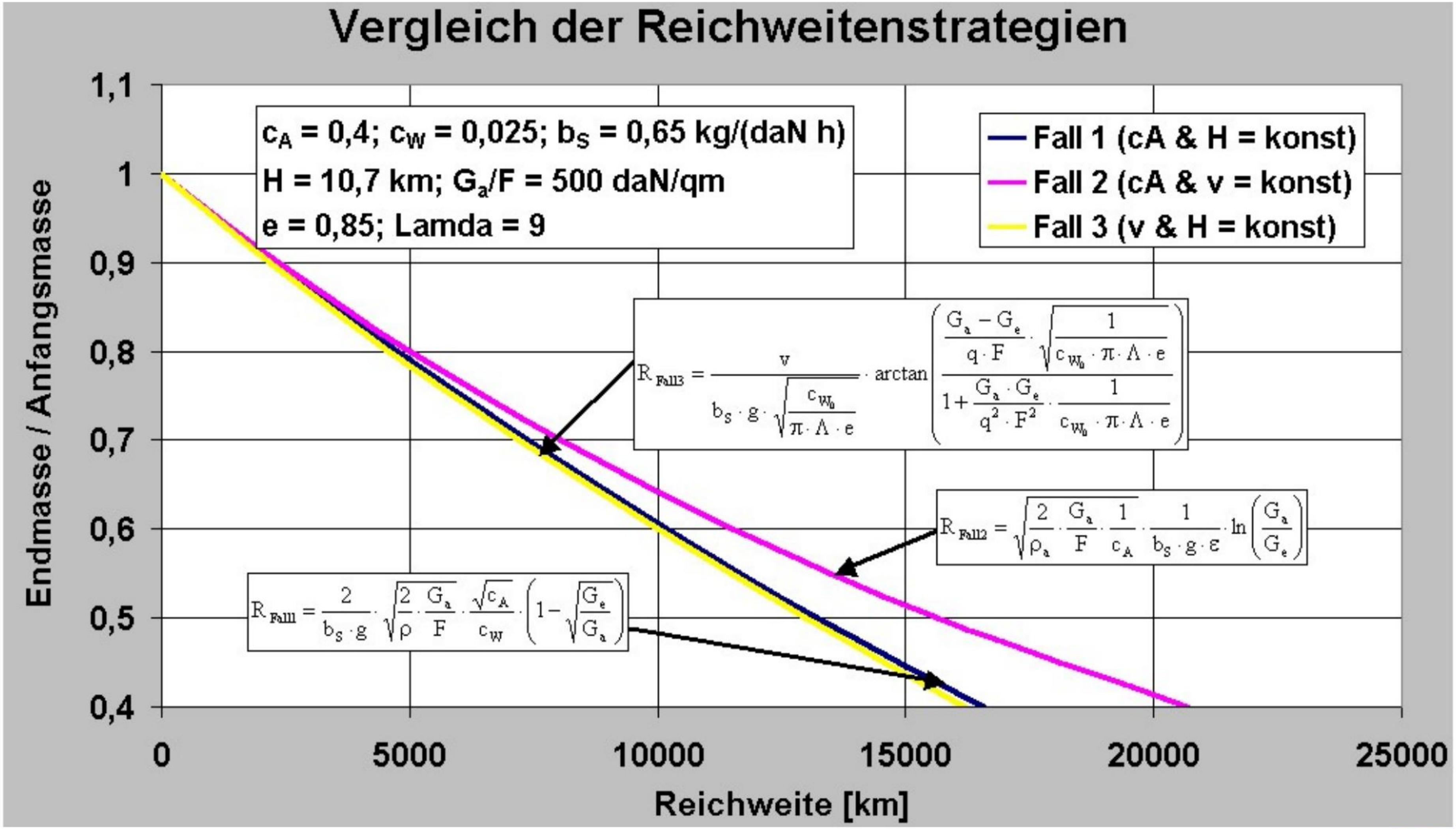
4.1.4 Comparison of reach strategies



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4.1.5 Range of the propeller aircraft

- For propeller propulsion, fuel consumption depends on the Engine power with the power-related fuel consumption in the unit (dm kg/kW · h) away:

$$\frac{K}{\text{engl}} \;\text{ÿ ÿ}\; b_N N$$

- With the differential track

$$dR\; v\; \text{ÿÿÿÿ}\, dt \qquad \frac{v}{b_N N} \;\text{ÿ}\; dm \;\text{ÿ}$$

- the range is

$$R \;\text{ÿ}\; \text{ÿ}_{G_e}^{G_a} \frac{v}{b_N N g} \;\text{ÿ}\; dG \;\text{ÿ}\;\text{ÿ}$$

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Range of the propeller aircraft • If the required power for unaccelerated flight W v

$$N \; ÿ \; \frac{ÿ}{ÿ} \qquad \text{and} \qquad W \; ÿ\,ÿ \; \frac{c_W}{c_A} \; G$$

• inserted, results

$$R \; ÿ \; \int_{G_e}^{G_a} \frac{ÿ}{b_N \, W \, g} \; ÿ\,ÿ \, dG \; ÿ \int_{G_e}^{G_a} \frac{ÿ}{b_N \, G \, g} \; ÿ \; \frac{c_A}{c_W} \; ÿ \; dG$$

• For the practical assumptions of constant fuel consumption, efficiency and lift coefficient follows one of the Breguet equation corresponding solution:

$$R \; ÿ \; \frac{ÿ}{b_N \; ÿe \; ÿg} \; ÿ \; ÿ \; \ln \; ÿ\,ÿ \frac{G_a}{G_e} \; ÿ\;ÿ\;ÿ\;ÿ$$

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4.1.5 Range of the propeller aircraft

- In contrast to jet-powered aircraft, the maximum range not at ÿ $\sqrt{c_A}/c_W$ ÿ $_{Max}$,

  ÿ $c_A/c_W\,_{max}$ ÿ ÿ ε_{min} .

- Fuel consumption is at its minimum at the best

  Efficiency and fuel load should be as high as possible.

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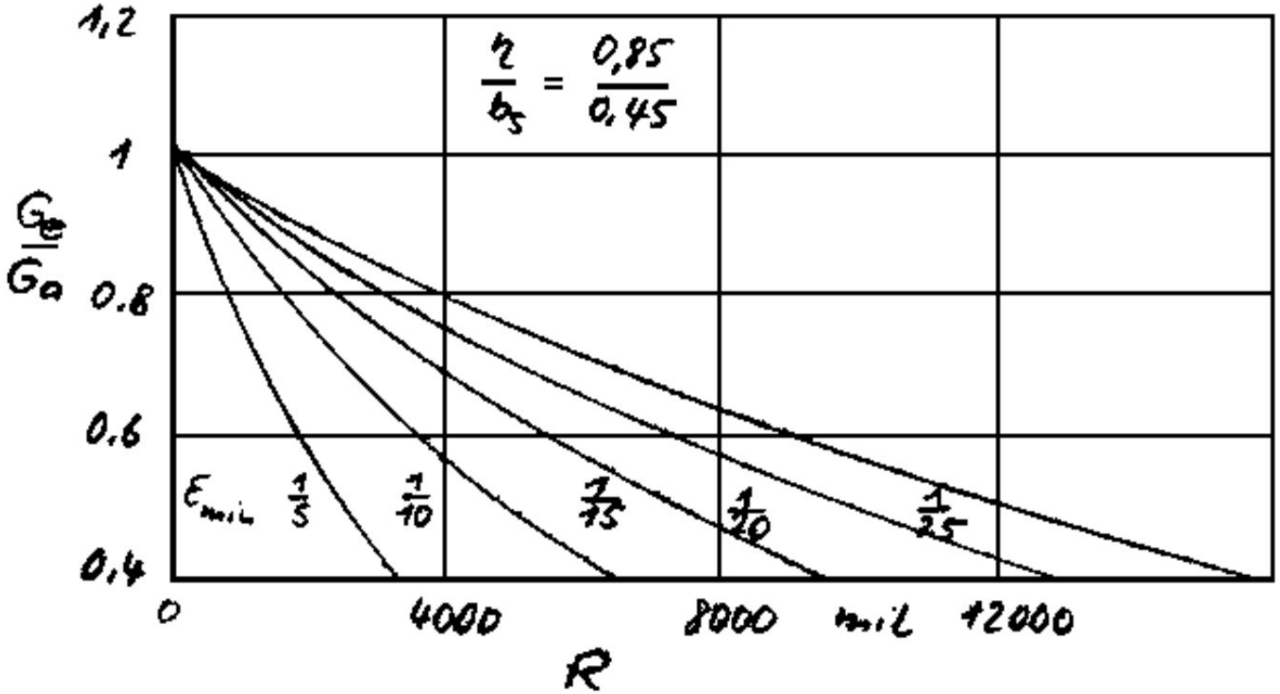
4.1.5 Range of the propeller aircraft



• **Compared to the TL drive, fuel consumption decreases more towards the end of the flight due to the reduced power requirement and lower weight.**

• **An increase in the final weight (eg due to greater structural weight or more equipment) requires a proportionally larger additional fuel weight if the design range is to remain the same.**

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4.2. Range calculation using the energy altitude method

- The energy level method can also be used to calculate the range.
  With the lower calorific value of the fuel H_u, the energy balance gives

$$G \, d\left(\frac{E}{G}\right) = \eta \, H_u \, dm - W \, v_k \, dt$$

– and then with v .dt = dR:

$$dR = \frac{\eta \, H_u}{W} \, dm_k - \frac{G}{W} \, d\left(\frac{E}{G}\right)$$

- With the relations A = G, dG = -dGK and $e = \frac{W}{A}$ becomes

$$dR = -\frac{1}{e}\left(\frac{\eta \, H_u}{G} \, \frac{dG}{G} + d\left(\frac{E}{G}\right)\right)$$

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4.2. Range calculation using the energy altitude method

• With ÿ ÿ $\frac{v}{b_s H_u}$ •

finally one obtains

R ÿ ÿ v / (e ÿ bg) ÿ ∫ (G_e … G_a) dG/G · 1/e ÿ ∫ (E/G_e … E/G_a) ÿ ÿ ÿ ÿ d E/G

• Integration assuming constant efficiency and constant glide ratio leads to the well-known Breguet range equation • This is now replaced by an energy-level

dependent term ÿ ÿ ÿ ÿ ÿ expanded:

R ÿ v / (bg ÿ e ÿ) ln (G_a / G_e) · 1/e E ÿ ÿ ÿ ÿ ÿ ÿ ÿ ÿ ÿ ÿ ÿ E/G_e … E/G_a

ÿ ÿ ÿ • The additional summand is only important if large changes in the energy level (speed, altitude) occur during cruise flight.

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4.3 Current (specific) range and flight time

- For the determination of the maximum flight duration or The weight-specific values for the current flight condition must be considered for the optimal range of weight and altitude-dependent flight speeds.

- The weight-related, current flight duration is determined using the equation of motion for the change in weight over time.

$$\text{ÿ ÿ} \; \frac{engl}{dG} \quad \frac{1}{b\,Ş\,g}$$

- The specific range or fuel efficiency (fuel

mileage):

$$\text{ÿ ÿ} \; \frac{dX}{dG} \quad \frac{v}{b\,Ş\,g}$$

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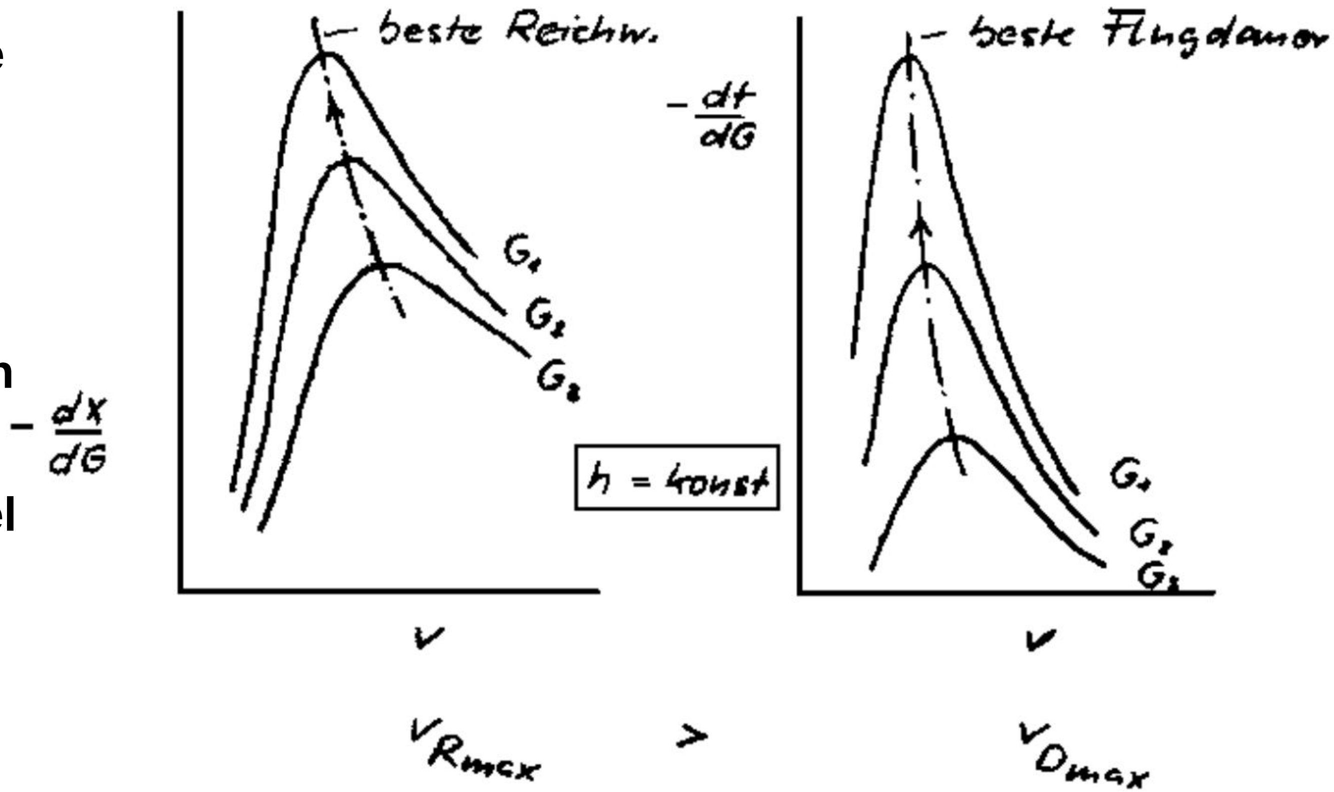
4.3 Current (specific) range and flight time

- Both the weight-related range and the weight-specific flight time depend on the flight altitude, the speed and the throttle level

- These variables are not independent of each other, but must be sufficient for each instantaneous weight of the level flight condition.



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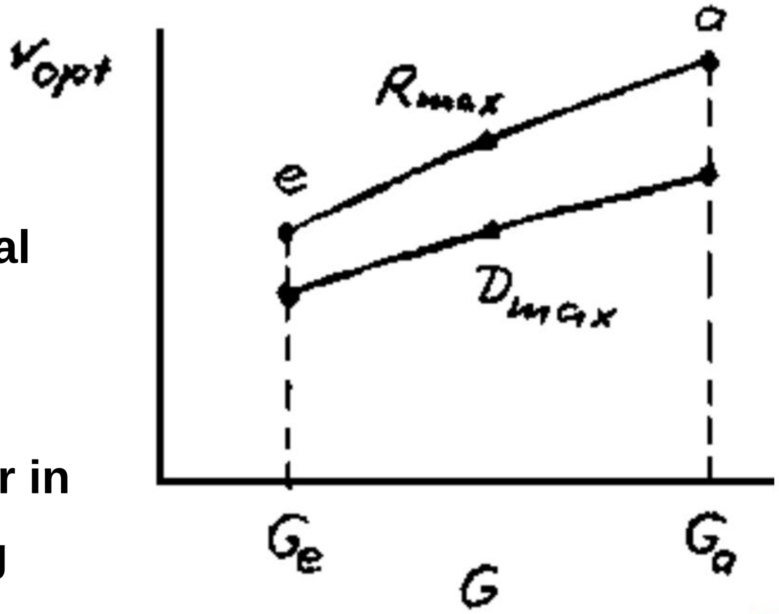
4.3 Current (specific) range and flight time

- **The maximum range speed is greater than the maximum flight time.**

- **This fact has a practical impact on the design of observation aircraft (e.g. aircraft for measurements in the atmosphere, aerial photography aircraft, holding patterns) and military tankers.**



- **The aircraft is designed for long flight durations and not, as with commercial transporters, for economical long-distance flights with minimal fuel consumption.**

- **The design surface loading must be lower in accordance with the optimum operating speed.**

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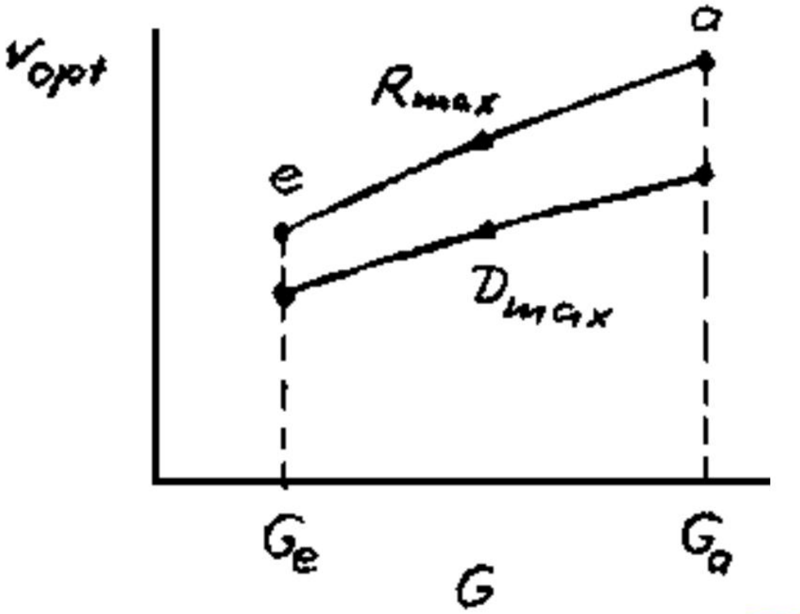
4.3 Current (specific) range and flight time

- **For each weight there is a speed that maximizes either the current range or the current flight time.**

- **These optimal flight speeds become smaller with the continuous weight loss due to fuel consumption.**



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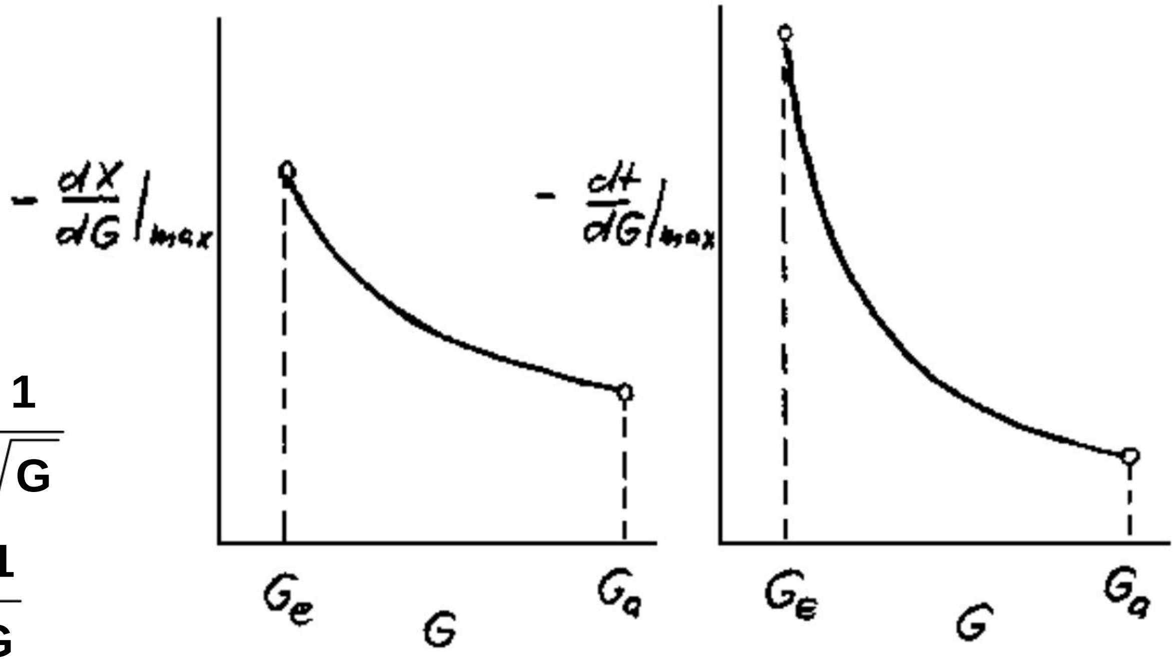
4.3 Current (specific) range and flight time

- In contrast, the corresponding maximum values for range and flight time increase along the flight path.

- For subsonic TL and ZTL aircraft, these dependencies are approximately:



$$\left(\frac{dX}{dG}\right)_{Max} \quad R_{max,mom} \sim \frac{1}{\sqrt{G}}$$

$$\left(\frac{\text{engl}}{dG}\right)_{Max} \quad D_{max,mom} \sim \frac{1}{G}$$

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4.3 Current (specific) range and flight time

- The prerequisite of correct horizontal flight with different diminishing climb angle g and constant flight altitude is practically non-existent.
- However, if we assume for simplicity that
  - the angle of climb remains sufficiently small so that cosg 1 ≈ and sing g apply, ≈ and
  - the weight change is much smaller than the resistance is,

Compared to the horizontal flight case, only the kinematic condition in the vertical direction changes and it applies

hv ÿ ÿ 0 .
ÿ ÿ g

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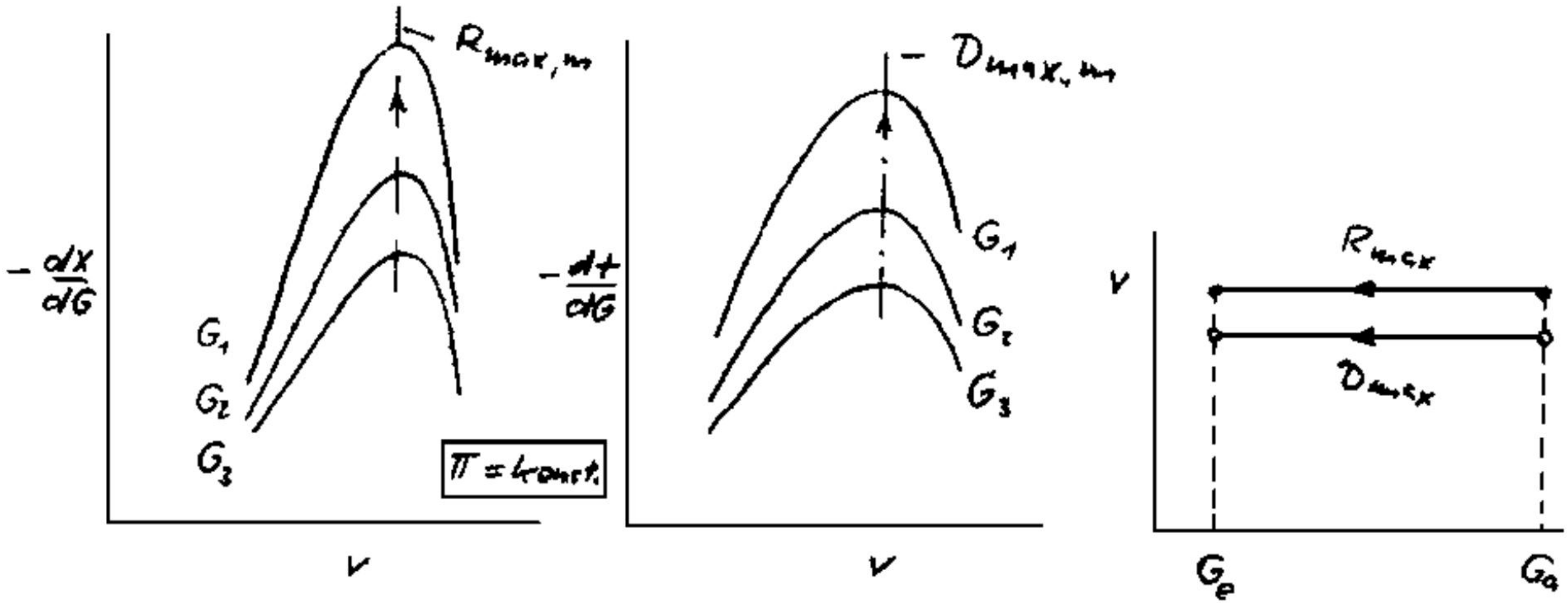
4.3 Current (specific) range and flight time

- **After eliminating the altitude, this results in the following figure in which again the influence of decreasing flight weight in the direction of the arrow is given parametrically.**



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4.3 Current (specific) range and flight time

- For jet-powered aircraft, when flying in the stratosphere
  - as well as $\left(\frac{dX}{dG}\right)_{Max}$ $\quad R_{max,mom} \sim \frac{1}{G}$
  - as well as $\left(\frac{engl}{dG}\right)_{Max}$ $\quad D_{max,mom} \sim \frac{1}{G}$

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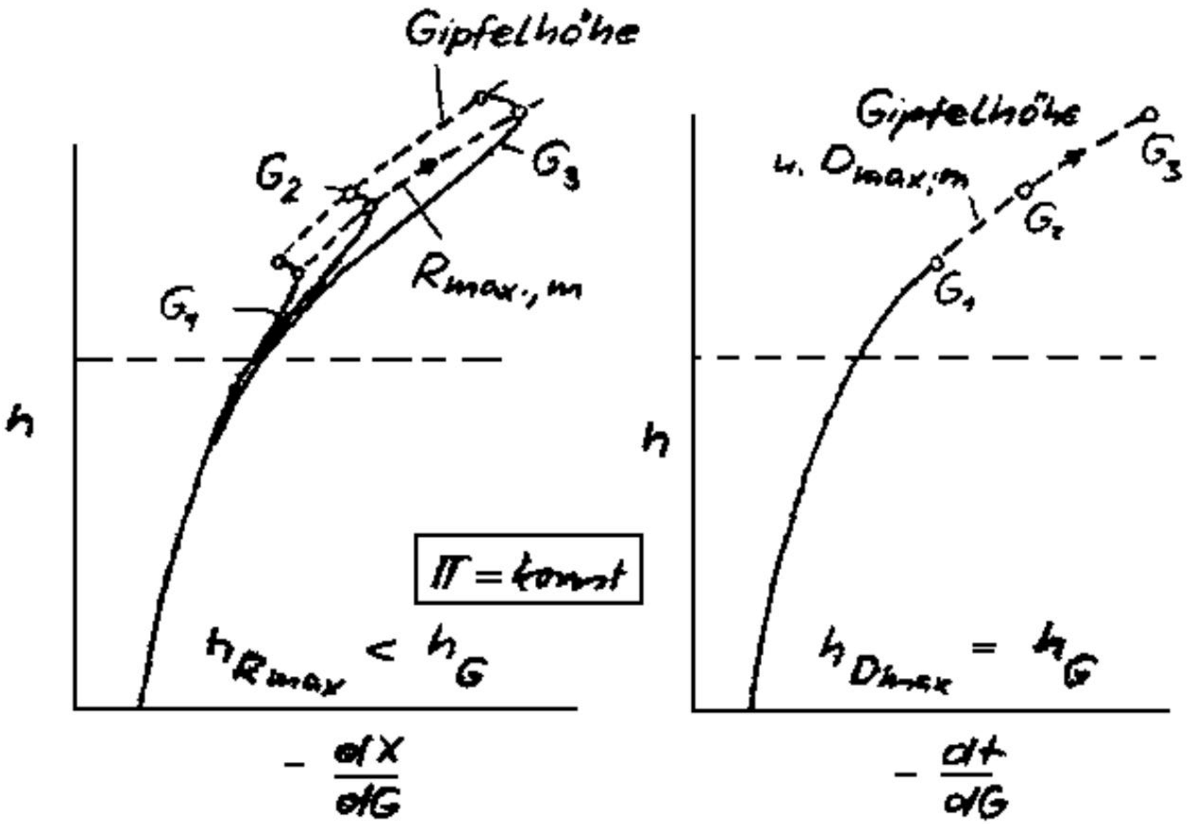
4.3 Current (specific) range and flight time

- **If the specific values are plotted against the height, we get you see the following picture:**



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4.3 Current (specific) range and flight time

- For each weight there is only one altitude where the current range tends towards the maximum range or the current flight time tends towards the maximum flight time.
- It can be shown that flight programs with a constant throttle level, which are flown either at the speed of the greatest range or the greatest flight time, are flown with a constant lift coefficient corresponding to the best glide ratio and thus with a constantly increasing flight altitude (cruise climb).

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4.3 Instantaneous (specific) range and flight duration •

Using the simplifying assumption of a quadratic drag polar and approximations for the efficiency of different engine types, the optimal speeds can be easily calculated.

- **The general solution of the range equation showed that the maximum range at $\sqrt{cA}/cW$ is achieved.**
- **The optimal lift coefficient corresponding to this condition value can be obtained by setting the derivative to zero: ÿ ÿ ÿ ÿ ÿ ÿ ÿ ÿ**

ÿ ÿ ÿ ÿ ÿ

$$\frac{\text{ÿ}\,\text{ÿ}\,\text{ÿ}\left(\frac{\sqrt{c_A}}{c_W}\right)\text{ÿ}\,\text{ÿ}\,\text{ÿ}\,\text{ÿ}\,\text{ÿ}\,\text{ÿ}\,\text{ÿ}}{\text{ÿ}c_A} \; \frac{\frac{\sqrt{c_A}}{c_{W0}\;\text{ÿ}\;\frac{c_A^2}{\text{ÿÿÿÿ}\;e}}}{\text{ÿ}c_A} \;\text{ÿ}\; 0$$

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4.3 Current (specific) range and flight time

- The execution of this differentiation leads to

$$c_{A_{opt}} \text{ ÿ } \sqrt{\frac{c_{W_0} \text{ ÿÿÿÿÿ } e}{3}}$$

and the corresponding Speed is:

$$v_{opt} \text{ ÿ } \sqrt{\frac{2G}{\text{ÿ}} \frac{}{F} \frac{1}{\sqrt{\frac{c_{W0} \text{ ÿÿÿÿÿ } e}{3}}}}$$

- With the induced

Drag coefficient cWi For the definition of the optimal polar point, it follows that here for e:

$$c_{W_i} \text{ ÿ } \frac{2c_A}{\text{ÿ ÿÿ ÿ} e} \text{ ÿÿ } \frac{1}{3} c_{W_0}$$

$$c \text{ ÿ ÿ}_{Wopt} \; \frac{4}{3} c_{W0}$$

$$e \text{ ÿ ÿ } opt \; 4 \sqrt{\frac{c_{W0}}{3 \text{ ÿÿÿÿÿ } e}}$$

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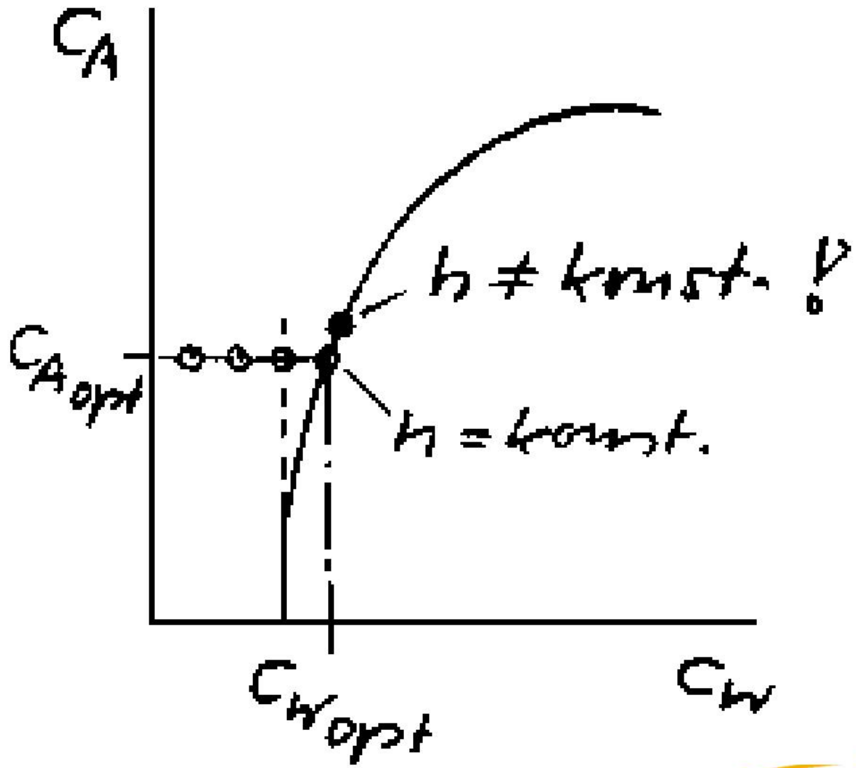
4.3 Current (specific) range and flight time

- The same result is also obtained via the reach Calculation according to the energy height method (4.2) if the range is to be maximum or (ÿ/ÿges) minimum.
- This consideration implicitly assumes that the thrust-specific consumption bs does not change with speed (i.e. the drive efficiency increases linearly with speed).



- This is only valid for TL drives!

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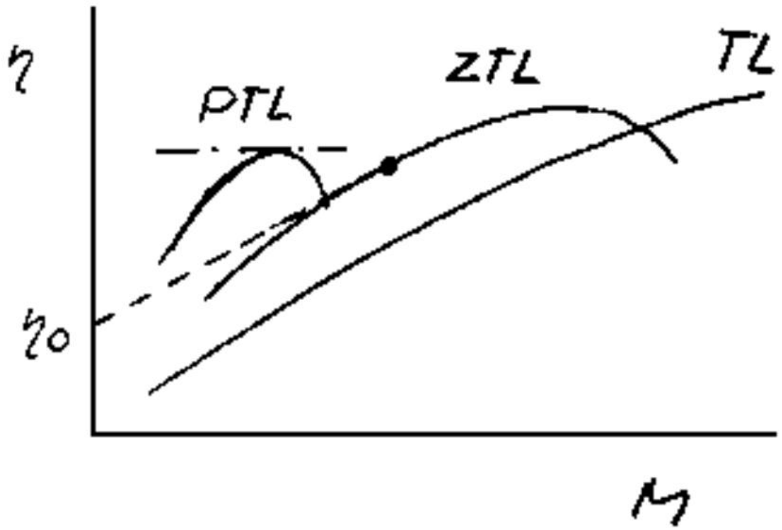
4.3 Current (specific) range and flight duration • The

efficiency curves of the various drive types can be approximated as follows:

$$ÿ_{total} \; ÿ\,ÿ \; \frac{ÿÿ}{ÿv} \; v$$ for TL engines,

$$ÿ_{total} \; ÿ\,ÿ \; ÿ_{0} \quad const$$ for PTL engines and

$$ÿ_{total} \; ÿ\,ÿÿ\,ÿ\,v\,0\,ÿ \; \frac{ÿ\,ÿ}{v}$$ for ZTL engines in the relevant area



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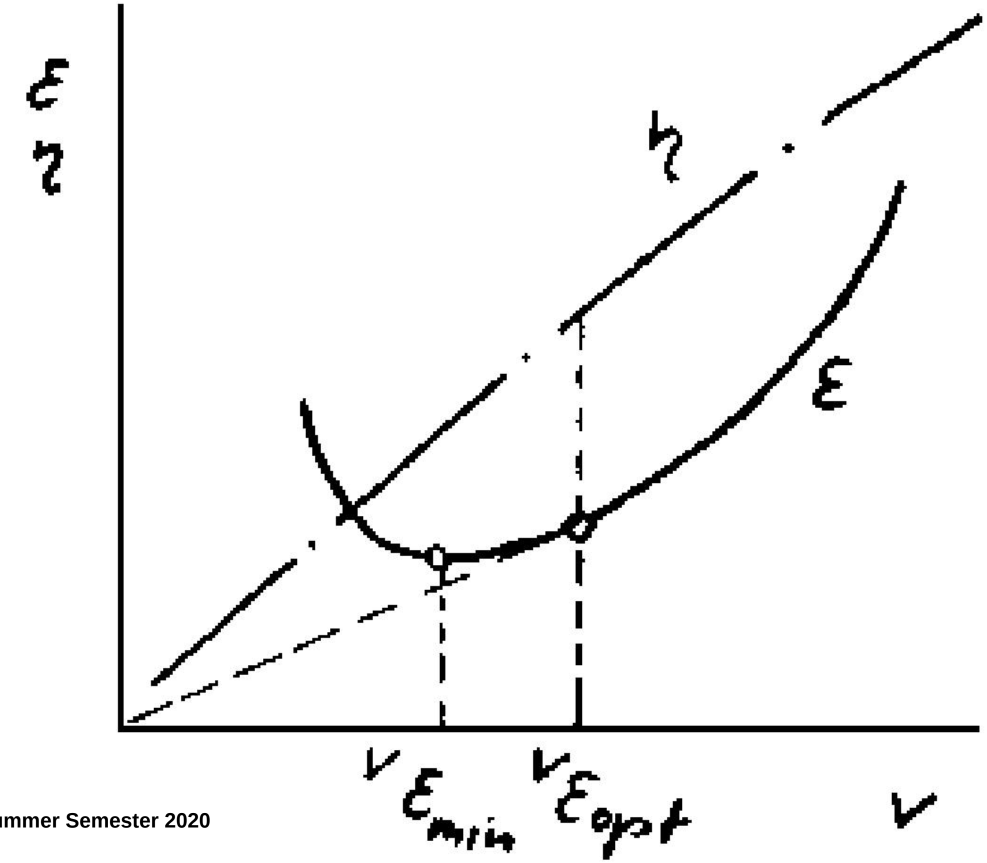
4.3.1 Range-optimal flight speed with TL drive

• The differentiation of (e/ÿges) leads for a TL drive to ÿ ÿ ÿ ÿ ÿ ÿ ÿ

$$\text{ÿ}\ \frac{\frac{e\,\text{ÿ}\,c_A\,\text{ÿ}}{\text{ÿ ÿ ÿtotal ÿ A ÿ ÿ ÿ ÿ ÿ}}}{\text{ÿ}\,c_A}\ \frac{\frac{\frac{c_{W0}}{c_A}\ \text{ÿ}\ \frac{c_A}{\text{ÿÿ}\,e}\ \text{ÿÿ}}{\frac{\text{ÿÿ}}{\text{ÿ}v}\ v}\ \text{ÿÿÿÿ ÿ}}{\text{ÿ}\,c_A}\ 0$$



• This differentiation leads again to the well-known Result

$$c_{A\,opt}\ \text{ÿ}\ \sqrt{\frac{c_{W_0}\ \text{ÿÿÿÿÿ}\ e}{3}}$$

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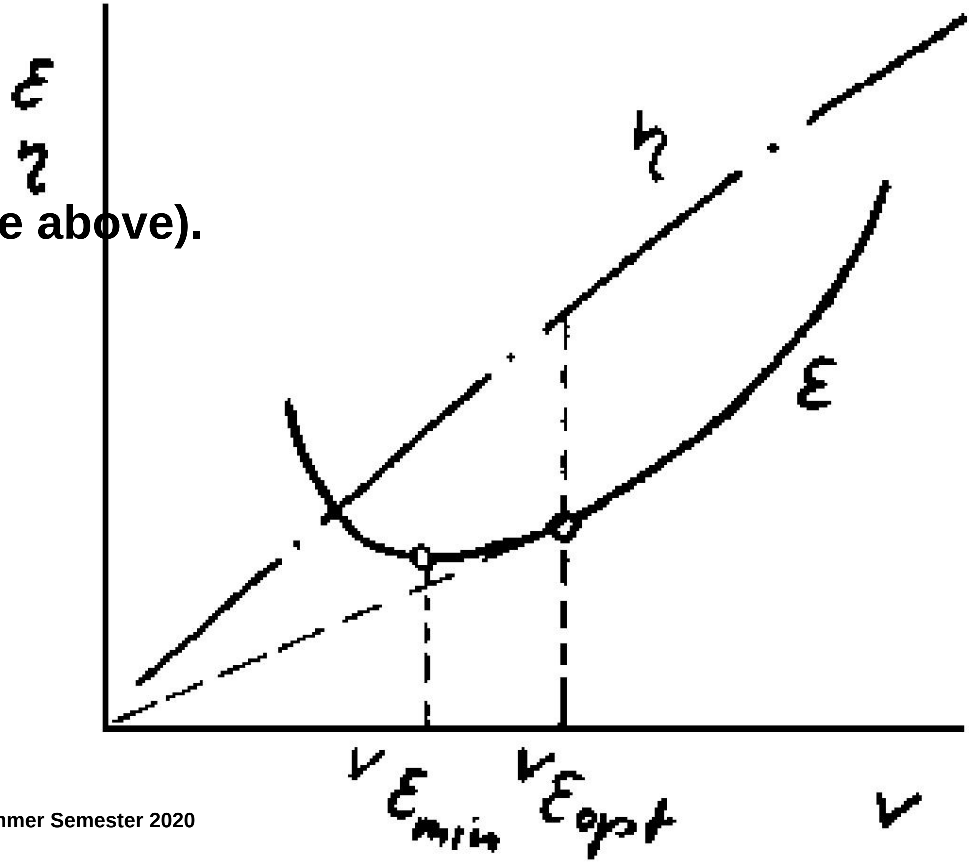
4.3.1 Range-optimal flight speed with TL drive

• This situation can be explained by the fact that the original tangent of the horizontal flight diagram is located at this point.

• The solution is therefore identical with the

assumption of a constant bs (see above).



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4.3.2 Range-optimal flight speed with PTL propulsion

- **For PTL drives, the derivation**

ÿ c ÿ $\frac{2c_A}{}$ ÿ ÿ

ÿ ÿ $\frac{W0 \text{ ÿÿ } e}{c_{A0}\text{ÿÿ}}$

$\frac{\text{ÿ ÿ ÿ ÿ} \qquad \text{ÿ ÿ ÿ ÿ}}{\text{ÿ } c_A}$ ÿ 0

- **The solution, solved for the optimal buoyancy coefficient is**

c_{Aopt} ÿ $\sqrt{W0 \text{ ÿÿÿÿÿ ce}}$

- **It follows that at this optimum point the induced resistance cWi** corresponds **to the zero resistance cW0 .**

- **This gives you the**

Optimal speed:

v_{opt} ÿ $\sqrt{\frac{2G}{\text{ÿ}} \cdot \frac{}{F} \cdot \frac{1}{\sqrt{W0 \text{ ÿÿÿÿÿ ce}}}}$

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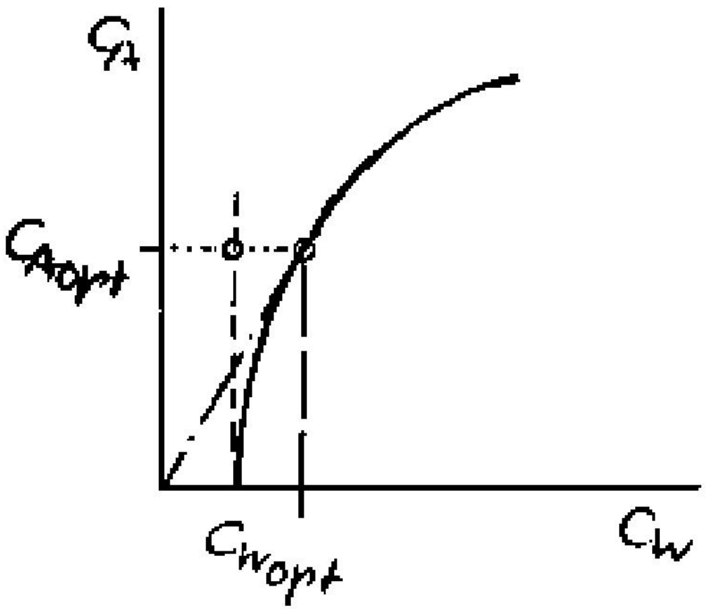
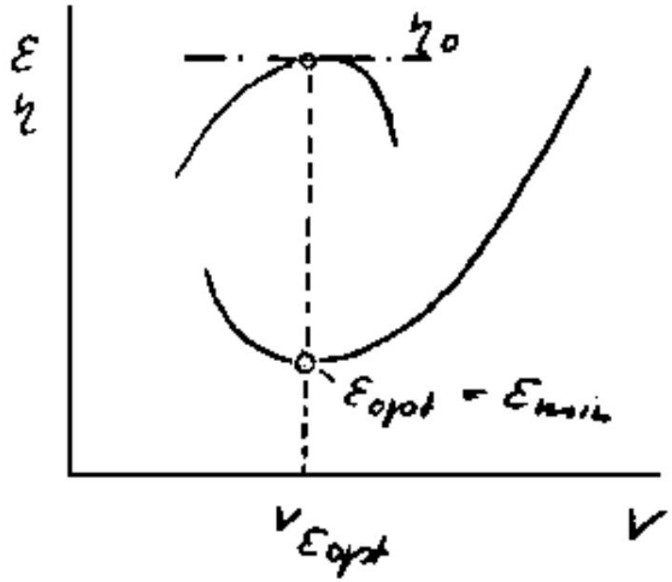
4.3.2 Range-optimal flight speed with PTL propulsion

- **The corresponding eopt is** $e\ \ddot{y}\ \ddot{y}\ opt\ 2\sqrt{\frac{c_{W0}}{\ddot{y}\ddot{y}\ddot{y}\ddot{y}\ \ e}}$

- **Graphically, this result can be interpreted as zero-point tangent to the drag polar or the minimum of cW/cA in the horizontal flight diagram.**





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Range-optimal flight speed with ZTL drive ÿ ÿ ÿ ÿ ÿ ÿ ÿ ÿ

- The same procedure is used for ZTL drives: The approach

$$\frac{c_A \,\text{ÿ ÿ}\, v_0 \,\text{ÿ ÿÿ ÿ ÿ ÿ ÿ ÿ}\sqrt{\frac{2G}{\text{ÿ}}\,\frac{}{F}\,\frac{1}{c_A}}}{c_{W0} \,\text{ÿ}\, \frac{2c_A}{\text{ÿÿÿÿ}\, e}} \;\text{ÿ}\; 0 \qquad \text{ÿ}\, c_A$$

leads to the solution

$$c_{A\,opt} \,\text{ÿ}\, \sqrt{c_{W0}\,\text{ÿÿÿÿÿ}\, e\, \frac{1\,\text{ÿ}\,\text{ÿ}_0/\text{ÿ}}{3 \cdot \text{ÿ}_0/\text{ÿ}}}$$

where the quotient is

$$\frac{c_{We}}{c_{W0}} \,\text{ÿ}\, \frac{1\,\text{ÿ}\,\text{ÿ}_0/\text{ÿ}}{3 \cdot \text{ÿ}_0/\text{ÿ}}$$

Resistance ratio corresponds.

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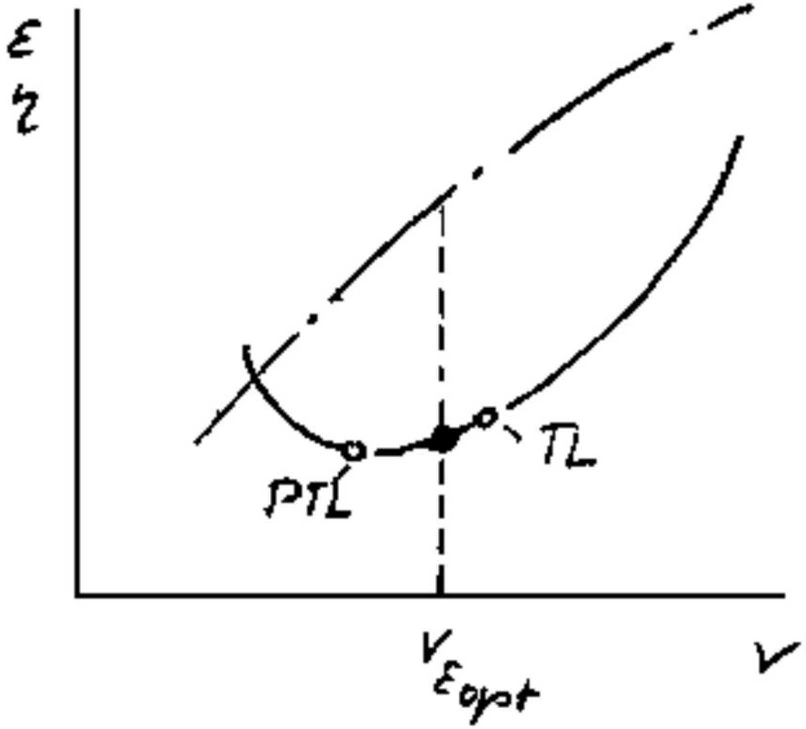
4.3.3 Range-optimal flight speed with ZTL drive

- The optimal speed
  you get accordingly with

$$v_{opt} \;\text{ÿ}\; \sqrt{\frac{2G}{\text{ÿ}} \cdot \frac{}{F} \cdot \frac{1}{\sqrt{c_{W_0}\;\text{ÿÿÿÿÿ}\;\text{ÿ}\;e\;\frac{1\;\text{ÿ}\;\text{ÿ}_0/\text{ÿ}}{3\cdot\text{ÿ}_0/\text{ÿ}}}}}$$

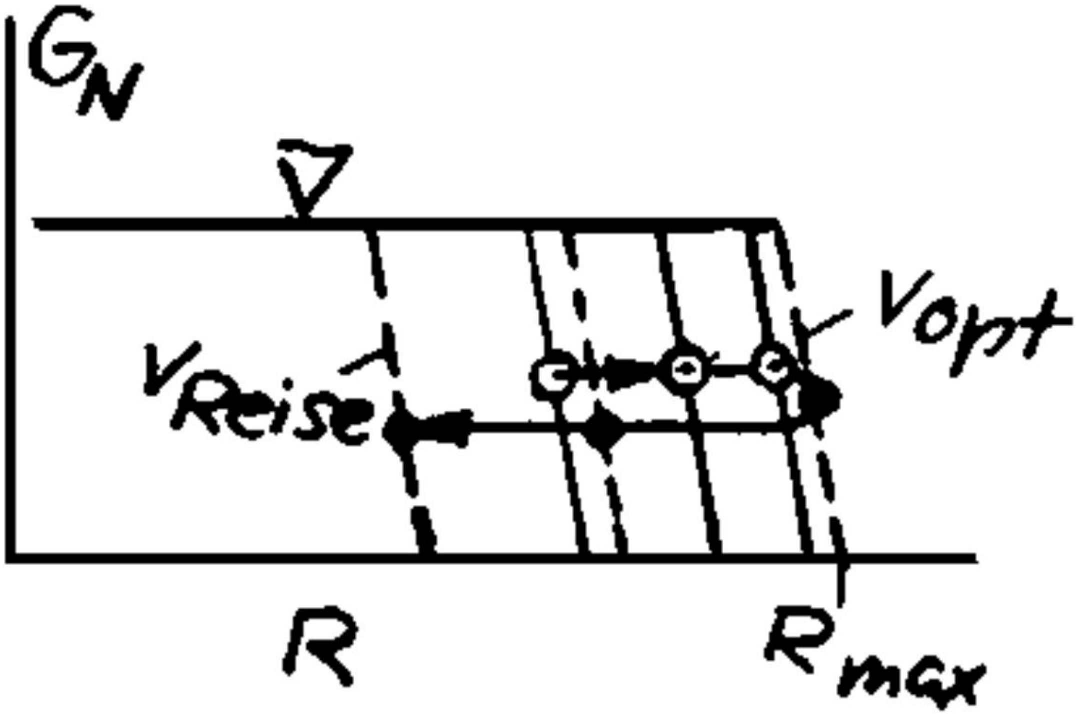


- The result, shown in the
  Horizontal flight diagram, shows, as expected, the position
  of the optimum point between those of the PTL and the TL drive.
- Since the efficiency ratio is a function of the flight speed
  speed, the solution can only be achieved with an
  iterative calculation.

G Flight performance

4.3.3 Range-optimal flight speed with ZTL drive

- **An iterative solution to determine the optimal flight speed speed is obtained by determining the payload-range diagram at constant fuel weight for different speeds.**



G Flight performance

4.3.4 Flight time with quadratic drag polar and PTL drive

- The maximum flight duration can also be determined directly using the simplified approach for the drag polar and the overall efficiency.
- The initial statement for a propeller drive is again the change in flight weight

$$\frac{dm}{\text{engl}} \; ÿ \; ÿ \; ÿ \, b_N N \, ÿ \, ÿ \, b_N W \, ÿ \, ÿ \, v \qquad \text{N total}$$

- Redesigned to improve engine efficiency and Force equilibria of unaccelerated flight expanded:

$$\text{engl} \; ÿ \; ÿ \; \frac{ÿ_{\text{total}}}{b_N W \, v g} \; ÿ \; ÿ \, dG \; ÿ \; ÿ \; \frac{ÿ_{\text{total}}}{b_N \; ÿ e \; ÿ \; g v \; G} \; ÿ \; ÿ \, dG \; ÿ \; ÿ \; \frac{ÿ_{\text{total}}}{b g_N} \cdot \frac{c_A^{3/2}}{c_W} \cdot \frac{1}{G^{3/2}} \cdot \sqrt{\frac{ÿÿ \, F}{2}} \cdot dG$$

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G Flight performance

4.3.4 Flight time with quadratic drag polar and PTL drive

- This gives you the flight duration

$$D = \int_{G_a}^{G_e} dt = \frac{\eta_{total}}{b_q} \cdot \frac{c_A^{3/2}}{c_W} \cdot \sqrt{2 F \rho} \cdot \left(\frac{1}{\sqrt{G_e}} - \frac{1}{\sqrt{G_a}} \right)$$

- From this it can be concluded that the flight time increases inversely proportionally $\sqrt{G/A}$ and that it is maximum when

if in point $A\; c_A^{3/2}/c_W$ is flown.

Flight duration with quadratic drag polar and PTL propulsion

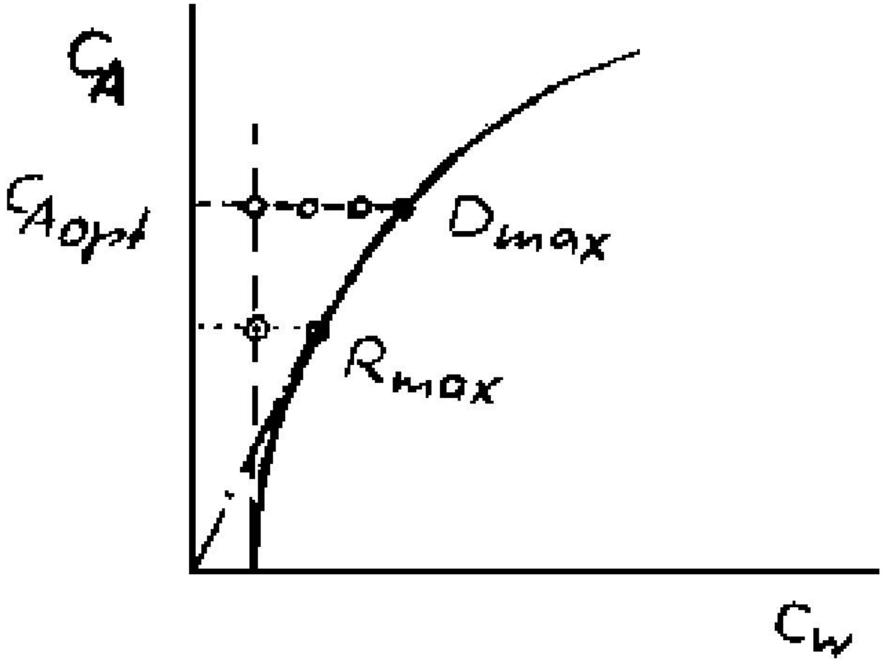
- If the derivative of the optimal condition is set to zero, we obtain

$$\frac{\partial\left(\frac{\sqrt{c_A^3}}{c_W}\right)}{\partial c_A} = 0$$

- This gives the optimal coefficient

$$c_{Aopt} = \sqrt{3 \cdot c_{W0} \cdot \pi \cdot \Lambda \cdot e}$$

where $c_{W0} = \frac{1}{3} c_{We}$ is.



- The speed is therefore

$$v_{opt} = \sqrt{\frac{2G}{\rho} \cdot \frac{1}{F} \cdot \frac{1}{\sqrt{3 \cdot c_{W0} \cdot \pi \cdot \Lambda \cdot e}}}$$

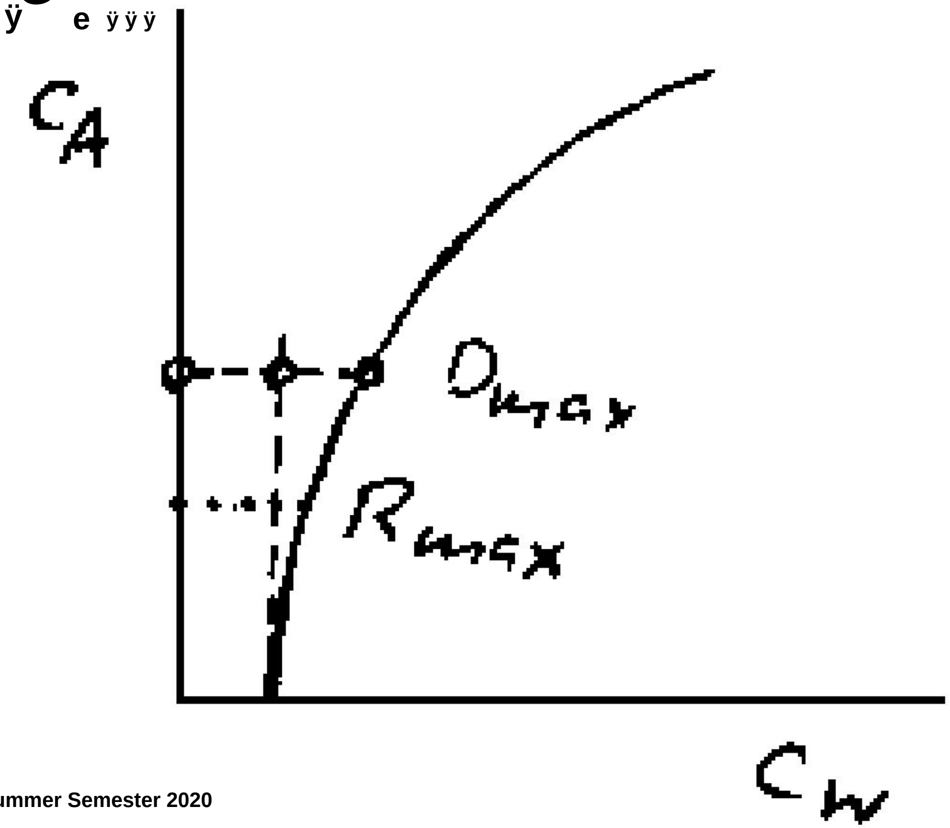
G Flight performance 4.3.5

Flight duration with quadratic drag polar and TL drive

- With the TL drive, the result is in the corresponding ÿ Way

$$D = \frac{1}{b\, g_s} \cdot \frac{c_A}{c_W} \cdot \ln\left(\frac{G_a}{G_e}\right)$$

- The flight duration is therefore independent of the flight altitude and is maximum at minimum cW/cA



G Flight performance

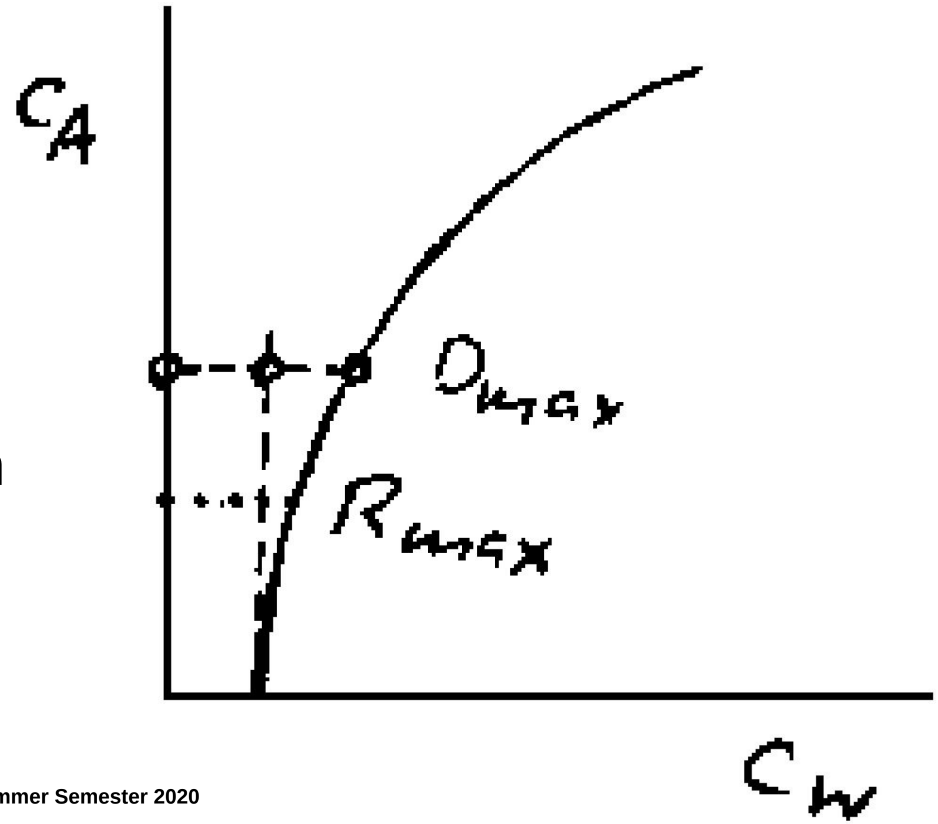
4.3.5 Flight time with quadratic drag polar and TL drive

- **This is given for the quadratic polar at**

$$c_{Aopt} \text{ ÿ } \sqrt{ce_{W0} \text{ ÿÿÿÿÿÿ}}$$

and the optimal speed

$$v_{opt} \text{ ÿ } \sqrt{\frac{2G}{\text{ÿ}} \text{ ÿ } \frac{}{F} \text{ ÿ } \frac{1}{\sqrt{ce_{W0} \text{ ÿÿÿÿÿ}}}}$$



- **This result is identical with that for the maximum range of the propeller aircraft.**

G Flight performance

4.3.6 Summary of range and flight time

	Optimal speed $V / V_{e_{min}}$			Aerodynamic quality E / E_{max} ÿ e_{min} / e			Optimal flight altitude		
drive	Ideal PTL ZTL	Ideal TL	Ideal	PTL	ZTL	Ideal TLS	Ideal PTL	ZTL	Ideal TLS
Max. Range	1 ÿ 3 1,32		$\sqrt[4]{\ }$ ÿ	1 ÿ		$\sqrt{3/4}$ ÿ 0.87	all	ÿ high	
Max. Flight duration	$1/\sqrt[4]{3}$ ÿ 0.76	ÿ 1		$\sqrt{3/4}$ ÿ 0.87	ÿ 1		deep	ÿ all	

- For the ZTL engine, solutions arise that lie between those of the PTL and the TL engines and depend on the bypass ratio.
- It is also noteworthy that a PTL-powered aircraft performs just like a glider in terms of its flight performance, but these conditions do not apply to horizontal flight, but to gliding flight.

G Flight performance

4.4 Range and flight time under wind influence

• The real operation of an aircraft takes place under meteorological conditions that are characterized by horizontal and vertical movements of the air. • The magnitude of the wind speed is so great, especially at altitudes close to the tropopause, that, for example, the flight time difference between the western and eastern passages of the Atlantic is usually at least one hour due to the jet stream if the flight route is optimized.

• The wind has no influence on the maximum flight time,

but on the range of a flight with a certain flight duration.

• For the typical range flight at constant altitude and constant lift coefficient, the following applies for a TL-powered ÿ Airplane ÿ

$$D = \frac{1}{bg} \cdot \frac{c_A}{c_W} \cdot \ln \left(\frac{G_a}{G_e} \right)$$

G Flight performance

4.4 Range and flight time under wind influence

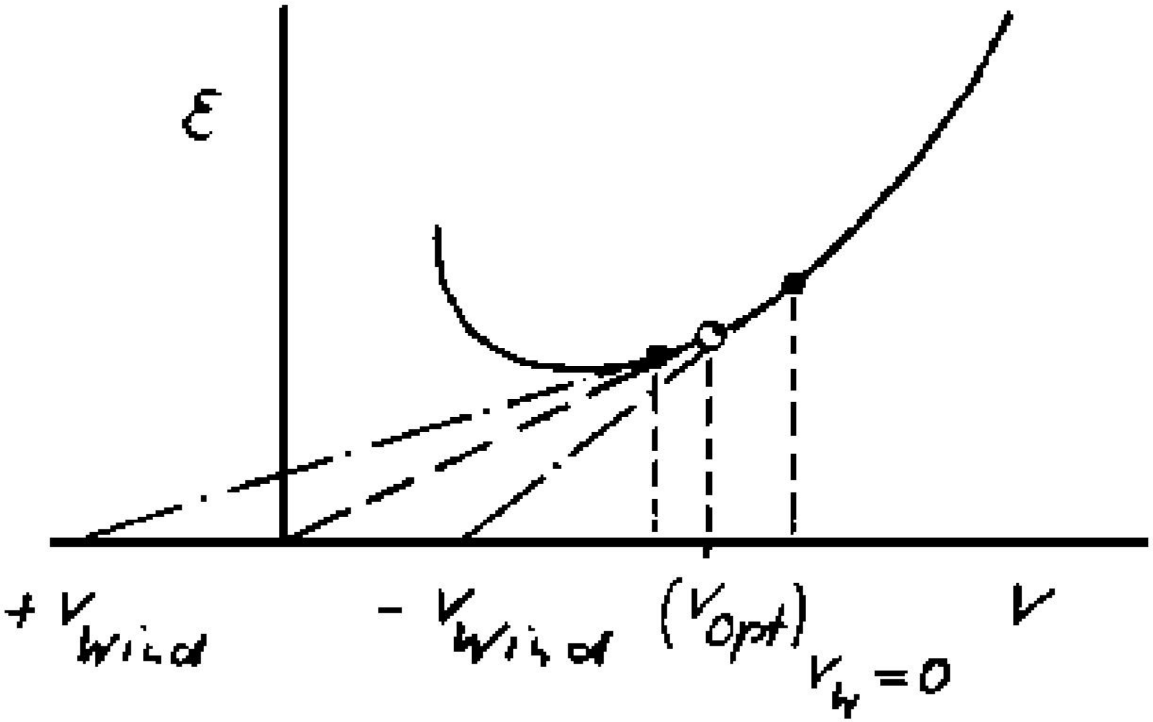
- **The range is extended by the additive wind term**

$$R \ \text{ÿ} \ \frac{2}{b g_{s}} \cdot \sqrt{\frac{2G}{\text{ÿ}} \cdot \frac{}{F}} \cdot \frac{\sqrt{c_A}}{c_W} \cdot \text{ÿ} \, 1 \ \sqrt{\frac{G_e}{G_a}} \, \text{ÿ ÿ ÿ} \quad v_{wind} \, D \, \text{ÿ}$$

- **The optimal flight speed is therefore a function of the wind speed.**

G Flight performance

4.4 Range and flight time under wind influence



- **The optimum speed can be obtained from the horizontal flight diagram using a tangent construction.**
- **For this purpose, the origin is increased by the magnitude of the wind speed. speed and the tangent is applied to the cW/cA curve.**

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G Flight performance

5 Dimensionless performance calculation

- The flight performance calculation (horizontal flight, climb, quasi-horizontal flight, turning flight) can be carried out for both the quadratic and any polar forms in dimensionless notation.

- In connection with the optimal flight conditions, some aspects of horizontal or quasi-horizontal flight for the parabolic polar are shown.

G Flight performance

5 Dimensionless performance calculation

• First, some dimensionless parameters are defined:

• **Load factor:** $n \text{ ÿ } \frac{A}{G}$

• **Dimensionless thrust:** $e \text{ ÿ } \frac{S_{E\,Max}}{G} \text{ ÿ } \frac{\text{current thrust}}{\text{minimum required thrust}}$

• **Dimensionless speed:** $u \text{ ÿ } \frac{v}{v_R}$

• **Maximum cA /cW:** $E_{Max} \text{ ÿ } \frac{1}{e_{min}} \text{ ÿ } \frac{1}{2 \cdot \sqrt{\frac{c}{W0 \text{ ÿÿÿÿ} e}}}$

• **Reference speed:** $v_R \text{ ÿ } \sqrt{\frac{2G}{\text{ÿ}} \cdot \frac{}{F}} \cdot \sqrt[4]{\frac{1}{c_{W0} \text{ ÿÿÿÿÿ } e}}$

G Flight performance

5 Dimensionless performance calculation

• The physical meaning of these values becomes apparent, if you take the polar

$$c_{W} \text{ ÿ } c_{W0} \text{ ÿ } c_{We}$$

by a quadratic approach and in Force dimensions converted

$$W \text{ ÿ } \frac{\text{ÿ}}{2} v^2 F c_{\text{ÿ}} \text{ ÿ ÿ } W_0 \text{ ÿ } \frac{c_A^2}{\text{ÿÿÿÿ e ÿ}} \text{ ÿ ÿÿ ÿ } \frac{\text{ÿ}}{2} C_{W_0} V^2 F \text{ ÿ ÿ } \frac{2 A^2}{\text{ÿÿÿÿÿÿ ÿ ev } F^2}$$

G Flight performance

5 Dimensionless performance calculation

- Using the abbreviations mentioned above, one then obtains

$$W \ddot{y} \frac{G}{2E_{Max}} \ddot{y}\, n\, 2 \ddot{y} \ddot{y} u \frac{2}{2u} \ddot{y}\ddot{y}\ddot{y} .$$

- The resistance has its minimum value at

$$W_{min} \ddot{y} \frac{G}{2 \cdot E_{Max}} ,$$

because the

factor – before the brackets cannot be zero and – in

the brackets will always be greater than or equal to zero.

The brackets are therefore $0 \ddot{y} u \ddot{y}^{2} \bullet \frac{2n}{2u}$ and thus for $u \ddot{y} n \sqrt{\ } .$

G Flight performance

5 Dimensionless performance calculation

- **Horizontal flight is characterized by n = 1. ÿ**
- **The resistance equation**

  The answer is:

$$ÿ\ ÿ \quad S \quad \frac{G}{2E_{Max}} \quad \frac{ÿ}{ÿ}\ ÿ\ ÿ u^{2} \quad \frac{1}{u^{2}\ ÿ} \quad 0$$

- **Solved for u,**

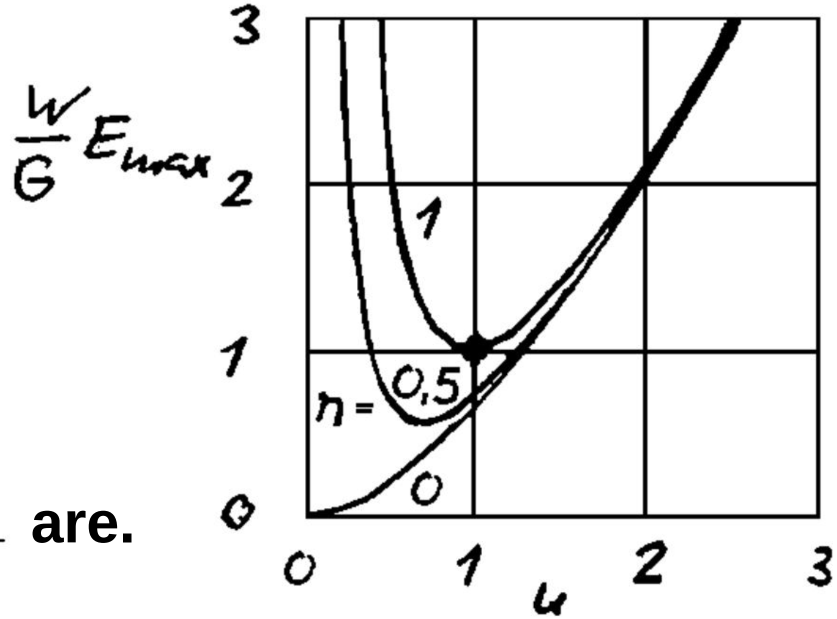
  a 4th order equation:

$$2 \text{ and } 2 \text{ to } 1\ 0\ ÿ\ ÿ\ ÿ \quad {}^{4}$$

- **Their two solutions are:**

$$u1 \quad ÿ\ ÿ\ ÿ \sqrt{zz1 \sqrt{\quad ^{2}}}$$

$$u2 \quad ÿ\ ÿ\ ÿ \sqrt{zz1 \sqrt{\quad ^{2}}}$$

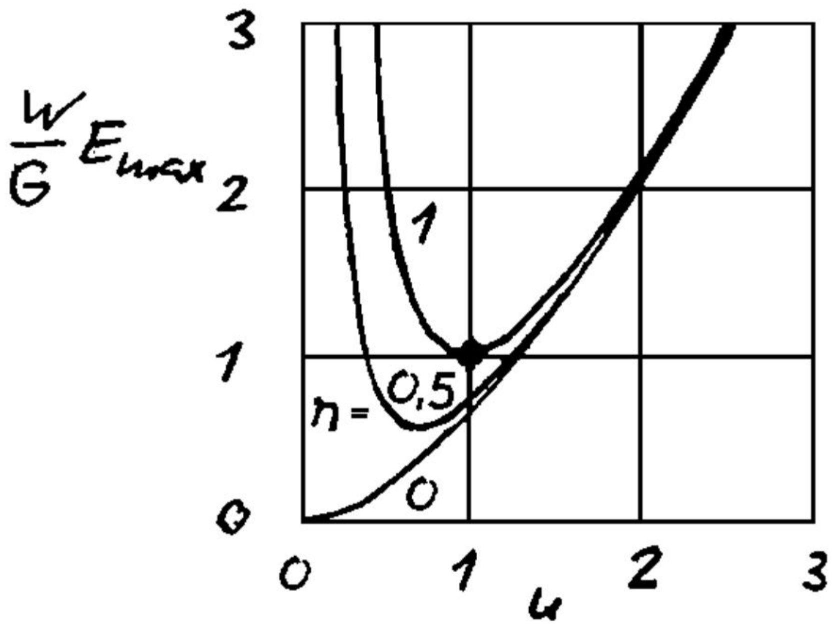


where u1 .u2 = 1 and $e\ ÿ\ \frac{SE_{Max}}{G}$ **are.**

G Flight performance

5 Dimensionless performance calculation

- **Of the two physically possible velocities, one is always greater and the other is always smaller than the velocity for minimum W/G.Emax .**

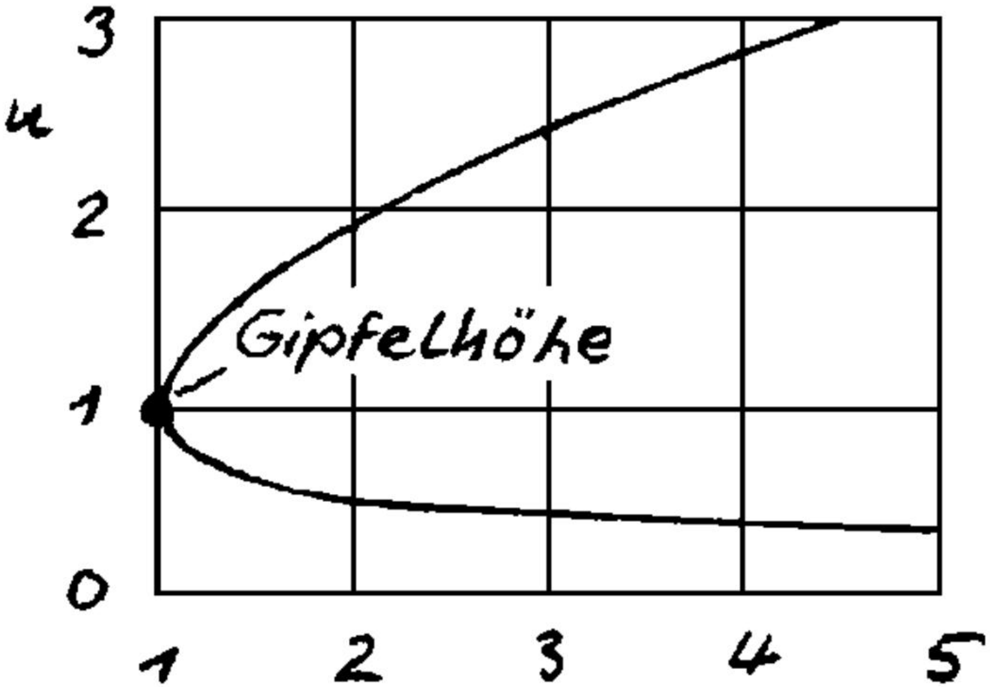


G Flight performance

5 Dimensionless performance calculation

- **From the solutions it is clear that a real solution at n = 1 only exists for z ÿ 1.**

- **The point (1, 1) therefore also indicates the summit height of the aircraft.**



z = W/G.Emax

G Flight performance

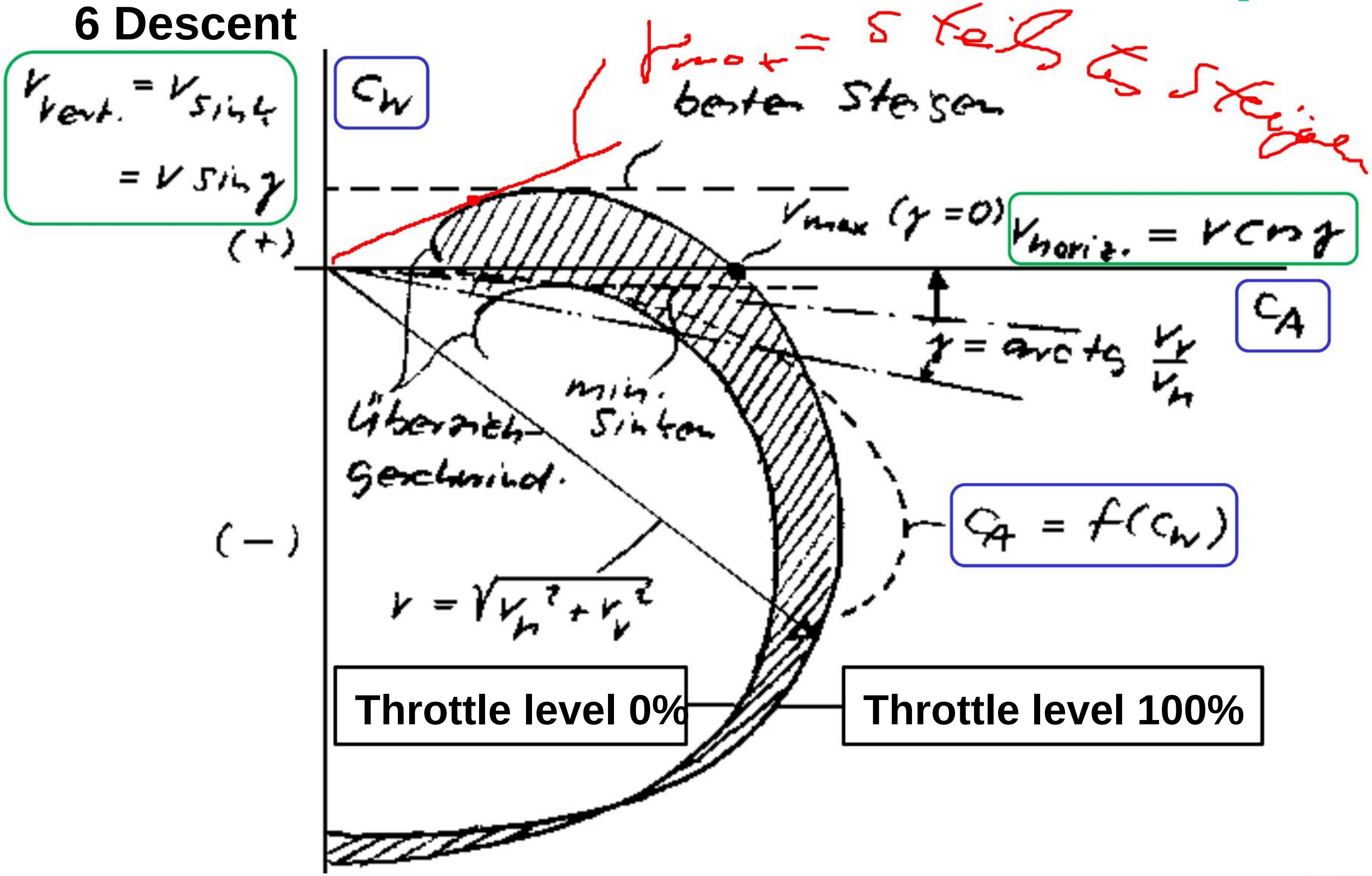
6 Descent

- **The consideration of the descent takes up the considerations of the climb, the thrust is varied between full thrust (throttle level 100%) and idle (throttle level 0%).**
- **In between, a wide spectrum of descents with variable thrust and variable speeds is possible.**
- **A propeller aircraft was chosen** for the following plot of the **vertical speed against the horizontal speed .**

- **Here the “windmilling drag” of the idling PTL drive is smaller than that at full load.**
- **Therefore, a higher dive speed is achieved at idle.**

G Flight performance

6 Descent



G Flight performance

6 Descent

- As already explained in the climb flight, descent rates can speeds up to the maximum design speed at not too large sliding angles below 15°

$$\frac{dH\ v}{dt\ G}\ \text{ÿ ÿ}\ —\ \text{ÿ}\,S_{existing}\quad S_{required}\ \text{ÿ}$$

and the orbit inclination angle

$$\tan\ g\ \text{ÿ}\ \frac{WS}{G}$$

which can also be determined by the energy height method is confirmed.

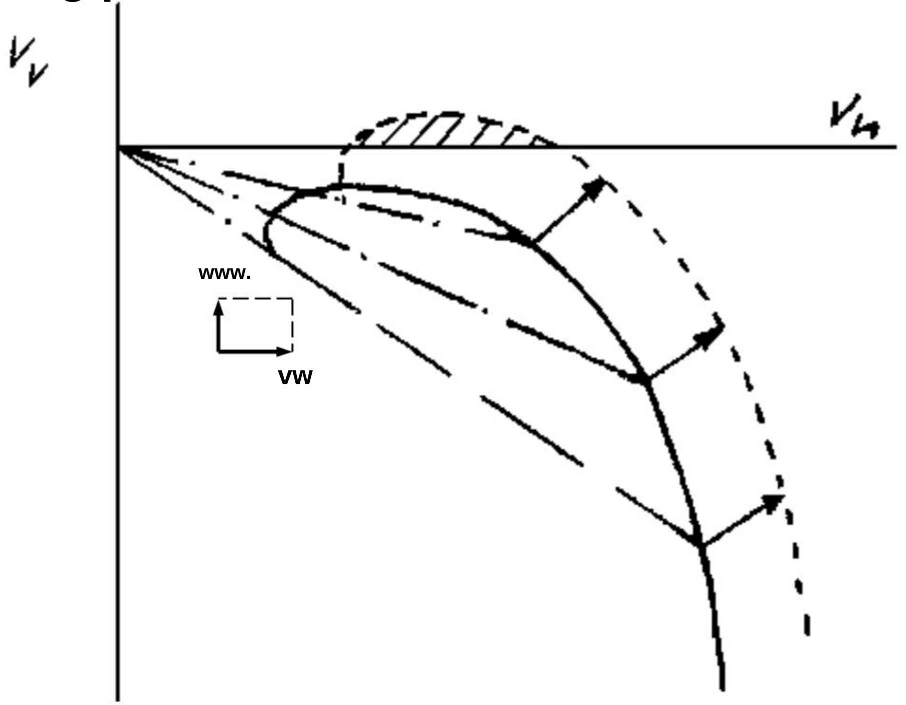
- Without thrust (power-off) the glide angle is naturally determined by e, because it is

$$\tan G\ \text{ÿ ÿ ÿ}\ \frac{S\ c}{G\ c}\quad \frac{W}{A}\quad e$$

G Flight performance

6 Descent

- For non-powered gliding (sailing), the Consideration of the descent under the influence of wind is particularly important.
- Here, with the help of vertical wind components, climb speeds can be achieved that make sustained gliding possible.



- The hatched area of the descent polar shown here indicates the flight range in which a climb is possible. • The horizontal offset results

from the horizontal wind component.

G Flight performance

6 Descent

- At emin the flattest glide, ie the greatest range in gliding flight, is achieved.
- During range flight this was the case for:

$$\frac{c_A^{3/2}}{c_W} \text{ ÿ } \text{Max.}$$

- The minimum sink rate occurs at a lower speed than at best gliding, since dH v

$$\frac{dH}{dt} \text{ ÿ ÿ ÿ ÿ ÿ } \frac{v}{G} S_{\text{required}} \quad \frac{N_{\text{required}}}{G}$$

becomes minimal when the required power for horizontal flight is at its minimum.

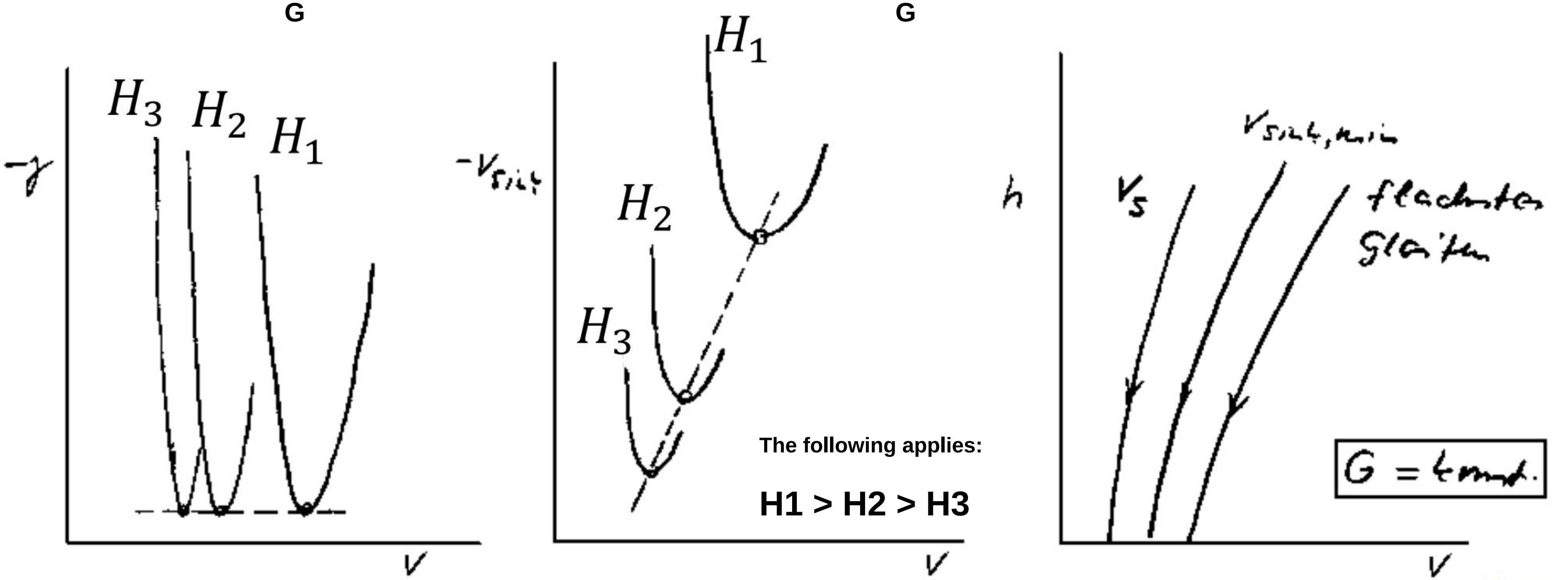
- For non-powered gliding (sailing), the Consideration of the descent under the influence of wind is particularly important.
- Vertical wind components (ww , thermals, slope and wave updrafts) enable climb speeds to be achieved which make sustained gliding possible.

G Flight performance

6 Descent

- **In dimensionless representation, assuming small Orbital inclination angle (sin g = g; cos g = 1) and vanishing thrust from the equations of motion**

ÿ g ÿ $\frac{W}{G}$ and v sin k ÿ ÿ ÿ g ÿ ÿ $\frac{W v}{G}$



G Flight performance

6 Descent

- As in the other cases, the optimal speeds increase with the flight altitude.

- In this case, the current range and flight time does not depend on fuel consumption, but rather:

$$\frac{dX1}{dH} \text{ ÿ } \frac{—}{G} \quad \text{and} \quad \frac{\text{engl}}{dH\ v} \text{ ÿ } \frac{1}{\text{sink}}$$

- The greatest range will be achieved at the minimum glide angle and the longest flight time with the minimum sink rate.

G Flight performance

6 Descent

- It is assumed that during the descent no fuel is consumed.
- This assumption is not entirely justified in practice, since usually a small amount of fuel is needed to maintain a flame in the combustion chamber.
- This is necessary for rapid availability of thrust when ending the descent or in emergencies. Restarting the engines would take too much time.

G Flight performance

6 Descent

- Using the quadratic polar, the unaccelerated Flight case in dimensionless form

$$W = \frac{G}{2 \cdot E_{Max}} \cdot \left(u^2 + \frac{1}{u^2} \right),$$

which results in the climbing angle

$$g = -\frac{W}{G} = -\frac{1}{2 \cdot E_{Max}} \cdot \left(u^2 + \frac{1}{u^2} \right)$$

and for the dimensionless sinking velocity

$$-\frac{v_{sink}}{v_R} = u \cdot G = \frac{1}{2 \cdot E_{Max}} \cdot \left(u^3 + \frac{1}{u} \right)$$

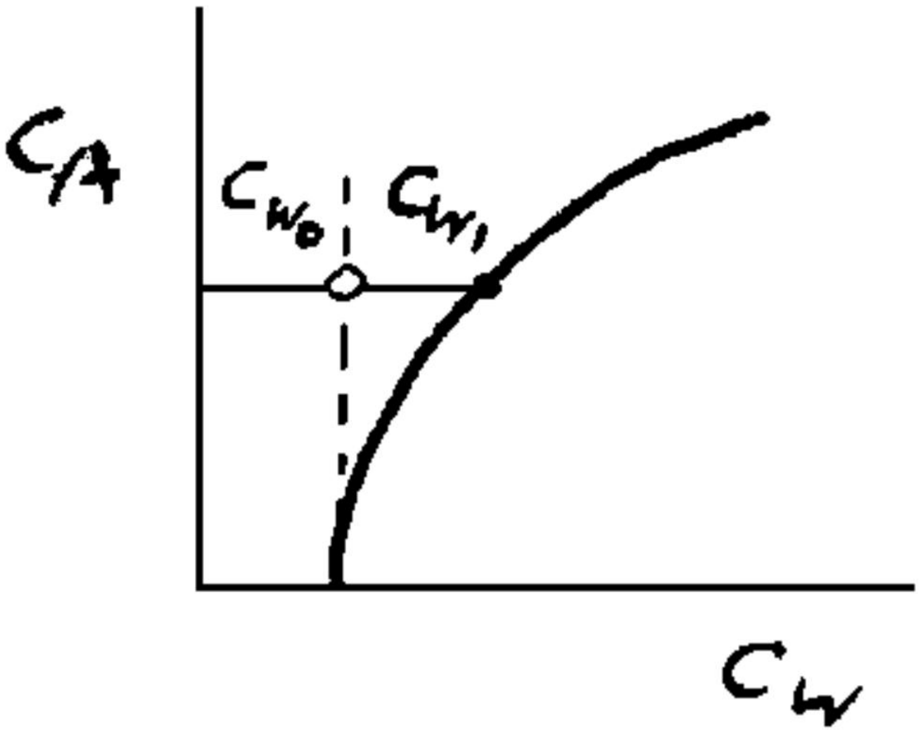
results.

G Flight performance

5 Dimensionless performance calculation

- **The flattest glide occurs - as is known - at emin, i.e. at**

$$\frac{c_{W_i}}{c_{W_0}} \text{ ÿ } 1 \quad \text{or at} \qquad \text{ÿ ÿ } G_{min} \quad \frac{1}{E_{Max}} \text{ ÿ } e_{min} \sim \sqrt{\frac{c_{W_0}}{\ddot{y}}} \qquad \text{instead of.}$$



G Flight performance

6 Descent

- Gliding with minimum sink rate results from

  the condition $\frac{ÿv_{sink}}{ÿu}$ ÿ 0 and leads to u ÿ opt $\frac{1}{\sqrt[4]{3}}$.

- This means the minimum

  Sinking speed: $ÿv_{sink} \sim \sqrt[4]{\frac{c_{W_0}}{ÿ^3}}$

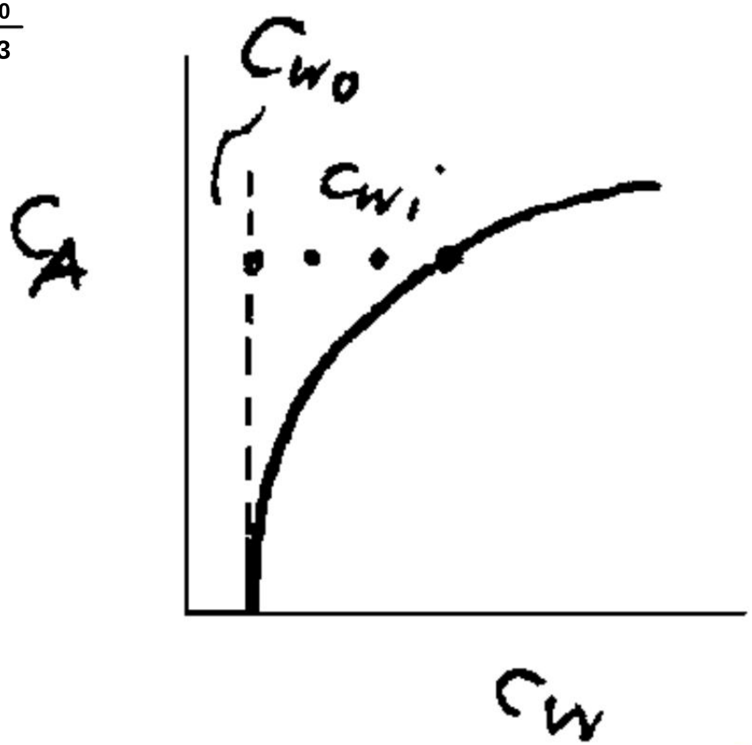
- The dimensionless speed for the

  minimum sinking speed is:

$$\frac{v_{sink}}{v_R} ÿ ÿ \frac{2}{3} \frac{3 \sqrt[4]{}}{E_{Max}}$$

- The resistance ratio is:

$$\frac{c_{W_i}}{c_{W_0}} ÿ 3.$$



G Flight performance

6 Descent

- As a conclusion, it can be stated that in the drive loose flight
  - the glide angle changes compared to zero drag ments than the rate of descent
  - the rate of descent is very sensitive to the Stretching is and
  - the speed of best glide is 32% higher than the speed of least sink.

G Flight performance

6 Descent

- **The current range and flight time are**

$$\frac{dX}{dH} \ddot{y}\ddot{y}\ddot{y} \frac{1}{G} \qquad \frac{2 \cdot u^{2}}{4\,1\ddot{y}\;u} \cdot E_{Max}$$

and

$$\frac{engl}{dH\;v} \ddot{y} \frac{1}{sink} \ddot{y} \frac{2 \cdot u\,E}{1\;\ddot{y}\;uv^{4}} \cdot \frac{Max}{R\,0} \cdot \sqrt{\frac{\ddot{y}}{\ddot{y}\;0}}$$

- v_{R0} **is the design speed at altitude zero.**

G Flight performance

6 Descent

- There are different descent strategies for unpowered descent from high altitudes: – Flight with a constant angle of attack – Flight with a constant speed.

- For a constant angle of attack, the range is

$$R \; ÿ \; ÿ_{H_a}^{H_e} \frac{dH\,u\,2}{G} \; ÿ \; \frac{2}{4\,1ÿ\;\;u} \; EHH_{Max} \; ÿ \;\; a \;\;\; e\,ÿ$$

- For optimal glide ratio conditions (u = 1) the following applies: e ÿ

$$REHH \quad max \; ÿ \; a$$

G Flight performance

6 Descent

- **For the flight duration with this strategy you get**

$$D \; ÿ \; \overset{H_e}{\underset{H_a}{ÿ}} \frac{dH}{v_{sink}} \; ÿ \; \frac{u\,E\,2}{1 \; ÿ \; uv^4} \cdot \frac{E_{Max}}{v_R} \cdot ÿ\,I H ÿ H_a ÿ \;\; ÿ \;\; e \; ÿ \; ÿ \qquad \text{with}\; I H ÿ \; ÿ \; ÿ_0^H \; ÿ \sqrt{\frac{ÿ}{ÿ_0}}$$

- **The dimensionless speed of the longest flight duration is:**

$$u \; ÿ_{opt} \;\; \frac{1}{\sqrt[4]{3}}$$

- **This gives the max. flight time:**

$$D_{Max} \; ÿ \; \frac{3}{2\,3\sqrt[4]{\;}} \cdot \frac{E_{Max}}{v_R} \cdot ÿ\,I H ÿ H_a ÿ \;\; ÿ \;\; e \; ÿ \; ÿ$$

G Flight performance

6 Descent

- The range is optimal at constant speed,

if

$$u = \frac{v}{v_R} = \frac{1}{\sqrt[4]{\left(\frac{\ddot{y}}{\ddot{y}_0}\right)_a \cdot \left(\frac{\ddot{y}}{\ddot{y}_0}\right)_e}}$$

- The flight duration is maximum for

$$\left(\frac{v}{v_R}\right)^2 \cdot \frac{\left(\frac{\ddot{y}}{\ddot{y}_0}\right)_a}{1 + \left(\frac{v}{v_R}\right)^4 \left(\frac{\ddot{y}}{\ddot{y}_0}\right)_a^2} \cdot \frac{\left(\frac{\ddot{y}}{\ddot{y}_0}\right)_a}{1 + \left(\frac{v}{v_R}\right)^4 \left(\frac{\ddot{y}}{\ddot{y}_0}\right)_e^2} \cdot \arctan \ddot{y} \frac{\left(\frac{v}{v_R}\right)^2 \left(\frac{\ddot{y}}{\ddot{y}_0}\right)_e \ddot{y}\ddot{y}\ddot{y} \left(\frac{\ddot{y}}{\ddot{y}_0}\right)_a}{1 + \left(\frac{v}{v_R}\right)^4 \left(\frac{\ddot{y}}{\ddot{y}_0}\right)_a \left(\frac{\ddot{y}}{\ddot{y}_0}\right)_e} = 0$$

G Flight performance

6 Descent

- The descent program with constant angle of attack results in greater ranges and flight times than the constant speed program.

- For a jet aircraft gliding from the altitude 40,000 ft to sea level, the difference is 11% in range and 8% in flight time.

G Flight performance

7 Curve flight

• Only stationary, horizontal turning flight is considered. •

The dimensioning of the thrust is, as previously considered, crucial for the aircraft's cruising and climbing performance.

• When turning, the thrust requirement also plays a major role in flight performance.

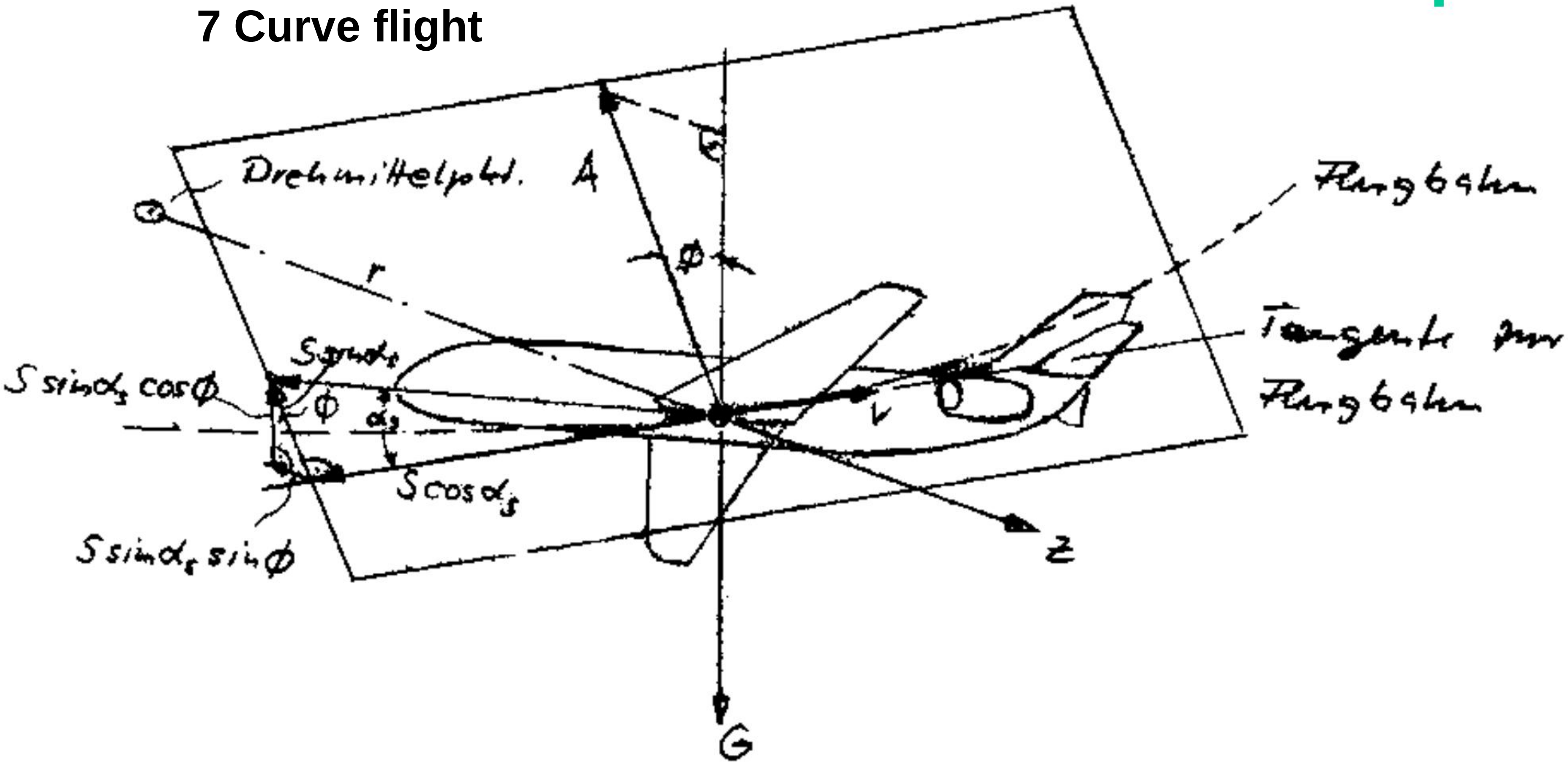
• Requirements for flight agility, especially in the area of high-performance aircraft, sometimes even determine the engine performance. But knowledge of the limitations of turning flight is also important for commercial use.

• For the performance mechanics considerations, the force equilibrium is assumed. •

Assuming that there is no yaw, i.e. that the surface through the longitudinal and vertical axes also includes the tangent to the flight path (coordinated turn), the following forces arise on the aircraft during turning flight:

G Flight performance

7 Curve flight



- **Assumption: No yaw, ie that the area through the longitudinal and vertical axes also includes the tangent to the flight path (coordinated turn)**

G Flight performance

7 Curve flight

- The following force equilibria apply:

ÿ V 0 ÿ A ÿ cos ÿ S ÿ sin ÿ ÿ ÿ ÿ ÿ ÿ ÿ ÿ ÿ cos ÿ G ÿ f ÿ ÿ ÿ ar ÿ s

ÿ H 0 ÿ A ÿ sin ÿ S ÿ sin ÿ ÿ ÿ ÿ ÿ ÿ ÿ ÿ sin ÿ Z ÿ f ÿ ÿ ar ÿ s

ÿ K Trajectory S cos W ÿ ÿ a ÿ ÿ ÿ s $\frac{G}{G}$ b

- The angle of inclination f follows from the centrifugal force Z and the triangle of mass forces and is

$$Z ÿ \frac{2 \text{ vm}}{r}$$

$$\tan f ÿ ÿ \frac{Z}{G} \frac{2 v}{gr}$$

- The load multiple is the quotient of the sum of the Buoyancy forces and weight:

$$n ÿ \frac{\text{AS sin ÿ ÿ as}}{G}$$

G Flight performance

7 Curve flight

- Inserting into the equation of the vertical force equilibrium gives:

$$n = \frac{1}{\cos e}$$

- For a coordinated turn, only the bank angle f (bank inclination angle) is responsible for the load on the aircraft. This tends towards infinity as the angle increases.

- With

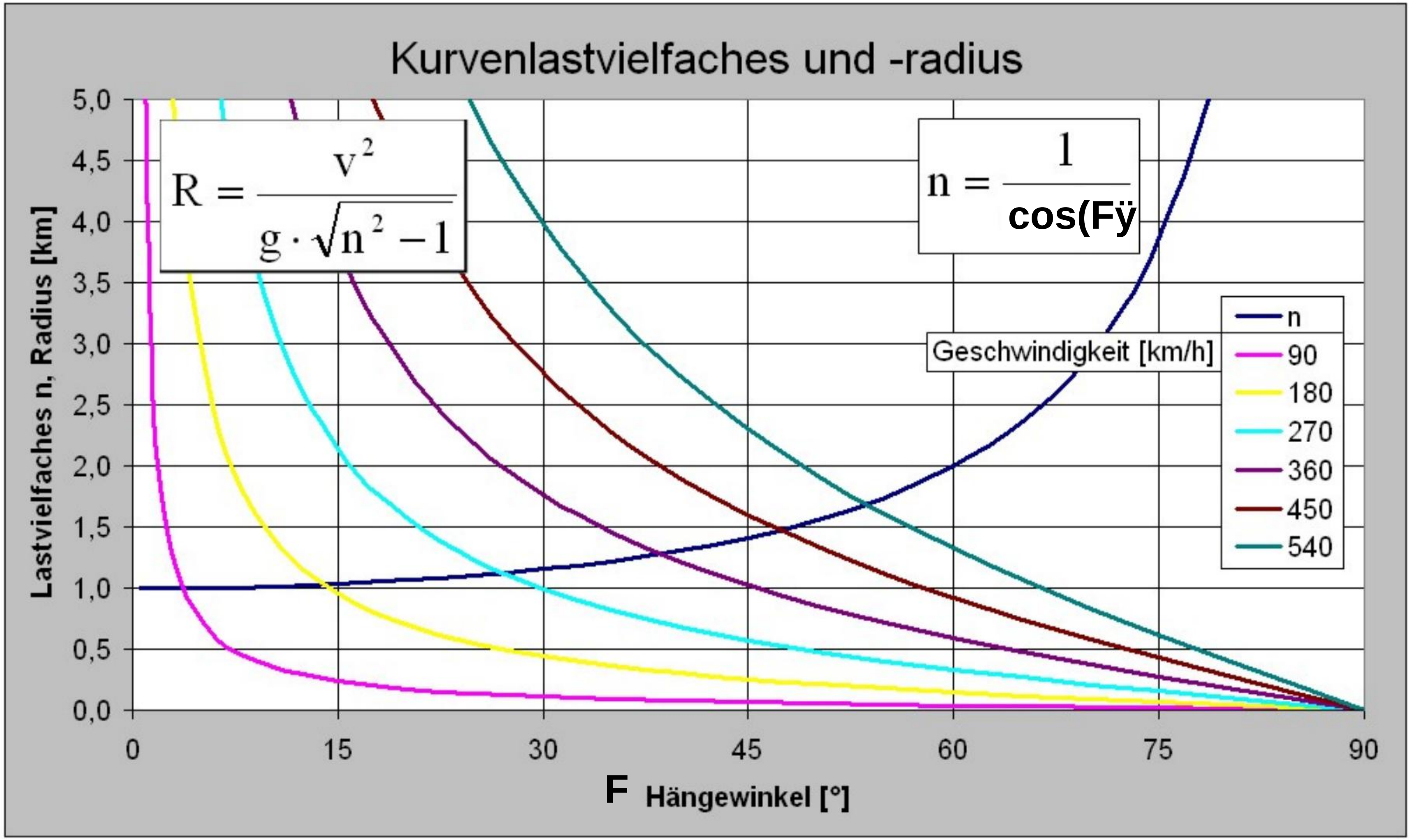
$$\cos f = \frac{1}{\sqrt{1 + \tan^2 e}}$$

- and the equation for the circle radius solved for the Centrifugal force of the circle radius follows:

$$r = \frac{v^2}{g \sqrt{n^2 - 1}}$$

G Flight performance

7 Curve flight



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G Flight performance

7 Curve flight

- Example:

  – safe load factor of 2.5 (see: vn diagram)

  – Travel speed of Ma = 0.8 at 11 km altitude • Result:

  – minimum circle radius: approx. 2.5 km

  – Transverse slope: approx. 66°

- For passengers of a commercial aircraft this would be Acceleration extremely uncomfortable

- In practice, much lower transverse inclinations are flown.

- This limitation does not apply to high-performance aircraft (aerobatics, military).

G Flight performance

7 Curve flight

- By introducing the lift coefficient into the equation for the vertical force equilibrium, the curve speed is obtained:

$$v ÿ \sqrt{\frac{2\ GÿS\ \cos\ \sin\ fÿ\ a_s\ ÿ}{c_A\ ÿÿÿÿ\ F\ \cos}}$$

- From the balance of forces along the flight path, taking into account taking a stationary circular flight at a constant altitude, for the

$S ÿ \frac{W}{\cos a_s}$ holds, the curve speed v is also:

$$v ÿ \sqrt{\frac{2G}{ÿ} ÿ \frac{}{F} ÿ \frac{1}{\cos_e} ÿ \frac{1}{ÿc_{A.W.}\ c\ \tan ÿ\ a_s\ ÿ}}$$

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G Flight performance

7 Curve flight

- It follows that the cornering speed for cross slopes angles of inclination that tend towards 90° tend towards infinity.
- However, this is not the case in practice, since the thrust required for this is not available and therefore the cornering speed is limited by the available thrust.

G Flight performance

7 Curve flight

- Determination of the curve radius by inserting this horizontal flight speed into the equation for the centrifugal force:

$$r \;\text{ÿ}\; \frac{2G}{g\ \text{Fÿÿÿnÿ f}} \;\text{ÿ}\; \frac{1}{\text{ÿcc tanÿ a A.W.} \quad {}_{s}\,\text{ÿ}}$$

- With maximum angle of attack, the smallest circle radius can be

tional flights.

- During cruise flight at high altitudes, this is limited by the Mach number

(Buffet). • At high angles of attack, the drag coefficient increases. With the available thrust, the equilibrium speed cannot be maintained.

- Similar to cruise flight, an investigation of the excess thrust in circling flight must also be carried out.

G Flight performance

7 Curve flight

- The minimum speed that can be achieved in stationary circular flight, which can be expressed as a function of the minimum speed in straight flight, is:

$$v_{s_e} \text{ ÿ } \frac{v_s}{\sqrt{\cos_e}}$$

- If the horizontal flight condition is removed, you can fly smaller curve radii.
- In extreme cases, the spiral trajectory of a roll upwards or downwards (spiral dive) can also be imagined as a circular flight.
- The most important parameter is again the existing thrust.

G Flight performance

7 Curve flight

- Assuming a constant thrust with speed (TL engine), the following results:

$$\frac{ÿr}{ÿf} ÿ 0$$

for the maximum bank angle

$$\cos f\, ÿ_{Max} \quad \frac{S ÿ sin_s}{G}$$

- This means that the bank angle is purely dependent on the orbit inclination and the thrust.

G Flight performance

7 Curve flight

- **Derivation of the descent and climb speed in curve flight from the general energy consideration:**

$$w \;\text{ÿ ÿ}\; \frac{dH}{\text{engl}} \; v\,SS \; \frac{\text{ÿ ÿ existing} \qquad \text{required ÿ}}{G}$$

- **There is a rate of descent when there is a thrust deficit and a rate of climb when there is a thrust excess.**

- **Assuming that the flight path is not more than 15° from the Horizontal deviates, results with the horizontal thrust form qc change sufficiently accurate** $S \;\text{ÿ}\; \frac{W}{\cos\ a_S} \;\text{ÿ}\; \frac{F\ \ W}{\cos\ a_S}$

- **… and for sink rate:**

$$\text{ÿ ÿ}\; \frac{dH}{\text{engl}} \quad v_{\text{sink}} \;\text{ÿ ÿ ÿ}\; \frac{v}{G} \;\text{ÿ ÿ ÿ}\; S \; \frac{\text{ÿ}\; C_W\, F^{2}}{2 \cos\ a_S} \;\text{ÿ}$$

G Flight performance

7 Curve flight

- The basis for the previous considerations was the Equilibrium of forces in vertical and horizontal directions as well as in the direction of the flight path.
- The circular flight performance of the aircraft can of course also be determined using the equations of motion

1) xv cos ÿ ÿ 0

2) yv sin ÿ ÿ 0

3) SW 0

4) A sin e $\frac{G}{G}$ v ÿÿ ÿ 0

5) A cos e ÿÿ G 0

6) G b S g ÿÿ 0

where x and y span the horizontal plane and ÿ is the yaw angle and its change over time is the yaw angle speed or the change in flight direction.

G Flight performance

7 Curve flight

• In addition to the three known force equilibrium relationships (3, 4 and 5), two geometric relationships (equations 1 and 2) are added, which include the yaw angle and the change in weight over time known from the range consideration (equation 6). • In addition, the yaw angle velocity has been introduced as

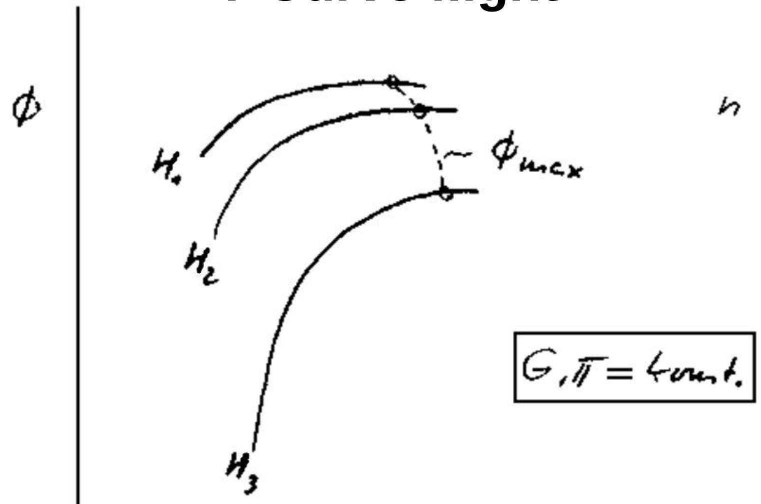
the ratio of the orbital velocity and the circle radius.

• For the sake of simplicity, it is assumed that the thrust tangential to the flight path and that the inertial forces in the flight path direction are neglected. With this assumption, the load factor

$$n = A/G.$$

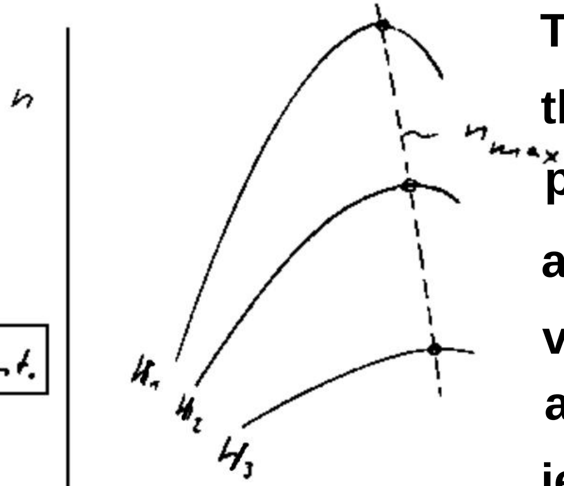
G Flight performance

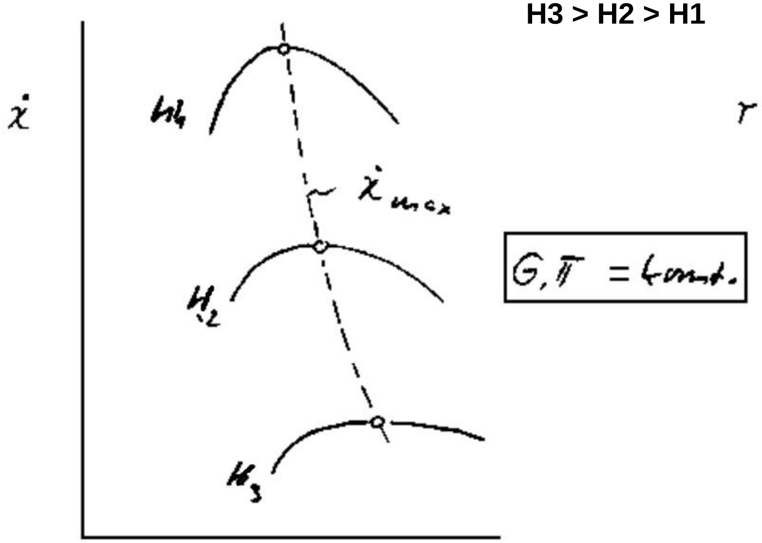
7 Curve flight

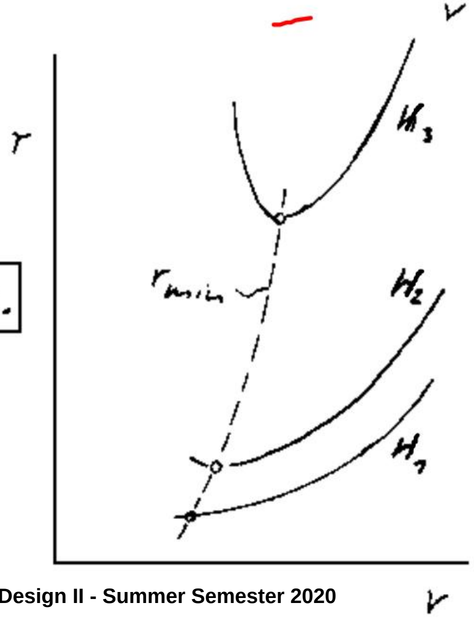


The following applies:

H3 > H2 > H1



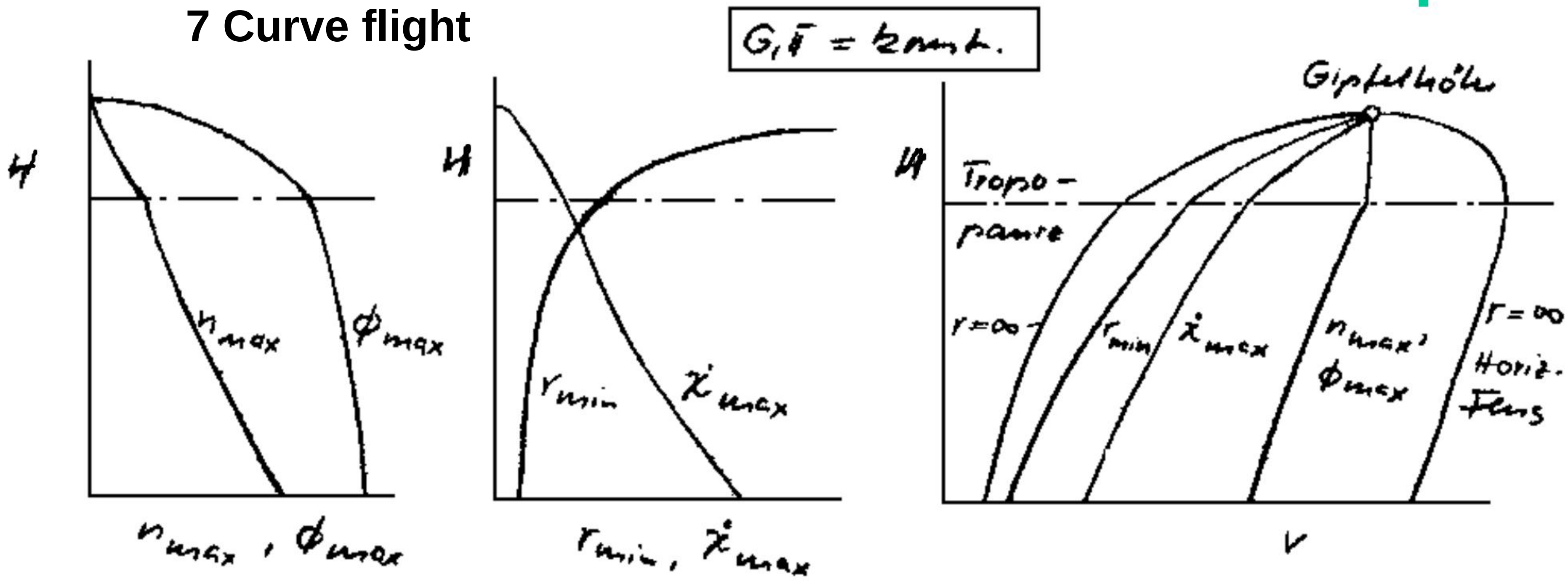




Typical solutions for the turning flight parameters bank angle, angular velocity, circle radius and load factor for a jet airliner characterize its maneuverability and show significant optima.

G Flight performance

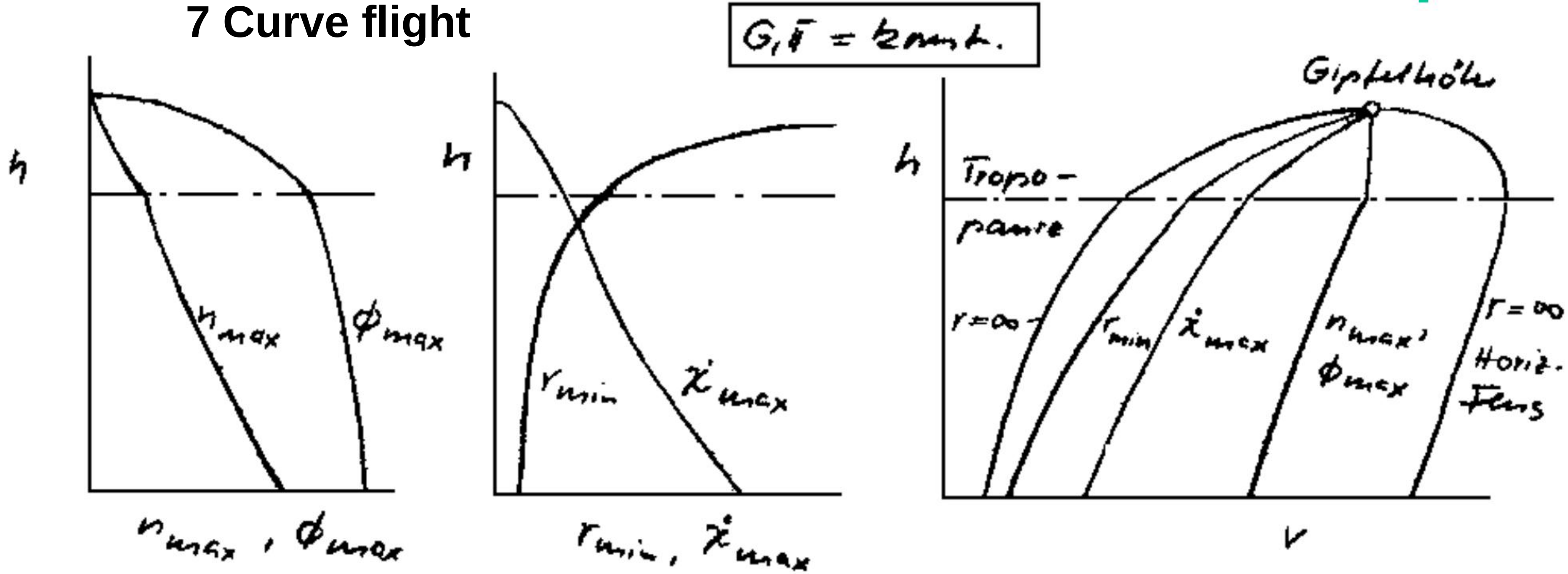
7 Curve flight



- **The optimum values entered in the flight range diagram (Hv diagram) show that the flight speed for the minimum circle radius is close to the minimum horizontal flight speed and is less than the speed of the largest direction change speed.**

G Flight performance

7 Curve flight



- All speeds increase with altitude and are in the
    - The maxima for

the load factor, the bank angle and the yaw rate decrease with increasing altitude, whereas the minimum circle radius increases and becomes infinite when the peak height is reached. At this point, circular flight is no longer possible!

G Flight performance

7 Curve flight

- Simple relationships can also be derived for turning flight using a quadratic drag polar.
- With the dimensionless parameters

$$n ÿ \frac{A}{G}, \quad e ÿ \frac{SE_{Max}}{G} \quad \text{as well as with} \quad u ÿ \frac{v}{v_R}$$

$$\text{and} \quad E_{Max} ÿ \frac{1}{2 ÿ \sqrt{\frac{c_{W0}}{ÿÿÿÿ \quad e}}} \quad \text{as well as} \quad v_R ÿ \sqrt{\frac{2G}{ÿ} ÿ \frac{}{F}} ÿ \sqrt[4]{\frac{1}{c_{W_0} \quad ÿÿÿÿÿ \quad e}}$$

is obtained from the rewritten equations of motion

3, 4 and 5:

$$2 \text{ to } ÿÿÿ \quad ^2 \quad \frac{n^2}{u^2} \quad 0;\ n \sin ÿ u f ÿ ÿ \quad \frac{ÿÿ v_R}{G} ÿ \quad 0;\ n \cos ÿ 1 ÿ 0 ÿ$$

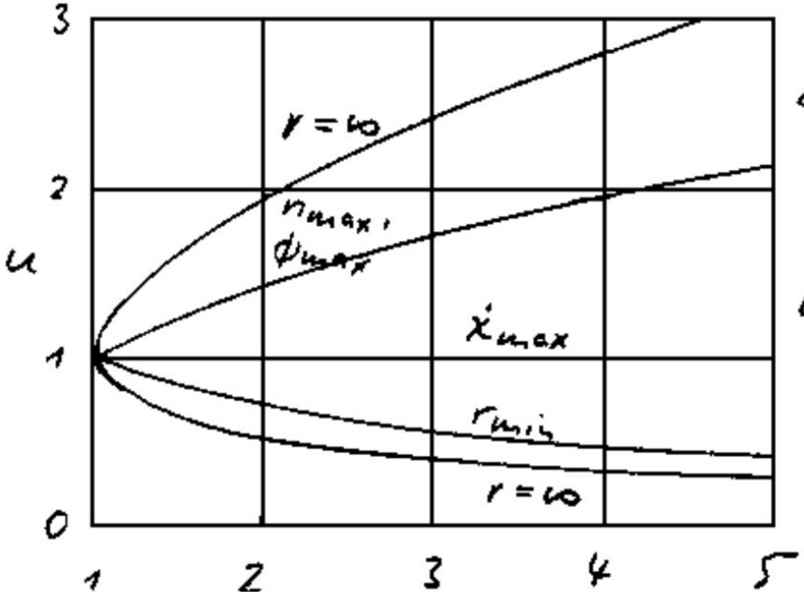
G Flight performance

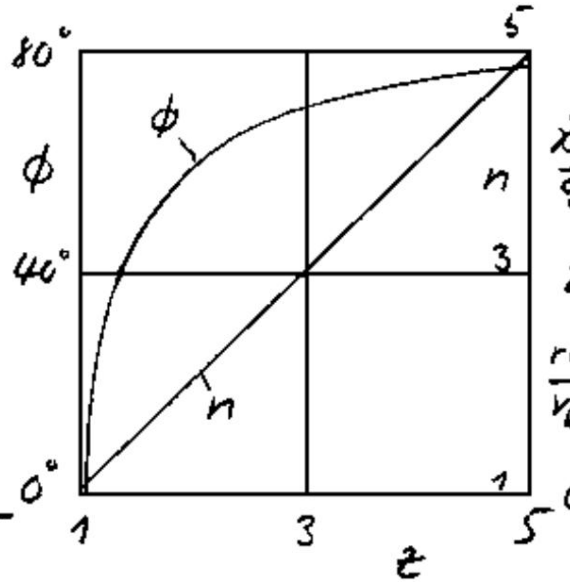
7 Curve flight

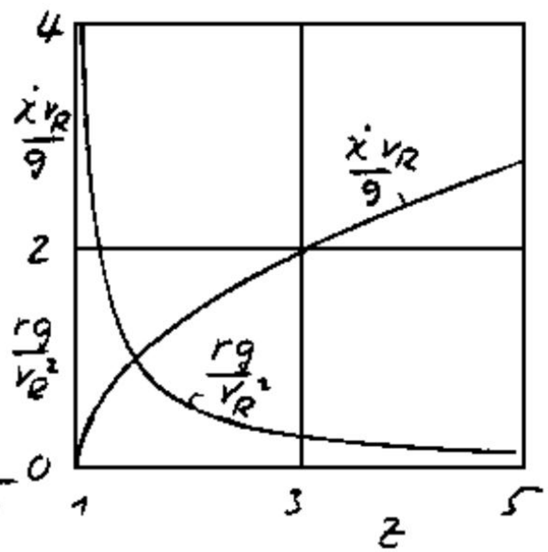
- **Load factor:** $now \quad \sqrt{2 \; \ddot{y}\,\ddot{y}\, to^{2}} \qquad \frac{v}{v_R} \sqrt{2 \cdot \frac{SE}{G}_{Max} \cdot \frac{v^2}{v_R^2}}$

- **Curve radius:** $\frac{rg}{v_R^2} \; \ddot{y} \; \frac{1}{\sqrt{n1^2}}$

- **Roll angle:** $\cos_{f} \ddot{y} \; \frac{1}{u\,2\,to \sqrt{\quad^{2}}}$







z ÿ ÿ SEG ( ) / $_{Max}$

G Flight performance

7 Turning

flight • You can see that the thrust must be greater than the resistance to be able to fly a circle with a limited radius and that both the roll angle and the load factor > 1.

• In general, the following must apply: $2 \; \frac{1}{2}$ ÿÿÿ z u ÿÿ $\frac{1}{2u}$ ÿ ÿÿ

• With n = 1 and f = 0 for a radius tending towards infinity, the solution for horizontal flight is

$2 \; \frac{1}{2}$ ÿÿÿ z u ÿÿ $\frac{1}{2u}$ ÿ ÿÿ

G Flight performance

7 Curve flight

- It is also clear that the wing loading has a major influence on the circle radius: the maneuverability of the aircraft increases as the wing loading decreases. • The optimal

wing loading is once again shown to be a compromise between the cruise requirement, whereby it should be as high as possible due to a minimum glide ratio, a minimum wing weight and the lowest gust load multiple, and the take-off and landing requirement as well as the requirement for sufficient maneuverability, whereby a small value is preferable.

G Flight performance

7 Curve flight

- **This representation can be used to determine the following Summarize phenomena:**
  - **Since z > 1 for any altitude below the summit altitude, the velocities must be for $n_{max} > \ddot{y}_{max} > r_{max}$.**
  - **All speeds are limited by the horizontal flight speed.**
  - **As thrust increases, cornering performance improves.**
  - **For a given thrust and weight, cornering performance is greatest near the ground.**

G Flight performance

7 Curve flight

- With increasing thrust or decreasing glide ratio or weight, the load factor occurring during optimal turning becomes larger.

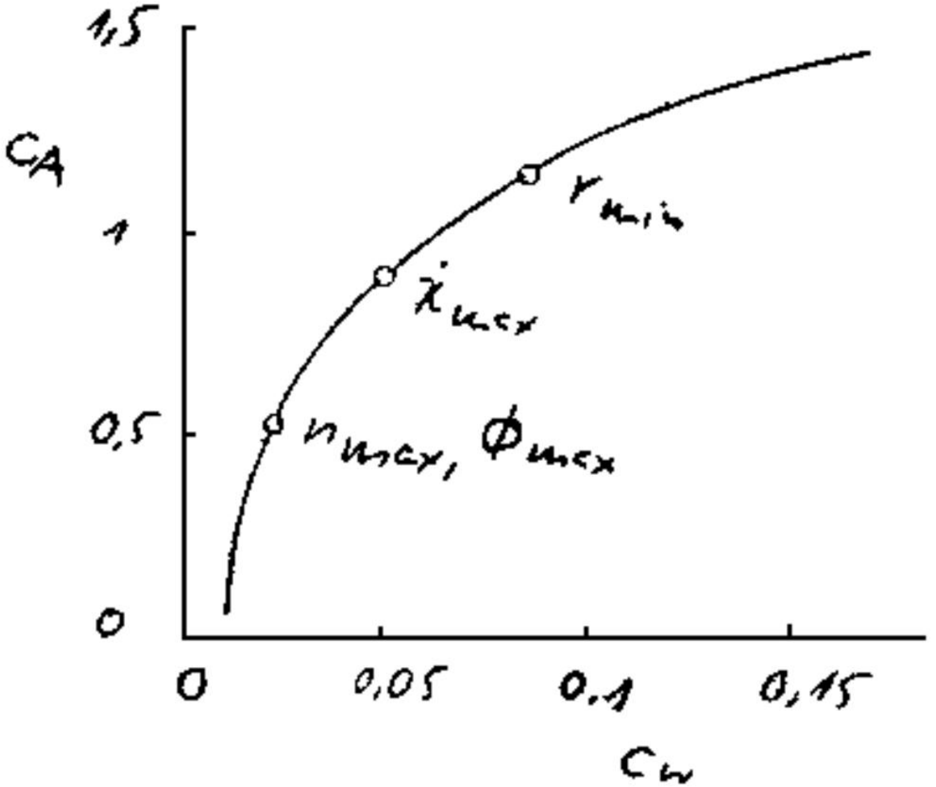
- For the minimum circle radius n ÿ 2 $\sqrt{}$ and the optimal Roll angle may become impossible due to structural overload.

- The curve with the maximum load factor is calculated with the Lift coefficient for the lowest glide ratio, the turn with the smallest turn radius is flown with a large lift coefficient that increases with increasing thrust.

- The quadratic polar is for the large Lift coefficients may be too inaccurate.

G Flight performance

7 Curve flight

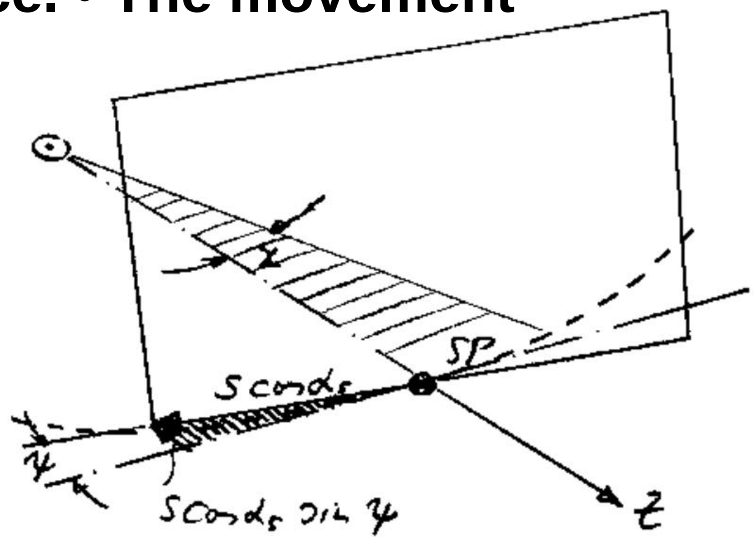


- Maneuverability can be increased by using high-lift aids (e.g. flaps, slats) can be improved, since the minimum circle radius increases proportionally to the square of the minimum speed.

G Flight performance

7 Curve flight

- It should be noted that another way to reduce the turning radius is to perform uncoordinated turns. • If the aircraft pushes its nose towards the center
of the turn, an additional thrust component results to compensate for the centrifugal force. • The movement of the aircraft changes.



equation in radial direction and Flight path direction and it results

tan ÿf ÿ aÿÿsin ÿÿcos $\frac{v^2}{gr}$ ÿ $\frac{S}{G}$ s

as well as r ÿ $\frac{2\ \text{GS sin cos ÿÿ ÿÿÿ}_s\ \text{e ÿ}}{\text{g ÿÿ F sin ÿÿ}_A\ \text{ÿ ÿ f ÿ ÿ}\frac{S}{G}\text{cos cos sin ÿÿ ÿÿ ÿ}_s}$ ÿ ÿ