Welcome to the course

Aircraft design II

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Overview

- The design task for an aircraft is also determined by the Performance requirements defined.
- These are defined differently for each area of application.
- For commercial aircraft, the focus is on the range with a given payload and the cruising speed.
- Also, dimensions such as climb rate and initial cruising altitude can be specified as design requirements.
- For combat and aerobatic aircraft, there are also For example, the maneuverability, which can be described by the achievable circle radius, as well as the acceleration capability are on the list of requirements.

G Flight performance
Overview

G Flight performance
Overview

- For commercial aircraft, the payload range diagram the most informative flight performance diagram.
- This was determined in the first design phase with simplifying assumptions.
- The take-off, climb and descent parts of the Flight mission simply neglected
- The following assumptions were made:
- During flight at design altitude, the aerodynamic conditions are constant (c_A , c_A /c_W), since only the average weight G_m was taken into account.
- However, a real flight mission consists of climb, cruise and descent flight phases, each with different flight conditions (altitude, speed, weight, etc.).

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Overview

- If the climb is taken into account when calculating the flight weight, it must be determined how an "optimal" climb should be carried out or which climb speed should "optimally" be flown.
- This raises the question of the optimal criteria for fastest climb, greatest climb angle and minimum consumption.
- For an accurate analysis of the cruise flight, the real conditions must be taken into account.
- This includes considering what an optimal cruise flight strategy should look like for different flight missions (longest flight duration, shortest flight time or flight with minimal fuel consumption) and what consequences this has for the calculation.

G Flight performance

Overview

- To calculate a real flight mission, the descent
  The proportion of the flight path can be included in the distance calculation and the advantage of low fuel consumption during unpowered gliding can be used. Here, too, the question of optimality criteria arises.
- Finally, this chapter also covers
  Flight performance statements for circling flight are derived and discussed. In particular, the increased thrust requirement in stationary turning flight is calculated.

G Flight performance

1 Equations of motion

- The flight performance of an aircraft can be determined in a general manner if its external forces (thrust, drag, lift, weight) are analyzed in the stationary, ie unaccelerated, state of horizontal, descent, climb and curve flight.
- The dynamic performance calculation takes into account accelerations perpendicular and parallel to the flight path in addition

to the forces mentioned. • For aircraft with subsonic speeds in the fligh In the altitude range, the dynamic effects (e.g. interception) can be captured by corrections to the stationary calculation.

- For supersonic aircraft, stationary observation alone is not sufficient ÿ dynamic performance analysis is necessary!

G Flight performance

1 Equations of motion

• For example, a subsonic aircraft can only reach the theoretical ceiling altitude (altitude at which the steady-state climb rate becomes zero) asymptotically, unless the weight is reduced (eg by jettisoning

ballast). • An SST with Ma = 2 to 3 at high altitudes can be achieved by Interception can easily bring the aircraft to a height greater than the stationary peak, whereby kinetic energy is partially converted into potential energy.

• Such a maneuver cannot, of course, be performed by a stationary

different perspectives.

• The differential equations of motion must be be evaluated.

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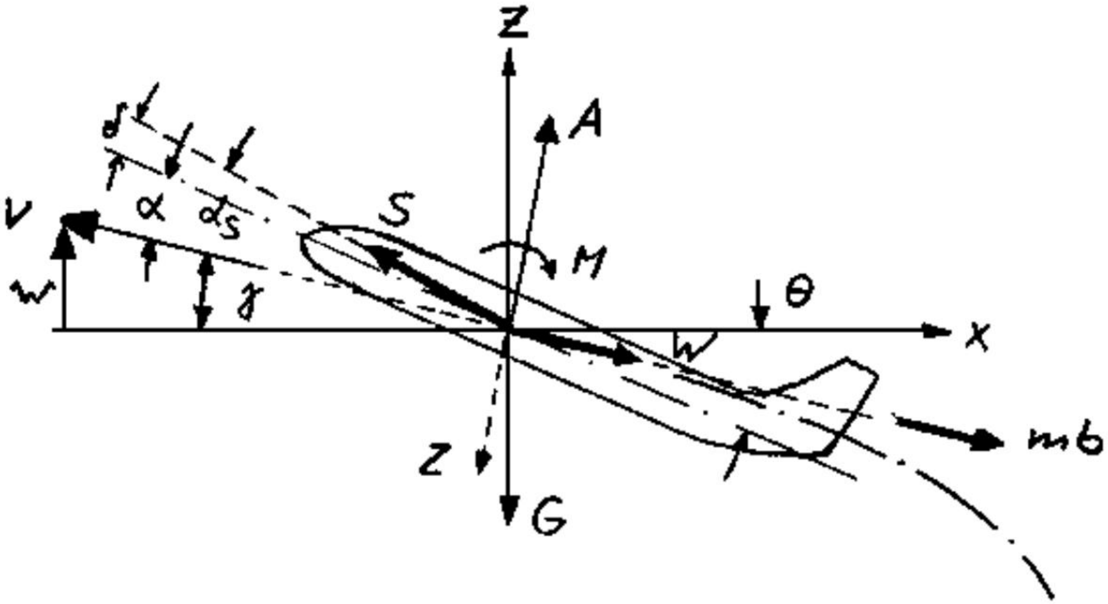
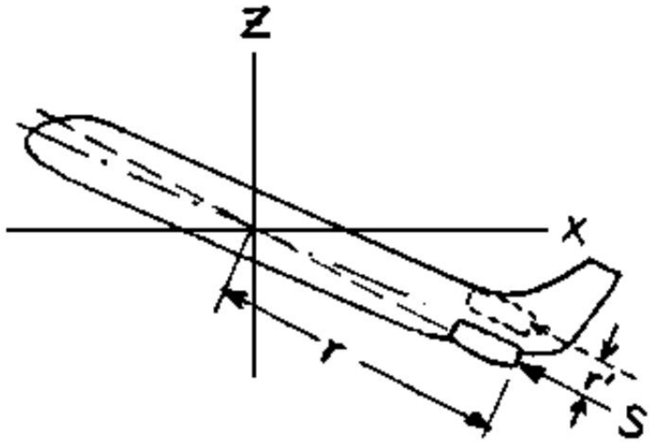
1 Equations of motion



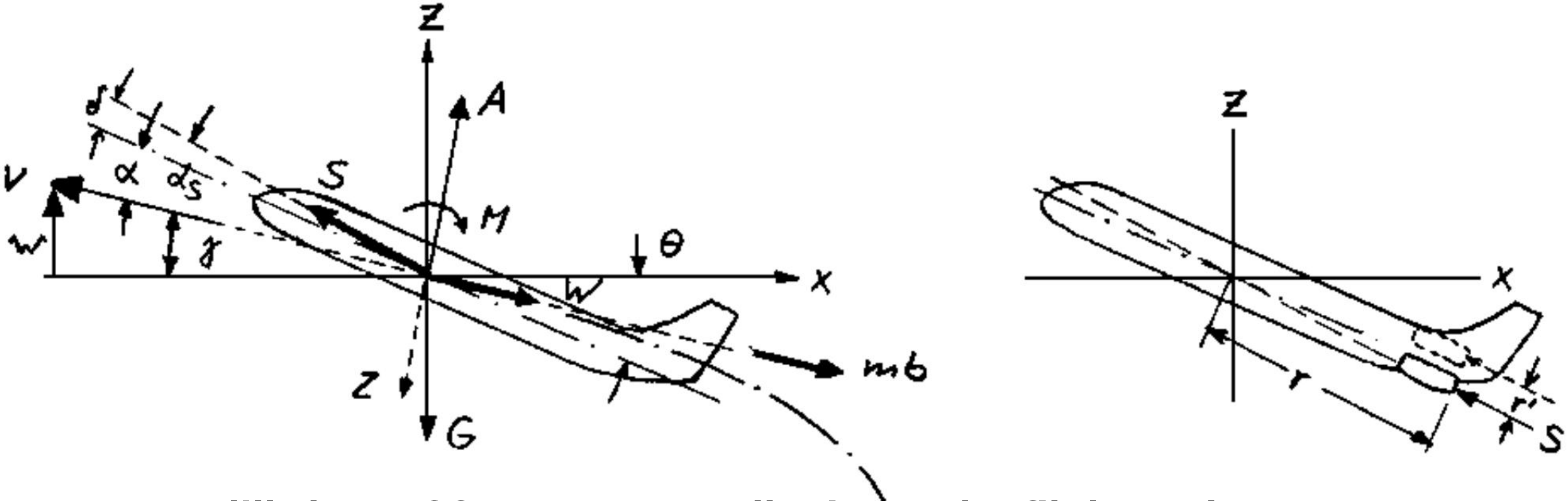


g Angle of inclination of the flight path (climbing angle) a

Angle of attack q Angle of the longitudinal axis of the fuselage (q = a + g) aS Angle between thrust vector and flight path d Angle between thrust vector and longitudinal axis of the fuselage (d = aS - a) r Horizontal distance of the thrust application point from the center of gravity r´ Vertical distance of the thrust application point from the center of gravity

G Flight performance

1 Equations of motion



- **Equilibrium of forces perpendicular to the flight path:**

AG cos ÿ ÿ ÿ ÿ S sin ÿ ÿ ÿ ÿ ÿ ÿ ÿ ÿ vÿ Z ÿ g ÿ a ÿ ÿ ÿ s gÿ $\frac{G}{G}$ b_n $\frac{G}{G}$

- **Parallel to the flight path:**

S cos ÿ ÿ a ÿ ÿ ÿ s ÿ G sin W g ÿ ÿ ÿ ÿ ÿ ÿ v $\frac{G}{G}$ b_t $\frac{G}{G}$ ÿ

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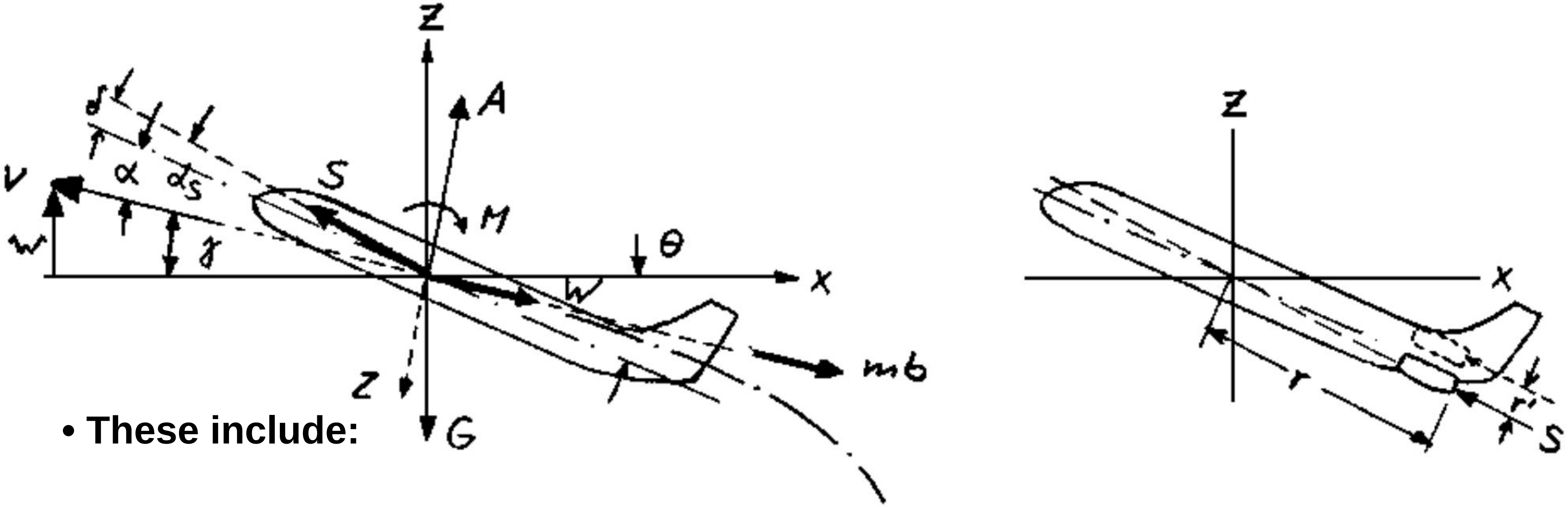
1 Equations of motion



• These include:

- dvb ÿ —— the tangential acceleration and dt

- b_n ÿ ÿ $\frac{v^2}{R}$ vÿÿ ÿ vÿ gÿ which points to the center of curvature of the

Path at distance R directed normal component with ÿ ÿ $\frac{v}{R}$

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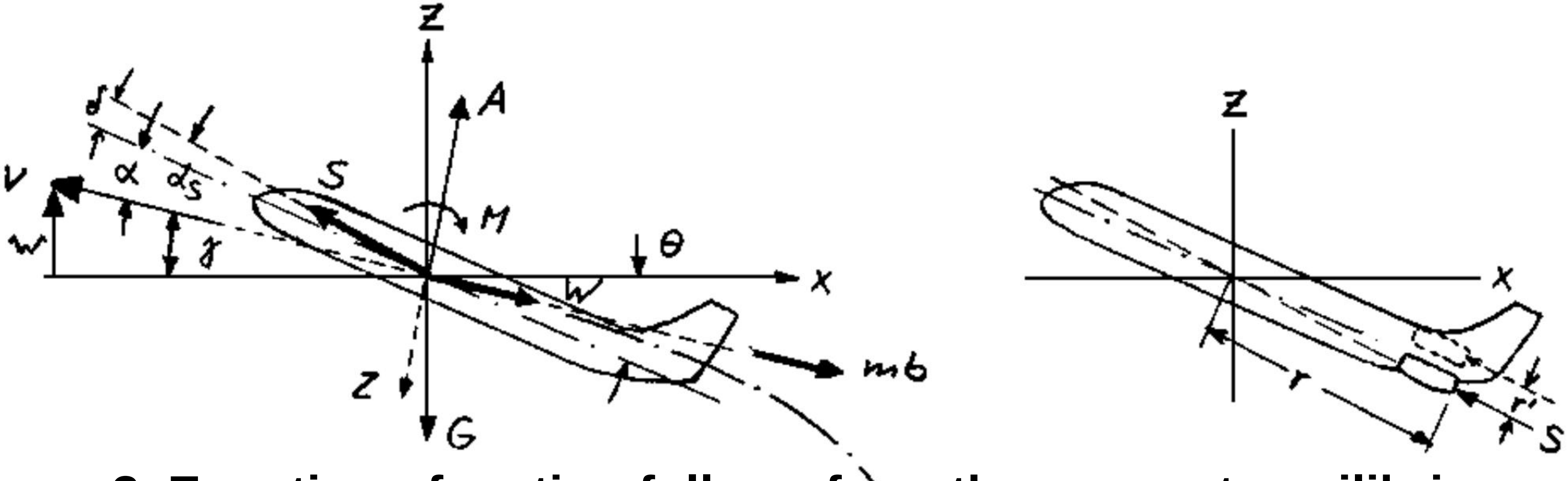
G Flight performance

1 Equations of motion



• 3. Equation of motion follows from the moment equilibrium:

$$\frac{G}{G}\, i_y^{2}\, M M \ddot{y} q\ \ddot{y}\ \ddot{y}\quad TW\ \ddot{y}$$

with the radius of gyration iy , This is derived from the relation I = m. iy ^{2} (same moment of inertia as Airplane).

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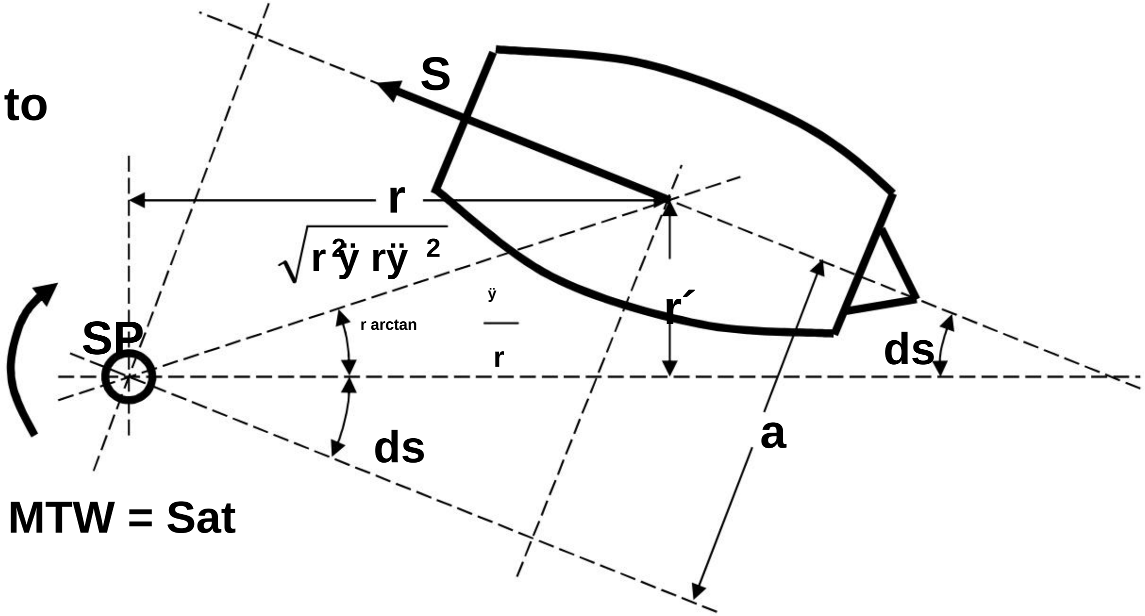
G Flight performance

1 Equations of motion

- The pitching moment due to thrust depends on the installation position of the engine.



MTW = Sat

MS rr sin arctan ÿ ÿ √ÿ ÿ ÿ ÿ ÿ ÿ 2 ÿ ÿ ÿ ÿ ÿ ÿ ÿ ÿ d 2 r/r ÿ s ÿ ÿ
TW

ÿ ÿ S r 1 ÿ ÿ √ ÿ ÿ ÿ ÿ r/r ÿ ÿ ÿ 2 ÿ ÿ ÿ ÿ ÿ ÿ ÿ ÿ ÿ ÿ ÿ ÿ ÿ d sin arctan r/r ÿ s ÿ ÿ

G Flight performance

1 Equations of motion

- This generally describes the longitudinal motion of the aircraft if engine thrust, aerodynamic forces, weight and moments of inertia are known.
- Assuming that all maneuvers considered below cause only slow rotations around the transverse axis and thus the contributions of the aerodynamic damping and the moment of inertia are negligible, the moment equation is no longer applicable. ÿ Only the two force equilibria remain.

G Flight performance

1 Equations of motion

- It applies to maneuvers with slow turns (quasi-stationary):

AG cos ÿÿÿÿÿ S sin ÿ ÿ a G / G

or with coefficients

ÿ / 2 vc² FG cos A ÿ ÿ g ÿ ÿ g G v S ÿ sin ÿÿÿ a 0 ÿ s

as well as

Sÿcos as ÿ G ÿsin g ÿ W ÿ ÿ g G v

or.

S ÿ cos G ÿ sin s ÿ ÿ ÿ g ÿ ÿ / 2 vc² F W ÿÿÿÿ G / G 0

G Flight performance

1 Equations of motion

- From the vertical force equilibrium, the Horizontal flight condition $G\ c\ q\ F\ \bar{A}$

for the lift coefficient $c_A \;ÿ\; \bar{c}_A \cos ÿgÿÿÿÿ \frac{v}{G}\, ÿ\, ÿ\, a \frac{S}{G}\sin$

- The term in brackets corresponds to the load factor n
- The thrust requirement at this equilibrium cA is given by the second equation of motion to

$$\frac{S}{G}\;ÿ\;\frac{1}{\cos ÿa_S}\;ÿ\;\frac{c_W}{\bar{c}_A}\;ÿ\;g\sin\;ÿ\;ÿ\;\frac{\dot{v}}{G}$$

- The quotient $\frac{c_W}{\bar{c}_A}$ corresponds to the aerodynamic efficiency of the aircraft.

G Flight performance

2 Stationary horizontal flight

- For stationary flight conditions, the accelerations and the resulting moment disappear.

- For stationary horizontal flight (cruise condition) the track inclination angle is also zero. • This means that the flight

speed is calculated from the vertical

Balance of power

$$v ÿ \sqrt{\frac{2 \; G S ÿÿ ÿ sin \quad ÿ \; ÿÿ_S}{ÿÿ \; ÿ \; c_A F}}$$

- and for the required thrust from the horizontal balance

$$S ÿ \frac{ÿÿ \; ÿ v^2 c_W F \quad ÿ}{2 \; cos \; ÿ \quad ÿ \quad ÿS}$$

G Flight performance

2 Stationary horizontal flight

- Inserting the speed into the thrust equation gives first

$$S \text{ ÿ } \frac{c_W G S \sin \text{ÿ ÿ ÿ ÿ } a \quad \text{ÿ ÿÿ}_S}{c_A \cos \text{ÿ } a \quad \text{ÿ} \quad _S\text{ÿ}}$$

- and after dividing by G and solving we get

$$\frac{S}{G} \text{ ÿ } \frac{c_W \text{ ÿÿÿ ÿ} 1 \frac{S}{G} \sin \text{ÿ} a \,_S \text{ÿÿ}}{c_A \text{ ÿ } \cos \text{ÿ} a \,_S \text{ÿ}}$$

$$\frac{S}{G} \text{ ÿ } \frac{c_W}{c_A \text{ ÿ } \cos \text{ÿ} a \,_S \text{ÿ}_W \text{ ÿ ÿ} c \quad \sin \text{ÿ} a \,_S \text{ÿ}}$$

G Flight performance 2

Stationary horizontal flight •

In the speed range of stationary horizontal flight cW . sin(aS) is very small compared to cA and cA . cos(aS) • Therefore

$$\frac{S}{G} \text{ ÿ ÿ } \frac{c_W}{c_A} \frac{1}{\cos \text{ÿ} a_S \text{ ÿ}}$$

as well as

$$v \text{ ÿ } \sqrt{\frac{2G}{\text{ÿ}} \cdot \frac{}{F} \cdot \frac{1}{Ac}}$$

• Assuming that the thrust S acts in the direction of the orbit and thus cos(aS) = 1, the well-known relationship for the

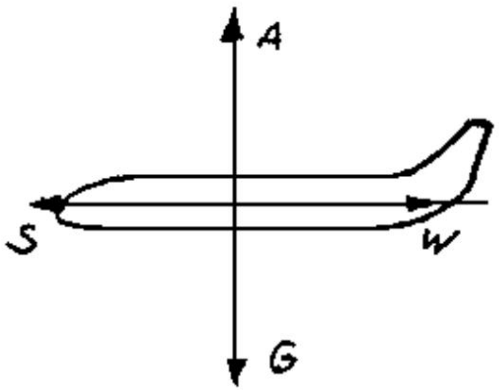
Thrust requirement:

$$\frac{S}{G} \text{ ÿ ÿ ÿ } \frac{c_W}{c_A}$$

G Flight performance

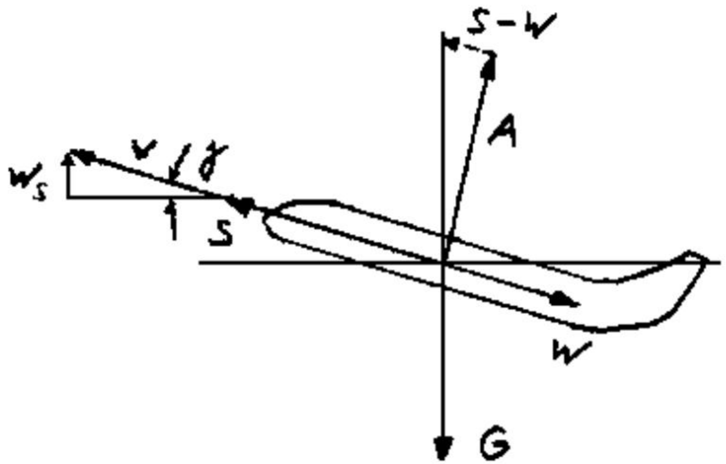
2 Stationary horizontal flight

Horizontal flight



$$\left.\begin{array}{l} A = G \\ S = W \end{array}\right\} \rightarrow v$$

Climb



$$\sin\gamma = \frac{S-W}{G} = \frac{w_s}{v}$$

$$w_s = \frac{S-W}{G}\,v$$

Thrust excess compared to Horizontal flight

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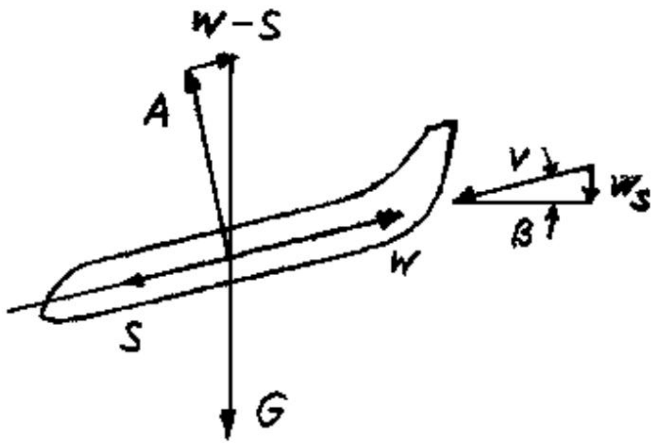
2 Stationary horizontal flight

descent

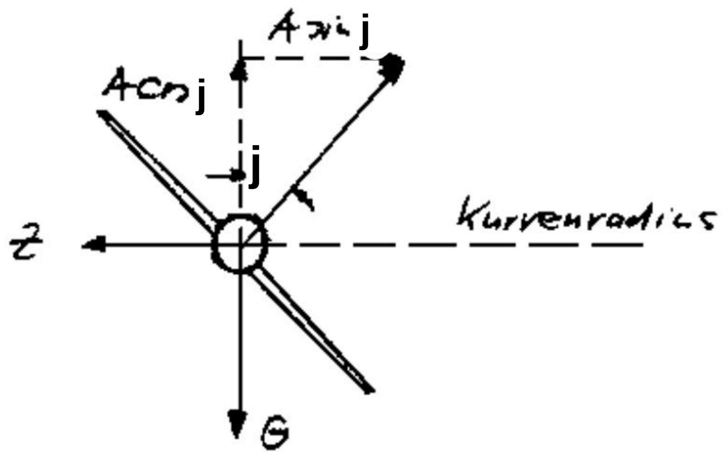


$$\sin\beta = \frac{W-S}{G} = \frac{w_s}{v}$$

$$w_s = \frac{W-S}{G}v$$

Thrust reduction compared to
Horizontal flight

Curving flight



$$A \sin j = Z = \frac{G}{g}\frac{v^2}{R}$$

$$R = \frac{G/F}{\frac{\rho}{2} g c_A \sin j}$$

G Flight performance

2.1 Horizontal flight diagram for jet aircraft • A

frequently used tool for performance analysis is the horizontal flight diagram

- Required thrust and available thrust are calculated here as a function of flight speed.
- With
  - a square resistance polar and • the settlement:

  “required thrust equals the resistance” results in:

ÿ ÿ S ÿ ÿ ÿ ÿ ÿ ÿ ÿ ÿ ÿ G required

$$\frac{c_W}{c_A} \qquad \frac{c_{W0}}{c_A} \qquad \frac{c_{W_p}}{c_A} \qquad \frac{c_{W_i}}{c_A}$$

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2.1 Horizontal flight diagram for jet aircraft

- **After inserting the aerodynamic coefficients and assuming that the average flight weight differs from the take-off weight by the fuel factor a, one obtains**

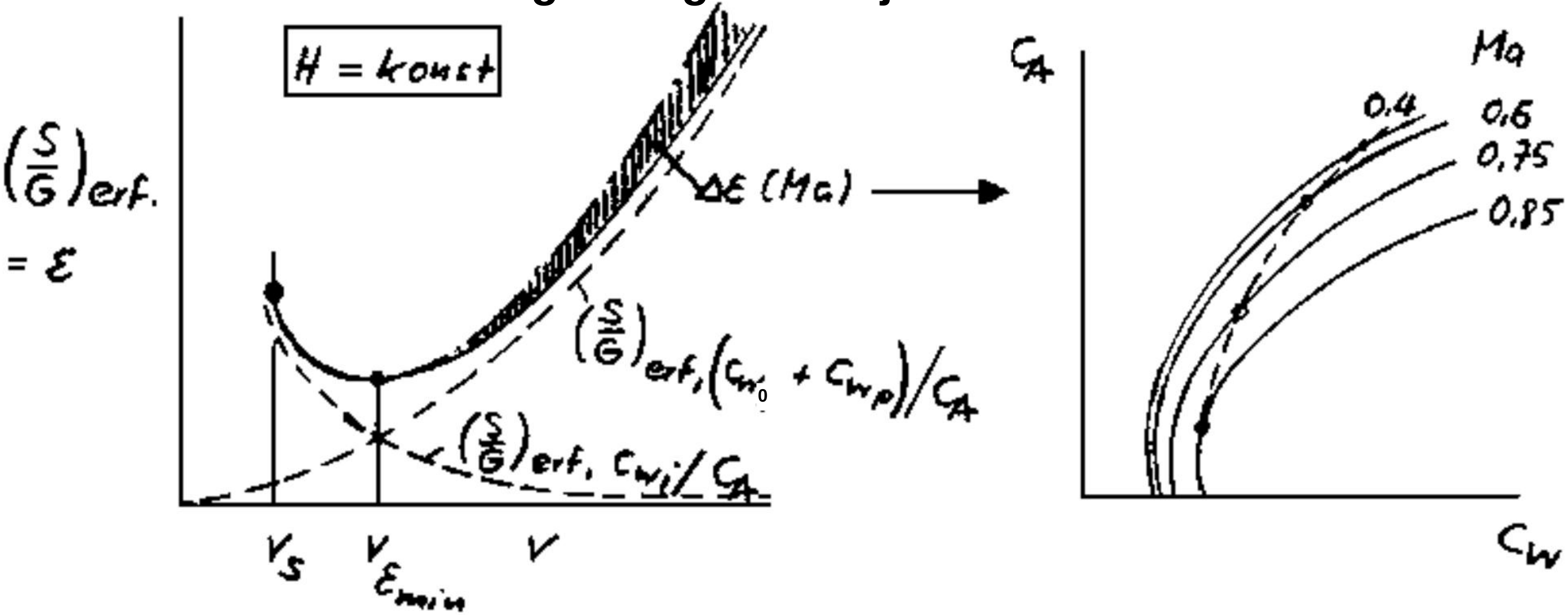
$$\frac{S_{required}}{G_A} ÿÿ\ a \left(\frac{ÿ \cdot v^2}{2 \cdot a\, G\, f\, A\, s} ÿ \frac{c_{W_p} \cdot ÿ \cdot v^2}{2 \cdot a\, G F\, A} ÿ \frac{2 \cdot a\, G F\, A}{ÿ\ ÿÿÿ\ ÿ\ ev\ ÿ \cdot {}^2} \right)$$

$$e_S\ ÿ\ c_{W_0} \cdot F$$

- **This equation represents weight-related thrust requirement for the incompressible case.**

G Flight performance

2.1 Horizontal flight diagram for jet aircraft



- **The individual resistance components are in their speed**
  - **The hatched area represents the increase**

due to the wave resistance determined using the compressible polars shown on the right.

G Flight performance

2.1 Horizontal flight diagram for jet aircraft

- The available thrust depends on the flight altitude, the speed, bypass ratio and throttle level.
- A fairly good approximation for the thrust map was already presented in Chapter C:

$$\frac{S}{S_0}\ \text{ÿÿ}\ D\ \text{ÿ}\left(\frac{\text{ÿ}}{\text{ÿ}_0}\right)\text{ÿ}^{\text{ÿÿ}}\ e^{\,0.35\,Ma\ \text{ÿ}\frac{p}{p_0}\text{ÿ}\ \text{ÿ}\sqrt{\text{ÿ}}\ \text{ÿ}}$$

- This includes the density and pressure conditions corresponding to the flight altitude and the throttle level in %.
- The following is a thrust map for a ZTL engine with Bypass ratio 6 and a throttle degree D = 0.8, which corresponds approximately to the cruise power.
- Other throttle states used are D = 1 for takeoff and D = 0.9 for maximum continuous climb power.

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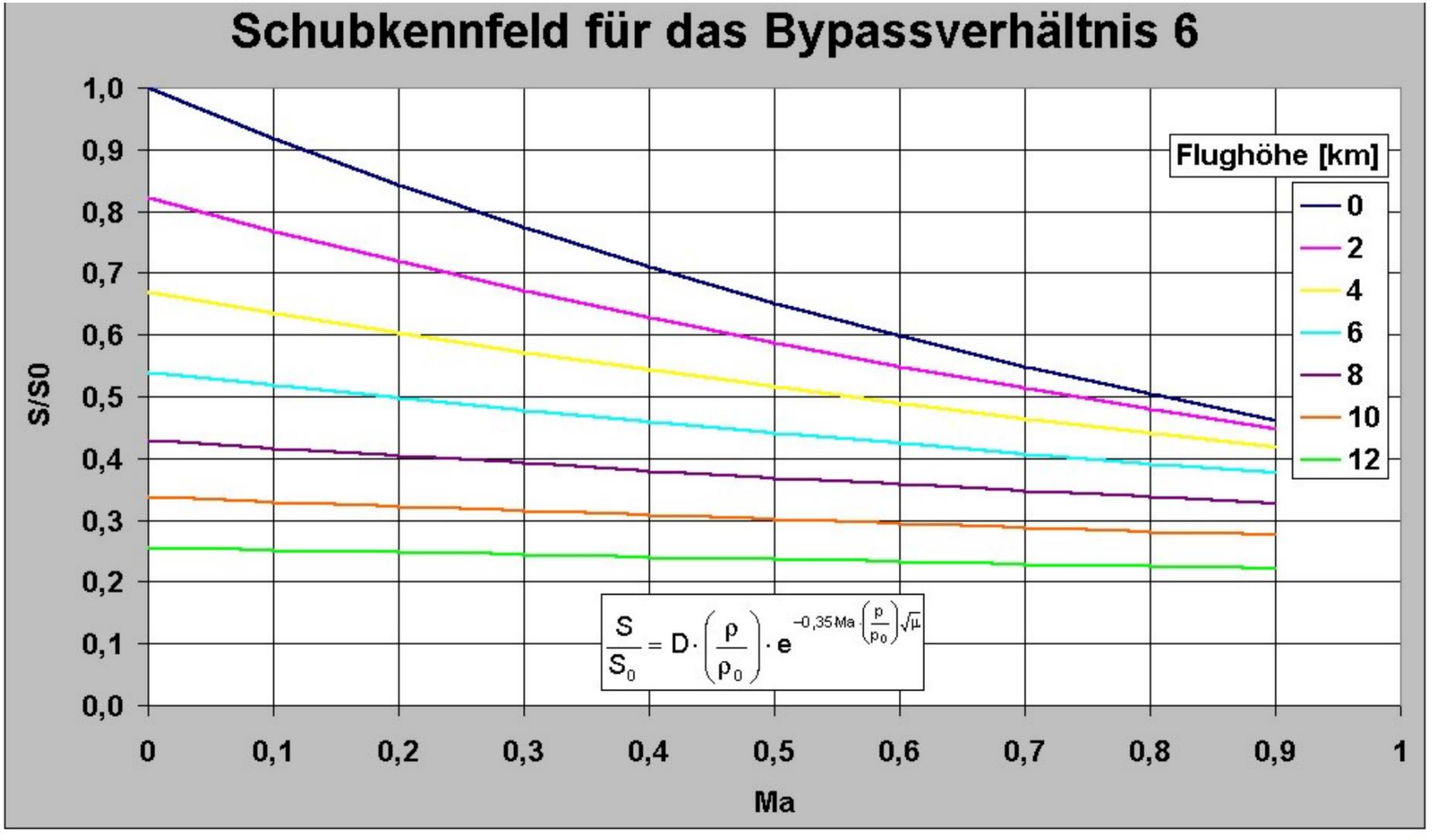
2.1 Horizontal flight diagram for jet aircraft



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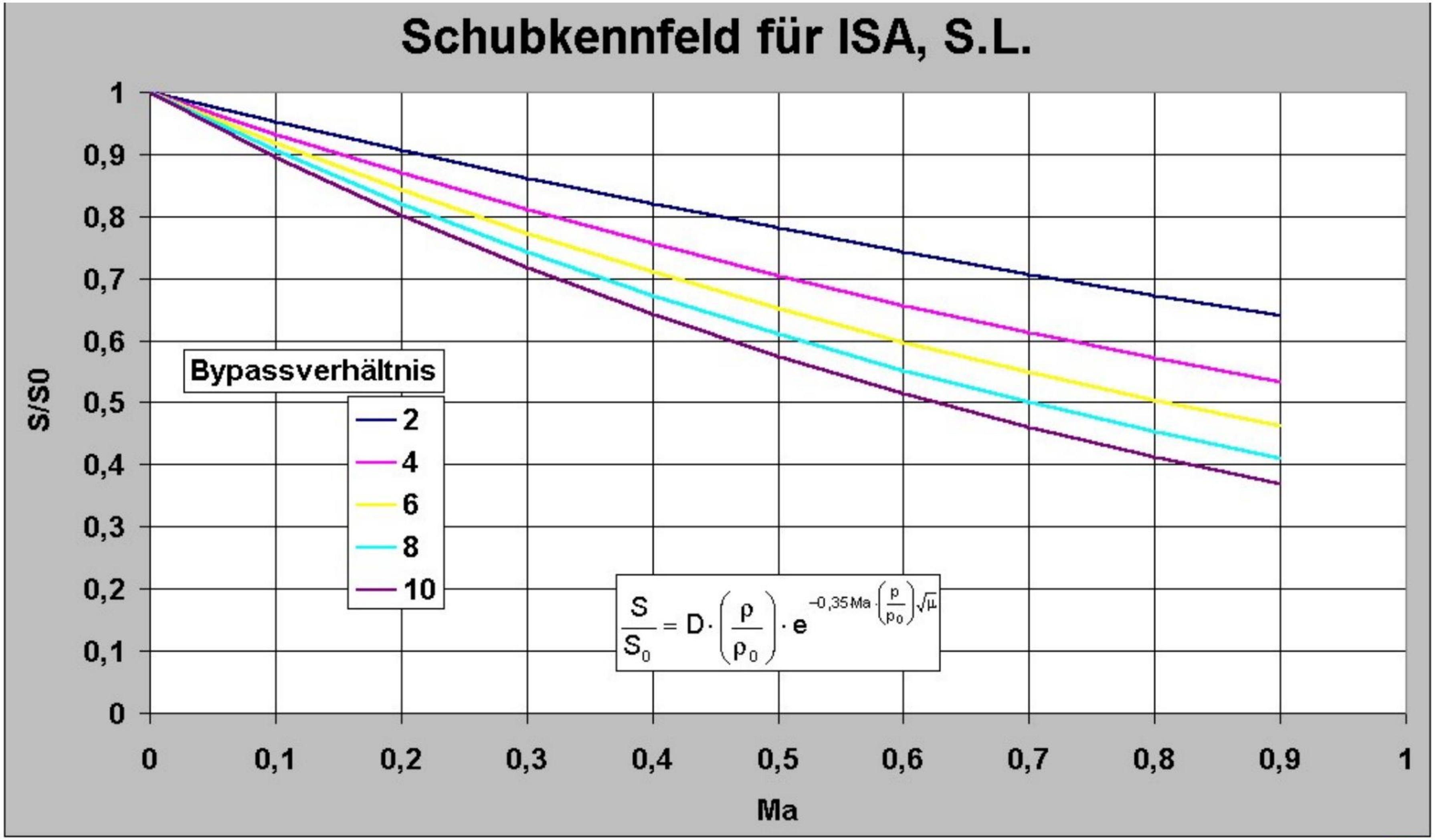
2.1 Horizontal flight diagram for jet aircraft



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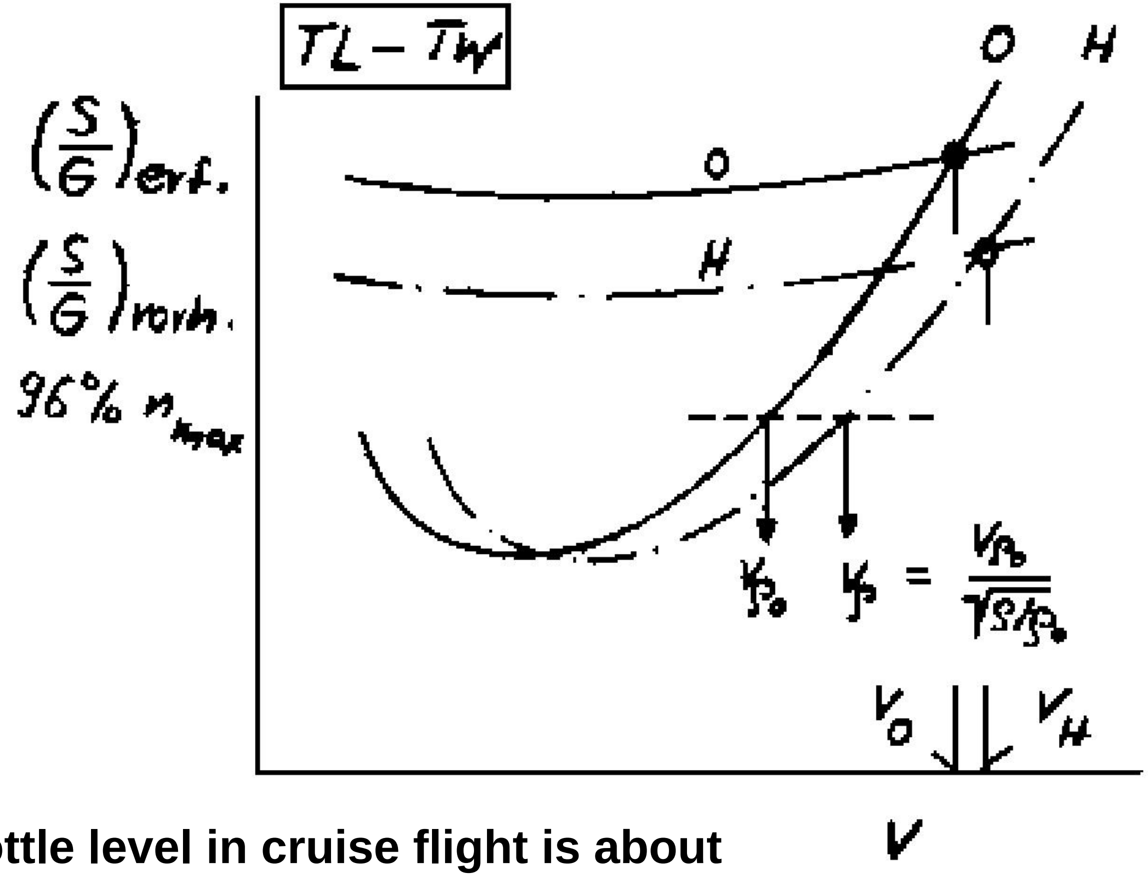
2.1 Horizontal flight diagram for jet aircraft

- The intersection point of the curves for the required thrust and those for the available thrust indicates the speed at which the conditions lift = weight and thrust = drag are met.



- A typical value for the throttle level in cruise flight is about 80%, which corresponds to an engine speed of 96% of the maximum speed.

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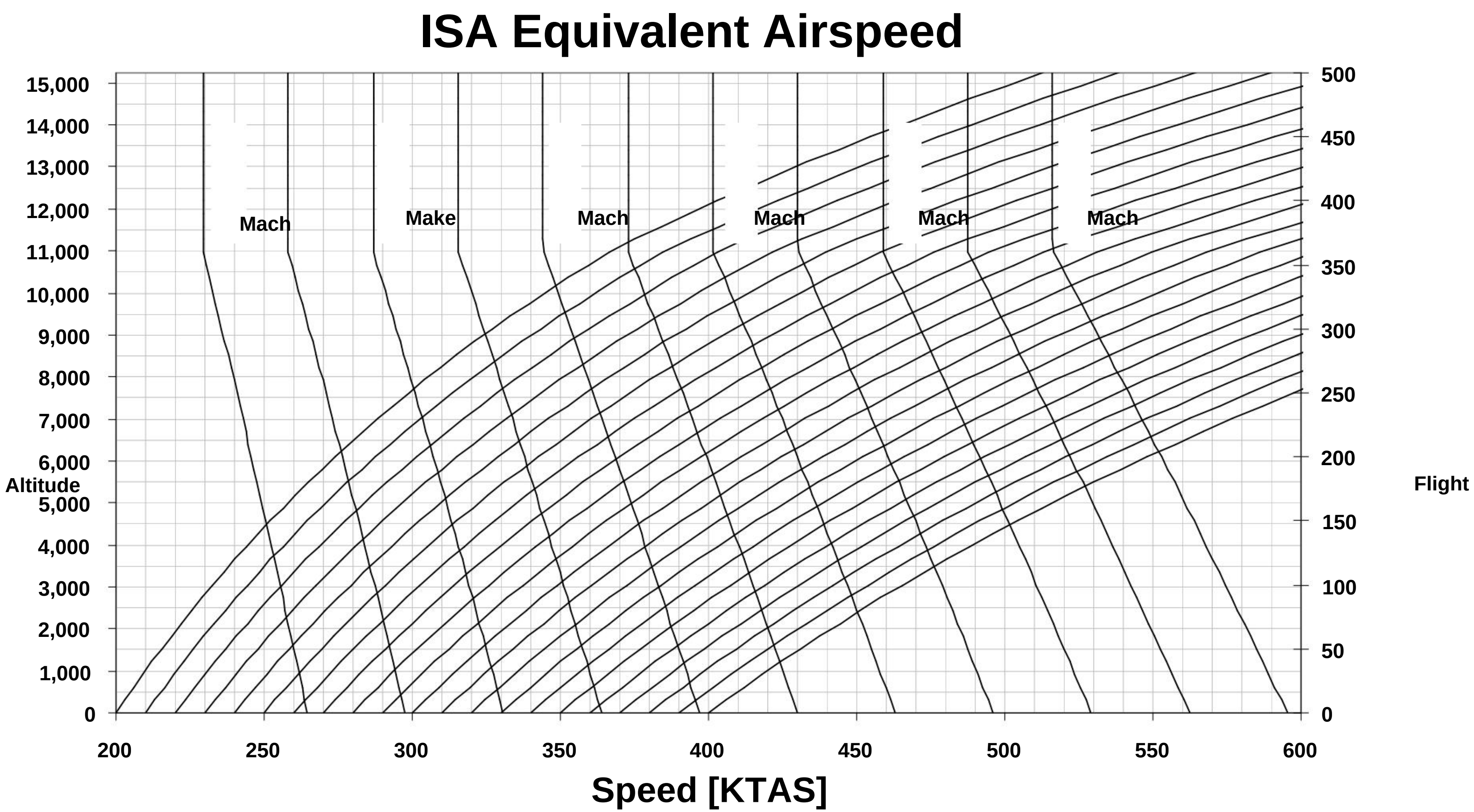
2.1 Horizontal flight diagram for jet aircraft



KTAS: knots true airspeed

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2.1 Horizontal flight diagram for jet aircraft

- The constant comparison airspeed lines (EAS – Equivalent Airspeed) incline to the right with increasing altitude, i.e. to higher true airspeeds (TAS).

- On the ground (altitude 0 m) the EAS corresponds to the TAS.

- The Mach number dependence with altitude leads to lines of constant Mach number inclined to the left.

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2.1 Horizontal flight diagram for jet aircraft

- Plotted against true air speed (TAS), thrust and drag curves depend on altitude.

- The distortion of the polar S/G due to increasing altitude can be described according to the following rule:

$$v = v_{H0} \cdot \sqrt{\frac{\rho_0}{\rho}}$$

- Justification: For constant cA and G the following applies:

$$\frac{v}{v_{H0}} = \frac{\sqrt{\frac{G}{F} \cdot \frac{2}{\rho} \cdot \frac{1}{c_A}}}{\sqrt{\frac{G}{F} \cdot \frac{2}{\rho_0} \cdot \frac{1}{c_A}}} = \frac{1}{\sqrt{\frac{\rho}{\rho_0}}}$$

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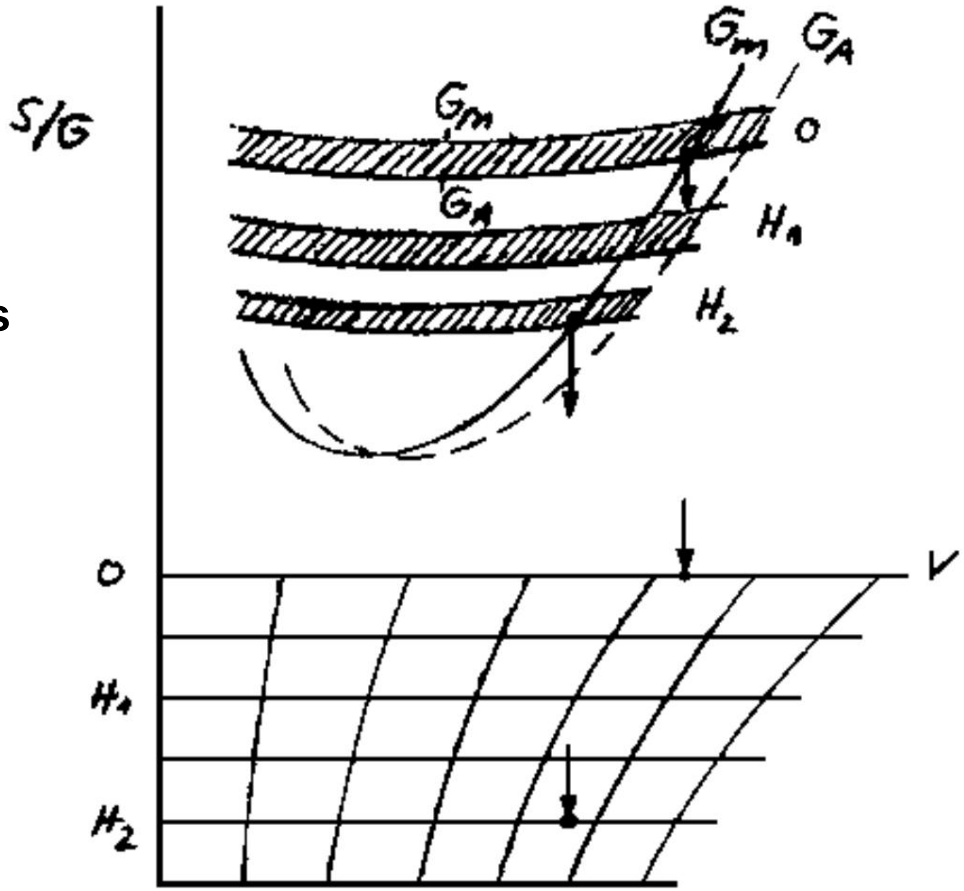
2.1 Horizontal flight diagram for jet aircraft

- **It is therefore possible to plot only one polar over the altitude-independent equivalent speed EAS and to correct the abscissa using this altitude function. • The display then looks like this: ÿ**



- **The respective value for the speed must then be read on the abscissa for the corresponding flight altitude.**

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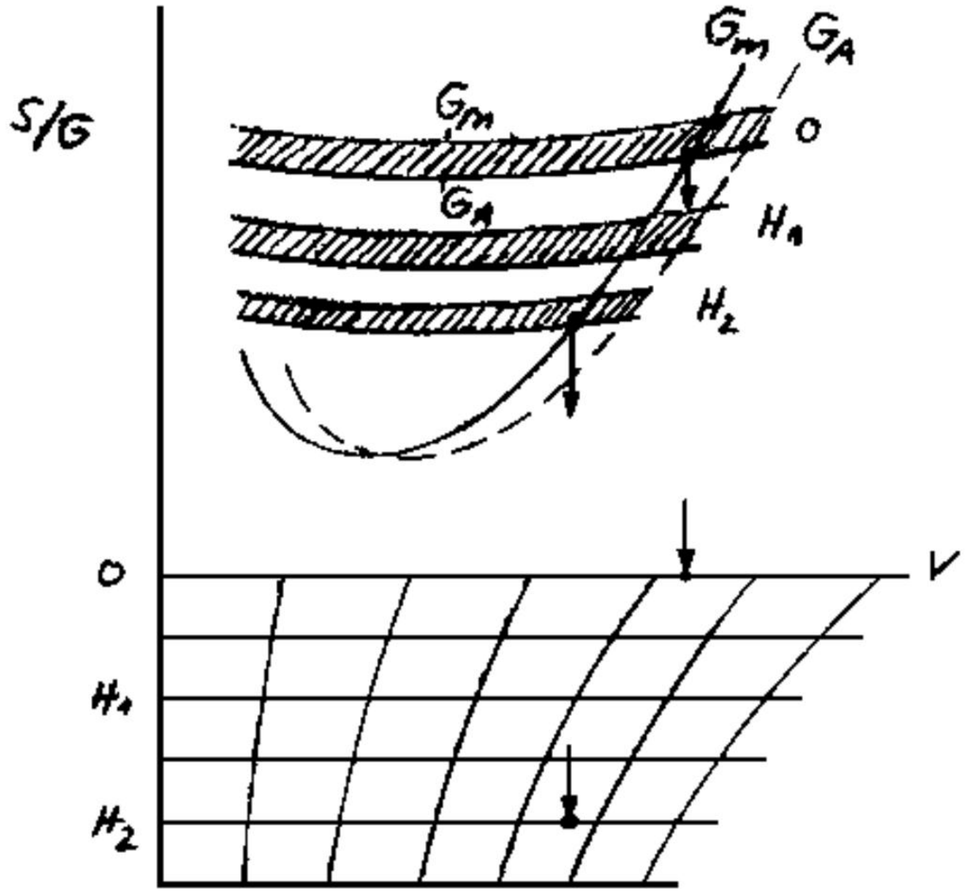
2.1 Horizontal flight diagram for jet aircraft

- With increasing flight weight:
  - the thrust decreases
    Weight ratio
  - shifts the
    Glide ratio polar to the right
- This results in a
  increased cruising speed with increasing flight weight.



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2.2 Horizontal flight diagram for propeller aircraft

- For propeller aircraft, thrust is not a suitable calculation size.
- Therefore, the required horizontal flight diagram and existing weight-specific power.
- Since the power is, the Nerf needs. ÿ Wÿ v ÿ Serf ÿ v Equation for the required thrust simply multiplied by the speed

$$S_{required} \text{ ÿÿ } c_W \frac{\text{ÿ}}{2} \text{ ÿ } v^2 F \text{ ÿ}$$

- One obtains a cubic dependence of the power on the Speed:

$$N_{required} \text{ ÿÿ } c_W \frac{\text{ÿ}}{2} \text{ ÿ } v^3 F \text{ ÿ}$$

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2.2 Horizontal flight diagram for propeller aircraft

- The transmitted power of controllable pitch propellers depends on the respective blade angle, density and degree of progress l (advance ratio).

- For the advance ratio l, n is the speed in 1/min and D is the diameter of the propeller. 60 v

$$l \ddot{y} \frac{v}{\ddot{y}\ddot{y} \quad D} \ddot{y} \frac{\ddot{y}}{\ddot{y}\ddot{y}\ \ddot{y}n\ D}$$

- The degree of progress is therefore the ratio of Flight speed to blade tip speed.

G Flight performance

2.2 Horizontal flight diagram for propeller aircraft

ÿ • Then ÿ ÿ l ÿ $N_{erd.}$ ÿ , which with the help of the wave

NBraking power of the engine (braking power) and the characteristic map of the propeller

efficiency. • Here too, the altitude conversion is to be carried out using the

Restrictions applicable:

$$N_{required} \text{ ÿ } \frac{N_{required\,H0}\text{ÿ}}{\sqrt{\frac{\text{ÿ}}{\text{ÿ}_0}}}$$

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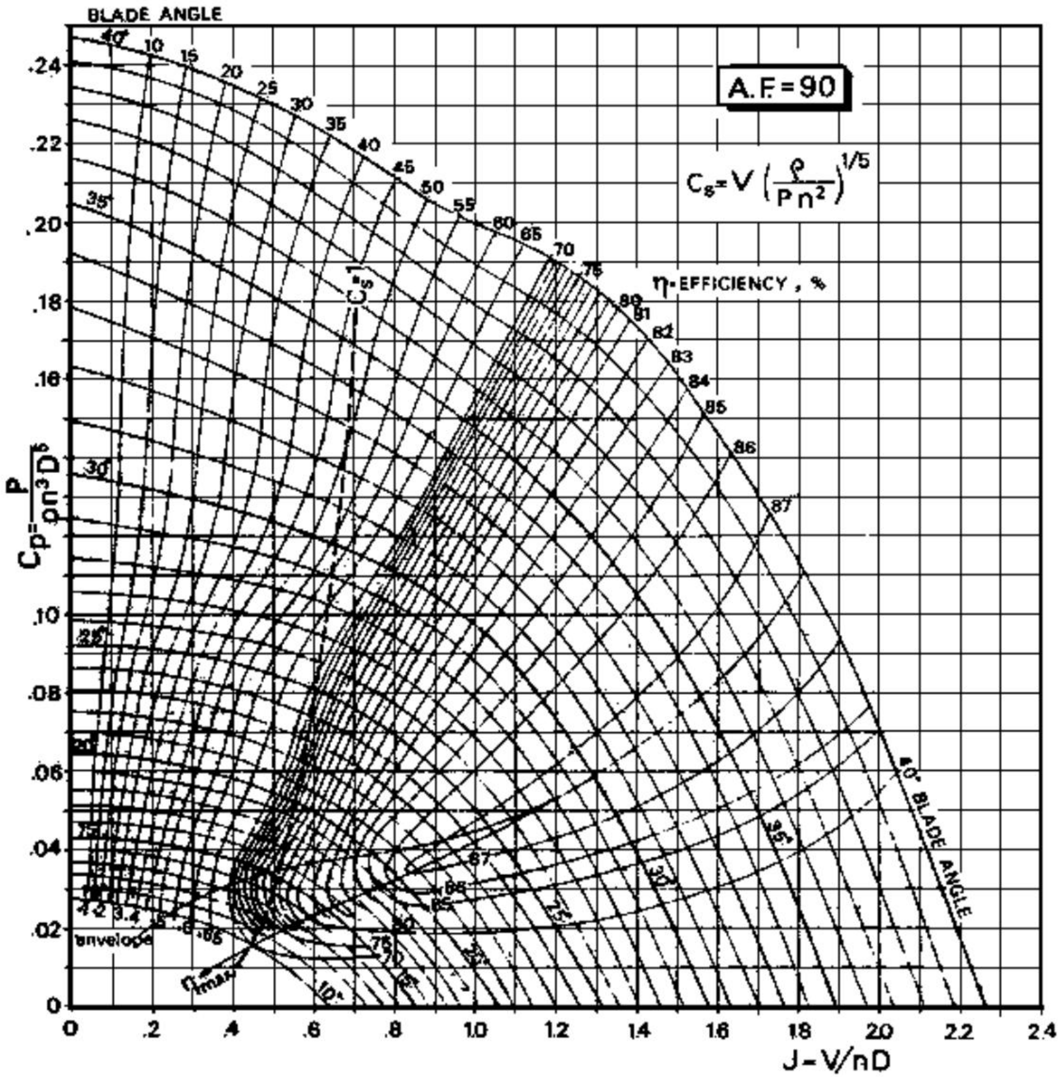
G Flight performance

2.2 Horizontal flight diagram for propeller aircraft



Activity Factor

$$AF = \frac{100{,}000}{16} \int_{0.15}^{1.0} \left(\frac{t}{D}\right)\left(\frac{r}{R}\right)^3 d\left(\frac{r}{R}\right)$$

Power Coefficient

$$c_P = \frac{P}{\rho n^3 D^5}$$

Thrust Coefficient

$$c_S = \frac{S}{\rho n^2 D^4}$$

Effectiveness Thrust

$$\eta = \frac{c_S}{c_P} J \qquad S = \frac{\eta P}{v}$$

Advance Ratio

J_{eff} ÿ hÿ 1ÿ ÿ J

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Propeller power

Determination of propeller performance using characteristic maps

- Characteristic maps for propellers were created by propeller manufacturers (e.g. Hamilton) and test facilities (NACA) on the basis of experimental investigations.

- With the help of mostly dimensionless parameters, the suitable propeller can be selected and/or its thrust behavior can be determined.

- The following parameters are required:
  - Advance Ratio J (= degree of progress l) $J = \lambda = \dfrac{v}{n \cdot D}$
  - Power Coefficient $c_P = \dfrac{P}{\rho \cdot n^3 \cdot D^5}$
  - Thrust Coefficient $c_T = \dfrac{T}{\rho \cdot n^2 \cdot D^4}$

Propeller power

Determining the propeller performance using characteristic maps • The following parameters are also required for this:

- **– Speed-Power Coefficient** $c_S = v\sqrt{\frac{\rho}{Pn^2}} = \sqrt{\frac{J^5}{c_P}}$
- **– Propeller Efficiency** $\eta_P = \frac{Tv}{P} = J\frac{c_T}{c_P}$

ÿ The Activity Factor - A measure of the propeller blade's usable energy:

$$AF ÿ \frac{100000}{16} \int_{0.15}^{1.0} \left(\frac{c}{D}\right)\left(\frac{r}{R}\right)^3 d\left(\frac{r}{R}\right) \qquad AF = \frac{10^5}{D^5}\int_{0,15R}^{R} cr^3 dr$$

where c is the local propeller blade chord. The AF is between 90 and 200. Typically: AF for sport aircraft ~90; for large turboprop aircraft ~140

Total Activity Factor (with number of leaves B): TAF ÿ B ÿ AF

Propeller power

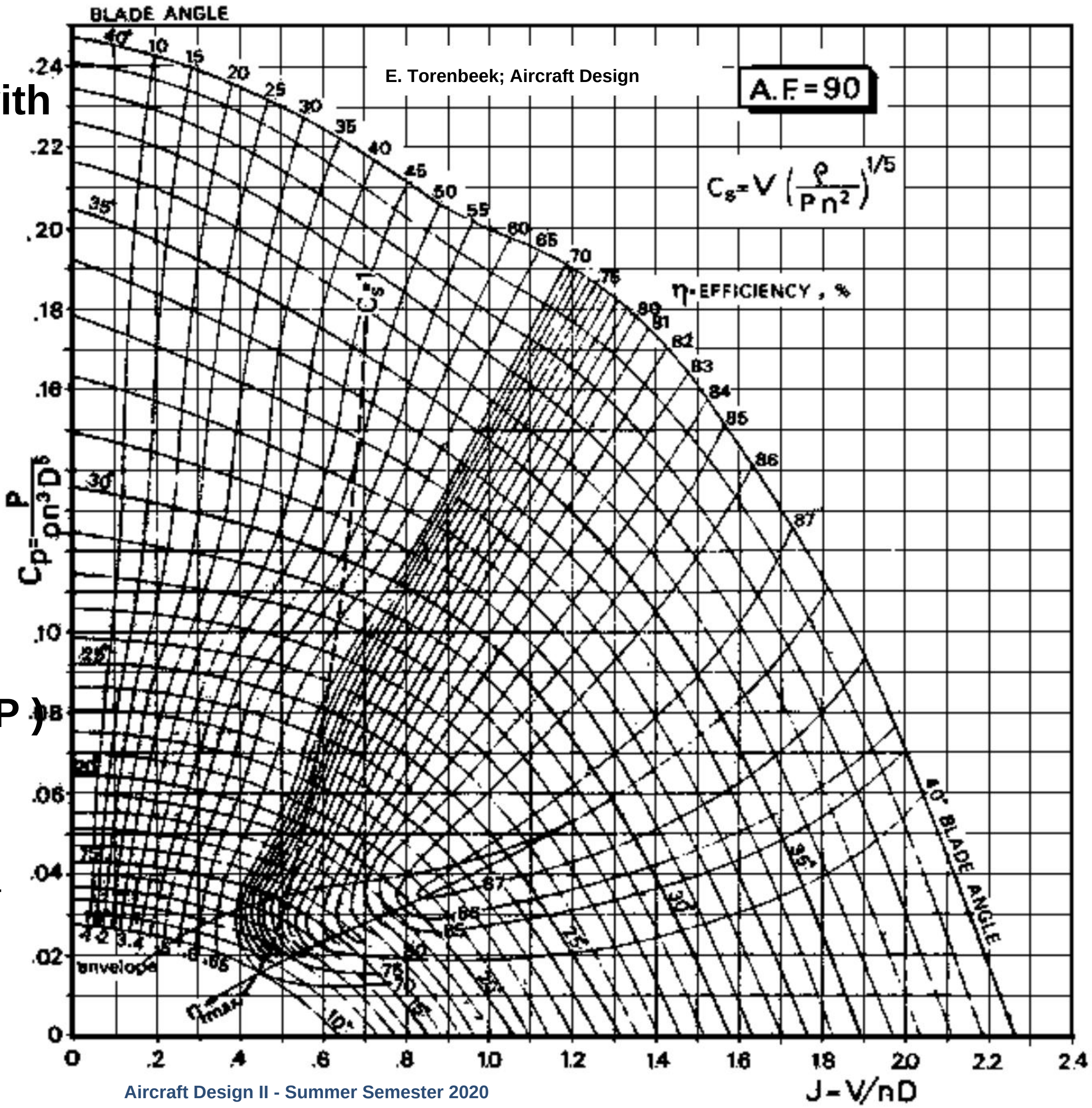
Determination of propeller performance with Help from characteristic fields

- Specifications:
  - Flight altitude
  - Flight speed
  - Engine speed
  - Prop. diameter

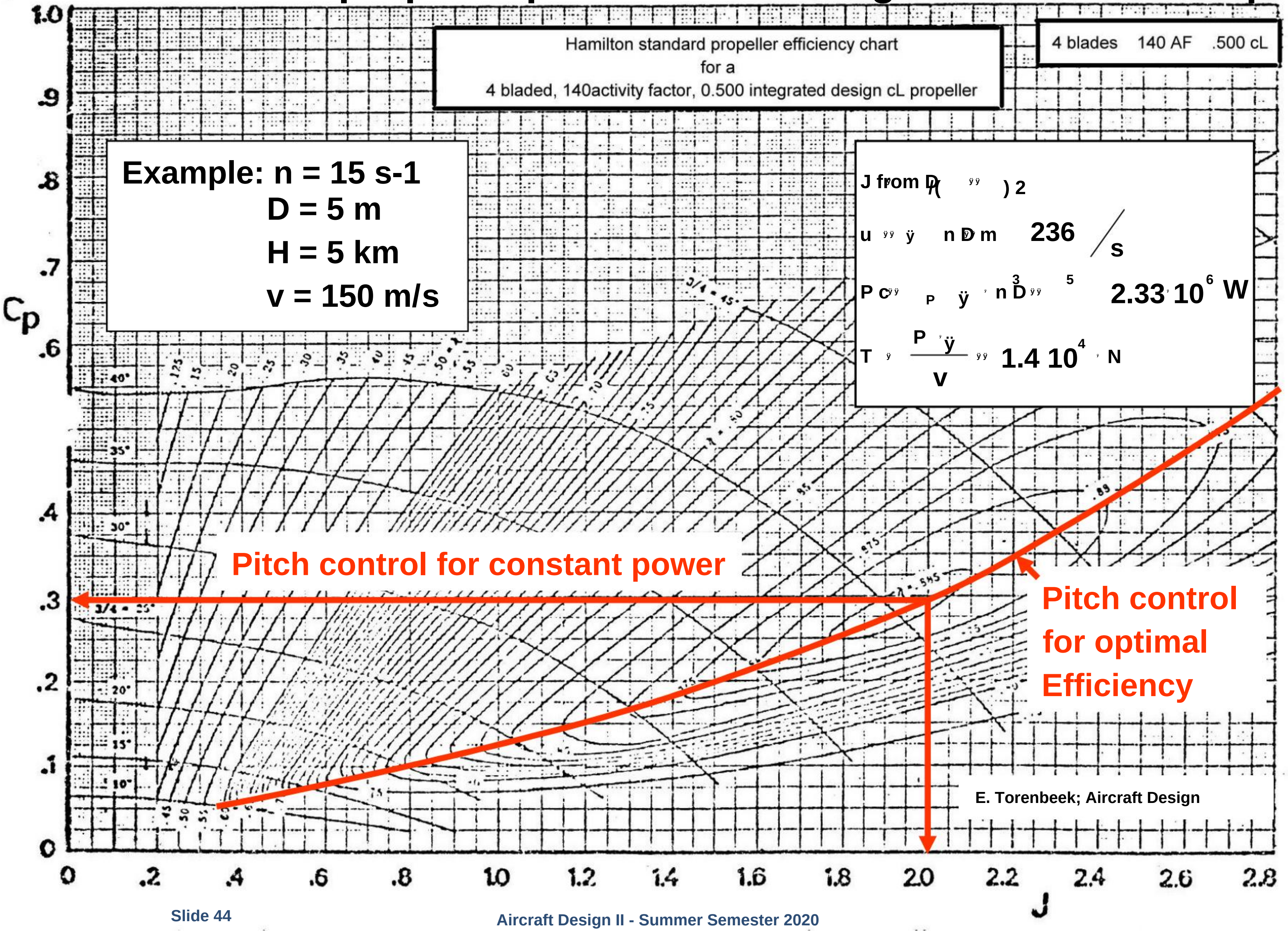
1. Calculation of J
2. Determination of cP at ÿmax (or ÿP from cP)
3. Calculate P from cP
4. Calculate T from

T cп D T ÿ 2 4

or T ÿ ÿ $\frac{P}{V}$ ÿ P



Determination of propeller performance using characteristic maps



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2.3 Thrust estimation from the travel demand

- For a quick decision on the thrust design for a given cruise speed or a quick determination of the speed for a given thrust, a helpful graphical representation can be derived.

- For simplification, the quadratic Polar equation is used and divided by the specified existing thrust.

- First of all, $c_W \text{ ÿ } \frac{S_{required}}{qF} \text{ ÿ ÿ} c_{W0} \text{ ÿÿ } \frac{c_A^2}{e}$

- This results in the thrust factor for stationary cruise flight with the simplifications

$$\text{ÿ ÿ } \frac{\text{ÿ}}{\text{ÿ}0} \text{ , } fc F_S \text{ ÿ ÿ } W0 \text{ , ÿ ÿ } \frac{b^2}{F} \text{ and } c_A^2 \text{ ÿ } \left(\frac{\text{ÿ } G \text{ ÿ}}{\text{ÿ } q F \text{ ÿ}} \right)^2$$

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2.3 Thrust estimation from cruise demand

- **Thrust factor for steady cruise flight: ÿÿÿ ÿ ÿÿ ÿ ÿ 2 ÿ**

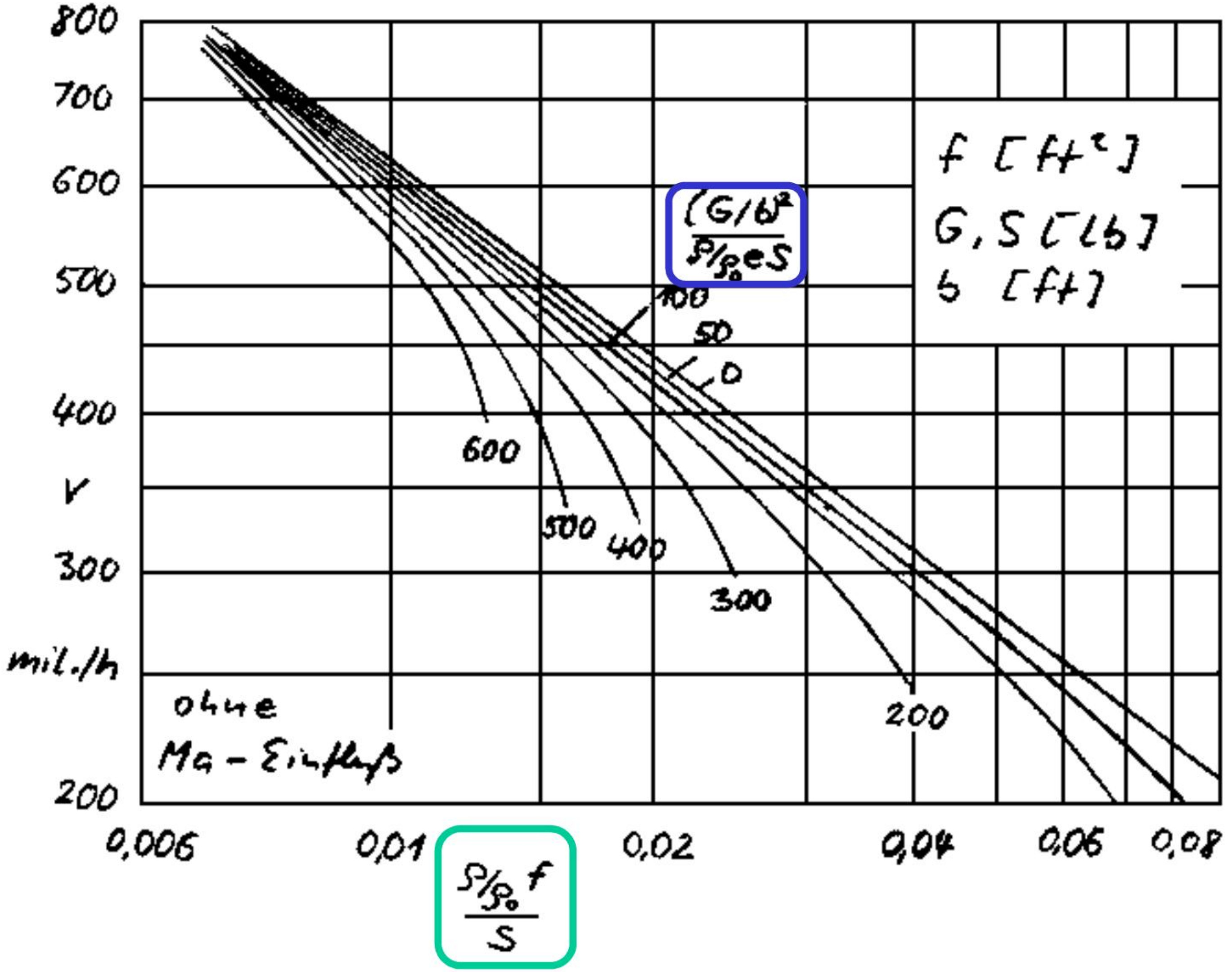
$$\frac{S_{required}}{S_{existing}} \text{ÿÿ} \; 1 \; \frac{\text{ÿÿ}_0}{\text{ÿÿ}} \boxed{\frac{e_S \, \text{ÿ}}{S_{existing} \, \text{ÿÿ}}} \; 2 \, \text{ÿÿ} \, v \; \frac{\text{ÿÿÿÿ} \, 2 \, \text{ÿÿÿÿÿ}}{\text{ÿÿÿÿÿ}_0} \text{ÿÿÿÿ} \boxed{\frac{G}{\text{ÿ} \, b \, \text{ÿ}} \text{ÿ}^2 \; \frac{1}{\text{ÿÿ} \, \text{ÿ} \, e \, S_{existing}} \, \text{ÿ} \, \text{ÿÿÿ}} \; \frac{1}{2 \, v}$$

- **If the square bracket expressions are evaluated parametrically for the speed, the result is the graphic on the following page.**

- **It can be used to easily determine the cruising speed or the required thrust for a given configuration (wing span) and flight altitude (ÿ) using estimates of the polar parameters (Oswald factor and damaging area).**

G Flight performance

2.3 Thrust estimation from the travel demand



G Flight performance

2.3 Thrust estimation from the cruise requirement

- For propeller aircraft, the parameters

ÿÿ f S / ÿÿ N Brake , ÿ ÿ ÿ ÿ ÿ ÿ b ² 1 / ÿÿÿe N ÿÿ Brake

and v can be used.

G Flight performance

2.4 Dimensionless horizontal flight diagram

• Assuming a quadratic drag polar, the horizontal flight diagram can also be created in dimensionless form.

• The reference values are the minimum ÿ = cW/CA and the associated speed v. • Minimum ÿ

occurs at the point where:

$$\frac{ÿÿ}{ÿv} ÿÿG \quad \frac{ÿW}{ÿv} ÿ \quad 0$$

• Approach to resistance:

Shared flat ÿ ÿ ÿ ÿ ÿ A v ÿ $\hat{}^{2}$ $\frac{B\hat{}}{v^2}$

• Using the well-known quadratic approximation of the Resistance polars:

$$c_W\ c_{W0}\ ÿ ÿ ÿ ÿ ÿ ÿ ÿ \frac{c_A^2}{e}$$

G Flight performance

2.4 Dimensionless horizontal flight diagram

- **It follows:**

$$2 \cdot \hat{A}\, v_{\text{ÿ}\, min} \;\text{ÿ}\; \frac{\hat{B} \cdot 2}{3\, v_{\text{ÿ}\, min}}$$

- **The constants are**

$$\hat{A} \;\text{ÿ}\; \frac{\text{ÿ}}{2} \cdot F\, c_{W0} \quad \text{and} \quad \hat{B} \;\text{ÿ}\; \left(\frac{\text{ÿ ÿ G}}{\text{ÿ ÿ ÿ ÿ b}} \right)^{2} \cdot \frac{2}{\text{ÿÿÿÿ e}}$$

- **Solving for the speed gives**

$$v_{min} \;\text{ÿ}\; \sqrt[4]{\frac{\hat{B}}{\hat{A}}}$$

- **The required thrust at this point is:**

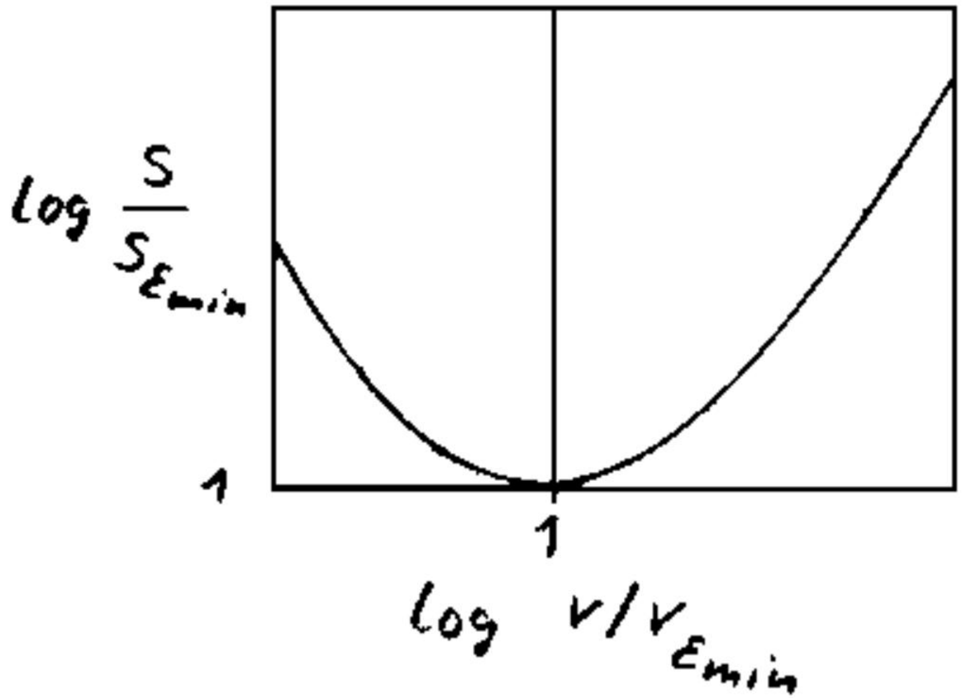
$$S_{min\, \text{ÿ}} \;\text{ÿ ÿ}\; \hat{A} \sqrt{\frac{\hat{B}}{\hat{A}}} \;\text{ÿ}\; \frac{\hat{B}}{\sqrt{\frac{\hat{B}}{\hat{A}}}}$$

G Flight performance

2.4 Dimensionless horizontal flight diagram

- **The following are not taken into account in the procedure described here using a quadratic resistance polar:**

  – the height-dependent Reynolds number influence (cW0) – the height-dependent course of the wave resistance increase (c



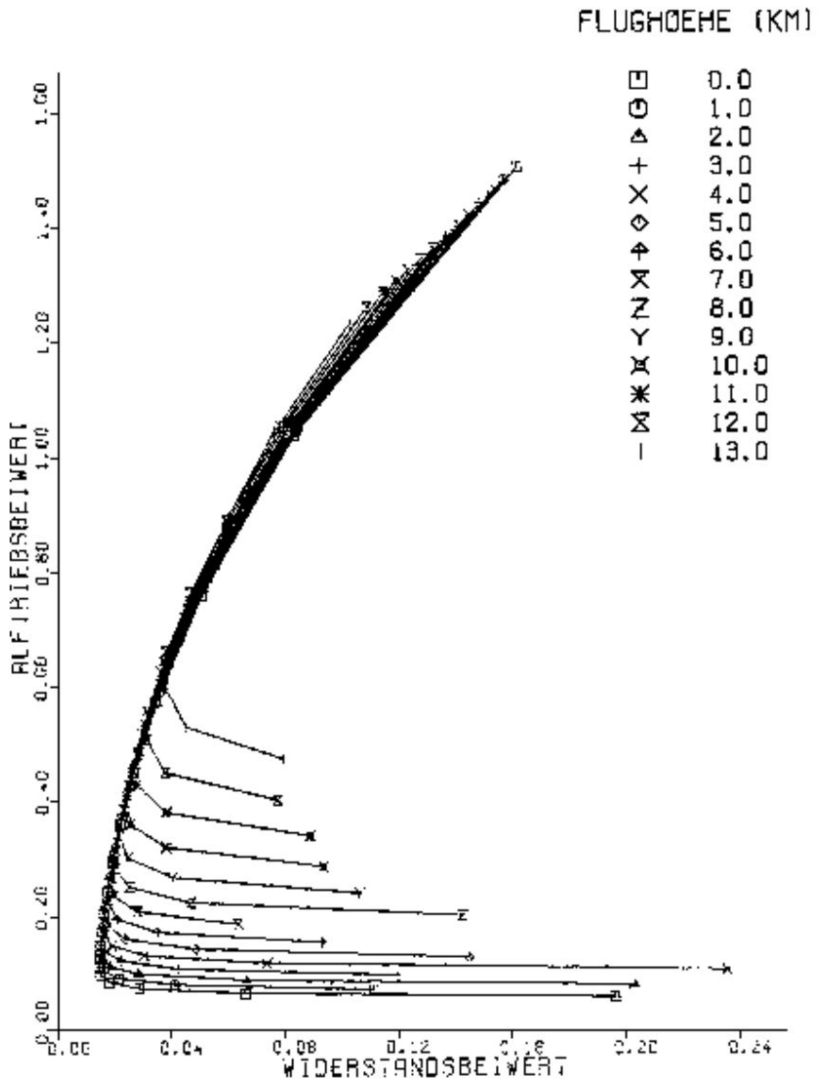
G Flight performance

2.4 Dimensionless horizontal flight diagram

- **Notes for aircraft operating at high subsonic speeds:**
  - **The off-design polars for different flight altitudes differ only slightly in the area of the design point or at lower flight speeds.**
  - **If you want to carry out a correct performance analysis, you have to determine a separate drag polar for each flight altitude. The above-mentioned distortion using the density ratio can then no longer be applied.**
- **However, for aircraft in the incompressible speed range, the simplified approach leads to useful results.**

G Flight performance

2.4 Dimensionless horizontal flight diagram



G Flight performance

3 Climb performance

- The horizontal flight diagram is the basis for a series of Further flight performance statements:
- It not only provides information about the speeds at which the equilibria of forces in horizontal and vertical directions are met, but it also provides information about the thrust excess and deficiency in the entire flight range.
- In addition to the intersection point, which indicates the cruising speed, there is also the point of equilibrium of forces in the low-speed range for higher flight altitudes.
- While a short-term disturbance of the balance in the While in cruise flight the cruise speed is automatically stabilized, since an increase in speed results in a reduction in thrust and thus in negative acceleration, this is not the case at the second equilibrium point.

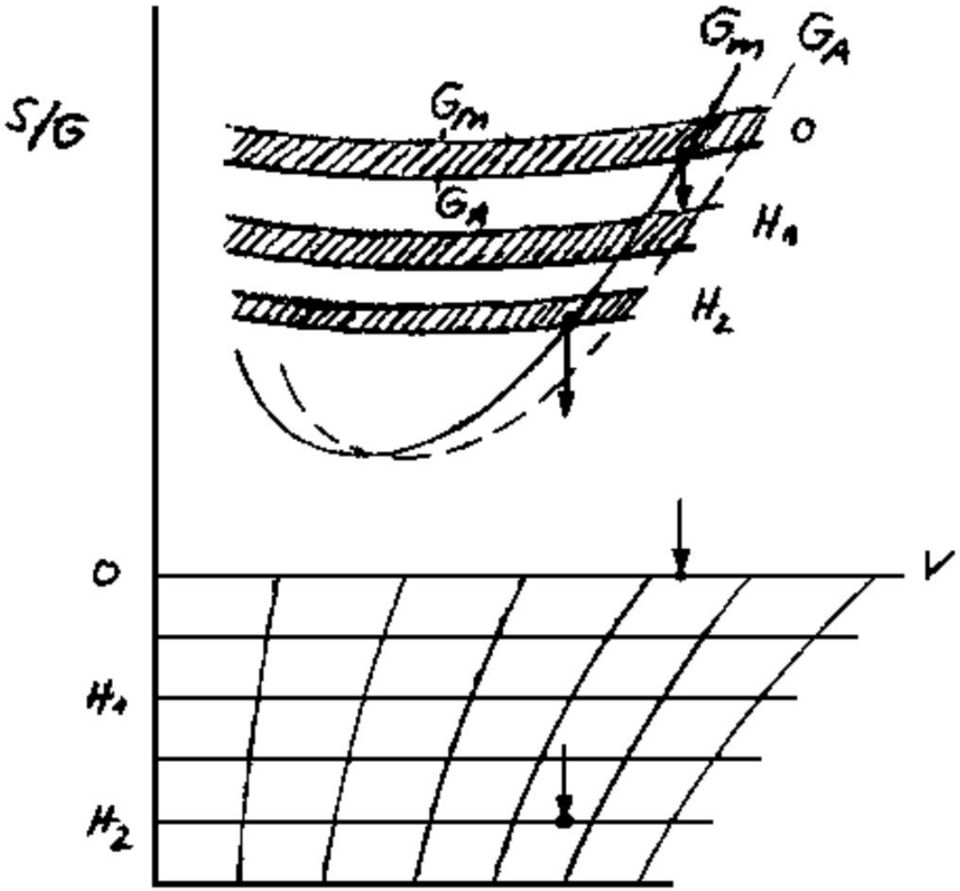
G Flight performance

3 Climb performance

- **Here a speed**

  An increase in speed leads to an increase in the excess thrust and thus to an increasing acceleration until the equilibrium state of the cruise condition is reached.

- **This point therefore represents an unstable equilibrium.**



G Flight performance

3.1 Quasi-stationary equations of motion

- For the quasi-stationary equations of motion of the flight
  The reference to the horizontal flight diagram becomes clear.

- Neglecting the thrust setting angle,

- Flight path geometry: xv ÿ cos ÿ ÿ_g ÿ ÿ 0

  hv ÿ sin ÿ ÿ_g ÿ ÿ 0

- Balance of power: SWG sin 0 ÿ ÿ ÿ_g ÿ ÿ

  AG ÿ cos ÿ ÿ_g ÿ ÿ 0

- Fuel consumption: GG0 ÿ ÿ K ÿ

$\dot{G}_K = g \cdot \dot{m}_K$

- The fuel consumption dGK /dt can be calculated as thrust-specific
  Express consumption: GK ÿ bs ÿ S ÿ g

G Flight performance

3.1 Quasi-stationary equations of motion

• From the equilibrium of forces and the angle of climb g follows:

sin ÿ ÿ ÿ ÿ ÿ ÿ ÿ wg / v   SW / G   s / G

• This corresponds to the difference between the curves for the available and the required thrust in the horizontal flight diagram. • The climb rate is then:

ÿ ÿ ÿ ÿ ÿ SWS ÿ ÿ ÿ ÿ ÿ ÿ ÿ ÿ ÿ ÿ ÿ ÿ • The climb consumption

wv ÿ ÿ

is calculated using the equation of motion in the vertical direction and the G G

equation of the change in mass: dG b S g

ÿ ÿ dG / ie   K / ie ÿ   dG K / engl   engl ÿ ÿ ÿ / ie   G K / w   s / w ÿ   b S g / ÿ S ÿ

v ÿ ÿ ÿ   ÿ ÿ G   ÿ ÿ

G Flight performance

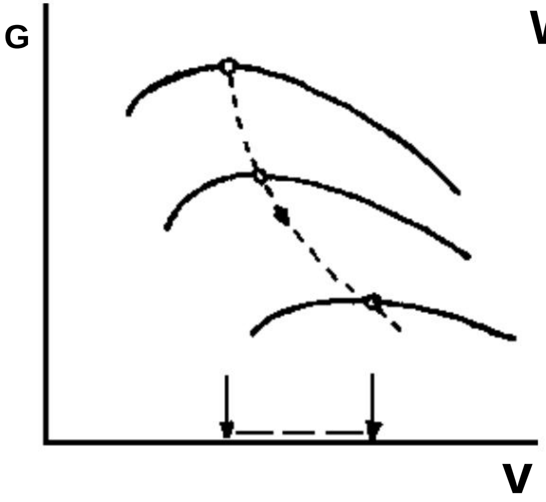
3.1 Quasi-stationary equations of motion

- The climb angle depends on the excess thrust.
- The climb rate and climb consumption depend on the power surplus, with the latter also taking into account the time-related consumption.

G Flight performance

3.1 Quasi-stationary equations of motion

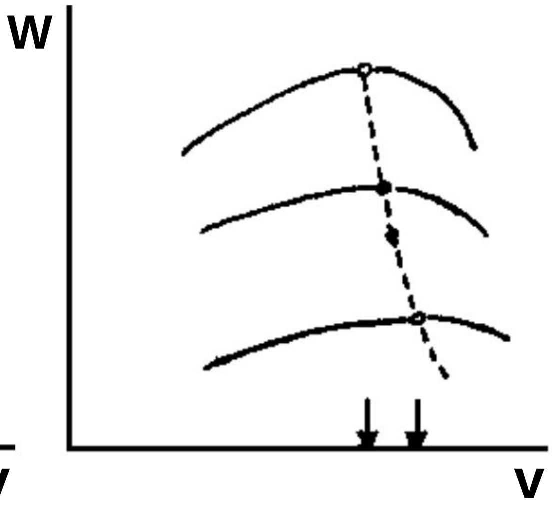
Steepest climb



$$\frac{(\text{ÿ ÿ} SW \quad)}{\text{ÿ} v} \text{ ÿ } 0$$

Excess
thrust ÿ max.

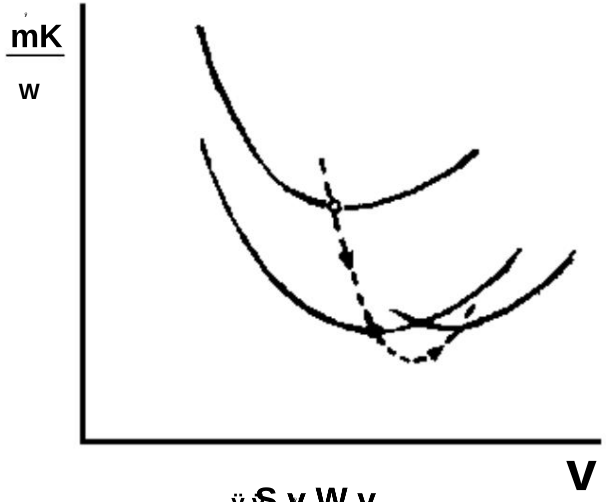
Fastest climb



$$\frac{(\text{ÿ ÿ ÿ ÿ} S\ v\ W\ v \quad)}{\text{ÿ} v} \text{ ÿ } 0$$

Power surplus
ÿ max.

Most economical climbing



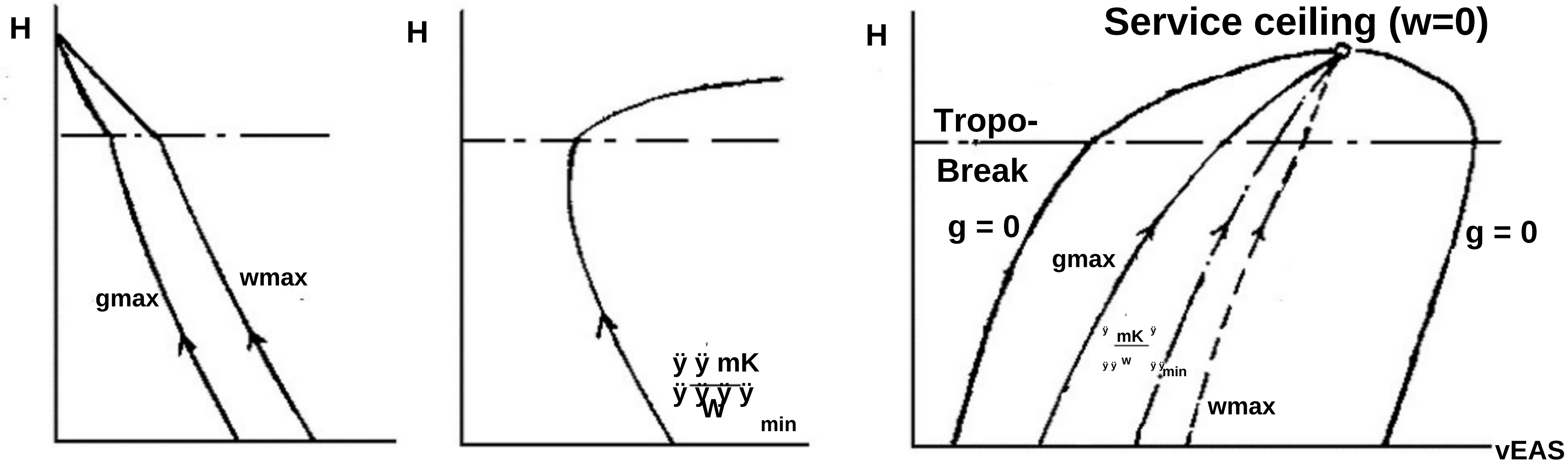
$$\frac{\text{ÿ ÿ ÿ} \; \frac{\text{ÿ ÿ} S\ v\ W\ v}{b_S S} \; \text{ÿ ÿ}}{\text{ÿ} v} \text{ ÿ } 0$$

Hourly consumption per
power surplus ÿ min.

G Flight performance

3.1 Quasi-stationary equations of motion

- **If this consideration is carried out for variable flight altitudes h and constant flight weight G, one already obtains essential statements about the realizable flight range:**



1. **Angle of climb and rate of climb decrease with increasing altitude and disappear at the theoretical ceiling altitude.**
2. **The fuel-efficient climb rate decreases up to the tropopause and increases sharply above it.**

G Flight performance

3.1 Quasi-stationary equations of motion

• **If this consideration is carried out for variable flight altitudes h and constant flight weight G, one already obtains essential statements about the realizable flight range:**

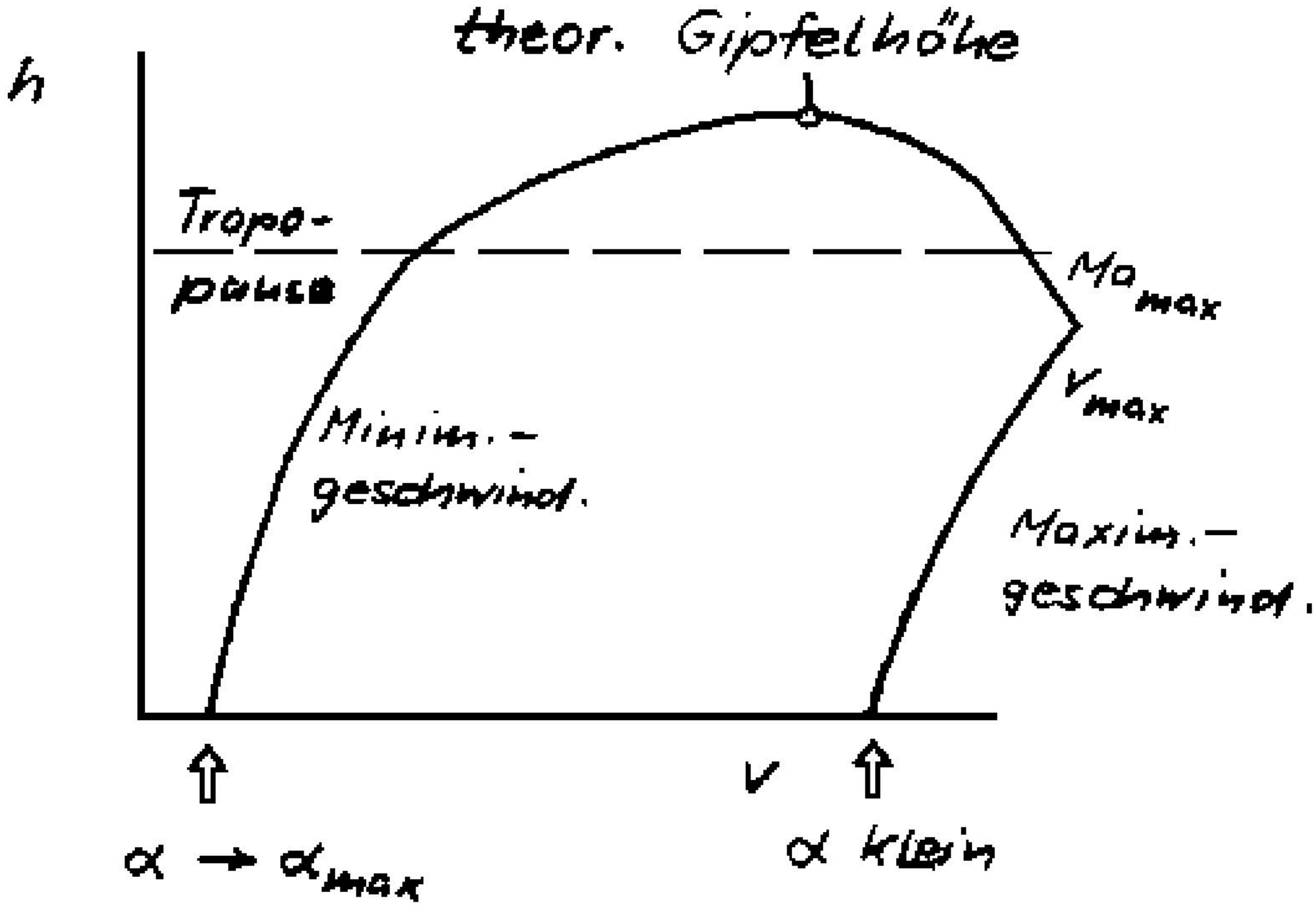
3. All speeds are given for the theoretical peak height identical.

4. The equilibrium speeds (g = 0) limit the horizontal flight range.

5. The highest rate of climb is achieved at a higher flight speed than the fuel-efficient climb speed, and this is greater than the speed of the maximum climb angle.

G Flight performance

3.1 Quasi-stationary equations of motion



G Flight performance

3.1 Quasi-stationary equations of motion

- The above hv diagram may only be interpreted in the range in which the aircraft can fly, ie that neither the minimum speed is undercut nor the maximum speed is exceeded.

- A flight area statement must therefore be superimposed.
- The maximum speed is determined by the maximum dynamic pressure that the structure can bear (maximum design speed) or by the Buffet limit (Mach number influence).
- The left boundary follows the EAS curve for the maximum lift coefficient.
- It therefore depends on the installed engine power or the Number of engines, whether this or the buffet limit limits the peak height!

G Flight performance

3.2 Climb performance with quadratic drag polar

- The climbing performance parameters can be determined using the Horizontal flight diagram can be graphical or numerical.
- An analytical solution is to be derived assuming a quadratic drag polar and a constant thrust (approximation for TL engines).

G Flight performance

3.2 Climb performance with quadratic drag polar

• The speed of the steepest climb is determined by the derivative of the excess thrust with respect to the lift coefficient.

• Set to zero and solved for the lift coefficient:

$$\frac{\ddot{y}\, d\, \ddot{y}\ddot{y}\ddot{y}\, \frac{\ddot{y}\, S}{G}\; \ddot{y} \;\ddot{y}\ddot{y}\;\ddot{y}}{dc_A} \;\ddot{y}\; \frac{d\,\ddot{y}\,\ddot{y}\,\frac{\ddot{y}\, S}{G}\,\frac{c_{W0}}{c_A}\,\frac{c_A}{\ddot{y}\ddot{y}\ddot{y}\ddot{y}\, e}\,\ddot{y}\;\ddot{y}\ddot{y}\ddot{y}}{dc_A}\;\ddot{y}\ddot{y}\; 0 \qquad \frac{c_{W0}}{2c_{A\,g\,max}} \;\; \frac{1}{\ddot{y}\ddot{y}\ddot{y}\ddot{y}\, e}$$

and consequently

$$c_{A\,g\,max} \;\ddot{y}\; \sqrt{W0\;\ddot{y}\ddot{y}\ddot{y}\ddot{y}\ddot{y}\; ec} \quad .$$

• The corresponding speed is then:

$$v\,\ddot{y}\,\ddot{y}_{G\,Max} \sqrt{\frac{G}{F\,c}\;\frac{2}{g\,\ddot{y}}\;\frac{1}{A_{Max}}}$$

G Flight performance

3.2 Climb performance with quadratic drag polar

- **The speed of maximum climb is ÿ ÿ ÿ ÿ**

analogous:

$$\frac{ÿ\, d\, ÿ\, ÿ\, ÿ \frac{ÿ\, S}{G}\; ÿ\; v\; ÿÿ\, ÿ}{dc_A}\; ÿ\; \frac{ÿ_d\, ÿ\, ÿ \frac{ÿ\, S}{G} \frac{c_{W0}}{c_A} \frac{c_A}{ÿÿÿÿ\, e}\; ÿ \sqrt{\frac{G}{F} \frac{2\,1}{ÿ} \frac{}{c_A}}}{dc_A}\; ÿ\; 0$$

- **After forming, the optimal lift coefficient is ÿ ÿ ÿ ÿ**

$$c_{A\,wmax}\; ÿ\; \frac{ÿÿÿÿ\, e}{2}\; ÿ \sqrt{\frac{12c_{W0}}{ÿÿÿÿ\, e}\; ÿ\; \frac{ÿ\, ÿ\, S\, ÿ}{ÿ\, ÿ\, ÿ\, ÿ\, ÿ\, ÿ\, G}^{2}}\; \frac{S}{G}\, ÿ\; ÿ$$

as well as for the speed

$$v_{wmax}\; ÿ \sqrt{\frac{G}{F} \frac{2\,1}{ÿ} \frac{}{c_{Aw\,max}}}$$

G Flight performance

3.2 Climb performance with quadratic drag polar

- The rate of increase to minimum consumption can only be determined numerically due to the complex nature of the consumption map.

- The derivative of the following term with respect to the Speed can be calculated:

v ÿ ÿS ÿ ÿ ÿ ÿ ÿ ÿ ÿ G / b S g ÿ ÿ s

- The altitude above ground during climb in an altitude interval ie The distance travelled RSteig results from the climbing triangle

$\frac{ie}{dR}$ ÿ tanÿ ÿ g through integration: R Climb ÿ ÿ h2 h1 i.e. / tan ÿ S/G ÿ ÿ ÿ ÿ ÿ ÿ

G Flight performance

3.2 Climb performance with quadratic drag polar

- The previous quasi-stationary consideration was carried out without inertia terms.
- This is equivalent to neglecting the influence of kinetic energy and assuming that the engine energy is used exclusively to increase the potential energy.
- With the tangential acceleration and without centripetal acceleration (intercepting arc curvature) we get

Sÿ W ÿ G ÿsin g ÿ ÿ v 0 G/G ÿ w ÿ V ÿsinÿgÿ with and vÿ ÿ dv/engl

as well as w ÿ ie/engl to SWG is ÿ1ÿg ÿ ÿ ÿ ÿ v dh/G dv/ÿ ÿÿ ÿ 0

G Flight performance

3.2 Climb performance with quadratic drag polar

- This means that the rate of climb

$$w = v \cdot \sin\gamma = \left(\frac{S - W}{G} \cdot v\right) \cdot \left(\frac{1}{1 + \frac{v}{G} \cdot \frac{dv}{ie}}\right)$$

- This is the well-known quasi-stationary solution with a correction factor for the conversion of kinetic energy into potential energy.
- For a climb with a constant lift coefficient, this correction can be neglected with sufficient accuracy for relatively slow aircraft (e.g. propeller aircraft, M = 0.5)
- The value of the factor is only about 0.85, ie the actual rate of climb reaches only 85% of the calculated rate of climb.

G Flight performance

3.2 Climb performance with quadratic drag polar

- For aircraft operating at high subsonic speeds (M = 1), the value of the factor is about 0.59 and should not be neglected.
- For supersonic aircraft (M = 2), a kinetic energy correction is highly recommended, since values of approximately 0.26 are applicable here.
- Since the change in speed due to altitude gain, the measure dv/dh, is not known in general, this consideration is not applicable in practice.

G Flight performance

3.3 Climb performance using energy altitude method

- The energy altitude method is more suitable for calculating climb performance. This takes into account the total energy during climb
- This also takes into account that part of the engine energy is converted into acceleration of the aircraft.
- The total energy is defined as the sum of the potential and the kinetic energy:

$$E = E_{pot} + E_{kin} = mgh + \frac{1}{2}mv^2 = G\left(h + \frac{v^2}{2g}\right) = \left(\frac{E}{G}\right)G$$

- The expression in brackets is the energy height or weight-specific total energy with the dimension of a length, which corresponds to the sum of geometric height and a height equivalent to the kinetic energy.

G Flight performance

3.3 Climb performance using energy altitude method

- Using the principle of conservation of energy E in time, the influence valley dt of the weight change due to the Determine fuel consumption:

$$dE\ S\ v\ dt\ W\ v\ dt \quad \frac{\partial E}{\partial G}\ dG \quad K$$

- The change in total energy is equal to the thrust energy minus the energy dissipation due to resistance and the energy change corresponding to the weight loss.

- The total differential of the total energy leads to:

$$dE\ d \quad \left(\frac{E}{G}\right) GG\ d \quad \left(\frac{E}{G}\right) \left(\frac{E}{G}\right) dG$$

- The weight change corresponds to the fuel consumption dG

G Flight performance

3.3 Climb performance using energy altitude method

• If the two equations above are linked and

with the weight change dG = -dGK follows

G d ÿ ÿ ÿ ÿ ÿ EEE ÿ ÿ ÿ ÿ ÿ GG • dG S v ÿ dt K ÿÿ ÿ ÿ E ÿ ÿ ÿ ÿ G dG W v dt K

Solving for the time derivative of the energy level gives

from this the basic equation for climbing:

d ES ÿ v ÿ W ÿ ÿ ÿ ÿ SW ÿ ÿ v

dt ÿ ÿ GGG • This again G

gives us the rate of climb known from the quasi-stationary

consideration, which now expresses the change in energy level

over time. • The change in the specific total energy

over time is therefore equal to the weight-specific power surplus.

G Flight performance

3.3 Climb performance using the energy altitude

method • The above correction factor for the kinetic energy can be

also derived from this consideration. The derivation of the energy level equation ÿ ÿ ÿ dd ÿ ÿ

ÿ ÿ h ÿ ÿ ÿ ÿ dt G dt ÿ

E 2 v ÿ dh dv v ÿgÿ ÿ dt dt

ÿ ÿ 2 g ÿ

inserted into the gradient equation

w ÿ ÿ ie / dt G gives ÿSW ÿ v ÿ v dv / g dt and with dv / dt ÿ ÿ dv / ie w

ÿ ÿ ÿ ÿ ÿ ÿ leads. dv

back to the equation w ÿ ÿSW v ÿ / G ÿ ÿ v 1 ÿ ÿÿ ÿ ÿ dh 1

ÿ g

G Flight performance

3.3 Climb performance using energy altitude method

• The energy level is only a function of the speed and altitude.

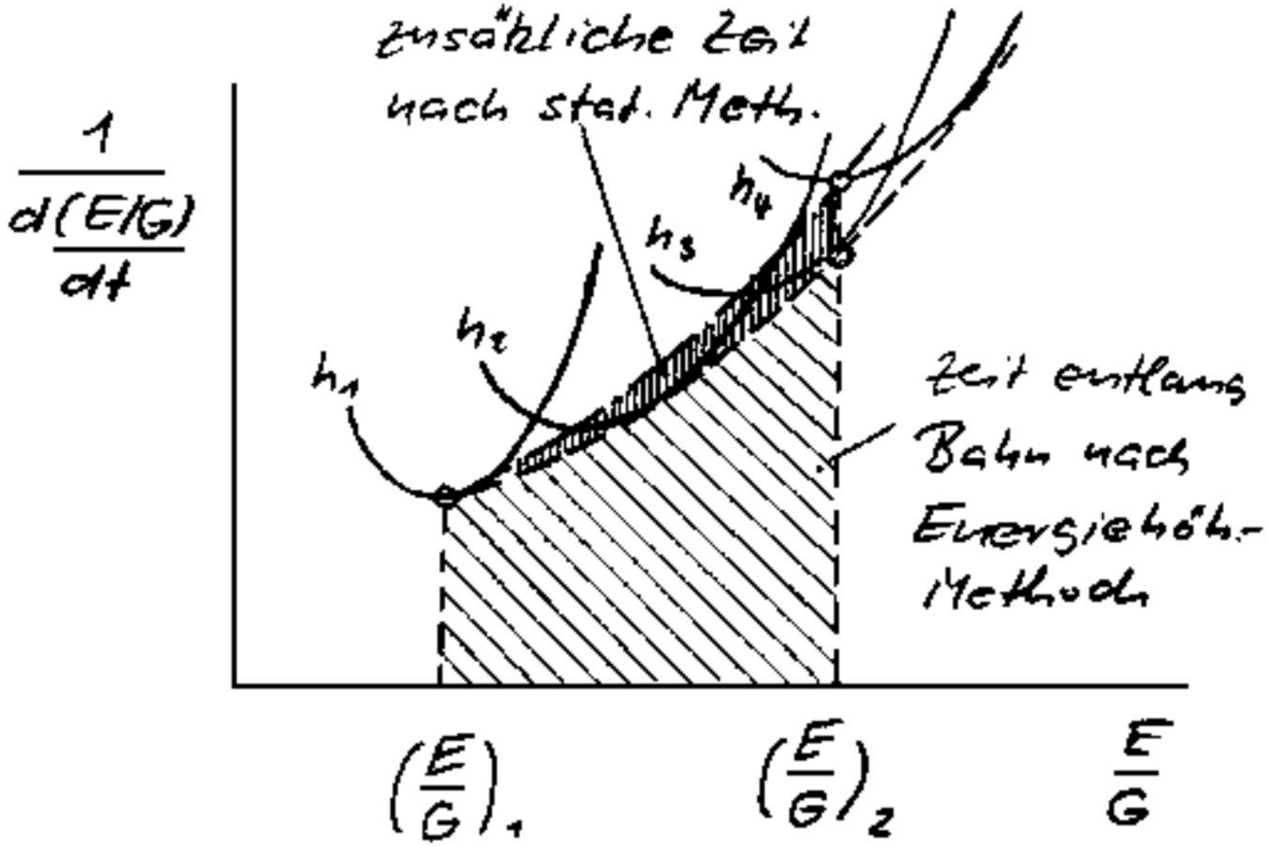
• The climb equation in energy notation does not include the change in speed with altitude and is therefore easy to use for the general case. The calculation of the climb performance parameters is thus considerably simplified. • To calculate the trajectory of minimum flight time to get from one combination of v and h to another, one can write the energy altitude (E/ G) as an independent variable after introducing

$$t = \int_{\left(\frac{E}{G}\right)_1}^{\left(\frac{E}{G}\right)_2} \frac{1}{\left(\frac{d\left(E/G\right)}{dt}\right)_{engl}} \, d\left(\frac{E}{G}\right)$$

G Flight performance

3.3 Climb performance using energy altitude method

- **The solution of the integral can be done graphically if the integrands (reciprocal power surplus) for several flight altitudes are plotted against the energy level.**



G Flight performance

3.3 Climb performance using energy altitude method

- The distance covered above ground during a climb, which will be added to the cruise distance, can be determined in a similar way.

- The above integrand is multiplied by the flight speed and one obtains

$$R_{Climb} = \int_{\left(\frac{E}{G}\right)_1}^{\left(\frac{E}{G}\right)_2} \frac{v}{\left(\frac{d\left(E/G\right)}{dt}\right)_{engl}} \, d\left(\frac{E}{G}\right)$$

- The path with minimum climb consumption is also easy to determine. Using the relationship derived above,

$$\frac{d\left(E/G\right)}{dG} = \frac{d\left(E/G\right)/dt}{dG/dt} = \frac{\left(S - W\right)v/G}{dG/dt} = \frac{\ldots}{gm} = \frac{\ldots}{g\, b_S\, S}$$

G Flight performance

3.3 Climb performance using energy altitude method

- With the relationship derived above

$$\frac{d\,E}{dG_K} = \frac{(S - W)\,v}{g\, b_S\, S}$$

- fuel consumption is

of the climb to

$$G_{k,st} = \int_{(E/G)_1}^{(E/G)_2} \frac{1}{\frac{d(E/G)}{dG_k}}\, d\left(\frac{E}{G}\right)$$

- This integral is again easy to solve in the above manner.
- Optimizing the climb fuel, as would be possible with this consideration, is generally not interesting, since the climb to the maximum operational altitude is completed in the shortest possible time due to the fact that the cruise consumption decreases with the current flight altitude but is m