Ana Beatriz Stahl

① $A(0,2,3)$ $B(2,1,1)$ $C(3,2,4)$ $D(1,3,5)$

a) $\vec{AB} = B - A = (2,-1,-2)$ $|\vec{AB}| = \sqrt{(2)^2+(-1)^2+(-2)^2} = \sqrt{4+1+4} = \sqrt{9} = 3$

$\vec{AD} = D - A = (1,1,2)$ $|\vec{AD}| = \sqrt{(1)^2+(1)^2+(2)^2} = \sqrt{1+1+2} = \sqrt{4} = 2$

$\vec{AB} \cdot \vec{AD} = |\vec{AB}| \cdot |\vec{AD}| \cdot \cos(\theta)$

$$\cos(\theta) = \frac{\vec{AB}\cdot\vec{AD}}{|\vec{AB}|\cdot|\vec{AD}|} = \frac{(2,-1,-2)\cdot(1,1,2)}{|(2,-1,-2)|\cdot|(1,1,2)|} = \frac{2-1-4}{3\cdot 2} = \frac{-3}{6} = \frac{-1}{2} = -0,5$$

$\theta = \cos^{-1} -0,5 = 120° \rightarrow \boxed{\theta = 120°}$

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b) $A_P = |\vec{AB} \times \vec{AD}| \rightarrow A_T = \frac{1}{2} A_P = \frac{1}{2} \cdot 3\sqrt{5} = \frac{3\sqrt{5}}{2}$ u.a. $\rightarrow \boxed{A_T = \frac{3\sqrt{5}}{2} \text{ u.a.}}$

$\vec{AB} = (2,-1,-2)$

$\vec{AD} = (1,1,2)$

$$\vec{AB} \times \vec{AD} = \left|\begin{array}{ccc|cc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ 2 & -1 & -2 & 2 & -1 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right| = (-2-(-2))\vec{i} + (-2-4)\vec{j} + (2-(-1))k$$

$\vec{AB} \times \vec{AD} = 0\vec{i} - 6\vec{j} + 3\vec{k} = (0,-6,3)$

$A_P = |(0,-6,3)| = \sqrt{0^2+(-6)^2+3^2} = \sqrt{45} = \sqrt{3^2\cdot 5} = 3\sqrt{5}$

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c) $V_P = |\vec{AB} \cdot (\vec{AC} \times \vec{AD})|$

$\vec{AB} = (2,-1,-2)$

$\vec{AC} = C - A = (3,0,1)$

$\vec{AD} = (1,1,2)$

$$V_P = \left|\begin{array}{ccc|cc} 2 & -1 & -2 & 2 & -1 \\ 3 & 0 & 1 & 3 & 0 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right| = (2\cdot0\cdot2 + (-1)\cdot1\cdot1 + (-2)\cdot3\cdot1) - (-2\cdot0\cdot1 + 2\cdot1\cdot1 + (-1)\cdot3\cdot2)$$

$V_P = -7 - (-4) = -3$

$V_T = \frac{1}{6} V_P = \frac{1}{6}(-3) = \frac{-3}{6} = \frac{-1}{2} \rightarrow \boxed{V_T = \frac{-1}{2}}$

② $r_1: \begin{cases} y = 2 + 3x \\ z = 5x - 1 \end{cases}$ $\quad r_2: \dfrac{x+5}{-2} = y - 3 = \dfrac{z+8}{3}$ $\quad P(1,5,7)$

$r_1: P_1(0, 2, -1)$

$P_2(2, 8, 9)$

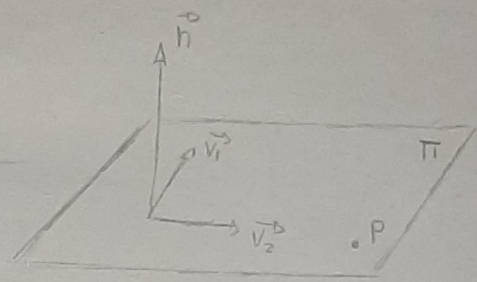
$\overrightarrow{P_1P_2} = P_2 - P_1 = (2, 6, 10)$

$r_1 = (1, 5, 4) + (2, 6, 10)t$

$\vec{v_1} = (2, 6, 10)$

$r_2 = (5, -3, 8) + (-2, 1, 3)t$

$\vec{v_2} = (-2, 1, 3)$



$\vec{n} = \vec{v_1} \times \vec{v_2}$

$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 6 & 10 \\ -2 & 1 & 3 \end{vmatrix} \begin{matrix} \vec{i} & \vec{j} \\ 2 & 6 \\ -2 & 1 \end{matrix}$$

$\vec{n} = (18 - 10)\vec{i} + (-20 - 6)\vec{j} + (2 - (-12))\vec{k}$

$\vec{n} = 8\vec{i} - 26\vec{j} + 14\vec{k} = (8, -26, 14)$

$ax + by + cz + d = 0$

$8x - 26y + 14z + d = 0 \longleftarrow P(\underset{x}{1}, \underset{y}{5}, \underset{z}{7})$

$8 \cdot 1 - 26 \cdot 5 + 14 \cdot 7 + d = 0$

$8 - 130 + 98 + d = 0$

$-24 + d = 0$

$d = 24$

Equação Geral da Reta:

$\boxed{8x - 26y + 14z + 24 = 0}$

(3) $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 3 \\ 3 & 9 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 2 & 4 \\ 6 & 4 & 1 \\ 1 & 8 & 9 \end{bmatrix}$

a)

$$\begin{array}{cc} & B \\ & \begin{bmatrix} 5 & 2 & 4 \\ 6 & 4 & 1 \\ 1 & 8 & 9 \end{bmatrix} \\ A & \\ \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 3 \\ 3 & 9 & 6 \end{bmatrix} & \begin{matrix} 25 & 30 & 25 \\ 43 & 48 & 40 \\ 75 & 90 & 75 \end{matrix} \end{array}$$

$AB = \begin{bmatrix} 25 & 30 & 25 \\ 43 & 48 & 40 \\ 75 & 90 & 75 \end{bmatrix}$

$$\boxed{AB^T = \begin{bmatrix} 25 & 43 & 75 \\ 30 & 48 & 90 \\ 25 & 40 & 75 \end{bmatrix}}$$

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b) $AB^T = \begin{bmatrix} 25 & 43 & 75 \\ 30 & 48 & 90 \\ 25 & 40 & 75 \end{bmatrix}$ $AB^{T^{-1}} = ?$

1º DETERMINANTE:

$\det = \begin{vmatrix} 25 & 43 & 75 \\ 30 & 48 & 90 \\ 25 & 40 & 75 \end{vmatrix} = 0$

2º MATRIZES DOS COFATORES:

$\begin{vmatrix} 48 & 90 \\ 40 & 75 \end{vmatrix} = 0$ $\begin{vmatrix} 30 & 90 \\ 25 & 75 \end{vmatrix} = 0$ $\begin{vmatrix} 30 & 48 \\ 25 & 40 \end{vmatrix} = 0$

$\begin{vmatrix} 43 & 75 \\ 40 & 75 \end{vmatrix} = 225$ $\begin{vmatrix} 25 & 75 \\ 25 & 75 \end{vmatrix} = 0$ $\begin{vmatrix} 25 & 43 \\ 25 & 40 \end{vmatrix} = -75$

$\begin{vmatrix} 43 & 75 \\ 48 & 90 \end{vmatrix} = 225$ $\begin{vmatrix} 25 & 75 \\ 30 & 90 \end{vmatrix} = 0$ $\begin{vmatrix} 25 & 43 \\ 30 & 48 \end{vmatrix} = -90$

$COF = \begin{vmatrix} 0 & 0 & 0 \\ -225 & 0 & 75 \\ 225 & 0 & -90 \end{vmatrix}$

$COF^T = \begin{vmatrix} 0 & -225 & 225 \\ 0 & 0 & 0 \\ 0 & 75 & -90 \end{vmatrix}$

3º DIVIDIR PELO DETERMINANTE:

$\boxed{\text{INVERSA: NÃO EXISTE}}$

4. a) $\begin{cases} 2x-3y+2z=2 \\ -x-2y-3z=5 \\ 5x-11y+3z=11 \end{cases}$

$$\left[\begin{array}{ccc|c} 2 & -3 & 2 & 2 \\ -1 & -2 & -3 & 5 \\ 5 & -11 & 3 & 11 \end{array}\right] \to \left[\begin{array}{ccc|c} -1 & -2 & -3 & 5 \\ 2 & -3 & 2 & 2 \\ 5 & -11 & 3 & 11 \end{array}\right] \times 2 \to \left[\begin{array}{ccc|c} -2 & -4 & -6 & 10 \\ 2 & -3 & 2 & 2 \\ 5 & -11 & 3 & 11 \end{array}\right] +$$

$$\left[\begin{array}{ccc|c} -1 & -2 & -3 & 5 \\ 0 & -7 & -4 & 12 \\ 5 & -11 & 3 & 11 \end{array}\right] \times 5 \to \left[\begin{array}{ccc|c} -5 & -10 & -15 & 25 \\ 0 & -7 & -4 & 12 \\ 5 & -11 & 3 & 11 \end{array}\right] + \left[\begin{array}{ccc|c} -1 & -2 & -3 & 5 \\ 0 & -7 & -4 & 12 \\ 0 & -21 & -12 & 36 \end{array}\right] \underset{\times(-3)}{\to} \left[\begin{array}{ccc|c} -1 & -2 & -3 & 5 \\ 0 & 21 & 12 & -36 \\ 0 & -21 & -12 & 36 \end{array}\right] +$$

$$\left[\begin{array}{ccc|c} -1 & -2 & -3 & 5 \\ 0 & -7 & -4 & 12 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

SPI $\begin{cases} -x-2y-3z=5 \\ -7y-4z=12 \to \boxed{y=\frac{12+4z}{-7}} \end{cases}$ $\boxed{z \in \mathbb{R}}$

$\to -x-2\left(\frac{12+4z}{-7}\right)-3z=5$

$-x=5+3z+2\left(\frac{12+4z}{-7}\right) \to \boxed{x=-5-3z-2\left(\frac{12+4z}{-7}\right)}$

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b) $\begin{cases} x_1+x_3=0 \\ x_2+x_4=1 \\ x_2+x_3=0 \\ x_3+x_4=1 \end{cases}$

$$\left|\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{array}\right| \times(-1) \left|\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & -1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{array}\right| + \left|\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 1 & 1 \end{array}\right| \times(-1)$$

$$\left|\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{array}\right| + \to \left|\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 2 & 2 \end{array}\right|$$

$\to x_1+1\cdot 0+0\cdot 1=0 \to x_1=0$

$\to x_2+1\cdot 1=1 \to x_2=0$

$\to x_3-1\cdot 1=-1 \to x_3=0$

$\to 2x_4=2 \to x_4=1$

$$\boxed{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}}$$