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① A(0,2,3) B(2,1,1) C(3,2,4) D(1,3,5)

a) $\vec{AB} = B - A = (2,-1,-2)$ $\quad |\vec{AB}| = \sqrt{(2)^2+(-1)^2+(-2)^2} = \sqrt{4+1+4} = \sqrt{9} = 3$

$\vec{AD} = D - A = (1,1,2)$ $\quad |\vec{AD}| = \sqrt{(1)^2+(1)^2+(2)^2} = \sqrt{1+1+2} = \sqrt{4} = 2$

$\vec{AB} \cdot \vec{AD} = |\vec{AB}| \cdot |\vec{AD}| \cdot \cos(\theta)$

$$\cos(\theta) = \frac{\vec{AB} \cdot \vec{AD}}{|\vec{AB}| \cdot |\vec{AD}|} = \frac{(2,-1,-2)\cdot(1,1,2)}{|(2,-1,-2)| \cdot |(1,1,2)|} = \frac{2-1-4}{3 \cdot 2} = \frac{-3}{6} = \frac{-1}{2} = -0,5$$

$\theta = \cos^{-1} -0,5 = 120° \rightarrow \boxed{\theta = 120°}$

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b) $A_p = |\vec{AB} \times \vec{AD}| \rightarrow A_T = \frac{1}{2} A_p = \frac{1}{2} \cdot 3\sqrt{5} = \frac{3\sqrt{5}}{2}$ u.a. $\rightarrow \boxed{A_T = \frac{3\sqrt{5}}{2} \text{ u.a.}}$

$\vec{AB} = (2,-1,-2)$

$\vec{AD} = (1,1,2)$

$$\vec{AB} \times \vec{AD} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & -2 \\ 1 & 1 & 2 \end{vmatrix} \begin{matrix} \vec{i} & \vec{j} \\ 2 & -1 \\ 1 & 1 \end{matrix} = (-2-(-2))\vec{i} + (-2-4)\vec{j} + (2-(-1))\vec{k}$$

$\vec{AB} \times \vec{AD} = 0\vec{i} - 6\vec{j} + 3\vec{k} = (0,-6,3)$

$A_p = |(0,-6,3)| = \sqrt{0^2+(-6)^2+3^2} = \sqrt{45} = \sqrt{3^2 \cdot 5} = 3\sqrt{5}$

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c) $V_p = |\vec{AB} \cdot (\vec{AC} \times \vec{AD})|$

$\vec{AB} = (2,-1,-2)$

$\vec{AC} = C - A = (3,0,1)$

$\vec{AD} = (1,1,2)$

$$V_p = \begin{vmatrix} 2 & -1 & -2 \\ 3 & 0 & 1 \\ 1 & 1 & 2 \end{vmatrix} \begin{matrix} 2 & -1 \\ 3 & 0 \\ 1 & 1 \end{matrix} = (2\cdot0\cdot2 + (-1)\cdot1\cdot1 + (-2)\cdot3\cdot1) - (-2\cdot0\cdot1 + 2\cdot1\cdot1 + (-1)\cdot3\cdot2)$$

$V_p = -7 - (-4) = -3$

$V_T = \frac{1}{6} V_p = \frac{1}{6}(-3) = \frac{-3}{6} = \frac{-1}{2} \rightarrow \boxed{V_T = \frac{-1}{2}}$

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② $r_1: \begin{cases} y = 2 + 3x \\ z = 5x - 1 \end{cases}$ $\quad r_2: \frac{x+5}{2} = y - 3 = \frac{z+8}{2}$ $\quad P(1,5,7)$

$r_1: P_1(1,5,4)$

$P_2(2,8,9)$

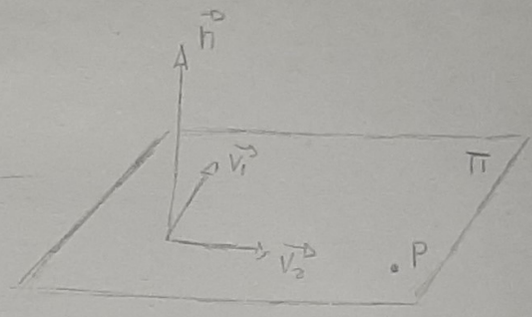
$\overrightarrow{P_1P_2} = P_2 - P_1 (1,3,5)$

$r_1 = (1,5,4) + (1,3,5)t$

$\vec{v_1} = (1,3,5)$

$r_2 = (5,-3,8) + (2,1,2)t$

$\vec{v_2} = (2,1,2)$



$\vec{n} = \vec{v_1} \times \vec{v_2}$

$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 3 & 5 \\ 2 & 1 & 2 \end{vmatrix} \begin{matrix} \vec{i} & \vec{j} \\ 1 & 3 \\ 2 & 1 \end{matrix}$$

$\vec{n} = (6-5)\vec{i} + (10-2)\vec{j} + (1-6)\vec{k}$

$\vec{n} = \vec{i} + 8\vec{j} - 5\vec{k} = (1,8,-5)$

$ax + by + cz + d = 0$

$x + 8y - 5z + d = 0 \leftarrow P(1,5,7)$

$x\ y\ z$

$1 + 8 \cdot 5 - 5 \cdot 7 + d = 0$

$1 + 40 - 35 + d = 0$

$6 + d = 0$

$d = -6$

Equação Geral da Reta:

$$\boxed{x + 8y - 5z - 6 = 0}$$

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③ $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 3 \\ 3 & 9 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 2 & 4 \\ 6 & 4 & 1 \\ 1 & 8 & 9 \end{bmatrix}$

a)

A	B		
	5	2	4
	6	4	1
	1	8	9
1 3 2	25	30	25
2 5 3	43	48	40
3 9 6	75	90	75

$AB = \begin{bmatrix} 25 & 30 & 25 \\ 43 & 48 & 40 \\ 75 & 90 & 75 \end{bmatrix}$

$AB^T = \begin{bmatrix} 25 & 43 & 75 \\ 30 & 48 & 90 \\ 25 & 40 & 75 \end{bmatrix}$

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b) $AB^T = \begin{bmatrix} 25 & 43 & 75 \\ 30 & 48 & 90 \\ 25 & 40 & 75 \end{bmatrix}$ $AB^{T-1} = ?$

1º DETERMINANTE:

$$\det = \begin{vmatrix} 25 & 43 & 75 \\ 30 & 48 & 90 \\ 25 & 40 & 75 \end{vmatrix} = 0$$

2º MATRIZES DOS COFATORES:

$\begin{vmatrix} 48 & 90 \\ 40 & 75 \end{vmatrix} = 0$ $-\begin{vmatrix} 30 & 90 \\ 25 & 75 \end{vmatrix} = 0$ $\begin{vmatrix} 30 & 48 \\ 25 & 40 \end{vmatrix} = 0$

$\begin{vmatrix} 43 & 75 \\ 40 & 75 \end{vmatrix} = 225$ $\begin{vmatrix} 25 & 75 \\ 25 & 75 \end{vmatrix} = 0$ $\begin{vmatrix} 25 & 43 \\ 25 & 40 \end{vmatrix} = -75$

$$COF = \begin{vmatrix} 0 & 0 & 0 \\ -225 & 0 & 75 \\ 225 & 0 & -90 \end{vmatrix}$$

$\begin{vmatrix} 43 & 75 \\ 48 & 90 \end{vmatrix} = 225$ $\begin{vmatrix} 25 & 75 \\ 30 & 90 \end{vmatrix} = 0$ $\begin{vmatrix} 25 & 43 \\ 30 & 48 \end{vmatrix} = -90$

$$COF^T = \begin{vmatrix} 0 & -225 & 225 \\ 0 & 0 & 0 \\ 0 & 75 & -90 \end{vmatrix}$$

3º DIVIDIR PELO DETERMINANTE:

INVERSA! NÃO EXISTE

4. a) $\begin{cases} 2x - 3y + 2z = 2 \\ -x - 2y - 3z = 5 \\ 5x - 11y + 3z = 11 \end{cases}$

$\left[\begin{array}{ccc|c} 2 & -3 & 2 & 2 \\ -1 & -2 & -3 & 5 \\ 5 & -11 & 3 & 11 \end{array}\right] \curvearrowright \left[\begin{array}{ccc|c} -1 & -2 & -3 & 5 \\ 2 & -3 & 2 & 2 \\ 5 & -11 & 3 & 11 \end{array}\right] \xrightarrow{\times 2} \left[\begin{array}{ccc|c} -2 & -4 & -6 & 10 \\ 2 & -3 & 2 & 2 \\ 5 & -11 & 3 & 11 \end{array}\right] \curvearrowright +$

$\left[\begin{array}{ccc|c} -1 & -2 & -3 & 5 \\ 0 & -7 & -4 & 12 \\ 5 & -11 & 3 & 11 \end{array}\right] \xrightarrow{\times 5} \left[\begin{array}{ccc|c} -5 & -10 & -15 & 25 \\ 0 & -7 & -4 & 12 \\ 5 & -11 & 3 & 11 \end{array}\right] \curvearrowright + \quad \left[\begin{array}{ccc|c} -1 & -2 & -3 & 5 \\ 0 & -7 & -4 & 12 \\ 0 & -21 & -12 & 36 \end{array}\right] \xrightarrow[\times(-3)]{} \left[\begin{array}{ccc|c} -1 & -2 & -3 & 5 \\ 0 & 21 & 12 & -36 \\ 0 & -21 & -12 & 36 \end{array}\right] \curvearrowright +$

$\left[\begin{array}{ccc|c} -1 & -2 & -3 & 5 \\ 0 & -7 & -4 & 12 \\ 0 & 0 & 0 & 0 \end{array}\right]$ SPI $\begin{cases} -x - 2y - 3z = 5 \\ -7y - 4z = 12 \rightarrow \boxed{y = \frac{12 + 4z}{-7}} \end{cases}$ $\boxed{z \in \mathbb{R}}$

$\hookrightarrow -x - 2\left(\frac{12+4z}{-7}\right) - 3z = 5$

$-x = 5 + 3z + 2\left(\frac{12+4z}{-7}\right) \rightarrow \boxed{x = -5 - 3z - 2\left(\frac{12+4z}{-7}\right)}$

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b) $\begin{cases} x_1 + x_3 = 0 \\ x_2 + x_4 = 1 \\ x_2 + x_3 = 0 \\ x_3 + x_4 = 1 \end{cases}$

$\left|\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{array}\right| \xrightarrow{\times(-1)} \left|\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & -1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{array}\right| \curvearrowright + \quad \left|\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 1 & 1 \end{array}\right| \xrightarrow{\times(-1)}$

$\left|\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{array}\right| \rightarrow \curvearrowright + \quad \left|\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 2 & 2 \end{array}\right|$

$\rightarrow x_1 + 1 \cdot 0 + 0 \cdot 1 = 0 \rightarrow x_1 = 0$

$\rightarrow x_2 + 1 \cdot 1 = 1 \rightarrow x_2 = 0$

$\rightarrow x_3 - 1 \cdot 1 = -1 \rightarrow x_3 = 0$

$\rightarrow 2x_4 = 2 \rightarrow x_4 = 1$

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

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