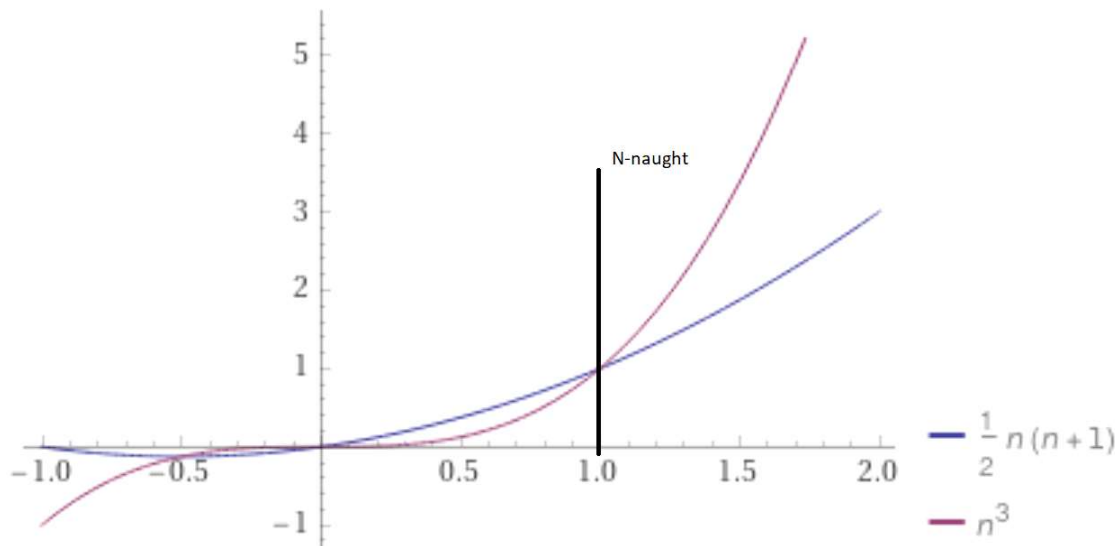


1:A  $n(n+1)/2 \in O(n^3)$  Answer: True.  $n(n+1)/2$  has the same or lower order of growth approximately where  $n$ -naught is shown on the graph.

$$n(n+1)/2 = 1/2n^2 + 1/2n \text{ multiply "1/2n" term by } n \text{ and combine} = n^2$$

$$n^2 \leq n^3$$

so  $n$ -naught = 1 and  $c = 1$



1:B  $n(n+1)/2 \in \Theta(n^2)$  Answer: True.

$$n(n+1)/2 = 1/2n^2 + 1/2n \text{ multiply "1/2n" term by } n \text{ and combine} = n^2$$

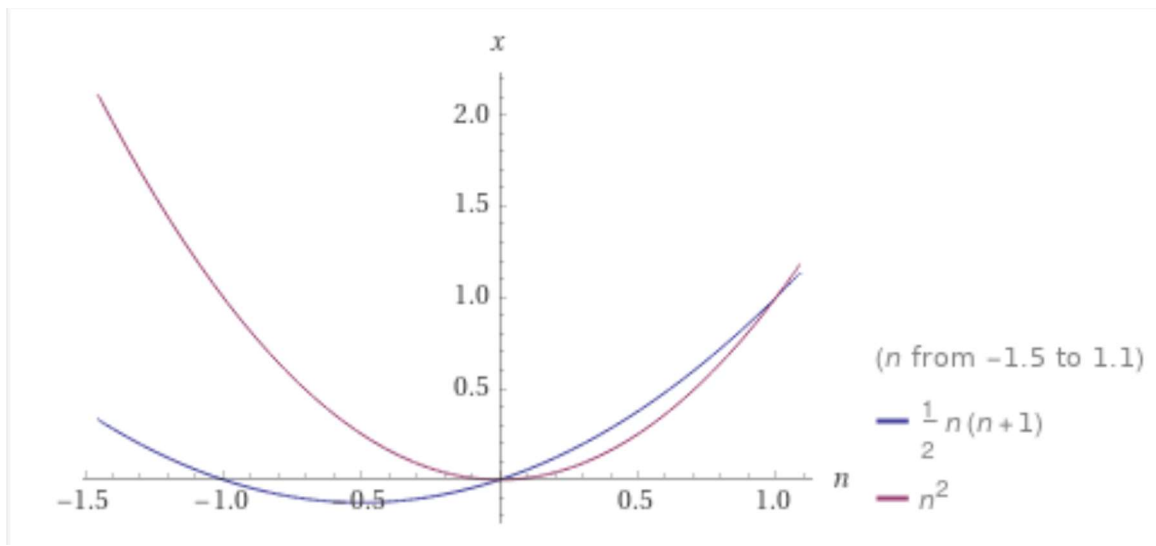
$$n^2 \leq n^2 \text{ exactly}$$

limit of  $(n(n+1)/2)/(n^2)$ , apply l hospitals

limit of  $(n+1/2)/n$  apply again

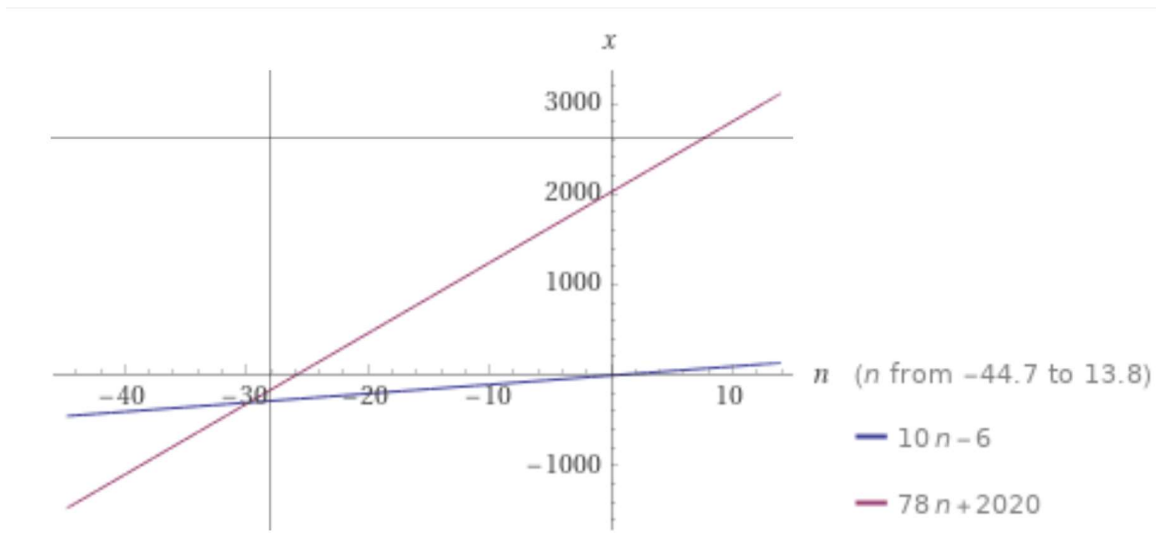
limit of  $(1/2) / 1 = 1/2$  so a constant

implies that  $n(n+1)/2 \in \Theta(n^2)$



1:C  $10n-6 \in \Omega(78n+2020)$  False.

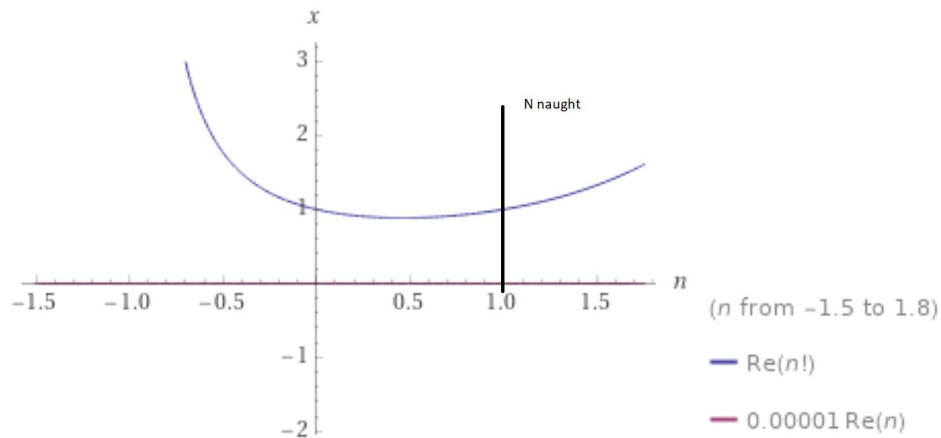
limit  $10n-6/78n+2020 = 10/78$  which is a constant so these functions have the same order of growth.



1:D  $n! \in \Omega(0.00001n)$  True

lmt  $n \rightarrow \infty$   $n! / 0.00001n = n \cdot (n-1) \cdot (n-2) \dots / 0.00001n = (n-1) \cdot (n+2) \dots / 0.0002$  applying limit is infinity

which implies that  $n! \in \Omega(0.00001n)$



2:A This function returns the difference between the max value and the min value in the array.

2:B The line "for i = 1 to n-1:"

2:C  $N-1$  times. The loop goes from 1 to  $n-1$  but has to cycle one more time for the value where it skips over the loop. So add 1 and get 1 to  $n$  or  $n-1$

2:D  $O(n)$

3:A The loop invariant is the sub array  $[i+1:j-1]$  has been reversed compared to the original array

3:B

Example  $n = 6$

Base Case: beginning of the loop.  $i = 2$  and  $j = 3$   $i+1 > j-1$  so array would be reversed of the original

Inductive case: Assume loop invariant is true for  $i$ th iteration. Assuming at the  $i$ th iteration the current array from  $[A_{i+1}:A_{j-1}]$  is already reversed. The body of the loop will swap the current  $i$  and  $j$  elements outside of the currently reversed array making a now  $[A_i:A_j]$  reversed array. Then  $i$  and  $j$  are decremented and incremented respectfully which means at the end of each loop cycle the loop invariant holds true.

Termination: When the loop terminates  $i = -1$  and  $j = n$  this combined with the loop invariant

tells us that the entire array from A0:An has been fully reversed.