Model

Goal: Start with the simplest, direct translation of (Thomas and Worrall, 1988), and update one aspect a meeting.

What to change?

- Shocks to water, not spot-market prices (add or switch?)
- Add upstream suppliers who also have contracts with downstream
- Upstream chooses a fraction of water to send
- Downstream not risk neutral
- Add covariance between upstream, downstream shocks (iid $\rightarrow$ perfect correlation (which is like a line))
- Renegotiation (apparently straightforward)

Setting

- Two towns $i \in \{u, d\}$ (upstream, downstream).
- Infinite horizon $t = 1, 2, \ldots, \infty$, discount factor δ.
- In each period, u is endowed with 1 unit of water that it can transfer downstream in return for ρ or keep to produce 1 unit of cassava which can be sold for spot-market price $p(s)$
- Finite set of states $s_t \in \{1, 2, \ldots, S\}$, $S \geq 2$ at each date. The state (i.i.d for now) determines the value of the cassava u could sell or d could buy on the spot market.
- Agents can either negotiate a contract (without commitment) at $t = 0$ or move to the spot market.

Assumption: Reneging puts an agent on the spot market forever.

Preferences:

- Upstream is risk averse, and gets utility $u = u(\rho) : [a, b] \to \mathbb{R}$ (differentiable, strictly increasing, strictly concave) from transfer ρ received.
- Downstream is risk-neutral.

States:

- The state s is identified by the spot-market price for the produced good, $p(s)$
- States are iid such that $a < p(1) < p(S) < b \leq 1$
- Probability of state s is π_s, for $\sum_{s=1}^{S} \pi_s = 1$

Because 1 unit of water produces 1 unit of cassava and $b \leq 1$, downstream always (weakly) wants water (check)

Expected spot price:

$$p^* := \sum_{s=1}^{S} \pi_s p(s)$$

Certainty-equivalent spot price:

$$p_* := u^{-1}\left(\sum_{s=1}^{S} \pi_s u\left(p(s)\right)\right)$$

Because utility is concave, $p^* > p_*$

Timing:

1) State s is observed by both parties
2) Upstream and downstream simultaneously decide whether to honor the contract (upstream transfers water, downstream pays transfer ρ) or renege (and take/pay spot-market prices $p(s)$)

For s_t the state in time t, define

- Contract $\mathcal{P}(\cdot)$ for each t and each history $h_t = (s_1, s_t, \dots, s_t)$
- The contract defines the transfer $\rho(h_t)$ from downstream to upstream as an infinite sequence $(\rho(h_t))_{t=1}^{\infty}$, for $\rho(h_t) \geq 0$ the transfer paid at date t, after history h_t

Define $U_t(h_t)$ as the expected utility gain of honoring vs. one-time reneging for upstream under contract $\mathcal{P}$, history h_t, and analogously $V_t(h_t)$ for downstream.

$$
\begin{aligned}
U_t(h_t;\mathcal{P}) = \\
& u\left(\rho(h_t)\right) - u\left(p(s_t)\right) + \quad \text{(SR gain)} \\
& \mathbb{E}\left\{ \sum_{j=t+1}^{\infty} \delta^{j-t} \left[u\left(\rho(h_j)\right) - u(p(s_j)) \right] | h_t \right\} \quad \text{(LR gain)}
\end{aligned}
\tag{1}
$$

If $u(\rho(h_t)) < u(p(s_t))$, upstream has a short-term incentive to renege

$$
\begin{aligned}
V_t(h_t;\mathcal{P}) = & \\
& p(s_t) - \rho(h_t) + \quad \text{(SR incentive)} \\
& \mathbb{E}\left\{ \sum_{j=t+1}^{\infty} \delta^{j-t} \left[p(s_j) - \rho(s_j) \right] | h_t \right\} \quad \text{(LR incentive)}
\end{aligned} \tag{1}
$$

If $\rho(h_t) > p(s_t)$, downstream has short-term incentive to renege and just buy cassava

The contract is self-enforcing if for all histories h_t the participation constraints hold for both up- and downstream.

$$U_t(h_t) \geq 0 \tag{2}$$

$$V_t(h_t) \geq 0 \tag{3}$$

Define $\Lambda(s_t)$ the set of contracts which satisfy (2) and (3) after history (h_{t-1}, s_t).

Constrained efficient contracts: Contracts which are self-enforcing and not Pareto dominated by other self-enforcing contracts.

Efficient contract $\mathcal{P}$: solves

$$\sup_{\mathcal{P}\in\Lambda(\mathbf{s_1})} \left\{ V_1(s_1;\mathcal{P}) | U_1(s_1;\mathcal{P}) \geq \hat{U}_1 \right\} \tag{4}$$

Varying $\hat{U}_1$ traces out the constrained Pareto frontier.

Proposition

An efficient contract has an interval of payments $[\underline{\rho}_s, \overline{\rho}_s]$ *for each state* s*. For any history* (h_t, s)*, the contract wage at* $t+1$ *satisfies:*

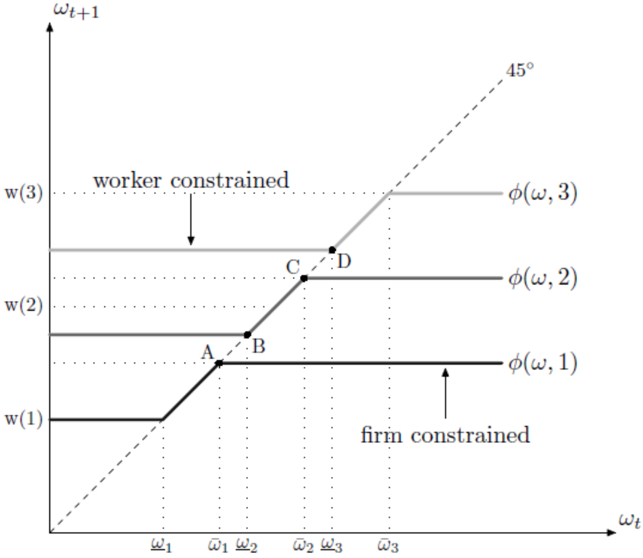
$$\rho(h_t, s) = \begin{cases} \overline{\rho}_s, & \textit{if } \rho(h_t) > \rho_s \\ \rho(h_t) & \textit{if } \rho(h_t) \in [\underline{\rho}_s, \overline{\rho}_s] \\ \underline{\rho}_s, & \textit{if } \rho(h_t) < \underline{\rho}_s \end{cases}$$

For any states $k > s$*,* $\rho_k > \rho_s$ *and* $\underline{\rho}_k > \underline{\rho}_s$*.*

Furthermore, $p(s) \in [\underline{\rho}_s, \overline{\rho}_s]$ *for all* $s \in \{1, 2, \ldots, S\}$ *with* $\underline{\rho}_1 = p(1)$ *and* $\rho_S = p(S)$

Figure 1

Example with 3 states where the intervals do not overlap.



Implications

1) **History Dependence:** Payments depend on the states (e.g. if $s_{t-1} = 1$ and $s_t = 2$, payment will be at B, $\underline{\rho}_2$. If $s_{t-1} = 3$ and $s_t = 2$, payment will be C, ρ_2)
2) **Amnesiac:** Once a constraint is hit, there's amnesia and the prior history is forgotten
3) **Convergence:** Eventually, ρ lands in a finite ergodic set (in Figure 1, A B C and D)
4) **Back-loading**

Thomas, J. and Worrall, T. (1988). Self-Enforcing Wage Contracts. *The Review of Economic Studies*, 55(4):541–553.