A mathematical modeling toolbox for ion channels and transporters across cell membranes

Shadi Zaheria, Fatemeh Hassanipoura,*

^aDepartment of Mechanical Engineering, The University of Texas at Dallas, Richardson, TX, 75080, USA

- The following supplementary material is from " A mathematical modeling toolbox for ion channels
- 2 and transporters across cell membranes" manuscript. It contains an overview of all equations
- 3 related to Ion channels, Pumps, Cotransporters, and Symporters, organized in a table form. The
- 4 detailed transporters along with the descriptions of their equatuons can be found from here.

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^{*}Corresponding author

4.3. Sodium Hydrogen Exchanger (NHE)

Sodium Hydrogen Exchanger (NHE)	Ref
Ammonium competitor	[34, 63]
$\mathbf{J_{Na^{+}}^{NHE}} = E_{t} \frac{g_{ENa}^{M} N a^{M} (g_{EH}^{N} H^{N} + g_{ENH4}^{N} N H_{4}^{N}) - g_{ENa}^{N} N a^{N} (g_{EH}^{M} H^{M} + g_{ENH4}^{M} N H_{4}^{M})}{R_{t} R_{t} R_{t} R_{t} R_{t} R_{t} R_{t}}$	
$\mathbf{J_{Na^+}^{NHE}} = E_t \frac{8ENa^{**}(8EH^{21} + 8ENH^{4})(8EH^{21} + 8ENH^{4})(8EH^{4} + 8ENH^{4})(8EH^{4})(8EH^{4} + 8ENH^{4})(8EH^{4} + 8ENH^{4})(8EH^{4})(8EH^{4} + 8ENH^{4})(8EH^{4} + 8ENH^{4})(8EH^{4} + 8ENH^{4})(8EH^{4} + 8ENH^{4})(8EH^{4})(8EH^{4$	
(149)	
$\mathbf{J_{H^{+}}^{NHE}} = E_{t} \frac{g_{EH}^{M} H^{M}(g_{ENa}^{N} N a^{N} + g_{ENH4}^{N} N H_{4}^{N}) - g_{EH}^{N} H^{N}(g_{ENa}^{M} N a^{M} + g_{ENH4}^{M} N H_{4}^{M})}{g_{ENa}^{M} N a^{M} + g_{ENH4}^{M} N H_{4}^{M}}$	
$\mathbf{J}_{\mathbf{H}^{+}}^{\mathbf{H}^{+}} = E_{t} \frac{E_{t}}{R_{M}R_{NN} + R_{N}R_{MM}}$	
(150)	
$\mathbf{J_{NH4^{+}}^{NHE}} = E_{t} \frac{g_{ENH4}^{M} N H_{4}^{M} (g_{ENa}^{N} N a^{N} + g_{EH}^{N} H^{N}) - g_{ENH4}^{N} N H_{4}^{N} (g_{ENa}^{M} N a^{M} + g_{EH}^{M} H^{M})}{R_{1} R_{2} R_{3} + R_{4} R_{3} R_{3}}$	
$\mathbf{J_{NH_4^+}} = E_t - R_M R_{NN} + R_N R_{MM}$	
(151)	
where	
$[E]_t = [E]_M + [ENa]_M + [EH]_M + [ENH_4]_M + [E]_N + [ENa]_N + [EH]_N + [ENH_4]_N$	
$Na^{M} = \frac{[Na]_{M}}{K_{Na}^{M}}, H^{M} = \frac{[H]_{M}}{K_{H}^{M}}, NH_{4}^{M} = \frac{[NH_{4}]_{M}}{K_{NH_{4}}^{M}} \mid Na^{N} = \frac{[Na]_{N}}{K_{Na}^{N}}, H^{N} = \frac{[H]_{N}}{K_{H}^{N}},$	
$NH_4^N = \frac{[NH_4]_N}{K_{NH_4}^N}$	
$R_{M} = 1 + Na^{M} + H^{M} + NH_{4}^{M} \mid R_{N} = 1 + Na^{N} + H^{N} + NH_{4}^{N}$ $R_{MM} = g_{ENa}^{M} Na^{M} + g_{EH}^{M} H^{M} + g_{ENH_{4}}^{M} NH_{4}^{M} \mid R_{NN} = g_{ENa}^{N} Na^{N} + g_{EH}^{N} H^{N} + g_{ENH_{4}}^{N} NH_{4}^{N}$	
No competitor $\frac{SENa^{11}}{SENH_4^{11}} + \frac{SENH_4^{11}}{SENH_4^{11}} + \frac{SENH_4^{11}}{SENH_4^$	[64, 65]
M N (27 M TN) N M (27 N TN)	
$\mathbf{J_{Na}^{NHE}} = E_t \frac{g_{ENa}^M g_{EH}^N (Na^M H^N) - g_{ENa}^N g_{EH}^M (Na^N H^M)}{R_M R_{NN} + R_N R_{MM}} $ (152)	
$\mathbf{J_{H}^{NHE}} = E_{t} \frac{g_{EH}^{M} g_{ENa}^{N} (H^{M} N a^{N}) - g_{EH}^{N} g_{ENa}^{M} (H^{N} N a^{M})}{R_{M} R_{NN} + R_{N} R_{MM}} $ (153)	
	F24 201
$([Na]_{M(bl)}[H]_{N(c)} - [Na]_{N(c)}[H]_{M(bl)})$	[34, 38]
$J_{NHE} = P_{NHE} \frac{([1/43]_{M(bl)}[1/1]_{N(c)} - [1/43]_{N(c)}[1/1]_{M(bl)})}{K_{Na}K_{H} \left(\left(1 + \frac{[Na]_{M(bl)}}{K_{Na}} + \frac{[H]_{M(bl)}}{K_{H}} \right) \left(\frac{[Na]_{N(c)}}{K_{Na}} + \frac{[H]_{N(c)}}{K_{H}} \right) \right)}$	
$K_{Na}K_H \left(\left(1 + \frac{1}{K_{Na}} + \frac{1}{K_H} \right) \left(\frac{1}{K_{Na}} + \frac{1}{K_H} \right) \right) $ (154)	
$+\left(1+\frac{[Na]_{N(c)}}{K_{Na}}+\frac{[H]_{N(c)}}{K_{H}}\right)\left(\frac{[Na]_{M(bl)}}{K_{Na}}+\frac{[H]_{M(bl)}}{K_{H}}\right)$	
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Table 29: The corresponding equations describing the flux and current transported via sodium hydrogen exchanger across the membrane