A mathematical modeling toolbox for ion channels and transporters across cell membranes

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The following supplementary material is from " A mathematical modeling toolbox for ion channels and transporters across cell membranes" manuscript. It contains an overview of all equations related to Ion channels, Pumps, Cotransporters, and Symporters, organized in a table form. The detailed transporters along with the descriptions of their equatuons can be found from here.

⋆This document is the result of the research project funded by the National Science Foundation.

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Voltage Gated Sodium Channel ($VGSC$, Na_v, $VONa$)	**Ref**
$$\boxed{I_{Na,Na_v} = g_{Na_v}^{max} m_{Na_v}^3 h_{Na_v} j_{Na_v}\Big(V_m - V_{Na,rev}^{M-N}\Big)} \quad (38)$$ $$\frac{dm_{Na_v}}{dt} = \frac{\bar{m}_{Na_v} - m_{Na_v}}{\tau_m} \quad (39)$$ $$\frac{dh_{Na_v}}{dt} = \frac{\bar{h}_{Na_v} - h_{Na_v}}{\tau_h} \quad (40)$$ $$\frac{dj_{Na_v}}{dt} = \frac{\bar{j}_{Na_v} - j_{Na_v}}{\tau_j} \quad (41)$$ $$\bar{m}_{Na_v} = \frac{1}{\left(1 + exp\left(\frac{-(V_m^{M-N} - V_{1/2,m\ Na_v}^{M-N})}{k_{m\ Na_v}}\right)\right)^2} \quad (42)$$ $$\bar{h}_{Na_v} = \frac{1}{\left(1 + exp\left(\frac{(V_m^{M-N} + V_{1/2,h\ Na_v}^{M-N})}{k_{h\ Na_v}}\right)\right)^2} \quad (43)$$ $$\bar{j}_{Na_v} = \frac{1}{\left(1 + exp\left(\frac{(V_m^{M-N} + V_{1/2,j\ Na_v}^{M-N})}{k_{j\ Na_v}}\right)\right)^2} \quad (44)$$ $$\tau_m = \alpha_m\, \beta_m \quad (45)$$ $$\tau_h = \frac{1}{\alpha_h + \beta_h} \quad (46)$$ $$\tau_j = \frac{1}{\alpha_j + \beta_j} \quad (47)$$	[14–16]

Table 3: The corresponding equations describing the ionic current transported via voltage gated sodium channels ($VGSC$s, Na_vs, $VONa$s) across the cell membrane (part 2/3 continued from previous page)

Voltage Gated Sodium Channel ($VGSC$, Na_v, $VONa$)	Ref
$$For\ all\ range\ of\ V_m : \begin{cases} \alpha_m = \frac{1}{1+exp\left(\frac{-(V_m^{M-N}+V_{1\alpha m})}{k_{\alpha m}}\right)} \\ \beta_m = \frac{A_{\beta m}}{1+exp\left(\frac{(V_m^{M-N}+V_{1\beta m})}{k_{\beta m}}\right)} + \frac{B_{\beta m}}{1+exp\left(\frac{(V_m^{M-N}-V_{2\beta m})}{k_{2\beta m}}\right)} \end{cases} \quad (48)$$ $$For\ \ V_m \geq -40 : \begin{cases} \alpha_h = 0 \\ \beta_h = \frac{A_{\beta_h}}{1+exp\left(\frac{-(V_m^{M-N}+V_{1\beta_h})}{k_{\beta_h}}\right)} \end{cases} \quad (49)$$ $$For\ \ V_m < -40 : \begin{cases} \alpha_h = A_{\alpha_h} exp\left(\frac{-(V_m+V_{\alpha_h}^{Nav})}{k_{\alpha_h}^{Nav}}\right) \\ \beta_h = A_{\beta_h}\ exp(a_{\beta_h}\ V_m) + B_{\beta_h} exp(b_{\beta_h} V_m) \end{cases} \quad (50)$$ $$For\ \ V_m \geq -40 : \begin{cases} \alpha_j = 0 \\ \beta_j = \frac{A_{\beta_j} exp(-a_{\beta_j}\ V_m)}{1 + exp\left(\frac{-(V_m^{M-N}+V_{2\beta_j})}{k_{\beta_j}}\right)} \end{cases} \quad (51)$$ $$For\ \ V_m < -40 : \begin{cases} \alpha_j = \frac{\left(A_{\alpha_j} exp(a_{\alpha_j} V_m) - B_{\alpha_j} exp(b_{\alpha_j} V_m)\right)(V_m + V_{1\alpha_j})}{1 + exp(\frac{V+V_{2\alpha_j}}{k_{\alpha_j}})} \\ \beta_j = \frac{A_{\beta_j} exp(a_{\beta_j}\ V_m)}{1 + exp\left(\frac{-(V_m^{M-N}+V_{2\beta_j})}{k_{\beta_j}}\right)} \end{cases} \quad (52)$$	
$$V_{Na,rev}^{M-N(a)} = \frac{RT}{z_{Na}F} ln\left(\frac{[Na]_{M(l)}}{[Na]_{N(i)}}\right) \quad (53)$$	[14–16]

Table 3: The corresponding equations describing the ionic current transported via voltage gated sodium channels ($VGSC$s, Na_vs, $VONa$s) across the cell membrane (part 3/3 continued from previous page)