A mathematical modeling toolbox for ion channels and transporters across cell membranes

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The following supplementary material is from " A mathematical modeling toolbox for ion channels and transporters across cell membranes" manuscript. It contains an overview of all equations related to Ion channels, Pumps, Cotransporters, and Symporters, organized in a table form. The detailed transporters along with the descriptions of their equatuons can be found from here.

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2. ATPase model

2.1. Sodium Potassium ATPase pump (Na-K ATPase)

Sodium Potassium ATPase pump (Na-K ATPase)	Ref
	[29–34]
$$\mathrm{ATP} + 3\,\mathrm{Na_M}^{+} + 2\,\mathrm{K_N}^{+} \rightleftharpoons \mathrm{ADP} + \mathrm{Pi} + 3\,\mathrm{Na_N}^{+} + 2\,\mathrm{K_M}^{+} \quad (100)$$	
$$\boxed{J_{Na^+}^{NakATpase} = J_{Na^+}^{NakATpase,max}\Big(\frac{[Na]_{M(i)}}{[Na]_{M(i)} + K_{Na_M}}\Big)^3\Big(\frac{[K]_{N(e)}}{[K]_{N(e)} + K_{K_N}}\Big)^2} \quad (101)$$	
$$\boxed{J_{K^+}^{NakATpase} = (\frac{-2}{3})J_{Na^+}^{NakATpase}} \quad (102)$$	
`where:`	[31]
$$K_{Nai} = K_{Na}^{NaK}(1 + \frac{[K]_i}{a_{NaK}}) \quad (103)$$	
$$K_{Ki} = K_{K}^{NaK}(1 + \frac{[Na]_e}{b_{NaK}}) \quad (104)$$	
	[35, 36]
$$\boxed{J_{Na}^{Pump} = J_{Na}^{NaKATPase,max}\left(\frac{[Na]_c}{[Na]_c + K_{Na}}\right)^3\left(\frac{[K]_{bl}}{[K]_{bl} + K_K}\right)^2} \quad (105)$$	
$$\boxed{J_{K}^{pump} + J_{NH4}^{pump} = -\frac{2}{3}J_{Na}^{pump}} \quad (106)$$	
$$\boxed{\frac{J_{NH4}^{pump}}{J_{K}^{pump}} = \frac{[NH4]_e}{K_{NH4}} \cdot \frac{K_K}{[K]_e}} \quad (107)$$	

Table 9: The corresponding equations describing the flux and current transported via sodium potassium ATPase pumps across the cell membrane

Sodium Potassium ATPase pump (Na-K ATPase)	Ref
$$\boxed{I_{NaK}^{M-N} = I_{NaK}^{max}\psi_{NaK}^{cyt}\left(\frac{[Na]_{cyt}^{1.5}}{[Na]_{cyt}^{1.5} + K_{m,Na,\alpha 1}^{1.5}}\right)\left(\frac{[K]_{out}}{[K]_{out} + K_{m,K}}\right)} \quad (108)$$	[2, 37]
$$\psi_{NaK}^{cyt} = \frac{1}{1 + 0.1245exp\left(-0.1\frac{V_m^{M-N}F}{RT}\right) + 0.365\ \sigma\ exp\left(\frac{-V_m^{M-N}F}{RT}\right)} \quad (109a)$$	
$$\sigma = \frac{1}{7}\left(\frac{[Na]_{out}}{67.3} - 1\right) \quad (109b)$$	
$$\boxed{J_{pump} = P_{pump}\left(\frac{[Na]_c}{[Na]_c + K_{Na}}\right)^3\left(\frac{[K]_{bl}}{[K]_{bl} + K_K}\right)^2 (a \times V_m^b + b)} \quad (110)$$	[8, 38]
$$\boxed{J_{NaKATPase} = P_{pump}\left(\frac{[Na]_i}{[Na]_i + K_{Na}^{NaK}}\right)^3\left(\frac{[K]_{bl}}{[K]_{bl} + K_K^{NaK}}\right)^2 (V_m^{i-bl} - V_{rev})} \quad (111)$$	[38]

Table 10: The corresponding equations describing the flux and current transported via Sodium Potassium ATPase pumps across the cell membrane