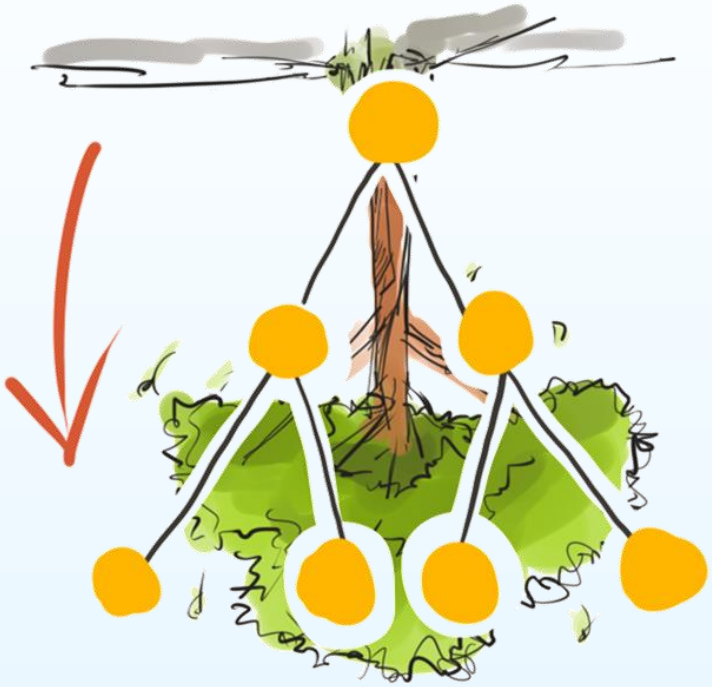

Lecture 10: Priority Queues

01204212 Abstract Data Types and Problem Solving

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MIKE LABORATORY

Outline

- Priority Queues
- Binary Heaps
- Application:
  - Huffman code

Priority Queues

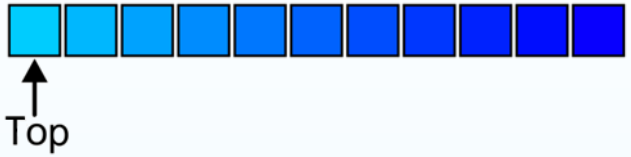
- With an abstract queue, objects are linearly ordered associated by first in, first out

- Like a normal queue, an abstract priority queue holds a collection of objects that each will be dequeued following a priority order
  - i.e., each object is associated with a priority, and the object having the highest priority will be dequeued first

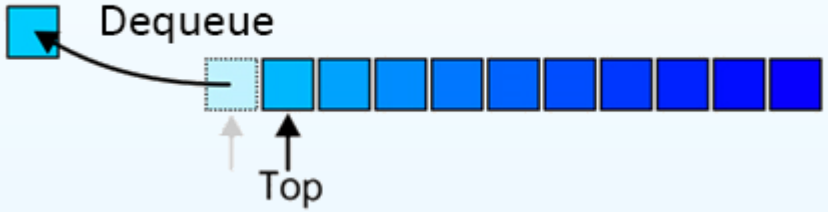
- With each enqueued object, we will associate a nonnegative integer (0, 1, 2, …) where:
  - The value 0 has the highest priority, and
  - The higher the number, the lower the priority

Operations

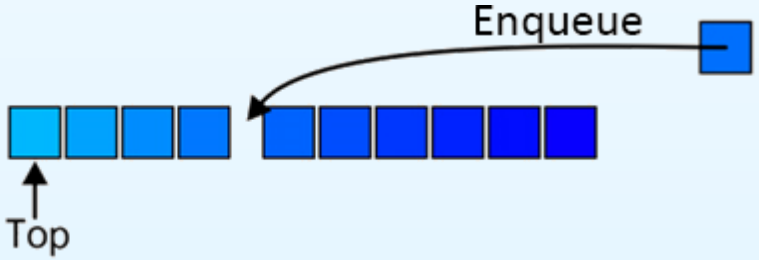
The top of a priority queue is the object with highest priority



Dequeuing from a priority queue removes the current highest priority object



Enqueuing places a new object into the appropriate place



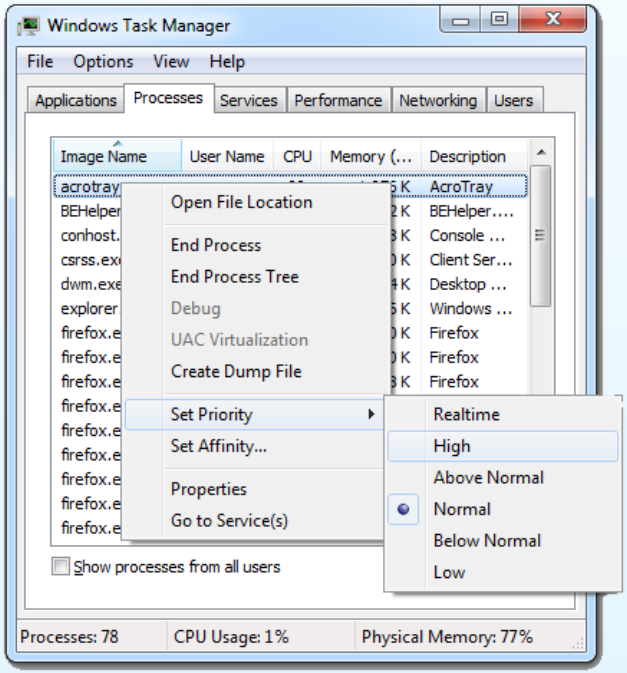
Lexicographical Priority

- Priority may also depend on multiple variables:
  - Two values specify a priority: (a, b)
  - A pair (a, b) has higher priority than (c, d) if
    - $a < c$, or
    - $a = c$ and $b < d$

- For example,
  - $(5,19)$, $(13,1)$, $(13,24)$, and $(15,0)$ all have higher priority than $(15,7)$

Applications

- Operating system needs to schedule jobs according to priority
  - Unix: `$ nice +15 ./a.out`
    - Reduces the priority of the execution of the routine `a.out` by 15
  - Windows: set in the Windows task Manager



Applications

- Doctors in emergency room take patients according to severity of injuries
- In traffic light, depending upon the traffic, the colors will be given priority
- Special service for members
  - e.g., airline passengers
- Huffman codes for data compression

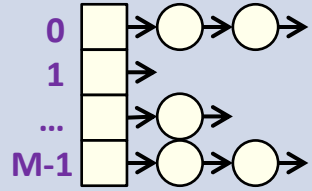
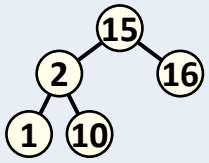
Design for Implementation

- In addition to the data values, we may need to store the priorities (i.e., keys) of each object
  - Remember, the smaller the number, the higher the priority

- Therefore, we want an ADT that can efficiently perform:
  - `find_min()`
  - `delete_min()`
  - `insert()`

Which ADT?

ADT Candidates

ADT	`find_min()`	`delete_min()`	`insert()`
1. Sorted List 	$O(1)$	$O(1)$	$O(n)$
2. Unsorted List 	$O(n)$	$O(n)$	$O(1)$
3. Multiple Queues 	$O(M)$	$O(M)$	$O(1)$
	Restrict the range of priority and require memory $\Theta(M+n)$		
4. BST/AVL Tree 	$O(\log n)$	$O(\log n)$	$O(\log n)$
	Look good but … are efficient for all finds, not just `find_min()`		

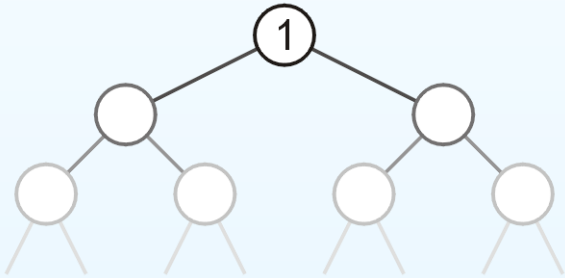
Better than a BST !!!

Can we do better with very limited requirements?

- Only need `find_min()`, `delete_min()`, and `insert()`
- Reduce the running time of some operations down to $\Theta(1)$

We will look at another ADT, called heap

- A tree with the highest priority object at the root
- We will focus on binary heaps
- Numerous other heaps exists:
  - d-ary heaps
  - Leftist heaps
  - Skew heaps
  - Binomial heaps
  - Fibonacci heaps
  - Bi-parental heaps



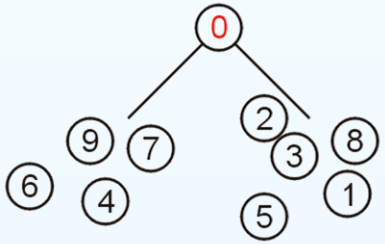
Outline

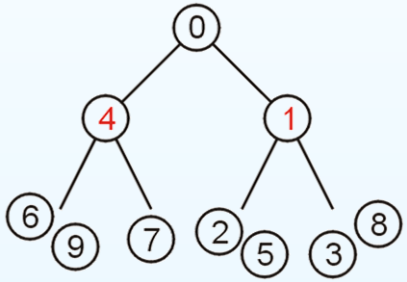
- Priority Queues
- Binary Heaps
- Application:
  - Huffman code

Definition: Binary Min Heaps

A non-empty binary tree is a min-heap if

- The key of the root is less than or equal to the keys associated with its left and right subtrees (if any)
- Both subtrees (if any) are also binary min-heaps



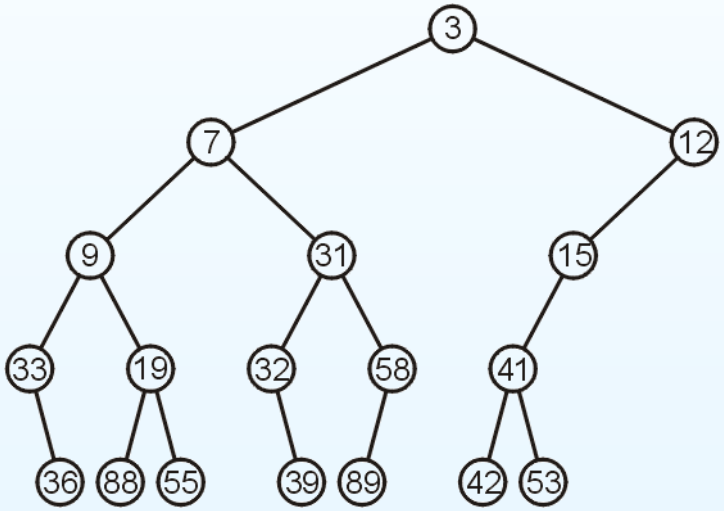


From the definition:

- A single node is a min-heap
- All keys in either subtree are greater than the key of root

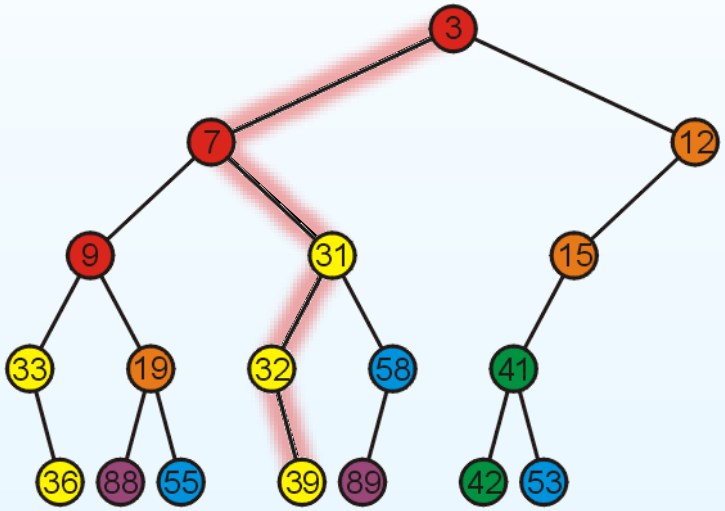
Example

A binary min-heap



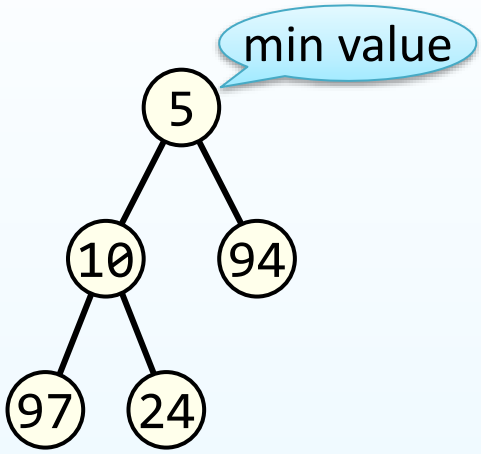
Heap Order Properties

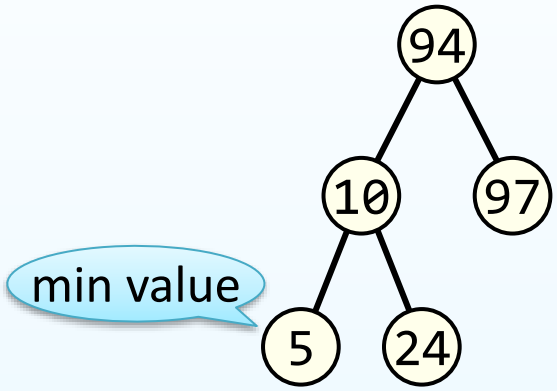
- Each path is sorted
  - e.g., $\langle 3, 7, 31, 32, 39 \rangle$
- There is no relationship between the elements in the two subtrees



Binary Heap vs. Binary Search Tree

A binary heap is NOT a binary search tree





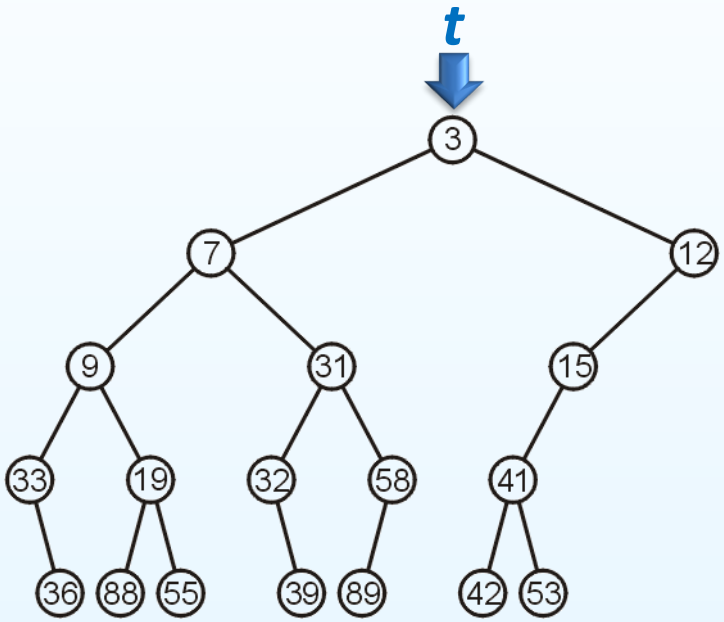
Binary min-heap: parent is less than both left and right children

Binary search tree: parent is greater than left child, but less than right child

The `find_min()` Operation

Return the minimum value in heap t

find_min(*t*)

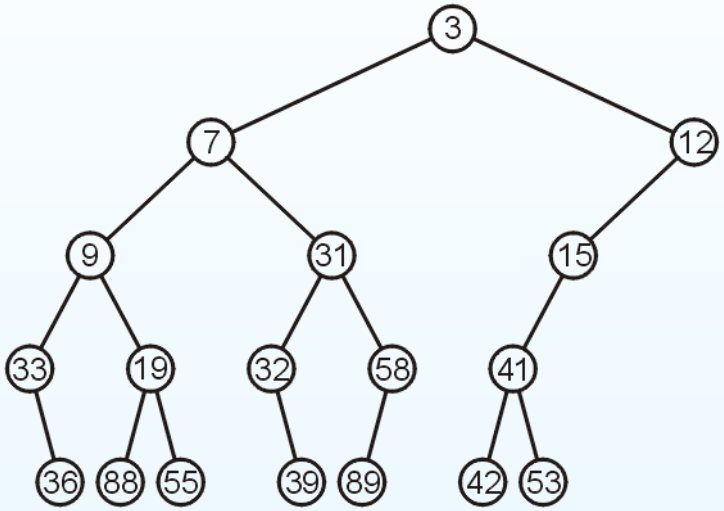


Running time:

We can easily return the value of the root in $\Theta(1)$

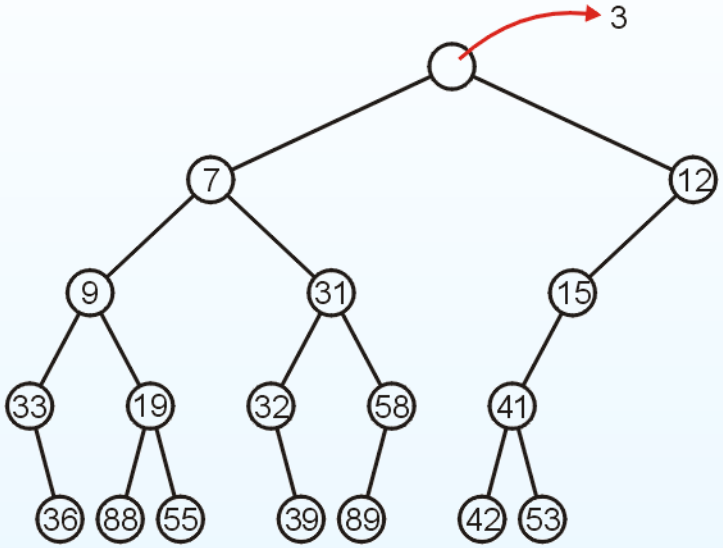
Deletion

We want to remove the minimum value from the heap:



Deletion

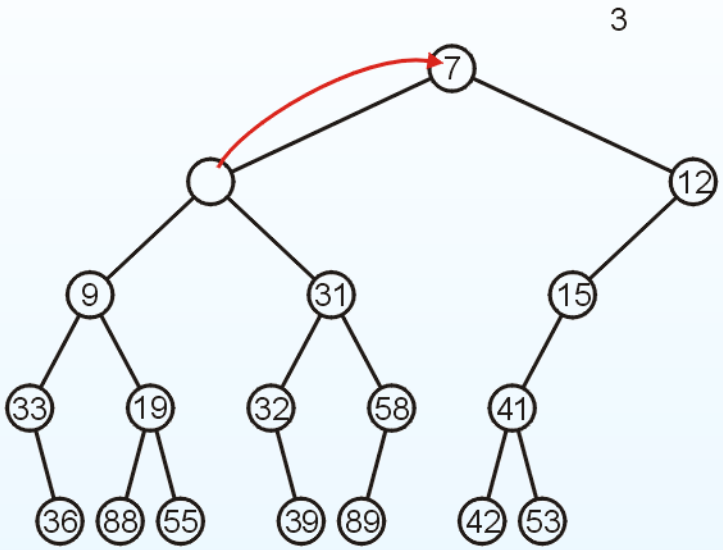
We want to remove the minimum value from the heap:



1. Remove 3

Deletion

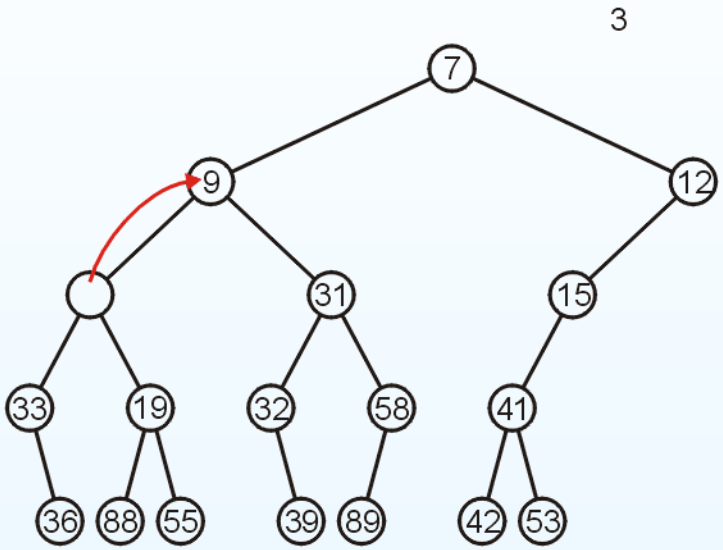
We want to remove the minimum value from the heap:



1. Remove 3
2. Promote 7 (the minimum of 7 and 12) to the root

Deletion

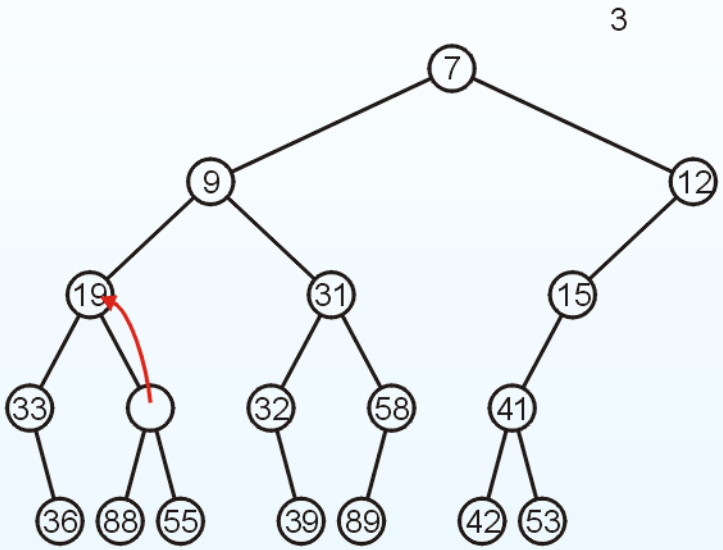
We want to remove the minimum value from the heap:



1. Remove 3
2. Promote 7 (the minimum of 7 and 12) to the root
3. Recursively, promote 9 (the minimum of 9 and 31)

Deletion

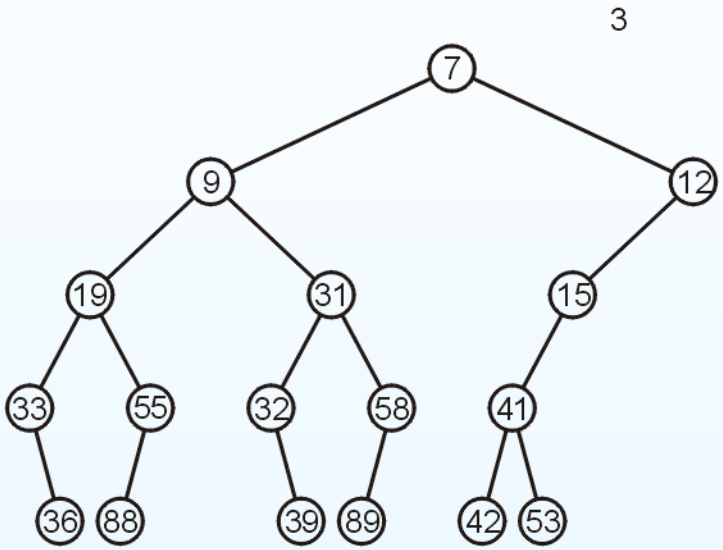
We want to remove the minimum value from the heap:



1. Remove 3
2. Promote 7 (the minimum of 7 and 12) to the root
3. Recursively, promote 9 (the minimum of 9 and 31)
4. Recursively, promote 19 (the minimum of 33 and 19)

Deletion

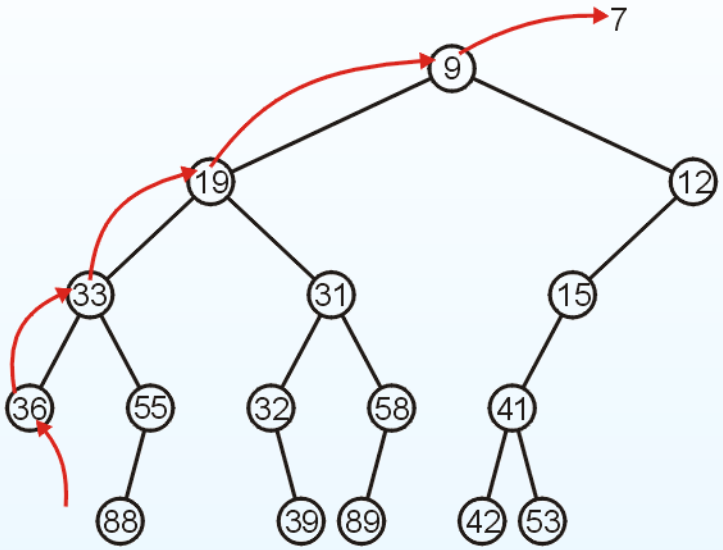
We want to remove the minimum value from the heap:



1. Remove 3
2. Promote 7 (the minimum of 7 and 12) to the root
3. Recursively, promote 9 (the minimum of 9 and 31)
4. Recursively, promote 19 (the minimum of 33 and 19)
5. Finally, 55 is a leaf node so that we promote it and delete that leaf

Deletion

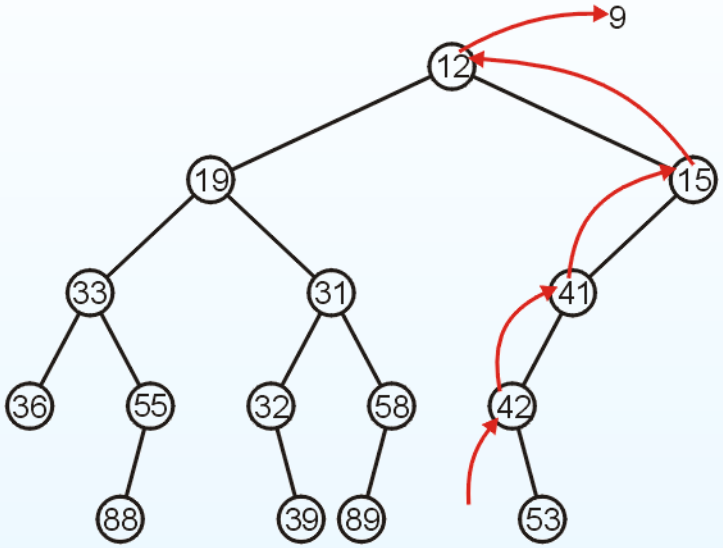
We want to remove the minimum value from the heap:



Repeat the deletion again, we can remove 7

Deletion

We want to remove the minimum value from the heap:



Again, to remove 9

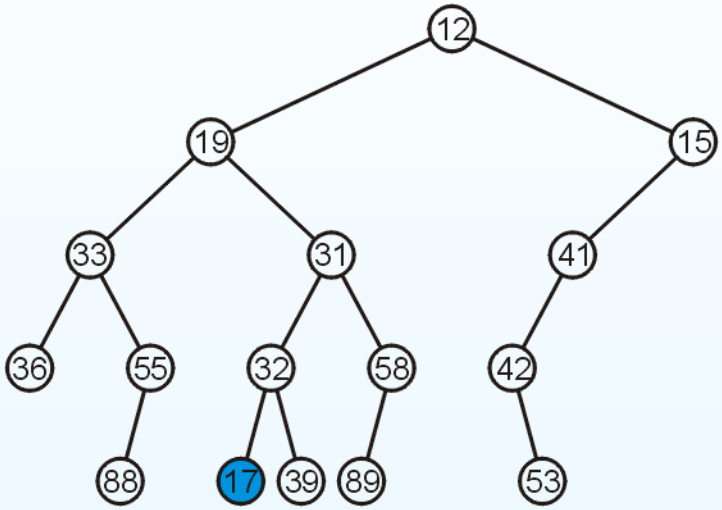
Insertion

Insert into a heap may be done either:

- At a leaf – move up if it is smaller than the parent
- At the root – move down to one of subtrees if it is larger

Insertion: Approach 1 (At a Leaf)

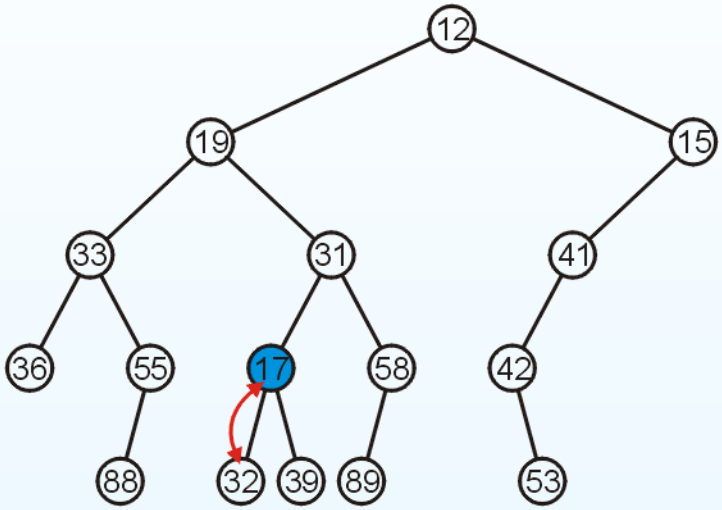
We want to insert 17 into the heap:



1. Select an arbitrary node to insert a new leaf node

Insertion: Approach 1 (At a Leaf)

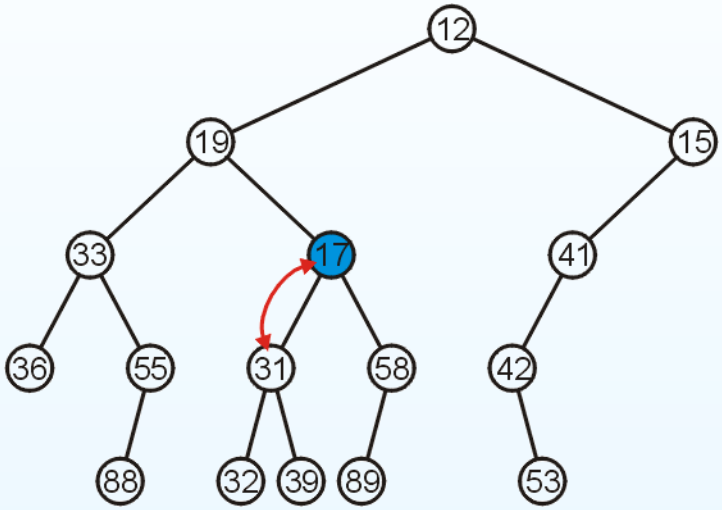
We want to insert 17 into the heap:



1. Select an arbitrary node to insert a new leaf node
2. The node 17 is less than its parent node 32, so we swap them

Insertion: Approach 1 (At a Leaf)

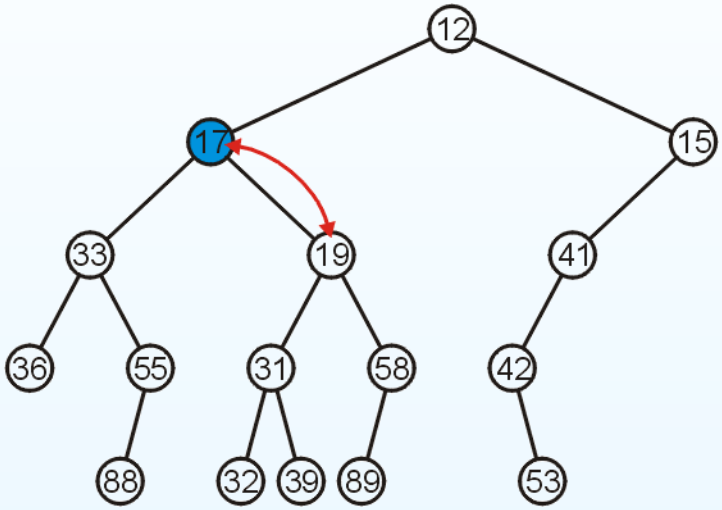
We want to insert 17 into the heap:



1. Select an arbitrary node to insert a new leaf node
2. The node 17 is less than its parent node 32, so we swap them
3. The node 17 is also less that the node 31; swap them

Insertion: Approach 1 (At a Leaf)

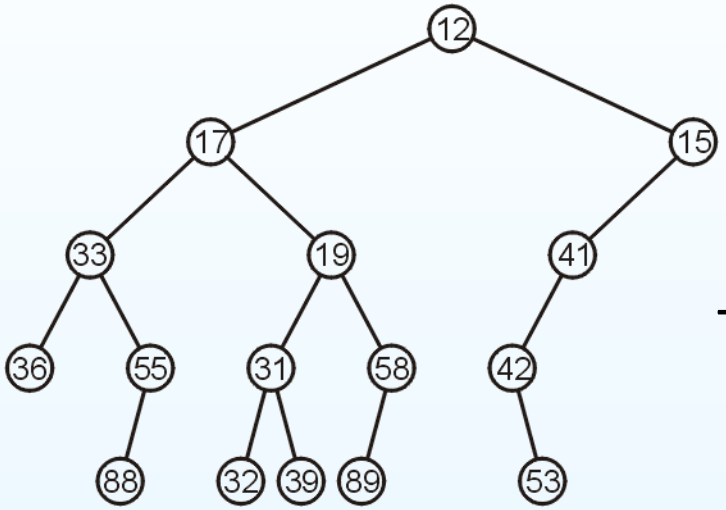
We want to insert 17 into the heap:



1. Select an arbitrary node to insert a new leaf node
2. The node 17 is less than its parent node 32, so we swap them
3. The node 17 is also less than the node 31; swap them
4. The node 17 is also less than the node 19; swap them

Insertion: Approach 1 (At a Leaf)

We want to insert 17 into the heap:



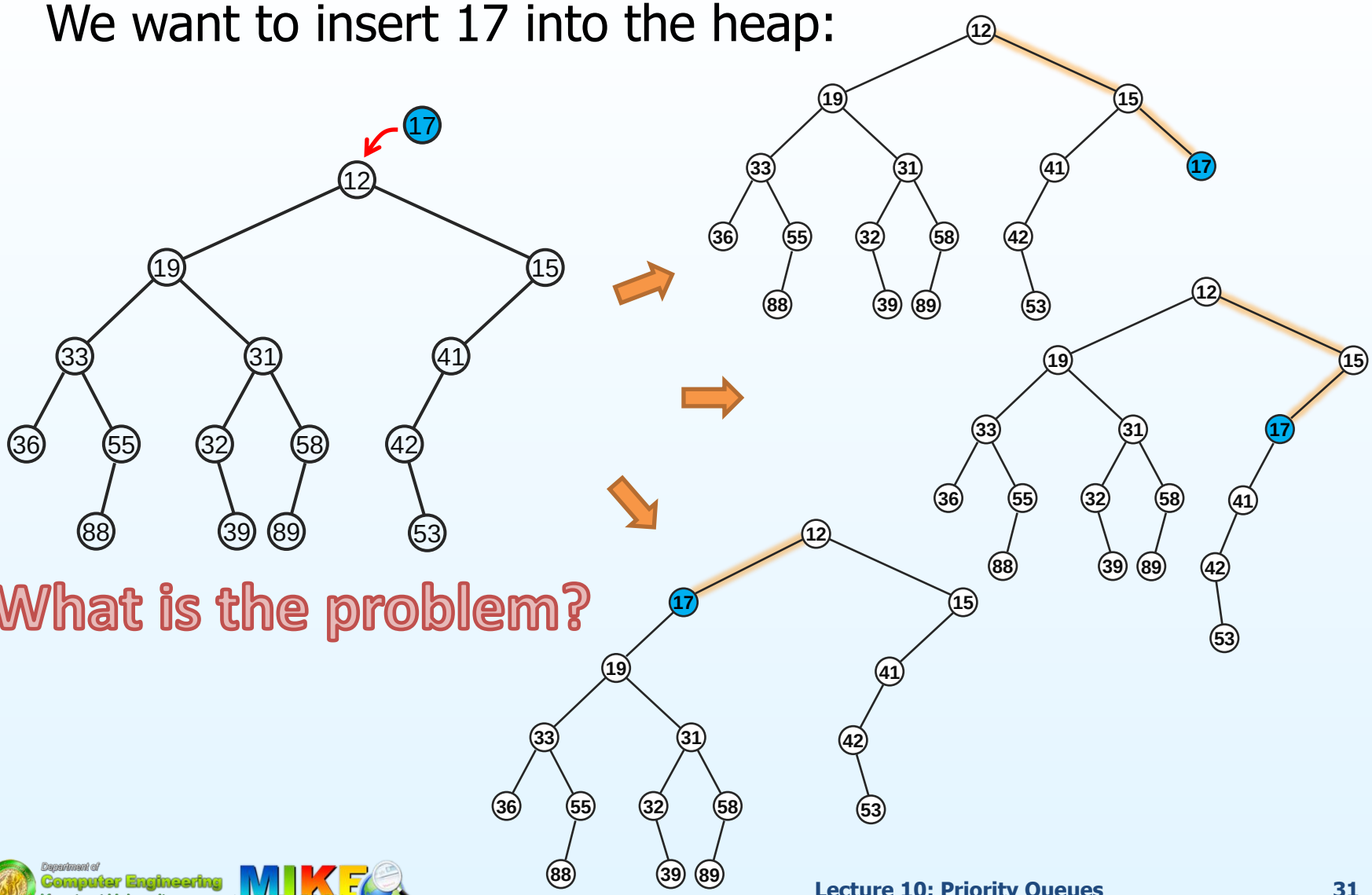
This process is called "percolation"

1. Select an arbitrary node to insert a new leaf node
2. The node 17 is less than its parent node 32, so we swap them
3. The node 17 is also less than the node 31; swap them
4. The node 17 is also less than the node 19; swap them
5. The node 17 is greater than the node 12, so we are finished

What is the problem?

Insertion: Approach 2 (At the Root)

We want to insert 17 into the heap:



Implementation

- With binary search trees, we introduced the concept of balance such as AVL trees

- However, to keep balance with binary heaps, there are various solutions:
  - Complete binary trees
  - Leftist heaps
  - Skew heaps

- We will look at using complete binary trees

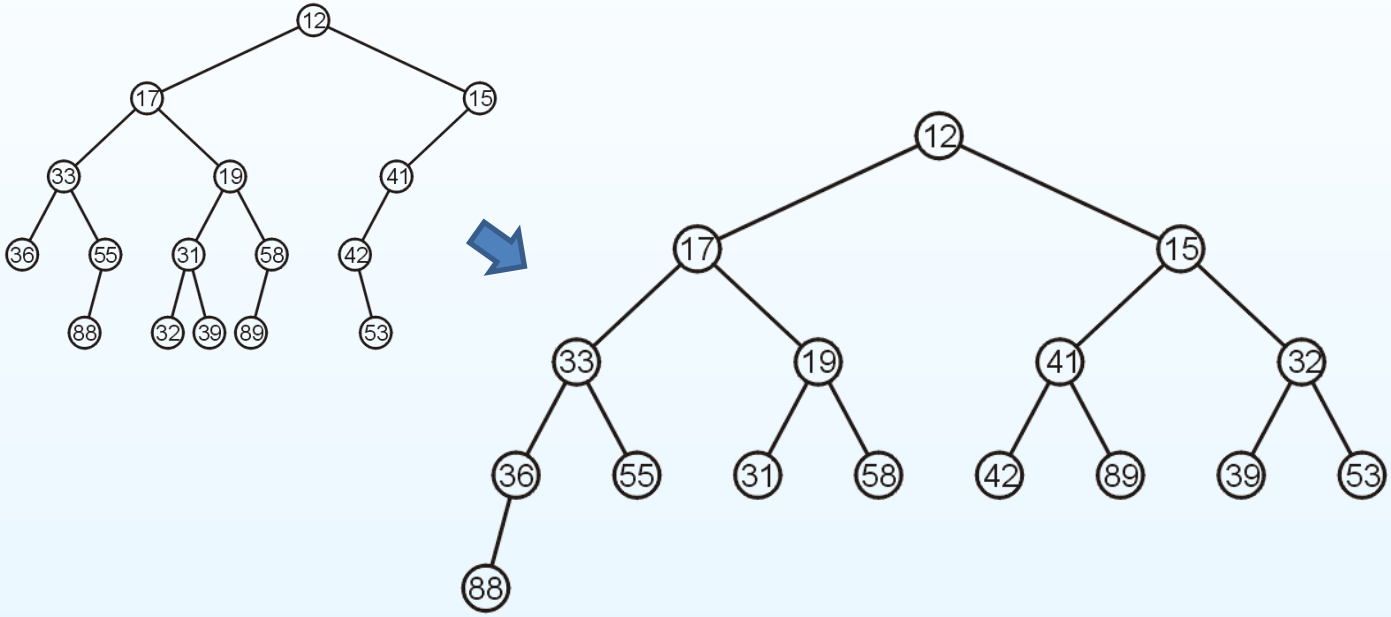
Complete Binary Heaps

- Based on the concept of complete binary trees, we can maintain the complete binary heap structure with minimal effort

- We have already seen that a complete tree is easily stored as an array
  - We can store a heap of size n as an array of size $\Theta(n)$

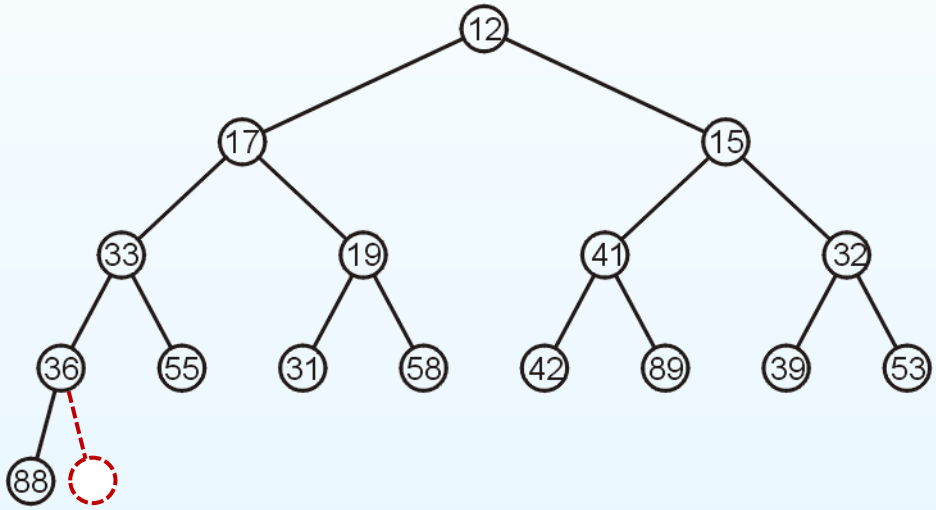
Complete Binary Heaps

For example, the previous heap may be represented as the following (non-unique) complete tree:



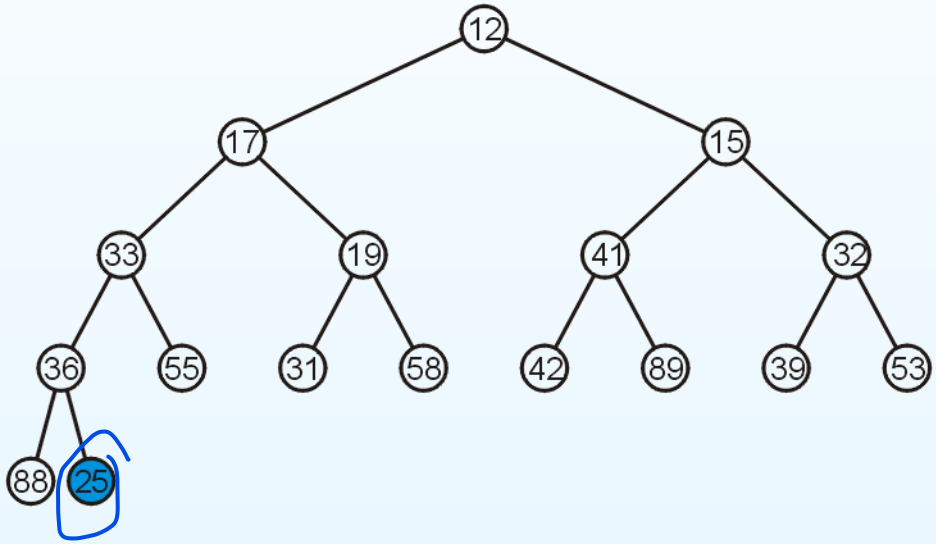
Complete Binary Heaps: Insertion

To insert, we need only place the new node as a leaf node in the appropriate location and percolate up



Complete Binary Heaps: Insertion

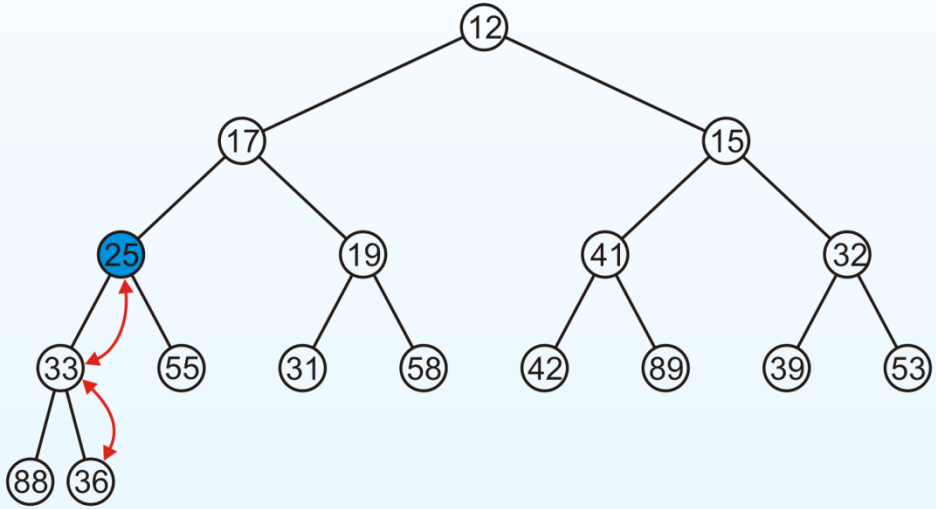
We will insert 25:



Complete Binary Heaps: Insertion

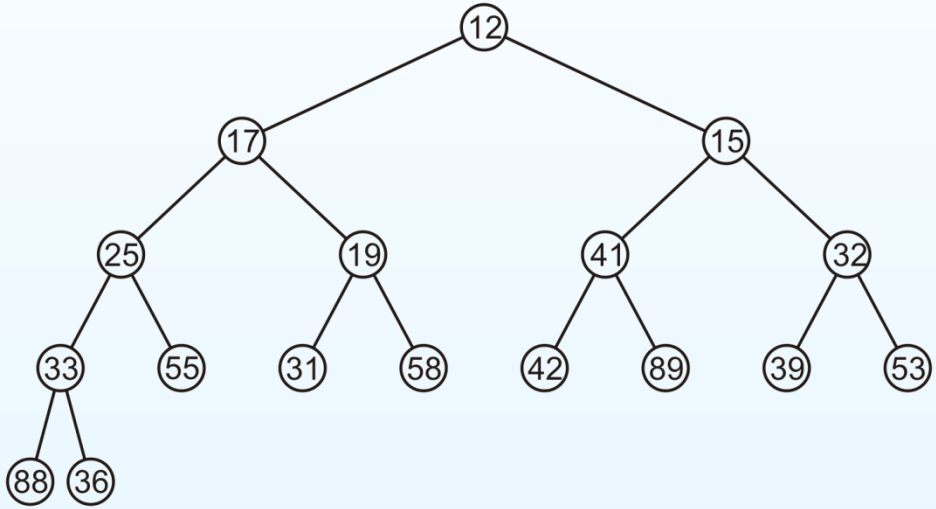
We will insert 25:

- Percolate 25 up into its appropriate location



Complete Binary Heaps: Deletion

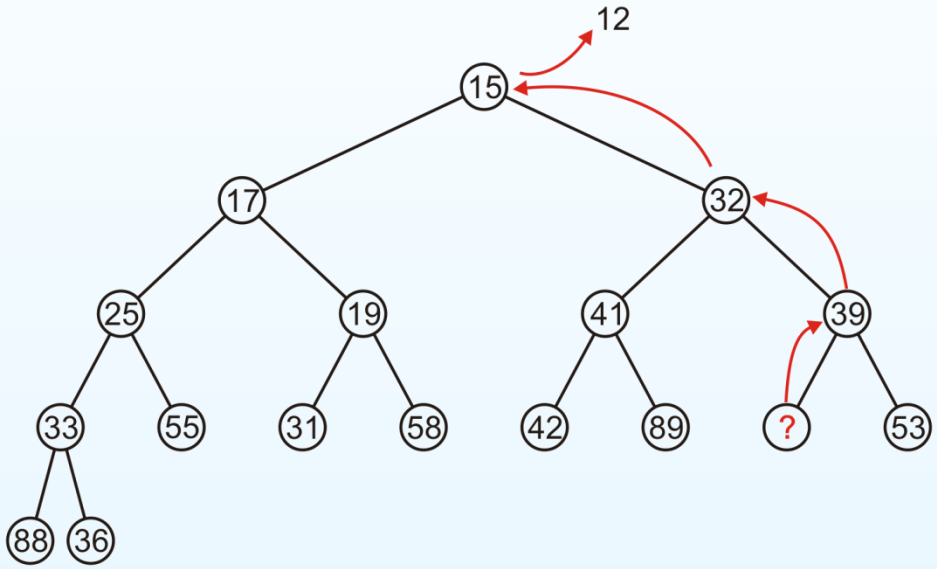
Support we want to delete the minimum value, i.e., 12:



Complete Binary Heaps: Deletion

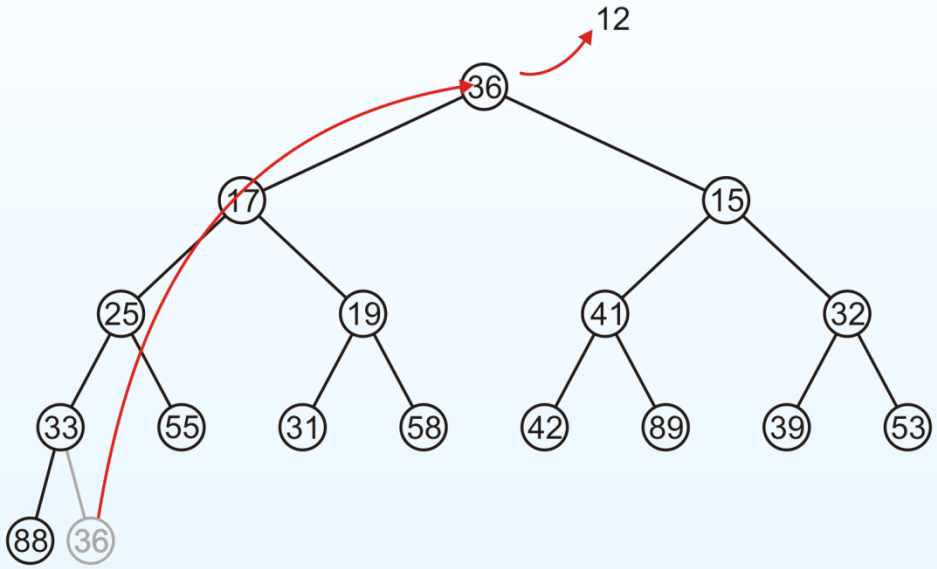
Support we want to delete the minimum value, i.e., 12:

- Percolating up will create a hole leading to a non-complete tree



Complete Binary Heaps: Deletion

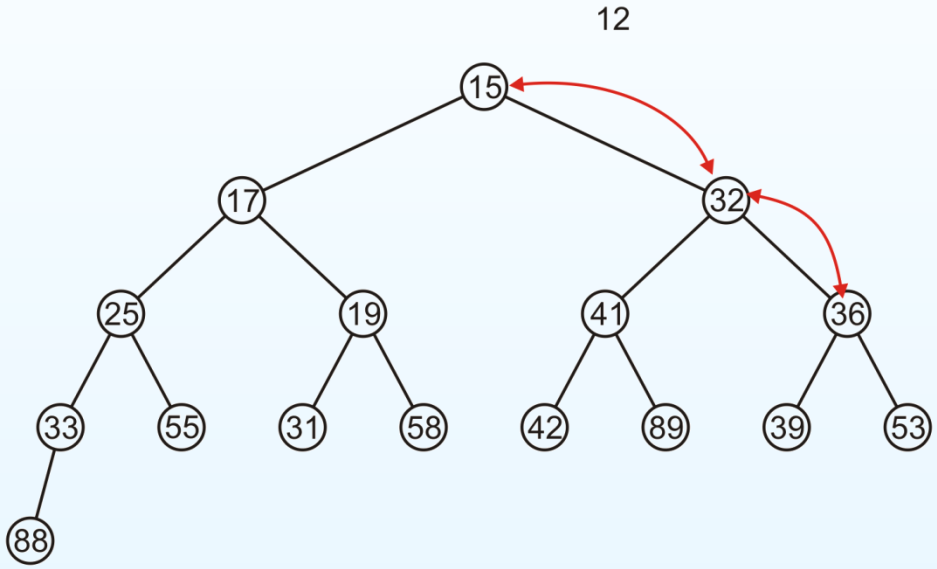
Therefore, to delete, we will copy the last element to the root and then percolate down



Complete Binary Heaps: Deletion

We delete the minimum value 12:

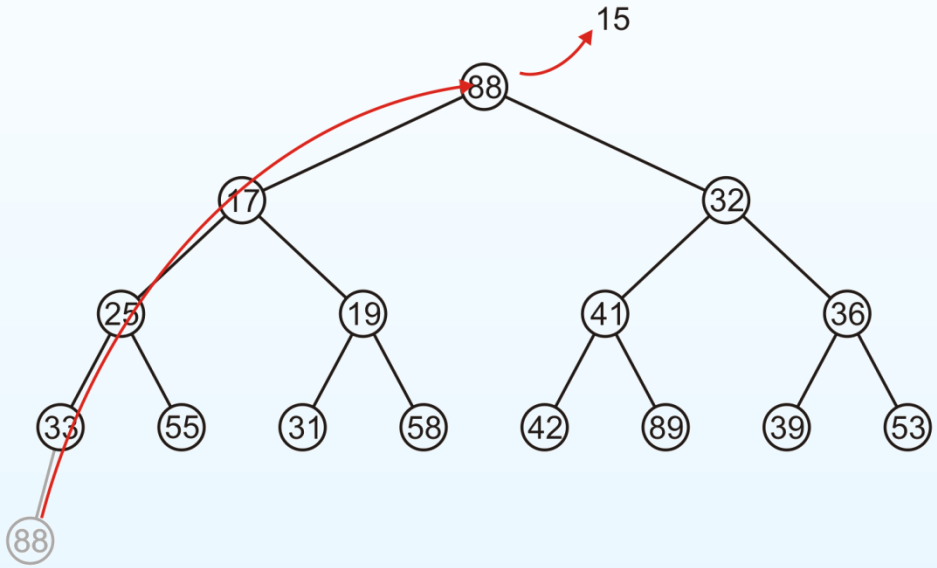
1. Copy 36 to the root
2. Percolate 36 down by swapping it with the smallest of its children



Complete Binary Heaps: Deletion

Again, delete 15:

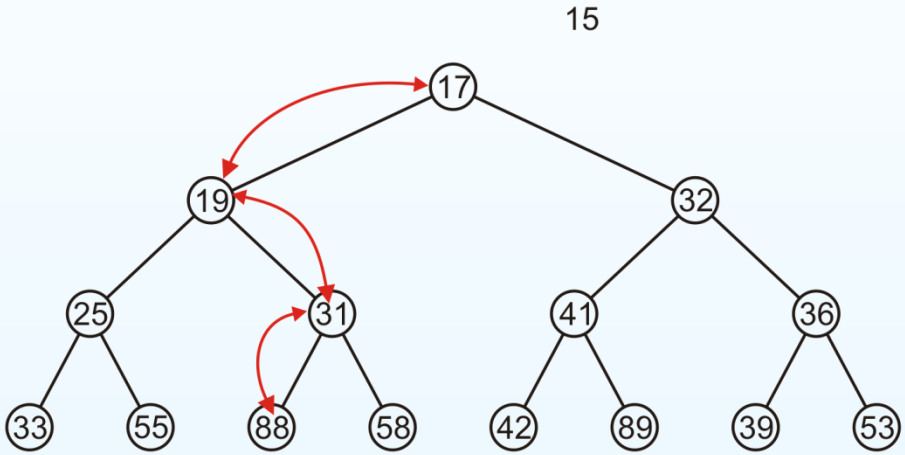
1. Copy 88 to the root



Complete Binary Heaps: Deletion

Again, delete 15:

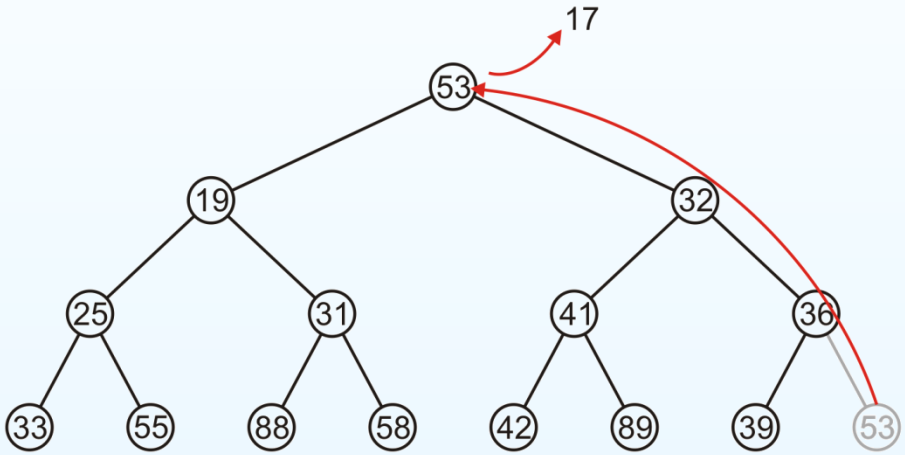
1. Copy 88 to the root
2. Percolate 88 down to the point where it has no children



Complete Binary Heaps: Deletion

Again, delete 17:

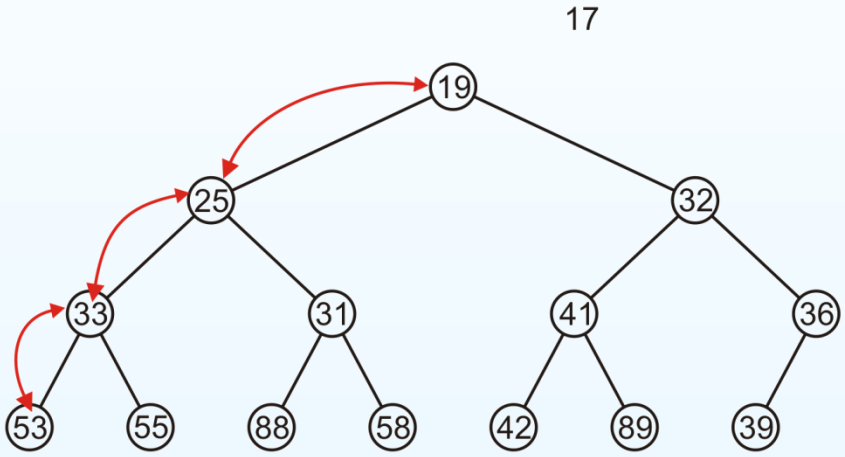
1. Copy 53 to the root



Complete Binary Heaps: Deletion

Again, delete 17:

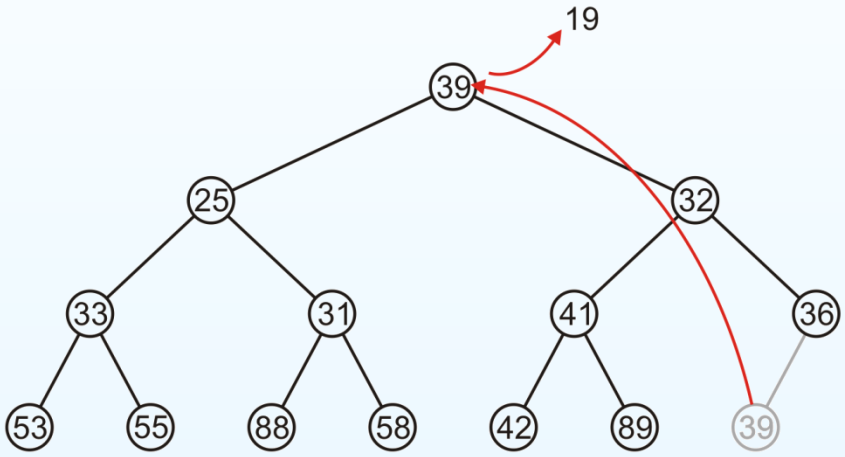
1. Copy 53 to the root
2. Percolate 53 down to the appropriate location



Complete Binary Heaps: Deletion

Again, delete 19:

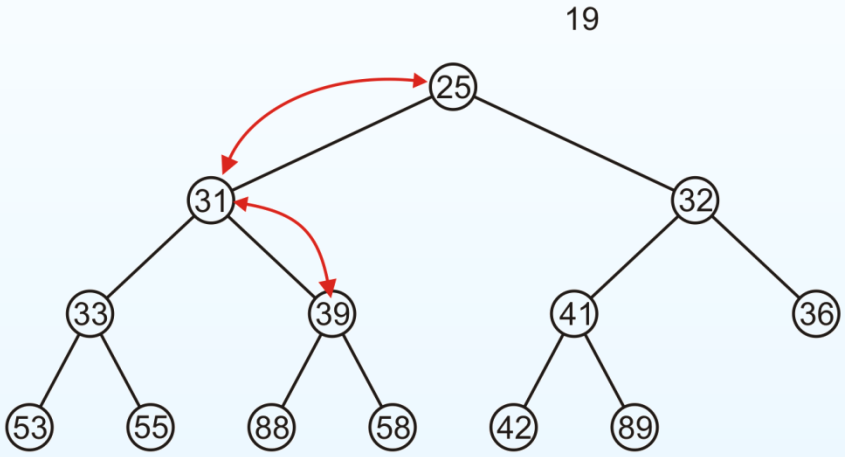
1. Copy 39 to the root



Complete Binary Heaps: Deletion

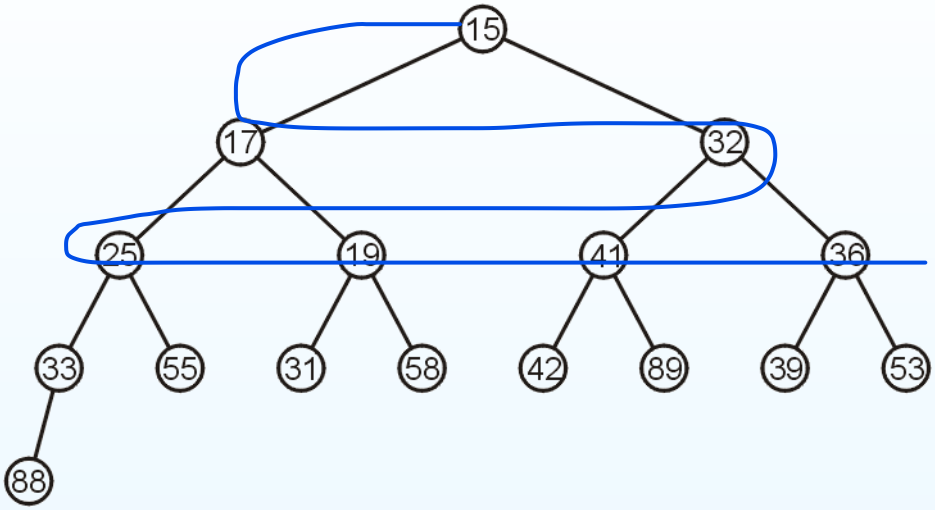
Again, delete 19:

1. Copy 39 to the root
2. Percolate 39 down to the appropriate location

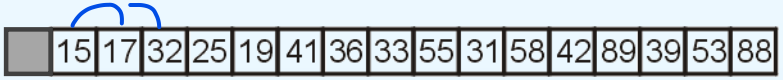


Array Implementation

For the heap:

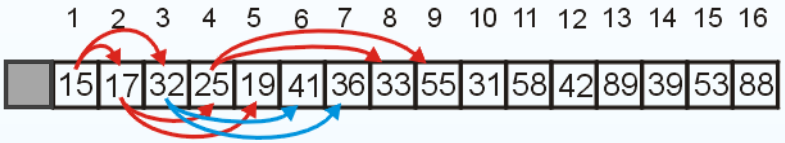


A breadth-first traversal yields:



Array Implementation

Recall that if we associate an index—starting at 1—with each element in the breadth-first traversal, we get:

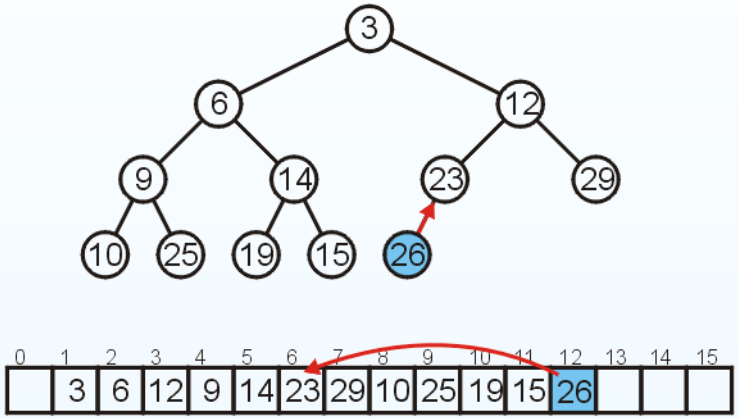


Given the element at index k, it follows that:

- The parent is at $\lfloor k/2 \rfloor$
- The children are at $2k$ and $2k + 1$

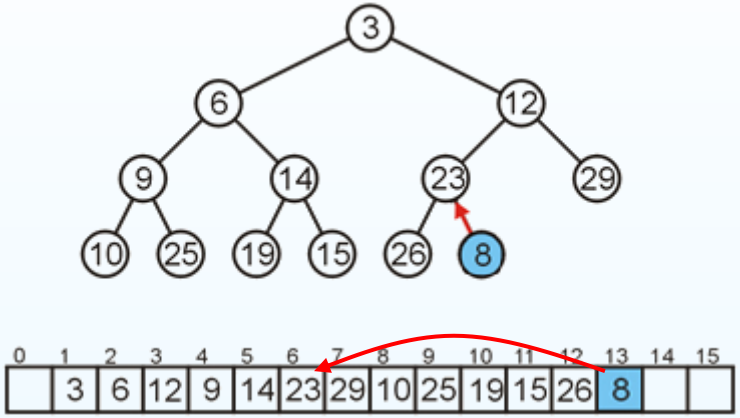
Array Implementation: Insertion

Inserting 26 requires no changes



Array Implementation: Insertion

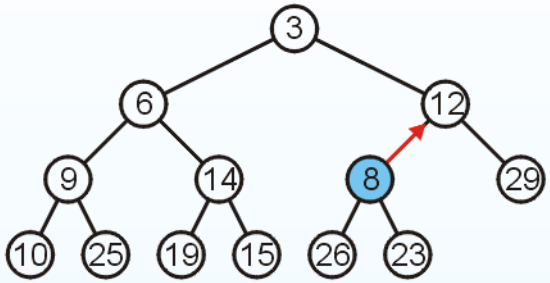
Inserting 8 requires percolations:

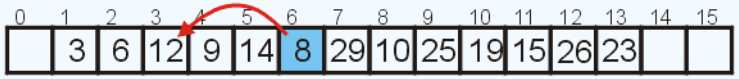


1. Swap 8 and 23

Array Implementation: Insertion

Inserting 8 requires percolations:

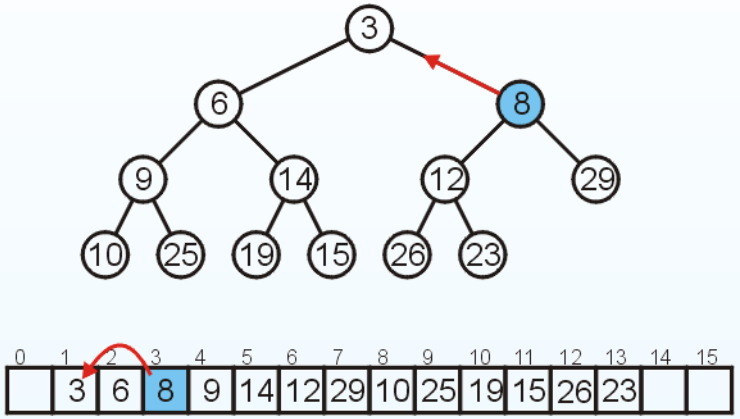




1. Swap 8 and 23
2. Swap 8 and 12

Array Implementation: Insertion

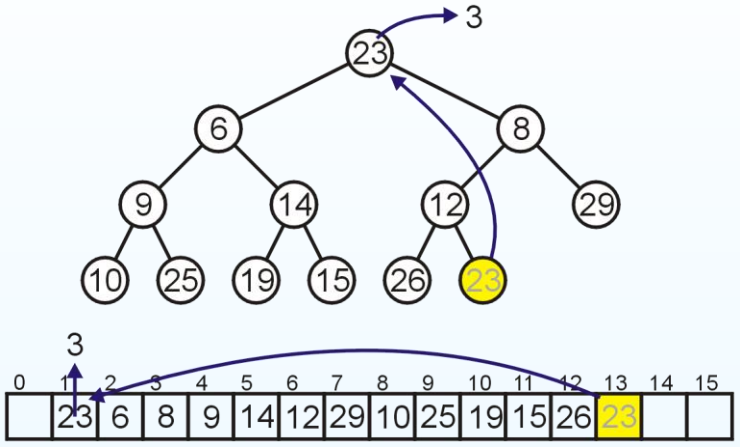
Inserting 8 requires percolations:



1. Swap 8 and 23
2. Swap 8 and 12
3. The node 8 is greater than its parent, so we are finished

Array Implementation: Deletion

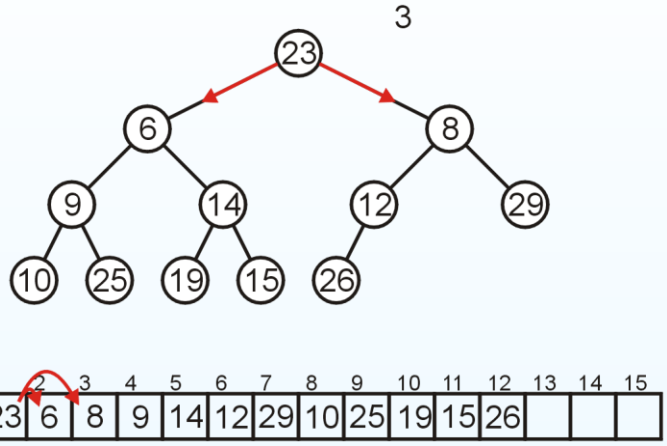
Removing the minimum value:



1. Copy the last element 23 to the root

Array Implementation: Deletion

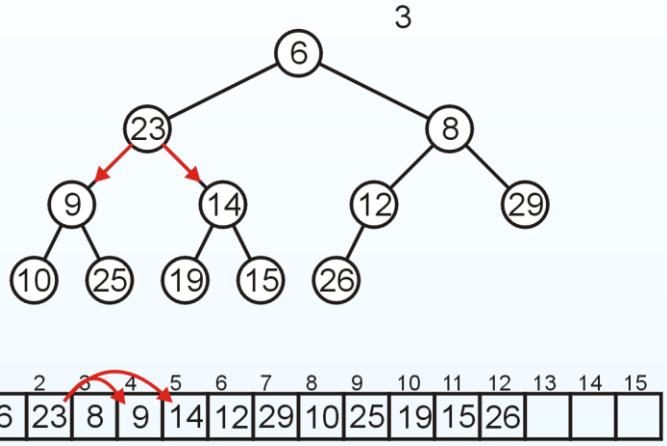
Removing the minimum value:



1. Copy the last element 23 to the root
2. Compare the node 23 (index 1) with its children (indices 2 and 3)
   - Swap 23 and 6

Array Implementation: Deletion

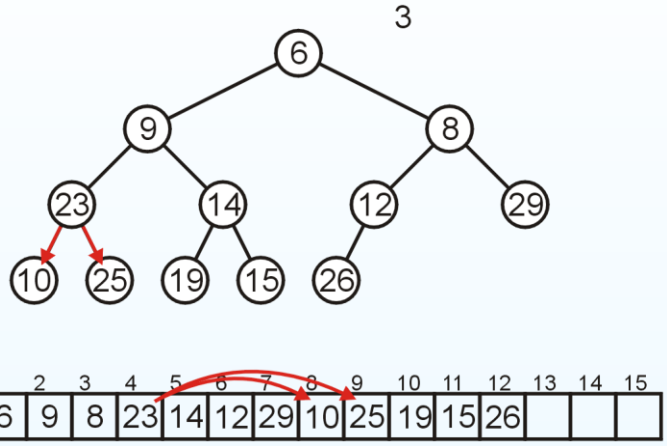
Removing the minimum value:



1. Copy the last element 23 to the root
2. Compare the node 23 (index 1) with its children (indices 2 and 3)
   - Swap 23 and 6
3. Compare the node 23 with its children: swap 23 and 9

Array Implementation: Deletion

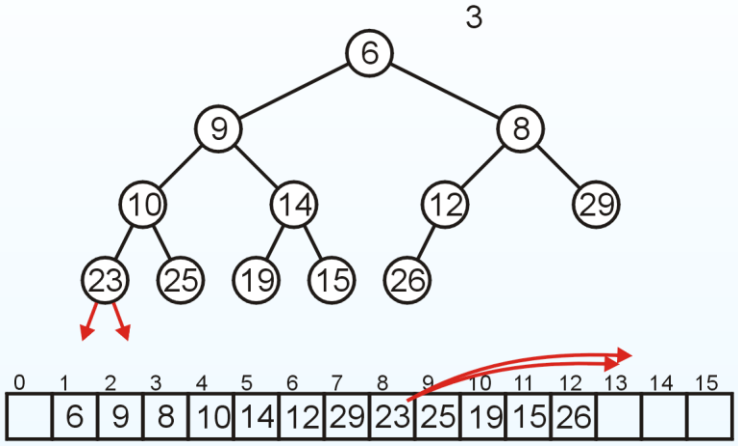
Removing the minimum value:



1. Copy the last element 23 to the root
2. Compare the node 23 (index 1) with its children (indices 2 and 3)
   - Swap 23 and 6
3. Compare the node 23 with its children: swap 23 and 9
4. Compare the node 23 with its children: swap 23 and 10

Array Implementation: Deletion

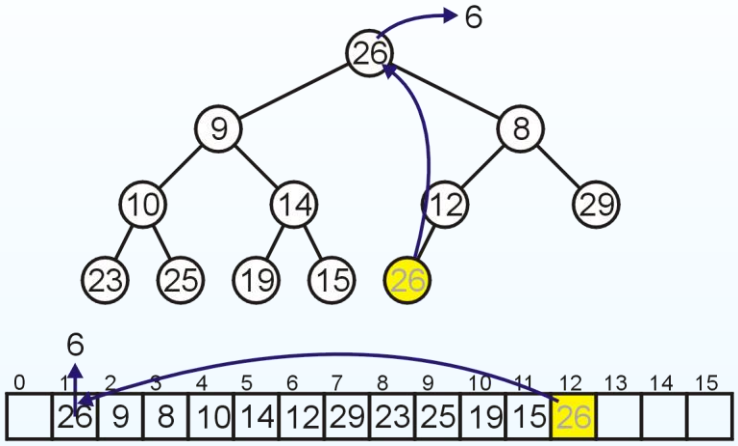
Removing the minimum value:



1. Copy the last element 23 to the root
2. Compare the node 23 (index 1) with its children (indices 2 and 3)
   - Swap 23 and 6
3. Compare the node 23 with its children: swap 23 and 9
4. Compare the node 23 with its children: swap 23 and 10
5. The node 23 is now the leaf node; stop the process

Array Implementation: Deletion

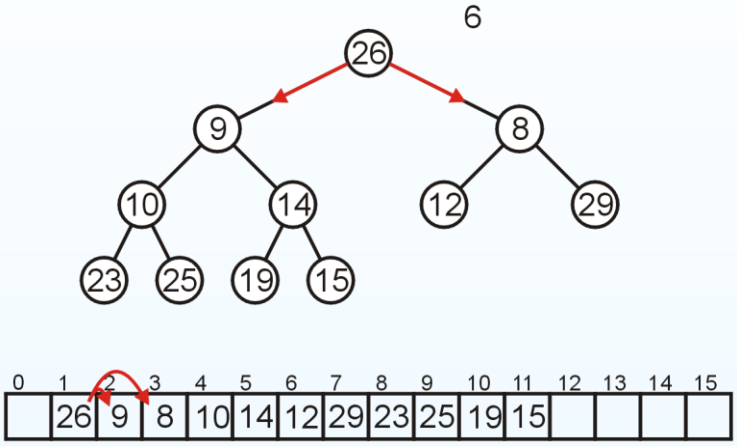
Again, removing the minimum value:



1. Copy the last element 26 to the root

Array Implementation: Deletion

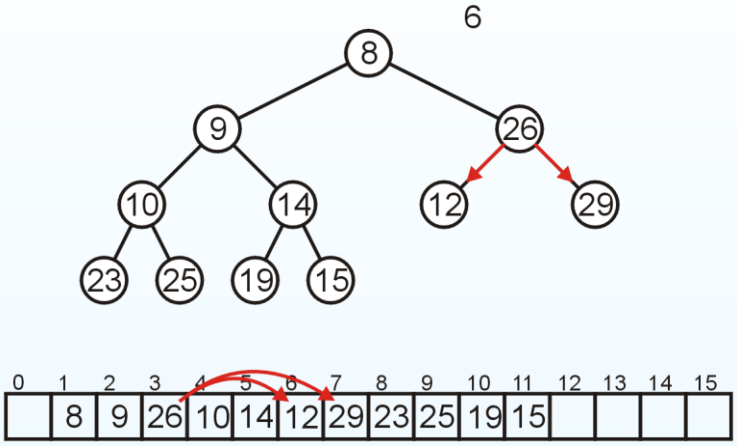
Again, removing the minimum value:



1. Copy the last element 26 to the root
2. Compare 26 with 9 and 8; swap 26 and 8

Array Implementation: Deletion

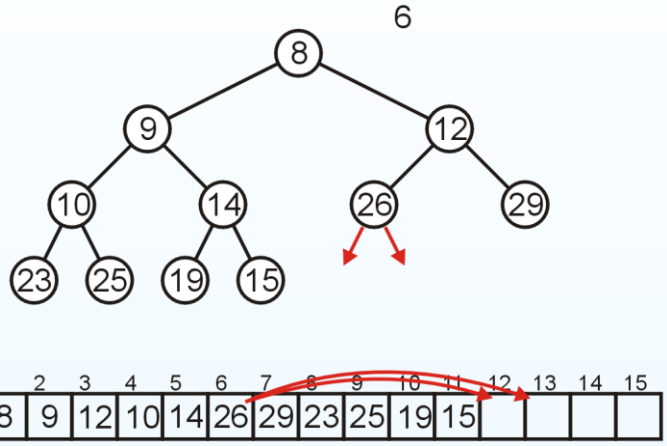
Again, removing the minimum value:



1. Copy the last element 26 to the root
2. Compare 26 with 9 and 8; swap 26 and 8
3. Compare 26 with 12 and 29; swap 26 and 12

Array Implementation: Deletion

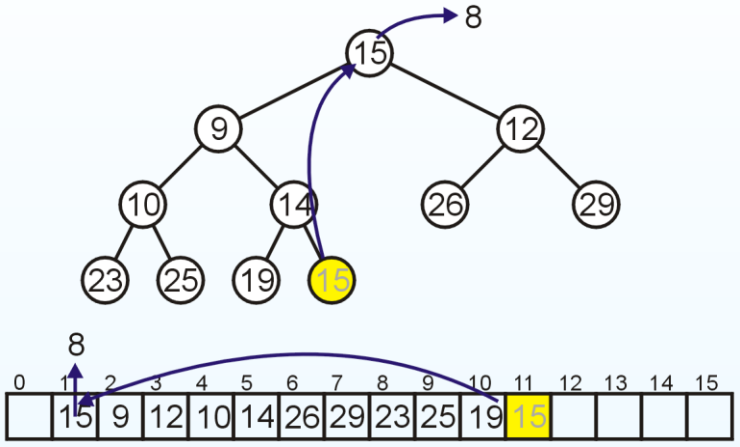
Again, removing the minimum value:



1. Copy the last element 26 to the root
2. Compare 26 with 9 and 8; swap 26 and 8
3. Compare 26 with 12 and 29; swap 26 and 12
4. The node 26 is now the leaf node; stop

Array Implementation: Deletion

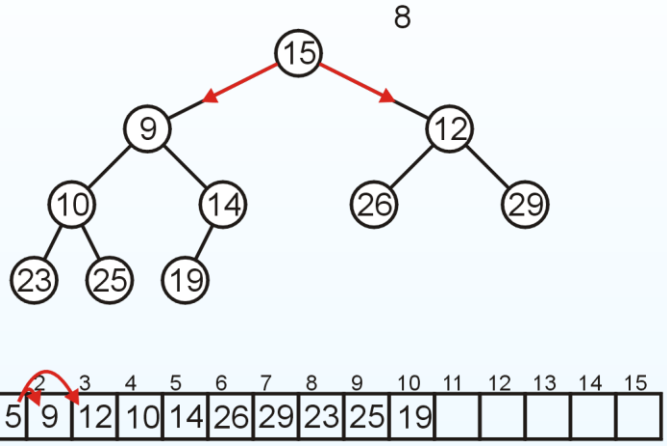
Again, removing the minimum value:



1. Copy the last element 15 to the root

Array Implementation: Deletion

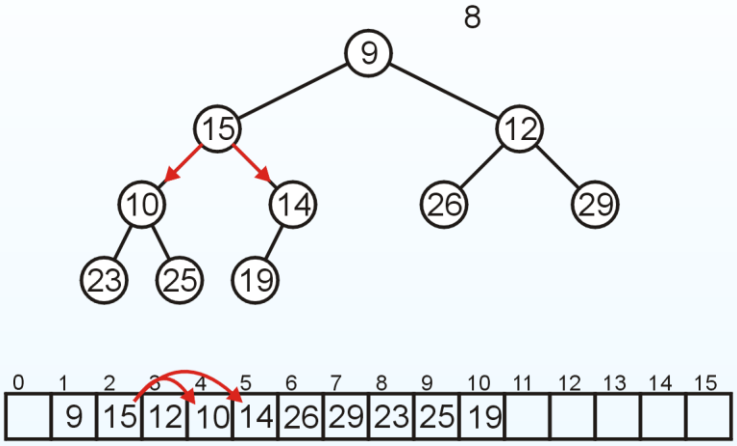
Again, removing the minimum value:



1. Copy the last element 15 to the root
2. Compare 15 with 9 and 12; swap 15 and 9

Array Implementation: Deletion

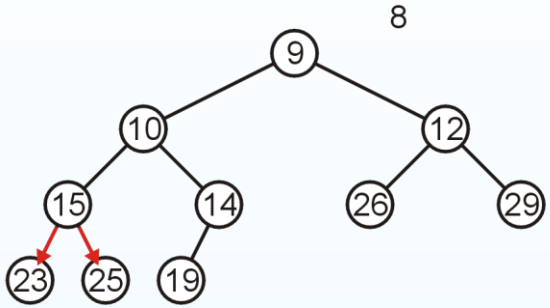
Again, removing the minimum value:

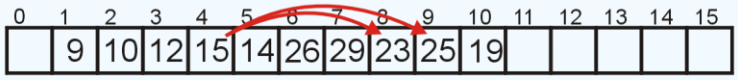


1. Copy the last element 15 to the root
2. Compare 15 with 9 and 12; swap 15 and 9
3. Compare 15 with 10 and 14; swap 15 and 10

Array Implementation: Deletion

Again, removing the minimum value:





1. Copy the last element 15 to the root
2. Compare 15 with 9 and 12; swap 15 and 9
3. Compare 15 with 10 and 14; swap 15 and 10
4. Compare 15 with 23 and 25; it is less than both so we stop

Running Time Analysis

With a complete binary heap,

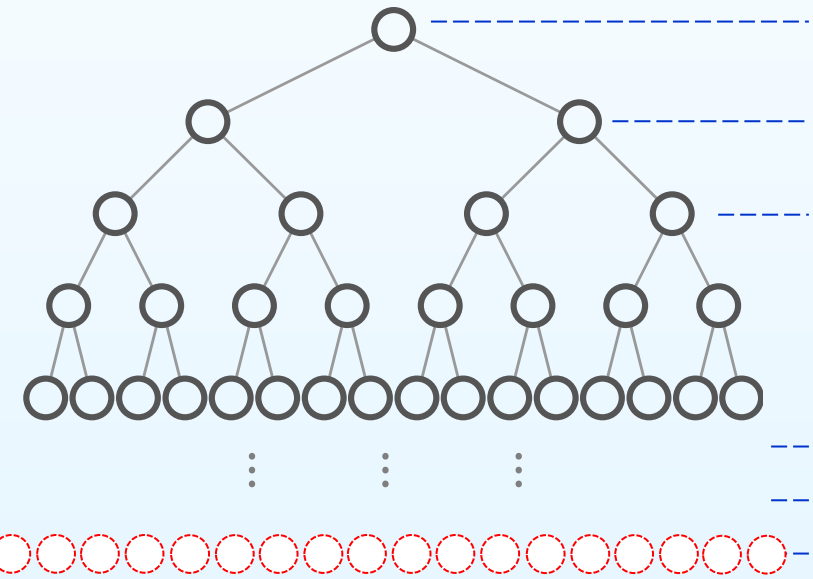
- The `find_min()` operation takes $\Theta(1)$
- The `delete_min()` operation takes $O(h) = O(\log n)$
  - Since we copy element (with some high value) in the lowest depth, it will likely be percolate down to the lowest depth

- The `insert()` operation takes $O(h)$ … Ummm >o<

Running Time Analysis: Insertion

- If we insert object greater than any object, the best-case = $\Omega(1)$
- If we insert object less than the root, the worst-case = $O(h)$
- How about insert any value on average?

- Inserting causes percolate up along the path to root
- Let the heap have n nodes



at depth 0, $\frac{n}{2^h} = 1$ node, h times

at depth 1, $\frac{n}{2^{h-1}} = 2$ nodes, $h-1$ times

at depth 2, $\frac{n}{2^{h-2}} = 4$ nodes, $h-2$ times

⋮ ⋮ ⋮

at depth $h-2$, there exist $n/8$ nodes, compare 3 times

at depth $h-1$, there exist $n/4$ nodes, compare 2 times

at depth h, there exist $n/2$ nodes, compare 1 time

The expected (average) work to insert $= \frac{1}{2} \cdot 1 + \frac{1}{4} \cdot 2 + \frac{1}{8} \cdot 3 + \cdots + \frac{1}{2^h} \cdot h = \sum_{i=1}^{h} \frac{i}{2^i}$

prob. to locate at h

Running Time Analysis: Insertion

$$\sum_{i=1}^{h} \frac{i}{2^i} = \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \cdots + \frac{h}{2^h} = x$$

$$\frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \cdots + \frac{h-1}{2^h} + \frac{h}{2^{h+1}} = \frac{1}{2}x$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots + \frac{1}{2^h} - \frac{h}{2^{h+1}} = \frac{1}{2}x$$

$$\frac{1 - \left(\frac{1}{2}\right)^{h+1}}{1 - \frac{1}{2}} - \frac{h}{2^{h+1}} = \frac{1}{2}x$$

$$x = \frac{4 \cdot 2^h - 2 - h}{2^h}$$

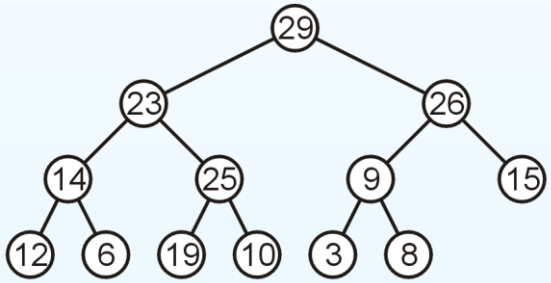
$$x = \lim_{n \to \infty} \frac{4n - 2 - \log n}{n}$$

the average-case for insertion = $\Theta(1)$

Binary Max Heaps

A binary max-heap is identical to a binary min-heap except that the parent is always larger than either of the children

For example, the same data as before stored as a complete binary max-heap yields:



0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	29	23	26	14	25	9	15	12	6	19	10	3	8		

Heaps and Priority Queues

A:

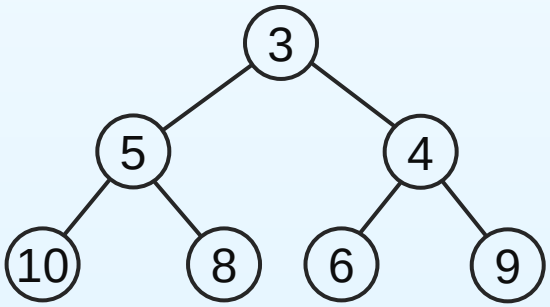
The smaller the number,
The higher the priority

B:

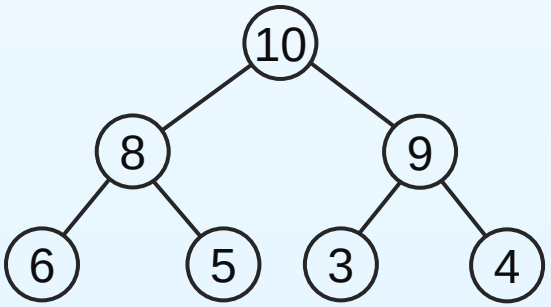
The larger the number,
The higher the priority

- Operations on an array may take $O(n)$
- While operations on a heap take $O(\log n)$

Min Heap



Max Heap



Outline

- Priority Queues
- Binary Heaps
- Application:
  - Huffman code

Problem

Suppose we want to encode a message "DEAACAAAAABA", constructed from 5 different symbols

How many bits are required?

- Use a fixed-length ASCII code:
  - Each symbol requires 8 bits
  - There are 12 symbols
  - So, the message requires 96 bits
- Use a fixed-length Unicode:
  - The message requires 192 bits

ASCII Table

Dec	Hex	Oct	Char	Dec	Hex	Oct	Char	Dec	Hex	Oct	Char	Dec	Hex	Oct	Char
0	0	0		32	20	40	[space]	64	40	100	@	96	60	140	`
1	1	1		33	21	41	!	65	41	101	A	97	61	141	a
2	2	2		34	22	42	"	66	42	102	B	98	62	142	b
3	3	3		35	23	43	#	67	43	103	C	99	63	143	c
4	4	4		36	24	44	$	68	44	104	D	100	64	144	d
5	5	5		37	25	45	%	69	45	105	E	101	65	145	e
6	6	6		38	26	46	&	70	46	106	F	102	66	146	f
7	7	7		39	27	47	'	71	47	107	G	103	67	147	g
8	8	10		40	28	50	(	72	48	110	H	104	68	150	h
9	9	11		41	29	51	)	73	49	111	I	105	69	151	i
10	A	12		42	2A	52	*	74	4A	112	J	106	6A	152	j
11	B	13		43	2B	53	+	75	4B	113	K	107	6B	153	k
12	C	14		44	2C	54	,	76	4C	114	L	108	6C	154	l
13	D	15		45	2D	55	-	77	4D	115	M	109	6D	155	m
14	E	16		46	2E	56	.	78	4E	116	N	110	6E	156	n
15	F	17		47	2F	57	/	79	4F	117	O	111	6F	157	o
16	10	20		48	30	60	0	80	50	120	P	112	70	160	p
17	11	21		49	31	61	1	81	51	121	Q	113	71	161	q
18	12	22		50	32	62	2	82	52	122	R	114	72	162	r
19	13	23		51	33	63	3	83	53	123	S	115	73	163	s
20	14	24		52	34	64	4	84	54	124	T	116	74	164	t
21	15	25		53	35	65	5	85	55	125	U	117	75	165	u
22	16	26		54	36	66	6	86	56	126	V	118	76	166	v
23	17	27		55	37	67	7	87	57	127	W	119	77	167	w
24	18	30		56	38	70	8	88	58	130	X	120	78	170	x
25	19	31		57	39	71	9	89	59	131	Y	121	79	171	y
26	1A	32		58	3A	72	:	90	5A	132	Z	122	7A	172	z
27	1B	33		59	3B	73	;	91	5B	133	[	123	7B	173	{
28	1C	34		60	3C	74	<	92	5C	134	\	124	7C	174	|
29	1D	35		61	3D	75	=	93	5D	135	]	125	7D	175	}
30	1E	36		62	3E	76	>	94	5E	136	^	126	7E	176	~
31	1F	37		63	3F	77	?	95	5F	137	_	127	7F	177	

Problem

Suppose we want to encode a message "DEAACAAAAABA", constructed from 5 different symbols

How many bits are required?

- Use a fixed-length user-defined code:
  - 2 bits are not enough for each symbol
  - At least 3 bits are required
  - So, the message with 12 symbols will requires 36 bits

Symbol	Code
A	000
B	001
C	010
D	011
E	100

Drawbacks of Fixed-Length Codes

- Wasted space
  - There are a lot of codes that are not used for symbols
  - Unicode uses twice as much space as ASCII
    - Inefficient for plain-text messages containing only ASCII characters

- **Potential solution:** use variable-length codes

Variable-Length Codes

- When frequency of occurrence is known,
  - Variable number of bits to represent characters
  - Short codes for characters that occur frequently
  - On average, the length of the encoded message is less than fixed-length encoding

- **Potential problem:** how do we know where one-character ends, and another begins?
  - Not a problem of number of bits is fixed

encoded message: 001011011100010

decoded message: BDDEC

Symbol	Code
A	000
B	001
C	010
D	011
E	100

Prefix Property

- The following code does not have prefix property

Symbol	Code
A	0
B	1
C	01
D	10
E	11

encoded message: 1110

decoded message: BBBA or BEA or BBD or ED

- The prefix property: character code is not prefix of another character

Symbol	Code
A	000
B	11
C	01
D	001
E	10

encoded message: 0100110110000 10

decoded message: CDEBAE

Problem

Suppose we want to encode a message "DEAACAAAAABA", constructed from 5 different symbols

How many bits are required?

- Use a variable-length code:

Symbol	Frequency	Code
A	8	0
B	1	~~01~~ ~~1~~ 10
C	1	110
D	1	1110
E	1	11110

decoded message: DEAACAAAAABA

encoded message: 11101111001100000100

Require 22 bits

Problem

Suppose we want to encode a message "DEAACAAAAABA", constructed from 5 different symbols

How many bits are required?

- Use a variable-length code:

Another possible code

Symbol	Frequency	Code
A	8	0
B	1	100
C	1	101
D	1	1101
E	1	1111

decoded message: DEAACAAAAABA

encoded message: 1101111100101000001000

Require 22 bits

Problem

Suppose we want to encode a message "DEAACAAAAABA", constructed from 5 different symbols

How many bits are required?

- Use a variable-length code:

Better code

Symbol	Frequency	Code
A	8	0
B	1	100
C	1	101
D	1	110
E	1	111

decoded message: DEAACAAAAABA

encoded message: 11011100101000001000

Require 20 bits

Huffman Coding Trees

- Binary tree
  - Each leaf contains symbol (character)
  - Label edge from node to left child with 0
  - Label edge from node to right child with 1

- Code for any symbol obtained by following path from the root to the leaf node containing symbol

- Code has the prefix property
  - Leaf node cannot appear on path to another leaf

Building a Huffman tree

- Find frequencies of each symbol occurring in message
- Begin with a forest of single node trees
  - Each contains symbol and its frequency
- Do repeatedly
  - Select two trees with the smallest frequency at the root
  - Produce a new binary tree with the selected trees as children and store the sum of their frequencies in the root
- Repetition end when there is one tree
  - This is the Huffman coding tree

Example

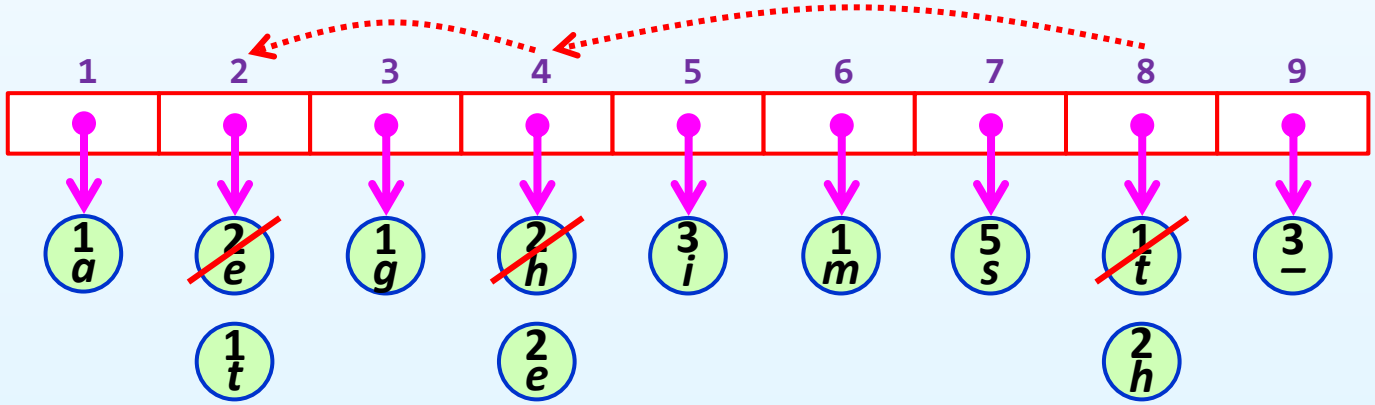
- Build the Huffman coding tree for the message:

this is his message

- Character frequencies

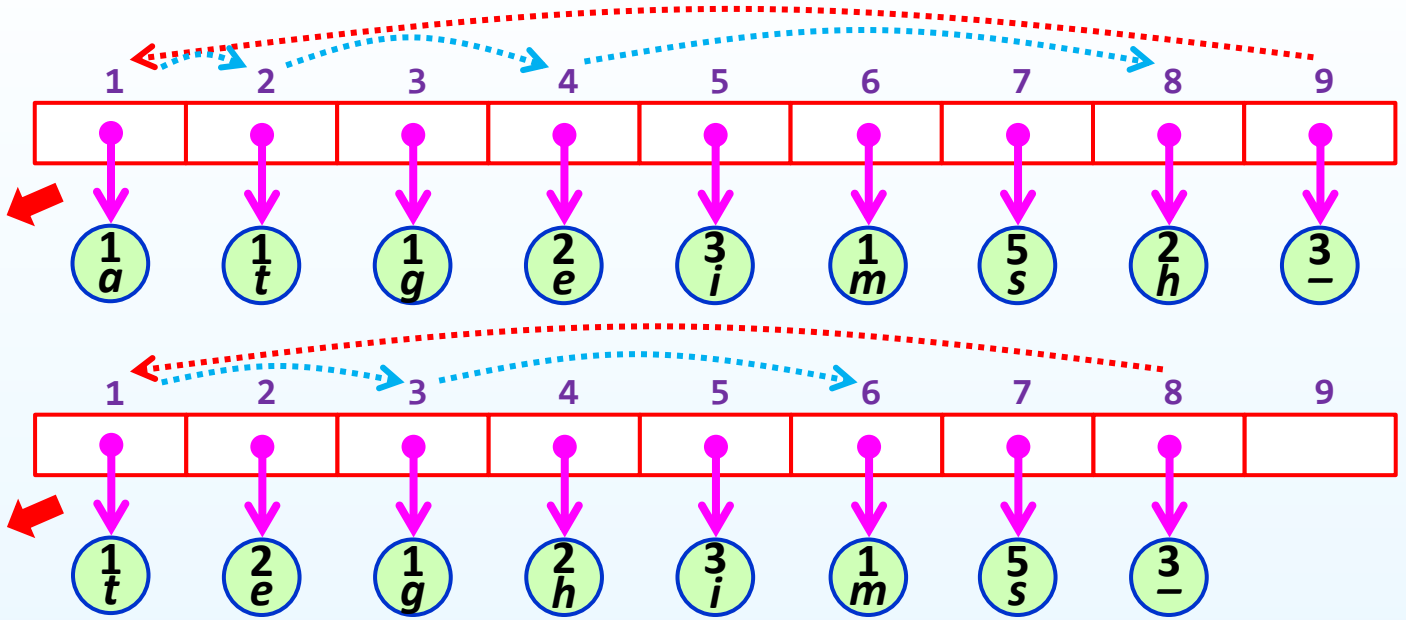
a	*e*	*g*	*h*	*i*	*m*	*s*	*t*	_
1	2	1	2	3	1	5	1	3

- Begin with a forest of single trees representing by a priority queue (min-heap)

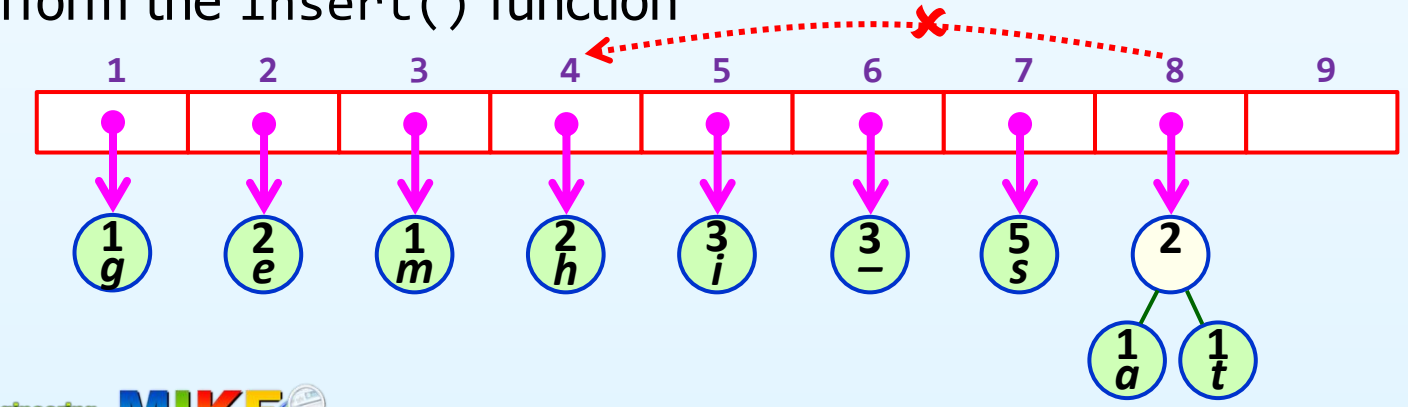


Example: Round 1

- Perform the `delete_min()` function 2 times

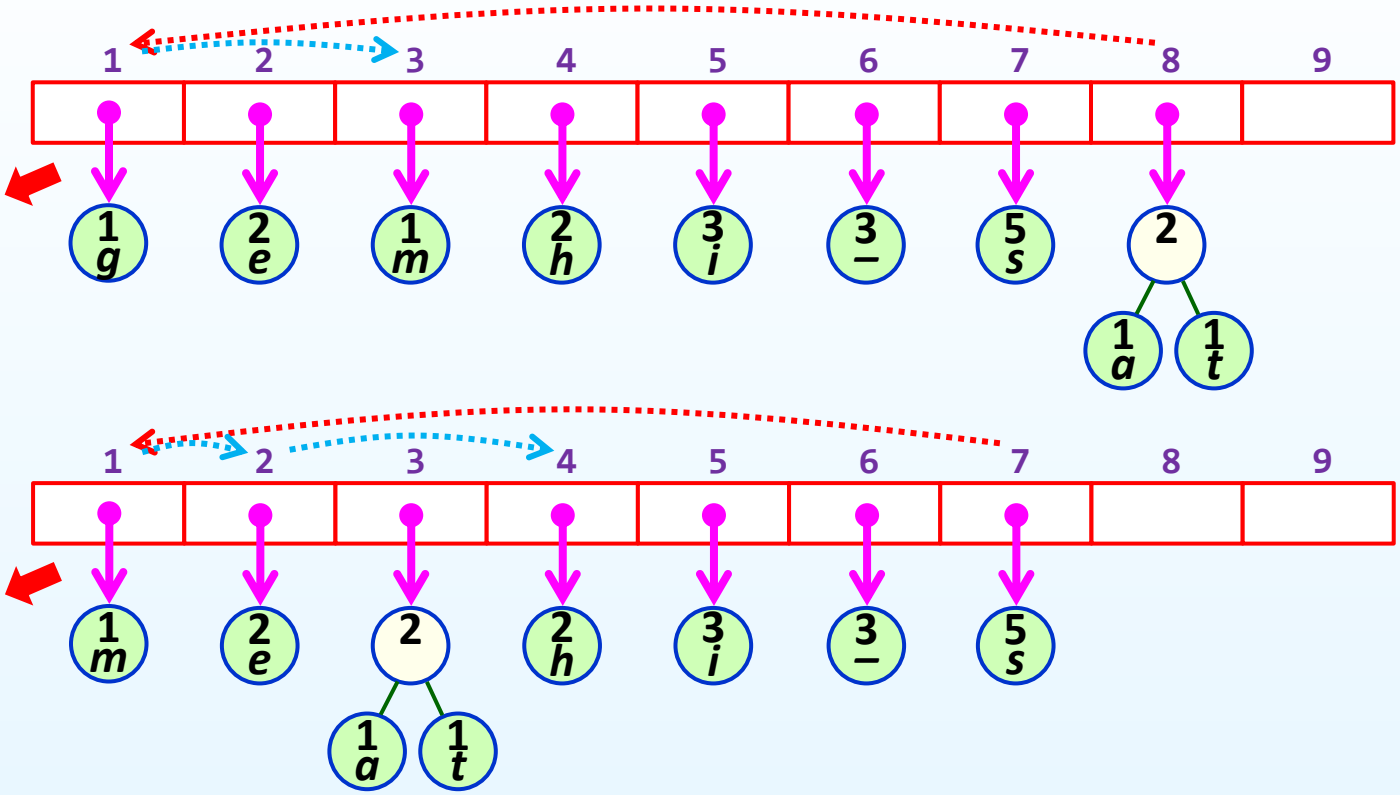


- Perform the `insert()` function



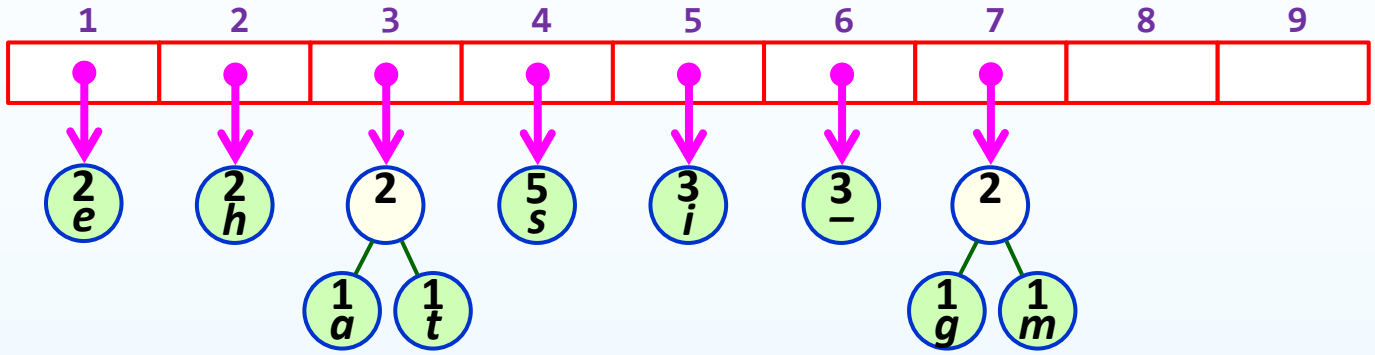
Example: Round 2

- Perform the `delete_min()` function 2 times



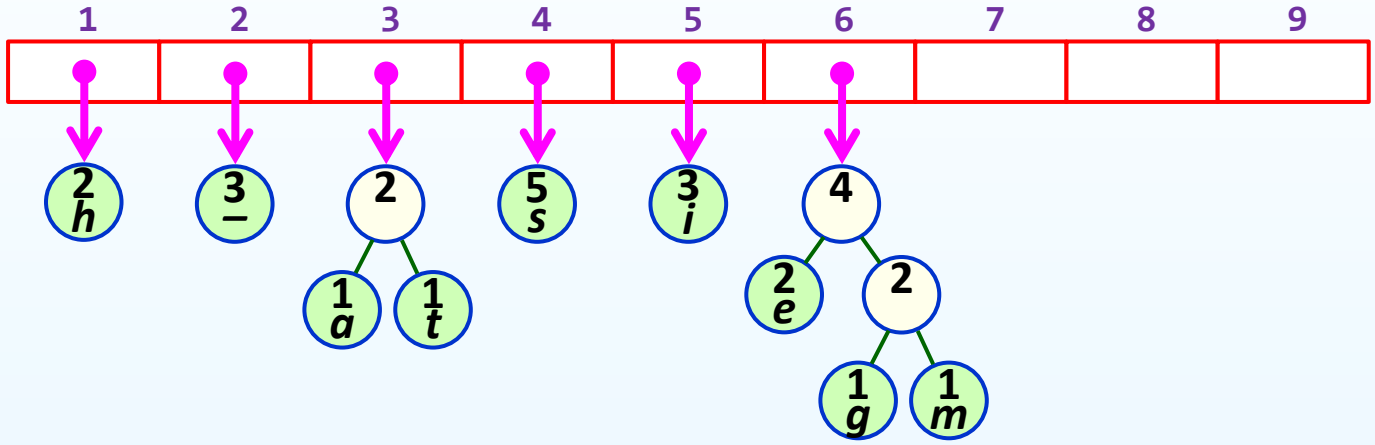
Example: Round 2

- Perform the `insert()` function



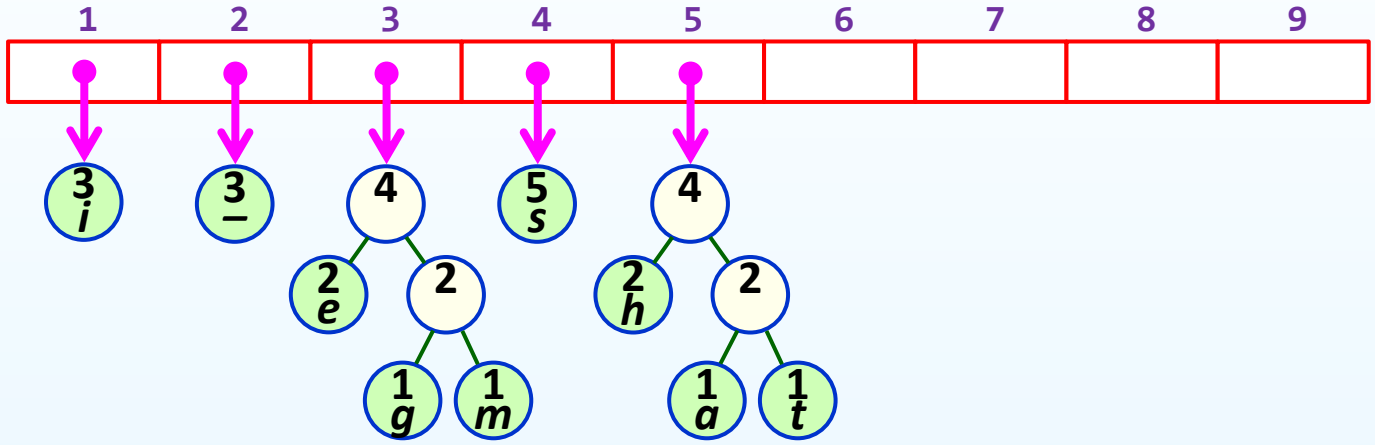
Example: Round 3

- Perform the `delete_min()` function 2 times
- Perform the `insert()` function



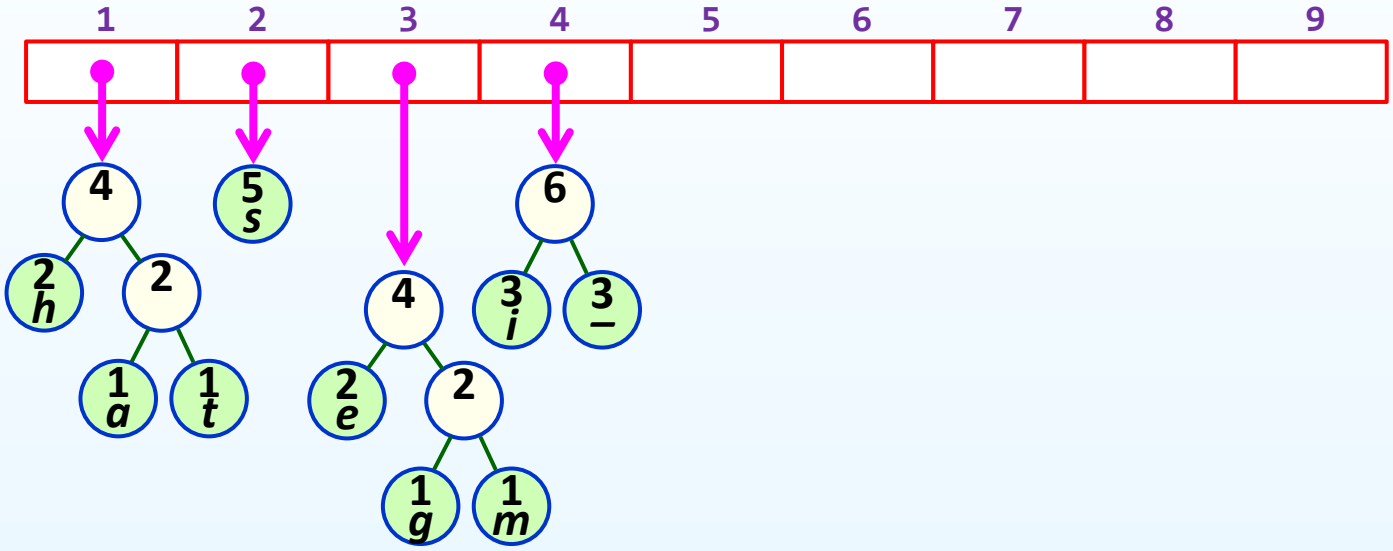
Example: Round 4

- Perform the `delete_min()` function 2 times
- Perform the `insert()` function



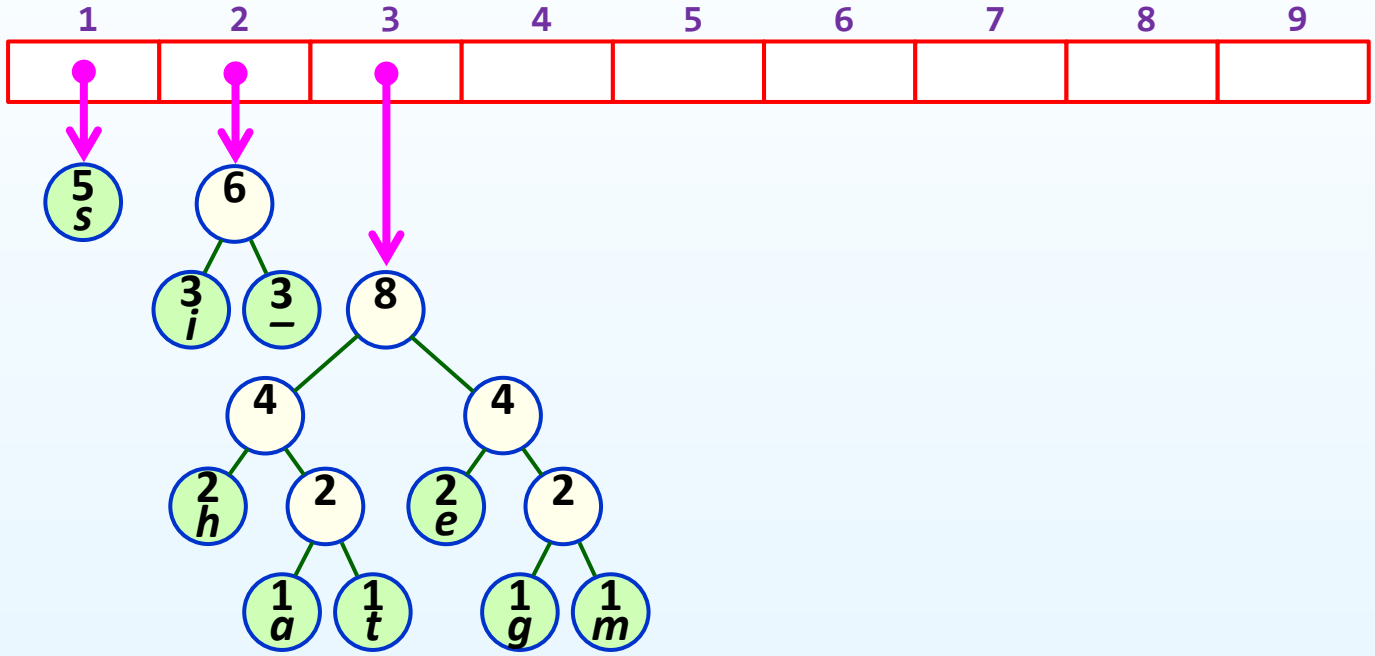
Example: Round 5

- Perform the `delete_min()` function 2 times
- Perform the `insert()` function



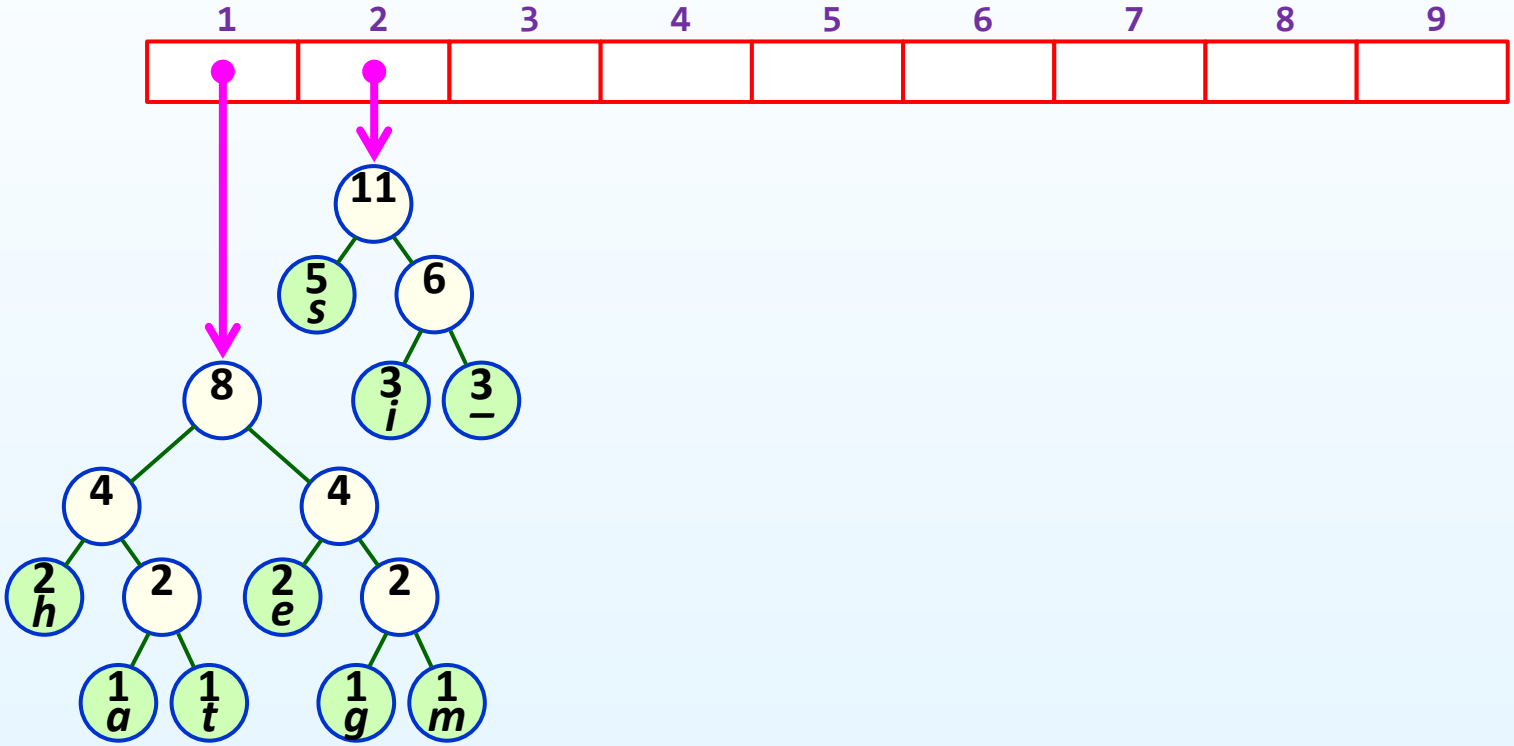
Example: Round 6

- Perform the `delete_min()` function 2 times
- Perform the `insert()` function



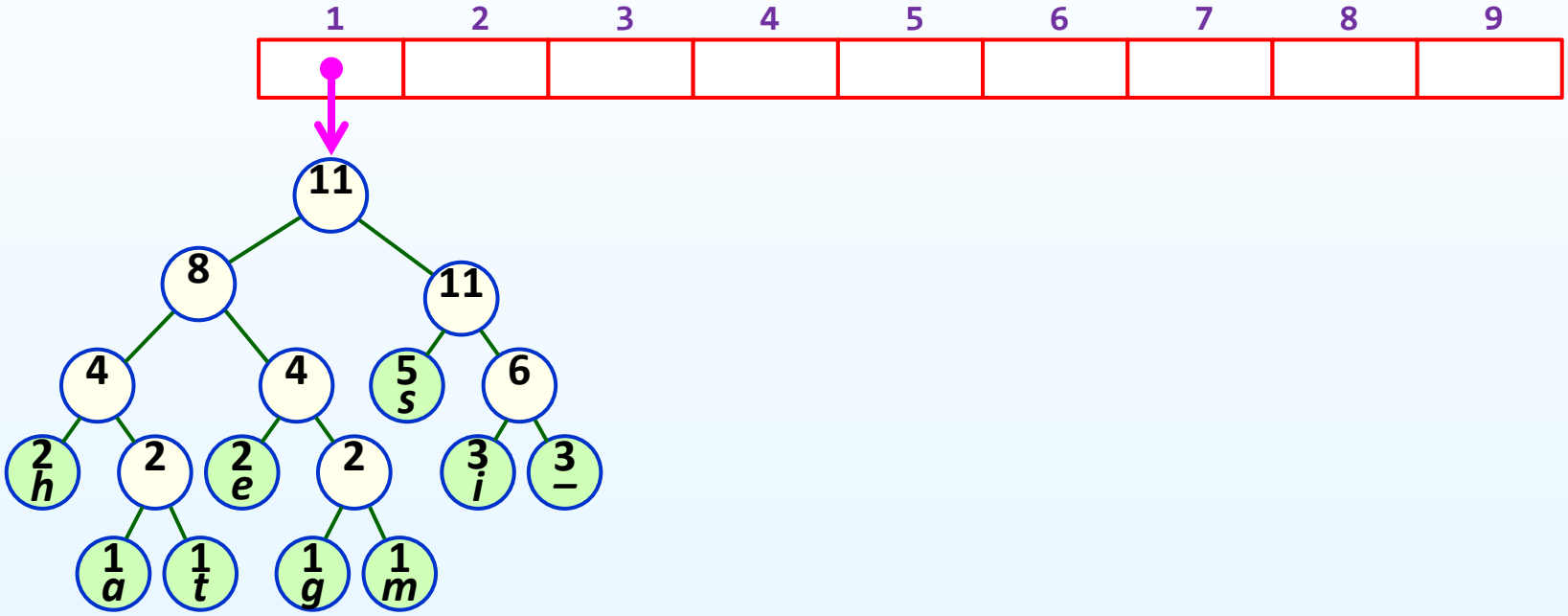
Example: Round 7

- Perform the `delete_min()` function 2 times
- Perform the `insert()` function



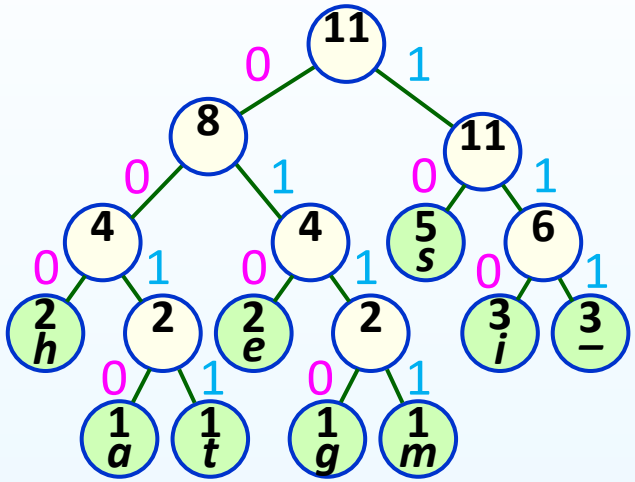
Example: Round 8

- Perform the `delete_min()` function 2 times
- Perform the `insert()` function



Example: Coding Table

- Finally, label edges from the node to its left and right children



Symbol	Frequency	Code
a	1	0010
e	2	010
g	1	0110
h	2	000
i	3	110
m	1	0111
s	5	10
t	1	0011
_	3	111

decoded message: *this is his message*

encoded message: 0011000110101111010111000110101110111010101000100110010

Any Question?