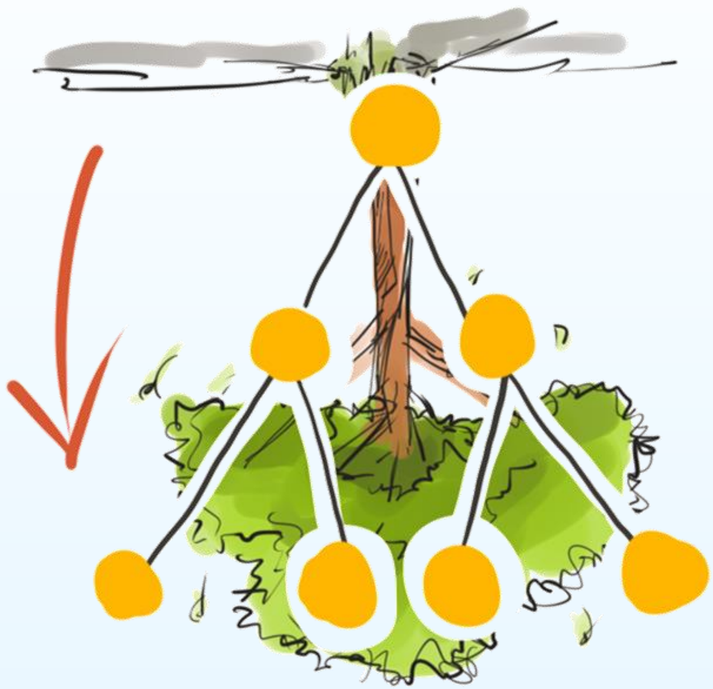

Lecture 11:
Hash ADT

01204212 Abstract Data Types and Problem Solving

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Outline

- Hash ADT
- Hash Functions
- Collisions
  - Separate Chaining
  - Open Addressing

The Need for Fast Retrieving Data

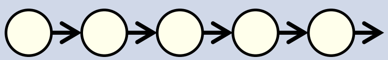
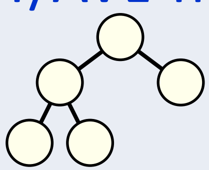
- In many applications, we need a system that fast stores and access the data

- For example: a simple system to students' GPA
  - Recall that each student has a unique 10-digit identifier so that we have:

    `(stdID_1,GPA_1),(stdID_2,GPA_2),…,(stdID_n,GPA_n)`
  - We need the fast `find()`, `insert()`, and `delete()` operations

Which ADT?

ADT Candidates

ADT	find()	insert()	delete()
1. List 	$O(n)$	$O(1)$	$O(n)$
2. BST/AVL Tree 	$O(\log n)$	$O(\log n)$	$O(\log n)$

- In real-world applications, n is typically between 100 and 100,000 (or more)
  - $\log n$ is between 6.6 and 16.6
- We need a designed ADT for all operations in $\Theta(1)$ time
  - However, we may incur other costs

Guess! Which data structure?

An Array-Implementation

Our goal:

- Store data so that all operations are $\Theta(1)$ time

Requirement:

- The memory requirement should be $\Theta(n)$
- Normally, the size is larger than the number of the data

In practice, we would like to

- Create an array of size M
- Store each of n objects in one of the M bins
- Have some means of determining the bin in which an object is stored

Example: Design for Implementation

- Consider 100 records of student ID with his GPA:

  `(stdID_1,GPA_1),(stdID_2,GPA_2),…,(stdID_100,GPA_100)`

  - We can use the student ID as the index of array

- Suppose "Joe" has the number 6310501234
  - Obviously, we cannot create an array of size 10^{10}

- We could create an array of size 1000:
  - How could you convert a 10-digit number into 3-digit number?
  - First three digits (631…) might cause a problem
  - The last three digits, however, are essentially random

- Therefore, Joe's GPA could be store in `gpa[234]=3.50`

Example: Design for Implementation

Question:

- What is the likelihood that a class of 100 students will have nobody with the same last three digits?

$$\frac{1000}{1000} \cdot \frac{999}{1000} \cdot \frac{998}{1000} \cdot \frac{997}{1000} \cdot \cdots \cdot \frac{901}{1000} \approx 0.005959$$

➡ Not very high

Example: Design for Implementation

- Consequently, we have a function that maps a student onto a 3-digit number:
  - The function is the modulus of 1000
  - We can store something in that location
  - Storing, accessing, and deleting it are $\Theta(1)$

- However, two or more students may map to the same number:
  - Joe has ID 6310501234 and GPA 3.5
  - Alma has ID 6310505234 and GPA 4.0

	⋮
231	
232	
233	
234	3.50
235	
236	
237	
238	
239	3.00
240	
	⋮

Hash Tables

- Objects are stored in an array, called a *table*
- The process of mapping an object or a number onto an integer in a given range is called *hashing*
- An event that multiple objects may hash to the same value is called a *collision*
- Therefore, hash tables use a hash function together with a mechanism for dealing with collisions

The Hash Process

Object

1.1 Pre-hashing an object into a 32-bit integer

1. Hashing

1.2 Mapping to an index $0, \ldots, M-1$ of array

Hash Value

2. Dealing with collisions

- Separate chaining
- Open addressing

Storing Data

Outline

- Hash ADT
- Hash Functions
- Collisions
  - Separate Chaining
  - Open Addressing

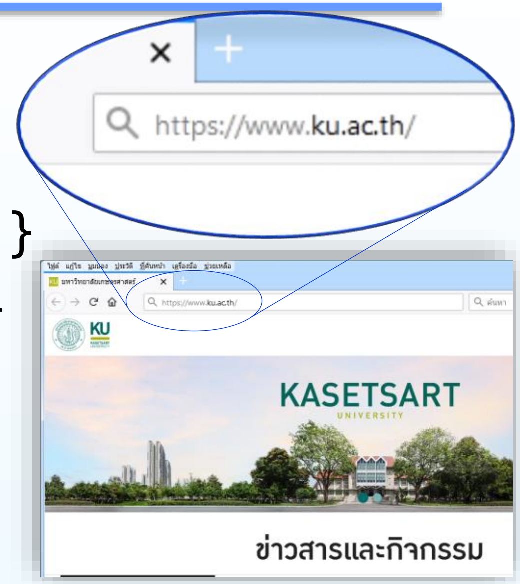
Hashing Schemes

- We want to store n objects in a table of M at a location computed from the key K

- Hash function
  - Method for computing table index from key
- Collision resolution strategy
  - How to handle two keys that hash to the same index

Example: Looking for an IP Address

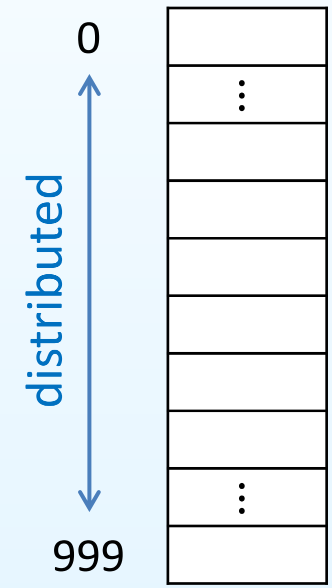
- Data records:
  - {"www.ku.ac.th", "158.108.216.5"}
  - {"www.eng.ku.ac.th", "158.108.215.137"}
  - {"www.cpe.ku.ac.th", "158.108.32.150"}
  - {"adt.mikelab.net", "158.108.32.156"}
  - …



- Can we directly index into the array as the followings?
  - table["www.ku.ac.th"] = "158.108.216.5"
  - table["www.eng.ku.ac.th"] = "158.108.215.137"
  - table["www.cpe.ku.ac.th"] = "158.108.32.150"
  - table["adt.mikelab.net"] = "158.108.32.156"
  - …

Mapping into Hash Table

- Need a fast hash function h to convert the element key (number or string) to an integer (hash value)
  - Then, use this value as an index of array
- Given an array `table` of size `1000`, we first calculate the index by:
  - h(`"www.ku.ac.th"`) ➡ `851`
  - h(`"www.eng.ku.ac.th"`) ➡ `291`
  - h(`"www.cpe.ku.ac.th"`) ➡ `441`
  - h(`"adt.mikelab.net"`) ➡ `705`
  - …



- Output of the hash function
  - Must always be less than size of array
  - Must be as evenly distributed as possible

Properties

Necessary properties of such a hash function h are:

- Should be fast
  - Ideally $\Theta(1)$
- Must be deterministic
  - Always return the same hash value for the same object
- Hash equal objects to equal values
  - $x = y \Longrightarrow h(x) = h(y)$
- Should be only a one-in-whole chance for hashing two random objects to the same hash value

Steps in Hash Function

A hash function may consist of two steps:

1. Pre-hash an object key into a 32-bit integer
   – The key can be number or string

2. Map the integer to an index of the array
   – Simply modulo the array size

1. Pre-hashing an Object Key

We design a deterministic hash function to calculate an arithmetic hash value from the relevant member variables of an object

We will look at the examples of arithmetic hash function for

- Rational numbers
  - e.g., (0,1), (1,2), (2,3), (99,100), …
- Strings
  - e.g., “apple”, “boy”, “cat”, “hello”, …

Example: Pre-hashing Rational Numbers

- Suppose we want to store rational numbers:
  - (0,1), (1,2), (2,3), (99,100), …
- Obviously, the array may not support tuples as indices
  - We need to first convert an individual into a 32-bit integer
- Each rational number is composed of the numerator (`numer`) and the denominator (`denom`)

Example: Pre-hashing Rational Numbers

Version #1

```
unsigned int pre_hash(int numer, int denom) {
  return (unsigned int)numer + (unsigned int)denom;
}
```

Problem:

– Rational numbers such as (1,2) and (2,1) have the same value

Example: Pre-hashing Rational Numbers

Version #2

```c
#include <stdio.h>
#define MULTIPLIER 429496751  // selected (large) prime
unsigned int pre_hash(int numer, int denom) {
  return (unsigned int)numer +
           MULTIPLIER*(unsigned int)denom;
}
int main(void) {
  printf("%u\n", pre_hash(0,1));
  printf("%u\n", pre_hash(1,2));
  printf("%u\n", pre_hash(2,1));
  printf("%u\n", pre_hash(2,4));
  printf("%u\n", pre_hash(99,100));
  return 0;
}
```

```
429496751
858993503
429496753
1717987006
2239
```

Arithmetic operations wrap on overflow

Problem:

– Rational numbers such as (1,2) and (2,4) have different value

Example: Pre-hashing Rational Numbers

Version #3

```c
#include <stdio.h>
#define MULTIPLIER 429496751  // selected (large) prime
unsigned int pre_hash(int numer, int denom) {
  int divisor = gcd(numer, denom);
  divisor = (denom >= 0)? divisor : -divisor;
  numer /= divisor;
  denom /= divisor;
  return (unsigned int)numer +
             MULTIPLIER*(unsigned int)denom;
}
int main(void) {
  printf("%u\n", pre_hash( 1, 2));
  printf("%u\n", pre_hash(-1, 2));
  printf("%u\n", pre_hash( 2,-4));
  printf("%u\n", pre_hash(-2,-4));
  return 0;
}
```

Need to be implemented

Make the denominator always positive

```
858993503
858993501
858993501
858993503
```

Example: Pre-hashing Strings

- Suppose we want to store words:
  - "apple", "boy", "cat", "hello", …
- Also, in C Program, a string cannot be an index of array
  - We need to first convert an individual into a 32-bit integer
- A string is simply an array of bytes
  - Each byte stores a value from 0 to 255
  - A hash function can be a function of these bytes

Example: Pre-hashing Strings

Version #1

```
unsigned int pre_hash(char *str) {
  unsigned int hash_value = 0;
  int          i = 0;

  for (i=0; i<strlen(str); i++)
    hash_value += str[i];
  return hash_value;
}
```

Problem:

- Words with the same characters hash to the same location: "from" and "form"
- Slow running time: $\Theta(n)$

Example: Pre-hashing Strings

Version #2.1

```
#define MULTIPLIER 12347  // selected prime
unsigned int pre_hash(char *str) {
  unsigned int hash_value = 0;
  int          i = 0;

  for (i=0; i<strlen(str); i++)
    hash_value = MULTIPLIER*hash_value + str[i];
  return hash_value;
}
```

Let the individual characters represent the coefficients of a polynomial in x:

$$p(x) = c_0 x^{n-1} + c_1 x^{n-2} + \cdots + c_{n-3} x^2 + c_{n-2} x + c_{n-1}$$

Example: Pre-hashing Strings

Version #2.2

```
#define MULTIPLIER 12347  // selected prime
unsigned int pre_hash(char *str) {
  unsigned int hash_value = 0;
  int          i = 0;

  for (i=1; i<=strlen(str); i*=2)
    hash_value = MULTIPLIER*hash_value + str[i-1];
  return hash_value;
}
```

We may pick some characters only:

– Use the characters in locations $2^k - 1$ for $k = 0, 1, 2, \ldots$

$0, 1, 3, 7, 15, \ldots$

2. Mapping to an Array Index

Suppose the array has size M, the easiest method is to return the value modulus M

```
unsigned int hash_map(unsigned int hv, unsigned int M) {
  return hv % M;
}
```

Note that the modulus operator % is relative slow, we may use the bitwise operators such as &, <<, or >> instead

Outline

- Hash ADT
- Hash Functions
- Collisions
  - Separate Chaining
  - Open Addressing

Collisions

- A collision occurs when two different object keys hash to the same value
  - For example, for a table size 17, the object keys 18 (`18%17=1`) and 35 (`35%17=1`) will hash to the same value
- We cannot store the data records in the same slot of the array

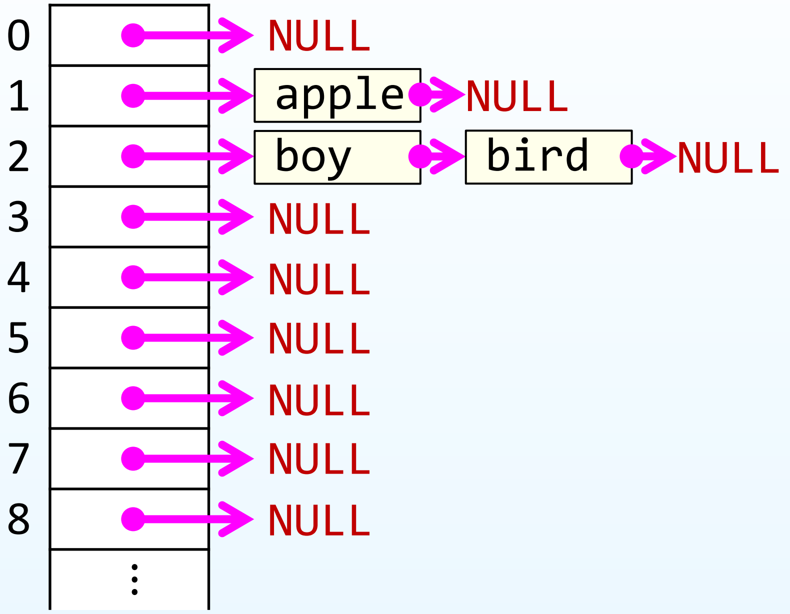
Collision Resolutions

1. Separate Chaining
   - Use data structure (such as a linked list) to store multiple objects that hash to the same slot

2. Open addressing (or probing)
   - Search for empty slots using a second function and store object in first empty slot that is found

1. Resolution by Separate Chaining

- Each hash table cell holds pointer to a linked list
- Collision: insert object into the linked list of same hash value
- To find an object: compute hash value, then find on the linked list
- Note that there are potentially as many as `table_size` lists



Why Lists?

- Can use List ADT for `find()`, `insert()`, and `delete()`
  - $O(l)$ running time where l is the number of elements in the particular chain

- Can also use Binary Search Trees
  - $O(\log l)$ time instead of $O(l)$
  - But the number of elements to search through should be small
  - Generally not worth the overhead of BSTs

Example

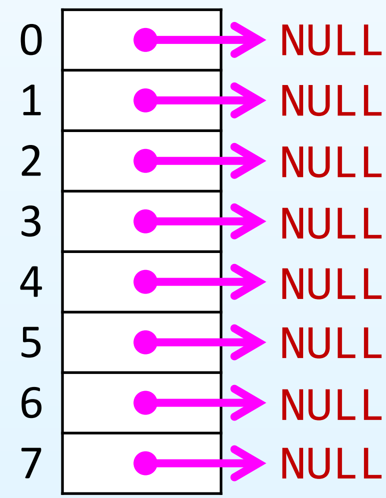
Let’s store words into a table and allow a fast look-up

Design: We will store strings and the hash value of a string will be the last 3 bits of the first character in the word

– E.g., the hash of “optimal” is based on ‘o’

a	01100001	b	01100010	c	01100011	d	01100100	e	01100101
f	01100110	g	01100111	h	01101000	i	01101001	j	01101010
k	01101011	l	01101100	m	01101101	n	01101110	o	01101111
p	01110000	q	01110001	r	01110010	s	01110011	t	01110100
u	01110101	v	01110110	w	01110111	x	01111000	y	01111001
z	01111010								

– We thus create an array of 8 pointers



Example: Hash Function

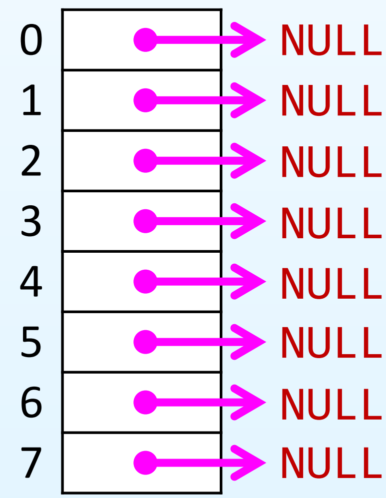
Let's store words into a table and allow a fast look-up

Design: Our hash function is

```c
unsigned int hash(char *str) {
  // the empty string "" is hashed to 0
  if (strlen(str) == 0)
    return 0;
  return str[0] & 7;
}
```

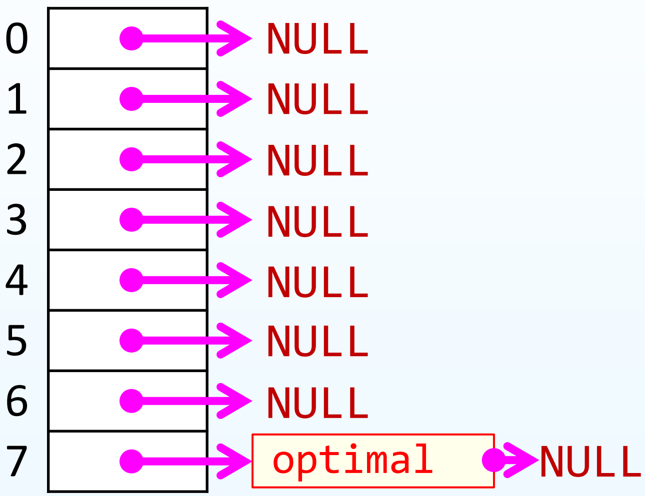
Bitwise operator (&)

```
'a':   01100001
  7:   00000111
       00000001
```



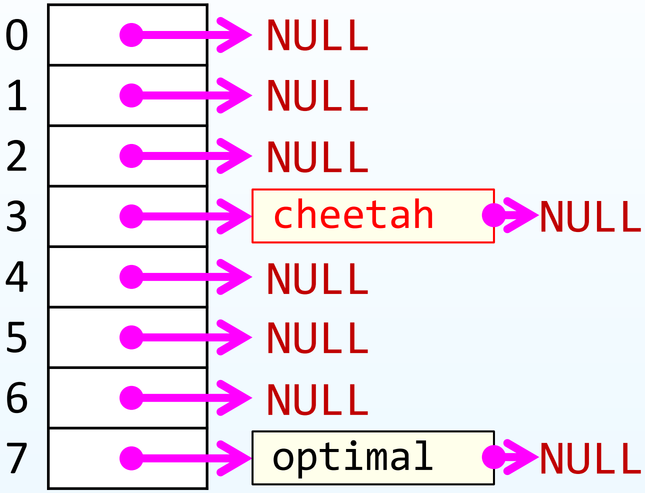
Example: `insert()`

- To do `insert("optimal")`, the first character `'o'` is entered into the bin `01101111 = 7`



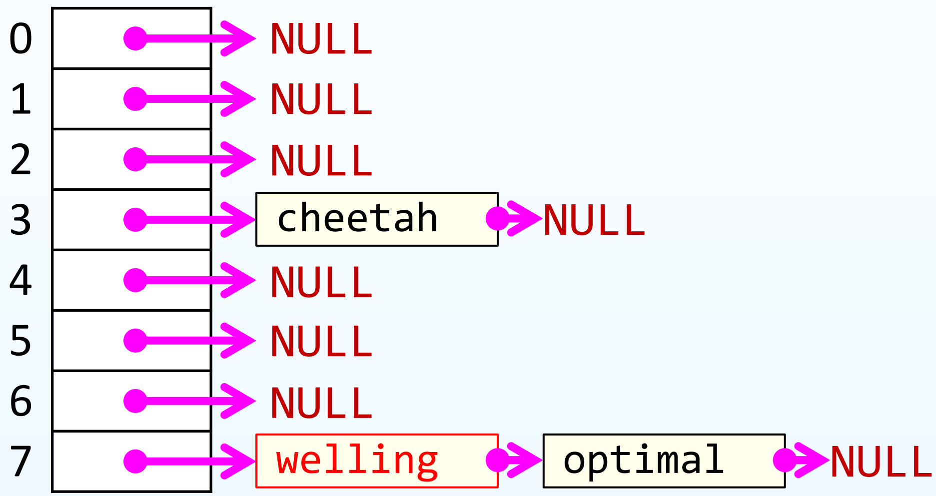
Example: insert()

- Similarly, `insert("cheetah")` will enter the word into the bin `01100011 = 3`



Example: `insert()`

- However, `insert("welling")` will enter the word into the same bin `01110111 = 7`

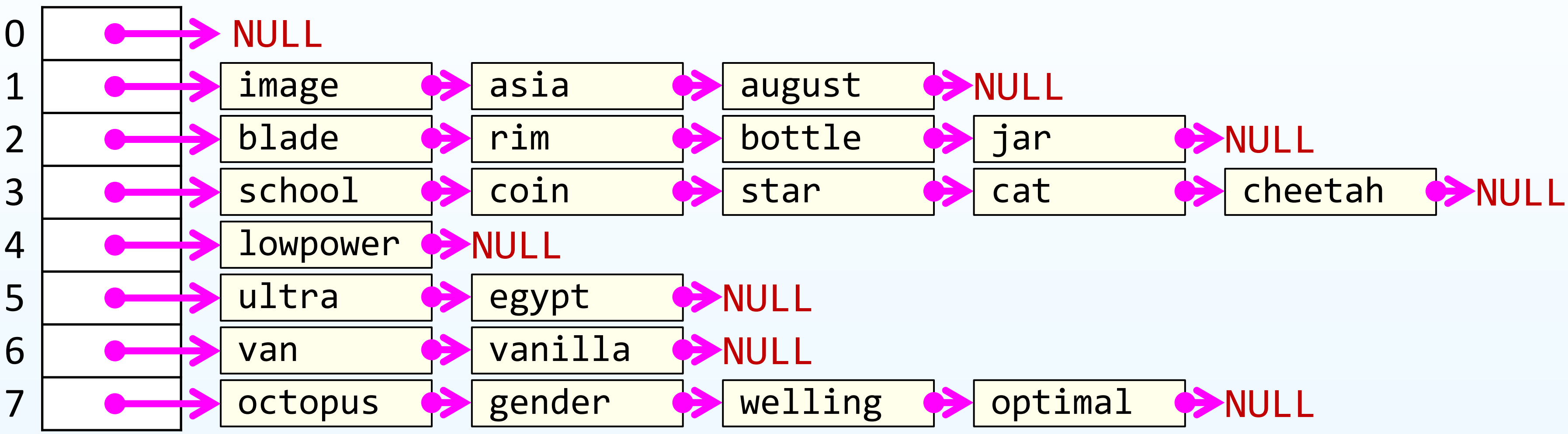


We will heuristically insert it at **front** of the linked list, why?

"The word accessed recently may be accessed again in the near future"

Example: `insert()`

- After 21 insertions, the linked lists are becoming rather long
  - We look for $\Theta(1)$ time, but must pay $O(l)$ for a linked list with l objects



Load Factor

- To describe the length of the linked lists, we define the load factor of the hash table:

$$\lambda = \frac{n}{M}$$

  This means the average number of objects per bin

- Right now, the previous insertions:
  - Each bin has $\lambda = \frac{21}{8} = 2.625$ objects on average

- If the load factor becomes too large, access time will increase to $\Theta(\lambda)$

Example: Doubling Size

Design: We double the table size by changing the hash function to use the last 4 bits of the first character

0	→ NULL
1	→ asia → august → NULL
2	→ blade → rim → bottle → NULL
3	→ school → coin → star → cat → cheetah → NULL
4	→ NULL
5	→ ultra → egypt → NULL
6	→ van → vanilla → NULL
7	→ gender → welling → NULL
8	→ NULL
9	→ image → NULL
10	→ jar → NULL
11	→ NULL
12	→ lowpower → NULL
13	→ NULL
14	→ NULL
15	→ octopus → optimal → NULL

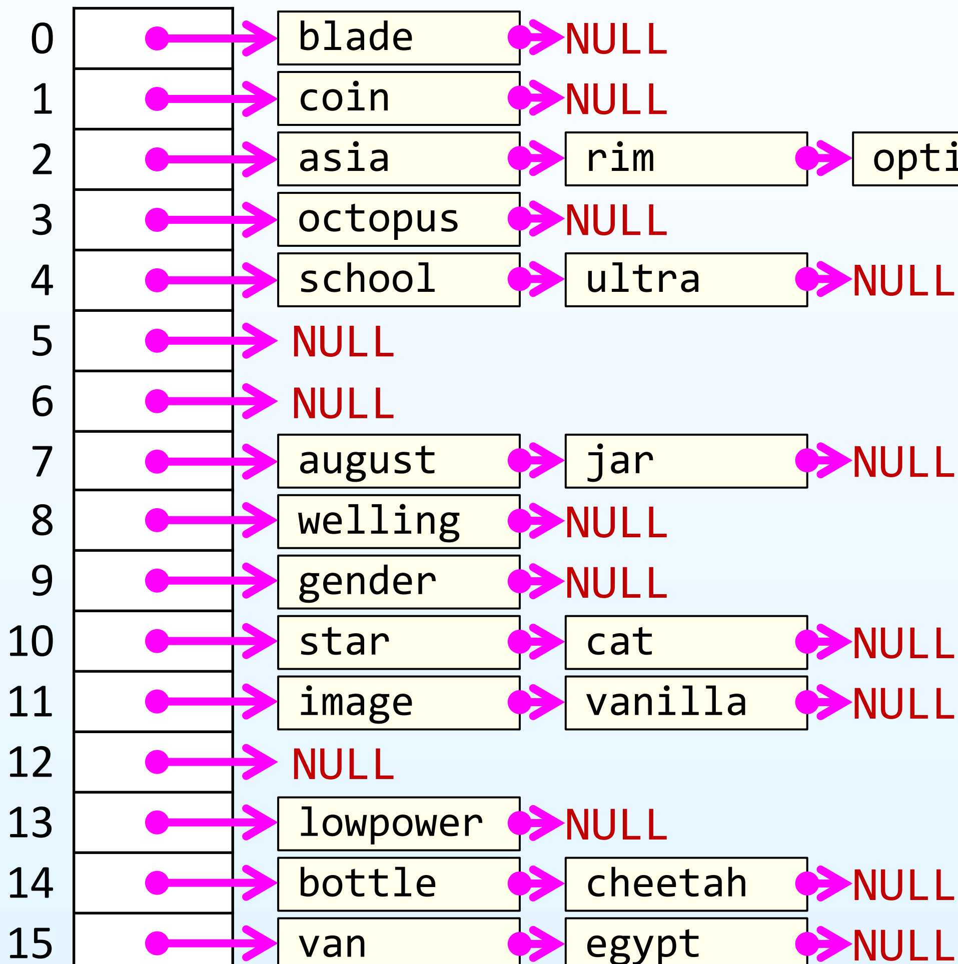
```
unsigned int hash(char *str) {
  // the empty string "" is hashed to 0
  if (strlen(str) == 0)
    return 0;
  return str[0] & 15;
}
```

Unfortunately, the distribution hasn't improved much although $\lambda = 1.3125$ decreases

➡ Significant clustering in bins 2 and 3 due to the choice of first character

Example: Choose a Good Hash Function

Design: Let's go back to the hash function defined previously




```c
unsigned int hash(char *str) {
  unsigned int prime = 12347;
  unsigned int hash_value = 0;
  int          i = 0;
  for (i=0; i<strlen(str); i++)
    hash_value = prime*hash_value + str[i];
  return hash_value % 16;
  // or return hash_value & 15;
}
```

Problems with Linked Lists

- One significant issue with chained hash tables using linked list
  - It requires extra memory
  - It uses dynamic memory allocation

Outline

- Hash ADT
- Hash Functions
- Collisions
  - Separate Chaining
  - Open Addressing

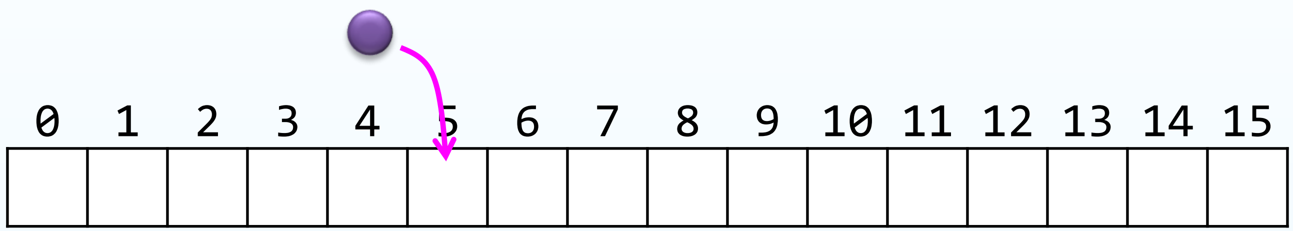
2. Resolution by Open Addressing

- No links, all object keys are in the table
  - Reduced overhead saves space
- We will define an implicit rule which tells us where to look next
  - When searching for x, check locations $h_1(x)$, $h_2(x)$, $h_3(x)$, … until either
    - x is found, or
    - we find an empty location (i.e., x is not present)
  - Various flavors of open addressing differ in which probe sequence they use

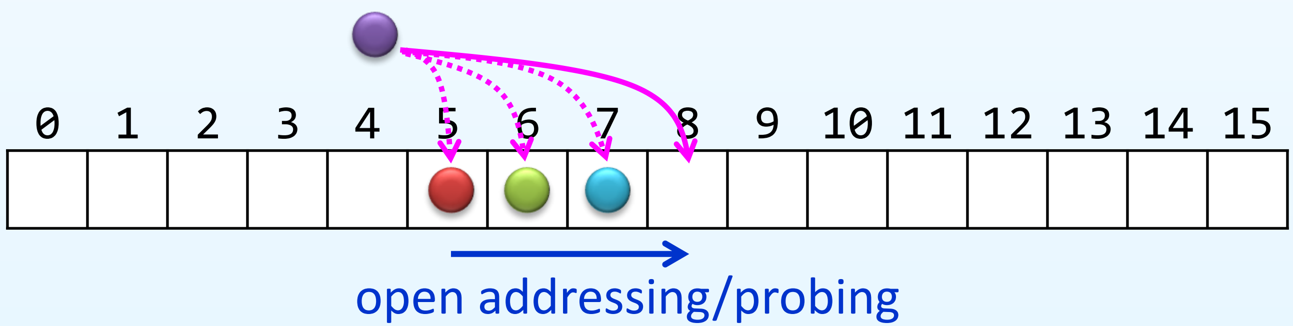
Open Addressing

Suppose an object hashes to bin 5

- If bin 5 is empty, we can insert the object into that entry



- Otherwise, we use an implicit rule to look for other unoccupied entry
  - **Rule:** continue searching until the first empty bin is found



  - However, we can only store as many objects as there are entries in the array, i.e., the load factor $\lambda < 1$

Open Addressing

- The implicit rule is then defined as:

$$h_i(x) = \big(hash(x) + F(i)\big) \bmod table\_size$$

for $i = 0, 1, 2, 3, \ldots$ and $F(0) = 0$

- $F(i)$ is the collision resolution function; some possibilities:
  - Linear probing: $F(i) = i$
  - Quadratic probing: $F(i) = i^2$
  - Double hashing: $F(i) = i \cdot hash_2(x)$

2.1 Linear Probing

- The easiest method to probe the bins of the hash table is to search forward linearly:

$$F(i) = i$$

- Assume we are inserting into bin k:
  - If bin k is empty, we immediately insert into that bin
  - Otherwise, check bin $k + 1$, $k + 2$, and so on, until an empty bin is found
    - If we reach the end of the array, we start at the front (bin 0)

Example: `insert()`

- Consider a hash table with $M = 16$ bins
- Given a 3-digit hexadecimal number:
  - The least-significant digit is the primary hash function
- Insert these numbers:

```
1    1    1    1
19A, 207, 3AD, 488, 5BA, 680, 74C, 826, 946, ACD, B32, C8B, DBE, E9C
```

0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
							207	488		19A			3AD		

Example: `insert()`

- Consider a hash table with $M = 16$ bins
- Given a 3-digit hexadecimal number:
  - The least-significant digit is the primary hash function
- Insert these numbers:

1 1 1 1 2

19A, 207, 3AD, 488, 5BA, 680, 74C, 826, 946, ACD, B32, C8B, DBE, E9C

$h_1(5\text{BA}) = (\text{A} + 1)\%16 = \text{B}$

collision occurs

0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
							207	488		19A	5BA		3AD		

Example: `insert()`

- Consider a hash table with $M = 16$ bins
- Given a 3-digit hexadecimal number:
  - The least-significant digit is the primary hash function
- Insert these numbers:

1	1	1	1	2	1	1	1							
19A,	207,	3AD,	488,	5BA,	680,	74C,	826,	946,	ACD,	B32,	C8B,	DBE,	E9C	

0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
680						826	207	488		19A	5BA	74C	3AD		

Example: `insert()`

- Consider a hash table with $M = 16$ bins
- Given a 3-digit hexadecimal number:
  - The least-significant digit is the primary hash function
- Insert these numbers:

1	1	1	1	2	1	1	1	4						
19A,	207,	3AD,	488,	5BA,	680,	74C,	826,	946,	ACD,	B32,	C8B,	DBE,	E9C	

collision occurs

+1 +2 +3

0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
680						826	207	488	946	19A	5BA	74C	3AD		

Example: `insert()`

- Consider a hash table with $M = 16$ bins
- Given a 3-digit hexadecimal number:
  - The least-significant digit is the primary hash function
- Insert these numbers:

1 1 1 1 2 1 1 1 4 2

19A, 207, 3AD, 488, 5BA, 680, 74C, 826, 946, ACD, B32, C8B, DBE, E9C

+1

0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
680						826	207	488	946	19A	5BA	74C	3AD	ACD	

Example: `insert()`

- Consider a hash table with $M = 16$ bins
- Given a 3-digit hexadecimal number:
  - The least-significant digit is the primary hash function
- Insert these numbers:

1	1	1	1	2	1	1	1	4	2	1	5	4	8
19A,	207,	3AD,	488,	5BA,	680,	74C,	826,	946,	ACD,	B32,	C8B,	DBE,	E9C

0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
680	DBE	B32	E9C			826	207	488	946	19A	5BA	74C	3AD	ACD	5BA

- We have completed all insertions:
  - The load factor is $\lambda = \frac{14}{16} = 0.875$
  - The average number of probes is $\frac{33}{14} \approx 2.36$

Example: Resizing the Array

- To double the capacity of the array, each value must be rehashed
  - We will use the least-significant five bits for hashing

```
unsigned int hash(int n) {
  return n & 31;
}
```

- Re-insert all previous numbers:

1	1	1	1	2	1	1	1	4	2	1	1	1	1
19A,	207,	3AD,	488,	5BA,	680,	74C,	826,	946,	ACD,	B32,	C8B,	DBE,	E9C

0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F	10	11	12	13	14	15	16	17	18	19	1A	1B	1C	1D	1E	1F
680						826	207	488	946		C8B	74C	3AD	ACD				B32								19A	5BA	E9C		DBE	

- After completing all insertions:
  - The load factor is $\lambda = \frac{14}{32} = 0.4375$
  - The average number of probes is $\frac{19}{14} \approx 1.36$

Example: `find()`

- Searching for membership is similar to insertions:
  - Start at the appropriate bin, and searching forward until
    1. The item is found,
    2. An empty bin is found ➡ not found, or
    3. We have traversed the entire array ➡ not found

- Searching for `5C8`

5C8 is not found

+1 +2

0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F	10	11	12	13	14	15	16	17	18	19	1A	1B	1C	1D	1E	1F
680						826	207	488	946	✗	C8B	74C	3AD	ACD				B32								19A	5BA	E9C		DBE	

Primary Clustering Phenomenon

- With more insertions, the contiguous regions (or clusters) get larger
  - ➡ This results in longer search times

0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F	10	11	12	13	14	15	16	17	18	19	1A	1B	1C	1D	1E	1F
680						826	207	488	946		C8B	74C	3AD	ACD				B32								19A	5BA	E9C		DBE	

- Above, we currently have three clusters
- There is a $\frac{5}{32} \approx 16\%$ chance that an insertion will fill bin A
  - Suppose 747 is inserted, two clusters will coalesce into one larger of length 9

2.2 Quadratic Probing

- Quadratic probing suggests moving forward by different amount:

$$F(i) = i^2$$

- Assume we are inserting into bin k:
  - If bin k is empty, we immediately insert into that bin
  - Otherwise, check bin $k + 1^2$, $k + 2^2$, and so on, until an empty bin is found
    - If we reach the end of the array, we start at the front (bin 0)

Will i^2 step all bins of the table?

```c
#include <stdio.h>
unsigned int hash(unsigned int n, unsigned int M) {
  return n % M;
}
void try_quadratic(unsigned int n, unsigned int M) {
  unsigned int init = hash(n, M);
  unsigned int i;
  for (i=0; i<M; i++)
    printf("%3d", (init + i*i) % M);
  printf("\n");
}
int main(void) {
  try_quadratic(0, 10);
  try_quadratic(0, 16);
  return 0;
}
```

```
  0  1  4  9  6  5  6  9  4  1
  0  1  4  9  0  9  4  1  0  1  4  9  0  9  4  1
```

Making M Prime

```
int main(void) {
  try_quadratic(0, 10);
  try_quadratic(0, 11);
  try_quadratic(0, 16);
  try_quadratic(0, 17);
  return 0;
}
```

```
 0  1  4  9  6  5  6  9  4  1
 0  1  4  9  5  3  3  5  9  4  1
 0  1  4  9  0  9  4  1  0  1  4  9  0  9  4  1
 0  1  4  9 16  8  2 15 13 13 15  2  8 16  9  4  1
```

Guarantee to iterate through $\left\lceil \frac{M}{2} \right\rceil$ entries

Problems:

– Modulus (%) is slow; cannot use bitwise operators (&, <<, or >>)

– Doubling the number of bins is difficult

Using the Powers of two $M = 2^m$

If $M = 2^m$ is powers of two, this guarantees that all M entries are visited:

$$F(i) = c_1 i + c_2 i^2$$

Since $i + i^2$ is always even, we can select $c_1 = c_2 = \frac{1}{2}$

Using $M = 2^m$

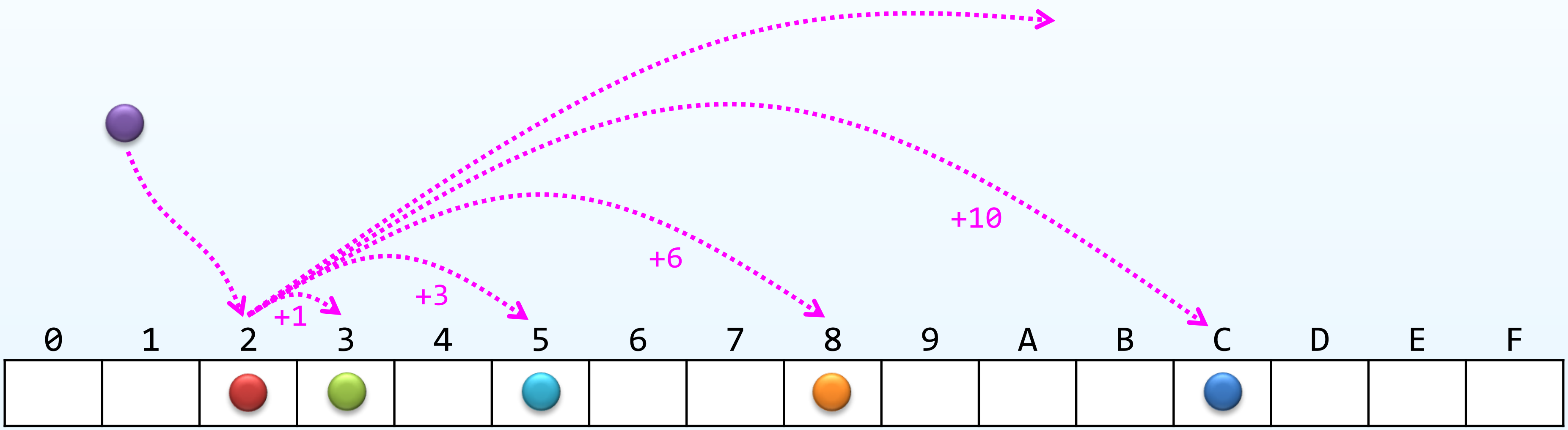
```c
#include <stdio.h>
unsigned int hash(unsigned int n, unsigned int M) {
  return n % M;
}
void try_quadratic(unsigned int n, unsigned int M) {
  unsigned int init = hash(n, M);
  unsigned int i;
  for (i=0; i<M; i++)
    printf("%3d", (init + (i + i*i)/2) % M);
  printf("\n");
}
int main(void) {
  try_quadratic(0, 10);  // not powers of two
  try_quadratic(0, 16);  // powers of two (2^4)
  return 0;
}
```

```
  0  1  3  6  0  5  1  8  6  5
  0  1  3  6 10 15  5 12  4 13  7  2 14 11  9  8
```

Example: `insert()` and `delete()`

- Consider a hash table with $M = 16$ bins
- Suppose try to insert at 2

$$h_i(x) = (2 + (i + i^2)/2)\ \%\ 16 \quad \text{for} \quad i = 0, 1, 2, \ldots$$



Secondary Clustering Phenomenon

- Weakness with quadratic problem:
  - It reverts to linear probing if many of the hash function is not random
  - Objects placed in the same bin will follow the same sequence

2.3 Double Hashing

- Double hashing suggests moving forward by a jump function:

$$F(i) = i \cdot hash_2(x)$$

- Assume we are inserting into bin k:
  - If bin k is empty, we immediately insert into that bin
  - Otherwise, check bin $k + hash_2(x)$, $k + 2 \cdot hash_2(x)$, and so on, until an empty bin is found
    - If we reach the end of the array, we start at the front (bin 0)

Example of Double Hashing

```c
#include <stdio.h>
#define PRIME 7
unsigned int hash(unsigned int n, unsigned int M) {
  return n % M;
}
void try_doublehash(unsigned int n, unsigned int M) {
  unsigned int init = hash(n, M);
  unsigned int jump = PRIME - hash(n, PRIME);
  unsigned int i;
  for (i=0; i<M; i++)
    printf("%3d", (init + i*jump) % M);
  printf("\n");
}
int main(void) {
  try_doublehash(0, 10);
  try_doublehash(0, 16);
  return 0;
}
```

```
  0  7  4  1  8  5  2  9  6  3
  0  7 14  5 12  3 10  1  8 15  6 13  4 11  2  9
```

Be Careful !!!

Must be careful about the second hash function $hash_2(x)$:

$$h_i(x) = \big(hash_1(x) + i \cdot hash_2(x)\big) \bmod table\_size$$

for $i = 0, 1, 2, 3, \ldots$

- Must never evaluate to zero
  - ➡ Otherwise, it will always hash the same value
- Must make sure that all bins can be probed
  - E.g., if $hash_1(x) = 3$, $hash_2(x) = 9$, and $M = 12$,
    then we can probe only the bins $(3 + 9)\%12 = 0$,
    $(3 + 18)\%12 = 9$, $(3 + 27)\%12 = 6$, $(3 + 36)\%12 = 3$,
    $(3 + 45)\%12 = 0$, and so on

Any Question?