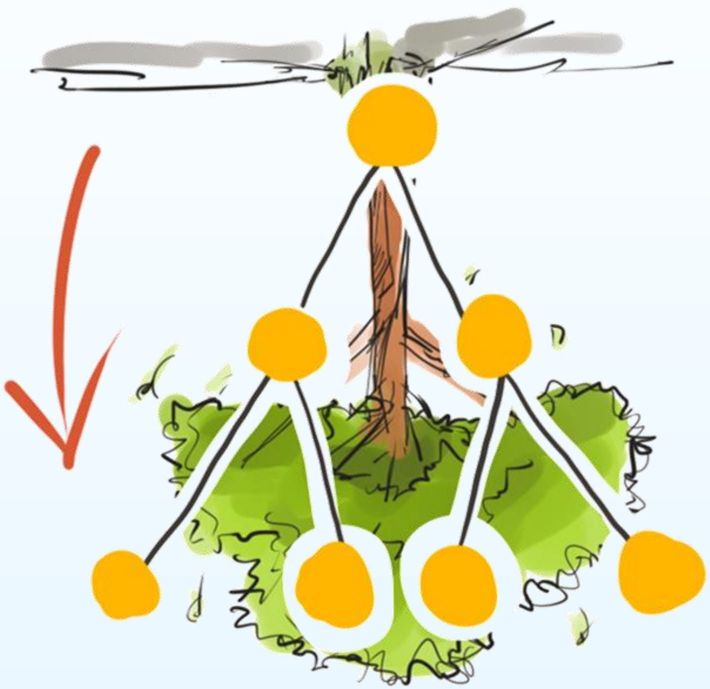

Lecture 7:
Tree ADT

01204212 Abstract Data Types and Problem Solving

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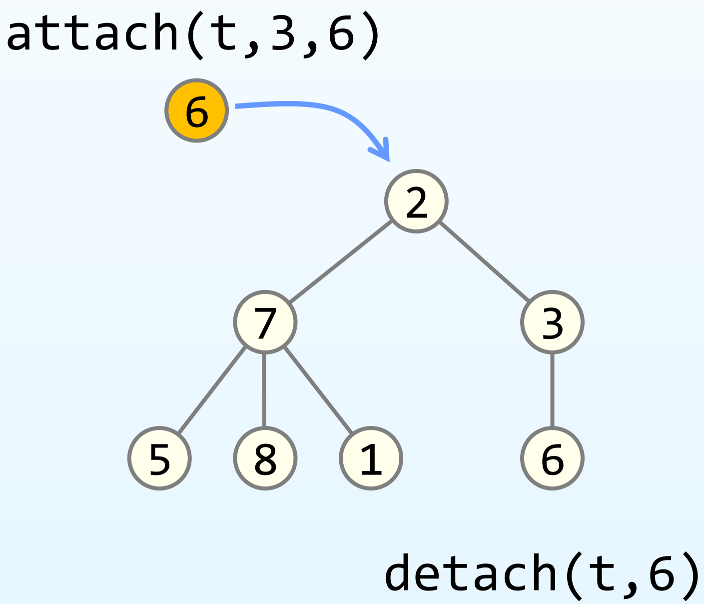
Outline

- Terminologies
- Implementation
- Tree Traversals
- Forests

What is a Tree ADT?

- Data:
  - A set of linked nodes (elements) that form a hierarchical tree structure with a root and its subtrees
  - This is a recursive definition
- Defined operations:
  - `attach(`*tree*`,` *parent*`,` *child*`)`
  - `detach(`*tree*`,` *node*`)`
  - `search(`*tree*`,` *node*`)`
  - `degree(`*tree*`,` *node*`)`
  - `is_root(`*tree*`,` *node*`)`
  - …



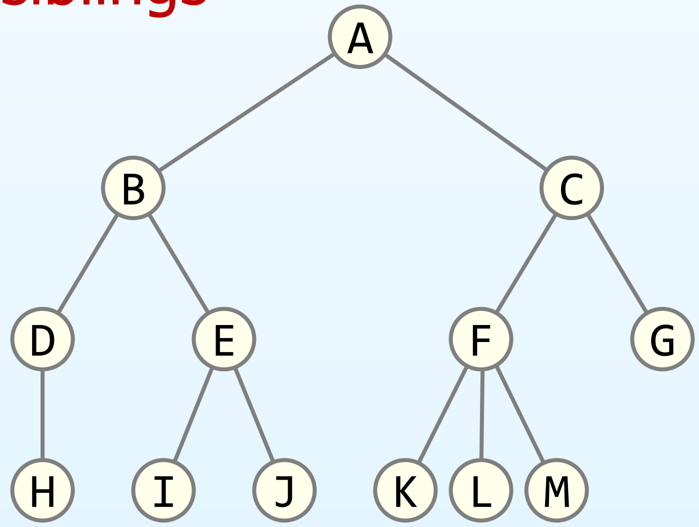
Terminology: Parent, Child, and Sibling

- A tree consists of nodes with a parent-child relation
- All nodes will have zero or more child(ren)

  e.g., B has two children: D and E
- All nodes, except the first node, have only one parent

  e.g., A is the parent of B
- Nodes with the same parent are siblings

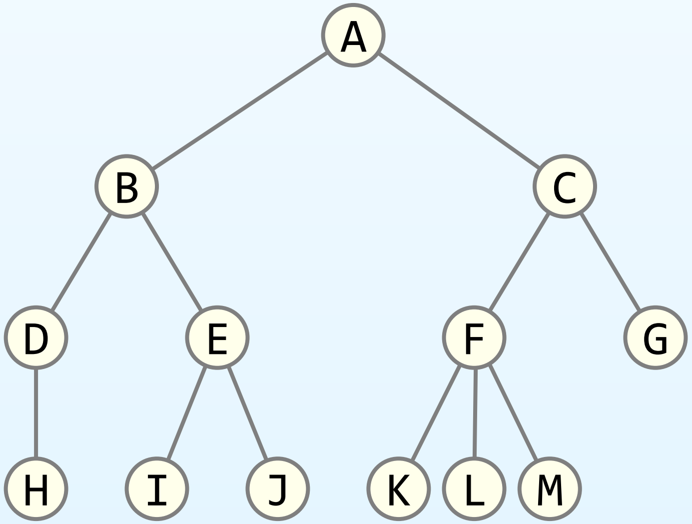
  e.g., B and C are siblings



Terminology: Root, Internal, and Leaf

- Root – node without parent
  e.g., A is the root of the tree
- Internal node – node with at least one child
  e.g., A, B, C, D, E, and F are internal nodes
- External node (a.k.a. leaf) – node without children
  e.g., G, H, I, J, K, L, and M are leaves

rooted tree

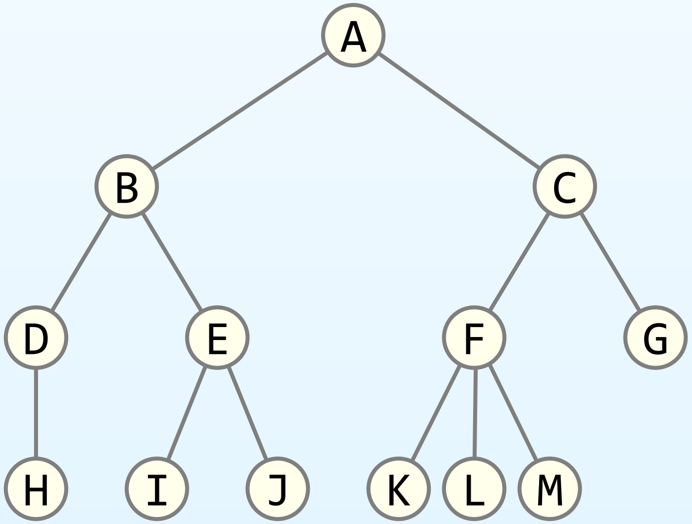


Terminology: Degree

- The degree of a node is defined as the number of its children

  e.g., `A` has degree 2

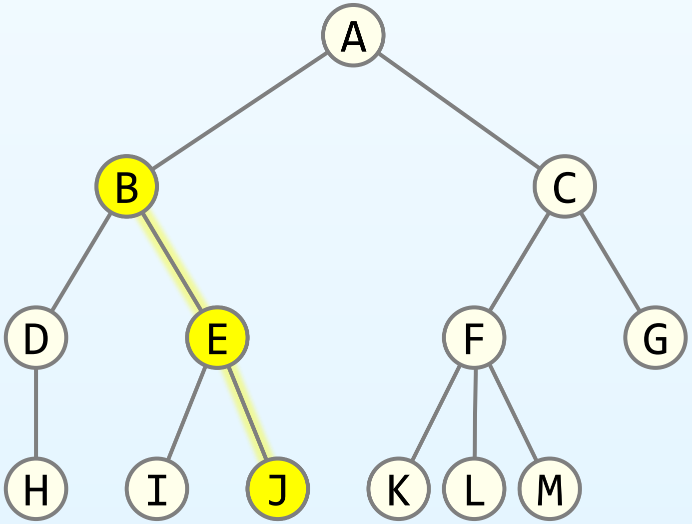
  All leaf nodes have zero-degree



Terminology: Path and Path Length

- A path is a sequence of nodes $\langle a_i, a_{i+1}, a_{i+2}, \ldots, a_j \rangle$ where a_{i+1} is a child of a_i
- The length of a path is defined as the number of links along that path

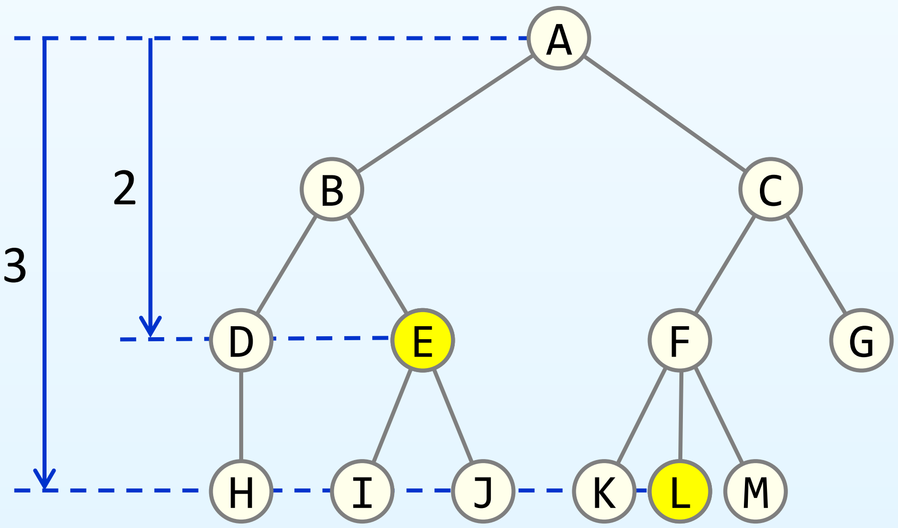
e.g., the path ⟨B, E, J⟩ has length 2



Terminology: Depth

- For each node in a tree, there exists a unique path from the root node to that node
- The length of this path is the depth of the node

e.g., E has depth 2
L has depth 3



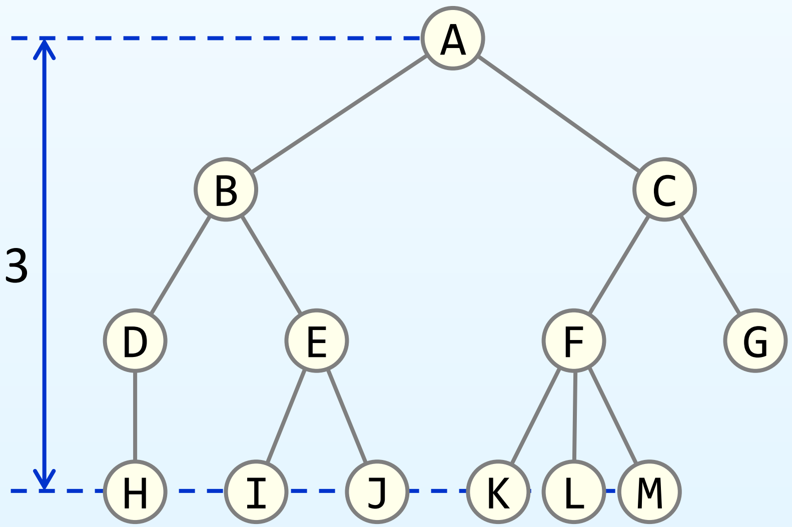
Terminology: Height

- The height of a tree is defined as the maximum depth of any node within the tree

  e.g., the height of the example tree is 3

- The height of a tree with one node is 0

- For convenience, we define the height of the empty tree to be -1

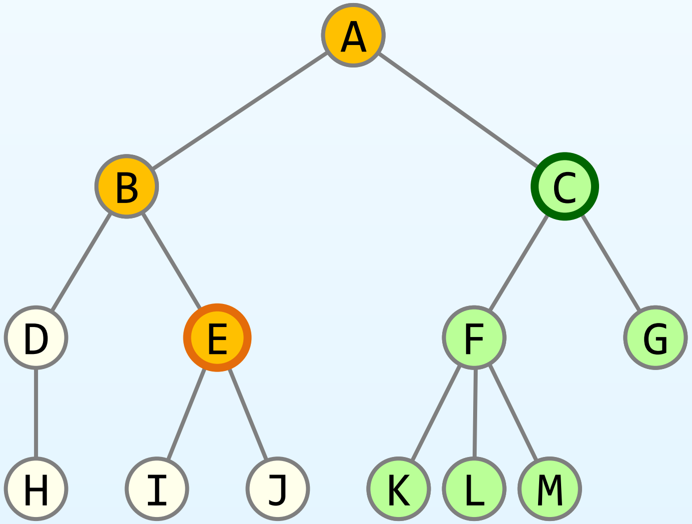


Terminology: Ancestor and Descendant

- Ancestor – the connected higher-level nodes
- Descendant – the connected lower-level nodes
- However, a node is both an ancestor and a descendant of itself

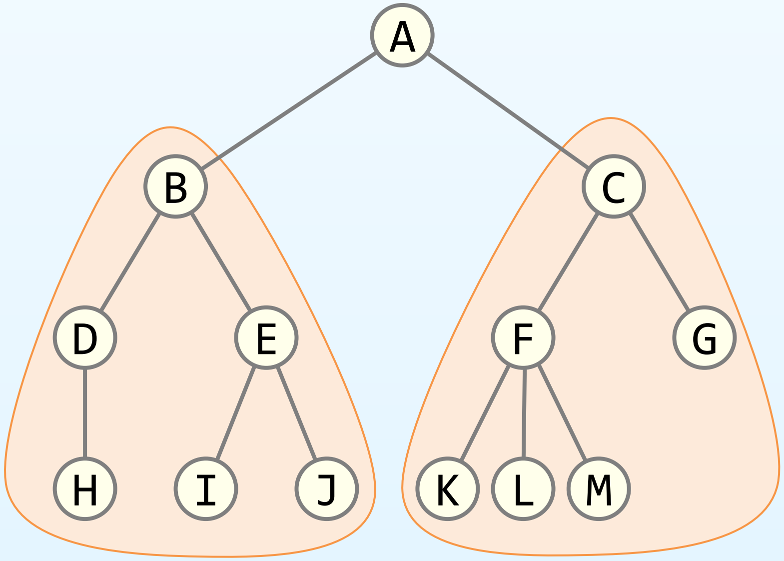
e.g., the ancestors of node **E** are A, B, and E

the descendants of node **C** are C, F, G, K, L and M



Terminology: Subtree

- Subtree is a part of tree consisting of a node and its descendants
- Another approach is to define a tree recursively:
  - (Base case) A zero-degree node is a tree
  - (Recursion) A node with degree n is a tree if it has n children and all of its children are disjoint trees—subtrees with no intersecting nodes



Important Properties

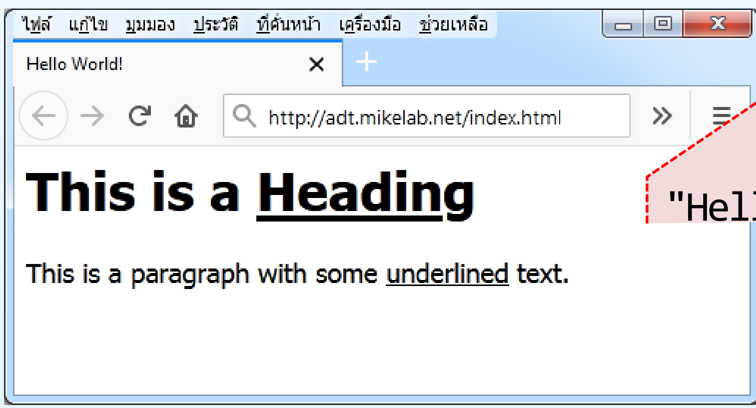
เส้นเชื่อม

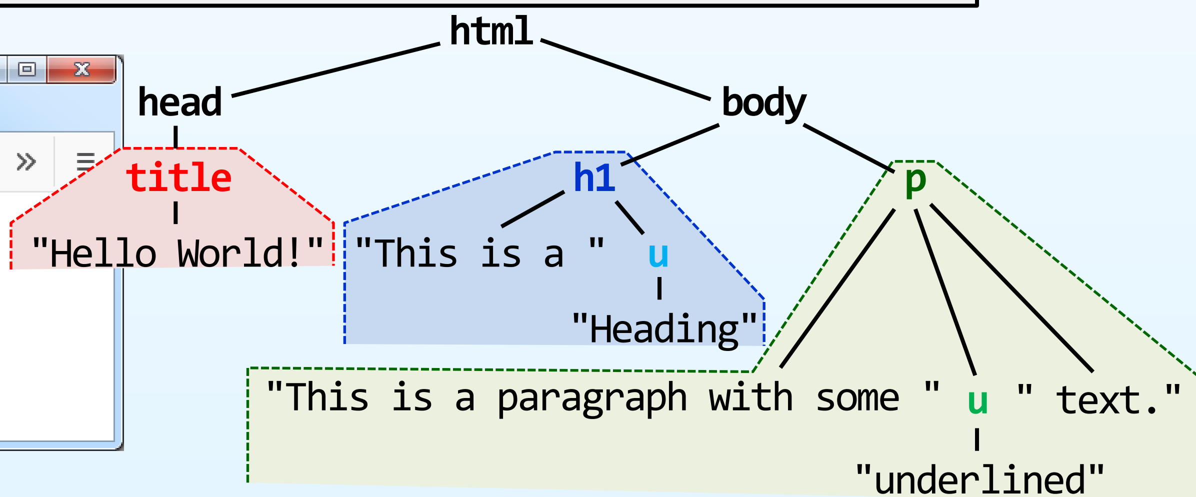
- A tree with n nodes always has $n - 1$ edges
- Any two nodes in a tree have at most one path between them

Application: HTML Structure

- The nested tags of HTML can be defined as a tree:

```
<html>
  <head>
    <title>Hello World!</title>
  </head>
  <body>
    <h1>This is a <u>Heading</u></h1>
    <p>This is a paragraph with some <u>underlined</u> text.</p>
  </body>
</html>
```

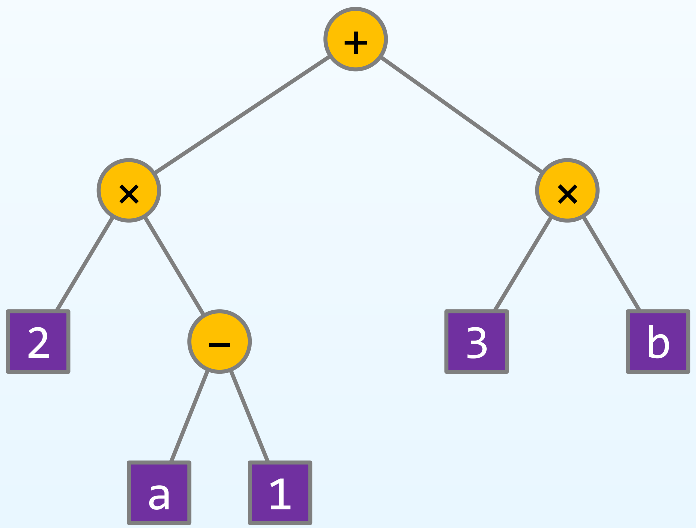




Application: Arithmetic Expression Tree

- A tree associated with an arithmetic expression
  - Internal nodes: operators
  - External nodes: operands

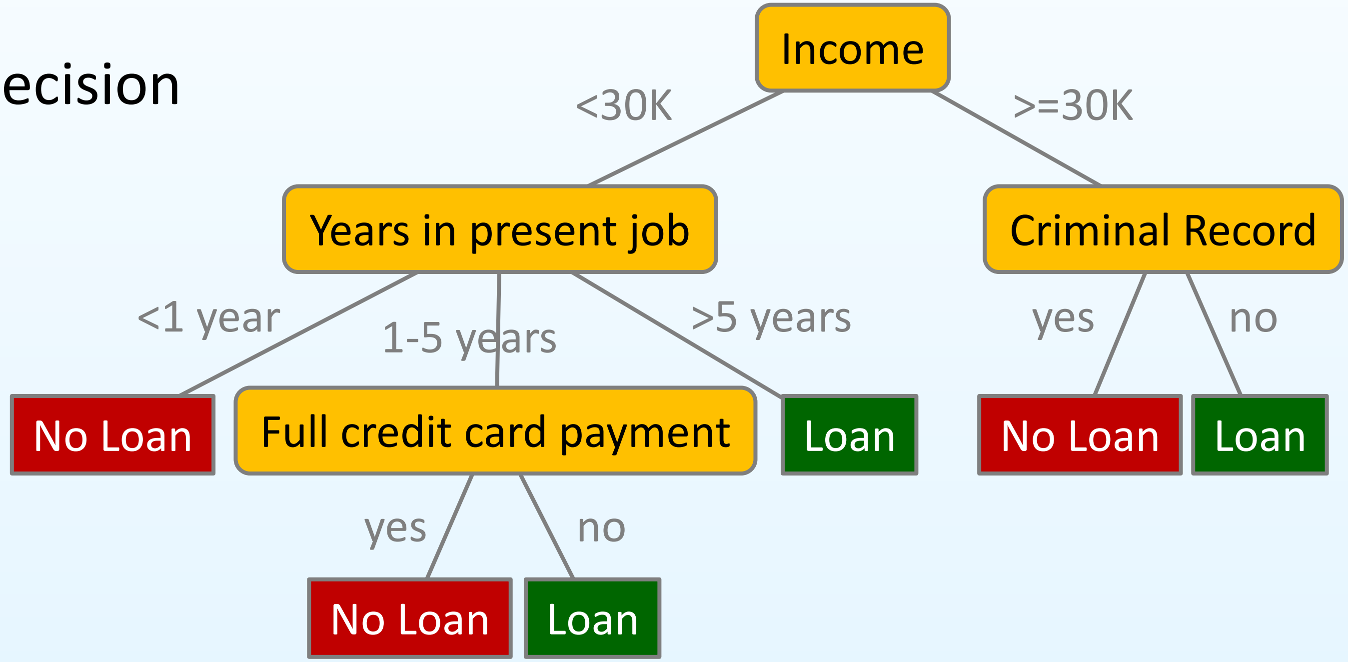
$(2\times(a-1)+(3\times b))$



Application: Decision Tree

- A tree associated with a decision process
  - Internal nodes: questions (with yes/no answer)
  - External nodes: decisions

Loan approval decision



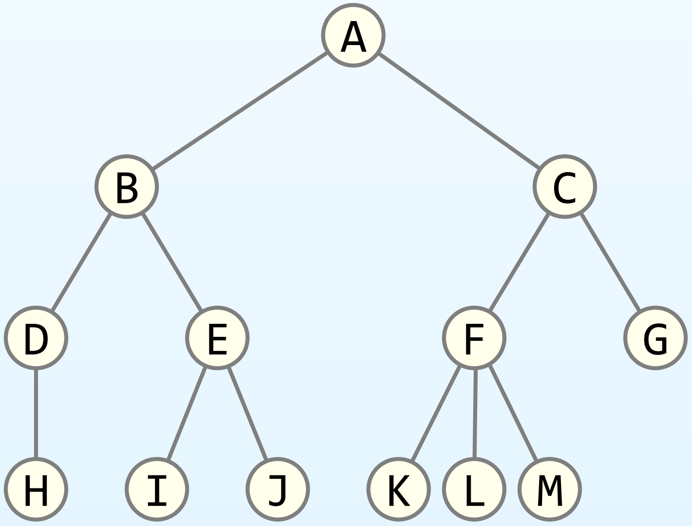
Outline

- Terminologies
- Implementation
- Tree Traversals
- Forests

Abstract Trees

- A hierarchical ordering of a finite number of objects may be stored in a tree data structure

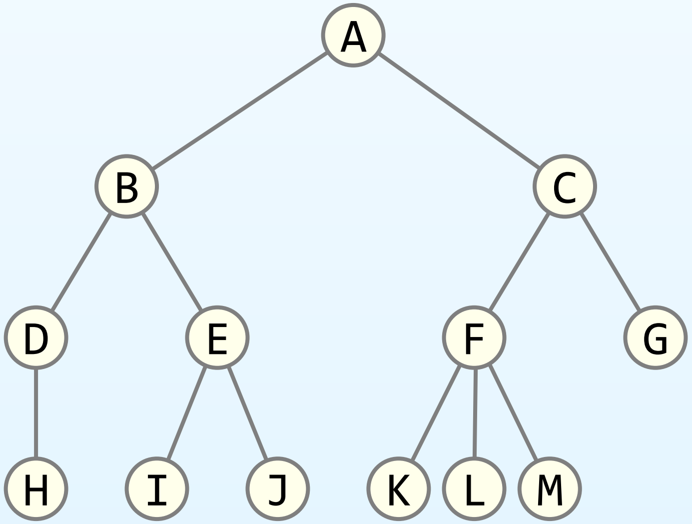
- Operations on a hierarchically stored container include:
  - Accessing the root
  - Given a node
    - Access its parent
    - Find the degree
    - Get a reference to a child
    - Attach a new subtree
    - Detach this subtree from its parent



Abstract Trees: Design

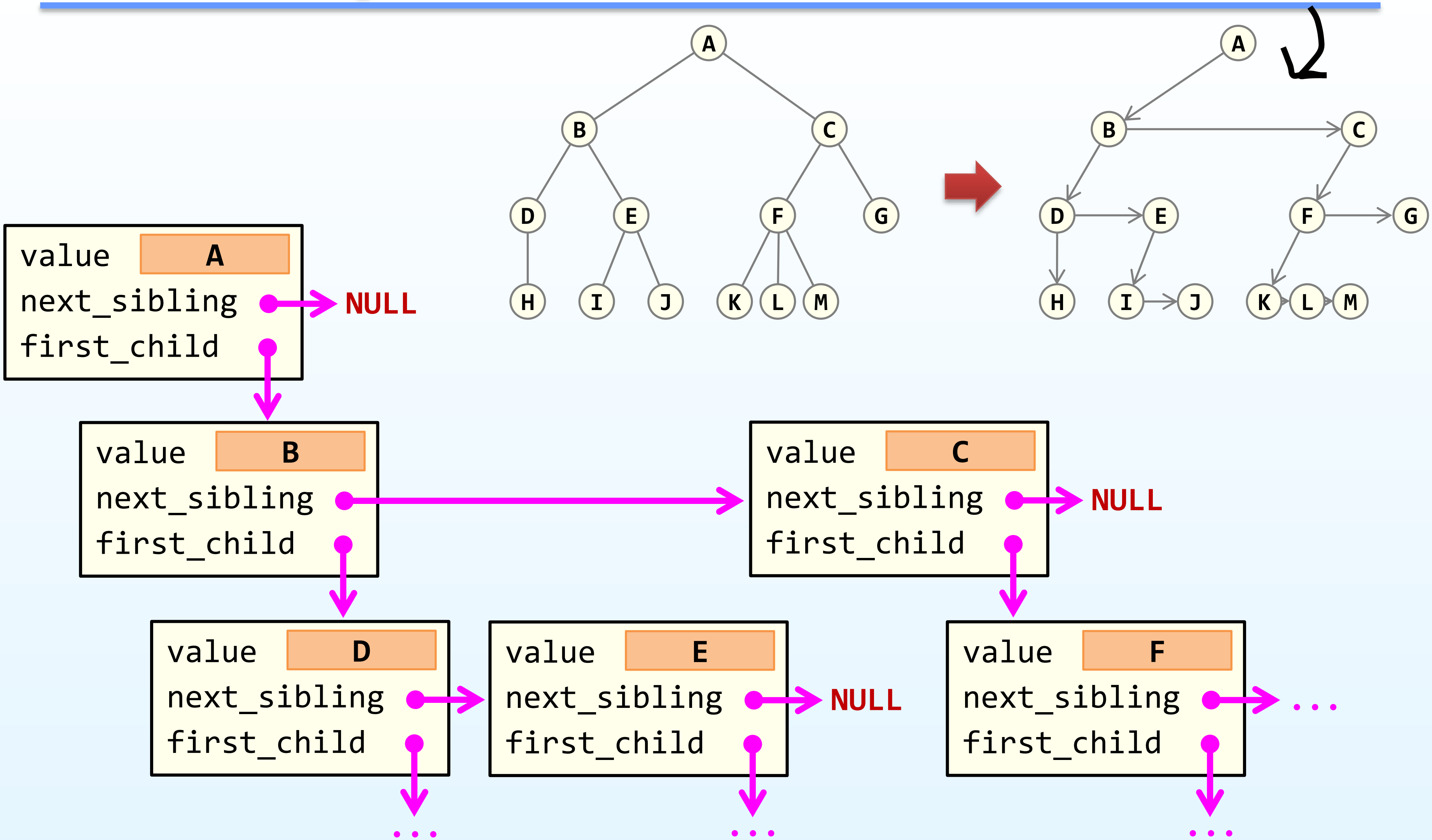
- A general tree does not strict the number of children
  - One possible pointer-based implementation

- What will be stored in an individual node?
  - A value
  - A pointer to next sibling
  - A pointer to the 1st child



Tree Implementation

implement code



Tree Implementation

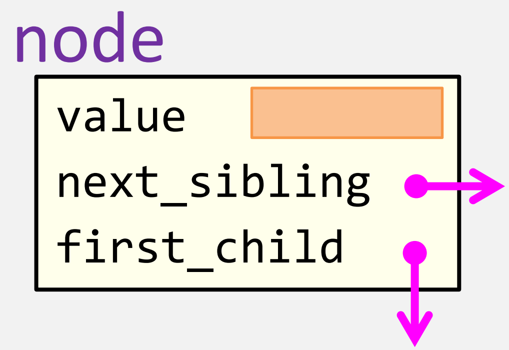
Assume that all data are English letters

```c
#include <stdio.h>
#include <stdlib.h>

typedef struct node {
  char value;
  struct node *next_sibling;
  struct node *first_child;
} node_t;

typedef node_t tree_t;

int main(void) {
  tree_t *t = NULL;
  return 0;
}
```



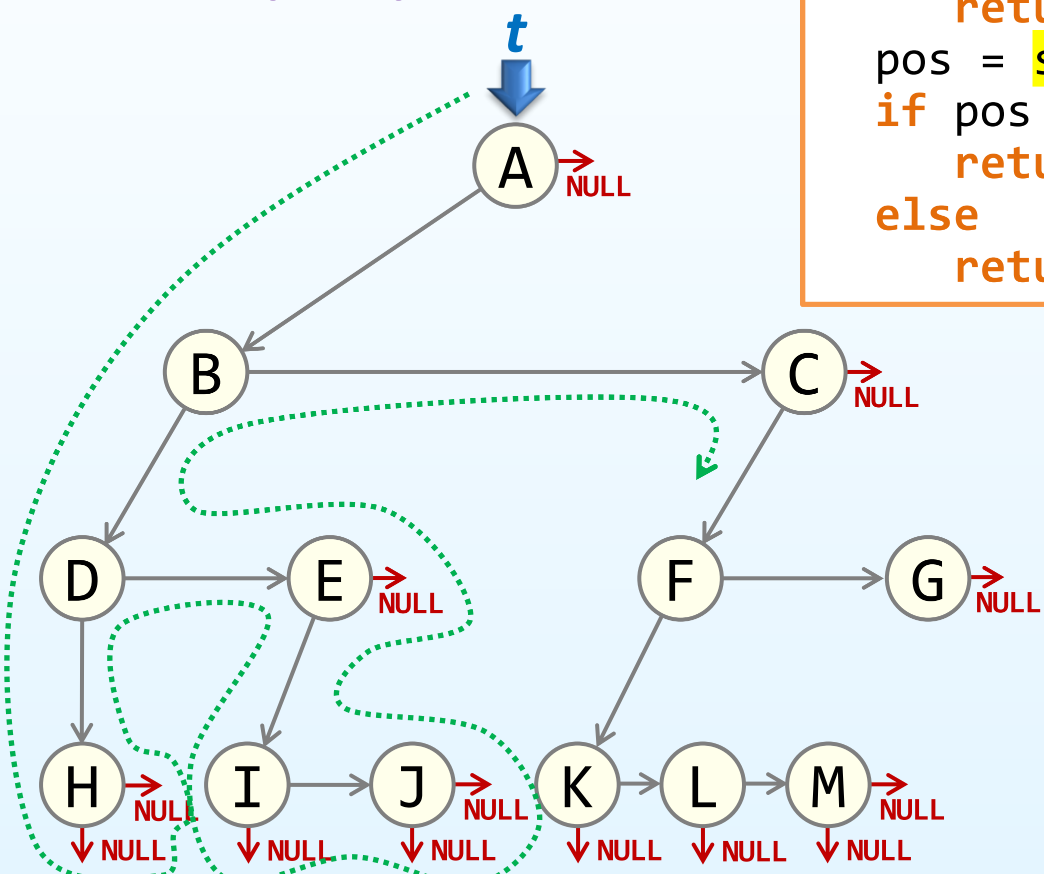
The `search()` Operation

Return position of node *v* in tree *t* if found, otherwise `NULL`

Algorithm: `search(t, v)`

```
if t->value == v  || t == NULL
    return t
pos = search(t->first_child, v)
if pos != NULL
    return pos
else
    return search(t->next_sibling, v)
```

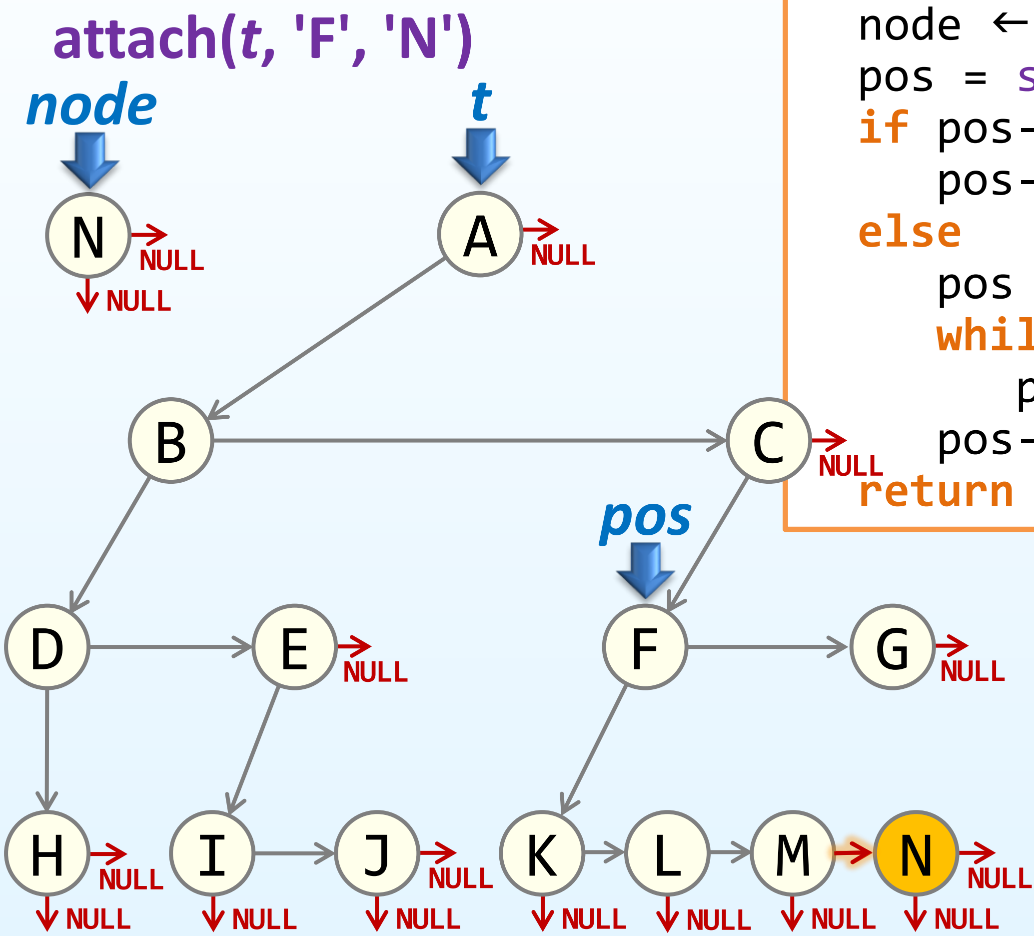
search(*t*, 'F')



What is the running time?

The `attach()` Operation

Insert a node/subtree *c* as a child of *p* in tree *t*




```
Algorithm: attach(t, p, c)
  node ← allocate a new node c
  pos = search(t, p)
  if pos->first_child == NULL
     pos->first_child = node
  else
     pos = pos->first_child
     while pos->next_sibling != NULL
        pos = pos->next_sibling
     pos->next_sibling = node
  return t
```

What is the running time?

Exercise 1: Other Operations

Implement the following functions for a rooted tree

- `detach(`*t*`,`*n*`)` – delete a node/subtree *n* from tree *t*
  - Return *t* after the deletion
- `degree(`*t*`,`*n*`)` – find the number of children of a node *n*
  - Return the number of children
- `is_root(`*t*`,`*n*`)` – check whether a node *n* is root
  - return 1 if that node is the root node, otherwise 0
- `is_leaf(`*t*`,`*n*`)` – check whether a node *n* is leaf
  - return 1 if that node is a leaf node, otherwise 0

Outline

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What is a traversal?

Once the objects are stored in a data structure,
how do we access them all?

- For an array or linked list, those objects can be accessed sequentially
  - ➡ $\Theta(n)$
- For a stack or queue, we can run multiple `pop()` or `dequeue()` operations
  - ➡ $\Theta(n)$
- However, how can we iterate through all the objects in a **tree** in an efficient manner
  - ➡ Require $\Theta(n)$ in running time

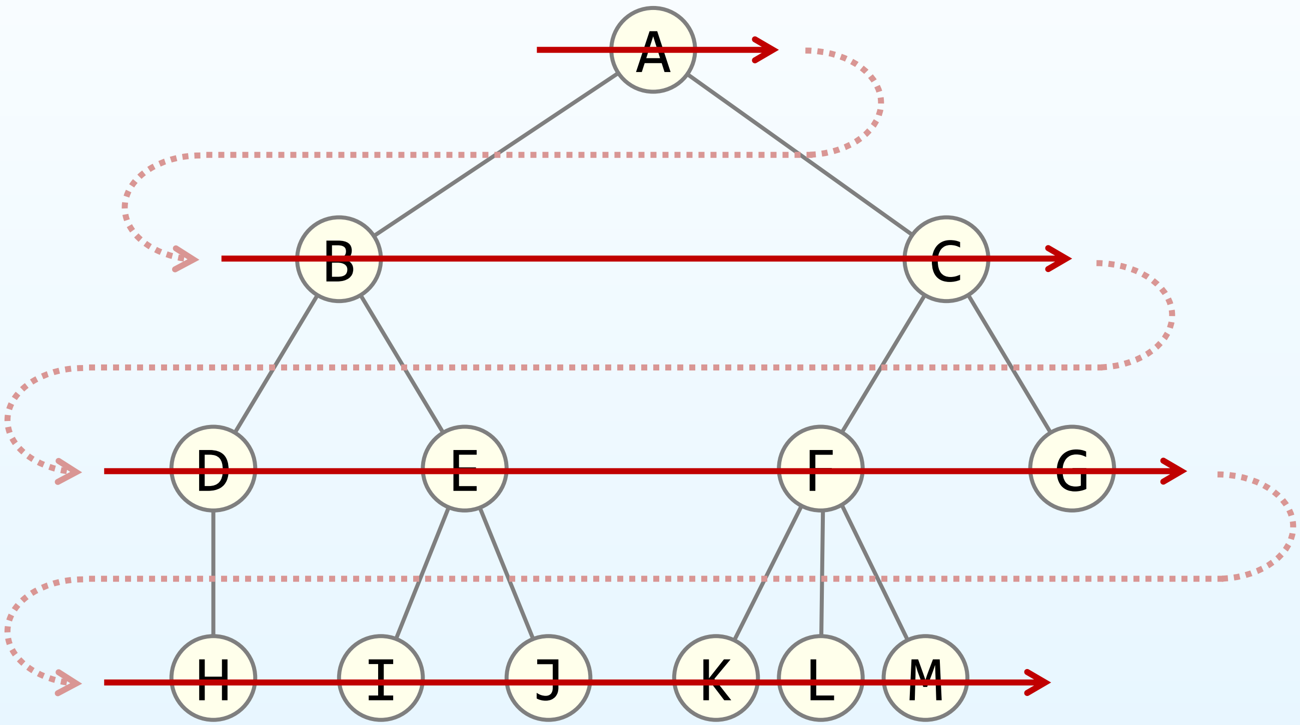
Types of Tree Traversal

Tree traversal algorithms can be classified broadly in two categories by the order in which the nodes are visited:

- Breadth-First Search (BFS)
- Depth-First Search (DFS)

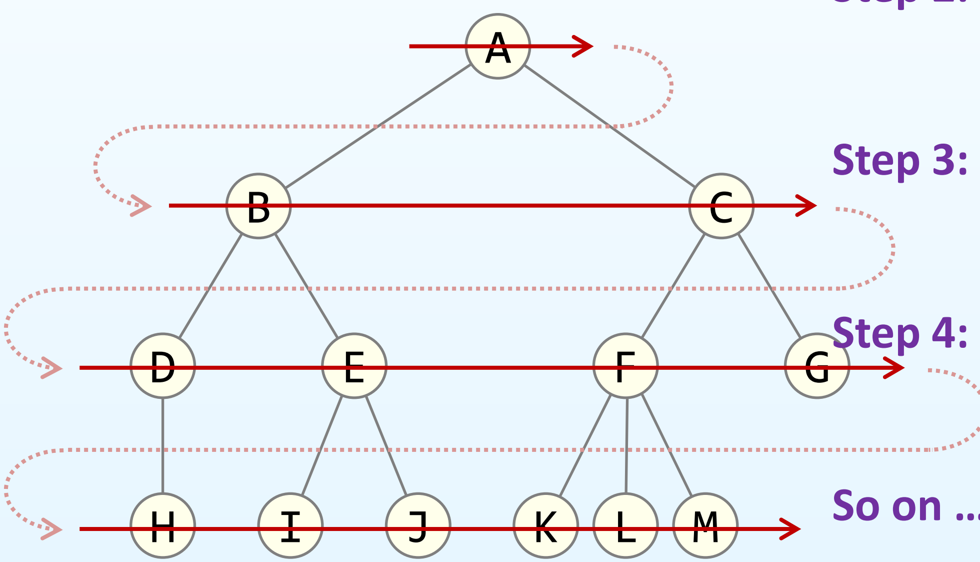
Breadth-First Search (BFS)

"It starts from the root node and visits all nodes of current depth before moving to the next depth of tree."



Breadth-First Search (BFS)

- BFS can be implemented using a queue



Algorithm for BFS

Step 1: enqueue({A})

A												

Step 2: dequeue() and then enqueue({B,C})

A	B	C										

Step 3: dequeue() and then enqueue({D,E})

A	B	C	D	E								

Step 4: dequeue() and then enqueue({F,G})

A	B	C	D	E	F	G						

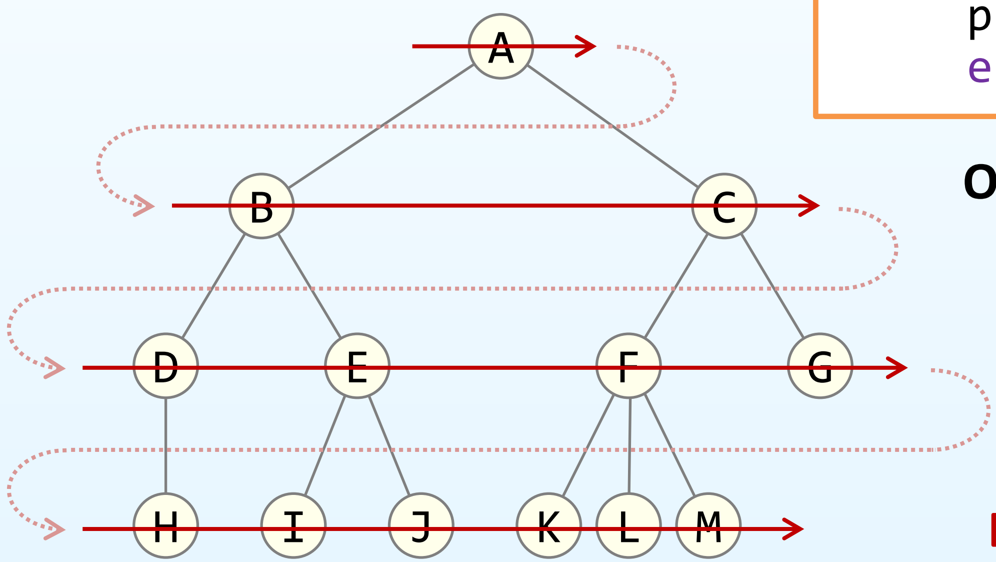
So on …

Output: A B C …

Breadth-First Search (BFS)

- BFS can be implemented using a queue

```
Algorithm: BFS(t)
  q ← allocate a queue
  enqueue(q, root)
  while q is not empty
     node ← dequeue(q)
     print(node)
     enqueue(q, node's all children)
```



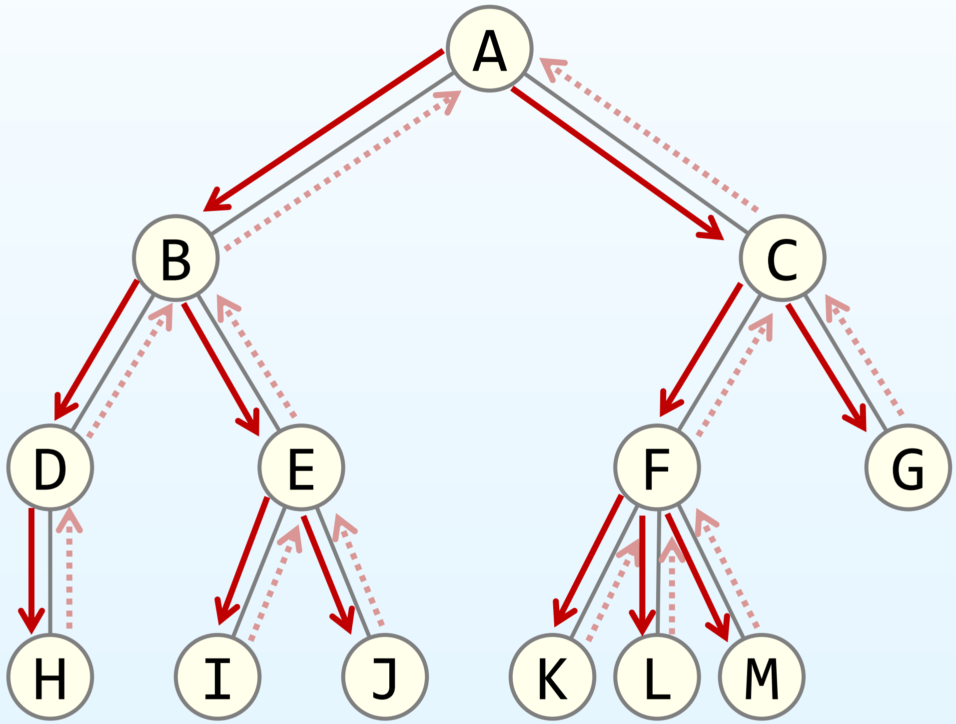
Output: A B C D E F G H I J K L M

What is the running time?

Drawback: Memory is potentially expensive – maximum nodes at a given depth

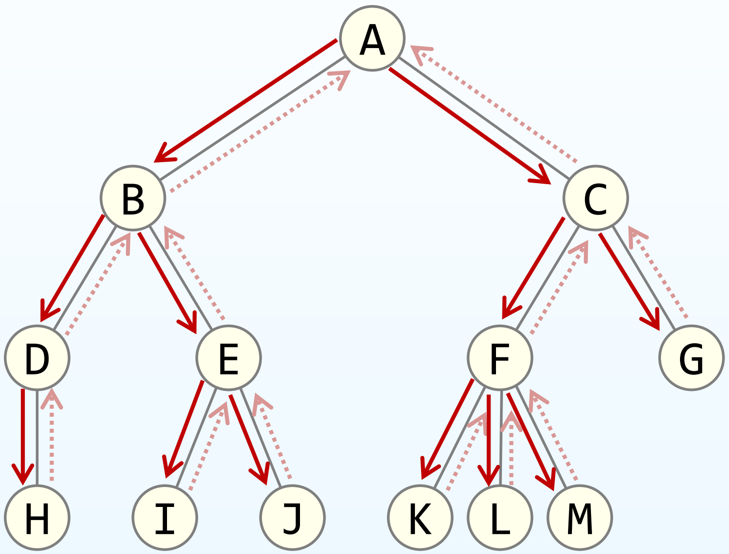
Depth-First Search (DFS)

"It starts with the root node and first visits all nodes of one branch as deep as possible before backtracking, it visits all other branches in a similar fashion."



Depth-First Search (DFS)

- DFS can be implemented using a stack



Output: A C G F M …

Algorithm for DFS

Step 1: push({A})

A												

Step 2: pop() and then push({B,C})

B	C											

Step 3: pop() and then push({F,G})

B	F	G										

Step 4: pop() and then push({})

B	F											

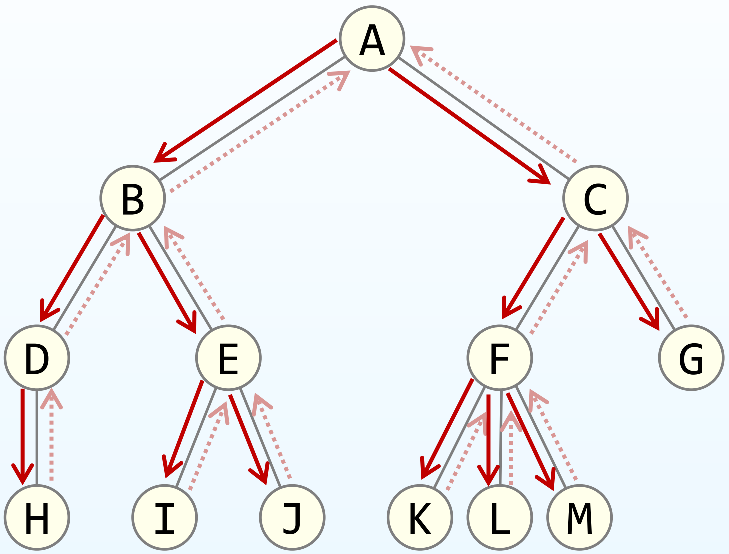
So on …

The sequence is not correct as expected!

What should we do?

Depth-First Search (DFS)

- DFS can be implemented using a stack



Output: A B D H E …

Algorithm for DFS

Step 1: push({A})

A												

Step 2: pop() and then push({C,B})

C	B											

Step 3: pop() and then push({E,D})

C	E	D										

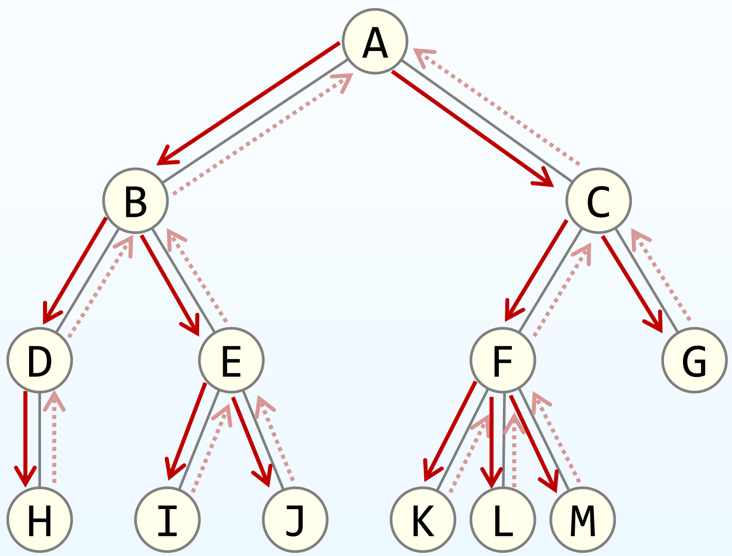
Step 4: pop() and then push({H})

C	E	H										

So on …

Depth-First Search (DFS)

- DFS can be implemented using a stack




```
Algorithm: DFS(t)
  s ← allocate a stack
  push(s, root)
  while s is not empty
     node ← pop(s)
     print(node)
     push(s, node's all children in
              reverse order)
```

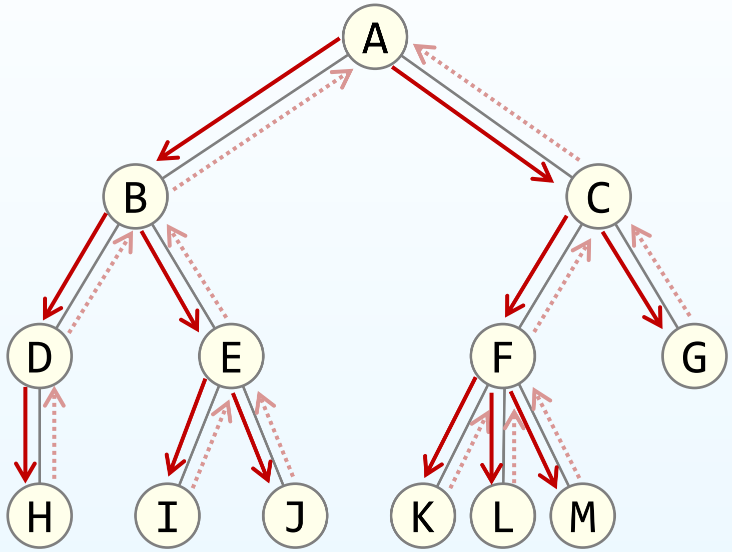
Output: A B D H E I J C F K L M G

What is the running time?

Note: The memory required is the height of the tree $\Theta(h)$

Depth-First Search (DFS)

- DFS can also be implemented using a recursion



We have already seen this approach!!!

Algorithm: DFS(t)

```
  node ← root of t
  print(node)
  DFS(node's first child)
  DFS(node's sibling)
```

Output: A B D H E I J C F K L M G

Applications of DFS

- Finding the height of a tree
- Printing a hierarchical structure
- Calculating the total value of subtrees

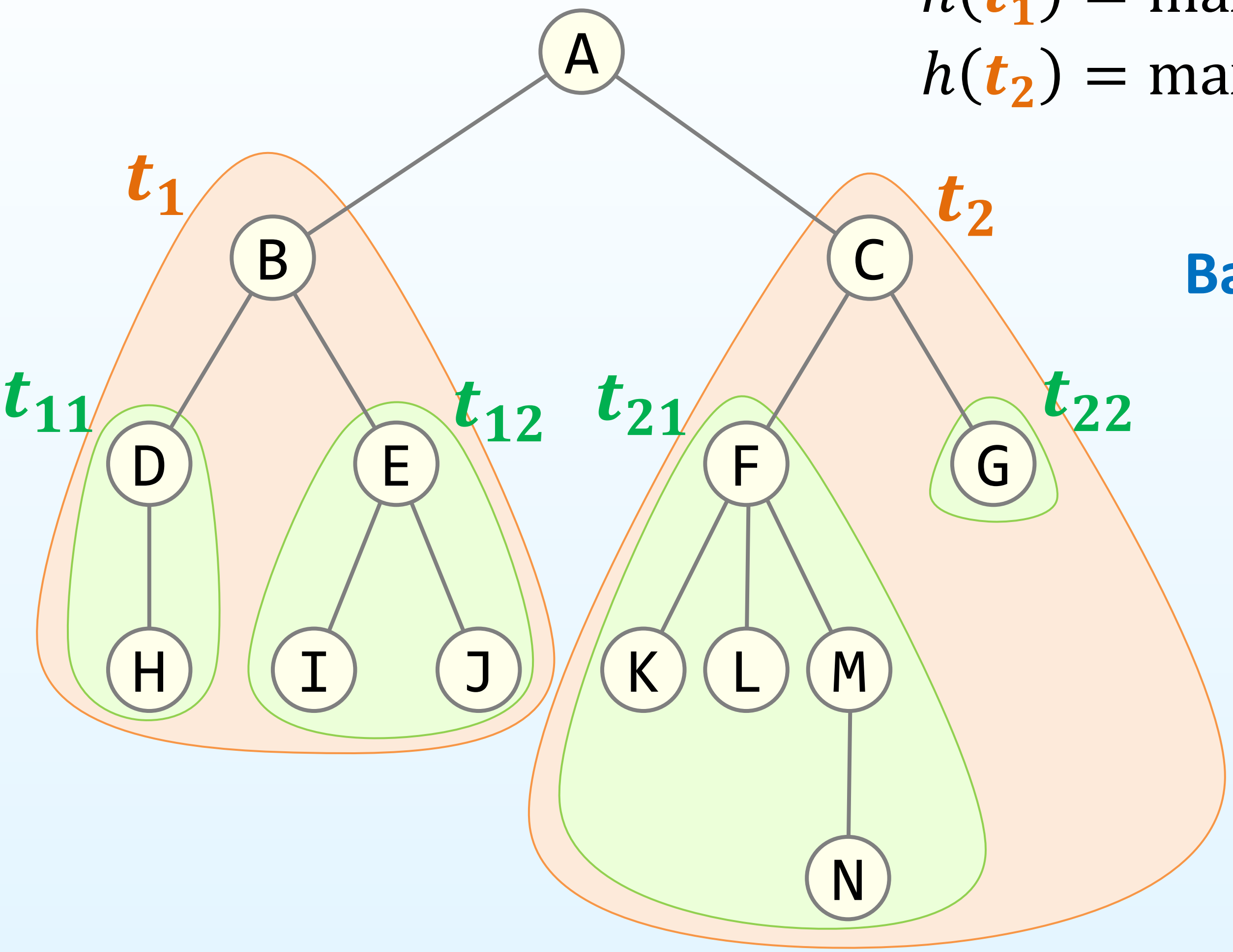
Finding the Height

$$h(\boldsymbol{t}) = \max\big(h(\boldsymbol{t_1}), h(\boldsymbol{t_2})\big) + 1$$

$$h(\boldsymbol{t_1}) = \max\big(h(\boldsymbol{t_{11}}), h(\boldsymbol{t_{12}})\big) + 1$$

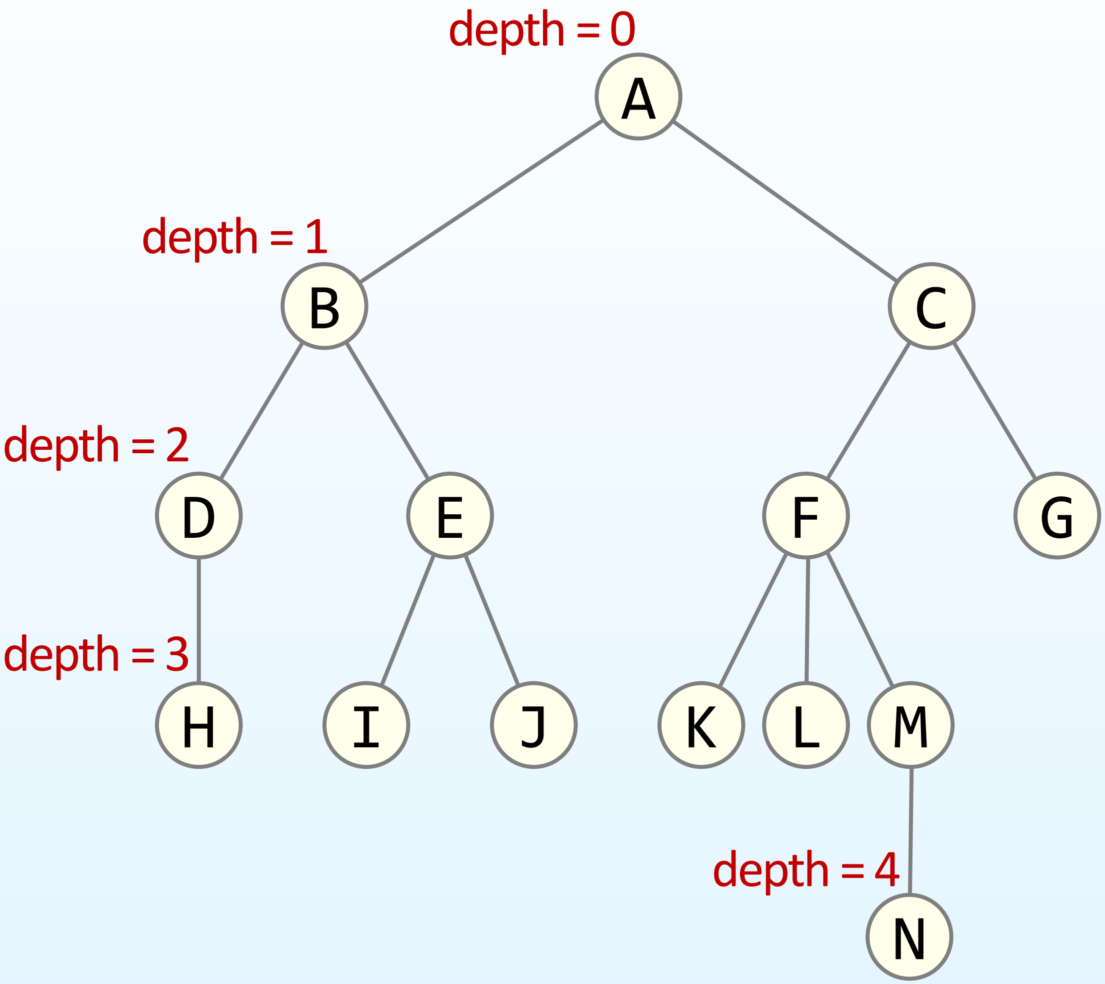
$$h(\boldsymbol{t_2}) = \max\big(h(\boldsymbol{t_{21}}), h(\boldsymbol{t_{22}})\big) + 1$$

$$\vdots$$



Base case ?

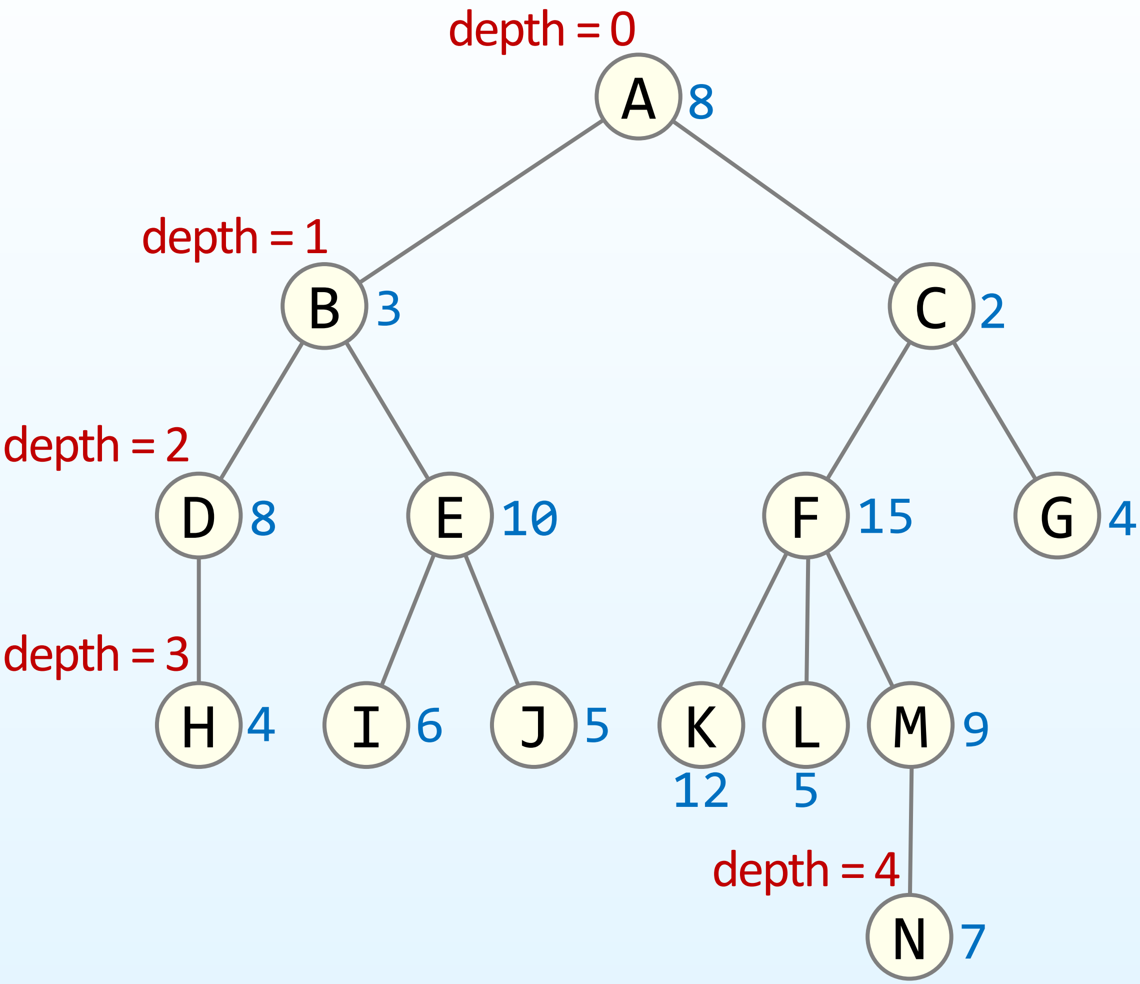
Printing a Hierarchical Structure



Output:

```
A
    B
        D
            H
        E
            I
            J
    C
        F
            K
            L
            M
                N
        G
```

Calculating the Total Value of Subtrees



Output:

```
         H(4) 4
      D(8) 12
         I(6) 6
         J(5) 5
      E(10) 21
   B(3) 36
         K(12) 12
         L(5) 5
            N(7) 7
         M(9) 16
      F(15) 48
      G(4) 4
   C(2) 54
A(8) 98
```

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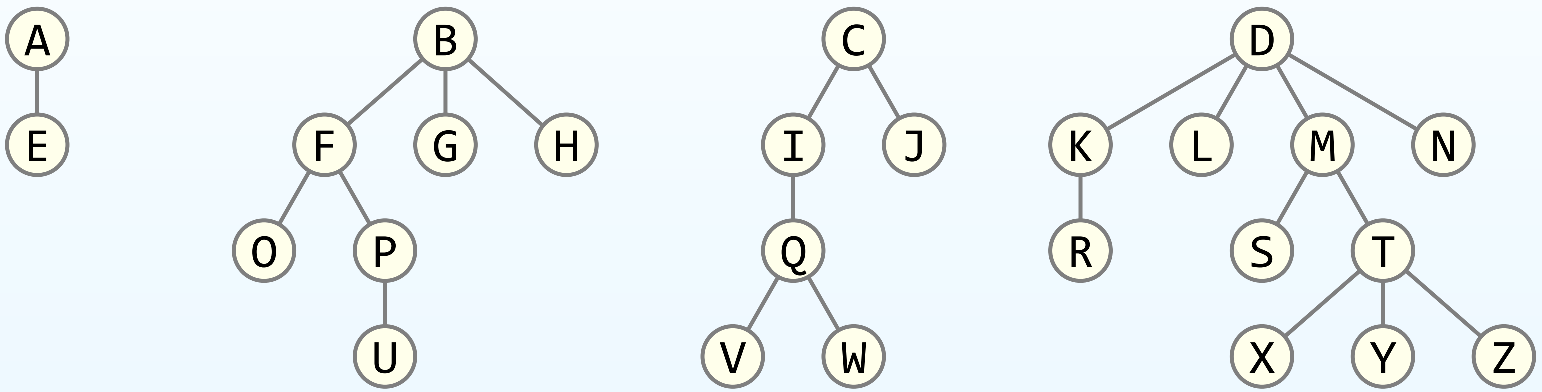
Definition

A Forest is a data structure that is a collection of disjoint rooted trees

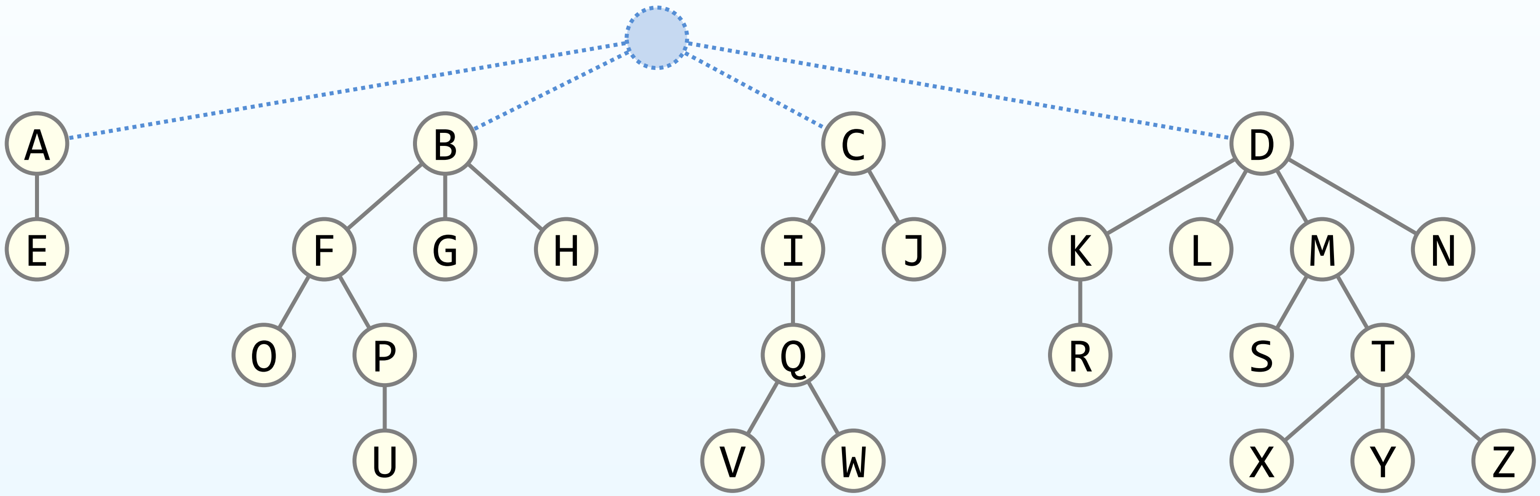


Note that:

- Any tree can be converted into a forest by removing the root node
- Any forest can be converted into a tree by adding a root node that has the roots of all the trees in the forest as children

Traversals

Traversals on forests can be achieved by treating the roots as children of a notional root

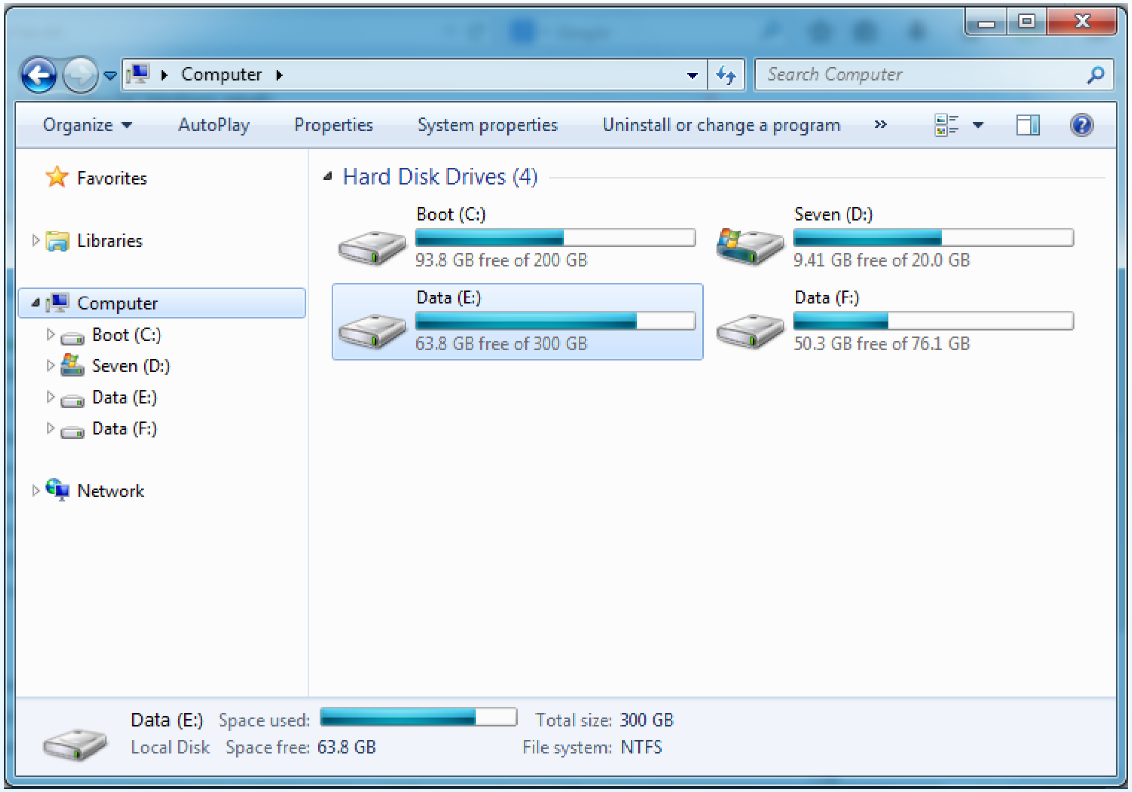


- Breadth-first traversal: A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
- Depth-first traversal: A E B F O P U G H C I Q V W J D K R L M S T X Y Z N

Application

In Windows, each drive form the root if its own directory structure

- Each of the directories is hierarchical—that is, a rooted tree



Any Question?