The study of the Calculus is one of the most powerful intellectual achievements of the human brain. ∅6.∅7 2∅25

What is the Calculus? that's a deep question!

Calculus is the most powerful Branch of math, which revolves around calculations involving varying quantities.

It provides a system of rules to calculate quantities → which cannot be calculated by applying any other branch of math.

Go further than just the Basic understanding of the fundamentals.

The ideas related to Calculus (the dev of Calc) appear throughout mathematical History. Spanning over more than 2∅∅∅ years!

06.09
2023

.. the credit of its invention goes to the mathematicians of the 17^{th} century
→ particularly to Newton, Leibniz → continues to (up to) the 19^{th} century

Augustin-Louis Cauchy (1789-1857)
→ introduces the concept of the limit

↳ this concept of the limit → Removed any doubts about the soundness of the Calculus → made it free from all confusion.

It was doubted by the greatest mathematicians of the 18^{th} century, yet, it was not only applied freely but great developments like diff eqs, diff geometry → so on were achieved.

"Calculus, which is the outcome of an intellectual struggle for such a long period of time, has proved to be the most beautiful intellectual achievement of the Human mind."

There are certain problems in mathem- 06.07
-atics, mechanics, physics, & many other 2025
branches of science

Which cannot be solved by ordinary methods of geometry or algebra alone.

Calculus uses ideas & methods from arithmetic, geometry, algebra, coordinated geometry, trig-trash & so on

↳ but most important the concept of the limit (notion of the limit) → which is a <u>new</u> idea which, lies at the foundation of Calculus.

Using this notion as a tool, the derivative of a function (which is a variable quantity) is defined as the limit of a particular kind.

Generally → Diff Calc provides a method for calculating "the rate of change."

of the value of the variable quantity. 06.07
2025

Integral Calculus → provides methods for calculating the total effect of such changes, under the given conditions.

The phrase rate of change mentioned → stands for the actual rate of change of a variable, NOT its average rate of change.

Phrases such as "rate of change", stationary point, root have precise mathematical meaning, agreed-upon all over the world.

↳ Understanding such words helps a lot in understanding the mathematics they convey.

∴ whereas algebra, geometry, & trig are the tools which are used in the study of the Calculus → they should not be confused with the subject of Calculus.

.. concept-oriented notes for systematic studies in diff. & integral Calculus → 6.07.2025

Concept of the limit → the foundation of the Calculus. → to provide the basic (solid) foundation of Calculus & develop the ability to select proper material, needed for their studies in any technical & scientific field, involving the Calculus.

.. important for clear exposition & for problems that will hook a student's interest.

Students see no reason why they should master tenuous ways of differentiating & integrating by hand when a calc/computer will do the job. We can still use those tools.

.. What they observe is that without basic understanding about the subject, solving diff & integration problems will be a futile exercise.

the word "Calculus" is an abbreviation for infinitesimal Calculus →

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Infinitesmal Calculus

Differential Calculus

Integral Calculus

two seperate but complimentary branches

fundamental theorem of Calculus

Brings these together, you already know how, but why?

So, what is Calculus? 06.07.2025

What does it calculate?

Is the Calculus different from other branches? (of math)

What type(s) of problems does the calculus address? (or handle)

i). What is the Calculus?

ii). What does it calculate?

iii) Why do teachers of physics & mathematics frequently advise you to 'learn Calculus seriously'?

iv) How is the Calculus more important & more useful than Algebra & trig / or any other branch(s) v). Why is Calculus more difficult to absorb than Algebra or trig? (is it really?) 06.07.2025

vi). Are there any problems faced in your day-to-day activities that can be solved more easily by the Calculus than Algebra/trig?/arithmetic

vii) Are there any problems which cannot be solved without calculus? (chaos)

viii) Why study the Calculus at all?

ix) Is Calculus different from other branches of math?

x) What type(s) of problems are handled of Calculus?

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- How does the Calculus begin?
  the subject of Calculus?
- How can we learn Calculus?
- What is Calculus about?

{{ Mathematician's delight }}
W.W Sawyer

This is not about learning the operations of Calculus faster

Calculus is the higher branch of math → which enters into the process of calculat-ing changing quantities (& certain properties)
, 2> Calculus → is said to be the
Mathematics of Change

.. So what is change?

So → what is that change & how it changes?

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↳ concept of function $y = f(x)$

wherein "y" is related to "x" through a rule 'f.'

You would say that 'y' is a function of x, by which you mean that "y" depends on "x".

You would say that "y" is a dependent variable depending on the value of x an independent variable.

It is clear (from that statement) that as the value of "x" changes, there results a corresponding changes in the value of "y" depending on the nature of the function "f" → OR the formula defining f.

The immense practical power of Calculus, is due to its ability to describe & predict the behavior of changing quantities' $\Rightarrow$ "y" & "x"

linear fns $\rightarrow$ such $y = mx + b$ } the amount of change in the value of x causes a proportionate change in the value of 'y.'

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But in other fns such as $y = x^2 - 5$, $y = x^3$, $y = x^4 - x^3 + 3$,

$$\underbrace{y = \sin x,\ y = 3e^x + x \ \ldots \text{etc}}_{\text{Not linear} \rightarrow \text{No proportionality exists}}$$

Here you will interest yourself in studying the behavior of the dependent variable

$y[=f(x)]$ with respect to the change in (the value of) the dependent variable "x"

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Basically → you want to find the

" the rate at which "y" changes with respect to "x."

Basically → you wish to find the rate at which "y" changes with respect to "x."

You know that every rate is the ratio of change that may occur in the quantities, which are related to one another through a rule

average rate is easy to calc → the avg rate at which the value of y changes when x is changed from x_1 to x_2

It can be easily checked that (for the non linear functions)

→ these average rate(s) are different between different values of x 06.07.2025

[thus, if $|x_2 - x_1| = |x_3 - x_2|$
$= |x_4 - x_3| \dots$
for all x_1, x_2, x_3, x_4

then you have $f(x_2) - f(x_1) \neq$
$f(x_3) - f(x_2) \neq f(x_4) - f(x_3)$
$\neq \dots$]

Thus, you get the rate of change of y is different in between different values of x.

Your interest lies in computing the rate of change of "y" at every value of "x."

> instantaneous rate of change of change of "y" with respect to "x" $\longrightarrow$

& you call it the rate "rate of change" of "y" with respect to "x." 06.07 2025

↳ it is called the derived function of "y" with respect to "x" & denoted by the symbol ↳ $y'\,[= f'(x)]$

The derived function $f'(x)$ is called the derivative of $y\,[= f(x)]$ with respect to x.

The equation $y' = f(x)$ tells that the derived function $f'(x)$ is also a function of x, derived (or obtained) from the original function $y = f(x)$

There is another (useful) symbol for the derived function denoted by dy/dx

its def not a ratio

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↳ it is a single unit

↳ the equation $y' = f'(x)$ gives you the instantaneous rate of change of "y with respect to x, for every value of "x", for which $f'(x)$ is defined

∴ To define the derivative formally & to compute it symbolically is the 'subject of Differential Calculus.

In the process of defining the derivative, various subtleties & puzzles will inevitiably arise.

You will use a systematic approach → it will not be too difficult to grasp the concept (of derivatives) with this approach.

∴ The relationship between $f(x)$ & $f'(x)$ is the main theme you will study what it means for $f'(x)$ to be the "rate function" of $f(x)$ & what each function says about the other. 06.07.2025

It is important to understand clearly "the meaning of the instantaneous rate of change of $f(x)$ with respect to x"

Now the 3rd question's answer?

There are certain problems in math & other branches of science(s), which cannot be solved by ordinary methods known to you in arthmetic, geometry & algebra alone.

∴ In the calculus you can study the propertie-s of a function without drawing its graph!

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It is important to be aware of the underlying presence of the curve of the given function.

Introduced to us by Decartes & Fermat → coordinate geometry

Consider the curve defined by the function

$$y = x^3 - x^2 - x$$

you know that the slope of this curve changes from point to point.

If you choose to find its slope @ $x=2$, then calculus alone can help you! in this case is 7

Calculus uses ideas & methods from arithmetic, geometry, algebra, coordinate geometry, trig ... so on, & also the notion of the limit!

.. The notion of the limit is the "new idea" that lies @ the foundation of Calculus

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Using the notion of the limit as a tool, the derivative of a function, is defined as the limit of a particular kind

{ it will be seen that the derivative of a func-tion is generally a new function

★ Calculus provides a system of rules for calculating changing quantities which cannot be calculated otherwise! ★

.. The concept of the $\lim_{x\to}$ is equally important & applicable in Integral Calculus

.. Calculus is the most beautiful & powerful achievement of the Human Brain.

Calculus is the most beautiful & powerful achievement ← read that again. 06.08.20 25

It has been developed over a period of more than 2000 years!

The idea of the derivative of a function is among the most important concepts in all of mathematics & it alone distinguishes Calculus from the other branches of mathematics.

The derivative & an integral have found many diverse uses.

Diff Calculus is a subject which can be applied to anything that moves. (or changes shapes / or has shape (a)) → good for electric lighting → wireless optics & of course thermodynamics → good for → questions about the greatest & smallest values a fn can take ...

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"Once the basic ideas of diff ca have been grasped, a whole world of problems can be tackled without great diff-iculty ~> it is a subject worth learning.

On the other hand, integral calculus considers the problem of determining a fn from the info about its rate of change.

Given a formula for the velocity of a body, as a function of time, you can integrate (use integral calc) to produce a formula that tells you how far the body has traveled from its starting point, at any instant.

It provides methods for the calculation of quantities such as areas & volumes of curvilinear shapes.

(->) It is also useful for the measurement of dimensions of mathematical curves

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2025

.. It provides methods for the calculation of quantities such as areas & volumes of curvilinear shapes.

It is also useful for the measurement of dimensions of mathematical curves.

.. The concepts basic to Calculus can be traced to → in uncrystallized form, to the time of the ancient Greeks (& Indus valley)

around 287 - 212 BCE

.. It is only in the 16th & early 17th centuries that mathematicians developed & refined techniques for determining tangents to curves & areas of plane regions.

Newton (1642 - 1727)
Leibniz (1646 - 1716) } invented* (not fully realized)

later they both uncovered the antiderivative → the integral → inverse process → , thus making possible the rule for evaluating definite integrals.

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There were many difficulties in the foundation of the Calculus.

∴ During the last 150 years, Calculus has matured bit by bit

French Mathematician Augustin-Louis Cauchy (1789 - 1857)

→ limit definition (which removed all doubts about the soundness of Calculus & made it free from all confusion.

∴ It was then that, Calculus had become, mathematically much as it is now!

at around the year 1930, the increasing use of calculus in engineering & science to learn Calculus. ₵6.₵8.2025

→ Back then the Calculus was considered an extremely difficult subject. Most students didn't find it to be groovy. → Because the basic Concepts of the Calculus & its interrelations with the other subjects were probably not conveyed or understood properly. ~~It's~~ NOT just some set of rules & formulas.

∴ don't rote memorize formulae (except for some of the "trig trash" → don't just apply them like some robot.

"Why study Calculus at all?" → Calculus from Graphical, Numerical & Symbolic points of View → Arnold Ostebee Paul Zorn

you learn it because
↳ it is → good for applications
higher math
good mental training
other major require it.
Jobs

Calculus is also among our deepest ∅6.∅8
& richest, farthest-reaching
& most beautiful intellectual achiev 2∅25
-ements

Co-ordinate geometry → the merging of geometry with algebra & helps in visualizing an equation as representing a curve & vice-versa

No co-ordinate Geometry → no calculus → thank you Descartes, watch flies on the ceiling.

You can do a lot in life without calculus → But you're missing out, You will never know what you're missing.

~ Intro → In less than 15 mins let yourself realize that calculus is capable of computing many quantities accurately, which cannot be calculated using any other branches of math.

you can begin by appreciating the fact $\Rightarrow$ consider a "non vertical" line that makes an angle "θ" with the positive direction of x-axis, & that $\theta \neq 0$.

06.08.2025

You say that the given line is "inclined" at an angle "θ" (or that the inclination of the given line is "θ")

The important idea here is "the slope of the given line" $\Rightarrow$ this is expressed by the trig ratio "$\tan \theta$"

• The slope of the line tells you that if you travel by "one unit," in the positive direction along the x-axis, then the number of units by which the height of the lines rises (or falls) is the measure of the slope.

Also remember that the "slope of a line" is a constant for that line. 06.08 2025

While the "slope of any curve" → changes from point to point & it is defined in terms of the slope of "the tangent line" existing there.

To find the slope of curve $y = f(x)$ @ any value of x, the "diff calculus" is the branch of Mathematics → which can be used even if you are unable to imagine the shape of the curve

Remember this → ★ Calculus demands the knowledge of Algebra → Geometry Coordinate geometry & trig, so on → (as prerequisite) ↝ But they do not form the subject of Calculus ~ Hence do not confuse this branch with others →

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Calc is a different Subject →

" The backbone of Calculus is the concept of the limit →

(Augustin - Louis Cauchy) 1789 - 1857

What you must know to learn Calculus → the concept of " $\underbrace{\text{derivative}}_{\text{the slope}}$ "

As an application of integral calc, the area under a curve $y = f(x)$ from $x = a$ to $x = b$, ↳ & the axis can be computed only by applying the integral calc.

★ ★ No other branch of mathematics is helpful in computing such areas with curved boundaries!

From Arithmetic to Algebra

06.08.2025

Numbers are symbols used for counting & measuring

Hindu-Arabic numerals 0, 1, 2, 3 ... 9 are grouped systematically in units, tens, hundreds, & so on, to solve problems containing numerical info.

Subject of Arithmetic → also involves an understanding of the structure of the number system & the facility to change numbers from 'one form to another' → e.g → the changing of fractions to decimals & vice versa → later discuss the real number line

recall some important subsets of real numbers → natural numbers or positive integers

set of natural numbers is denoted

$$\mathbb{N} = \{1, 2, 3, 4, 5, \ldots\}$$

Rember $\emptyset$ is a whole number but is not a real number

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The Set of Integers

All natural numbers → their negatives & zero when considered together, form "the set of integers" denoted by $\mathbb{Z}$

→ thus $\mathbb{Z} = \{\ldots, -3, -2, -1, \emptyset, 1, 2, 3, \ldots\}$

The Set of Rational Numbers

The numbers of the form p/q where p & q are integers → & the denominator $q \neq \emptyset$, form the set of <u>rational numbers</u>, denoted by $\mathbb{Q}$

such as → $\frac{3}{5}$, $\frac{-7}{9}$, $\frac{8}{-15}$, $\frac{\emptyset}{15}$, $\frac{9}{1}$, $\frac{-121}{-12}$, $\frac{16}{2}$ ... so on → all rational numbers

a) Zero is a rational number 06.08.2025

↳ but division by Zero is not defined

$5/0$, $0/0$ are meaningless expressions

b) All integers are rational numbers, but the converse is <u>not</u> true

c) Positive rational numbers are called fractions

Generally fractions are used to represent the parts of a given quantity, under consider-ation

↳ Thus $3/7$ tells you that a given quantity or an object is divided into seven equal parts & three parts are under consideration

A fraction is also used to express a ratio. Thus, $2:5$ is also written as $2/5$ & similarly $12:5$ is written as $12/5$.

Since the ratio of two natural numbers can be greater than $\underline{1}$, all positive rational numbers are called fractions."

Thus → fractions (by suggestion) could be classified more meaningfully
↳ as follows

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• When both numerator & denominator are positive integers, the fraction is known as a simple, common or vulgar

• A complex fraction is one in which either the numerator or the denominator or both are fractions

$$\frac{3}{\left(\frac{7}{5}\right)} \qquad \frac{\frac{5}{9}}{2} \qquad \frac{\frac{7}{3}}{\frac{11}{4}}$$

If the numerator is less than the denominator, the fraction is called a proper fraction

such as → $\frac{4}{7}, \frac{3}{5}, \frac{1}{4}$

06.08.25

- If the numerator is greater than denominator, the fraction is called an improper fraction
- A unit fraction is a special proper fraction, whose numerator is 1

★ A fraction is said to be in lowest terms → if the only common factor of the numerator & denominator is 1.

Thus, $\frac{3}{4}$ is in the lowest terms, but 6/8 is not → since 6 & 8 are divisible by 2, or a common factor of 2, which is not 1

You say that $\frac{a}{b}$, $\frac{2a}{2b}$, $\frac{3a}{3b}$ ...

→ all belong to the same family of fractions

↳ as described by $\frac{a}{b}$

$\{$ You use fractions in lowest terms to describe the family of fractions 06.08.23

You define the set of all fractions by

$$\star \quad F = \left\{ \frac{a}{b} \;\middle|\; a, b \in N \right\} \leftarrow \text{Remember this}$$

The Set of IRRATIONAL Numbers

There are numbers that cannot be expressed in the form $\frac{p}{q}$, where p & q are ints $\rightarrow$ These are called irrational numbers $\rightarrow$ the set is denoted by Q' q prime?

e.g $\rightarrow$ $\sqrt{2}$, $\sqrt{5}$, $6\sqrt{3}$, $7\sqrt{11}$

e, π, $1.101001\ldots$, $5.710710010\ldots$

The set of IReal Numbers

06.08.2025

The set of rational Numbers together with the set of irrational numbers

form the set of real numbers, denoted by $\mathbb{R}$

.. Remember $\sqrt{-1}$, $\sqrt{-7}$ are not represented as real numbers

Arithmetic & Algebra

Four fundemental operations →

namely → addition, subtraction, multiplication & division

which are performed on the set of natural numbers to make new numbers, namely, the number 0, negative ints, & rational numbs.

→ For the formation of irrational numbers you have to go beyond the 4 fund-

- amental arithmetic operations given ↑ $\phi 6.\phi 8,$ $2\phi 25$

The study of, Algebra involves the study of equations & a number of other problems that developed out of the theory of equations.

--> It is in connection with the solution of algebraic, equations that negative numbers, fractions, & rational numbers were developed. The number "ϕ" could enter the family of numbers only after negative numbers were developed.

In arithmetic, you deal with numbers that have 1 (single) definite value.

On the other hand → in Algebra you deal with symbols such as x, y, z --> so on, which represent variable quantities & those like a, b, c ... etc which may have any value(s) → that you could assign to (them).

Hence these symbols are called (represent) variable quantities & are hence called variables. 06.08.2025

You may operate with all these symbols as numbers without assigning to them any particular numerical value.

Note → both numbers & letters are symbols (used to solve various problems)

You can say that traditional Algebra is generalization of arithmetic. (same symbols used)

But before you get into Algebra → it is useful to recall → some more subsets of real numbers → this will be needed in various discussions.

Even & Odd Numbers →

Every int' is exactly divisible by 2, is called an even number otherwise it is odd.

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Thus, an even number is of the form $2n$, where is n is an int.

An odd number is of form $(2n \pm 1)$. If number "a" is even, then, $(a \pm 1)$ is odd, & vice/versa

yes, this means that $\emptyset$ is an even int!

Factors → Natural numbers that exactly divide a given int are called the factors of that num.

I, the factors of 12 are 1, 2, 3, 6, 12,

You also say that 12 is a multiple of 1, 2, 3, 4, 6, & 12

Similarly, the factors of 6 are 1, 2, 3, & 6 & the factors of zero are all natural numbers.

06. 08. 2025

Remember 0 is not a factor of any number

Prime & Composite Numbers

a natural number has exactly two unique factors (the number itself & 1)

this is called a prime number → again a prime is a natural number that has exactly two unique factors (the number itself & 1)

a Natural Number that has 3 or more factors is called a composite number.

Some examples of prime numbers are

2, 3, 5, 7, 11, 13, 17, 19, ...

& so on →

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- Each prime number, except 2, is odd

- The number 1 is neither prime nor composite.

Six is a composite number since it has 4 factors, namely 1, 2, 3, & 6

A given natural number can be uniquely expressed a product of primes.

Co Prime Numbers

two Natural numbers are said to be coprime (or relatively prime) to each other if they have no common factor except 1

→ e.g. 8 & 25 are coprime to one another.

★

→ All prime numbers are coprime to each other

H.C.F

06.08.2025

Highest Common Factor

HCF is the Highest Common Factor of 2 or more (natural) numbers → is the greatest number which divides each of them exactly.

2> AKA Greatest Common Divisor (G.C.D)

2> the highest common factor of any two prime numbers (or coprime numbers)

(LCM) least Common Multiple

The least Common Multiple of 2 or more natural numbers is the smallest number which is exactly divisible by each of them

To find the LCM of two or more natural numbers → you find prime factors.

If two (or more nums) have a factor in common →

you select it once. This is done for each such common factor & the remaining factors from each number are taken as they are.

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The product of all these factors taken together → gives the LCM of the given numbers.

(Product of two Numbers = their H.C.F × their L.C.M)

Continuous Variables & Arbitrary Constants

A variable → is a changing quantity, usually denoted by a (x, y, z) which takes on any one of the possible values, in an interval, is called a variable.

While Constants → a, b, c, d are used as arbitrary constants.

∴ In Arbitrary Constants → though there is no restriction to the numerical values a letter may represent, it is understood that in the same piece of work, it keeps the same value → througout.

06.08.2025

e.g. → expression ⇒ $f(x) = ax^2 + bx + c \quad (0 \le x \le 5)$

is a continuous variable in the interval $[0, 5]$ & a, b, c are arbitracy constants

The Language of Algebra →

- An Algebriac expression
- factors, coefficients, index/exponent (or power) of a quantity
- Positive & neg terms
- like & unlike terms

- process involving add, sub, divide, multiply ~ are Algebriac expressions

- Removal & insertion Brackets
- Simplification of an Algebriac expression
- Polynomials & related concepts

06.08
2025

Polynomials →

a polynomial is x is an expression of the form

$$p(x) = a_n \cdot x^n + a_{n-1} \cdot x^{n-1} + \dots + a_1 \cdot x + a_0$$

where $a_0, a_1, a_2 \dots a_n$ are real numbers called coefficients of $p(x)$ & n in x^n is non-negative integer.

Usually, you write a polynomial in either descending powers of x or ascending powers of x ↓

The form of a polynomial written in this way is called the standard form. 06.08.2025

From the definition of a polynomial, it is clear that polynomials are special types of algebriac expressions involving only finite number of terms & one var

Degree of a polynomial

The exponent, in the highest degree term of a non zero polynomial is called a degree of the polynomial.

if $a_n \neq 0 \rightarrow$ then n (in x^n) is the degree of the polynomial

thus $\rightarrow 3x^5 + 2x^3 - x + 7$ is 5

the degree of $(3/2)y^3 - \sqrt{2y} - 1$ is 3

A polynomial having only one term is called a "monomial"

06.08, 2025

The Zero Polynomial

→ now you know that a polynomial having all coefficients as zero is called "the zero polynomial" is unique & it is denoted by the symbol "0"

The degree of a "zero polynomial" is not defined.

↳ $0 = 0x = 0 \cdot x^5 \ldots = 0 \cdot x^{107}$ & so on

There are all zero polynomials & obviously, their degree cannot be defined

→ in what follows → a polynomial will mean a nonzero polynomial (in a single variable) with real coefficients

Polynomials Behave like Ints

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There are many properties that are possessed by ints that are also possessed by polynomials.

thus → if $p(x)$ & $q(x)$ are two polynomials, then the expression $\frac{p(x)}{q(x)}$, where $q(x)$ is a none zero polynomial, is called a rational expression

•• A rational expression must be expressed in its lowest terms, by canceling the common factors in the numerator & denominator.
--> you have to learn the process of factorization of a polynomial.

Factors of a Polynomial → a polynomial $g(x)$ is called a factor of polynomial $p(x)$ if $g(x)$ divides $p(x)$ exactly

→ on dividing $p(x)$ by $g(x)$ you get $\phi 6. \phi 8. 25$ zero as the remainder.

Division Algorithm (or Procedure) for polynomials

⇒ $p(x)$ by a polynomial $g(x)$

let the quotient be $q(x)$ & the remainder be $r(x)$

⇒ you have

$$p(x) = g(x) \cdot q(x) + r(x)$$

where either $r(x) = \phi$

or degree of $r(x)$ < degree of $g(x)$

Remember → When a polyn. $p(x)$ is divided by a linear polynomial $(x - \alpha)$ then the remainder is a constant, which may be zero or nonzero. The value of the remainder can be obtained by applying the remainder theorem.

Remainder Theorem → If a polynomial $p(x)$ is divided by a linear polyn. $(x - \alpha)$ then the remainder $p(\alpha)$ 06.08.2025

↳ this theorem can be easily proved using the division algo

Again → If $p(x)$ is divided by $(x+\alpha)$, then the remainder $= p(-\alpha)$.

Similarly, when $p(x)$ is divided by $(ax+b)$ then the remainder $= p(-b/a)$

It is sometimes possible to express a polynomial as a product of other polyn. each of degree ≥ 1

e.g → $x^3 - x^2 + 9x - 9 = (x-1)\cdot(x^2+9)$;

$3x^2 - 6x - 9 = 3(x^2 - 2x - 3) = 3(x-3)(x+1)$

Value of a polyn & Zeros of a Polynomial 06.08.2025

--> you know that for every real value of x, a polynomial has a real value.

let $p(x) = 3x^4 - 2x^3 + x + 5$.

Then, for $x = 1$ → you have $p(1) = 7$

& for $x = 0$, $p(0) = 5$

An important aspect of the study of a polynomial is to determine those values of x for which $p(x) = 0$

Such values of x are called zeros of the polyn. is to determine those values of x for which $p(x) = 0$

Such values of x are called zeros of the polyn. $p(x)$ Consider the quadratic polyn. $q(x) = x^2 - x - 6$ →

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∴ It may be seen that $q(3) = 0$ & $q(-2) = 0$. If $x = a$ is a zero of the polynomial $p(x)$ then $(x-a)$ is a factor of $p(x)$.

The factor theorem of Algebra

-- what is the fundamental theorem of Algebra?

The factor theorem helps in finding the (linear factors of a polynomial, provided such factors exist.

∴ Remember there are no standard methods available for finding linear factors of polyn.(s) of a higher degrees. → except in very special cases.

Every quadratic polyn. can have @ most two zeros, a cubic polyn. @ most three zeros etc...

Some polyn(s) do not have any real zeros. In other words → there may be no real number "x" for which the value of the polynomial becomes $\emptyset$. e.g. → there is no real number "x" for which x^2+3 will be zero. $\emptyset6.\emptyset8.2\emptyset25$

So how to determine the zeros of a given polyn. $p(x)$

⇝ so how to solve the equation $p(x)=\emptyset$?

Polynomial Equations & their Solutions (Roots)

If $p(x)$ is a quadratic polynomial, then the equation $p(x)=\emptyset$ is called a quadratic equation. If $p(x)$ is a cubic polynomial, then the corresponding equation $p(x)=\emptyset$ is called a quadratic equation

↓

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If $p(x)$ is a cubic polyn., then the corresponding equation $p(x) = 0$ is called a cubic equation, & so on.

If the numbers α & β are two zeros of the quadratic polynomial $p(x) \Rightarrow$ you say that α & β are the two roots of the corresponding quadratic equation $p(x) = 0$

The fundamental theorem of Algebra states that a nonzero n^{th} degree polynomial eq. has at most n roots, in which some roots may be repeated roots.

Thus, starting from the concepts of an algebriac expression you have revised the concepts of polynomials, zeros of a polynomial & the solution of simple polynomial equations

Algebra As a Language of Thinking

06.08.2025

you already know that algebra has a set of rules; but you should not feel satisfied to have learnt algebra merely as a set of rules. It is more important to have some understanding of:

- What is algebra all about?
- How does it grow out of arithemtic?
- And how is it used to convey concepts of arithmetic e.g →

3^2 is 1 bigger than $2 \cdot 4$

4^2 is 1 bigger than $3 \cdot 5$

5^2 is 1 bigger than $4 \cdot 6$

This means → that the square of any number (natural) is $\underline{1}$ bigger than the result of multiplying two numbers of which one is less by $\underline{1}$ & the other is more by one than the given number.

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Thus, you should guess that 87^2 would be 1 bigger than $86 \cdot 88$

The general result is → conveniently stated in the language of algebra.

let n be any natural number → then "the number before n" will be written as $(n-1)$ & "the number after n" is $(n+1)$. You shall now say, n^2 is $\underline{1}$ bigger than $(n-1)(n+1)$, or completely in symbols

↳ $$\underbrace{n^2 = 1 + (n-1)(n+1)}$$

this equation only holds not only for natural numbers but also for <u>all</u> numbers

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But also for all numbers!

It expresses what you guessed @ by looking at particular results in arithmetic.
The beauty of algebra lies in its utility. →

Here, it enables you to prove that your guess its correct. By the usual procedures of Algebra, you can simplify the expression on the right-hand side of

$$\left(n^2 = 1 + (n-1)(n+1) \right)$$

& see that it equals the left-hand side.

In Algebra itself → you often pass from particular results to more general ones.
Such as this → $n^2 - 1 = (n-1)(n+1)$

you see $\rightarrow n^2 - 1 = n^2 - 1^2 = (n-1)(n+1)$ 06.08.2025

↳ in general → you have

$$a^2 - b^2 = (a-b)(a+b)$$

or $a^2 = (a-b)(a+b) + b^2$

which becomes more general than the one expressed

as $\rightarrow n^2 = 1 + (n-1)(n+1)$

you can make use of $a^2 = (a-b)(a+b) + b^2$

This result is more general than

$$n^2 = 1 + (n-1)(n+1)$$

make of $a^2 = (a-b)(a+b) + b^2$ e.g →

$$27^2 = (27-3)(27+3) + 3^2$$

$$= (24 \cdot 30) + 9 = 729$$

also → 06.08.2025

$$103 \cdot 97 = (100+3)(100-3)$$

$$= (100)^2 - 3^2 = 10000 - 9$$

$$= 9991$$

consider the following products →

$$(x+3)(x+4) = x^2 + 7x + 12$$

$$= x^2 + (3+4)x + 3 \cdot 4$$

$$(x+5)(x+3) = x^2 + 3x + 5x + 15$$

$$= x^2 + 8x + 15$$

$$= x^2 + (5+3)x + 5 \cdot 3$$

so →

$$\underbrace{(x+a)(x+b) = x^2 + (a+b)x + a \cdot b}_{\text{notice the general expression.}}$$

Algebra is the Best Language for Thinking About Laws

06.08.2025

x:	0	1	2	3	4	5	...
y:	0	2	4	6	8	10	...

it's easy to guess the law lies behind this table. The law behind this → $y = 2x$ while this one is $y = x^2$

x:	0	1	2	3	4	5	...
y:	0	1	4	4	16	25	...

It is far easier to see what the formula $y = 2x^2 - 5x + 7$ → than to see the formula in the form of words.

★ $\boxed{\text{Induction}}$ In math, it is not always wise to proceed by analogy & draw conclusions. The process of reasoning from some particular results to general one is called "induction".

Induction begins by observation. 06.08.
2025

You observe particular result(s) & use your intuition to arrive at a tentative conclusion -- tentative, because it is an educated guess or a conjecture. It may be true or false. If the general result is provided by systematic deductive reasoning, then it is accepted as true. → while the result (on the other hand) will be considered false → if you are able to show a counter example where the conjecture fails.

Remember that, a conjecture remains a conjecute no matter how many examples you (we) can find to support it.

Pierre de Fermat (1601 - 1665) observed that:

$$\left(2^{2^1}+1\right)=\left(2^2+1\right)=5 \rightarrow \text{is a prime}$$

$$\left(2^{2^2}+1\right)=\left(2^4+1\right)=17 \text{ is a prime}$$

$$\left(2^{2^3}+1\right)=\left(2^8+1\right)=257 \text{ is a prime}$$

→ Pierre de Fermat → conjectured that 06.08.2025

$\left(2^{2^n} + 1\right)$ is a prime number for every natural number n & had challenged the Mathematicians of his day to prove otherwise.

It was some years later Euler (1707 - 1783) showed that

$$\left(2^{2^5} + 1\right) = \underbrace{4,294,967,297}_{\text{which is a prime!}}$$

it is divisible by 641

another example →
you observe the absolute values of the coefficients of various terms in each of the following factorization are equal to 1

$$x^1 - 1 = (x - 1);$$

$$x^2 - 1 = (x-1)(x+1)$$

$$x^3 - 1 = (x - 1)(x^2 + x + 1);$$

$$x^4 - 1 = (x-1)(x+1)(x^2+1)$$

$$x^5 - 1 = (x - 1)(x^4 + x^3 + x^2 + x + 1)$$

→ so the conjecture is now that when $x^n - 1$ (n, a natural number) is expressed into factors, with integer coefficients, none of the coefficients is greater than 1) is an absolute value!

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All attempts to prove this general statement failed → i.e. until 1941 → Ivanov came up with a counter example.

He found that → one of the factors of $x^{105} - 1$ violates the conjecture. This factor is a polynomial of degree 48 as given →

$$x^{48} + x^{47} + x^{46} - x^{43} - x^{42} - 2x^{41} - x^{40} - x^{39} + x^{36} + x^{35} + x^{34} + x^{33} + x^{32}$$
$$+ x^{31} + x^{28} - x^{26} - x^{24} - x^{22} - x^{20} - x^{17} + x^{16} + x^{15} + x^{14} + x^{12} - x^{9} - x^{8}$$
$$- 2x^{7} - x^{6} - x^{5} + x^{2} + x + 1$$

in math → you have several such conjectures → that have remained conjectures → because there is NO proof!

→ ~~even if~~ though literally thousands of examples have been found in support of them ~~06.08.25~~

Having employed intuition & arrived @ a conjecture, the very difficult task of proving the conjecture begins.

If the conjecture is in the form of a statement, say $P(n)$ → involving natural numbers, a method of proof is provided by the

↳ the principle of mathematical induction

↳ $P(n)$ represent the statements: (i) $n(n+1)$ is even or (ii) $3^n > n$, or (iii) n^3+n is divisible by 3 or (iv) $2^{3n}-1$ is divisible by 7 etc...

An Important Result: The Number of Primes Is Infinite

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There is no known formula that relates successive primes to successive integers.

Therefore, it is not possible to use the principle of mathematical induction to prove this result.

Algebra provides a simple method to prove it.

∴ An indirect approach is needed

ALGEBRA AS THE SHORTHAND OF MATHEMATICS

Algebra can be compared to writing shorthand in ordinary life.

<u>It can be used either to make statements</u> or

<u>to give instructions in a concise form</u>

Mathematical statements → in ordinary language can be translated into algebriac statements & similarly statements in Algebra, can be translated into ordinary language. 06.08.2025

More → Notations in Algebra → Equivalent Statements in Alg.

Statement → (i) Think of a number add 7 to it & double the result → $2(x+7)$

choose a num. multiply it by 5, add 2 square this expression, & divide the result by 8 → $\dfrac{(5x+2)^2}{8}$

• Algebra puts mathematical statements in a small space. The statement is shorter to write, easier to read, quicker to say, & simplar to understand, than the corresponding sentence in ordinary English.

When you say that Algebra is a language you mean that it has its own words & symbols for expressing what might →

otherwise be expressed in ordinary language such as French or Spanish

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~ you do not look at Algebra from this point of view. For you algebra is a special Kind of language for the following two Reasons →

a) Algebra is concerned primarily with statement(s) about numbers, items, symbols, or quantities.

b) The language of Algebra uses symbols in place of words.

e.g to discuss about a class of numbers (say a class of natural numbers) you could say →

let "α" be any natural number.

Thereafter, in the entire discussion whenever he wishes to refer to an arbitrary natural number, he will use the letter of α & thus save words & space.

∴ any statements about α applies to all natural numbers.

{ NOTATIONS IN ALGEBRA }

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An Important difference between the notation of arithmetic & Algebra is as follows →

In arithmetic → the product of 3 & 5 → is written 3×5, in Algebra → the product of a & b may be written in any of the forms $a \times b$, $a \cdot b$, ab

- Product of first n natural numbers is given by ⇝ $n! = n \cdot (n-1) \cdot (n-2) \cdot (n=3) \ldots 3 \cdot 2 \cdot 1$

e.g ⇝ $7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

- Numbers of permutations (arrangements) of n different things taken r at a time → given by ↓

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- Number of permutations (arrangements) of n different things taken r at a tim is given by →

$${}^{n}P_r = \frac{n!}{(n-r)!} \rightarrow {}^{5}P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!}$$

$$= \frac{\text{Product of first 5 natural numbers}}{\text{Product of first 2 natural numbers}}$$

$$= \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = 60$$

$${}^{n}P_n = \frac{n!}{(n-r)!} = \frac{n!}{0!} = \frac{\text{Product of first } n \text{ natural numbers}}{\text{Product of first "zero" natural numbers}}$$

$= n!$ which then follows that $0! = 1$ (this is taken as the def of $0!$)

- number of combinations of n different items taken at r at a time; is given by

$${}^{n}C_r = \frac{n!}{r!(n-r!)} \rightsquigarrow$$

e.g $\rightarrow$ $${}^7C_3 = \frac{7!}{3!(7-3)!} = \frac{7!}{(3!)(4)!}$$

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$$= \frac{7!}{(3!)(4)!} = \frac{7.6.5.4.3.2.1}{(3.2.1)(4.3.2.1)} = 35$$

$${}^nC_r = {}^nC_{(n-r)}, \quad {}^nC_0 = 1, \quad {}^nC_n = 1$$

★ With all these notations $\rightarrow$ n is a natural number & r is a whole number, with $n \geq r$

•• When one has mastered the language of Algebra $\rightarrow$ & has grasped the ideas & reasoning, does one 'appreciate the mathematical' symbolism.

★★ It is a relatively modern invention & mathematicians should be complimented for 'designing "symbols" & "notations" out of necessity.

★ It is important to realize that, while all the languages of the world are diff from one another, the language of Algebra is a common one!

Expressions & Identities in Algebra

06.09.25

∴ The basic function of Algebra is to convert expressions into more useful ones.

e.g. → $\sum_{k=1}^{n} k = \sum n = 1 + 2 + 3 + 4 + \ldots + n$

was converted by Gauss to the more useful form $(n(n+1)/2)$

Prove it? The method may not be obvious & yet a simple idea does the trick →

let $S = 1 + 2 + 3 + 4 + \ldots + (n-1) + n$ (3)

↓ Also, $S = n + (n-1) + (n-2) + \ldots + 2 + 1$ (4)

Adding corresponding terms → 3 & 4 → you get →

$2S = (n+1) + (n+1) + (n+1) \ldots$ (n times)

$= n(n+1)$ → Hence, $S = (n(n+1)/2)$

observe in this proof → it is important to realize that by simple means you have converted the cumbersome expression to a simpler & readily computable expression. 06.09.2025

→ using Algebra → many such expressions can be obtained

.. → $\sum n^2 = 1^2 + 2^2 + 3^2 + 4^2 + \dots + n^2$

$$= \frac{n(n+1)(2n+1)}{6}$$

• $\sum n^3 = 1^3 + 2^3 + 3^3 + 4^3 + \dots + n^3$

$$= \frac{n^2(n+1)^2}{4}$$

[note → $\sum n^3 = \left(\sum n\right)^2$]

• $a + ar + ar^2 + ar^3 + \dots + ar^{n-1}$

$$= \frac{a(1-r^n)}{(1-r)}, \quad (r < 1) \quad = \frac{a(r^n - 1)}{(r-1)}, \quad (r > 1)$$

Sometimes $\rightarrow$ possible that a question 06.09.
you may have has two answers $\rightarrow$ at 2025
which at first sight appear different
but which are actually the same.

$\hookrightarrow$ This can be checked by simplifying both expressions
- An important part of Algebra therefore consists in learning how to express any result in the simplest form.

Algebriac identities $\rightarrow$ Σ methods available for factorizing polynomials ~ are a helpful in simplifying expressions

- $(x+y)(x-y) = x^2 - y^2$

Thus $\rightarrow$ $(a+b)(a-b) = a^2 - b^2$

- $(x+y)^2 = (x+y)(x+y) = x^2 + xy + xy + y^2$

$= x^2 + 2xy + y^2$

$= x^2 + y^2 + 2xy$

So $\rightarrow$ $a^2 + b^2 = (a+b)^2 - ab$ ø6.ø9 2ø25

- $(x-y)^2 = x^2 + y^2 - 2xy$

Thus $\rightarrow$ $a^2 + b^2 = (a-b)^2 + 2ab$,

$(a+b)^2 + (a-b)^2 = 2(a^2+b^2)$,

&

$(a+b)^2 - (a-b)^2 = 4ab$

- $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$

Thus $\rightarrow$ $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$,

or $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

- $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

Thus $a^3 - b^3 = (a-b)^3 + 3ab(a-b)$,

or $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

many useful identities can be obtained (from these two expressions $\rightarrow$ $(a \pm b)^3$ & $(a \pm b)^2$

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$$\frac{a^3 + b^3}{a^2 + b^2 - ab} = (a + b)$$

$$\frac{a^3 - b^3}{a^2 + b^2 + ab} = (a - b)$$

$$\frac{(a+b)^2 + (a-b)^2}{(a^2 + b^2)} = \frac{2(a^2 + b^2)}{(a^2 + b^2)} = 2$$

$$\frac{(a+b)^2 - (a-b)^2}{ab} = \frac{4ab}{ab} = 4$$

observe →

$$\left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2$$

$$\left(a - \frac{1}{a}\right)^2 = a^2 + \frac{1}{2} - 2$$

$$\therefore \left(a + \frac{1}{2}\right)^2 - \left(a - \frac{1}{a}\right)^2$$

$$= 4$$

- $(a+b+c)^2$

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$= (a+b+c)(a+b+c)$

$= \underline{a}^2 + \underline{ab} + ac + \underline{ab} + b^2 + bc + ac + bc + c^2$

$a^2 + 2ab + \underline{ac} + b^2 + bc + \underline{ac} + bc + c^2$

$a^2 + 2ab + 2ac + \underline{b}^2 + \underline{bc} + \underline{bc} + c^2$

$a^2 + 2ab + 2ac + b^2 + 2bc + c^2$

$a^2 + b^2 + c^2 + 2ab + 2\cancel{ac}^{\,ca} + 2bc$

$a^2 + b^2 + c^2 + 2(ab + ac + bc)$

should be (ca)

- $a^3 + b^3 + c^3 - 3abc = a^3 + b^3 + c^3 - 3a - 3b - 3c$

~~$(a+b+c)(a^2+b^2+c^2-3a-3b-3c)$~~

$a+b+c\,(a^2+b^2+c^2)\left(-\dfrac{3abc}{a+b+c}\right)$?

$$(a+b+c)(a^2+b^2+c^2) - \frac{3(abc)}{a+b+c}$$

$0\,6.$ $0\,9.$ 25

$$(a+b+c)(a^2+b^2+c^2) - \frac{3a-3b-3c}{(a+b+c)}$$

$$-3a-3b-3c \cdot (a+b+c)$$

$$-3a^2-3ab-3ac-3ab-3b^2-3bc-3ac-3bc-3c^2$$

$$\frac{3(abc)}{a+b+c}$$

Wrong!

$$a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$$

• If $a+b+c=0$ (Review your Algebra)

then $\boxed{a^3+b^3+c^3=3abc}$

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so if $a^3 + b^3 + c^3 = 3abc$

then

$$\cancel{(a+b+c)\left(a^2+b^2+c^2-\left(a^3+b^3+c^3\right)\right)}$$

$$\cancel{(a+b+c)\left(a^2+b^2+c^2-a^3-b^3-c^3\right)}$$

- $$\frac{1}{a\cdot b} = \frac{1}{b-a}\left[\frac{1}{a} - \frac{1}{b}\right]$$

- If n is a natural number, then the expansion

$$(x+y)^n = \underbrace{{}^nC_0\, x^n + {}^nC_1\, x^{n-1}\cdot y + {}^nC_2\, x^{n-2}\cdot y^2 + \dots + {}^nC_n\, y^n}_{\text{Binomial}}$$

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↑ Binomial expansion → where x & y can be any real numbers

→ This expansion has $(n+1)$ terms

→ The general term is of the form ${}^nC_r x^{n-r} y^r$ & it is the $(r+1)$th term in the expansion

→ In each term → the sum of the indices of x & y is n

• If m is a negative integer or a rational number, then the binomial expansion is

$$(b+x)^m = b^m + mb^{m-1}x\ \frac{m(m-1)}{2!} b^{m-2}x^2 + \dots$$

$$+ \frac{m(m-1)(m-2)\dots(m-r+1)}{r!} b^{m-r} x^r \dots$$

"provided $|x| < b$

Remark 1. 06.09.2025

Note that the coefficients $m, \frac{m(m-1)}{2!}$ so on $\rightarrow$ look like combinatorial coefficients $\Rightarrow$ ${}^nC_0, {}^nC_1, {}^nC_2, \ldots, {}^nC_r \rightarrow$ so on

However $\Rightarrow$ recall that ${}^nC_r$ is defined for natural number n & whole number r (with $n \geq r$) & such has no meaning in other cases

Remark 2.

When m is a negative integer or a rational number, there are infinite number of terms in the expansion of $(b+x)^m$

Remark 3. The following results are very useful & can be easily obtained by using the expansion

- $\dfrac{1}{1+x} = (1+x)^{-1} = 1 - x + x^2 - x^3 + \ldots ; |x| < 1$

- $\dfrac{1}{(1+x)^2} = (1+x)^{-2}$

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$= 1 - 2x + 3x^2 - 4x^3 + \ldots ; |x| < 1$

- $\dfrac{1}{1-x} = (1-x)^{-1} = 1 + x + x^2 + x^3 + \ldots ; |x| < 1$

- $\dfrac{1}{(1-x)^2} = (1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \ldots ; |x| < 1$

Operations Involving Negative Numbers

A good deal of machinery of Elementary algebra is concerned with the solution of equations involving unknowns. The "simple machinary" can lead directly to useful results in numerous other types of problems.

★★ The Most difficult item in Algebra is that devoted to operations involving negative numbers → The difficulty is two fold

i) Why introduce negative?

ii) Why does multiplication of two negative numbers (or divisions of a negative number by another negative number) yield a positive number?

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Lets see, shall we?!

This is important → In fact, it is in connection with the solution of equations, that both questions can be answered

note → that if you do not accept negative numbers then even a simple equation → like $2x + 5 = 0$ cannot be solved
~ what about $\sqrt{-1}$ now consider →

$$7x - 5 = 10x - 11$$

To solve → you can transpose the terms in two ways so, that the unknowns are on one side & knows on the other side → but then →

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$$11 - 5 = 10x - 7x$$

or $6 = 3x$ so $x = 3$

also you get

$$7x - 10x = -11 + 5$$

$$-3x = -6$$

$$x = \frac{-6}{-3} = \frac{(-1)\,6}{(-1)\,3} = \frac{(-1)}{(-1)}\,\frac{6}{3} = \frac{(-1)}{(-1)} \cdot 2$$

also $\Rightarrow$ $\frac{-6}{-3} =$

$$(-1)\cdot 6 \cdot \frac{(-1)}{3} \left[\because \frac{1}{-3} = \frac{1}{(-1)(3)} = (-1)\,\frac{1}{3} = \frac{(-1)}{3} \right]$$

$$= (-1)\cdot(-1)\cdot\frac{6}{3} = (-1)\cdot(-1)\cdot 2$$

Now in order that the solution of the Equation $(7x - 5) = 10x - 11$ should be the same

it is necessary $\frac{(-1)}{(-1)} = 1$

Division By Zero

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So, why can't you divide by zero?

The answer lies in Algebra!

While in Arithmetic (or in generally in Algebra) the operation of division is defined in terms of the operation of multiplication.

This → according to the existing rule, the division of an arbitrary number "a" by another number "b" means that to find a number x such that

$$a \cdot \frac{1}{b} = x \quad \text{where} \quad b \neq 0$$

$$b \cdot x = a$$

or → You should see what happens if division by Zero is permitted. If $b = 0$ then you must consider the following 2 cases →

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(i) when $a \neq \emptyset$, ε

(ii) when $a = \emptyset$

Case (i) $\Rightarrow$ when you try to solve the equation you get $b \cdot x = a$, (where $b = \emptyset$, but $a \neq \emptyset$)

you get $\rightarrow$ $\emptyset \cdot x = a$

It follows that $a = \emptyset$, which against our assumption that $\rightarrow a \neq \emptyset$. This situation arises because there is no number x, which could be multiplied by "$\emptyset$" to get a fixed (none-zero) number "a". It follows that if a nonzero number is divided by zero than you get a meaningless result!

Case (ii) You try to solve the equation

you get $\rightarrow$ $b \cdot x = a$ (where $b = \emptyset$

$\emptyset \cdot x = \emptyset$ and $a = \emptyset$)

↳ then you get $\emptyset \cdot x = \emptyset$

$\emptyset 6.\emptyset 9$
$2\emptyset 25$

Unfortunately, this is true. Here any number x satifies this equation. See this consequence of this situation.

If division by zero is permitted, then you get from the equation $\emptyset \cdot x = \emptyset$, $x = \frac{\emptyset}{\emptyset}$

same → from $\emptyset \cdot y = \emptyset$, you get $y = \frac{\emptyset}{\emptyset}$

, where x, y, are all different (nonzero) numbers it follows that $\frac{\emptyset}{\emptyset} = x = y = z$ which means that all different numbers are equal!

Thus, if $a = \emptyset$ & $b = \emptyset$, then you have $\frac{a}{b} = \frac{\emptyset}{\emptyset}$ & it represents any number whatever you choose!

But mathematicians require that division of "a" by "b" should yield a unique as a result. But this again not achieved

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So → You observe that division by zero leads to meaningless results & hence is not permitted in math.