A Bayesian approach to phase angles

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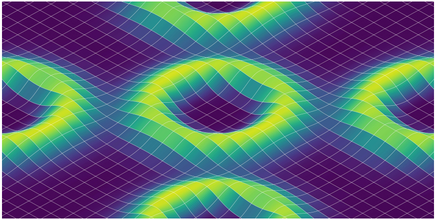
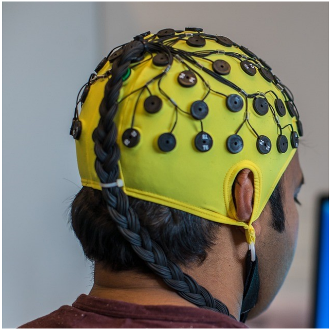
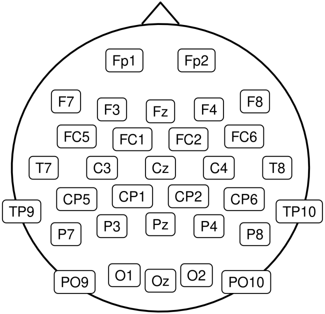


Data and experiment

EEG experiment





EEG traces

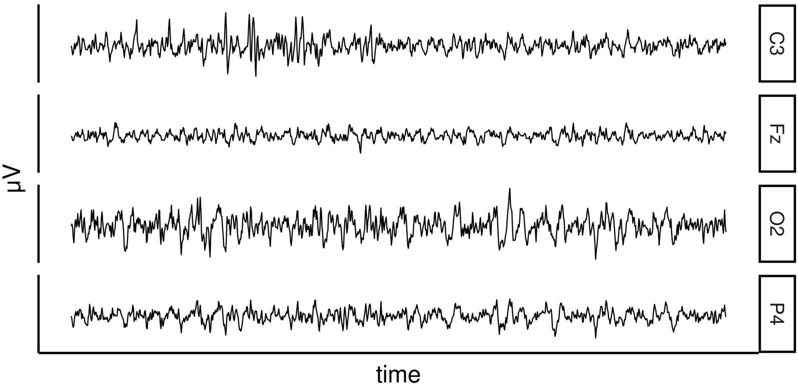


Figure 1: One second of EEG signal

Data from [Burroughs et al., 2021].

Phrases, sentences and syllables

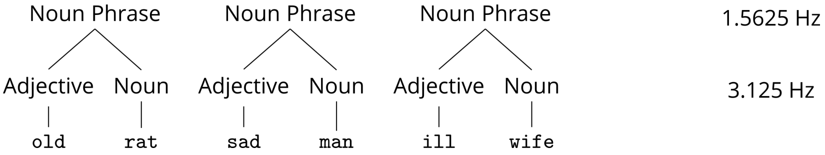


Figure 2: Frequency-tagged phrases and syllables

The signal is in the 'tagged' frequency in the response!

Phase coherence graphically

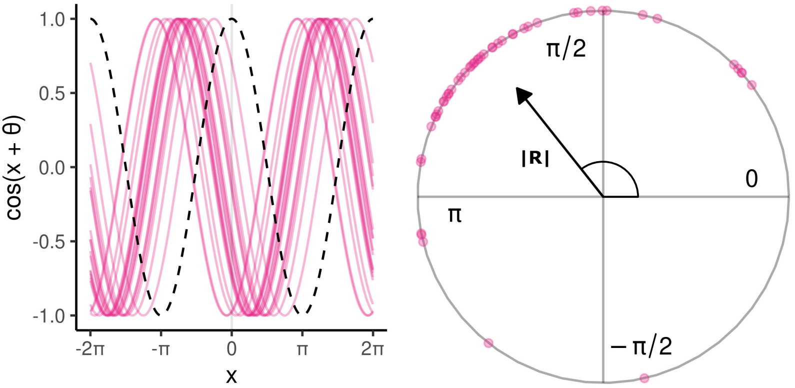


Figure 3: Phase coherence.

$$R(f, \phi) = \frac{1}{K} \sum_k e^{i\theta_{fk\phi}} \quad (1)$$

ITPC headcaps

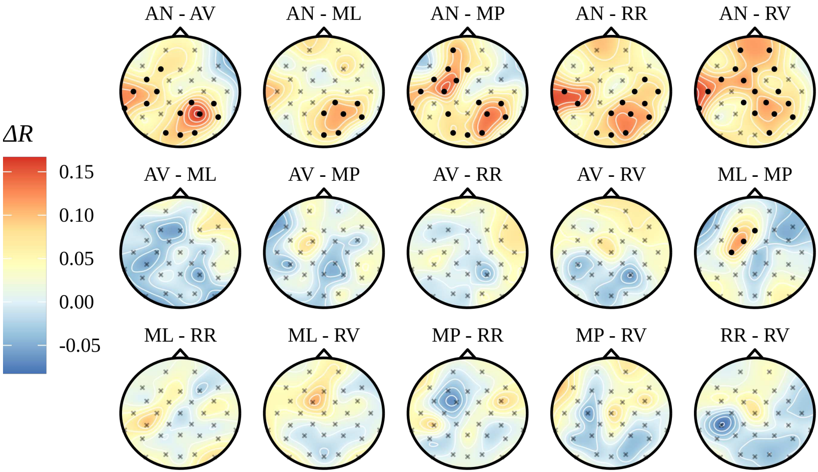


Figure 4: ITPC differences by electrodes using Cluster-based permutation[1].

[1] [Maris and Oostenveld, 2007]

ITPC headcaps

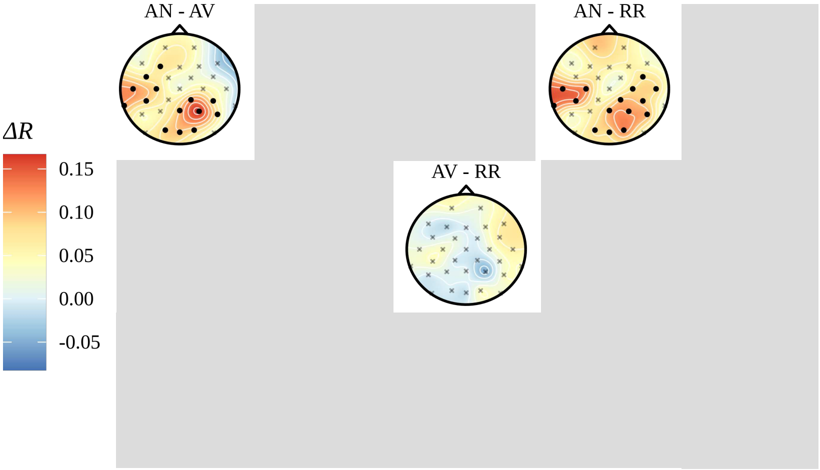


Figure 5: Apparent structure arising between ungrammatical conditions.

Inter-trial phase coherence

The ITPC hides the individual items inside a two stage analysis:

$$\text{items} \to \text{ITPC} \to \text{statistical analysis} \tag{2}$$

This means that the item in the statistical analysis is not a trial.

Solution: construct a Bayesian model that works directly on the phase angles.

Modelling phase angles

Wrapped Cauchy distribution

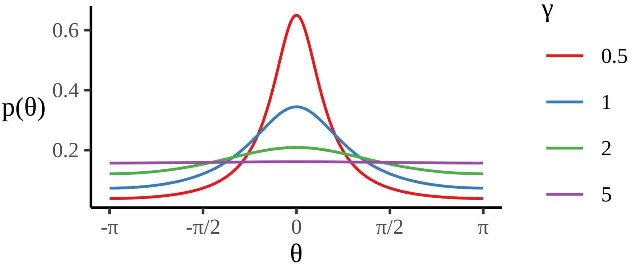


Figure 6: Wrapped Cauchy density.

$$p(\theta|\mu,\gamma) = \frac{1}{2\pi}\frac{\sinh(\gamma)}{\cosh(\gamma) - \cos(\theta-\mu)} \tag{3}$$

A prior for the mean phase

$$p(\mu) \sim \text{Uniform}(-\pi, \pi), \quad p(r) \sim ? \tag{4}$$

Mean resultant in Stan

```
parameters{
    ...
    vector[2] xy_unconstr[N];
}
transformed parameters{
    ...
    vector[N] mu;
    mu = atan2(xy_unconstr[:,2], xy_unconstr[:,1]);
}
```

A prior for the radius

$$p(\mu) \sim \text{Uniform}(-\pi, \pi), \quad p(r) \sim \text{Gamma}(10, 10) \tag{5}$$

Radius in Stan

```
transformed parameters{
    vector[N] vector_length;
    ...
    vector_length[i] = sqrt(dot_self(xy_unconstr[i]))
}
model{
    vector_length ~ gamma(10,10);
    target += -log(vector_len) // Jacobian adjustment
}
```

Prior geometry

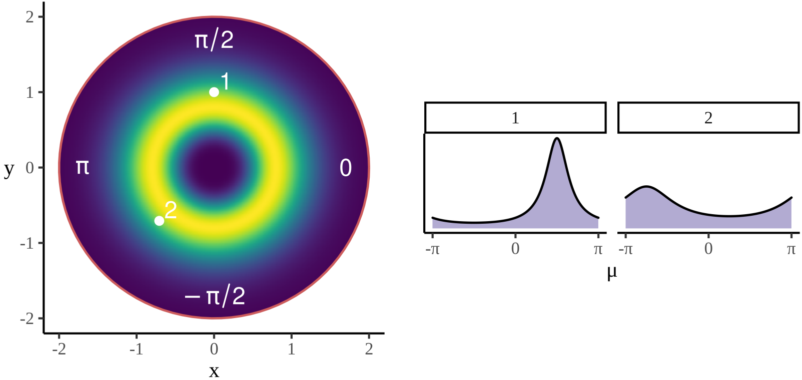


Figure 7: Prior geometry (target).

Prior considerations

Similar ideas have been proposed on the Stan forums, motivated by problems with the built in *`unit_vector`*[2,3] type.

We can verify similar benefits with this alternative prior:

- higher effective sample size
- fewer problems with divergences
- smaller tree-depth resulting in much faster sampling

Questions:

- distributional choices for the radius
- generalisations to higher dimensions

[2] *https://discourse.mc-stan.org/t/divergence-treedepth-issues-with-unit-vector/8059*
[3] *https://discourse.mc-stan.org/t/a-better-unit-vector/26989/17*

Target density

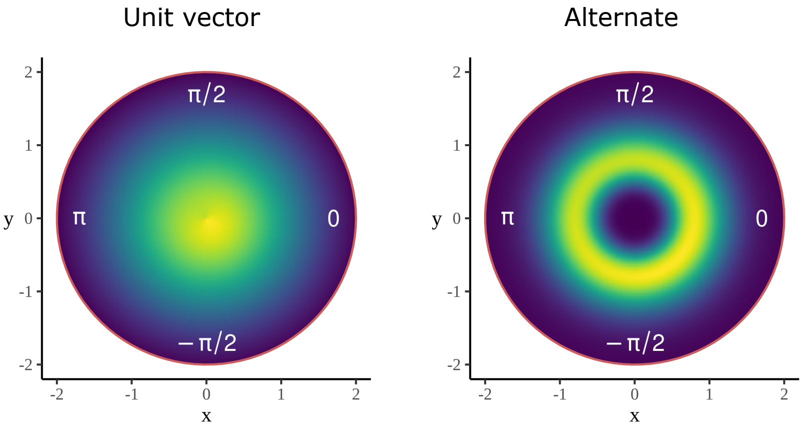


Figure 8: Target density when the likelihood has a high circular variance.

Posterior samples

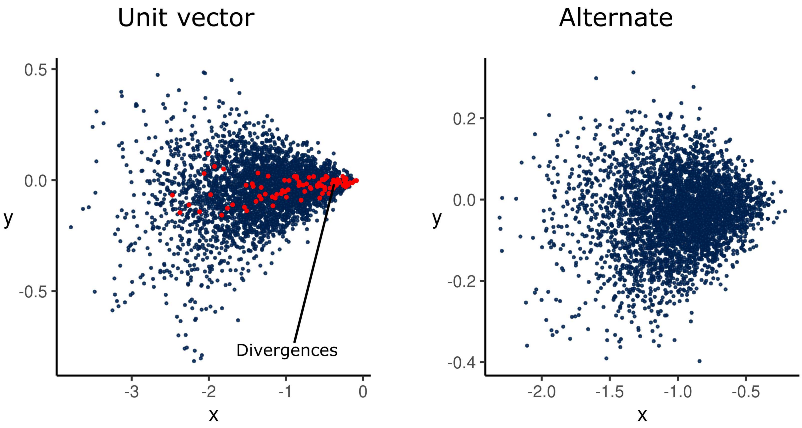


Figure 9: Posterior samples when the likelihood has a low circular variance.

Circular variance

The variance (circular) of the wrapped Cauchy distribution is:

$$S = 1 - e^{-\gamma} \tag{6}$$

so it's scale γ gamma is a function of the circular variance

$$\gamma = -\log 1 - S \tag{7}$$

Linear model

S is related to other parameters such a condition and participant number through a logistic regression:

$$\text{logit}(S_{pce}) = \alpha_c + \beta_{pc} + \delta_{ce} \tag{8}$$

- α fixed effects of condition
- β random effects for participants
- δ random effects for electrodes

Results

Bayes headcaps

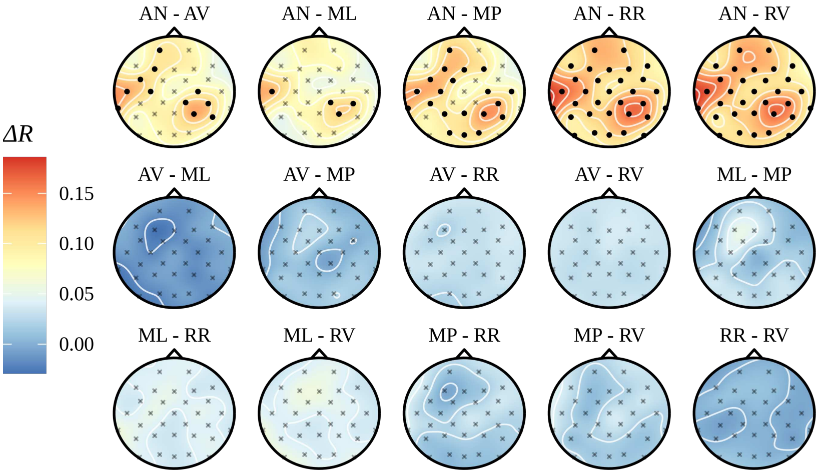


Figure 10: Posterior mean with highest density intervals.

ITPC headcaps (again)

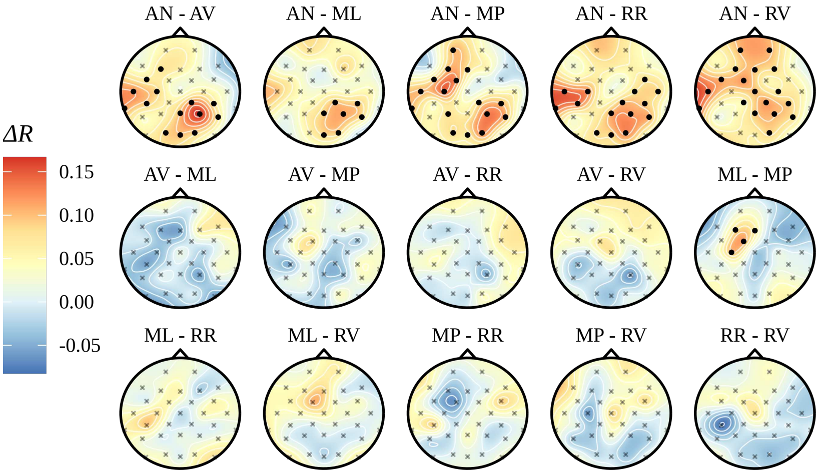


Figure 11: ITPC differences by electrodes using Cluster-based permutation[4]

[4] [Maris and Oostenveld, 2007]

Random activity (simulation)

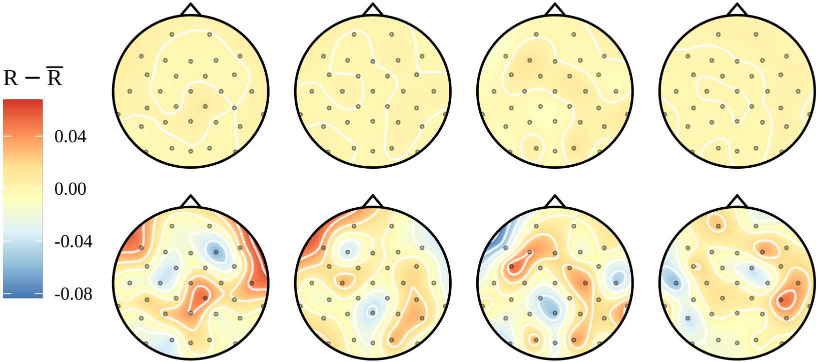


Figure 12: Bayesian estimates (top) and ITPC estimates (bottom)

Data efficiency

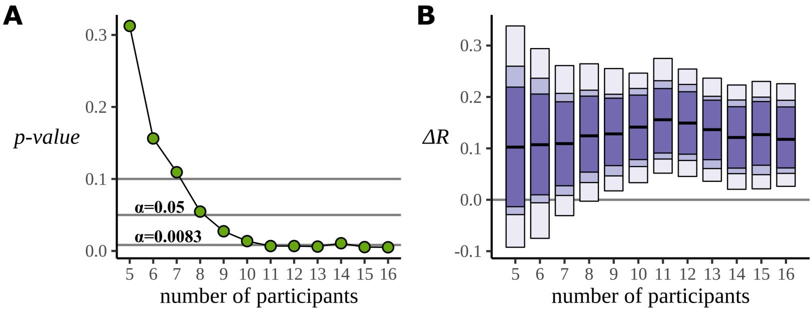


Figure 13: The Bayesian model detects the signal in fewer participants

Conclusions

Benefits of the Bayesian model:

- more efficient use of the data
- descriptive at all levels of the data
- cleaner result
- inference is not independent and information is shared

Preprint:

"Bayesian analysis of phase data in EEG and MEG"
https://psyarxiv.com/2vcsy/

References i

Burroughs, A., Kazanina, N., and Houghton, C. (2021).
Grammatical category and the neural processing of phrases.
Scientific Reports, 11(1):1–10.

Maris, E. and Oostenveld, R. (2007).
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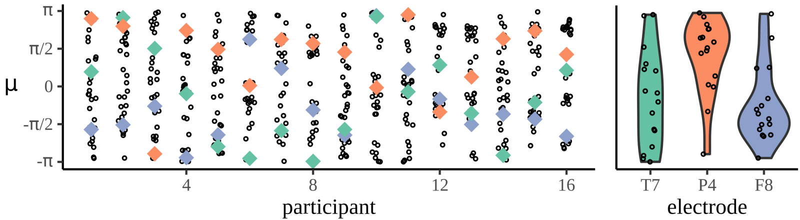

Figure 14: Mean phase angles are uniformly distributed.