

Exercice 3

$\exists x \in \mathbb{R}^3$ tel que $\lambda \neq 0$ et $Ax = x \Leftrightarrow$ l'up de A .

$$x_A = x_{A^{-1}} \cdot \alpha_{A^{-1}}$$

$\text{et } x_A^{(\lambda)} = x_A \cdot \frac{(\lambda)-1}{\lambda} \lambda^3 \det(A)$

$$= -x_{A^{-1}} \left(\frac{1}{\lambda} \right) \lambda^3 \det(A)$$

$$\Rightarrow \det(A) = \det(A^{-1})$$

$$\ln(A) = \ln(A^{-1})$$

$$\lambda^3 - \ln(A)x^2 + bx - \det(A) = \left(\frac{1}{\lambda} \right)^3 \ln(A^{-1}) + \frac{b}{\lambda} - \det(A^{-1})$$

$$\lambda^3 - \ln(A)\lambda^2 + bx - \det(A) = -\det(A) + \ln(A^{-1})\det(A)x - b\lambda^2\det(A) + \lambda^3$$

$$\text{Ensuite, } b = \ln(A^{-1})\det(A) - \ln(A)\det(A)$$

$$\det(A) = \det(A^{-1}) = \frac{1}{\det(A)} \Rightarrow \det(A) = 1 \Rightarrow \boxed{\det(A) = 1}$$

$$\text{et donc } x_A = \lambda^3 - \ln(A)\lambda^2 + \ln(A) - 1$$

$$x_A(1) = 1^3 - \ln(A) + \ln(A) - 1 = 0$$

l'up de A