

Exercice 3

$\lambda \in \mathbb{R}^3$ tel que $\lambda \neq 0$ et $A\lambda = \lambda$ (λ est un v.p. de A).

$$\lambda_A = \lambda_{A^T} = \lambda_{A^{-1}} \quad \text{et} \quad \lambda_A = \lambda_{A^{-1}} \left(\frac{1}{\lambda} \right)^3 \lambda^3 \det(A) \\ = -\lambda_{A^{-1}} \left(\frac{1}{\lambda} \right) \lambda^3 \det(A)$$

$$\Rightarrow \det(A) = \det(A^{-1})$$

$$\ln(A) = \ln(A^{-1})$$

$$\lambda^3 - \ln(A)\lambda^2 + b\lambda - \det(A) = \left(\left(\frac{1}{\lambda} \right)^3 - \ln(A^{-1}) + \frac{b}{\lambda} - \det(A^{-1}) \right) \lambda^3 \det(A)$$

$$\lambda^3 - \ln(A)\lambda^2 + b\lambda - \det(A) = -\det(A) + \ln(A^{-1})\det(A)\lambda - b\lambda^2\det(A) + \lambda^3$$

En identifiant, $b = \ln(A^{-1})\det(A) - \ln(A)\det(A)$

$$\det(A) = \det(A^{-1}) = \frac{1}{\det(A)} \Rightarrow \det(A) = 1 \Rightarrow \det(A) = 1$$

$\det(A) \geq 0$

et donc $\lambda_A = \lambda^3 - \ln(A)\lambda^2 + \ln(A) - 1$

$$\lambda_A(1) = 1^3 - \ln(A) + \ln(A) - 1 = 0$$

1 v.p. de A