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SPECULATIVE BUBBLES, CRASHES AND RATIONAL EXPECTATIONS

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Self ending speculative bubbles, i.e., speculative bubbles followed by market crashes, are consistent with the assumption of rational expectations. More generally, speculative bubbles may take all kinds of shapes. Detecting their presence or rejecting their existence is likely to prove very hard.

This note makes the following two points: Self ending speculative bubbles in asset markets, i.e., speculative bubbles which end in market crashes, are consistent with the assumption of rational expectations.

Speculative bubbles may take all kinds of shapes. Detecting their presence or rejecting their existence is likely to prove very hard. [1]

(A) These points are made most easily by using the standard asset market model: [2]

$$x_t = aE(x_{t+1} | \Omega_t) + Z_t , \qquad a \in (0, 1), \quad t = 0, ..., \infty. \tag{1}$$

Various interpretations are possible: This may be the reduced form of a money market equilibrium. x_t stands for the logarithm of the price level and Z_t for (a scalar times) the logarithm of nominal money. [See Flood and Garber (1980) among others.]

This may be an arbitrage equation giving the value of a share: x_t stands for the price of a share, Z_t for (a scalar times) the dividend.

This may finally be the equilibrium condition of a materials market, of the gold

[1] This note derives from reading Flood and Garber (1980) who introduce the concepts and terminology used here. They statistically cannot reject the hypothesis of no speculative bubbles during the German hyperinflation. This note in effect argues that they only consider one particular form of speculative bubbles.

[2] It is assumed that this equation entirely characterizes the market equilibrium, that there are no transversality or boundary conditions in addition to (1).

market for example. x_t stands for the price and Z_t for (a scalar times) the existing stock.

In all cases, Ω_t denotes the information set available at time t.

Following Flood and Garber, define $\{Z_t\}_{t=0,\dots,\infty}$ as the 'market fundamentals'. In order to concentrate on speculative bubbles, I assume that market fundamentals are constant,

$$Z_t = \bar{Z} \quad \forall t, \qquad E(Z_{t+i}|\Omega_t) = \bar{Z} \quad \forall t, i \geqslant 0,$$

so that

$$x_t = aE(x_{t+1}|\Omega_t) + \bar{Z}. \tag{2}$$

It is clear that (2) admits $x_t = \bar{x} \equiv \bar{Z}/(1-a)$ as a solution. It is also clear that for any other solution

$$\lim_{i\to\infty} E(x_{t+i}|\Omega_t) - \bar{x} = \lim_{i\to\infty} a^{-i}(x_t - \bar{x}) = \infty.$$

Thus any other solution is such that the expectation of x_t explodes; if this possibility is a priori excluded, we are left with a unique solution. Some of the other solutions seem however to capture what is usually meant by a speculative bubble:

(B) Consider the following description of the current gold market: 'As the price [of gold] enters the stratosphere, the risks become extraordinary. If you look down the edge from here, it's a long way down' (Clayton Yeutter, President of the Chicago Mercantile Exchange, *Time,* October 1, 1979).

Consider now the following class of solutions to (2): $x_0 = a_0$ and the behavior of x_t, $t > 0$, is described by the transition matrix

$$\begin{array}{l|cc|} & x_t - \bar{x} \neq 0 & x_t - \bar{x} = 0 \\ \hline x_{t+1} - \bar{x} = (a\pi)^{-1}(x_t - \bar{x}) & \pi & 0 \\ x_{t+1} - \bar{x} = 0 & 1-\pi & 1 \\ \hline \end{array} \tag{3}$$

π is the probability that x_{t+1} will be different from $\bar{x}$ if x_t is different from $\bar{x}$. Thus, in each period, there is a probability π that the bubble remains and a probability $(1-\pi)$ that the bubble ends and the market crashes, returning to market fundamentals. Although we still have

$$\lim_{i\to\infty} E(x_{t+i}|\Omega_t) = \infty \quad \text{if} \quad x_t \neq \bar{x},$$

the bubble will end with probability one; its expected duration, unconditional or conditional is $(1-\pi)^{-1}$. If $\pi = 1$, then $x_t = \bar{x} + a_0 a^{-t}$. This is the type of bubble considered by Flood and Garber. For this bubble to be consistent with rational expectations, it must however never end: if it ends at time T, then the 'true' π cannot be one at time T. If $\pi < 1$, then as long as the bubble remains, $x_t = a_0(a\pi)^{-t}$. During the duration of the bubble, x_t is growing faster than in the case above because asset holders have to be compensated for the probability of a 'crash'.

There is clearly no reason to limit ourselves to this class of solutions. The following extensions also appear to capture certain aspects of speculative bubbles: π may be time or state dependent. The probability that the bubble ends may well be thought by market participants to be a function of how long the bubble has existed. If π increases for some period of time, x_t will be growing at a decreasing exponential rate; if π decreases, then the higher probability of a crash leads to an acceleration of x_t while the bubble lasts.

There may be more than two states at each time t; in particular rather than 'crashing', x_t may decrease to a level higher than $\bar{x}$; this seems to capture what is meant by 'consolidations' in speculative markets for example.

It should be clear by now that eq. (2) is consistent with an infinity of self ending bubbles.

(C) By relating the behavior of asset markets as described by (1) to the real sector, it is easy to construct models in which speculative bubbles may have real effects. Consider the following (admittedly ad hoc) 'animal spirit-rational expectation' model:

$$q_t = aE(q_{t+1} | \Omega_t) + \pi_t \,,$$

$$\pi_t = b_0 + b_1 y_t \,, \qquad y_t = c_0 + c_1 q_t \,,$$

where q_t is the stock market, π_t profit, y_t output. The first equation is the same as (1); the second relates profit to output; the third relates output to aggregate demand and aggregate demand to wealth. (The price level and interest rates are implicitly assumed constant.) This gives

$$q_t = \alpha_0 E(q_{t+1} | \Omega_t) + \alpha_1 \,,$$

where

$$\alpha_0 = a(1 - b_1 c_1)^{-1} \,, \qquad \alpha_1 = (b_0 + b_1 c_0)(1 - b_1 c_1)^{-1} \,.$$

This is of the same form as (2). Thus if the solution is of form (3), this will generate a boom in the stock market, output and profit, followed by a market crash, a fall in output and profit. (There is no claim that the 1929 stock market crash and subsequent depression can be explained by the above model.)

Reference

Flood, R. and P. Garber, 1980, Market fundamentals versus price-level bubbles: The first tests, Journal of Political Economy, forthcoming.