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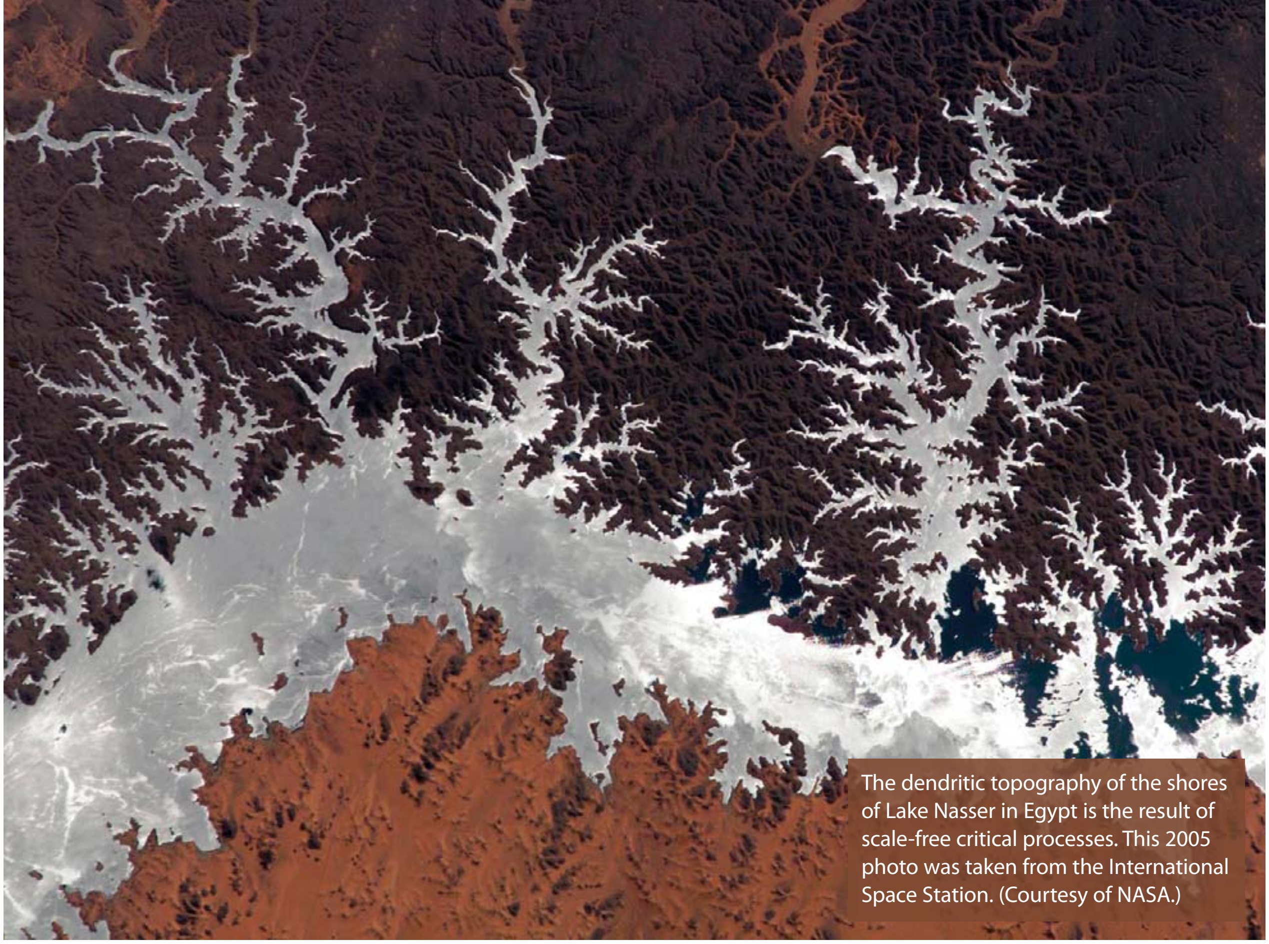

The dendritic topography of the shores of Lake Nasser in Egypt is the result of scale-free critical processes. This 2005 photo was taken from the International Space Station. (Courtesy of NASA.)

Theory of dynamic critical phenomena

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New mathematical approaches have extended physicists' understanding of magnets, superfluids, and other complex systems.

When a system is brought to a critical phase transition, such as the gas–liquid critical point where the density difference between liquid and gas disappears, or the Curie point of a ferromagnet where the spontaneous magnetization disappears, many of its properties exhibit singular behavior. Beginning with Johannes van der Waals's work in the 19th century,[1] analyses of critical phenomena have largely focused on static properties, such as free energies, equilibrium expectation values and linear responses to time-independent perturbations. In classical statistical mechanics, static properties are determined by the equal-time correlation functions. However, critical singularities also occur in dynamic properties, such as multi-time correlation functions, responses to time-dependent perturbations, and transport coefficients. Those properties cannot be derived from the equilibrium distribution. A different approach is needed.

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In the 1960s and early 1970s, major advances occurred in the theory of critical phenomena. Important ideas that emerged included the introduction of critical exponents to describe how various static quantities diverge or go to zero as one approaches a critical point and the introduction of scaling laws, which lead to relations among the various exponents.[2] Renormalization-group methods gave a means for understanding scaling laws and gave methods for calculating critical exponents, at least approximately.[3] For that achievement, Kenneth Wilson was awarded the 1982 Nobel Prize in Physics. Importantly, those ideas led to an understanding that the static critical behavior of various systems could be divided into what were termed universality classes. The classes are sensitive to such features as the symmetry of the order parameter or the spatial dimension of the system, but they are independent of other microscopic details of the Hamiltonian, within a broad range.

As progress was made in the theory of static critical phenomena, physicists realized that ideas of scaling and universality classes, as well as renormalization group methods, could also be applied to dynamic properties.[4–6] The review article "Theory of dynamic critical phenomena," published in 1977 in *Reviews of Modern Physics*, provided a summary of the status of those theories[7] and promoted a classification scheme that remains in use today.

Two systems belonging to the same static universality class may belong to different classes of dynamic phenomena. That important distinction is true even away from a critical point. For example, both the classical Heisenberg ferromagnet and the antiferromagnet on a simple cubic lattice have essentially identical thermodynamic properties: One can map the antiferromagnet onto the ferromagnet simply by changing the signs of the spin vectors on one of the sublattices. However, the antiferromagnet has a dynamic property, a spectrum of spin waves, whose frequency is linear in the wavevector at long wavelengths, whereas the ferromagnet's spectrum is quadratic.

In general, the low-frequency dynamic properties of a system not at a critical point can be characterized by a hydrodynamic theory. Such a theory describes fluctuations of the conserved quantities and any additional slow variables that may occur when the equilibrium state has a spontaneously broken symmetry. The form of the theory depends sensitively on symmetry and on the Poisson brackets, or quantum mechanical commutation relations, among the slow variables. The universality classes for dynamic properties near a critical point depend on those features and on the parameters that affect the static critical properties, such as the spatial dimension.

An important quantity characterizing any dynamic universality class is the dynamic critical exponent z. It is defined so that at the critical point, the characteristic frequency for fluctuations of the order parameter at wavevector k is proportional to k^z, for small k. In some cases, the dynamic exponent z can be directly related to the static exponents.

For example, for the Heisenberg ferromagnet in three dimensions, theory[5] predicts $z = (5 - \eta)/2$, where η is a static critical exponent whose value is about 0.035. For the antiferromagnet, one has simply $z = 3/2$. By contrast, in the model of an Ising-like ferromagnet that interacts with an external heat bath, one finds that $z = 2 + x$, where x cannot be related to static exponents. It can be shown that $x \geq 0$, but its value for three dimensions ($d = 3$) is unknown. What is known is that a renormalization group calculation[6,8] near the case of four dimensions finds a small nonzero x, given to lowest order in an expansion in $4 - d$ by $x = 0.0134(4 - d)^2$.

Experimentally, the most accurate studies of critical behavior have been made at the superfluid–normal transition of liquid helium-4. Scaling theory[4,9] predicts that the thermal conductivity λ should diverge here as $\lambda \sim (\xi C_p)^{1/2}$, where $\xi \sim 1/(\delta T)^{0.67}$ is the correlation length of the order parameter at a temperature difference δT above the critical point, and C_p is the specific heat at constant pressure, which has a sharp cusp maximum at the transition point. Experiments agree well with the prediction over four decades of δT.

In recent years, interest has shifted to critical behavior at, or near, a zero-temperature phase transition, where quantum effects play a decisive role.[10] There, static and dynamic quantities are intimately mixed, and many new phenomena are encountered. Nevertheless, ideas such as dynamic scaling, universality classes, and the dynamic exponent z continue to figure prominently in the quantum regime.

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