Singular Value Decomposition

Let A be an $m \times n$ matrix. Then there exist orthogonal matrices $U_{m\times n}$ and $V_{n\times n}$ such that

$$A = U\Sigma V^T$$

where

$$\Sigma = \begin{bmatrix} \sigma_1 & 0 & \dots & 0 & 0 & 0 \\ 0 & \sigma_2 & \dots & 0 & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & \dots & \sigma_r & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where

$$\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_r > 0$$

This implies that you can always write a matrix as

$$A = \sigma_1 u_1 v_1^T + \sigma_2 u_2 v_2^T + ... + \sigma_r u_r v_r^T$$

Reverse engineering of genetic networks

Here we assume that we are at or near steady state, which implies that the dynamics are described by a system of linear ordinary differential equations.

$$x_t'(t) = -\lambda x_i(t) + \sum_{j=1}^{N} w_{ij} x_j(t) + b_i(t) + \zeta_i(t)$$

where

$$1 \leq i \leq N$$