1. What is the formula for the height of a bar in a relative frequency histogram?

Solution: Suppose the left and right end points of the bar are L and R and $X_1, X_2, \ldots, X_n$ are the data points, then the height of the bar is

$$H = \frac{\#\{L < X_i \leq R\}}{n \times (R - L)}$$

2. What is the area of one bar in a relative frequency histogram?

Solution: It represents the proportion of a sample of data that fall between the lower and upper limits defined by that bar.

3. What is the sample mean of the following data $X_1 = 2, X_2 = 5, X_3 = 7, X_4 = 10$?

Solution: 6

4. What is the formula for computing the sample variance?

Solution:

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (y_i - \bar{y})^2$$

5. What is the difference between the population mean and the sample mean?

Solution: The sample mean is a statistic (function) computed from a sample of data collected from a population. Because the sample is random, the *sample mean* is a random variable. The population mean is a constant, usually unknowable because we can almost never calculate an average over the *entire* population.

6. What proportion of data are *generally* expected to fall in the range $(\mu - 3\sigma, \mu + 3\sigma)$?

Solution: 100% by the empirical rule.

7. When is the empirical rule a good approximation of reality?

Solution: When the data are sampled from some kind of normal distribution.

8. How would you use `R` to calculate the standard deviation of data collected in `R` vector `d`?

Solution: `sd(d)`

9. You have six tickets labeled 1 through 6, what `R` command would give you the realization of a random experiment to sample 3 tickets *without* replacement?

Solution: `sample(x=1:6, size=3)`

10. What does it mean for two events to be mutually exclusive?

Solution: A and B are mutually exclusive means $A \cap B = \varnothing$.

11. Name two subsets of the positive integers that are mutually exclusive.

Solution: $A = \{1\}$ and $B = \{2\}$. Perhaps less trivial is the sets of even and odd integers.

12. Use DeMorgan's law to obtain an alternative expression for the complement of $P = \{\text{prime numbers}\}$ and $O = \{\text{odd numbers}\}$, i.e. $\overline{P \cap O}$.

Solution: Not {prime numbers} or not {odd numbers}, i.e. $\overline{P} \cup \overline{O}$.

13. Identify one simple event of the random experiment of spinning a dial with four equal sized outcomes colored red, blue, green, and yellow.

Solution: Yellow.

14. Name one of the axioms of probability.

Solution:

1. $P(E) \geq 0$ for any event E.
2. $P(S) = 1$
3. $E_1, E_2, \ldots$ are mutually exclusive events, then

$$P(E_1 \cup E_2 \cup \cdots) = \sum_{i=1}^{\infty} P(E_i)$$

15. You are trying to choose a new color for your apartment walls. If you have to choose from 10 colors and you ask 5 friends for their recommendation, what is the chance that 2 or more people will agree on a color if they have random color preferences?

Solution: We need to find the probability that all 10 friends choose different colors, which is

$$\frac{P_5^{10}}{10^5} = 0.3024$$

16. To promote a new estimation method you have developed, you need to demonstrate it on a sample dataset. How many choices do you have if there are datasets from 6 different disciplines and each dataset may consist of 100, 200, or 300 samples?

Solution: Application of mn-rule: $6 \times 3 = 18$ choices.

17. You have to choose a color, texture, and finish for a remodeling project. If there are 10 colors, 4 textures, and 3 finishes *and* there is a distinct effect if you add the color or the texture first, how many different outcomes are possible?

Solution: $2! \times 10 \times 4 \times 3 = 240$ You need to multiply the mn-rule result by 2! for the possible orderings of the color and texture.

18. There are 3 top-scorers in every class. There are 24 students in this class. How many different sets of first, second, and third top-scorers can the class produce?

> **Solution:** $P_3^{24} = 24 \times 23 \times 22 = 12,144$

19. There are 5 grades, A, B, C, D and F possible. If I assign 5 A's, 9 B's, 9 C's, and 1 D, how many ways could the class be assigned grades?

> **Solution:**
> $$C_{5,9,9,1}^{24} = \frac{24!}{5!9!9!1!} = 39264345120$$

20. If there are 5 A's, 9 B's, 9 C's, and 1 D in the class, and I form a team of size 5, what is the probability all 5 A's end up on the team?

> **Solution:**
> $$\frac{\binom{5}{5}\binom{9}{0}\binom{9}{0}\binom{1}{0}}{\binom{24}{5}} = \frac{5 \times 4 \times 3 \times 2}{24 \times 23 \times 22 \times 21 \times 20} \approx 2.352720e - 05$$

21. What is the probability one person ends up with exactly 6 candies if I distribute 12 candies randomly to 24 people?

> **Solution:**
> $$\frac{\binom{12}{6}23^6}{24^{12}} \approx 3.745451e - 06$$

22. What is the probability a person has the flu and tests positive if 30% of all tests are come back positive and the probability a person has the flu given they test positive is 0.95?

> **Solution:**
> $$P(F \cap P) = P(F \mid P)P(P) = 0.95 \times 0.3 = 0.285$$

23. If the probability someone gets greater than 90% on midterm II is 0.17 and 0.04 on midterm III, what is the probability you get greater than 90% on both if midterm grades are independent?

> **Solution:**
> $$0.17 \times 0.04 = 0.0068$$

24. If there are 20 cards left in a deck (after dealing the first 32) and 5 are face cards, what is the probability that all 5 face cards end up in the same hand if 2 more hands of 10 cards are dealt?

> **Solution:** Use counting and mn-rule.
> $$\frac{\binom{2}{1}\binom{5}{5}\binom{15}{5}}{\binom{20}{10}} \approx 0.03250774$$

25. The probability of getting the flu during one season is 0.01. The probability of getting serious symptoms given you get the flu is 0.02. The probability of dying given you get serious symptoms is 0.1. What is the probability you die from flu in the year?

> **Solution:** Use multiplication rule.
> $$0.01 \times 0.02 \times 0.1 = 0.00002$$

26. There are three rooms, A, B, and C. You start in room A. The probability of moving to room B from A is 0.1. The probability of moving to room C from A is 0.2. The probability of moving outside from room A is 0.3. The probability of moving outside from room B is 0.4. What is the probability you move from room A to outside by passing through just two doors?

> **Solution:** Use the addition rule over all partitions of the two-door exit event.
> $$0.1 \times 0.3 + 0.2 \times 0.4 = 0.11$$

27. The probability you get influenza A in a season is 0.01. The probability you get influenza B is 0.02 in a season. The probability you get both in one season is 0.001 (poor you). What is the probability you get sick with flu during the season?

> **Solution:** Use addition rule
>
> $$P(A \cup B) = 0.01 + 0.02 - 0.001 = 0.029$$

28. As above, what is the probability get influenza A but not influenza B?

> **Solution:** $P(A \cap \overline{B}) = P(A) = P(A \cap B) = 0.01 - 0.001 = 0.009$

29. The probability a tree survives its first year and produces 0 applies is 0.5, the probability a tree survives its first year and produces 1 applie is 0.1. Similarly, the probability of survival and 2 applies is 0.05 and the probability of survival and 3 or more applies is 0.05. What is the probability the tree survives its first year?

> **Solution:** Use the law of total probability for the partition $0, 1, 2, \geq 3$ apples.
>
> $$0.5 + 0.1 + 0.05 + 0.05 = 0.7$$

30. Define a partition of the sample space of all 5 card hands from a 52 card deck.

> **Solution:** Many choices. You could partition hands into all hands with 1 ace and all others.

31. Suppose lane 1 takes 60% of cards coming eastbound through an intersection and all other eastbound cars use lane 2. If the probability of getting in an accident in the intersection is 0.01 for lane 1 and 0.001 for lane 2, then what is the probability of an accident occurring to an eastbound car while passing through the intersection.

> **Solution:** Use the law of total probability.
>
> $$0.01 \times 0.6 + 0.001 \times 0.4 = 0.0064$$

32. If 60% of candidates are female and 40% male and two candidates are chosen, the first one randomly, and the second one of the opposite sex with probability 70%, then was is the probability that the first one selected was male given that the second one was female.

> **Solution:**
>
> $$P(M_1 \mid F_2) = \frac{P(F_2 \mid M_1)P(M_1)}{P(F_2 \mid M_1)P(M_1) + P(F_2 \mid F_1)P(F_1)} = \frac{0.7 \times 0.4}{0.7 \times 0.4 + 0.3 \times 0.6} \approx 0.6086957$$

33. Which of the following is a random variable?

    1. I draw a random color and announce it.
    2. I draw a random number between 1 and 10 and announce it is even or odd.
    3. I draw a random widget and read off the serial number from the bottom.

> **Solution:** The third one is a random number because it is the only one that is a number!

34. What is the probability mass function for the number of typos on a random page if there are 120 pages total with 30 containing 1 typo, 10 containing 2 typos, and 1 containing 4?

> **Solution:**
>
> $$\begin{array}{cc} p(0) = \frac{120-41}{120} = \frac{79}{120} & p(1) = \frac{30}{120} = 0.25 \\ p(2) = \frac{10}{120} = \frac{1}{12} & p(3) = \frac{1}{120} \end{array}$$

35. What is

$$\sum_{i=1}^{8} \binom{8}{i} (0.5)^8$$

> **Solution:** Use sum of binomial probabilities over sample space.
>
> $$\sum_{i=1}^{8} \binom{8}{i} (0.5)^8 = 1 - (0.5)^8 \approx 0.9960938$$

36. Given Y has pmf

$$\begin{array}{ll} p(0) = \frac{1}{3} & p(1) = \frac{1}{4} \\ p(2) = \frac{1}{4} & p(3) = \frac{1}{6} \end{array}$$

what is $E[Y]$?

Solution: $E[Y] = \frac{1}{4} + \frac{2}{4} + \frac{4}{6} = \frac{9+8}{18} = \frac{17}{18}$

37. What is $V(Y)$ if the pmf is $p(1) = \frac{1}{4}, p(2) = \frac{1}{2}$, and $p(3) = \frac{1}{4}$?

Solution: It is easy to see that $E[Y] = 2$, so

$$V(Y) = \frac{1}{4} + \frac{4}{2} + \frac{9}{4} - 2^2 = \frac{18-16}{4} = \frac{1}{2}$$

38. If Y has pmf $p(1) = \frac{1}{4}, p(2) = \frac{1}{2}$, and $p(3) = \frac{1}{4}$ and

$$g(y) = \begin{cases} 1, & y \geq 3 \\ 0, & \text{otherwise} \end{cases}$$

then what is $E[g(Y)]$?

Solution: $E[g(Y)] = \frac{1}{4}$

39. If $Y \sim \text{Normal}\left(\frac{1}{4}, 6\right)$ and $X \sim \text{Beta}(3,1)$, then what is $E[2Y + X]$?

Solution: $2 \times \frac{1}{4} + \frac{3}{4} = \frac{5}{4}$

40. The weight W of a truck has pfm $f(w) = \frac{1}{26}$ for $w = \{15, 16, \ldots, 40\}$ tons. The number of trucks Y crossing a bridge has Poisson distribution with mean 50 per hour. What is the total expected weight passing over the bridge in 1 hour?

Solution:

$$E[W] = E[YW] = E[Y]E[W] = 50 \times \frac{15 + 16 + \cdots + 40}{26} = 1375 \text{ tons per hour}$$

41. If $X \sim \text{Gamma}(3,2)$ is the weight in tons of vehicles crossing a bridge and the stress on a support is given by $S = 4X^2 + 3$, what is the expected stress?

Solution: $E[S] = 4E[X^2] + 3 = 4 \times 3 \times 4 \times 2^2 + 3 = 195$

42. What is the pmf of a Bernoulli(p) random variable?

Solution: $p(y) = p^y(1-p)^{1-y}$

43. What is the expectation of a Bernoulli(p) random variable?

Solution: $E[Y] = 1 \times p + 0 \times (1-p) = p$

44. What is the variance of a Bernoulli(p) random variable?

Solution: $V(Y) = 1 \times p - p^2 = p(1-p)$

45. Name one assumption of a Binomial random experiment.

Solution:

1. There are n trials, where n is fixed.
2. Each trial results in a success or failure.
3. The probability of success is a constant p.
4. The trials are independent.
5. The interesting result is the total number of successes.

46. How would you generate 1000 Binomial$(100, 0.3)$ random variables?

Solution: `rbinom(n=1000, size=100, prob=0.3)`

47. If $Y \sim \text{Binomial}\left(3, \frac{1}{2}\right)$, what is $P(Y = 3)$?

Solution:

$$P(Y=3)=\binom{3}{3}\left(\frac{1}{2}\right)^3=\frac{1}{8}$$

48. If $Y \sim \text{Binomial}\left(4, \frac{1}{4}\right)$, what is $P(2 \leq Y \leq 4)$?

Solution:

$$P(2 \leq Y \leq 4) = 1 - P(Y=0) - P(Y=1) = 1 - \left(\frac{3}{4}\right)^4 - 4\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^3 \approx 0.2617188$$

49. What is the approximate 95% confidence interval for $Y \sim \text{Binomial}\left(20, \frac{1}{2}\right)$?

Solution: Use the empirical rule. We'll need the expectation and variance.

$$E[Y] = 10 \quad V(Y) = 20 \times \frac{1}{4}$$

Then, the limits are

$$(10 - 2\sqrt{5}, 10 + 2\sqrt{5})$$

50. What is the mle for λ if $X_1, \ldots, X_n \sim \text{Exponential}(\lambda)$ are independent observations.

Solution: The pdf of an exponential with mean λ is

$$f(x) = \frac{e^{-x/\lambda}}{\lambda}$$

The joint distribution of all observations $X_1, \ldots, X_n$ is

$$f(x_1, \ldots, x_n)\frac{e^{-\sum_{i=1}^n x_i/\lambda}}{\lambda^n}$$

The log likelihood is

$$\ln f(x_1, \ldots, x_n) = -\frac{1}{\lambda}\sum_{i=1}^n x_i - n\ln\lambda$$

We take the derivative with respect to λ and set it to 0.

$$\frac{d}{d\lambda} \ln f(x_1, \ldots, x_n) = \frac{1}{\lambda^2} \sum_{i=1}^{n} x_i - \frac{n}{\lambda} = 0$$

with solution

$$\lambda = \frac{\sum_{i=1}^{n} x_i}{n} = \bar{X}$$

51. What is the pmf of $Y \sim \text{Geometric}(p)$?

Solution: $f(y) = p(1-p)^{y-1}$

52. If each each attempted sale costs \$10 and each realized sale additionally costs \$10, what is the expected cost to get the first sale if 90% of attempts are failures.

Solution: Let $Y \sim \text{Geometric}(0.10)$, then the cost is

$$E[10(Y-1)+20] = 10E[Y] + 10 = \frac{10}{0.1} + 10 = 110$$

53. How would you compute $\phi_{0.9}$ for a Geometric(0.3) distribution?

Solution: `qgeom(0.9, prob=0.3)`

54. If it takes 5 minutes to pose a question, we get correct answers 70% of the time, and we continue until 10 correct answers are obtained, how long, on average, will it take us?

Solution: First, we recognize the number of questions Y for the 10th correct answer follows

$$Y \sim \text{negBinomial}(10, 0.7)$$

The amount of time is $T = 5Y$, so

$$E[T] = 5E[Y] = \frac{5 \times 10}{0.7} \approx 71.43$$

55. If $Y \sim$ negBinomial$(10, 0.7)$, which has mean 14.3 and variance 6.12, within what range do you approximately expect to find 100% of the outcomes?

Solution: Again, the request for approximate intervals calls us to use the empirical rule. The limits are

$$(14.3 - 3 \times \sqrt{6.12}, 14.3 + 3 \times \sqrt{6.12})$$

56. If there are 1000 bikes reigstered on campus and 750 are mountain bikes, what is the exact probability that 2 of 5 tickets going to registered bikes go to mountain bikes?

Solution:

$$\frac{\binom{750}{2}\binom{250}{3}}{\binom{1000}{5}}$$

In other words, a Hypergeometric$(1000, 2, 3)$.

57. Identify one pair of distributions that are approximately equal and the conditions when they are equal.

Solution: We have several choices. When n is large and p is not too small or too large, then

$$\text{Binomial}(n, p) \approx \text{Normal}(np, np(1-p))$$

When N is large, then

$$\text{Hypergeometric}(N, n, r) \approx \text{Binomial}\left(n, \frac{r}{N}\right)$$

When n is large and p is small, then

$$\text{Binomial}(n, p) \approx \text{Poisson}(np)$$

58. Name the following distribution

$$p(y) = \frac{e^{-1}}{y!}$$

Solution: $Y \sim \text{Poisson}(1)$

59. What is the value of the following sum?

$$\sum_{i=0}^{\infty} \frac{(\ln 3)^i}{3i!}$$

Solution: By some arrangement, we recognize the pmf of Poisson($\ln 3$) distribution.

$$\sum_{i=0}^{\infty} \frac{(\ln 3)^i}{3i!} = \sum_{i=0}^{\infty} \frac{(\ln 3)^i e^{-\ln 3}}{i!} = 1$$

60. If 1 in 1000 cycles of a traffic light result in a crash, what is the probability of no crashes in 10000 cycles?

Solution: The number of crashes $Y \sim \text{Binomial}(10000, 0.001)$, so $P(Y = 0) = \binom{10000}{0}(0.999)^{10000} = 4.517335e - 05$. We could also recognize that the number of trials 10000 is large with crash probability 0.001 small, so $Y \sim \text{Poisson}\left(\frac{10000}{1000}\right)$ and $P(Y = 0) \approx e^{-10} = 4.539993e - 05$.

61. What would be an appropriate distribution to model the number of airline flights in a year that experience a mechanical malfunction that must be reported to the FAA?

Solution: Presumably there are many flights per year, but there is only a small probability of "success", so a Poisson distribution would be appropriate. In this case Binomial doesn't work because the number of trials is not known beforehand. The Poisson approximation holds regardless.

62. If $Y_1 \sim \text{Poisson}(3)$ and $Y_2 \sim \text{Poisson}(6)$ are independent, what is $Y_1 + Y_2$?

Solution: $Y_1 + Y_2 \sim \text{Poisson}(9)$

63. The number of people you encounter in time arise as a Poisson process with mean 100 per year. If each person has an independent chance 10% of being more intelligent than you, what is the distribution of the number of people you meet that are smarter than you?

> **Solution:** Poisson(100×0.1)

64. What is $P(X = x)$ if X is discrete?

> **Solution:** It is the pmf value $p(x)$.

65. What is $P(X = x)$ if X is continuous?

> **Solution:** 0

66. What is the cdf in two dimensions?

> **Solution:** $F(x, y) = P(X \leq x, Y \leq y)$

67. Where is the cdf undefined?

> **Solution:** Everywhere.

68. Where is the conditional pdf undefined?

> **Solution:** $f(x|y)$ is undefined wherever $f(y) = 0$.

69. What is the appropriate relationship?

$$F(3) \quad F(5)$$

Solution: $F(3) \leq F(5)$

70. What is the range of $f(y)dy$?

Solution: Since $f(y)dy \approx P(y - dy/2 < Y \leq y + dy/2)$, then $f(y)dy \in [0, 1]$.

71. Draw the cdf for $p(0) = \frac{1}{2}$, $p(1) = \frac{1}{4}$, and $p(2) = \frac{1}{4}$.

Solution: If you need help with this, let me know.

72. What is the appropriate relationship?

$$F(\phi_p) \quad p$$

Solution: $F(\phi_p) \geq p$

73. How would you use `r` to find the 5% and 95% quantiles of a standard normal distribution?

Solution: `qnorm(0.05)` and `qnorm(0.95)`

74. If

$$f(x) = \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-(x/\lambda)^k}, x \geq 0$$

then write down an integral for $E[X(X-1)]$.

Solution:

$$E[X(X-1)] = \int_0^\infty x^2 f(x)dx - \int_0^\infty x f(x)dx$$

or

$$E[X(X-1)] = \int_0^\infty (x-1)\left(\frac{x}{\lambda}\right)^k e^{-(x/\lambda)^k} dx$$

75. Given there are 10 maple trees sprouting somewhere on a 1000 square foot patch, from seeds dropped randomly by birds flying overhead. What is a good distribution for the locations of the seedlings?

> **Solution:** Uniform on the 1000 square foot patch, whatever its shape, so $f(x,y) = \frac{1}{1000}$ for (x,y) in the patch.

76. If $X \sim \text{Uniform}(10, 40)$, what is $P(X > 30 \mid X > 20)$?

> **Solution:** $\frac{P(X>30)}{P(X>20)} = \frac{1/3}{2/3} = \frac{1}{2}$

77. People in a study are put on 3 possible exercise regimens. One of those regimens involves x hours of exercise and it is found that the random weight loss has pdf

$$f(y) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[\frac{(y - 3x - 1)^2}{2\sigma^2}\right]$$

What is $E[Y]$?

> **Solution:** We recognize a normal distribution and $E[Y] = 3x + 1$.

78. If $Y \sim \text{N}(3, 6)$, how do you form the standard normal Z from Y?

> **Solution:** $\frac{Y-3}{\sqrt{6}}$

79. If $Y \sim \text{N}(60, 9)$ is the amount of time it take a student to take an exam, how much timeshould I schedule to insure 97.5% of the students have time to complete the exam?

> **Solution:** The upper quantile of a standard normal is 1.96, which we'll approximate by 2.
>
> $$\frac{Y - 60}{3} = 2 \quad \Rightarrow \quad Y = 66$$

80. What is $\int_0^\infty y^4 e^{-y} dy$?

Solution: This is the gamma function, so the answer is 4!

81. The random variable $Y \sim \text{Gamma}(n, \beta)$ can be thought of as $Y = X_1 + \cdots + X_n$, where X_i are independent exponential random variables with mean β. Use this to prove that $V(Y) = n\beta$.

Solution: The last theorem we learned tells us that

$$V(Y) = V(X_1 + \cdots + X_n) = \sum_{i=1}^{n} V(X_i) + 2 \sum_{1 \le i < j \le n} \text{Cov}(X_i, X_j) = \sum_{i=1}^{n} \beta = n\beta$$

where the covariance terms are 0 because of independence and $V(X_i) = \beta$.

82. If $X \sim \text{Exponential}(3)$ is the time it takes to answer a question, how many questions should I ask if I want the exam done in an average 120 minutes?

Solution: The time it takes to finish the exam is $T = X_1 + X_2 + \cdots + X_n$, where n is unknown, where we assume X_i are independent. By linearity of expectation, we have

$$E[T] = \sum_{i=1}^{n} E[X_i] = 3n$$

Since we want $E[T] = 120$, then $n = \frac{120}{3} = 40$.

83. If you are shown a U-shaped Beta distribution on the interval $[0, 1]$, what are the appropriate relationships between the parameters

$$\alpha \quad \beta \qquad\qquad \alpha \quad 0$$

Solution: $\alpha = \beta$ for a perfectly symmetric U-shape. $\alpha < 1$ to get the U-shape.

84. What is another name for the Beta$(1, 1)$ distribution?

Solution: Standard Uniform, Uniform$(0, 1)$

85. What is C if $f(x) = Cx^2e^{-x/2}$?

Solution: We recognize a gamma distribution and the coefficient is $\frac{1}{\beta^\alpha \Gamma(\alpha)} = \frac{1}{2^3 \times 2!}$.

86. Find $E[Y^n]$ if $Y \sim \text{Poisson}(\lambda)$.

Solution: The moment generating function for Poisson is $m(t) = e^{\lambda(e^t-1)}$. We see $\frac{dm(0)}{dt} = \lambda e^{\lambda(e^t-1)+1}\big|_{t=0} = \lambda$ and $\frac{d^2m(0)}{dt^2} = \lambda(\lambda+1)e^{\lambda(e^t-1)+2}\big|_{t=0} = \lambda(\lambda+1)$. You can continue, but the pattern seems obvious and we conclude

$$E[Y^n] = \lambda(\lambda+1)\cdots(\lambda+n-1)$$

87. If Y has mgf $m_Y(t)$, what is mgf $Z = aY + b$?

Solution: $m_Z(t) = e^{bt}m_Y(at)$

88. What is the distribution of Y if $m(t) = \frac{0.3e^t}{1-0.7e^t}$

Solution: $Y \sim \text{Geometric}(0.3)$

89. What is the distribution of Y if $m(t) = \frac{1-e^{30t}}{30t}$?

Solution: $Y \sim \text{Uniform}(0, 30)$

90. Random variable Y is what function of what random variable if $m(t) = e^{3t}(1-\beta t)^{-\alpha}$?

Solution: $Z \sim \text{Gamma}(\alpha, \beta)$ and $Y = Z + 3$.

91. Random variable Y is what function of what random variable if $m(t) = \frac{pe^{3t}}{1-e^{3t}(1-p)}$?

Solution: $Z \sim$ Geometric(p) and $Y = 3Z$

92. Find the integral needed for computing $E[Y]$ when (X, Y) is uniform over the triangle with corners $(-1, 0), (0, 1), (1, 0)$.

Solution:

$$\int_{-1}^{0}\int_{0}^{x+1} y dx dy + \int_{0}^{1}\int_{0}^{1-x} y dx dy$$

93. Are X and Y independent if their joint pdf is

$$f(x,y) = \frac{1}{2n}\left(1 + \frac{x^2+y^2}{n}\right)^{-(n+2)/2}$$

Solution: No. The pdf cannot be factored $f(x)f(y)$ as required.

94. Are X and Y independent if the joint pdf is $f(x, y) = ax^2 + 2axy + y^2$ for $1 \leq x, y \leq 3$?

Solution: No, again the joint pdf cannot be factored.

95. Are X and Y independent if the joint pdf is $f(x, y) = cxy - ay + bcx - ab$ for $1 \leq x, y \leq 3$?

Solution: Yes, because $f(x, y) = (cx - a)(y + b)$ can be factored and the domain is finite rectangle.

96. If $f(x, y) = e^{-x-y}$ for $x, y \geq 0$, what is the integral to compute $P(X - 1 < Y < X)$?

Solution:

$$\int_{0}^{1}\int_{0}^{x} e^{-x-y} dy dx + \int_{1}^{\infty}\int_{x-1}^{x} e^{-y-x} dy dx$$

97. Name the marginal distributions if $f(x, y) \propto ye^{x/3}$ for $0 \leq y \leq 1$ and $x \geq 0$.

Solution: $Y \sim \text{Beta}(2, 1)$ and $X \sim \text{Exponential}(3)$.

98. If X and Y are independent Gamma$(3, 1/2)$ distributions, what is $E[4XY + 9]$?

Solution: $E[4XY + 9] = 4E[X]E[Y] + 9 = 4\left(\frac{3}{2}\right)^2 + 9 = 18$

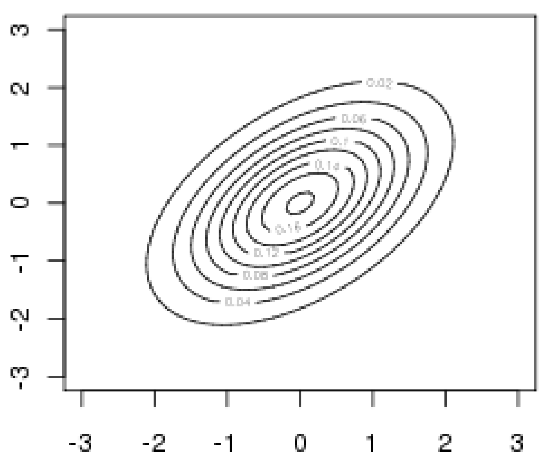
99. If $f(x, y) = Cx^3(1 - x)^2 y^4$ for $0 \le x, y \le 1$, what is $\text{Cov}(X, Y)$?

Solution: 0 because X and Y both have independent Beta distributions.

100. If X and Y have marginal exponential(β) distributions and $\text{Cov}(X, Y) = \alpha$, then what is $\text{Corr}(X, Y)$?

Solution: $\text{Corr}(X, Y) = \frac{\text{Cov}(X,Y)}{\sqrt{\text{Var}(X)\text{Var}(Y)}} = \frac{\alpha}{\beta^2}$

101. If (X, Y) has correlation 0.7 and (W, Z) has correlation -0.3, which is plotted below?



Solution: (X, Y)