Probabilistic boolean networks

How does uncertainty propagate through a boolean network?

Consider a boolean network $G(V, F)$ containing n genes $x_1, x_2, ..., x_n$, and an initial joint probability distribution

$$p(x), x = (x_1...x_n \in \{0, 1\}^n$$

$$Pr[f_1(x) = i_1, f_2(x) = i_2, ..., f_n(x) = i_n]$$

$$f_1(x) = i_1, f_2(x) = i_2, ..., f_n(x) = i_n]$$

$$= \bigcup_{k \leq n} O \times O...x\{f_k(x) = i_k\} \times O.. \times O$$

$$\sum_{x \in A} Pr(x)$$

$$A = \{x \in 0, 1^n | f_k(x) = i_k\}$$

The sum implicitly defines an iterative map

$$p^{(t+1)} = \psi(p^{(t)})$$

One can show that if you write $p^{(t)} = (p_1^{(t)} p_2^{(t)} ... p_n^{(t)})$ then

$$p^{(t+1)} = p^{(t)} P$$

(What is P equal to?)

The k-means algorithm

We assume that we have n data points $\{a_j\}_{j=1}^n \in R^m$. Let

$$\Pi = \{\pi_i\} i = 1^k$$

denote a partition of A.

$$\pi_j = \{v | a_v \text{belongs to the jth cluster}\}$$

Let the "mean" (centroid) of the j^{th} cluster be

$$m_j = \frac{1}{n_j} \{\sum_{v \in \pi_j} a_v\}$$

where n_j is the number of elements of π_j.

The "tightness" (coherence) of the cluster π_j is defined as

$$q_j = \sum_{v \in \pi_j} ||a_v - m_j||^2$$

The quality of clustering can be measured as the overall coherence of all clusters.

$$Q(\Pi) = \sum_{j=1}^{k} q_j$$