1.

(i) One can graphically determine S by looking at the excursion graph (E_n as a function of n) and identifying the highest point. Let l be the value of n associated with the highest point, and let k be the largest value of n less than l such that $E_k = 0$. S then is $S_k S_{k+1} ... S_{l-1} S_l$.

(ii) If we set p to a particular value, we can find x_c by solving the following equation.

$$\begin{aligned}
e^{-ke^{-\lambda x_c}} &= 1 - p \\
-ke^{-\lambda x_c} &= ln(1 - p) \\
e^{-\lambda x_c} &= -\frac{ln(1-p)}{k} \\
-\lambda x_c &= ln\left(-\frac{ln(1-p)}{k}\right) \\
x_c &= -\frac{ln\left(-\frac{ln(1-p)}{k}\right)}{\lambda}
\end{aligned}$$

(iii) We can obtain x_c^* simply by multiplying x_c by ρ. Recall that we defined λ as follows.

$$\lambda : \sum_{j=1}^{r} p_j e^{\lambda s_j} = 1$$

If we now define $t_i = \rho s_i$ then we have the following.

$$\lambda' : \sum_{j=1}^{r} p_j e^{\lambda' t_j} = 1$$

These equations give us the following relationship between λ and λ'.

$$\lambda' = \frac{\lambda}{\rho}$$

We can then identify x_c^* using the same method as in (ii).

$$\begin{aligned}
e^{-ke^{-\lambda' x_c^*}} &= 1 - p \\
-ke^{-\lambda' x_c^*} &= ln(1 - p) \\
e^{-\lambda' x_c^*} &= -\frac{ln(1-p)}{k} \\
-\lambda' x_c^* &= ln\left(-\frac{ln(1-p)}{k}\right) \\
x_c^* &= -\frac{ln\left(-\frac{ln(1-p)}{k}\right)}{\lambda'} \\
x_c^* &= -\frac{ln\left(-\frac{ln(1-p)}{k}\right)}{\frac{\lambda}{\rho}} = \rho\left(-\frac{ln\left(-\frac{ln(1-p)}{k}\right)}{\lambda}\right) \\
x_c^* &= \rho x_c
\end{aligned}$$

(iv) The λ term can be interpreted as a scale factor since it provides a direct, linear relationship between scoring schemes that are scalar multiples of each other. If the relationship between 2 scoring schemes can be found, then scores and significance levels can be easily transformed from one scheme to the other using the λ term.

(v) By the derivation below, doubling the size of the sequence increases the p-level threshold by 2.

$$\begin{aligned}
S'_p &= \frac{ln2N}{\frac{ln2}{2}} + x_c \\
&= \frac{lnN}{\frac{ln2}{2}} + \frac{ln2}{\frac{ln2}{2}} + x_c \\
&= \frac{lnN}{\frac{ln2}{2}} + 2 + x_c \\
&= \left(\frac{lnN}{\frac{ln2}{2}} + x_c \right) + 2 \\
&= S_p + 2
\end{aligned}$$

(vi) Consider the following transformation of the originally given inequality.

$$\begin{aligned}
S &> \frac{lnN}{\lambda} + x \\
\lambda S &> lnN + \lambda x \\
\lambda S - lnK &> lnN + \lambda x - lnK \\
\frac{\lambda S - lnK}{ln2} &> \frac{lnN + \lambda x - lnK}{ln2} \\
S' &> y
\end{aligned}$$

The left hand side of the inequality is the normalized score, and the right hand side represents y. We can solve the equation for y in terms of x.

$$\begin{aligned}
\frac{lnN + \lambda x - lnK}{ln2} &= y \\
lnN + \lambda x - lnK &= yln2 = ln2^y \\
\lambda x &= ln\left(\frac{2^y K}{N}\right) \\
x &= \frac{1}{\lambda} ln\left(\frac{2^y K}{N}\right)
\end{aligned}$$

Now, if we place x in the formula for the Poisson parameter we get the following result.

$$\begin{aligned}
Ke^{-\lambda\left(\frac{1}{\lambda} ln \frac{2^y K}{N}\right)} &= Ke^{-ln\frac{2^y K}{N}} \\
&= K\left(\frac{N}{2^y K}\right) \\
&= \frac{N}{2^y}
\end{aligned}$$