- S: stoichiometric matrix
- *nullspace* of S: v such that $Sv = 0$
- *left nullspace* of S: y such that $yS = 0$

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \dots \\ y_m \end{bmatrix}, x = \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_m \end{bmatrix}$$

If $y^T S = 0$, then

$$y^T \frac{dx}{dt} = (y^T S)v = 0 \rightarrow \frac{d}{dt}(y^T x) = 0$$

which implies that the value $y^T x$ is constant.

$$S = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \\ -1 & 0 & -1 \\ 1 & 0 & 1 \end{bmatrix}$$

If we want to identify all possible conservation relations associated with the "glycolysis network", we need to find all y for which $y^T S = 0$. If $y^T S = 0$, then $(y^T S)^T = 0^T$, which implies that $S^T y = 0^T$.

$$V = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}, S = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

$$\frac{d}{dt}x_1 = v_1 - v_2$$

$$\frac{d}{dt}x_2 = v_2 - v_3$$

$$\frac{d}{dt}x_3 = v_4 - v_2$$

$$\frac{d}{dt}x_4 = v_2 - v_4$$

Elementary flux modes, extreme pathways

All v that satisfy $Sv = 0$ can be written as

$$v = c_1 v_1 + c_2 v_2 + \dots + c_r v_r$$