Cortical Columns as Amplistwistor Cascades: A Recursive Field-Theoretic Account of Cortical Computation

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Abstract

This paper develops a unified mathematical model of cortical computation in which semantic representation emerges from recursive field dynamics. The theoretical substrate consists of a relativistic tri-field system comprising a scalar potential field, a vector inference field, and an entropy field. These fields interact through nonlinear differential equations defined on a geometric manifold. Local cortical transformations are implemented by *amplistwistors*, a class of nonlinear operators generalizing complex-analytic amplitwists to high-dimensional fields. Temporal evolution is driven by a recursion operator that performs multiscale semantic refinement, while global synchronization arises from stationary eigenmodes of the cortical sheet, interpreted as cymatic resonance. The resulting model portrays cortical computation as a resonance-gated sequence of nonlinear transformations, consistent with electrophysiological studies demonstrating hierarchical temporal processing and with ultrafast fMRI evidence for standing-wave cortical modes. Mathematical derivations establish the well-posedness of the field equations, the approximation capacity of amplistwistor cascades, and the spectral properties of resonance modes.

1 Introduction

Cortical computation unfolds across multiple temporal and spatial scales. Sensory responses in primary regions arise within tens of milliseconds, whereas higher-order processes such as syntactic parsing and semantic integration may extend across intervals of several hundred milliseconds. Recent studies using electrocorticography have demonstrated that the temporal evolution of neural activity during language comprehension exhibits a graded hierarchy, with deeper representational transformations occurring systematically later in time. Complementary findings from ultrafast functional magnetic resonance imaging have revealed that

the cortex supports macroscale standing-wave eigenmodes that oscillate coherently across distant regions without requiring direct propagating signals. Together, these results suggest that cortical computation is neither strictly feedforward nor purely recurrent, but emerges from the interaction of recursive local transformations and global synchronizing structures.

The present work develops a mathematical theory of this interaction. The core substrate is a system of coupled fields—a scalar field representing semantic potential, a vector field representing directed inference, and an entropy field modulating uncertainty. These fields evolve through relativistic dynamic equations that encode local interactions, nonlinearity, and stochastic perturbation. Local cortical processing is realized via *amplistwistors*, operators that amplify, rotate, and project field quantities in a manner extending the geometric derivative formalism of complex analysis. Temporal unfolding of computation is governed by a recursive update operator, derived as a discrete-time semigroup acting on the tri-field system. Global synchronization is maintained by eigenmodes of the cortical geometry; these modes, observed directly through ultrafast fMRI, constitute a basis for large-scale oscillatory activity and provide the temporal scaffolding for local recursive dynamics.

The aim of this paper is to formulate these components rigorously, establish their mathematical properties, and show how they collectively yield a unified account of cortical computation. The exposition proceeds by introducing the field equations, analyzing their structural properties, defining the amplistwistor operators, proving approximation theorems, and deriving global resonance modes as solutions to the Laplace–Beltrami eigenvalue problem. The paper then integrates these components into a single evolution equation and discusses the implications for cognitive neuroscience.

2   Mathematical Preliminaries

The theory is formulated on a smooth, compact Riemannian manifold $(\mathcal{M}, g)$ of dimension n, representing the semantic or functional space across which representations are distributed. The manifold is equipped with metric tensor g_{ij}, Levi-Civita connection ∇_i, and Laplace–Beltrami operator

$$\Delta f = \nabla_i \nabla^i f.$$

Fields defined on $\mathcal{M}$ evolve in time $t \in \mathbb{R}_{\geq 0}$. We denote the scalar field by $\Phi : \mathcal{M} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$, the vector field by $\mathbf{v} : \mathcal{M} \times \mathbb{R}_{\geq 0} \to T\mathcal{M}$, and the entropy field by $S : \mathcal{M} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$.

The function space for each field is taken to be a Sobolev space $H^k(\mathcal{M})$ of order $k \geq 2$, ensuring sufficient differentiability for the operators under consideration. The tri-field system is represented by the vector

$$\Psi = (\Phi, \mathbf{v}, S).$$

We consider dynamics of the form

$$\frac{\partial \Psi}{\partial t} = F(\Psi) + \eta,$$

where F is a nonlinear differential operator and η is a stochastic field.

The existence and uniqueness of solutions will be addressed in later sections.

3   The Relativistic Scalar Vector Plenum

We now derive the field equations governing the evolution of Φ, $\mathbf{v}$, and S. These equations are motivated by three principles: diffusion of semantic potential, flow-driven transport of inference, and regulation of uncertainty.

The scalar potential obeys

$$\dot{\Phi} = c_1 \Delta \Phi + c_2 \Phi \left(\nabla \times \mathbf{v} \right) - c_3 S \Phi + \eta_\Phi,$$

where $c_1, c_2, c_3 > 0$ are constants and η_Φ is zero-mean noise. The first term represents diffusion, the second a nonlinear coupling with vector vorticity, and the third an entropy-weighted decay.

The vector field satisfies

$$\dot{\mathbf{v}} = c_4 \nabla \Phi + c_5 \nabla \times \mathbf{v} - c_6 S \mathbf{v} + \eta_v.$$

The gradient term attracts the vector field toward regions of higher semantic potential, the curl term sustains local rotations, and the final term damps flow under high entropy.

The entropy evolves according to

$$\dot{S} = c_7 |\nabla \cdot \mathbf{v}| + c_8 \Phi^2 - c_9 S + \eta_S.$$

The divergence term reflects the intuition that compressive or expansive flow informs uncertainty, the quadratic term captures the entropy-reducing effect of strong semantic potential, and the decay term ensures stability.

We will show that under reasonable assumptions on the coefficients and noise terms, the coupled system admits global solutions.

4   Well-Posedness of the Tri-Field System

To establish the mathematical legitimacy of the field-theoretic model, it is necessary to show that the coupled evolution equations for Φ, $\mathbf{v}$, and S admit well-defined solutions. The objective is to demonstrate that the system constitutes a well-posed initial-value problem in

the sense of Hadamard: a solution exists, is unique, and depends continuously on the initial data. Throughout this section we impose the assumption that $\mathcal{M}$ is compact, connected, and without boundary, ensuring that elliptic operators such as Δ and $\nabla\times$ have discrete spectra and that Sobolev embeddings are compact.

We begin by rewriting the tri-field dynamics concisely as

$$\frac{\partial \Psi}{\partial t} = \mathcal{L}\Psi + \mathcal{N}(\Psi) + \eta, \tag{1}$$

where $\Psi = (\Phi, \mathbf{v}, S)$, $\mathcal{L}$ is a linear differential operator defined by

$$\mathcal{L}(\Phi, \mathbf{v}, S) = \Big(c_1\Delta\Phi, \ c_5\nabla\times\mathbf{v}, \ -c_9 S\Big),$$

and $\mathcal{N}(\Psi)$ is the nonlinear operator

$$\mathcal{N}(\Psi) = \Big(c_2\Phi(\nabla\times\mathbf{v}) - c_3 S\Phi, \ c_4\nabla\Phi - c_6 S\mathbf{v}, \ c_7|\nabla\cdot\mathbf{v}| + c_8\Phi^2\Big).$$

The stochastic forcing η consists of square-integrable fields satisfying the usual martingale conditions.

4.1 Local Existence and Uniqueness

We now prove a local existence theorem for the system.

Let $\Psi_0 \in H^k(\mathcal{M})$ for $k > n/2 + 1$. Then there exists a time $T > 0$ and a unique solution $\Psi \in C([0,T], H^k(\mathcal{M}))$ to the tri-field system.

Proof. The proof proceeds by applying the Picard iteration argument to the mild form of the equation. We write

$$\Psi(t) = e^{t\mathcal{L}}\Psi_0 + \int_0^t e^{(t-s)\mathcal{L}}\mathcal{N}(\Psi(s))\,ds + \int_0^t e^{(t-s)\mathcal{L}}\eta(s)\,ds.$$

The operator $e^{t\mathcal{L}}$ is a contraction on $H^k(\mathcal{M})$ due to the smoothing properties of the heat semigroup and the boundedness of the curl term on compact manifolds. The nonlinear operator $\mathcal{N}$ is locally Lipschitz in $H^k(\mathcal{M})$ because multiplication and composition are bounded operations in Sobolev spaces of order $k > n/2 + 1$. Thus the mapping defined by the right-hand side is a contraction for sufficiently small T, establishing existence and uniqueness. $\square$

4.2 Global Existence via Energy Estimates

Local solutions can be extended globally in time if uniform a priori bounds can be established. We construct an energy functional

$$E(t) = \|\Phi(t)\|_{H^k}^2 + \|\mathbf{v}(t)\|_{H^k}^2 + \|S(t)\|_{H^k}^2.$$

Differentiating and applying standard Sobolev inequalities yields

$$\frac{dE}{dt} \leq -\lambda E(t) + C E(t)^{3/2} + \|\eta(t)\|^2,$$

where $\lambda > 0$ depends on the decay terms c_3 and c_9.

Suppose the initial data Ψ_0 satisfies $E(0) < \infty$. Then for sufficiently small nonlinear coefficients c_2, c_4, c_7, c_8, or under a smallness assumption on initial data, the solution to the tri-field system exists for all $t \geq 0$.

Proof. The differential inequality above is of Riccati type. If $CE(0)^{1/2} < \lambda$, then $E(t)$ remains uniformly bounded for all time. Standard continuation arguments extend the local solution globally. $\square$

This establishes the mathematical well-posedness of the field substrate.

5 Energy Functionals and Semantic Stability

Energy methods not only establish global existence but also allow the definition of notions such as semantic stability. We define a generalized Lyapunov functional

$$\mathcal{V}(\Psi) = \int_{\mathcal{M}} \left(\alpha_1 \Phi^2 + \alpha_2 |\mathbf{v}|^2 + \alpha_3 S^2 \right) d\mu,$$

where $\alpha_i > 0$ and $d\mu$ is the Riemannian measure.

If the entropy coupling coefficients satisfy $c_3 > 0$ and $c_9 > 0$, then $\mathcal{V}$ is a Lyapunov functional: $\dot{\mathcal{V}} \leq 0$ in expectation.

Proof. Differentiating under the integral sign and using the field equations produces terms of the form $-c_3\alpha_1\Phi^2 S$ and $-c_9\alpha_3 S^2$, both nonpositive. The remaining terms integrate to zero due to divergence theorems on compact manifolds. $\square$

Thus the entropy field acts to stabilize the dynamics by damping extreme deviations in both Φ and $\mathbf{v}$.

6 Invariant Sets and Attractor Structure

The existence of a Lyapunov functional implies that solutions flow toward invariant sets of reduced energy. Let

$$\mathcal{A} = \{\Psi \mid \dot{\mathcal{V}}(\Psi) = 0\}.$$

The set $\mathcal{A}$ consists of the field configurations satisfying:

1. $\nabla \Phi = 0$,

2. $\nabla \times \mathbf{v} = 0$,

3. $S = 0$.

Proof. The only nonpositive contributions to $\dot{\mathcal{V}}$ arise from entropy damping terms. Equality requires both $S = 0$ and $\nabla \cdot \mathbf{v} = 0$, eliminating divergence-driven entropy production. The remaining curl and gradient terms follow from the structure of $\mathcal{N}$. $\qquad\square$

The invariant set corresponds to "predictive silence" states in which semantic potential is harmonic, flows are divergence-free, and uncertainty vanishes.

This completes the mathematical foundation for the RSVP field substrate.

7 Amplistwistors as Local Nonlinear Operators

The recursive structure of cortical computation requires local operators capable of shaping the tri-field system through amplification, rotation, projection, and nonlinear mixing. Motivated by the geometric derivative formalism of complex analysis—where the derivative of a holomorphic function acts as an *amplitwist*, simultaneously scaling and rotating infinitesimal vectors—we generalize this notion to a class of operators defined on the semantic field $\Psi = (\Phi, \mathbf{v}, S)$.

An *amplistwistor* is defined as a smooth map

$$\mathcal{A} : H^k(\mathcal{M}) \to H^k(\mathcal{M})$$

having the decomposition

$$\mathcal{A}(\Psi) = \pi\Big(\alpha(\Psi)\,\tau(\Psi)\Psi\Big),$$

where each component is defined as follows.

The *amplitude operator* α is a positive, smooth functional

$$\alpha : H^k(\mathcal{M}) \to C^\infty(\mathcal{M}, \mathbb{R}_{>0}),$$

that rescales field magnitudes according to semantic salience or contextual gain.

The *twist operator* τ is a smooth mapping

$$\tau : H^k(\mathcal{M}) \to \mathrm{SO}(d),$$

acting pointwise on the field representation embedded in $\mathbb{R}^d$. This operator generalizes the notion of a local rotation or shear in representation space.

Finally, the *projection operator* π is a smooth, idempotent map

$$\pi^2 = \pi,$$

projecting field values onto semantically admissible submanifolds or entropy-reducing directions.

These components collectively define a rich family of nonlinear transformations that operate locally yet generate substantial global effects through recursive composition.

8 Infinitesimal Generators and Linearization

To study the local behavior of amplistwistors, we compute their Fréchet derivatives. Let $\mathcal{A}$ be an amplistwistor and consider a variation $\Psi + \epsilon Y$. Expanding,

$$\mathcal{A}(\Psi + \epsilon Y) = \mathcal{A}(\Psi) + \epsilon\, D\mathcal{A}(\Psi)[Y] + \mathcal{O}(\epsilon^2).$$

The derivative decomposes naturally:

$$D\mathcal{A}(\Psi)[Y] = D\pi(\Xi)[D\alpha(\Psi)[Y] \cdot \tau(\Psi)\Psi \,+\, \alpha(\Psi)D\tau(\Psi)[Y]\Psi \,+\, \alpha(\Psi)\tau(\Psi)Y],$$

where $\Xi = \alpha(\Psi)\tau(\Psi)\Psi$. The dominant term is the action of τ on Y, indicating that infinitesimal updates are governed by a generalized "twisted" derivative.

This linearization motivates the continuous-time analog:

$$\frac{\partial \Psi}{\partial t} = \mathcal{G}(\Psi) = \lim_{\epsilon \to 0} \frac{\mathcal{A}_\epsilon(\Psi) - \Psi}{\epsilon},$$

where $\mathcal{A}_\epsilon$ denotes a one-parameter family of amplistwistors. In Section 5, this will serve as the generator of the recursion semigroup.

9 Recursive Composition and Expressive Capacity

Recursive composition of amplistwistors is defined by

$$\Psi_{k+1} = \mathcal{A}(\Psi_k), \qquad \Psi_0 = \Psi(t),$$

yielding a discrete sequence of field refinements. For a finite set of amplistwistors $\{\mathcal{A}_1, \ldots, \mathcal{A}_N\}$, we consider the expressive class

$$\mathcal{F}_N = \left\{\mathcal{A}_{i_m} \circ \cdots \circ \mathcal{A}_{i_1} \,\middle|\, m \in \mathbb{N}, \ i_j \in \{1, \ldots, N\}\right\}.$$

This section establishes that $\mathcal{F}_N$ is a universal approximator for continuous maps on compact subsets of $H^k(\mathcal{M})$.

9.1 Approximation on Function Spaces

We recall a classical result: if σ is a non-polynomial activation function, then finite superpositions of the form $\sum_i a_i \, \sigma(w_i \cdot x + b_i)$ approximate continuous functions on compact subsets of $\mathbb{R}^n$ arbitrarily well. The operators (α, τ, π) in amplistwistors play analogous roles to (a_i, w_i, b_i, σ), but in infinite-dimensional function spaces.

9.2 Density Lemma

We begin with a key lemma.

Let $K \subset H^k(\mathcal{M})$ be compact and convex. Suppose the projection operator π has dense range in $H^k(\mathcal{M})$, and suppose τ spans a dense subset of $\mathrm{SO}(d)$ as Ψ varies over K. Then finite compositions of amplistwistors are dense in $C(K, H^k(\mathcal{M}))$.

Proof. The map $\Psi \mapsto \alpha(\Psi)\tau(\Psi)\Psi$ contains the pointwise operations of scaling, rotation, and linear mixing of components. Under the density assumption, arbitrary linear combinations of field values can be generated. Projection then permits nonlinear shaping, creating a composition scheme analogous to ridge-function networks. The Stone–Weierstrass theorem extends to Banach-valued continuous functions, yielding density. $\square$

9.3 Universal Approximation Theorem

We now state the main theorem of this section.

[Universal Approximation] Let $K \subset H^k(\mathcal{M})$ be compact and let $\varepsilon > 0$. For any continuous operator $F : K \to H^k(\mathcal{M})$, there exists a finite composition of amplistwistors $\mathcal{A}_{i_1}, \ldots, \mathcal{A}_{i_m}$ such that

$$\sup_{\Psi \in K} \|F(\Psi) - \mathcal{A}_{i_m} \circ \cdots \circ \mathcal{A}_{i_1}(\Psi)\| < \varepsilon.$$

Proof. By the density lemma, the algebra generated by amplistwistors is dense in $C(K, H^k(\mathcal{M}))$. The operator F lies in this space, and thus can be approximated arbitrarily closely by finite compositions of amplistwistors. $\square$

This theorem establishes that amplistwistors possess full expressive capacity for local cortical computation, mirroring the approximation capabilities of deep neural networks but grounded in differential geometry.

10 Stability of Amplistwistor Cascades

The recursive application of nonlinear operators poses the risk of instability or divergence. To ensure meaningful computation, we require sufficient conditions under which amplistwistor cascades remain bounded.

Suppose $\alpha(\Psi)$ is uniformly bounded above and below by positive constants, $\tau(\Psi)$ is orthogonal, and π is a contraction. Then the composition sequence $\Psi_{k+1} = \mathcal{A}(\Psi_k)$ remains bounded for all $k \geq 0$.

Proof. Orthogonality of τ ensures $\|\tau(\Psi)\Psi\| = \|\Psi\|$. Boundedness of α implies $\|\alpha(\Psi)\tau(\Psi)\Psi\| \leq C\|\Psi\|$. Contractivity of π then yields $\|\Psi_{k+1}\| \leq C\|\Psi_k\|$. Iteration preserves the bound. $\square$

Hence amplistwistors serve as stable local transformers capable of repeated application without leading to divergence.

This completes the mathematical theory of amplistwistors.

11 Eigenmodes of the Cortical Manifold

The cortex may be modeled as a compact, oriented Riemannian manifold $(\mathcal{M}, g)$ with smooth boundary (or none). The intrinsic geometry of $\mathcal{M}$ supports a natural Laplace–Beltrami operator

$$\Delta_g : C^\infty(\mathcal{M}) \to C^\infty(\mathcal{M}),$$

defined by

$$\Delta_g f = \mathrm{div}_g(\nabla_g f).$$

The spectrum of $-\Delta_g$ is discrete, nonnegative, and admits the eigendecomposition

$$-\Delta_g u_m = \lambda_m u_m, \qquad m = 0, 1, 2, \ldots,$$

where $0 = \lambda_0 < \lambda_1 \leq \lambda_2 \leq \ldots$, with $\lambda_m \to \infty$. The eigenfunctions $\{u_m\}$ form a complete orthonormal basis for $L^2(\mathcal{M}, d\mathrm{vol}_g)$.

Physically, these eigenfunctions represent spatial standing waves on the cortical surface. Experimental work in ultrafast fMRI demonstrates that large-scale functional connectivity

patterns admit precisely such a decomposition, with frequencies determined by macroscopic anatomical geometry.

12 Time-Dependent Spectral Modes

To understand the oscillatory character of cortical activity, we examine wave-like solutions to a second-order dynamical operator acting on fields defined over $\mathcal{M}$. Consider the equation

$$\frac{\partial^2 X}{\partial t^2} + \gamma \frac{\partial X}{\partial t} + \mathcal{L}X = F(\Psi, t),$$

where $\mathcal{L}$ is an elliptic operator such as $-\Delta_g + V$, γ is a damping constant, and $F(\Psi, t)$ is a forcing term depending on RSVP fields. Expanding X in the eigenbasis:

$$X(\mathbf{r}, t) = \sum_{m=0}^{\infty} a_m(t)\, u_m(\mathbf{r}),$$

substituting into the PDE and projecting onto each mode via orthogonality yields a collection of decoupled ODEs:

$$\ddot{a}_m + \gamma \dot{a}_m + \lambda_m a_m = f_m(t),$$

where

$$f_m(t) = \int_{\mathcal{M}} F(\Psi, t) u_m(\mathbf{r})\, d\mathrm{vol}_g.$$

The homogeneous solutions are

$$a_m(t) = C_1 e^{-\gamma t/2} e^{i\sqrt{\lambda_m - \gamma^2/4}\, t} + C_2 e^{-\gamma t/2} e^{-i\sqrt{\lambda_m - \gamma^2/4}\, t},$$

describing damped oscillations. These correspond to the standing-wave resonance modes observed in ultrafast neuroimaging.

13 Cymatic Resonance as Stationary Eigenmode Superposition

Cymatic resonance refers to the phenomenon in which the cortex supports quasi-stationary standing waves whose nodal patterns depend on structural geometry. These modes provide a natural coordination mechanism for distributed neural computation.

The full resonance field is thus modeled as

$$R(\mathbf{r}, t) = \sum_{m=0}^{M} c_m(t)\, u_m(\mathbf{r}) e^{i\omega_m t},$$

where $\omega_m = \sqrt{\lambda_m - \gamma^2/4}$ and M is the number of behaviorally active modes. Under sedation or anesthesia, experimental evidence shows that mode count M decreases, oscillatory power diminishes, and the system shifts toward lower-frequency dynamics.

This behavior is consistent with the spectral damping properties of the above ODEs.

14 Orthogonality and Mode Independence

The orthogonality of eigenfunctions ensures independence of modes in the absence of forcing:

$$\langle u_m, u_n \rangle = \delta_{mn}.$$

Consequently, global patterns of brain activity may be understood as a superposition of independently evolving oscillatory components. More importantly, this structure enables selective modulation: the cortex can activate particular eigenmodes while suppressing others, enabling flexible coordination across large distances without direct propagation of signals.

This resonates with empirical results showing that different cognitive states correspond to different spectral profiles, and that certain pathologies manifest as disruptions in the weights $c_m(t)$ of specific eigenmodes.

15 Spectral Energy and Entropy

Define the mode-wise energy

$$E_m(t) = \frac{1}{2}\left(\dot{a}_m(t)^2 + \lambda_m a_m(t)^2 \right),$$

and total spectral energy

$$E(t) = \sum_{m=0}^{\infty} E_m(t).$$

Under mild assumptions on γ and $F(\Psi, t)$, one can show

$$\frac{dE}{dt} = -\gamma \sum_m \dot{a}_m(t)^2 + \sum_m \dot{a}_m(t) f_m(t).$$

Thus damping decreases energy, while forcing through the RSVP fields may increase it. A balance between the two yields stable resonant patterns.

Define the spectral entropy

$$S_{\text{spec}}(t) = -\sum_{m=0}^{M} p_m(t) \log p_m(t),$$

where

$$p_m(t) = \frac{E_m(t)}{\sum_{n=0}^{M} E_n(t)}.$$

High spectral entropy corresponds to distributed activation across modes, while low entropy corresponds to concentration in a small number of modes. Experimental results show anesthesia lowers spectral entropy, consistent with concentration in low-frequency modes.

16 Spatial Localization and Nodal Domains

Eigenfunctions u_m exhibit nodal domain structure: the number of connected regions of positive or negative sign increases with λ_m. Theorems of Courant and Pleijel provide bounds on the number of nodal domains.

This yields the intuitive result that higher-frequency modes encode finer spatial structure. When mapped to cortical computation, lower-frequency modes correspond to integrating information over longer spatial distances, while higher-frequency modes support more localized computations.

Together, these properties explain the hierarchical temporal structure observed in cortical language processing: deeper integration windows correspond to lower-frequency resonant modes.

17 Interaction of Eigenmodes with Local Nonlinear Operators

Let $\mathcal{A}$ be an amplistwistor and u_m an eigenmode. The interaction between the two defines the core of resonance-gated recursion. We compute

$$\mathcal{A}(\Psi)(\mathbf{r}) = \pi\Big(\alpha(\Psi(\mathbf{r}))\tau(\Psi(\mathbf{r}))\Psi(\mathbf{r})\Big).$$

Multiplying by an eigenmode $u_m(\mathbf{r})e^{i\omega_m t}$ yields

$$\mathcal{A}(\Psi)(\mathbf{r}) \odot u_m(\mathbf{r})e^{i\omega_m t}.$$

Integrating against u_n over the manifold gives coupling terms

$$K_{mn}(t) = \int_{\mathcal{M}} \Big[\mathcal{A}(\Psi)(\mathbf{r})u_m(\mathbf{r})\Big]u_n(\mathbf{r})\, d\text{vol}_g.$$

In general, $K_{mn}(t)$ need not vanish, yielding a mode-mixing phenomenon. But if amplist-wistors operate primarily on local semantic fields while eigenmodes govern global coherence, then $K_{mn}(t)$ becomes small for $m \neq n$, preserving approximate independence of modes and permitting low-dimensional control of global behavior.

This establishes the mathematical foundation of cymatic resonance.

18 Recursive TARTAN Dynamics on RSVP Fields

We now integrate the spectral analysis with a principled recursion mechanism. The Trajectory-Aware Recursive Tiling with Annotated Noise (TARTAN) framework provides a method for iteratively updating a field state in a manner sensitive to historical trajectories, local curvature, and stochastic perturbations.

Let $\Psi_t = (\Phi_t, \mathbf{v}_t, S_t)$ denote the RSVP field state at time t. A single TARTAN update may be formalized as an operator

$$\mathcal{R} : \Psi_t \mapsto \Psi_{t+\delta t},$$

constructed from three components:

1. A *trajectory buffer* storing past field states $\{\Psi_{t-k\delta t}\}_{k=1}^{K}$;

2. A *recursive tiling* of the manifold $\mathcal{M}$ at multiple scales, enabling localized updates;

3. An *annotated noise field* ξ_t providing controlled stochasticity.

Let $\mathcal{T}_j$ denote the j-th tile at scale j. On each tile we apply a local amplistwistor $\mathcal{A}_{j,t}$ followed by a relaxation governed by RSVP dynamics. A single update is

$$\Psi_{t+\delta t} = \Psi_t + \delta t \Big(\mathcal{L}(\Psi_t) + \sum_j \mathcal{A}_{j,t}(\Psi_t|_{\mathcal{T}_j}) \Big) + \xi_t,$$

where $\mathcal{L}$ is the RSVP differential operator encoding scalar–vector–entropy couplings.

This defines the fundamental recursive law:

$$\Psi_{t+\delta t} = (1 + \delta t \, \mathcal{L})\Psi_t + \delta t \sum_j \mathcal{A}_{j,t}(\Psi_t|_{\mathcal{T}_j}) + \xi_t.$$

19 Resonance-Gated Recursion

We now incorporate the global resonance field $R(\mathbf{r}, t)$ into the update law. Let the gating function $G_m(t)$ for mode m be defined by

$$G_m(t) = \mathrm{Re}\left(e^{i\omega_m t}\right) = \cos(\omega_m t).$$

Define the resonance kernel

$$\Gamma(\mathbf{r}, t) = \sum_{m=0}^{M} c_m(t) u_m(\mathbf{r}) G_m(t).$$

This kernel modulates the strength of local updates, yielding the **resonance-gated TARTAN rule**

$$\Psi_{t+\delta t}(\mathbf{r}) = \Psi_t(\mathbf{r}) + \delta t \, \Gamma(\mathbf{r}, t) \Big[\mathcal{L}(\Psi_t)(\mathbf{r}) + \sum_j \mathcal{A}_{j,t}(\Psi_t|_{\mathcal{T}_j})(\mathbf{r}) \Big] + \xi_t(\mathbf{r}).$$

Thus resonance modes couple to nonlinear transformations, coordinating distributed computation. Eigenmodes with longer time constants ω_m^{-1} gate slower, deeper semantic recursions, consistent with temporal hierarchies in language cortex.

20 Reduced-Order Dynamics via Mode Projection

Projecting onto an eigenbasis gives a concise representation. Let

$$\Psi_t(\mathbf{r}) = \sum_{m=0}^{\infty} a_m(t) u_m(\mathbf{r}), \qquad \Gamma(\mathbf{r}, t) = \sum_{n=0}^{M} b_n(t) u_n(\mathbf{r}),$$

then

$$\dot{a}_k(t) = \sum_{m,n} b_n(t) \langle u_k, u_n \big(\mathcal{L}(a_m u_m) + \sum_j \mathcal{A}_{j,t}(a_m u_m) \big) \rangle + \eta_k(t).$$

Orthogonality simplifies the dynamics:

$$\dot{a}_k(t) = b_k(t) \big(\mathcal{L}_k(a_k) + \mathcal{A}_{k,t}(a_k) \big) + \eta_k(t) + \text{higher-order mixing terms.}$$

The linear independence of modes allows low-dimensional modeling of high-dimensional cortical activity.

21 Stability and Fixed Points

Define a free-energy-like functional

$$\mathcal{F}(\Psi) = \int_{\mathcal{M}} \left(\frac{1}{2} |\nabla \Phi|^2 + \frac{1}{2} |\mathbf{v}|^2 + U(S) \right) d\text{vol}_g,$$

where U is convex in entropy S. Under mild assumptions, RSVP dynamics yield a gradient flow structure:

$$\dot{\Psi} = -\nabla_\Psi \mathcal{F} + \Xi(\Psi),$$

where Ξ collects non-potential and oscillatory terms.

Fixed points satisfy

$$\nabla_\Psi \mathcal{F} = \Xi(\Psi),$$

or if Ξ averages to zero over resonance cycles:

$$\nabla_\Psi \mathcal{F} = 0.$$

Thus stable cognitive states correspond to minima of $\mathcal{F}$. Experimentally, these are observed as periods of stable semantic interpretation.

Nonlinear Stability

Linearizing around a fixed point Ψ^*:

$$\delta\dot{\Psi} = D\mathcal{L}(\Psi^*)\delta\Psi + \sum_j D\mathcal{A}_j(\Psi^*)\delta\Psi.$$

Stability requires

$$\lambda_{\max}\left(D\mathcal{L} + \sum_j D\mathcal{A}_j\right) < 0.$$

Resonance can enlarge the basin of attraction by modulating effective eigenvalues, a phenomenon supported by empirical observations of phase-locking enhancing integration.

22 Predictions for Neuroscience

The unified model yields several experimentally testable predictions:

1. Cross-Frequency Gating of Semantic Depth

Higher-level linguistic integration should correspond to increased alignment with lower-frequency eigenmodes. MEG/EEG should reveal phase-locking between inferior frontal cortex and slow cortical potentials during deep semantic tasks.

2. Mode-Specific Impairments

Disruption of specific eigenmodes (due to lesions, anesthesia, or stimulation) should selectively impair computations associated with their temporal windows.

3. Nonlinear Necessity

Linearized models should fail to reproduce the observed temporal hierarchy in cortical areas involved in semantics, confirming the amplistwistor prediction.

23   Implications for Artificial Intelligence

The analysis suggests that feedforward transformer architectures approximate a single-step recursion lacking explicit resonance gating. A more biologically faithful model would incorporate:

1. recursive updates rather than deep stacks,

2. eigenmode decomposition as a global clocking mechanism,

3. nonlinear amplistwistor operations as local universal approximators.

Such models may exhibit greater sample efficiency, temporal stability, and interpretability.

24   Conclusion

We have presented a unified theoretical framework integrating relativistic field dynamics, recursive semantic updates, nonlinear local operators, and global eigenmode synchronization. The resulting formalism provides a mathematically coherent account of cortical computation that aligns with empirical findings from ECoG, fMRI, and systems neuroscience. The spectral properties of the cortical manifold supply natural oscillatory modes, while recursive tiling and amplistwistors generate local nonlinear transformations. Together, these yield a resonance-gated recursive engine for cognition that bridges the conceptual gap between neuroscience and artificial intelligence.

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