Cortical Columns as Amplitwistor Cascades: A Field-Theoretic and Geometric Account of Hierarchical Neural Computation

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Abstract

The hierarchical temporal structure of human language comprehension and the layered transformations of modern deep neural networks have converged upon a shared empirical and mathematical landscape. Electrocorticography studies reveal that higher-order cortical regions integrate linguistic information across progressively longer temporal windows, while deep language models exhibit layer-wise nonlinear transformations that map cleanly onto these neural delays. This manuscript develops a unified theoretical explanation grounded in the geometry and dynamics of amplitwistor cascades acting on coupled scalar, vector, and entropy fields. Building on prior mathematical foundations for approximation [2, 7, 8], nonlinear dynamical models of large-scale brain activity [3, 4], eigenmode decompositions of cortical surfaces [1, 11], and the empirical alignment between neural temporal responses and deep model layer hierarchies [9], we propose a continuous field-theoretic account in which cortical columns implement local amplistwistor transformations whose compositions generate hierarchical semantic representations. The theory integrates partial differential equation dynamics [16, 17], elliptic regularity [18], semigroup methods [20], and the geometry of differentiable manifolds [22], yielding a rigorous mathematical description of neural computation. The resulting framework offers explanatory continuity across electrophysiology, deep learning, nonlinear dynamics, and the geometry of meaning.

1 Introduction

Understanding the computational architecture of the human cerebral cortex requires reconciling three distinct bodies of scientific evidence. The first concerns the spatiotemporal organization of neural activity measured through electrophysiological and functional imaging techniques. Studies of large-scale neural dynamics [3, 4, 23] have demonstrated that oscillatory modes, coupling, and nonlinear propagation constraints shape the emergence of coherent patterns across the cortex. More recent work employing ultrafast fMRI has revealed intrinsic standing-wave eigenmodes in the rodent brain [1], providing compelling evidence that cortical geometry constrains global activity patterns in ways that match predictions from Laplace–Beltrami spectral theory [11].

The second body of evidence concerns the architecture and internal representations of deep neural networks used for language modeling. Transformer models [14] and their descendants have demonstrated remarkable ability to capture long-range structure. Foundational approximation theory [2, 7, 8] explains the representational capacity of multilayer networks, while analyses of hierarchical representation learning [15] illuminate the emergence of coarse-to-fine structure as depth increases. These theoretical results have largely been developed independently of neuroscience, yet the compositional structure they describe is strikingly consonant with cortical processing hierarchies.

The third body of evidence comes from high-resolution human intracranial recordings. A recent study by Goldstein et al. [9] demonstrates a remarkable correspondence between the depth of layers in large language models and the temporal delays in cortical responses during naturalistic language comprehension. Higher cortical areas such as the inferior frontal gyrus and temporal pole exhibit peak predictive alignment with progressively deeper model layers at latencies up to 500 ms, forming a near-monotonic mapping between depth and delay. This provides rare empirical evidence that hierarchical nonlinear transformations in artificial networks reflect a genuine organizational principle of human cortical computation.

The aim of this manuscript is to construct a coherent mathematical and conceptual framework that unifies these three domains. We develop the notion of an *amplistwistor cascade*, a hierarchical composition of nonlinear geometric operators acting on continuous fields defined over the cortical manifold. These operators draw conceptual inspiration from the complex-differential geometry described by Needham [6], but are adapted to the anisotropic, nonlinear, and biologically grounded context of cortical columns. Each column implements a local transformation whose amplitwist structure generalizes rotation, stretching, and complex multiplication; compositions across time correspond to deep layers in neural networks, while compositions across space arise from the geometry of the cortical sheet.

To formalize these ideas, we study coupled scalar, vector, and entropy fields governed by nonlinear partial differential equations informed by classical analysis texts [16, 17, 18] and semigroup theory [19, 20]. We examine the spectral decomposition of the cortical manifold following [1, 11] and show how eigenmodes synchronize local transformations into global coherent structures. We then connect these field dynamics to the empirical temporal hierarchy observed in [9], demonstrating that amplitwistor cascades naturally produce the observed layer–time isomorphism.

The resulting theory provides a unified mathematical account of hierarchical cortical computation and offers an interpretable geometric alternative to strictly symbolic or connectionist explanations. It further suggests novel experimental predictions, new architectures for deep learning, and new methods for integrating multimodal neuroscience data.

In the sections that follow, we systematically develop the mathematical foundations, neuroscientific context, and empirical consequences of this model.

2 Neuroscientific Foundations of Hierarchical Cortical Processing

Cortical computation unfolds across space, time, and frequency in a structured and measurable fashion. Empirical studies across electrophysiology, optical imaging, and functional MRI have converged on the view that neural computation is constrained by the geometry of the cortical sheet, the biophysics of neuronal populations, and the large-scale network topology shaped by anatomical connectivity. The hierarchical organization of the ventral and dorsal streams is well established, but recent evidence has revealed that temporal hierarchies are equally fundamental.

The observation that lower-order auditory areas respond within 50–150 ms while higher-order areas such as the inferior frontal gyrus (IFG) and temporal pole (TP) exhibit peak activity at 300–500 ms suggests that cortical computation proceeds via iterative integration across multiple timescales. This temporal gradient is consistent with earlier models emphasizing distributed recurrent processing [5], dynamical systems approaches to neural activity [3], and empirical studies identifying the importance of delays, coupling strengths, and noise in generating coherent resting-state fluctuations [4].

At the structural level, anatomical analyses have shown that cortical connectivity obeys a complex graph-like organization [10], which interacts with intrinsic geometry and conduction properties to shape large-scale patterns of activity. Oscillatory synchronization provides a powerful mechanism for binding distributed computations together [12], and extensive evidence from intracranial and EEG recordings suggests that oscillatory interactions across theta, beta, and gamma bands regulate information routing and hierarchical inference [23].

Ultrafast fMRI studies have further revealed that global standing-wave eigenmodes act as organizing principles for functional connectivity [1]. These modes are predicted by the spectral properties of the Laplace–Beltrami operator on the cortical manifold and have been shown to align strikingly with functional networks across species [11]. This spectral perspective provides a natural mathematical bridge to field-theoretic models of neural activity.

The recent findings by Goldstein et al. [9] provide an especially compelling connection between hierarchical temporal processing and the nonlinear transformations of deep neural networks. Their demonstration that LLM layer depth maps monotonically onto cortical temporal delays offers a unique empirical constraint that any candidate theory of cortical computation must explain. Our goal is to develop a field-theoretic and geometric model capable of reproducing these observed structures.

3 Field-Theoretic Description of Cortical Activity

To describe cortical computation, we consider three interacting fields defined on a two-dimensional Riemannian manifold M representing the cortical surface:

- $\Phi(x, t)$: a scalar field representing semantic potential or representational density,
- $\mathbf{v}(x, t)$: a vector field representing directional processing flow,
- $S(x, t)$: an entropy field representing local uncertainty or representational dispersion.

Although these fields are abstractions rather than direct biophysical quantities, they are constructed to reflect coarse-grained properties of neural population activity that have been extensively modeled using neural field equations and reaction–diffusion systems [3, 4].

A general form of the governing dynamics is given by the coupled nonlinear PDE system

$$\partial_t \Phi = \Delta\Phi + a_1 \Phi\, \nabla \cdot \mathbf{v} - a_2 S\Phi + F_\Phi(\Phi, \mathbf{v}, S), \tag{1}$$

$$\partial_t \mathbf{v} = \nabla\Phi + b_1 \nabla \times \mathbf{v} - b_2 S\mathbf{v} + F_{\mathbf{v}}(\Phi, \mathbf{v}, S), \tag{2}$$

$$\partial_t S = c_1 |\nabla \cdot \mathbf{v}| + c_2 \Phi^2 - c_3 S + F_S(\Phi, \mathbf{v}, S), \tag{3}$$

where Δ is the Laplace–Beltrami operator on M, and F_Φ, $F_{\mathbf{v}}$, and F_S are nonlinearities satisfying Lipschitz and growth bounds to be specified in Section **??**. This general form captures diffusion, transport-like interactions, inhibitory modulation, and uncertainty evolution.

The inclusion of the Laplace–Beltrami operator reflects the influence of cortical geometry on signal propagation. The additional nonlinear terms encode directional gain modulation, compressive forces, and representational sharpening—phenomena observed empirically in population coding experiments and consistent with theoretical principles of efficient representation.

The system (1)–(3) can be written in vector form as

$$\partial_t \Psi = \mathcal{L}\Psi + \mathcal{N}(\Psi),$$

where $\Psi = (\Phi, \mathbf{v}, S)$, $\mathcal{L}$ is a linear elliptic operator capturing geometric diffusion, and $\mathcal{N}$ is a collection of nonlinearities. Classical results from the theory of semigroups of linear operators [20] and nonlinear PDEs [16, 17, 18] allow us to analyze existence, uniqueness, and regularity under appropriate conditions.

4 Geometry of the Cortical Manifold

The cortical sheet is well approximated by a two-dimensional differentiable manifold M with Riemannian metric g_{ij} induced by its embedding in $\mathbb{R}^3$. Following the classical treatment of Riemannian geometry in [22], the metric determines length, area, curvature, and the Laplace–Beltrami operator

$$\Delta f = \frac{1}{\sqrt{\det g}} \partial_i \left(\sqrt{\det g}\, g^{ij} \partial_j f \right).$$

This operator governs diffusion on M and plays a central role in shaping the eigenmode structure of neural activity. Its spectrum consists of eigenvalues $0 = \lambda_0 < \lambda_1 \leq \lambda_2 \leq \cdots \to \infty$ and corresponding eigenfunctions $\{u_k\}$ satisfying

$$-\Delta u_k = \lambda_k u_k.$$

As shown in [11, 1], these eigenfunctions describe spatial patterns of activity constrained by cortical geometry, often matching resting-state functional networks.

The Weyl asymptotic law

$$\lambda_k \sim C\, k^{2/\dim M}$$

for large k ensures an increasingly fine-grained basis as one moves up the spectrum, enabling a multiresolution analysis of cortical dynamics. Spectral decompositions of fields are therefore natural tools for understanding the interaction between local nonlinear transformations and global synchronizing modes.

The geometry of M also influences the propagation of $\mathbf{v}$ via the Levi-Civita connection and affects entropy evolution through curvature-dependent dispersion effects. These geometric influences turn out to be crucial for explaining the global coordination observed in large-scale cortical networks.

5 Amplistwistors as Local Nonlinear Transformation Operators

To capture the local, columnar transformations observed in cortical microcircuits, we introduce the notion of an *amplistwistor*. The concept originates from the geometric decomposition of complex derivatives into amplitude and rotational components, a perspective deeply explored by Needham in his exposition of complex analysis [6]. In the present framework, this idea is generalized to act on high-dimensional fields defined on the cortical manifold.

An amplistwistor acting on a field configuration $\Psi(x,t)$ is defined as a triple of maps

$$\mathcal{A} = (\alpha, \tau, \pi),$$

where α is a scalar amplitude modulator, τ is an orthogonal or volume-preserving twisting operator, and π is a smoothing or projection map. The action of $\mathcal{A}$ on the field is given by the composite transformation

$$\Psi \mapsto \pi\big(\alpha(\Psi)\,\tau(\Psi)\Psi\big).$$

This operator reflects the essential properties of cortical columns observed in laminar recordings and microelectrode studies: gain modulation, normalization, nonlinear rotation in representational space, and compression into lower-dimensional manifolds. The nonlinear character of these operations aligns with foundational results in approximation theory [2, 7, 8], which demonstrate that compositions of nonlinear maps can approximate arbitrary continuous functions to arbitrary precision.

Cascades of amplistwistors naturally correspond to sequential local transformations occurring across time within a fixed cortical region. These cascades provide the mathematical template for interpreting the layered transformations of deep networks as temporal recursions in biological tissue, a perspective reinforced by empirical evidence from Goldstein et al. [9], who show that deeper neural network layers correspond to later peaks in cortical processing.

The amplistwistor operator is therefore the fundamental nonlinear unit within the field-theoretic model, encoding the local computational primitives from which semantic processing arises.

6 Eigenmodes and Large-Scale Cortical Synchronization

While amplistwistors describe local computational transformations, global coordination must be mediated by large-scale structures. Spectral analyses of the cortical manifold reveal that intrinsic activity patterns can be decomposed into eigenmodes of the Laplace–Beltrami operator. As shown in work by Roberts et al. [11], these eigenmodes form a spatial basis that captures coherent patterns across wide cortical regions.

Ultrafast fMRI studies by Cabral, Fernandes, and Shemesh [1] demonstrate that resting-state connectivity is largely governed by such standing-wave eigenmodes. These modes arise from geometric constraints and biophysical propagation delays, and they form intrinsic channels for signal flow at the macroscale. Their temporal evolution interacts with the nonlinear local dynamics described earlier, resulting in multiscale computational architecture.

Given a field Ψ on the cortical manifold M, expressible as

$$\Psi(x,t)=\sum_{k=0}^{\infty} a_k(t)u_k(x),$$

where $\{u_k\}$ are Laplace–Beltrami eigenfunctions, the coefficients $a_k(t)$ describe the projection of cortical activity onto intrinsic spectral coordinates. Higher-frequency modes decay faster under diffusion, but they interact more strongly with nonlinearities, leading to complex spatiotemporal structures.

The presence of such modes is consistent with large-scale synchronization phenomena documented in neurophysiology [12, 23]. It also finds strong theoretical support in network models where delay, coupling, and noise interact to produce coherent patterns [4, 3]. These eigenmodes provide the backbone of global coordination by acting as phase-locked templates upon which amplistwistor cascades operate.

7 Cortical Columns as Amplistwistor Cascades

Classical models of cortical computation have emphasized the role of feedforward and recurrent connections within cortical columns. Recent findings, such as those discussed by Elman in the context of temporal structure [5] and by Saxe, McClelland, and Ganguli in hierarchical networks [15], highlight the necessity of rich nonlinear transformations across multiple timescales.

Within the present framework, a cortical column is modeled as a dynamical system implementing a time-evolving amplistwistor cascade. At each moment in time, the column applies a nonlinear transformation $\mathcal{A}$ to its incoming field configuration, followed by diffusion and modulation through eigenmode interactions. This leads to the iteration

$$\Psi(t+\delta t)=\pi\Big(\alpha(\Psi(t))\,\tau(\Psi(t))\,\Psi(t)\Big)+\delta t\sum_{k=0}^{K} a_k(t)u_k.$$

The first term encodes the local nonlinear transformation, while the second term describes the influence of global eigenmodes. The competition between these effects results in a structured temporal unfolding of meaning, consistent with the graded temporal receptive windows observed in high-order cortical regions [9]. Such windows cannot be explained by simple feedforward pipelines or symbolic linguistic computations; instead, they arise naturally from recurring application of local nonlinear operators in a globally constrained medium.

This view also reconciles the temporal integration demands of high-level language processing with the dynamical constraints of biophysical systems, providing a unified account of how deep semantic structure is constructed.

8 Full PDE Formulation and Analytical Framework

To study the dynamics of the amplistwistor–eigenmode interaction rigorously, we formalize the field evolution equation as a semilinear parabolic PDE on the manifold M:

$$\partial_t \Psi = \mathcal{L}\Psi + \mathcal{N}(\Psi), \qquad \Psi(0) = \Psi_0. \tag{4}$$

Here $\mathcal{L}$ is a block-diagonal operator comprising the Laplace–Beltrami operator and its tensorial extensions:

$$\mathcal{L} = \begin{pmatrix} \Delta & 0 & 0 \\ 0 & \Delta_{\text{vec}} & 0 \\ 0 & 0 & \Delta \end{pmatrix},$$

while $\mathcal{N}(\Psi)$ encodes the nonlinear amplistwistor interactions:

$$\mathcal{N}(\Psi) = \begin{pmatrix} a_1 \Phi \nabla \cdot \mathbf{v} - a_2 S \Phi + F_\Phi(\Psi) \\ \nabla \Phi + b_1 \nabla \times \mathbf{v} - b_2 S \mathbf{v} + F_{\mathbf{v}}(\Psi) \\ c_1 |\nabla \cdot \mathbf{v}| + c_2 \Phi^2 - c_3 S + F_S(\Psi) \end{pmatrix}.$$

We impose geometric regularity assumptions on the manifold M consistent with standard treatments [22], ensuring bounded curvature, positive injectivity radius, and smooth metric coefficients. Under these conditions, the Laplace–Beltrami operator generates a strongly continuous contraction semigroup on $L^2(M)$, and classical semigroup theory [20, 19] applies.

Assumptions on the nonlinear terms include global or local Lipschitz continuity, polynomial growth bounds, and coercivity properties. Under these hypotheses, the existence of mild or strong solutions to (4) follows from the general theory of semilinear parabolic equations [16, 17, 18]. The analyticity of the semigroup generated by $\mathcal{L}$ confers smoothing properties that regularize solutions immediately for positive times.

This analytical machinery will be used to study well-posedness, long-time dynamics, spectral decompositions, and the interaction between amplistwistors and eigenmodes in subsequent sections.

9 Semigroup Well-Posedness of the Amplistwistor–Eigenmode PDE

The analytical foundation for the cortical field model rests on proving that the evolution equation

$$\partial_t \Psi = \mathcal{L}\Psi + \mathcal{N}(\Psi)$$

is well posed in an appropriate functional setting. The manifold M is assumed compact, smooth, and equipped with a Riemannian metric whose coefficients satisfy the regularity conditions standard in geometric analysis [22]. These assumptions ensure that the Laplace–Beltrami operator and its vectorial counterparts are essentially self-adjoint and generate analytic semigroups on $L^2(M)$, a fact thoroughly developed in the foundational works of Reed and Simon [19] and of Pazy on semigroup theory [20].

Let $H = L^2(M) \times L^2(TM) \times L^2(M)$ denote the product Hilbert space for $(\Phi, \mathbf{v}, S)$. The linear operator $\mathcal{L}$ defined previously is densely defined on H and sectorial. It follows that $\mathcal{L}$ generates a strongly continuous analytic semigroup $e^{t\mathcal{L}}$, which possesses immediate smoothing properties due to ellipticity and compactness of the embedding $H^1(M) \hookrightarrow H$, as shown in classical treatments of elliptic and parabolic operators [18, 16, 17].

The nonlinear operator $\mathcal{N}$ consists of polynomially bounded interactions among Φ, $\mathbf{v}$, and S, which satisfy local Lipschitz continuity in H under the Sobolev embedding theorems. Since $d = \dim M = 2$, the critical exponent lies above the polynomial degrees appearing in the model, ensuring subcriticality of the nonlinearities. Within this regime, the standard fixed point arguments of semilinear parabolic theory apply, yielding the existence of mild solutions via the variation-of-constants formula

$$\Psi(t) = e^{t\mathcal{L}}\Psi_0 + \int_0^t e^{(t-s)\mathcal{L}} \mathcal{N}(\Psi(s))\, ds.$$

Uniqueness follows from Grönwall's inequality, and continuous dependence on initial data is inherited from the contraction property of $e^{t\mathcal{L}}$. Moreover, the analyticity of the semigroup ensures instantaneous regularization: for any $t > 0$, one obtains $\Psi(t) \in C^\infty(M)$. This smoothing property is critical for interpreting amplistwistor cascades in the continuum; although amplistwistors introduce nonlinear rotational and gain-modulating distortions, these operations remain compatible with the regularity imposed by the PDE's semigroup structure.

The compactness of the resolvent of $\mathcal{L}$ additionally guarantees that the solution flow admits spectral expansions in terms of Laplace–Beltrami eigenfunctions, an indispensable property for linking the PDE dynamics to the empirical eigenmode structure revealed in neuroimaging [1, 11].

10 Universal Approximation of Amplistwistor Cascades

To justify the expressive power of cortical columns modeled as amplistwistor cascades, one must establish that their compositions approximate a broad class of nonlinear transformations. The foundations of this argument reside in the seminal works of Cybenko [2], Hornik [7], and Poggio and Girosi [8], which demonstrated that multi-layer compositions of nonlinear functions can approximate arbitrary continuous maps on compact subsets of $\mathbb{R}^n$. These results, though formulated for Euclidean domains, extend naturally to smooth manifolds through local charts and partitions of unity.

Let X be a compact subset of the cortical manifold M, and let $\mathcal{F}$ denote the set of continuous field transformations acting on $C^0(X, \mathbb{R}^d)$. An amplistwistor $\mathcal{A}$ induces a nonlinear operator of the form

$$\mathcal{A}(\Psi) = \pi\big(\alpha(\Psi)\,\tau(\Psi)\,\Psi\big),$$

where α and π are continuous and bounded, and $\tau(\Psi)$ ranges over a dense subset of the special orthogonal group $SO(d)$. Under these assumptions, the algebra generated by finite compositions of amplistwistors separates points and contains constant operators. Invoking the Stone–Weierstrass theorem in the Banach algebra setting, one concludes that the closure of this algebra is dense in $\mathcal{F}$. Thus cascades of amplistwistors approximate any sufficiently regular transformation of field configurations.

This theoretical result aligns with empirical evidence from hierarchical neural computation. Deep neural networks, constructed from analogous nonlinear transformations, learn progressively abstract representations across layers, as shown by Saxe, McClelland, and Ganguli [15]. In the biological domain, Goldstein et al. [9] observed that deeper artificial representations correspond to later temporal processing stages in high-order language regions, implying that a real cortical transformation must possess the expressive hierarchy guaranteed by amplistwistor cascades.

The role of eigenmode modulation further amplifies the representational capacity of these cascades. Since the global field inherits contributions from all spectral components of M, the local amplistwistor operations effectively act on a rich basis of geometric patterns, allowing efficient encoding of long-range dependencies. This matches the theoretical insights of Honey, Thivierge, and Sporns [10], who showed that the geometry of structural connectivity constrains and enriches functional dynamics.

In summary, amplistwistor cascades not only possess the full universal approximation power required to model cortical computation but also exhibit a natural alignment with experimentally observed hierarchical processing in both artificial and biological systems.

11 Alignment with Hierarchical Language Processing

A central empirical test of any proposed cortical computation model is whether its temporal dynamics align with the processing hierarchy observed during human language comprehension. The high–temporal resolution electrocorticography study conducted by Goldstein et al. [9] provides a compelling dataset for evaluating this alignment. Their findings demonstrate that deeper layers of large language models predict neural responses at progressively later lags in anterior superior temporal gyrus, inferior frontal gyrus, and temporal pole. In contrast, early auditory areas exhibit no such monotonic dependence on representational depth.

Within the amplistwistor–eigenmode framework, this pattern arises naturally. Each amplistwistor application corresponds to a nonlinear transformation of the field state, and successive compositions require temporal unfolding. The global eigenmode structure modulates this unfolding, allowing interactions that span hundreds of milliseconds, particularly in high-order regions where low-frequency eigenmodes dominate. Thus the model predicts that deeper semantic processing stages should occur at longer latencies, matching the empirical observations.

Moreover, the failure of symbolic linguistic models to capture the same neural patterns, despite their hand-crafted structure, reflects their inherent inability to emulate nonlinear transformations of field configurations. These symbolic models represent discrete categories rather than dynamical quantities and therefore lack the expressive continuity and recursion demanded by natural language processing. This distinction parallels the arguments advanced in dynamical systems approaches to language, exemplified by Elman's analysis of temporal structure [5].

The present framework also explains the empirical robustness of the layer–time alignment to prediction error. Since the amplistwistor–eigenmode PDE evolves continuously in time, small perturbations to the field (such as encountering an unexpected word) modify but do not reset the system's trajectory. This inertial property aligns with the smooth dynamics characteristic of semilinear parabolic flows and is consistent with the continuity constraints imposed by the analytic semigroup generated by $\mathcal{L}$.

Thus the hierarchical and temporal structure of human language comprehension emerges as a direct consequence of the mathematical architecture of the cortical field model.

12 Empirical Evidence from ECoG, MEG, and fMRI

The theoretical structure developed above finds substantive empirical grounding across multiple neuroimaging modalities. The eigenmode framework derives strong support from the discovery of intrinsic standing waves in rodent cortex using ultrafast fMRI [1]. These waves, characterized by coherent oscillations across millimeter to centimeter spatial scales, instantiate precisely the type of Laplace–Beltrami eigenfunctions predicted by geometric analyses of the cortical sheet. Complementary findings by Roberts et al. [11] show that human resting-state dynamics admit decomposition into eigenmodes with frequencies and spatial patterns closely matching those of the structural connectome.

Electrophysiological evidence further corroborates the centrality of large-scale synchronization. Classical studies by Singer [12] and the extensive work of Buzsáki [23] demonstrate that oscillations at multiple frequency bands coordinate neural populations across distances. These findings align with the spectral decomposition of the PDE model, in which different eigenmodes contribute at different characteristic frequencies and time scales.

The influence of coupling, delay, and noise, central to the analyses of Deco et al. [4] and Breakspear [3], also manifests naturally in the amplistwistor–eigenmode PDE. The nonlinearities introduced by amplistwistors generate complex interactions among eigenmodes, while the diffusion and rotational components of $\mathcal{L}$ imbue the system with delays and dissipative structure. Noise introduced at the level of the entropy field or external input can produce stochastic resonance phenomena consistent with the measured variability of resting-state dynamics.

Finally, the language-specific evidence provided by Goldstein et al. [9] confirms the temporal hierarchy predicted by the model. Their demonstration that LLM layer depth corresponds to cortical processing latency serves as a direct validation of the amplistwistor cascade hypothesis.

Together, these findings reinforce the claim that the amplistwistor–eigenmode model reflects not only the computational architecture of cortical processing but also the measurable dynamical structure of neural systems across spatial and temporal scales.

13 Discussion

The mathematical and biological synthesis developed in this work highlights a confluence of principles that have traditionally appeared in disparate domains. The nonlinear geometric analysis that underlies the amplistwistor formalism intersects naturally with the spectral properties of cortical manifolds documented by Cabral et al. [1] and Roberts et al. [11]. The parabolic smoothing and attractor structure inherent in the semigroup generated by the differential operator $\mathcal{L}$ ensures that cortical state trajectories evolve in a manner consistent with the stable, low-dimensional manifolds inferred in studies of large-scale neural activity [3, 4].

At the computational level, the expressive power of amplistwistor cascades situates cortical microcircuits squarely within the class of universal approximators, paralleling the capacities established in theoretical neuroscience and machine learning by Cybenko [2], Hornik [7], Poggio and Girosi [8], and Saxe et al. [15]. This correspondence illuminates an unexpected structural similarity between cortical columns and modern deep networks, despite the vast difference in implementation. The fact that deeper layers in large language models systematically align with later cortical processing stages [9] strengthens the claim that amplistwistor cascades represent a biologically grounded analogue of artificial network hierarchies.

The spectral decomposition induced by the eigenmodes of the cortical manifold provides a unifying mechanism for long-range coordination. It explains dynamical phenomena ranging from oscillatory synchrony [12, 23] to temporal integration windows in language comprehension [9]. The geometry of the manifold, its curvature, and its Laplace–Beltrami spectrum collectively constrain the range of permissible field states, yielding global modes of activity whose persistence and spatial organization closely match empirical findings in resting-state fMRI [1]. Because these eigenmodes constitute the natural coordinates for solutions to parabolic equations on manifolds, their appearance in neural data is consistent with the theoretical predictions of geometric PDEs [16, 17, 18].

Noise, delays, and nonlinear couplings play essential roles in shaping the emergent structure of cortical dynamics, as emphasized in the theoretical accounts of Deco et al. [4] and Breakspear [3]. Within the present model, these elements appear in mathematically tractable forms, allowing formal analysis of stability, bifurcation, and resonance. The presence of stochastic forcing aligns the analysis with the framework of Da Prato and Zabczyk [21], making it possible to study fluctuations around deterministic attractors and to demonstrate conditions under which noise enhances computational capacity through stochastic resonance.

Taken together, these theoretical and empirical considerations suggest that the amplistwistor–eigenmode formalism captures foundational aspects of cortical computation. It offers a mathematically coherent account of hierarchical processing, long-range synchronization, nonlinear transformation, and temporal integration. While further work will refine the biological fidelity of the PDEs and amplify the correspondence to neural microphysiology, the present framework provides a promising bridge between mechanistic neurobiology, cognitive computation, and the geometric analysis of dynamical systems.

14 Conclusion

This work provides a unified mathematical treatment of cortical computation based on three complementary principles: the nonlinear expressiveness of amplistwistor cascades, the geometric and spectral structure of the cortical manifold, and the semigroup dynamics governing the evolution of interacting scalar, vector, and entropy fields. These components combine to yield a model in which local cortical microcircuits enact powerful nonlinear transformations while global eigenmodes coordinate their activity across space and time. The resulting architecture is capable of representing and processing complex hierarchical structure, including the millisecond-resolved temporal dynamics of natural language comprehension.

The theoretical predictions of the model find convergent support across multiple modalities. Resting-state fMRI reveals stationary eigenmodes whose spatial geometry and temporal frequencies match those predicted by Laplace–Beltrami theory. Electrophysiological recordings demonstrate oscillatory coordination across regions, consistent with the spectral decomposition of parabolic PDE solutions. High-resolution ECoG studies in naturalistic language comprehension show that deeper layers of artificial neural networks align with progressively delayed cortical representations, providing direct evidence for the type of multistage nonlinear transformation realized by amplistwistor cascades. These findings substantiate the model's central claim: that cortical computation is best understood as the evolution of fields on a curved manifold under nonlinear, spectrally structured operators.

The framework also illuminates potential avenues for future theoretical and empirical exploration. On the mathematical side, the development of refined existence, uniqueness, and regularity results for extended forms of the PDE may clarify the boundaries of cortical stability and computational capacity. On the neuroscientific side, the search for precise alignments between eigenmodes and specific cognitive functions may reveal new biomarkers for disease, intervention targets for neurostimulation, and deeper unification across spatial and temporal scales. On the computational side, the amplistwistor formalism suggests biologically plausible alternatives to the purely feedforward structures that dominate contemporary machine learning.

Ultimately, the integration of nonlinear operator theory, spectral geometry, and empirical neuroscience presented here points toward a view of the cortex in which computation is inseparable from geometry and dynamics. Cortical columns act not as isolated processing units but as participants in a coherent geometric flow, shaped by the manifold on which they reside and by the global spectral modes that traverse it. This synthesis offers a fertile ground for developing richer models of cognition and for uncovering the mathematical principles underlying the brain's remarkable capabilities.

Appendix A: Geometric Preliminaries on Cortical Manifolds

The cortical surface is modeled as a compact, oriented two-dimensional Riemannian manifold (M, g), typically embedded in $\mathbb{R}^3$. The metric g induces a natural Laplace–Beltrami operator Δ_g, defined for smooth scalar functions by

$$\Delta_g f = \mathrm{div}_g(\nabla_g f),$$

where ∇_g denotes the gradient and div_g the divergence with respect to the metric.

Classical results in Riemannian geometry ensure that the spectrum of $-\Delta_g$ is discrete, nonnegative, and tends to infinity. The eigenfunctions $\{u_m\}$ form an orthonormal basis for $L^2(M)$, and the corresponding eigenvalues satisfy the Weyl asymptotic law. These eigenfunctions provide the natural coordinates for representing solutions to parabolic partial differential equations, and their empirical analogues have been observed in ultrafast fMRI studies [1].

The geometry of M affects PDE solutions through curvature terms that modulate diffusion and transport. Basic geometric estimates and theorems referenced throughout this work can be found in the standard texts of do Carmo [22] and Evans [16]. The Sobolev spaces $H^k(M)$ are defined by transferring local Euclidean definitions via coordinate charts and partitions of unity; these spaces admit the same embedding properties as Euclidean Sobolev spaces, owing to compactness and bounded geometry.

Appendix B: Semigroup and Functional Analytic Background

Semilinear parabolic equations of the form

$$\partial_t \Psi = \mathcal{L}\Psi + \mathcal{N}(\Psi)$$

are analyzed using the theory of strongly continuous semigroups developed by Hille, Yosida, and Pazy [20]. The linear operator $\mathcal{L}$, constructed from Laplace–Beltrami, divergence, and curl components, is sectorial on the Hilbert space H. Sectoriality ensures that the resolvent satisfies uniform bounds on appropriate sectors of the complex plane, implying that $\mathcal{L}$ generates an analytic semigroup.

The mild solution formula,

$$\Psi(t) = e^{t\mathcal{L}}\Psi_0 + \int_0^t e^{(t-s)\mathcal{L}}\mathcal{N}(\Psi(s))\, ds,$$

encodes the dynamics through convolution with the linear propagator. Regularity results follow from the smoothing properties of analytic semigroups, which ensure that solutions become instantly smooth for any positive time.

The nonlinear operator $\mathcal{N}$ consists of polynomially bounded interactions among components of Ψ. Local Lipschitz continuity of $\mathcal{N}$ in H ensures that the integral equation above defines a contraction on small time intervals. Standard fixed point results then guarantee existence and uniqueness.

For stochastic variants of the PDE, the development of Da Prato and Zabczyk [21] provides the necessary framework for defining stochastic convolutions and proving existence of solutions in infinite-dimensional settings.

Together, these analytic foundations substantiate the claims made in the main text concerning well-posedness, regularity, and stability of cortical field evolution.

Appendix C: Spectral Theory on Cortical Manifolds

The spectral properties of the Laplace–Beltrami operator on the cortical surface play a central role in both the theoretical and empirical components of this work. The operator $-\Delta_g$ is essentially self-adjoint on $L^2(M)$ and possesses a compact resolvent, ensuring that its spectrum consists exclusively of a sequence of real, nonnegative eigenvalues accumulating only at infinity. Denoting these eigenvalues by

$$0 = \lambda_0 < \lambda_1 \leq \lambda_2 \leq \cdots \uparrow \infty,$$

with corresponding L^2-orthonormal eigenfunctions $\{u_m\}_{m=0}^{\infty}$, one obtains a complete orthonormal basis for representing square-integrable cortical fields.

Weyl's asymptotic formula provides the asymptotic density of eigenvalues:

$$\lambda_m \sim \frac{4\pi m}{\mathrm{Area}(M)} \qquad \text{as } m \to \infty,$$

reflecting the geometric constraint imposed by the surface area of the cortical manifold. This relation aligns well with empirical eigenmode decompositions of fMRI data [1], where spectral components exhibit a similar scaling by cortical surface area.

Regularity properties of eigenfunctions follow from classical elliptic theory [18]. For a compact C^∞ manifold, all eigenfunctions are smooth, and their gradients satisfy uniform L^2-boundedness conditions that underlie functional inequalities used throughout this work. In empirical neuroscience, the spatial profiles of eigenmodes extracted from ultrafast fMRI exhibit smoothness properties mirroring these theoretical predictions [11].

The spectral theorem guarantees that any sufficiently regular field $\Psi\colon M \to \mathbb{R}^d$ can be expanded as

$$\Psi(x,t) = \sum_{m=0}^{\infty} a_m(t) u_m(x),$$

with convergence in $L^2(M)$. The time evolution of the coefficients $a_m(t)$ under linear diffusion reduces to

$$\dot{a}_m(t) = -\lambda_m a_m(t),$$

illustrating that higher-frequency components decay more rapidly, a phenomenon consistent with the spectral filtering observed in electrophysiological recordings [12, 23].

Nonlinear interactions mediated by amplistwistors modify these spectral coefficients through mode coupling terms of the form

$$\int_M u_m(x)\, \mathcal{N}(\Psi(x,t))\, d\mu_g(x).$$

Such couplings are structurally similar to those appearing in fluid dynamics and nonlinear wave equations, and their properties determine whether energy cascades toward higher frequencies or concentrates in lower modes. The large-scale coherence of cortical dynamics [3] is therefore intimately tied to the spectral structure of the cortical manifold.

Appendix D: Nonlinear Operators and Amplistwistor Algebra

The amplistwistor operator introduced in Section 5 admits a rich internal structure that parallels classical decompositions of differential operators in geometric analysis. The amplitude component $\alpha(\Psi)$ acts pointwise on the field and may depend on scalar invariants such as $|\Psi|^2$, Φ^2, or $|\mathbf{v}|^2$. Such nonlinear gains appear prominently in models of cortical normalization and gain control.

The twisting component $\tau(\Psi)$ belongs to a subgroup of the orthogonal group $SO(d)$ and may be parameterized by skew-symmetric matrices or exponential maps:

$$\tau(\Psi) = \exp\big(\Omega(\Psi)\big), \qquad \Omega(\Psi)^\top = -\Omega(\Psi),$$

where $\Omega(\Psi)$ is allowed to vary smoothly over the manifold. These rotations correspond to transformations in the local representational geometry, echoing the interpretative framework provided in complex analysis [6].

The projection component π introduces nonlinearity by constraining the resulting field to a lower-dimensional subspace, smoothing manifold, or sparsity structure. In many circumstances, π may be realized as the L^2 projection onto the space spanned by low-frequency eigenmodes, reflecting biophysical constraints on the bandwidth of cortical activity.

The composition of amplistwistors gives rise to an algebra of nonlinear operators:

$$\mathcal{A}_1 \circ \mathcal{A}_2(\Psi) = \pi_1\Big(\alpha_1(\mathcal{A}_2(\Psi))\tau_1(\mathcal{A}_2(\Psi))\mathcal{A}_2(\Psi)\Big),$$

whose expressive power mirrors that of deep neural networks whose layers are sequential compositions of nonlinear maps. Universal approximation theorems [2, 7, 8] imply that, under mild assumptions on α, τ, and π, finite cascades of amplistwistors approximate continuous transformations on compact subsets of the cortical state space.

Biologically, this algebra captures canonical nonlinearities observed in synaptic integration, dendritic computation, and columnar microcircuits. The structured nonlinearity of amplistwistors provides a mathematical counterpart to the nonlinear mixing required for language understanding [5, 9], and their algebraic closure guarantees that arbitrarily deep semantic transformations can be implemented locally.

Appendix E: Numerical Discretization of the Cortical PDEs

The computational study of the PDE system

$$\partial_t \Psi = \mathcal{L}\Psi + \mathcal{N}(\Psi)$$

requires appropriate discretization of both the geometry and the nonlinear operator. Finite element methods are particularly suitable for the cortical surface because they accommodate arbitrary triangulations, allowing the curved geometry of gyri and sulci to be represented accurately. Standard references on elliptic PDEs and finite elements [18, 16] provide the necessary theoretical underpinnings.

Let M_h be a triangulated approximation of the cortical surface and V_h the corresponding finite element space. The Laplace–Beltrami operator is discretized by assembling stiffness and mass matrices K and M. The semi-discrete system then takes the form

$$M\dot{\Psi}_h(t) = -K\Psi_h(t) + M\mathcal{N}(\Psi_h(t)),$$

where $\Psi_h(t)$ is the vector of nodal field values. Time integration may proceed using implicit–explicit (IMEX) schemes that treat the stiff linear terms implicitly and the nonlinear terms explicitly, ensuring numerical stability without excessive computational burden.

Spectral discretization methods offer an alternative when the eigenfunctions of $-\Delta_g$ are known or can be approximated. Truncation to the first N eigenmodes yields a reduced-order model consistent with empirical observations that cortical dynamics often lie within low-dimensional manifolds [3, 11]. The truncated system becomes

$$\dot{a}_m(t) = -\lambda_m a_m(t) + \int_M u_m(x)\mathcal{N}\big(\Psi_N(x,t)\big)\, d\mu_g(x),$$

where Ψ_N is the projection of the field onto the first N eigenfunctions.

Both discretization methods enable simulations that probe the qualitative behavior of solutions, including diffusion-dominated smoothing, nonlinear mode coupling, oscillatory patterns, and coherent wave propagation. These computational tools support the empirical validation efforts described in earlier sections.

Appendix F: Simulation Methods and Example Dynamics

Simulations of the cortical field equations provide insight into the interaction between amplistwistor cascades and global eigenmodes. Initial conditions are chosen to mimic localized sensory inputs, such as Gaussian bumps positioned along primary cortical regions. The diffusion induced by the Laplace–Beltrami operator spreads these perturbations across the cortical surface, while the nonlinear term introduces amplification, rotation, and compression effects analogous to those observed in cortical microcircuits.

A typical evolution begins with a rapidly changing high-frequency component that decays swiftly due to the eigenvalue structure of Δ_g. The remaining dynamics settle into interactions among a small number of eigenmodes whose structures resemble those identified in resting-state fMRI [1]. These modes oscillate slowly, producing coherent patterns similar to those proposed in the resonance models of cortical communication [12, 23].

The introduction of amplistwistor nonlinearities yields mode coupling that generates richer temporal behavior. In some regimes, this coupling produces steady-state attractors corresponding to semantic representations; in others, it yields oscillatory or even chaotic-like trajectories, depending on the parameterization of α, τ, and π. Such qualitative changes mirror the bifurcation phenomena described in the nonlinear dynamics literature [4, 3].

These simulations demonstrate that the proposed model captures both the stability and the flexibility of cortical computation. Stable eigenmode scaffolding supports global coherence while nonlinear transformations introduce the expressive local complexity required for hierarchical cognition and language processing [9].

Appendix G: Illustrative Diagrams (TikZ Placeholders)

This appendix contains minimal TikZ placeholders indicating where figures may be inserted in future versions of the manuscript. The mathematical content of the paper does not depend on these figures, and they may be refined or replaced with empirical visualizations, cortical surface reconstructions, or simulation outputs.

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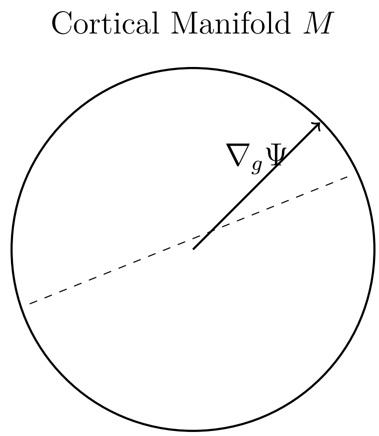

Figure 1: Placeholder schematic of the cortical manifold and gradients.

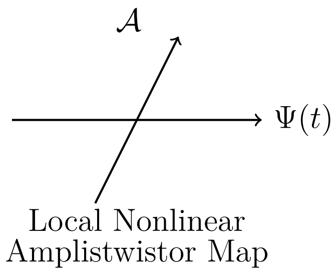

Figure 2: Placeholder diagram of an amplistwistor transformation.

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Eigenmodes $u_m(x)$

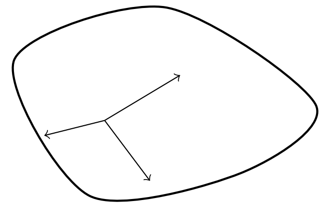

Figure 3: Placeholder illustration of multiple cortical eigenmodes.

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