Unidimary Numbers:
Base $\frac{3}{2}$, the $3 \to 2$ Box, and Examples

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1 What Are Unidimary Numbers?

In ordinary positional notation, we choose an integer base b and write every nonnegative integer N in the form

$$N = \sum_{k=0}^{m} d_k b^k,$$

where the digits d_k are integers in $\{0, 1, \ldots, b-1\}$.

Unidimary numbers are the same idea carried over to a *non–integer base*. Here the base is:

$$b = \frac{3}{2},$$

so we are working in "base one and a half." Remarkably, every nonnegative integer still has a finite positional representation, but now the digits are restricted to $\{0, 1, 2\}$:

$$N = \sum_{k=0}^{m} d_k \left(\frac{3}{2}\right)^k, \qquad d_k \in \{0, 1, 2\}.$$

This system was explored by Jim Propp using a $2 \leftarrow 3$ exploding dot machine. We refer to this representation as the *unidimary system*.

2 The $3 \to 2$ Exploding Box Rule

The most intuitive way to generate unidimary representations is the "exploding dots" or $3 \to 2$ *box* rule.

Place N dots in the rightmost box (box 0). Repeatedly apply:

> Whenever a box contains 3 or more dots, remove 3 dots and add 2 dots one box to the left.

The value is conserved because:

$$3 \cdot \left(\frac{3}{2}\right)^{k} = 2 \cdot \left(\frac{3}{2}\right)^{k+1}$$

Once no box contains more than 2 dots, the digit in each box gives the coefficient of that power of 3/2.

3 Powers of $\frac{3}{2}$3/2

$$\left(\frac{3}{2}\right)^0 = 1, \qquad \left(\frac{3}{2}\right)^1 = 1.5, \qquad \left(\frac{3}{2}\right)^2 = 2.25,$$
$$\left(\frac{3}{2}\right)^3 = 3.375, \qquad \left(\frac{3}{2}\right)^4 = 5.0625, \qquad \left(\frac{3}{2}\right)^5 = 7.59375,$$
$$\left(\frac{3}{2}\right)^6 = 11.390625$$

4 Example 1: Representing 10

Using Exploding Boxes

- Start with 10 dots in box 0.
- 3 explosions: 33 6 dots to box 1; 1 dot left in box 0.
- 2 explosions in box 1 4 dots to box 2.
- 1 explosion in box 2 2 dots to box 3, 1 dot left.

Final state:

$$\text{Box 3: } 2, \quad \text{Box 2: } 1, \quad \text{Box 1: } 0, \quad \text{Box 0: } 1 \Rightarrow (2101)_{3/2}$$

Check Algebraically

$$2 \cdot 3.375 + 1 \cdot 2.25 + 0 + 1 = 6.75 + 2.25 + 1 = 10$$

5 Example 2: Representing 14

Explode in box 0 explode box 1 explode box 2 final state:

$$\text{Box 3: } 2, \quad \text{Box 2: } 2, \quad \text{Box 1: } 1, \quad \text{Box 0: } 2 \Rightarrow (2122)_{3/2}$$

$$2 \cdot 3.375 + 1 \cdot 2.25 + 2 \cdot 1.5 + 2 = 14$$

6 Example 3: Representing 12

$$(2120)_{3/2} = 2 \cdot 3.375 + 1 \cdot 2.25 + 2 \cdot 1.5 = 6.75 + 2.25 + 3.0 = 12$$

7 Decimal Interpretation and Integer Preservation

Each digit in a unidimary expansion multiplies a rational place value, but the result is always an integer. Why?

Because each $3 \to 2$ explosion is an exact identity:

$$3 \cdot \left(\frac{3}{2}\right)^k = 2 \cdot \left(\frac{3}{2}\right)^{k+1}$$

Hence, the system preserves value — just redistributes it.

8 Beta Expansions and Irrational Bases

Unidimary numbers are part of the larger class of *beta expansions*. For any $\beta > 1$, numbers can be written as:

$$x = \sum_{k=0}^{\infty} d_k \beta^{-k}, \quad d_k \in \mathbb{Z}$$

Famous cases: - Base-ϕ (golden ratio) - Base-e - Base-$\frac{3}{2}$ (unidimary)

These systems appear in symbolic dynamics, ergodic theory, and aperiodic tilings.

9 Arithmetic in Unidimary Notation

You can perform addition and subtraction digit-wise, then apply explosions:

$$(2101)_{3/2} + (2)_{3/2} = (2120)_{3/2} \Rightarrow 10 + 2 = 12$$

Digits may exceed 2 trigger explosion: - 3 becomes 0, with 2 carried left - Borrowing for subtraction is similar (reverse the rule)

Multiplication and division are harder; symbolic simulation or lookups help.

10 Lookup Table (0–20)

Decimal	Unidimary
0	0
1	1
2	2
3	210
4	211
5	212
6	2100
7	2101
8	2102
9	2120
10	2101
11	2121
12	2120
13	21010
14	2122
15	21011
16	21012
17	21200
18	21201
19	21202
20	21210

11 Relation to Field-Theoretic Frameworks

This toy system mirrors dynamics in RSVP and TARTAN:

- **Invariant-preserving local dynamics** (like energy flow) - **Multiscale recursion** via positional left-shift - **Compression** via explosion stabilization (complexity-minimizing) - **Bridging discrete and continuous** — analytic symbolic equivalence

This aligns with lamphrodynamic entropy descent, symbolic approximation, and semantic dynamics in RSVP.

Appendix: Python Converter for Unidimary

```
def to_unidimary(n):
    boxes = [0] * 32
    boxes[0] = n
    for i in range(len(boxes)):
        while boxes[i] >= 3:
            boxes[i] -= 3
            boxes[i+1] += 2
```

```
    digits = boxes[:]
    while digits and digits[-1] == 0:
        digits.pop()
    return ''.join(str(d) for d in reversed(digits)) or '0'

# Try:
for i in range(21):
    print(f"{i}  {to_unidimary(i)}")
```

This script simulates the $3 \to 2$ explosion machine and prints unidimary codes.

12 Conclusion

Unidimary numbers offer a whimsical but mathematically exact alternative to ordinary base systems. Despite using fractional place values, they preserve integer meaning and allow intuitive "explosion"–based conversion.

Their structural elegance connects them to symbolic dynamics, multiscale computation, and field-theoretic analogies — making them not only a pedagogical curiosity, but a portal into deeper structural mathematics.