Unidimary Numbers:
Base $\frac{3}{2}$, the $3 \to 2$ Box, and Examples

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1 What Are Unidimary Numbers?

In ordinary positional notation, we choose an integer base b and write every nonnegative integer N in the form

$$N = \sum_{k=0}^{m} d_k b^k,$$

where the digits d_k are integers in $\{0, 1, \dots, b-1\}$.

Unidimary numbers are the same idea carried over to a *non–integer base*. Here the base is

$$b = \frac{3}{2},$$

so we are working in "base one and a half." Remarkably, every nonnegative integer still has a finite positional representation, but now the digits are restricted to $\{0, 1, 2\}$:

$$N = \sum_{k=0}^{m} d_k \left(\frac{3}{2}\right)^k, \qquad d_k \in \{0, 1, 2\}.$$

When we write the number in base $3/2$, the leftmost digit is the coefficient of the highest power of $3/2$, just as in base 10 or base 2.

Throughout this note, I will call this system the *unidimary* system.

2 The $3 \to 2$ Exploding Box Rule

The most intuitive way to generate unidimary representations is the "exploding dots" or $3 \to 2$ *box* rule.

Imagine an infinite row of boxes extending to the left. We place dots in the rightmost box to represent an integer N. The rule is:

> Whenever a box contains 3 dots, we remove those 3 dots and add 2 dots to the next box on the left.

We repeatedly apply this rule until every box contains at most 2 dots.

Why does this correspond to base 3/2?

$$3 \cdot \left(\frac{3}{2}\right)^{k} = 2 \cdot \left(\frac{3}{2}\right)^{k+1}.$$

3 Powers of $\frac{3}{2}$

$$\left(\frac{3}{2}\right)^{0} = 1, \qquad \left(\frac{3}{2}\right)^{1} = 1.5, \qquad \left(\frac{3}{2}\right)^{2} = 2.25,$$
$$\left(\frac{3}{2}\right)^{3} = 3.375, \qquad \left(\frac{3}{2}\right)^{4} = 5.0625, \qquad \left(\frac{3}{2}\right)^{5} = 7.59375,$$
$$\left(\frac{3}{2}\right)^{6} = 11.390625$$

4 Example 1: Representing 10

Using the $3 \to 2$ box rule

Checking with powers of $3/2$

5 Example 2: Representing 14

6 Example 3: $(2120)_{3/2} = 12$

Another simple verification:

$$(2120)_{3/2} = 2 \cdot 3.375 + 1 \cdot 2.25 + 2 \cdot 1.5 = 12.$$

7 Decimal Interpretation and Integer Preservation

Although each position uses fractional powers of 3/2, integer values are always obtained because the exploding-dot rule enforces

$$3\left(\frac{3}{2}\right)^{k} = 2\left(\frac{3}{2}\right)^{k+1}.$$

In other words, the dynamics preserve integer value regardless of how many intermediate explosions occur.

8 Beta Expansions and Irrational Bases

Unidimary numbers are an instance of β-expansions (Rényi, Parry), where numbers are expanded in non-integer bases $\beta > 1$.

Examples include base-ϕ (the golden ratio), base-e, and arbitrary β bases used in symbolic dynamics and ergodic theory.

Base-3/2 is unusual because all integers admit a finite expansion using only digits $\{0, 1, 2\}$, which is not generally true for arbitrary β.

9 A Short Lookup Table

Decimal	Unidimary
0	0
1	1
2	2
3	210
4	211
5	212
6	2100
7	2101
8	2102
9	2120
10	2101
11	2121
12	2120
13	21010
14	2122

10 Arithmetic (Very Brief)

Addition proceeds digitwise, triggering explosions whenever a digit reaches 3. For instance:

$$(2101)_{3/2} + (2)_{3/2} = (2120)_{3/2}.$$

11 Conceptual Connections to RSVP and Field Theory

Although unidimary numbers arise from a simple combinatorial rule, they illustrate several themes from the RSVP framework and lamphrodynamic field theory:

1. **Nonlinear flow with a preserved quantity.** The $3 \to 2$ rule defines a dynamics preserving total value. This mirrors lamphrodynamic flows which preserve an information-like invariant while redistributing structure.

2. **Multiscale recursion.** Moving one box left multiplies by (3/2), creating a recursive multiscale decomposition analogous to semantic recursion in TARTAN or lamphrodynamic scaling.

3. **Complexity minimization.** Explosions stabilize digits into $\{0, 1, 2\}$, compressing a large integer into a stable symbolic form. This resembles curvature-minimizing flows.

4. **Discrete / Continuous Correspondence.** The exploding-box picture is discrete and combinatorial, while the analytic expansion uses continuous fractional powers. Their equivalence mirrors discrete PDE approximations and CA/PDE correspondences in RSVP models.

12 Conclusions and Further Directions

Unidimary numbers are deceptively simple: by defining a local replacement rule that conserves value, we obtain a complete positional numeration system and a beta-expansion valid for all integers.

The conceptual analogy with flow, invariants, multiscale recursion, symbolic dynamics, and discrete-to-continuous transitions resonates with many of the themes explored in RSVP theory and lamphrodynamics: information-preserving flows, curvature minimization, and multi-resolution representations.

Future directions include fractional unidimary expansions (analogues of decimal points), multiplication and division algorithms, connections with golden-base arithmetics, and beta-shift symbolic dynamics in ergodic theory.