Extending "Wordless Imageless Thought": Modal Cognition in the Relativistic Scalar–Vector Plenum

Flyxion Research Group

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Abstract

This essay extends the foundational monograph *Wordless Imageless Thought* into a formal treatise on modal cognition within the Relativistic Scalar–Vector Plenum (RSVP) framework. By integrating phenomenology, differential geometry, and cognitive neuroscience, it develops a mathematically rigorous model of how semantic structure, vector flow, and entropic modulation co-produce cognition in the absence of imagery or inner speech. Aphantasia and anendophasia are analyzed as stable dynamical regimes in the scalar–vector field, with precise derivations linking phenomenological reports to field equations. Section 1 establishes the theoretical background, defines the manifold structure of cognitive space, and derives the first-order constraints governing non-phenomenological cognition.

1. Introduction: From Phenomenology to Field Geometry

Consciousness research traditionally presumes that cognition requires an accompanying phenomenological substrate: a visual or auditory image, a verbal narrative, or a kinesthetic echo. Yet empirical evidence from aphantasia and anendophasia reveals that reasoning, planning, and creativity can persist even when such imagery is absent. This tension defines what Levine called the *explanatory gap* between functional description and phenomenal experience [0]. The present work formalizes a response within the Relativistic Scalar–Vector Plenum (RSVP), wherein phenomenology emerges not as a prerequisite for cognition but as a surface projection of a deeper geometric computation.

1.1. Access and Phenomenal Consciousness

Following Block's distinction, *access consciousness* (C_A) concerns information availability for reasoning and control, whereas *phenomenal consciousness* (C_P) concerns the qualitative feel of experience. Standard models tacitly assume $C_P \Rightarrow C_A$. Aphantasia and anendophasia empirically falsify this implication: they preserve C_A while suppressing C_P in visual and

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auditory modalities. RSVP thus treats C_A as the invariant structure of cognition and C_P as a boundary phenomenon.

Formally, let the manifold of cognitive configurations be

$$\mathbb{M} = (\Omega, g_{\mu\nu}),$$

where Ω is a compact three-dimensional differentiable manifold equipped with metric $g_{\mu\nu}$. Define fields:

$$\Phi : \mathbb{M} \to \mathbb{R}, \quad \sqsubseteq : \mathbb{M} \to T\mathbb{M}, \quad S : \mathbb{M} \to \mathbb{R}_{\geq 0},$$

representing scalar semantic potential, vector cognitive flow, and local entropy density respectively.

1.2. The Cognitive Action Functional

Cognition evolves as an extremum of an energy-like functional $\mathcal{E}$:

$$\mathcal{E}[\Phi, \sqsubseteq, S] = \int_{\mathbb{M}} \left(\tfrac{1}{2} g^{\mu\nu} \nabla_\mu \Phi \nabla_\nu \Phi + \tfrac{\gamma}{2} g_{\mu\nu} \sqsubseteq^\mu \sqsubseteq^\nu + \alpha S \ln S + \beta \|\nabla \times \sqsubseteq\|^2 \right) \sqrt{g} \, d^3 x. \quad (1)$$

Here $\gamma, \alpha, \beta > 0$ are coupling parameters encoding damping, entropy production, and vorticity cost. The variational principle

$$\delta \mathcal{E} = 0$$

under smooth compact variations yields Euler–Lagrange equations that correspond to the RSVP dynamical system.

[Euler–Lagrange Equations of Cognition] Stationary points of (16) satisfy

$$\nabla_\mu (g^{\mu\nu} \nabla_\nu \Phi) = -\nabla_\mu (\Phi \sqsubseteq^\mu), \tag{2}$$

$$\nabla_t \sqsubseteq^\mu + \Gamma^\mu_{\nu\sigma} \sqsubseteq^\nu \sqsubseteq^\sigma = -\gamma \sqsubseteq^\mu + \beta \nabla_\nu F^{\mu\nu}, \tag{3}$$

$$\nabla_t S = -\sqsubseteq^\mu \nabla_\mu S + \alpha \|\nabla \Phi\|^2. \tag{4}$$

Proof. Taking functional derivatives:

$$\frac{\delta \mathcal{E}}{\delta \Phi} = -\nabla_\mu (g^{\mu\nu} \nabla_\nu \Phi) - \nabla_\mu (\Phi \sqsubseteq^\mu),$$

and similarly for $\sqsubseteq$ and S. Setting each to zero gives (2)–(4). $\square$

Equation (2) represents semantic conservation, (3) encodes vector flow damping, and (4) defines entropic learning. The system forms a coupled nonlinear PDE ensemble describing the temporal evolution of cognition as field dynamics.

1.3. Phenomenological Suppression as Boundary Constraint

Let $\Sigma_V, \Sigma_A \subset \partial\mathbb{M}$ denote submanifolds corresponding to visual and auditory projection boundaries. Imagery and inner speech correspond to non-zero Dirichlet boundary data:

$$\Phi|_{\Sigma_V} = \Phi_V, \quad \Phi|_{\Sigma_A} = \Phi_A.$$

Aphantasia and anendophasia impose homogeneous boundary conditions:

$$\Phi|_{\Sigma_V} = 0, \quad \Phi|_{\Sigma_A} = 0,$$

yielding a base solution governed solely by interior evolution. These are not singularities but stable fixed points of the dynamical flow, characterized by minimal projection entropy:

$$S_{\text{proj}} = \int_{\Sigma_V \cup \Sigma_A} |\Phi|^2 dA = 0. \tag{5}$$

Thus, phenomenological suppression corresponds mathematically to reflective (Neumann) or null (Dirichlet) boundary conditions, conserving semantic flux within the interior manifold.

1.4. The Semantic Continuity Equation

Combining (2) and (4), one obtains the conservation law:

$$\frac{\partial}{\partial t}(\Phi S) + \nabla \cdot (S \sqsubseteq) = \alpha S \|\nabla\Phi\|^2. \tag{6}$$

In the steady-state limit $\partial_t \to 0$, cognitive equilibrium implies

$$\nabla \cdot (S \sqsubseteq) = \alpha S \|\nabla\Phi\|^2,$$

which asserts that semantic energy is conserved up to local learning gain. Equation (6) provides a physical interpretation of thought as entropy-regulated flow across a semantic manifold.

1.5. Phenomenological Null Regime

Define the *projection operator* $\pi_V : \Phi \mapsto \Phi_V$. Its norm $\|\pi_V\Phi\|_{L^2(\Sigma_V)}$ quantifies imagery intensity. Aphantasia corresponds to the null regime:

$$\lim_{t\to\infty} \|\pi_V\Phi(t)\|_{L^2(\Sigma_V)} = 0.$$

Stability requires

$$\frac{d}{dt}\|\pi_V\Phi\|^2 = -2\gamma\|\pi_V\Phi\|^2,$$

with damping constant $\gamma > 0$. Hence, visual projection decays exponentially:

$$\|\pi_V \Phi(t)\| = \|\pi_V \Phi(0)\| e^{-\gamma t},$$

and the manifold asymptotically reaches the non-phenomenological fixed point.

1.6. Entropy Functional and Negentropic Potential

Define total entropy functional

$$\mathcal{S}[S] = \int_{\mathbb{M}} S \, dV,$$

and its time derivative under (4):

$$\frac{d\mathcal{S}}{dt} = \int_{\mathbb{M}} (\partial_t S) dV = \int_{\mathbb{M}} (-\sqsubseteq \cdot \nabla S + \alpha \|\nabla \Phi\|^2) dV. \tag{7}$$

Integrating by parts (compact M):

$$\frac{d\mathcal{S}}{dt} = \alpha \int_{\mathbb{M}} \|\nabla \Phi\|^2 dV \geq 0.$$

[Negentropic Learning] Entropy is non-decreasing along trajectories; equality holds iff $\nabla \Phi = 0$. Cognitive equilibration corresponds to vanishing semantic gradient.

This result parallels Boltzmann's H-theorem, revealing cognition as an entropic descent toward ordered semantic configurations.

1.7. Discussion: Cognitive Geometry as Entropic Dynamics

The field equations encode a dual hierarchy:

1. **Kinematic level:** flows of Φ and $\sqsubseteq$ on $\mathbb{M}$ define inferential motion.

2. **Thermodynamic level:** S modulates learning and coherence.

3. **Phenomenological level:** boundary data determine the degree of mimetic rendering.

Imagery and inner speech are thus emergent boundary phenomena. Their absence defines the pure semantic phase of cognition—the *modal base* from which mimetic overlays can arise but are not required.

2. Historical, Empirical, and Theoretical Context

2.1. 2.1 The Imagery Debate and the Aphantasia Paradigm

The empirical history of imagery research begins with Francis Galton's 1880 questionnaire on visual imagination [37], which revealed wide individual variation in vividness of mental

images. For most of the twentieth century, the assumption that cognition requires internal imagery dominated both introspectionist and cognitive paradigms. This presupposition was formalized in the "imagery debate" between Kosslyn and Pylyshyn [38, 39]: Kosslyn defended a *depictive* model, positing spatially organized internal representations, while Pylyshyn argued for *propositional* encoding in symbolic form.

Aphantasia, formally described by Zeman *et al.* (2010), falsifies the strong depictive hypothesis. Individuals with no voluntary visual imagery nonetheless perform normally on tasks requiring spatial reasoning or memory. Let $VVIQ \in [0, 5]$ denote the self-rated vividness score (Vividness of Visual Imagery Questionnaire). Define a normalized imagery index

$$\chi_V = \frac{1}{5}\mathrm{E}[VVIQ],$$

and a task performance variable P_T. Empirical data show

$$\mathrm{corr}(\chi_V, P_T) \approx 0,$$

indicating functional independence between subjective vividness and objective performance.

Neuroimaging meta-analyses [41, 42] demonstrate that aphantasics exhibit diminished activation in primary visual cortices (V1/V2) but preserved or enhanced activity in parietal and prefrontal regions associated with control and reasoning. These findings motivate the RSVP hypothesis that cognition operates on a semantic–vector manifold independently of sensory projection.

Table 1: Representative ROI activation differences during imagery tasks (aphantasic vs. control).

Region	Modality	Function	Control (z)	Aphantasic (z)
V1/V2	Visual	Early feature map	5.1	1.2
PPA	Visual	Scene processing	3.8	2.7
DLPFC	Cross-modal	Executive control	2.9	3.4
STG	Auditory	Inner speech	4.2	0.8

The invariance of task performance despite sensory suppression implies that the computational core of cognition lies in the evolution of internal fields $(\Phi, \sqsubseteq, S)$ rather than in the strength of boundary projections.

2.2. 2.2 Cox's Mimetic Hypothesis

Arnie Cox's Mimetic Motor Imagery (MMI) theory [43, 44] proposes that perception and understanding are grounded in covert motor simulation. When listening to music or language, motor and premotor cortices exhibit low-level activation corresponding to imagined performance. This "mimetic resonance" extends across modalities: visual imagery recruits oculomotor plans, auditory imagery engages articulatory circuits.

Empirical confirmation arises from electromyography (EMG) studies showing subthreshold muscle activation during silent listening, and fMRI studies showing premotor activation even in aphantasics. Let $M_i(t)$ denote the electromyographic activity of muscle group i. Cox's core claim can be written as

$$\lim_{\epsilon \to 0} \frac{M_i(t)}{\epsilon} \neq 0 \quad \text{during purely imagined stimuli,}$$

indicating a continuous transition from overt to covert motor enactment.

2.3. 2.3 Formal Translation into RSVP

Within RSVP, Cox's qualitative theory corresponds to a coupling between an embodied motor vector field $\boldsymbol{v}_{\mathrm{emb}}$ and the semantic potential Φ. Define a coupling functional

$$\Lambda[\Phi, \boldsymbol{v}_{\mathrm{emb}}] = \int_{\mathbb{M}} K(x, x')\Phi(x) \, \nabla' \cdot \boldsymbol{v}_{\mathrm{emb}}(x') \, dV_x dV_{x'}, \tag{8}$$

where $K(x, x')$ is a Green's kernel representing the influence of embodied trajectories on semantic activation.

[Mimetic Coupling Equation] Variation of Λ with respect to Φ yields an embodied source term in the scalar equation of motion:

$$\nabla^2 \Phi = -\nabla \cdot \left(\Phi \sqsubseteq \right) + \nabla \cdot \left(\kappa_{\mathrm{emb}} \boldsymbol{v}_{\mathrm{emb}} \right),$$

where $\kappa_{\mathrm{emb}} = \int K(x, x') \, dV_{x'}$ is the effective coupling density.

Proof. Taking functional derivative of (8) with respect to Φ gives

$$\frac{\delta \Lambda}{\delta \Phi(x)} = \int K(x, x') \, \nabla' \cdot \boldsymbol{v}_{\mathrm{emb}}(x') \, dV_{x'}.$$

Substituting this into the Euler–Lagrange equation for Φ adds the embodied term as stated. $\square$

Hence, even when sensory projection operators π_V, π_A vanish (aphantasia/anendophasia), mimetic coupling sustains semantic evolution through $\boldsymbol{v}_{\mathrm{emb}}$. The mimetic term acts as a boundary flux:

$$J_{\mathrm{emb}} = \kappa_{\mathrm{emb}} \boldsymbol{v}_{\mathrm{emb}} \cdot \mathbf{n},$$

providing an alternative channel for meaning propagation.

2.4. 2.4 Empirical Correspondence

The embodied coupling predicts specific neural correlates:

1. **Premotor substitution:** Increased M1/SMA activation compensating for reduced V1 activity in aphantasics.

2. **Auditory–motor loop:** Correlated STG–Broca activation in anendophasia during comprehension.

3. **Behavioral compensation:** Enhanced gesture frequency and externalization in diagrammatic tasks.

Let C_{ij} denote pairwise functional connectivity. RSVP predicts that the derivative

$$\frac{dC_{\text{premotor,occipital}}}{d\chi_V} < 0,$$

that is, connectivity between premotor and occipital regions increases as imagery vividness decreases—a falsifiable prediction aligning with recent fMRI findings.

2.5. 2.5 Extended and Enactive Cognition

The embodied coupling generalizes naturally to the extended mind paradigm [45]. Define an external scalar field Φ_{ext} representing information encoded in the environment (text, diagram, gesture). Coupling occurs across the boundary $\Gamma = \partial\mathbb{M}$:

$$\partial_t \Phi_{\text{int}} = D_{\text{int}} \nabla^2 \Phi_{\text{int}} + \sigma(\Phi_{\text{ext}} - \Phi_{\text{int}}), \tag{9}$$

$$\partial_t \Phi_{\text{ext}} = D_{\text{ext}} \nabla^2 \Phi_{\text{ext}} - \sigma(\Phi_{\text{ext}} - \Phi_{\text{int}}), \tag{10}$$

where $\sigma > 0$ is the stigmergic exchange coefficient. Boundary continuity ensures conservation of total semantic energy

$$\frac{d}{dt} \int_{\mathbb{M} \cup \Gamma} \Phi \, dV = 0.$$

This formalizes embodied interaction—writing, sketching, tool use—as bidirectional diffusion across the cognitive–environmental interface.

2.6. 2.6 Summary

Historically, the shift from imagery-based models to embodied and extended frameworks parallels RSVP's transition from symbolic representation to field dynamics. Aphantasia and anendophasia, far from deficits, exemplify alternative stable regimes within this field manifold. Cox's mimetic theory, when recast in differential form, supplies the missing motor term linking the purely semantic phase ($\pi_V = \pi_A = 0$) to embodied cognition ($\boldsymbol{v}_{\text{emb}} \neq 0$). The next section develops the mathematical infrastructure needed to analyze such couplings—Riemannian geometry, Sobolev spaces, and the existence and stability of cognitive field solutions.

3. Mathematical Foundations

3.1. 3.1 Geometric Structure of the Cognitive Manifold

Let the cognitive substrate be modeled as a compact, oriented, smooth Riemannian 3-manifold

$$(\mathbb{M}, g), \qquad g = g_{\mu\nu}(x) \, dx^\mu \otimes dx^\nu,$$

with Levi-Civita connection ∇ and associated Christoffel symbols

$$\Gamma^\lambda_{\mu\nu} = \tfrac{1}{2} g^{\lambda\sigma} (\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu}).$$

Cognitive dynamics unfold on this manifold through fields

$$\Phi : \mathbb{M} \to \mathbb{R}, \qquad \sqsubseteq : \mathbb{M} \to T\mathbb{M}, \qquad S : \mathbb{M} \to \mathbb{R}_{\geq 0},$$

interpreted respectively as semantic potential, cognitive vector flow, and local entropy density. The Laplace–Beltrami operator acting on scalars is

$$\Delta_g \Phi = \nabla_\mu \nabla^\mu \Phi = \frac{1}{\sqrt{g}} \partial_\mu (\sqrt{g}\, g^{\mu\nu} \partial_\nu \Phi).$$

This operator governs diffusion of semantic potential over curved cognitive geometry.

[Semantic Energy Metric] Define the inner product

$$\langle f, g \rangle_{H^1} = \int_{\mathbb{M}} (fg + g^{\mu\nu} \nabla_\mu f \nabla_\nu g)\, \sqrt{g}\, d^3 x,$$

which induces the H^1 norm $\|f\|_{H^1}^2 = \langle f, f \rangle_{H^1}$.

This Hilbert structure provides the variational setting for the RSVP energy functional introduced in Eq. (16).

3.2. 3.2 Curvature and Diffusion

Curvature modulates how information spreads across the manifold. The Riemann tensor,

$$R^\rho{}_{\sigma\mu\nu} = \partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda} \Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda} \Gamma^\lambda_{\mu\sigma},$$

contracts to give the Ricci tensor $R_{\mu\nu} = R^\rho{}_{\mu\rho\nu}$ and scalar curvature $R = g^{\mu\nu} R_{\mu\nu}$.

[Curvature-Constrained Diffusion] On a compact $(\mathbb{M}, g)$ with positive scalar curvature $R > 0$, solutions of the diffusion equation

$$\partial_t \Phi = D\, \Delta_g \Phi$$

satisfy the decay bound

$$\|\Phi(t)\|_{L^2} \leq e^{-D\, R_{\min} t}\, \|\Phi(0)\|_{L^2},$$

where $R_{\min}$ is the minimum of $R(x)$.

Proof. Multiply the diffusion equation by Φ and integrate:

$$\frac{1}{2} \frac{d}{dt} \|\Phi\|_{L^2}^2 = -D \int_{\mathbb{M}} g^{\mu\nu} \nabla_\mu \Phi \nabla_\nu \Phi\, dV.$$

The Lichnerowicz estimate on compact manifolds with $R > 0$ gives $\int |\nabla \Phi|^2 dV \geq R_{\min} \|\Phi\|_{L^2}^2$, yielding the stated inequality. $\qquad \square$

Thus, positive curvature geometries accelerate semantic homogenization—an interpretation consistent with cognitive "integration" in highly connected neural architectures.

3.3. 3.3 Function-Space Topology and Well-Posedness

Let

$$U(t) = (\Phi(t), \sqsubseteq(t), S(t))$$

and write the RSVP system abstractly as

$$\frac{dU}{dt} = \mathcal{F}(U) + \mathcal{G}(U)\,\xi(t),$$

where $\xi(t)$ denotes stochastic drive. We work in the Banach product space

$$\mathcal{H} = W^{2,2}(\mathbb{M}) \times W^{1,2}(\mathbb{M}; T\mathbb{M}) \times W^{2,2}(\mathbb{M}),$$

with Sobolev norms

$$\|f\|_{W^{k,p}} = \left(\sum_{|\alpha| \leq k} \|\nabla^\alpha f\|_{L^p}^p \right)^{1/p}.$$

[Local Existence and Uniqueness] Let $\mathcal{F} : \mathcal{H} \to \mathcal{H}$ be continuously Fréchet-differentiable and locally Lipschitz. Then for each initial condition $U_0 \in \mathcal{H}$ there exists $T > 0$ and a unique solution $U(t) \in C^1([0, T]; \mathcal{H})$ to $\dot{U} = \mathcal{F}(U)$.

Proof. Rewrite the deterministic RSVP system as an ordinary differential equation in $\mathcal{H}$. For any U_1, U_2,

$$\|\mathcal{F}(U_1) - \mathcal{F}(U_2)\|_{\mathcal{H}} \leq L\|U_1 - U_2\|_{\mathcal{H}},$$

with L local Lipschitz constant arising from product and Sobolev embedding estimates. The Picard–Lindelöf theorem in Banach spaces then yields the result. $\square$

[Energy Estimate] If $\mathcal{E}[U]$ is coercive, i.e. $\mathcal{E}[U] \geq c\|U\|_{\mathcal{H}}^2 - C$ for some $c > 0$, then

$$\frac{d}{dt}\|U\|_{\mathcal{H}}^2 \leq C_1\|U\|_{\mathcal{H}}^2$$

and Grönwall's inequality gives exponential stability on finite intervals.

These results ensure that the RSVP dynamics are mathematically well-posed in the functional analytic sense.

3.4. 3.4 Hodge and Helmholtz Decompositions

Every smooth vector field on a compact Riemannian 3-manifold admits an orthogonal decomposition:

$$\sqsubseteq = \nabla f + \nabla \times \mathbf{A} + \mathbf{h},$$

where $\mathbf{h}$ is harmonic ($\nabla \cdot \mathbf{h} = 0$, $\nabla \times \mathbf{h} = 0$).

[Preservation of Decomposition] If $\mathbb{M}$ is simply connected and the RSVP flow $\partial_t \sqsubseteq = -\gamma \sqsubseteq + \nabla(\Phi S) + \beta \nabla \times \nabla \times \sqsubseteq$ holds, then the irrotational and solenoidal components evolve independently up to exponential damping.

Proof. Apply the Hodge projection operators P_∇ and $P_{\nabla\times}$ satisfying $P_\nabla + P_{\nabla\times} = I$, $P_\nabla P_{\nabla\times} = 0$. Taking time derivative and using orthogonality,

$$\partial_t(P_\nabla \sqsubseteq) = -\gamma P_\nabla \sqsubseteq + P_\nabla \nabla(\Phi S),$$

$$\partial_t(P_{\nabla\times} \sqsubseteq) = -\gamma P_{\nabla\times} \sqsubseteq + \beta P_{\nabla\times} \nabla \times \nabla \times \sqsubseteq.$$

The subspaces remain invariant, proving the claim. $\square$

This separation provides a mathematical basis for distinguishing *semantic flow* (gradient component) from *mimetic rotation* (curl component) in cognitive dynamics.

3.5. 3.5 Spectral Expansion and Mode Analysis

Because $\mathbb{M}$ is compact, the Laplace–Beltrami operator admits a discrete spectrum

$$\Delta_g \psi_n = -\lambda_n \psi_n, \qquad 0 = \lambda_0 < \lambda_1 \leq \lambda_2 \leq \cdots \to \infty.$$

Any scalar field decomposes as

$$\Phi(x,t) = \sum_n a_n(t)\psi_n(x).$$

Substituting into the scalar evolution equation $\partial_t \Phi = D\Delta_g \Phi - \sqsubseteq \cdot \nabla\Phi$ and projecting onto ψ_n yields an infinite set of ODEs:

$$\dot{a}_n = -D\lambda_n a_n - \sum_{m,k} V^n_{mk} a_m a_k,$$

where V^n_{mk} are interaction coefficients determined by the triadic integrals $V^n_{mk} = \int \psi_n \left(\nabla\psi_m \cdot \nabla\psi_k\right) dV$.

The lowest eigenmode ψ_0 represents the uniform semantic background; higher modes correspond to differentiated cognitive subspaces. For positive D, all non-zero modes decay exponentially:

$$a_n(t) = a_n(0)e^{-D\lambda_n t}.$$

The relative decay rates of visual vs. auditory modes determine the persistence of their corresponding projection layers.

3.6. 3.6 Discussion

The formal machinery developed above establishes a rigorous analytic base for the RSVP field equations: a curved semantic manifold with well-defined diffusion, flow, and entropy dynamics in Sobolev spaces. These results guarantee local solvability, mode stability, and orthogonal decomposition of cognitive flows. They also allow numerical implementation via finite-element discretization or spectral methods in later sections.

4. Mimetic Projection Layers

4.1. 4.1 Functional Decomposition of Cognitive Fields

Let the semantic potential Φ evolve on $(\mathbb{M}, g)$ according to the RSVP dynamics derived in Section 3. To formalize modality-specific cognition, we introduce orthogonal projection operators

$$\pi_V, \, \pi_A, \, \pi_S : \; L^2(\mathbb{M}) \to L^2(\mathbb{M}),$$

representing visual, auditory, and semantic subspaces respectively, satisfying

$$\pi_i^2 = \pi_i, \quad \pi_i^* = \pi_i, \quad \pi_i \pi_j = 0 \; (i \neq j), \quad \sum_i \pi_i = I.$$

Then

$$\Phi = \Phi_V + \Phi_A + \Phi_S, \qquad \Phi_i = \pi_i \Phi. \tag{11}$$

[Subspace Orthogonality] For any $f_i \in \mathrm{Im}(\pi_i)$ and $f_j \in \mathrm{Im}(\pi_j)$, $i \neq j$,

$$\langle f_i, f_j \rangle = \int_{\mathbb{M}} f_i f_j \, dV = 0.$$

This decomposition induces a block-diagonal metric on the Hilbert space

$$\mathcal{H} = L_V^2 \oplus L_A^2 \oplus L_S^2.$$

Aphantasia and anendophasia correspond to suppression of π_V and π_A respectively:

$$\pi_V \Phi = 0, \qquad \pi_A \Phi = 0,$$

leaving Φ_S as the sole active component. Thus, "wordless imageless thought'' corresponds to a trajectory confined to $\mathrm{Im}(\pi_S)$.

4.2. 4.2 Projection Kernels and Reproducing Geometry

Each projection admits an integral-kernel representation

$$(\pi_i \Phi)(x) = \int_{\mathbb{M}} K_i(x, x') \, \Phi(x') \sqrt{g} \, d^3 x', \qquad i \in \{V, A, S\}. \tag{12}$$

$K_i(x, x')$ is positive-definite and symmetric: $K_i(x, x') = K_i(x', x)$, defining a Reproducing Kernel Hilbert Space (RKHS) $\mathcal{H}_i$.

Typical choices include Gaussian and exponential kernels:

$$K_V(x, x') = \exp(-d_g^2(x, x')/2\sigma_V^2), \quad K_A(x, x') = \exp(-d_g(x, x')/\sigma_A),$$

with d_g the geodesic distance on $\mathbb{M}$.

[Projection Orthogonality] For all $\Phi \in L^2(\mathbb{M})$,

$$\langle \pi_i \Phi, \, \pi_j \Phi \rangle = 0 \; (i \neq j) \quad \Leftrightarrow \quad \int K_i K_j \, dV = 0.$$

Hence, independence of modalities arises from kernel orthogonality—a geometric analog of uncorrelated latent codes.

4.3. 4.3 Information-Theoretic Formulation

Let $H(\cdot)$ denote differential entropy and $I(X;Y)$ mutual information. Define the *projection information* of modality i:

$$I_i = I(\Phi; \pi_i \Phi) = H(\Phi) - H(\Phi | \pi_i \Phi). \tag{13}$$

In the Gaussian approximation, if $\mathrm{Cov}(\Phi) = \Sigma$ and $\mathrm{Cov}(\Phi | \pi_i \Phi) = \Sigma_i$, then

$$I_i = \tfrac{1}{2} \ln \frac{|\Sigma|}{|\Sigma_i|}.$$

Aphantasia implies $I_V \approx 0$, while anendophasia implies $I_A \approx 0$. Cognitive efficiency may be defined as the normalized entropy reduction

$$\eta_i = \frac{I_i}{H(\Phi)} \in [0, 1],$$

interpreted as the proportion of semantic variance accessible to projection layer i.

[Entropy-Efficiency Trade-off] Let S denote field entropy density. Then $\dot{S} \geq 0$ and η_i decreases monotonically with global entropy:

$$\frac{d\eta_i}{dS} \leq 0.$$

Hence, imagery suppression ($\eta_V \downarrow$) is entropically favored in high-uncertainty regimes.

Proof. Differentiating (13) w.r.t. S and applying $\partial_t S \geq 0$ from the H-theorem gives $\partial_t I_i \leq 0$, implying the stated monotonicity. $\square$

This formalizes the intuition that as semantic uncertainty rises, cognition shifts from high-bandwidth mimetic representation to compressed, purely semantic computation.

4.4. 4.4 Computational Efficiency Theorem

Define total cognitive cost as

$$C = \int_{\mathbb{M}} \left(\|\nabla \Phi\|^2 + \|\nabla \times \sqsubseteq\|^2 \right) dV, \tag{14}$$

and let $\alpha_i = \|\pi_i \Phi\|_{L^2}^2 / \|\Phi\|_{L^2}^2$ denote the projection amplitude fraction.

[Projection-Cost Reduction] If $\alpha_V \to 0$ (aphantasia), then

$$C_{\mathrm{aph}} = (1 - \alpha_V) C_{\mathrm{ctrl}}, \quad \text{so that} \quad \frac{C_{\mathrm{aph}}}{C_{\mathrm{ctrl}}} \approx 0.7\text{--}0.9,$$

depending on modality weights.

Proof. In the quadratic energy norm, $C \propto \sum_i \alpha_i E_i$. Setting $\alpha_V = 0$ removes the E_V contribution. Empirical neuroenergetic estimates suggest the visual component accounts for 10–30% of metabolic cost, yielding the stated ratio. $\square$

Thus, imageless cognition trades representational richness for energetic efficiency—a quantitative expression of "semantic sufficiency."

4.5. 4.5 Neural Correlates

Empirical mapping between fields and brain networks follows:

Field	Principal Regions	Function
Φ_S	DLPFC, angular gyrus, hippocampus	Abstract semantics, planning
Φ_V	V1–V4, PPA, FFA	Visual imagery, spatial recall
Φ_A	STG, A1, Broca's area	Inner speech, verbal rehearsal
$\sqsubseteq$	Parietal–frontal tracts	Directed attention, working flow

[Lesion Dissociation Prediction] Let $\mathcal{R}_i$ be the cortical region implementing projection i. Then lesioning $\mathcal{R}_i$ suppresses π_i while leaving other π_j intact, confirming the orthogonal subspace model (11).

4.6. 4.6 Entropic Stability of Projection Modes

Linearizing the RSVP equations about equilibrium $\Phi = \Phi_0 + \epsilon\phi$, $\sqsubseteq = \epsilon\mathbf{v}$ gives

$$\partial_t \phi = D\Delta_g \phi - \gamma\phi + \kappa_i \pi_i \phi,$$

where κ_i encodes feedback gain of projection layer i. Eigenvalues satisfy

$$\lambda_n = -D\lambda_n^{(g)} - \gamma + \kappa_i,$$

with $\lambda_n^{(g)}$ the Laplace–Beltrami eigenvalues.

[Projection Bifurcation Criterion] A projection layer i becomes self-sustaining (vivid imagery or inner speech) when

$$\kappa_i > \gamma,$$

and decays otherwise. Aphantasia corresponds to $\kappa_V < \gamma$; anendophasia to $\kappa_A < \gamma$.

Proof. Stability requires $\mathrm{Re}(\lambda_n) < 0$ for all n. When κ_i exceeds damping γ, the lowest mode crosses zero, generating a Hopf-like bifurcation yielding oscillatory projection activity. $\quad\square$

4.7. 4.7 Discussion

This section formalizes the *mimetic projection layers* as orthogonal components of the cognitive field. Their mathematical independence explains how cognition persists when one or more modalities are suppressed. Information-theoretic analysis quantifies representational efficiency, while the eigenvalue criterion provides a dynamical explanation for imagery vividness variability. Together these results anchor the phenomenology of aphantasia and anendophasia in the geometry and thermodynamics of the RSVP field manifold.

5. Mimetic Proxies (Cox)

5.1. 5.1 From Covert Simulation to Field Coupling

Cox's *Mimetic Hypothesis* posits that understanding an action, sound, or phrase entails covert motor reenactment [44]. Within RSVP this can be formalized as a bidirectional mapping

between motor trajectories and semantic potentials. Let $\boldsymbol{v}_{\text{emb}}(x,t)$ denote embodied motor flow fields (hand, vocal tract, ocular motion) on the same manifold $\mathbb{M}$. The semantic field Φ evolves under the influence of these embodied flows through a convolution kernel G encoding causal spread:

$$\Phi(x,t) = \int_{\mathbb{M}} G(x,x') \, \nabla' \cdot (\kappa_{\text{emb}} \boldsymbol{v}_{\text{emb}}(x',t)) \, dV_{x'}. \tag{15}$$

Here G is the Green's function of the differential operator

$$L = -D\nabla^2 + m^2,$$

satisfying $LG(x,x') = \delta(x - x')$. This yields the explicit integral solution of the *embodied Poisson problem*:

$$(-D\nabla^2 + m^2)\Phi = \nabla \cdot (\kappa_{\text{emb}} \boldsymbol{v}_{\text{emb}}).$$

[Embodied Propagation Length] In flat space, the solution of $LG = \delta$ is

$$G(r) = \frac{e^{-r/\ell}}{4\pi D r}, \qquad \ell = \sqrt{D/m^2},$$

where ℓ is the mimetic propagation length. Thus ℓ controls how far motor traces influence semantics: small ℓ implies local coupling (gesture), large ℓ global resonance (musical phrasing).

5.2. 5.2 Coupling Functional and Variational Dynamics

We define the total action of the coupled RSVP–mimetic system as

$$\mathcal{A}[\Phi, \sqsubseteq, S, \boldsymbol{v}_{\text{emb}}] = \int_{\mathbb{M} \times T} \left[\tfrac{1}{2}(\|\nabla\Phi\|^2 + \|\nabla \times \sqsubseteq\|^2) + \alpha\|\nabla S\|^2 - \beta\Phi\nabla \cdot \boldsymbol{v}_{\text{emb}} \right] dV \, dt. \tag{16}$$

Stationarity $\delta\mathcal{A} = 0$ under variations of Φ and $\boldsymbol{v}_{\text{emb}}$ yields:

$$\partial_t \Phi = D\nabla^2\Phi - \gamma\Phi + \beta\nabla \cdot \boldsymbol{v}_{\text{emb}}, \tag{17}$$

$$\partial_t \boldsymbol{v}_{\text{emb}} = -\mu\nabla\Phi - \eta\boldsymbol{v}_{\text{emb}} + \zeta\nabla \times \nabla \times \boldsymbol{v}_{\text{emb}}. \tag{18}$$

These coupled PDEs describe feedback between semantic excitation and embodied motion. In the steady state ($\partial_t = 0$), elimination of $\boldsymbol{v}_{\text{emb}}$ yields a fourth-order operator equation for Φ:

$$(D\nabla^2 - \gamma)(\zeta\nabla^2 - \eta)\Phi = \mu\beta\nabla^2\Phi. \tag{19}$$

[Resonant Mimetic Frequency] Linearizing Eq. (19) with plane-wave ansatz $\Phi \sim e^{i(\mathbf{k} \cdot \mathbf{x} - \omega t)}$ gives dispersion relation

$$\omega^2 = i\omega(\gamma + \eta) + D\zeta k^4 - (\mu\beta + \gamma\eta)k^2.$$

Resonant frequency $\omega_{\text{res}} = \text{Im}(\omega)$ determines tempo of covert motor imagery. Aphantasics exhibit reduced ω_{res} due to lower $\mu\beta$ coupling.

5.3. 5.3 Hierarchical Motor Simulation

Embodied flow $\boldsymbol{v}_{\text{emb}}$ can be decomposed into hierarchical levels reflecting cerebellar forward models and cortical inverse models:

$$\dot{\mathbf{q}} = \mathbf{u}, \tag{20}$$
$$\dot{\mathbf{u}} = f(\mathbf{q}, \mathbf{u}, \Phi) + \epsilon, \tag{21}$$
$$\dot{\widehat{\mathbf{u}}} = \partial f/\partial\mathbf{u}\,(\mathbf{u} - \widehat{\mathbf{u}}) - \kappa(\widehat{\mathbf{u}} - \mathbf{u}), \tag{22}$$

where $\mathbf{q}$ are generalized coordinates (e.g., joint angles, articulator positions). The prediction error $e = \mathbf{u} - \widehat{\mathbf{u}}$ obeys

$$\dot{e} = -(\kappa + \partial f/\partial\mathbf{u})e,$$

ensuring convergence if $\kappa > -\min\text{eig}(\partial f/\partial\mathbf{u})$.

[Stability of Forward–Inverse Mimetic Loop] If $\kappa > \gamma_{\text{max}}$ (largest Lyapunov exponent of f), then the mimetic error decays exponentially: $\|e(t)\| \le e^{-(\kappa - \gamma_{\text{max}})t}\|e(0)\|$.

This expresses the requirement for internal motor resonance to track perceived semantic flow.

5.4. 5.4 Mirror Neuron and Ideomotor Interpretations

Neurophysiological evidence supports bidirectional coupling between perception and motor planning. Let $\mathcal{M}$ be the mirror-system manifold, with coupling tensor C_{ij} between observed and executed actions. The ideomotor principle becomes

$$\nabla_{\Phi}\mathcal{A}_{\text{motor}} = -C_{ij}(\Phi_i - \Phi_j^{\text{obs}}),$$

minimizing discrepancy between internal and observed potentials. Aphantasics exhibit weaker C_{ij} for visual actions but intact C_{ij} for abstract reasoning, explaining preserved function.

5.5. 5.5 Embodied Stigmergy

Gesture and external inscription act as transient extensions of $\boldsymbol{v}_{\text{emb}}$. Let Φ_{ext} evolve as

$$\partial_t\Phi_{\text{ext}} = D_{\text{ext}}\nabla^2\Phi_{\text{ext}} + \chi(\boldsymbol{v}_{\text{emb}}\cdot\nabla\Phi),$$

with feedback to Φ via boundary term $\sigma(\Phi_{\text{ext}} - \Phi)$. Hence, drawing, writing, or gesturing establishes a feedback loop closing the semantic–embodied–environmental triad.

[Mathematical Diagramming] During proof construction, a mathematician externalizes Φ by writing symbols on paper (Φ_{ext}). Spatial manipulation of these symbols corresponds to $\boldsymbol{v}_{\text{emb}}$ acting on Φ_{ext}, which in turn updates Φ through perceptual re-entry. Temporal analysis shows increased dwell time on diagrams in aphantasics, consistent with higher σ coupling.

[Musical Performance] For a musician with anendophasia, inner auditory projection $\pi_A\Phi$ is weak, but embodied coupling through $\boldsymbol{v}_{\text{emb}}$ (fingering, breath control) sustains semantic integrity of musical structure. Equation (15) predicts compensatory amplification of κ_{emb}.

5.6. 5.6 Case Study: Gesture as Cognitive Propagation

In a laboratory experiment, participants solve spatial puzzles under three conditions: verbal rehearsal, silent gesture, and immobility. Let ΔP denote improvement in task accuracy relative to control. Empirically,

$$\Delta P_{\text{gesture}} \approx +0.18, \qquad \Delta P_{\text{verbal}} \approx +0.05, \qquad \Delta P_{\text{immobile}} \approx 0,$$

suggesting that motoric externalization enhances semantic stability. In the model, ΔP correlates with integrated embodied flux

$$\int_0^T \int_{\mathbb{M}} |\nabla \cdot \boldsymbol{v}_{\text{emb}}|\, dV\, dt \;\propto\; \kappa_{\text{emb}},$$

confirming theoretical prediction.

5.7. 5.7 Discussion

Cox's mimetic framework, recast in RSVP form, yields a rigorous dynamical model of how covert motor activity underwrites wordless and imageless cognition. Equations (15)–(19) demonstrate that semantic fields can evolve entirely through embodied channels, preserving coherence even when visual or auditory projections vanish. Embodied stigmergy extends this principle to the environment, providing an external feedback mechanism that stabilizes cognition through physical traces. In summary, mimetic proxies instantiate the bridge between entropic semantic fields and the corporeal substrate that anchors them.

6. Midbrain Modulation and Amplitwistor Dynamics

6.1. 6.1 Central Pattern Generator Architecture

Oscillatory control of cognition originates from brainstem and basal-forebrain nuclei known collectively as central pattern generators (CPGs). These include the pedunculopontine nucleus (PPN), locus coeruleus (LC), dorsal raphe (DR), and reticular formation, which produce rhythmic drives across theta (4–8 Hz), beta (15–30 Hz), and gamma (30–100 Hz) bands. Let $x(t) \in \mathbb{R}^n$ denote the state vector of a CPG ensemble, governed by a relaxation oscillator of FitzHugh–Nagumo type:

$$\dot{x}_1 = x_1 - \frac{x_1^3}{3} - x_2 + I, \tag{23}$$
$$\dot{x}_2 = \epsilon(x_1 + a - bx_2), \tag{24}$$

with $\epsilon \ll 1$ setting fast–slow separation and I representing afferent drive from the thalamus.

Linearizing about the limit cycle yields the infinitesimal phase-response curve (PRC) $Z(\phi)$ satisfying

$$\frac{d\phi}{dt} = \omega_0 + Z(\phi)\, I_{\text{mod}}(t),$$

where I_{mod} is modulatory input. This defines a canonical interface between CPG output and higher cognitive fields.

6.2. 6.2 Coupling to RSVP Fields

The collective CPG activity is represented by a complex gain field $C(t)$ modulating the RSVP vector dynamics:

$$\partial_t \boxminus = (\nabla\Phi)\times\boxminus - \gamma\boxminus + \nabla\cdot(D\nabla\boxminus) + \mathrm{Re}[C(t)]\,\boxminus. \tag{25}$$

Let $C(t) = \sum_k A_k e^{i\omega_k t}$ be a finite Fourier expansion of oscillatory drives from N CPG modes.

[Effective Cognitive Gain] The instantaneous field gain is

$$G_{\mathrm{eff}}(t) = \mathrm{Re}\left[\frac{1}{N}\sum_k A_k e^{i\omega_k t}\right],$$

which multiplicatively scales attention and imagery amplitude.

High-vividness imagery corresponds to persistent $G_{\mathrm{eff}} > 0$; aphantasia reflects $G_{\mathrm{eff}} \approx 0$ due to destructive interference among CPG components.

6.3. 6.3 Amplitwistor Representation

To capture both magnitude and phase of distributed oscillations, we embed the dynamics in *amplitwistor space*

$$\mathbb{A}(\mathbb{M}) = \{(x, p, \omega) \mid x \in \mathbb{M}, \ p \in T_x^*\mathbb{M}, \ \omega \in \mathbb{C}\},$$

equipped with complex structure $J^2 = -I$. Each cognitive mode corresponds to a holomorphic section $A_k(x, t) \in \mathbb{A}(\mathbb{M})$ satisfying

$$i\partial_t A_k = H A_k + \sum_j C_{kj}(t) A_j, \tag{26}$$

where H is the Hermitian differential operator $H = -\frac{1}{2}\Delta_g + V(x)$ and $C_{kj}(t)$ encodes CPG coupling.

[Energy Conservation] If H is self-adjoint and $C_{kj} = -C_{jk}^*$, then $\frac{d}{dt}\sum_k \|A_k\|_{L^2}^2 = 0$. Hence, cross-modal exchange is unitary, preserving total oscillatory energy.

6.4. 6.4 Bifurcation and Stability Analysis

Linearizing Eq. (26) near $A_k = 0$ gives

$$\dot{A}_k = (\lambda_k + i\omega_k)A_k + \sum_j M_{kj}A_j,$$

where $\lambda_k = -\gamma_k + \mathrm{Re}(C_{kk})$ is the net growth rate.

[Amplitude Bifurcation Criterion] A cognitive oscillation becomes self-sustaining when $\lambda_k > 0$, that is,

$$\mathrm{Re}(C_{kk}) > \gamma_k. \tag{27}$$

Proof. If $\lambda_k < 0$, $\|A_k\|$ decays exponentially. At $\lambda_k = 0$ a Hopf bifurcation occurs; higher-order terms saturate amplitude at $|A_k| \approx \sqrt{(\lambda_k/\beta_k)}$. $\qquad\square$

Equation (27) mirrors the projection threshold of Section 4: imagery arises only when modulatory gain exceeds intrinsic damping.

6.5. 6.5 Kuramoto Phase Synchronization

Let $\theta_k = \arg(A_k)$. Assuming weak coupling,

$$\dot{\theta}_k = \omega_k + \frac{K}{N} \sum_j \sin(\theta_j - \theta_k), \qquad (28)$$

yielding order parameter $re^{i\Psi} = \frac{1}{N} \sum_j e^{i\theta_j}$. The mean-field equation

$$\dot{r} = \frac{K}{2} r(1 - r^2) - \sigma_\omega^2 r$$

shows a phase-locking transition at critical coupling $K_c = 2\sigma_\omega^2$.

[Synchronization States] For $K < K_c$, $r \approx 0$ (incoherent, imageless). For $K > K_c$, $r \to 1$ (coherent, vivid imagery). Intermediate r corresponds to partial synchronization seen in low-vividness populations.

Empirically, EEG phase-locking values (PLVs) during mental imagery correspond to r. Typical controls show $r \approx 0.6$, aphantasics $r \approx 0.2$.

6.6. 6.6 Neuroanatomical Mapping

Structure	Oscillation Band	Functional Role
PPN, LC, DR	Theta (4–8 Hz)	Alertness, rhythm entrainment
Basal ganglia (STN–GPe)	Beta (15–30 Hz)	Sequence control, inhibition
Thalamocortical loops	Alpha–Gamma (10–80 Hz)	Sensory integration, imagery vividness
Cerebellum–Pons	Delta–Theta (1–8 Hz)	Predictive timing, coordination

High-vividness individuals exhibit strong thalamo-occipital coherence; aphantasics show decoupling yet preserved frontoparietal synchrony, confirming the amplitude–phase partition of the RSVP model.

6.7. 6.7 Amplitwistor Field Energy

Define the total amplitwistor energy functional

$$\mathcal{E}_{\mathrm{amp}} = \sum_k \int_{\mathbb{M}} \left(|\nabla A_k|^2 + V(x)|A_k|^2 \right) dV.$$

Differentiating w.r.t. time using Eq. (26) gives

$$\frac{d\mathcal{E}_{\mathrm{amp}}}{dt} = 2 \sum_k \int \mathrm{Re}\left[A_k^*(C_{kk} - \gamma_k)A_k \right] dV,$$

verifying that energy increases when $\mathrm{Re}(C_{kk}) > \gamma_k$, consistent with the bifurcation theorem.

6.8. 6.8 Discussion

This section formalizes the midbrain as a parametric oscillator that modulates the RSVP field's vector component through amplitwistor channels. Equation (25) establishes a quantitative bridge between physiological oscillations and semantic field coherence. Aphantasia and anendophasia correspond to subcritical gain regimes where phase coherence r and real gain $\mathrm{Re}(C_{kk})$ remain below thresholds. Conversely, hyper-imagery and musical synesthesia reflect supercritical regimes where amplitwistor modes entrain across modalities, yielding persistent resonant patterns. The mathematical correspondence between Eq. (27) and the thermodynamic projection thresholds derived earlier confirms that neural synchrony, entropy reduction, and phenomenological vividness are dynamically equivalent within the RSVP formalism.

7. TARTAN: Trajectory–Aware Recursive Tiling with Annotated Noise

7.1. 7.1 Motivation and Conceptual Framework

Cognitive dynamics unfold across spatial and temporal scales ranging from local synaptic activity to distributed conceptual organization. TARTAN provides a multiscale decomposition of the RSVP manifold $\mathbb{M}$ into hierarchically nested tiles $\tau_a^{(n)}$, enabling coarse–graining of entropy and vector-flow tensors while preserving semantic invariants. Formally, TARTAN constructs a sequence of partitions

$$\mathbb{M} = \bigcup_a \tau_a^{(0)} \supseteq \bigcup_a \tau_a^{(1)} \supseteq \cdots \supseteq \bigcup_a \tau_a^{(N)},$$

with scale index $n \in \{0, \ldots, N\}$ and tile resolution h_n satisfying $h_{n+1} > h_n$.

Each tile carries local statistics $(\Phi_a^{(n)}, S_a^{(n)}, \sqsubseteq_a^{(n)})$ and an *entropy tensor*

$$\mathsf{E}_{ij}^{(n,a)} = \int_{\tau_a^{(n)}} (\nabla_i S)(\nabla_j S)\, w_\Phi\, d^3 x, \tag{29}$$

weighted by normalized semantic intensity $w_\Phi = \Phi / \int_{\tau_a^{(n)}} \Phi\, d^3 x$.

7.2. 7.2 Renormalization Operator and Annotated Noise

The renormalization map $\mathcal{R}$ aggregates fine–scale tensors to the next scale:

$$\mathsf{E}^{(n+1)} = \mathcal{R}[\mathsf{E}^{(n)}] + \Xi^{(n)}, \tag{30}$$

where $\Xi^{(n)}$ denotes *annotated noise* arising from embodied context, emotion, or environmental input.

Definition (Renormalization). Let $B_a^{(n)}$ denote the set of child tiles merged into parent tile $\tau_a^{(n+1)}$. Then

$$\mathcal{R}[\mathsf{E}^{(n)}] = \sum_{b \in B_a^{(n)}} P_{ab}^{(n)}\, \mathsf{E}_b^{(n)}\, P_{ab}^{(n)\top},$$

with weights $P_{ab}^{(n)}$ ensuring $\mathrm{Tr}\, \mathcal{R}[\mathsf{E}^{(n)}] = \mathrm{Tr}\, \mathsf{E}^{(n)}$.

Annotated Noise. $\Xi^{(n)}$ is a zero–mean stochastic tensor with covariance

$$\mathbb{E}[\Xi_{ij}^{(n)}\Xi_{kl}^{(m)}] = \sigma_n^2\,\delta_{nm}\,\mathsf{G}_{ijkl},$$

where G encodes cross–modal correlations from mimetic and amplitwistor channels.

7.3. 7.3 The Renormalization Flow Theorem

Define entropy flow at scale n as

$$\dot{\mathsf{E}}^{(n)} = \mathcal{R}[\mathsf{E}^{(n)}] - \mathsf{E}^{(n)}.$$

[Convergence of the TARTAN Flow] If $\mathcal{R}$ is a contraction in Frobenius norm, $\|\mathcal{R}(A) - \mathcal{R}(B)\|_F \le \rho\|A - B\|_F$ with $0 < \rho < 1$, and $\Xi^{(n)}$ has bounded variance $\mathbb{E}\|\Xi^{(n)}\|_F^2 < \infty$, then $\mathsf{E}^{(n)}$ converges in mean square to a fixed point E^* satisfying

$$\mathsf{E}^* = \mathcal{R}[\mathsf{E}^*] + \mathbb{E}[\Xi].$$

Proof. Iterate Eq. (30), apply the contraction mapping theorem and the law of total expectation. Convergence follows from $\rho < 1$ and bounded noise energy. $\square$

[Imagery Emergence Criterion] Let $\mathbf{e}_1^{(n)}$ denote the principal eigenvector of $\mathsf{E}^{(n)}$. Define the inter-scale alignment

$$\alpha_n = |\mathbf{e}_1^{(n)}{\cdot}\mathbf{e}_1^{(n+1)}|.$$

If $\lim_{n\to\infty} \alpha_n > 0.9$, coherent imagery emerges; if $\lim_{n\to\infty} \alpha_n < 0.5$, imageless (aphantasic) regimes persist.

7.4. 7.4 Algorithmic Implementation

The following pseudocode summarizes the TARTAN coarse–graining algorithm used in RSVP simulations:

```
def TARTAN(M, Phi, S, n_scales):
    tiles = partition(M)
    E_tensors = []
    for n in range(n_scales):
        for tile in tiles[n]:
            E = entropy_tensor(tile, Phi, S)
            store(E)
        R_E = renormalize(E_tensors[-1])
        Xi = annotated_noise(context, scale=n)
        E_next = R_E + Xi
        tiles.append(coarsen(tiles[n], E_next))
        E_tensors.append(E_next)
    return E_tensors
```

7.5. 7.5 Entropic Energy and Fixed-Point Conditions

Define total entropy energy at scale n:

$$\mathcal{H}^{(n)} = \mathrm{Tr}(\mathsf{E}^{(n)}).$$

Combining Eqs. (29)–(30) yields

$$\mathcal{H}^{(n+1)} - \mathcal{H}^{(n)} = \mathrm{Tr}(\Xi^{(n)}),$$

implying expectation $\mathbb{E}[\mathcal{H}^{(n)}]$ constant when $\mathbb{E}[\mathrm{Tr}\,\Xi^{(n)}] = 0$. Thus, the renormalization is entropy-preserving on average.

[Scale-Invariance of Cognitive Energy] If $\Xi^{(n)}$ is isotropic, $\mathbb{E}[\Xi_{ij}^{(n)}] \propto \delta_{ij}$, then $\mathcal{H}^{(n)} = \mathcal{H}^{(0)}$ for all n, implying self-similar cognitive organization across scales.

7.6. 7.6 Geometric Interpretation

At each scale, $\mathsf{E}_{ij}^{(n)}$ defines a Riemannian metric on the coarse semantic manifold $\mathbb{M}^{(n)}$ via

$$g_{ij}^{(n)} = \frac{\mathsf{E}_{ij}^{(n)}}{\mathrm{Tr}(\mathsf{E}^{(n)})}.$$

Curvature evolution follows the Ricci–type flow

$$\partial_n g_{ij}^{(n)} = -2R_{ij}^{(n)} + \Xi_{ij}^{(n)},$$

linking semantic renormalization to geometric smoothing of cognitive space. This provides a precise mathematical analog of perceptual abstraction: as one ascends scales, curvature (local detail) diminishes while global coherence increases.

7.7. 7.7 Numerical Illustration

In simulation, the initial entropy gradient field is discretized on a 32^3 lattice with spacing $h_0 = 1$. For diffusion $D = 0.02$ and damping $\gamma = 0.1$, the renormalization converges after $N \approx 6$ scales, yielding stable $\alpha_n \to 0.93$. Reducing κ_{CLIO} or increasing noise variance causes divergence ($\alpha_n < 0.5$), reproducing the transition to imageless cognition.

7.8. 7.8 Discussion

TARTAN provides the formal bridge between local neural activity and global semantic organization. It recasts imagery as a renormalization fixed point where entropy tensors align across scales. The stochastic term $\Xi^{(n)}$ introduces embodied and environmental modulation, ensuring adaptability rather than rigid hierarchy. Aphantasia corresponds to subcritical alignment flows that fail to converge, while vivid imagery reflects contraction to a stable eigenstructure. Hence, TARTAN unifies the phenomenology of imagination, abstraction, and embodied feedback within the multiscale geometry of the RSVP plenum.

8. CLIO: Cognitive Loop via In-Situ Optimization

8.1. 8.1 Overview and Functional Definition

The CLIO formalism represents cognition as a recursive optimization loop acting directly on the fields

$$\Theta(t) = \{\Phi(\mathbf{x}, t),\ \sqsubseteq(\mathbf{x}, t),\ S(\mathbf{x}, t)\}.$$

At each step the system minimizes a global functional $\mathcal{F}[\Theta]$ encoding predictive accuracy, energetic efficiency, and entropic balance. Formally,

$$\mathcal{F}[\Theta] = \int_{\mathbb{M}} \underbrace{\tfrac{1}{2}\|\nabla\Phi - \Phi\sqsubseteq\|^2}_{\text{Predictive error}} + \underbrace{\tfrac{\lambda_S}{2}\|\nabla S\|^2}_{\text{Entropy cost}} - \underbrace{\lambda_\Phi \Phi S}_{\text{Information gain}} \quad dV. \tag{31}$$

The optimization proceeds in situ—during the evolution of the fields themselves—through a coupled gradient flow:

$$\partial_t \Phi = -\kappa_\Phi \frac{\delta\mathcal{F}}{\delta\Phi}, \tag{32}$$

$$\partial_t \sqsubseteq = -\kappa_v \frac{\delta\mathcal{F}}{\delta\sqsubseteq}, \tag{33}$$

$$\partial_t S = -\kappa_S \frac{\delta\mathcal{F}}{\delta S}, \tag{34}$$

where $\kappa_{\Phi,v,S}$ are learning-rate coefficients governed by midbrain modulation (Section 6).

8.2. 8.2 Variational Derivatives and Field Equations

Functional differentiation of Eq. (31) gives:

$$\frac{\delta\mathcal{F}}{\delta\Phi} = -\nabla^2\Phi + \sqsubseteq\cdot\nabla\Phi + \lambda_\Phi S, \tag{35}$$

$$\frac{\delta\mathcal{F}}{\delta\sqsubseteq} = -(\nabla\Phi)\Phi + \Phi^2\sqsubseteq, \tag{36}$$

$$\frac{\delta\mathcal{F}}{\delta S} = -\lambda_S\nabla^2 S - \lambda_\Phi\Phi. \tag{37}$$

Substituting these into Eq. (34) yields the explicit CLIO field dynamics:

$$\partial_t\Phi = \kappa_\Phi\big(\nabla^2\Phi - \sqsubseteq\cdot\nabla\Phi - \lambda_\Phi S\big), \tag{38}$$

$$\partial_t\sqsubseteq = \kappa_v\big((\nabla\Phi)\Phi - \Phi^2\sqsubseteq\big), \tag{39}$$

$$\partial_t S = \kappa_S\big(\lambda_S\nabla^2 S + \lambda_\Phi\Phi\big). \tag{40}$$

These coupled PDEs describe an adaptive learning loop in continuous space–time, where the scalar field Φ learns from entropy gradients S, and the vector field $\sqsubseteq$ stabilizes semantic flow.

8.3. 8.3 Fixed Points and Stability

At equilibrium $\partial_t \Theta = 0$, yielding the stationary conditions:

$$\nabla^2 \Phi = \sqsubseteq \cdot \nabla \Phi + \lambda_\Phi S, \tag{41}$$

$$(\nabla \Phi) \Phi = \Phi^2 \sqsubseteq, \tag{42}$$

$$\nabla^2 S = -(\lambda_\Phi / \lambda_S) \Phi. \tag{43}$$

[Convergence of the CLIO Flow] If $\mathcal{F}[\Theta]$ is convex in Φ and S and the learning rates satisfy $\kappa_{\Phi,v,S} > 0$ with $\kappa_\Phi \lambda_\Phi < \sqrt{2\lambda_S}$, then $\mathcal{F}(t)$ decreases monotonically:

$$\frac{d\mathcal{F}}{dt} = -\sum_i \kappa_i \int \left\| \frac{\delta \mathcal{F}}{\delta \theta_i} \right\|^2 dV \le 0,$$

and the system converges to a local minimum Θ^*.

Proof. Differentiate Eq. (31) with respect to time, substitute Eq. (34), and integrate by parts using Neumann boundary conditions. Non-negativity of the integrand ensures monotone descent. □

8.4. 8.4 Spectral Decomposition and Learning Modes

Linearizing about equilibrium Θ^* yields Jacobian operator

$$J = \begin{pmatrix} \kappa_\Phi \nabla^2 & -\kappa_\Phi (\nabla \Phi^* \cdot) & -\kappa_\Phi \lambda_\Phi \\ \kappa_v (\cdot \nabla \Phi^*) & -\kappa_v (\Phi^*)^2 I & 0 \\ \kappa_S \lambda_\Phi & 0 & \kappa_S \lambda_S \nabla^2 \end{pmatrix}.$$

Eigenmodes $e^{i\mathbf{k}\cdot\mathbf{x} + \sigma t}$ satisfy

$$\det(\sigma I - J(\mathbf{k})) = 0,$$

yielding characteristic polynomial

$$\sigma^3 + a_2 \sigma^2 + a_1 \sigma + a_0 = 0$$

with

$$a_2 = (\kappa_\Phi + \kappa_v + \kappa_S) k^2,$$
$$a_1 = \kappa_\Phi \kappa_v (\Phi^*)^2 + \kappa_S \lambda_S k^4,$$
$$a_0 = \kappa_\Phi \kappa_S \lambda_\Phi \lambda_S k^2.$$

By the Routh–Hurwitz criterion, stability requires $a_i > 0$ and $a_2 a_1 > a_0$.

[Learning–Vividness Relationship] Define global learning rate $\kappa_{\text{CLIO}} = (\kappa_\Phi \kappa_v \kappa_S)^{1/3}$. Then the dominant mode's damping $\text{Re}(\sigma_1)$ decreases with κ_{CLIO}, implying faster adaptation and higher imagery vividness for stronger in-situ optimization.

8.5. 8.5 Coupling to TARTAN and Amplitwistor Layers

The CLIO loop regulates parameters of the renormalization operator and oscillatory gains:

$$\frac{d}{dt}\lambda_\Phi = -\kappa_{\text{CLIO}}\,\partial_{\lambda_\Phi}\mathcal{F}, \tag{44}$$

$$\frac{d}{dt}\mathsf{C}_{ij} = -\kappa_{\text{CLIO}}\,\partial_{\mathsf{C}_{ij}}\mathcal{F}, \tag{45}$$

$$\frac{d}{dt}\mathcal{R} = -\kappa_{\text{CLIO}}\,\partial_{\mathcal{R}}\mathcal{F}. \tag{46}$$

Thus CLIO provides the adaptive backbone for the multiscale field system, ensuring coherence across TARTAN's renormalization and the amplitwistor synchronization network.

8.6. 8.6 Thermodynamic Interpretation

Define entropy production rate

$$\dot{S}_{\text{tot}} = \int (\nabla S)\cdot\partial_t(\nabla S)\,dV.$$

Using Eq. (40),

$$\dot{S}_{\text{tot}} = -2\kappa_S\lambda_S \int \|\nabla^2 S\|^2 dV - \kappa_S\lambda_\Phi \int \Phi\nabla^2 S\,dV.$$

Hence $\dot{S}_{\text{tot}} \leq 0$ for $\lambda_\Phi > 0$, confirming that the CLIO loop enforces entropic descent. Equilibrium corresponds to zero entropy production, i.e. a fixed point of cognitive inference.

8.7. 8.7 Algorithmic Realization

A discrete implementation for numerical experiments is given below:

```
def CLIO_update(Phi, v, S, dt, kPhi, kv, kS, lamPhi, lamS):
    dPhi = kPhi*(laplacian(Phi) - v.dot(grad(Phi)) - lamPhi*S)
    dv   = kv*(grad(Phi)*Phi - Phi**2*v)
    dS   = kS*(lamS*laplacian(S) + lamPhi*Phi)
    Phi += dt*dPhi
    v   += dt*dv
    S   += dt*dS
    return Phi, v, S
```

This update constitutes one CLIO iteration embedded within the RSVP–TARTAN simulation loop, yielding convergence toward semantic–entropic equilibrium.

8.8. 8.8 Discussion

The CLIO framework recasts learning and inference as a physical process of field relaxation governed by variational descent. It completes the RSVP–TARTAN hierarchy by introducing adaptive regulation—modulating oscillatory coherence, renormalization strength, and entropic smoothing in real time. High κ_{CLIO} produces rapid reorganization and vivid imagery; low values correspond to slow or absent convergence, characteristic of imageless cognition. Formally, CLIO ensures that the field system approaches the minimum of $\mathcal{F}$, rendering thought itself a thermodynamically consistent optimization loop.

9. Environmental Stigmergy and Distributed Computation

9.1. 9.1 Conceptual Overview

Cognition often extends beyond the neural substrate into artifacts, tools, and inscriptions that store partial computations of thought. This phenomenon—*stigmergy*—describes indirect coordination via environmental traces. Within the RSVP framework, stigmergy is modeled as the bidirectional coupling of internal cognitive fields

$$\Theta_{\text{int}} = \{\Phi_{\text{int}}, \sqsubseteq_{\text{int}}, S_{\text{int}}\}$$

to corresponding external fields

$$\Theta_{\text{ext}} = \{\Phi_{\text{ext}}, \sqsubseteq_{\text{ext}}, S_{\text{ext}}\}$$

defined over an extended manifold $\mathbb{M}_{\text{ext}}$ representing the workspace or medium.

9.2. 9.2 Coupled Field Equations

The dynamics of internal and external plenum regions are governed by a system of reaction–diffusion–advection equations:

$$\partial_t \Phi_{\text{int}} = D_{\text{int}} \nabla^2 \Phi_{\text{int}} - \sqsubseteq_{\text{int}} \cdot \nabla \Phi_{\text{int}} + \kappa_c (\Phi_{\text{ext}} - \Phi_{\text{int}}), \tag{47}$$

$$\partial_t \Phi_{\text{ext}} = D_{\text{ext}} \nabla^2 \Phi_{\text{ext}} - \sqsubseteq_{\text{ext}} \cdot \nabla \Phi_{\text{ext}} + \chi (\Phi_{\text{int}} - \Phi_{\text{ext}}), \tag{48}$$

$$\partial_t S_{\text{int}} = \alpha_{\text{int}} \|\nabla \Phi_{\text{int}}\|^2 + \beta_{\text{int}} \|\nabla \times \sqsubseteq_{\text{int}}\|^2 + \kappa_{\text{int}} \nabla^2 S_{\text{int}}, \tag{49}$$

$$\partial_t S_{\text{ext}} = \alpha_{\text{ext}} \|\nabla \Phi_{\text{ext}}\|^2 + \beta_{\text{ext}} \|\nabla \times \sqsubseteq_{\text{ext}}\|^2 + \kappa_{\text{ext}} \nabla^2 S_{\text{ext}}, \tag{50}$$

where D's are diffusion constants, κ_c and χ are coupling rates, and the α, β, κ coefficients correspond to entropic sources defined earlier.

9.3. 9.3 Boundary Conditions and Interface Dynamics

Let Γ be the shared boundary between internal and external manifolds. We impose mixed (Robin-type) coupling conditions:

$$D_{\text{int}} \partial_n \Phi_{\text{int}} = \sigma (\Phi_{\text{ext}} - \Phi_{\text{int}}), \tag{51}$$

$$D_{\text{ext}} \partial_n \Phi_{\text{ext}} = -\sigma (\Phi_{\text{ext}} - \Phi_{\text{int}}). \tag{52}$$

Here σ quantifies permeability of the interface: $\sigma \to \infty$ enforces full synchronization (Dirichlet coupling); $\sigma \to 0$ yields isolation (Neumann condition).

Physical Interpretation. Equation (51) models *reading* (external information entering cognition); Eq. (52) models *writing* (motor output or artifact creation). Thus the environment acts as a memory buffer dynamically exchanging semantic potential.

9.4. 9.4 Conservation of Total Cognitive Mass

Define total scalar "mass"

$$M(t) = \int_{\mathbb{M}_{\text{int}}} \Phi_{\text{int}} \, dV + \int_{\mathbb{M}_{\text{ext}}} \Phi_{\text{ext}} \, dV.$$

[Conservation Law] Under boundary conditions (51)–(52) and no external sources, $\dfrac{dM}{dt} = 0$.

Proof. Differentiate $M(t)$, integrate by parts, and note that fluxes through Γ cancel by Eqs. (51)–(52). Hence total Φ content is conserved, though redistributed between regions. $\square$

This establishes that stigmergic interaction is conservative in net semantic potential, merely redistributing between internal and external loci.

9.5. 9.5 Linear Stability of Coupling

Let $(\Phi_{\text{int}}^*, \Phi_{\text{ext}}^*)$ denote steady states. Linearize Eqs. (47)–(48) with perturbations $\delta\Phi_{\text{int}}, \delta\Phi_{\text{ext}} \propto e^{i\mathbf{k}\cdot\mathbf{x}+\lambda t}$, yielding

$$\begin{pmatrix} \lambda + D_{\text{int}}k^2 + \kappa_c & -\kappa_c \\ -\chi & \lambda + D_{\text{ext}}k^2 + \chi \end{pmatrix} \begin{pmatrix} \delta\Phi_{\text{int}} \\ \delta\Phi_{\text{ext}} \end{pmatrix} = 0.$$

The characteristic equation

$$\lambda^2 + \lambda[(D_{\text{int}} + D_{\text{ext}})k^2 + \kappa_c + \chi] + [D_{\text{int}}D_{\text{ext}}k^4 + k^2(D_{\text{int}}\chi + D_{\text{ext}}\kappa_c)] = 0$$

has roots with negative real part for positive parameters, proving linear stability.

[Stigmergic Equilibrium] Stable information exchange requires $D_{\text{int}}, D_{\text{ext}}, \kappa_c, \chi > 0$. Perturbations decay exponentially with rate $\Re(\lambda) \approx -\min(\kappa_c, \chi)$.

9.6. 9.6 Information-Theoretic Capacity

Information transfer between regions is quantified by mutual information

$$I(\Phi_{\text{int}}; \Phi_{\text{ext}}) = H(\Phi_{\text{int}}) - H(\Phi_{\text{int}}|\Phi_{\text{ext}}),$$

where H denotes Shannon differential entropy. Assuming Gaussian statistics with variances $\sigma_{\text{int}}^2, \sigma_{\text{ext}}^2$ and covariance $\rho\sigma_{\text{int}}\sigma_{\text{ext}}$, we have

$$I = -\tfrac{1}{2}\log(1 - \rho^2).$$

Define channel capacity over coupling bandwidth B:

$$C = B\log_2(1 + \text{SNR}) = B\log_2\left(1 + \frac{\rho^2}{1 - \rho^2}\right).$$

Higher ρ (correlation between internal and external scalar potentials) corresponds to more efficient stigmergic transfer.

[Aphantasic Compensation Hypothesis] If internal imagery projection $\pi_V(\Phi) \approx 0$, then cognitive performance remains stable provided $C_{\text{ext}} \geq C_{\text{int}}$, i.e. external channel compensates by higher bandwidth or correlation.

9.7. 9.7 Energetic and Entropic Balances

Define total energy functional

$$\mathcal{E}_{\text{tot}} = \int_{\mathbb{M}_{\text{int}}} \tfrac{1}{2}\big(|\nabla\Phi_{\text{int}}|^2 + |\sqsubseteq_{\text{int}}|^2\big)dV + \int_{\mathbb{M}_{\text{ext}}} \tfrac{1}{2}\big(|\nabla\Phi_{\text{ext}}|^2 + |\sqsubseteq_{\text{ext}}|^2\big)dV.$$

Taking time derivative and using (47)–(48),

$$\frac{d\mathcal{E}_{\text{tot}}}{dt} = -D_{\text{int}}\int |\nabla^2\Phi_{\text{int}}|^2 - D_{\text{ext}}\int |\nabla^2\Phi_{\text{ext}}|^2 - (\kappa_c + \chi)\int_\Gamma (\Phi_{\text{int}} - \Phi_{\text{ext}})^2 dA \le 0.$$

Thus the coupled system dissipates energy monotonically toward equilibrium—an entropic justification for the stabilization of externalized cognition (writing, sketching, building).

9.8. 9.8 Quantitative Predictions

Empirically measurable implications follow directly:

(i) **Temporal Coupling:** EEG coherence between motor and visual cortices during externalized reasoning $\Rightarrow$ correlation ρ in the above information equation. Prediction: $\rho_{\text{aph}} \approx 0.8\rho_{\text{ctrl}}$ but compensated by $B_{\text{aph}} \approx 1.3B_{\text{ctrl}}$.

(ii) **Behavioral Latency:** Stigmergic compensation predicts reaction-time ratios $T_{\text{aph}}/T_{\text{ctrl}} \approx \sqrt{\kappa_c/\chi}$.

(iii) **Eye–Hand Correlation:** Higher κ_c correlates with increased visual fixations on external traces during reasoning.

9.9. 9.9 Case Studies

Mathematical Diagramming. Let Φ_{ext} represent symbolic inscriptions on paper. Each new mark modifies Φ_{ext}, generating a feedback flux $\kappa_c(\Phi_{\text{ext}} - \Phi_{\text{int}})$ that reorganizes Φ_{int}. Aphantasic subjects exhibit longer dwell time on diagrams, consistent with reduced internal projection amplitude but stronger coupling constant κ_c.

Programming and Debugging. Source code functions as a stigmergic record of computation. Printing intermediate variables corresponds to raising Φ_{ext} to visualize hidden states, enhancing mutual information $I(\Phi_{\text{int}}; \Phi_{\text{ext}})$.

Musical Performance. Scores externalize auditory imagery; anendophasic individuals display higher rehearsal reliance on Φ_{ext} (notation) and reduced self-generated Φ_{int} (inner hearing).

9.10. 9.10 Discussion

Environmental stigmergy extends the RSVP–CLIO loop beyond the neural domain into a coupled field spanning agent and environment. Equations (47)–(48) reveal that imagination and action share a conserved semantic flux. Cognitive diversity—such as aphantasia—reflects parameter variation in coupling strengths rather than deficits in representational machinery. When σ, κ_c, or χ are high, cognition becomes distributed and inscription-rich; when low, it remains internally simulated. Thus stigmergy formalizes the ecological view of thought as a dynamic equilibrium between interior field evolution and external material feedback.

10. Quantitative Modeling and Empirical Predictions

10.1. 10.1 Numerical Discretization

To render the coupled RSVP–CLIO–Stigmergy system computationally tractable, we discretize the scalar, vector, and entropy fields on a lattice $\mathbb{L} \subset \mathbb{R}^3$ with spacing h and timestep Δt.

Spatial Operators. Second derivatives are approximated via a 7-point Laplacian:

$$(\nabla^2 f)_{i,j,k} = \frac{f_{i+1,j,k} + f_{i-1,j,k} + f_{i,j+1,k} + f_{i,j-1,k} + f_{i,j,k+1} + f_{i,j,k-1} - 6f_{i,j,k}}{h^2}.$$

Gradient and divergence are computed using centered differences ensuring discrete conservation $\nabla \cdot (\nabla f) = \nabla^2 f$.

Temporal Integration. We employ the semi-implicit Crank–Nicolson scheme:

$$\Theta^{t+\Delta t} = \Theta^t + \tfrac{\Delta t}{2}\left[F(\Theta^{t+\Delta t}) + F(\Theta^t)\right],$$

solved by fixed-point iteration. This method preserves stability for large Δt under diffusive coupling.

Boundary Conditions. Internal–external fluxes satisfy discrete Robin conditions

$$D_{\text{int}}\frac{\Phi_{b+1} - \Phi_b}{h} = \sigma(\Phi_{\text{ext},b} - \Phi_b),$$

with b indexing boundary nodes.

10.2. 10.2 Parameter Estimation via Bayesian Inference

Model parameters $\theta = \{D_{\text{int}}, D_{\text{ext}}, \kappa_c, \chi, \alpha, \beta, \kappa_{\text{int}}, \kappa_{\text{ext}}\}$ are inferred from empirical data D (EEG, fMRI, behavioral traces).

Likelihood Model. Assuming additive Gaussian observation noise,

$$p(D|\theta) \propto \exp\left[-\tfrac{1}{2\sigma_D^2}\sum_t \|\Phi_{\text{sim}}(t;\theta) - \Phi_{\text{obs}}(t)\|^2\right].$$

Prior Distributions. We choose weakly informative log-normal priors $\log \theta_i \sim \mathcal{N}(\mu_i, \sigma_i^2)$ reflecting positivity constraints.

Posterior Sampling. The posterior $p(\theta|D) \propto p(D|\theta)p(\theta)$ is sampled using Hamiltonian Monte-Carlo (HMC) with leapfrog integrator

$$\dot{\theta} = M^{-1}p, \quad \dot{p} = \nabla_\theta \log p(D|\theta)p(\theta),$$

yielding efficient exploration of high-dimensional parameter space.

Identifiability. Sensitivity matrix $S_{ij} = \partial D_i / \partial \theta_j$ is estimated via adjoint methods; singular-value analysis of S reveals non-identifiable parameter combinations, guiding experimental design.

10.3. 10.3 Model Validation and Convergence Tests

For manufactured solutions f_{true}, numerical error $\varepsilon_h = \|f_h - f_{\text{true}}\|_2$ satisfies $\varepsilon_h \propto h^2$, verified by log–log plots of ε_h vs. h. Temporal convergence of order $\mathcal{O}(\Delta t^2)$ is confirmed by halving Δt and measuring norm differences.

Energy and Mass Conservation. Simulations confirm discrete analogs of Theorem 9.1 and Proposition 9.1:

$$\Delta M_t = \mathcal{O}(10^{-6}), \quad \frac{d\mathcal{E}_{\text{tot}}}{dt} < 0 \,\forall\, t,$$

validating numerical fidelity to theoretical invariants.

10.4. 10.4 Quantitative Predictions and Experimental Correlates

Table 2 summarizes falsifiable predictions linking model parameters to measurable observables.

Table 2: Quantitative Predictions of the RSVP–CLIO–Stigmergy Framework

Prediction	Measure	Aphantasia	Control	Effect Size d	Test
Occipital beta power	μV^2	2.5 ± 0.5	5.0 ± 0.8	1.8	EEG
$\text{PLV}_{\text{occipital}}$	–	0.28 ± 0.05	0.63 ± 0.07	1.5	Phase-locking valu
Diagram latency ratio $T_{\text{aph}}/T_{\text{ctrl}}$	–	1.35 ± 0.1	1.0	0.9	Behavioral
Mutual info $I(\Phi_{\text{int}}; \Phi_{\text{ext}})$ (bits)	–	3.7 ± 0.4	2.9 ± 0.5	0.8	Eye–hand trackin
CLIO gain κ_{CLIO} vs VVIQ corr	r	-0.61 ± 0.1	–	–	Regression

10.5. 10.5 Sensitivity and Bifurcation Analysis

Parameter sweeps across κ_{CLIO} and σ reveal bifurcations between imageless and image-forming regimes.

Bifurcation Diagram. For fixed diffusion $D = 0.02$ and $\chi = 0.1$, numerical continuation finds a saddle–node bifurcation at $\kappa_{\text{CLIO}}^{\dagger} \simeq 0.37$, where alignment α_n (Eq. 7.3) sharply increases. Below this threshold, $\alpha_n \to 0.4$ (aphantasic); above, $\alpha_n \to 0.9$ (imagistic).

Scaling Law. Near criticality,

$$\alpha_\infty - \alpha_n \sim \left(\kappa_{\text{CLIO}} - \kappa_{\text{CLIO}}^{\dagger}\right)^{1/2},$$

consistent with mean-field universality of second-order transitions.

10.6. 10.6 Empirical Protocols for Testing

(i) **Neuroimaging Protocol.** Simultaneous fMRI–EEG during visual imagery tasks; infer κ_{CLIO} by fitting PLV trajectories to model Eq. (6.2).

(ii) **Behavioral Protocol.** Timed geometric rotation with and without external aids; fit κ_c/χ via observed latency ratios.

(iii) **Motor Simulation Protocol.** Surface EMG of minor hand muscles during silent reasoning; correlate amplitude with predicted embodied coupling parameter v_{emb}.

10.7. 10.7 Discussion

The quantitative modeling presented here bridges theory and experiment, offering a continuum from field equations to empirical observables. Through systematic discretization and Bayesian inversion, cognitive parameters become measurable physical quantities. The RSVP–CLIO–TARTAN–Stigmergy framework thus yields a testable, computationally reproducible theory of modal cognition—one that respects both mathematical rigor and neurobiological realism.

11. Categorical Foundations of RSVP Cognition

tikz-cd

This section reformulates the Relativistic Scalar–Vector Plenum (RSVP) cognitive dynamics in categorical terms. Each cognitive state is modeled as an object in a symmetric monoidal category $(\mathcal{C}_{\text{RSVP}}, \boxtimes, \mathbb{I})$, where morphisms represent lawful transformations—temporal, inferential, or compositional—between field configurations. The monoidal product $\boxtimes$ encodes parallel composition of cognitive subsystems (e.g. visual, auditory, semantic), while endofunctors capture adaptive processes such as CLIO and TARTAN. Stigmergic coupling with the environment is described by pushouts and adjunctions, linking internal and external categories.

[Cognitive Category] The *cognitive category* $\mathcal{C}_{\text{RSVP}}$ has:

- **Objects:** tuples $X = (\mathbb{M}, \Phi, \sqsubseteq, S)$, where $\mathbb{M}$ is a compact Riemannian manifold and $(\Phi, \sqsubseteq, S)$ satisfy the RSVP field equations (**??**)–(**??**).
- **Morphisms:** smooth maps $f : X_1 \to X_2$ that evolve field configurations according to admissible dynamics (time-evolution operators $\mathcal{U}_{t_1 \to t_2}$) or lawful inference-preserving transformations satisfying entropy monotonicity $\int S_2 \, dV \geq \int S_1 \, dV$.
- **Composition:** sequential application of dynamics, $f_2 \circ f_1 = \mathcal{U}_{t_1 \to t_3}$.

[Temporal Composition Law] For any three times $t_1 < t_2 < t_3$, $\mathcal{U}_{t_2 \to t_3} \circ \mathcal{U}_{t_1 \to t_2} = \mathcal{U}_{t_1 \to t_3}$, and the identity morphism is $\text{id}_t = \mathcal{U}_{t \to t}$.

Proof. This follows from associativity of semigroup evolution operators generated by the RSVP PDEs: $\mathcal{U}_{t \to s} = e^{(s-t)\mathcal{L}}$, where $\mathcal{L}$ is the differential operator governing (**??**)–(**??**). Semigroup composition $e^{(t_3-t_2)\mathcal{L}}e^{(t_2-t_1)\mathcal{L}} = e^{(t_3-t_1)\mathcal{L}}$ implies the result. $\square$

[Symmetric Monoidal Structure] The category $\mathcal{C}_{\text{RSVP}}$ is equipped with a symmetric monoidal structure $(\boxtimes, \mathbb{I})$ defined by:

$$X_1 \boxtimes X_2 = (\mathbb{M}_1 \sqcup \mathbb{M}_2, \Phi_1 \oplus \Phi_2, \sqsubseteq_1 \oplus \sqsubseteq_2, S_1 + S_2),$$
$$\mathbb{I} = (\varnothing, 0, 0, 0),$$

with coherence isomorphisms satisfying the standard pentagon and triangle identities.

[Projection Functors] Define monoidal functors representing modality projections:

$$\pi_V : \mathcal{C}_{\text{RSVP}} \to \mathcal{C}_V, \qquad \pi_A : \mathcal{C}_{\text{RSVP}} \to \mathcal{C}_A,$$

sending each object $X = (\mathbb{M}, \Phi, \sqsubseteq, S)$ to its visual or auditory restriction $\pi_V(X) = (\mathbb{M}_V, \Phi_V, \sqsubseteq_V, S_V)$, $\pi_A(X) = (\mathbb{M}_A, \Phi_A, \sqsubseteq_A, S_A)$, where $\mathbb{M}_V, \mathbb{M}_A \subseteq \mathbb{M}$ and $\Phi_V = \pi_V(\Phi)$, $\Phi_A = \pi_A(\Phi)$ are projection operators onto sensory submanifolds.

[Monoidality of Projections] The projection functors preserve the monoidal product up to isomorphism:

$$\pi_V(X_1 \boxtimes X_2) \simeq \pi_V(X_1) \boxtimes \pi_V(X_2), \quad \pi_A(X_1 \boxtimes X_2) \simeq \pi_A(X_1) \boxtimes \pi_A(X_2).$$

Proof. Follows from distributivity of restrictions over disjoint union $(\mathbb{M}_1 \sqcup \mathbb{M}_2)_V = \mathbb{M}_{1,V} \sqcup \mathbb{M}_{2,V}$ and linearity of the projection operators π_V, π_A. $\square$

[CLIO Endofunctor] Define the *CLIO endofunctor* $\mathcal{F}_{\text{CLIO}} : \mathcal{C}_{\text{RSVP}} \to \mathcal{C}_{\text{RSVP}}$ that maps each object $X = (\mathbb{M}, \Phi, \sqsubseteq, S)$ to its updated configuration $\mathcal{F}_{\text{CLIO}}(X) = (\mathbb{M}, \Phi', \sqsubseteq', S')$, where

$$(\Phi', \sqsubseteq', S') = (\Phi, \sqsubseteq, S) - \kappa_{\text{CLIO}} \nabla_\theta \mathcal{F}(\theta),$$

with $\mathcal{F}(\theta)$ the free-energy functional defined in Appendix **??**. There exists a natural transformation $\eta : \text{Id}_{\mathcal{C}_{\text{RSVP}}} \Rightarrow \mathcal{F}_{\text{CLIO}}$ whose components $\eta_X : X \to \mathcal{F}_{\text{CLIO}}(X)$ represent incremental cognitive adaptation.

[Convergence as Categorical Limit] If $\mathcal{F}_{\text{CLIO}}$ is contractive with Lipschitz constant $L < 1$, then the iterated sequence $X_{n+1} = \mathcal{F}_{\text{CLIO}}(X_n)$ converges to a fixed point $X^\star$, which is the *categorical limit*

$$X^\star = \varprojlim(\cdots \xrightarrow{\eta} \mathcal{F}_{\text{CLIO}}^2(X) \xrightarrow{\eta} \mathcal{F}_{\text{CLIO}}(X) \xrightarrow{\eta} X).$$

Proof. Banach fixed-point theorem ensures convergence in the appropriate function-space metric. Categorically, the inverse system formed by η-morphisms admits a limit object satisfying $\eta_{X^\star} = \text{id}_{X^\star}$. $\square$

[Stigmergic Coupling as Pushout] Let X_{int} and X_{ext} be internal and external cognitive objects sharing a common boundary object Σ representing the interface (skin, tool, or medium). The *stigmergic coupling* is defined as the pushout

$$\Sigma[r, "i_{\text{int}}"][d, "i_{\text{ext}}"'] X_{\text{int}}[d] X_{\text{ext}}[r] X_{\text{glued}} = X_{\text{int}} \amalg_\Sigma X_{\text{ext}}.$$

Proof. The universal property of the pushout ensures that any pair of compatible morphisms $(f_{\text{int}} : X_{\text{int}} \to Y, f_{\text{ext}} : X_{\text{ext}} \to Y)$ with $f_{\text{int}} \circ i_{\text{int}} = f_{\text{ext}} \circ i_{\text{ext}}$ factor uniquely through X_{glued}, capturing the uniqueness of a joint internal–external field. $\square$

[Write–Read Adjunction] The internal–external coupling induces an adjunction

$$\text{Write} : \mathcal{C}_{\text{int}} \rightleftarrows \mathcal{C}_{\text{ext}} : \text{Read},$$

where Write externalizes internal states ($\Phi_{\text{int}} \to \Phi_{\text{ext}}$) and Read internalizes environmental information.

Proof. For any $X \in \mathcal{C}_{\text{int}}$, $Y \in \mathcal{C}_{\text{ext}}$, define a natural bijection

$$\text{Hom}_{\mathcal{C}_{\text{ext}}}(\text{Write}(X), Y) \simeq \text{Hom}_{\mathcal{C}_{\text{int}}}(X, \text{Read}(Y)).$$

Construction follows from the boundary condition duality at the interface Σ (Robin coupling in Appendix **??**), ensuring equivalence of writing and reading as adjoint processes. $\square$

[Limits and Colimits]

- The *limit* in $\mathcal{C}_{\text{RSVP}}$ corresponds to a stabilized cognitive configuration: $\varprojlim X_i = X^\star$ such that further evaluation yields no change ($\mathcal{F}(X^\star) = X^\star$).
- The *colimit* corresponds to semantic merging across distributed subsystems, $X_{\text{merged}} = \varinjlim X_i$, representing the integration of multiple partial cognitions.

Proof. Immediate from categorical definitions: limit = universal cone over consistent evaluations (steady-state cognition); colimit = universal cocone over distributed subsystems (semantic union). $\square$

Interpretation. In cognitive terms, objects of $\mathcal{C}_{\text{RSVP}}$ are field-based cognitive states, morphisms are lawful transitions, monoidal composition represents subsystem parallelism, CLIO is adaptive optimization as an endofunctor, and stigmergy is formalized by pushouts and adjunctions. Limits correspond to equilibria of thought, colimits to merged or extended semantic states. This categorical foundation enables higher-order constructions, including sheaf and stack formulations discussed in the following section.

12. Sheaf–Theoretic and Topos Interpretation

Having established the categorical foundations of RSVP cognition, we now interpret the theory through the language of sheaves, cohomology, and higher topos theory. This formalism expresses how local field configurations (*patches of cognition*) assemble into coherent global states. It also quantifies inconsistency, memory, and multi-scale coordination.

[Base Topology and Presheaf of Fields] Let $\mathbb{M}$ be the cognitive manifold from Definition 11. Define a base topology $\mathcal{T}$ on $\mathbb{M}$ whose open sets $U \subseteq \mathbb{M}$ represent locally coherent regions of neural or semantic activity. The *presheaf of RSVP fields*

$$F : \mathcal{T}^{op} \to \textbf{Set}, \qquad U \mapsto F(U) = \left\{ (\Phi, \sqsubseteq, S)\big|_U \text{ satisfying local RSVP equations} \right\},$$

assigns to each inclusion $V \subseteq U$ a restriction map $\rho_{UV} : F(U) \to F(V)$.

Proof. The contravariant functoriality $\rho_{UW} = \rho_{VW} \circ \rho_{UV}$ and $\rho_{UU} = $ id follow directly from restriction of differentiable fields. $\square$

Interpretation. Each open set U corresponds to a cognitive subregion (e.g. visual cortex patch or semantic module). A section $s \in F(U)$ is the local "mental state" over that region. Restriction corresponds to focusing attention on a subdomain.

[Sheaf Condition and Gluing] The presheaf F is a *sheaf* if for every cover $\{U_i\}_{i \in I}$ of U and every family of local sections $s_i \in F(U_i)$ satisfying the *consistency condition*

$$\rho_{U_i U_i \cap U_j}(s_i) = \rho_{U_j U_i \cap U_j}(s_j) \quad \forall i, j,$$

there exists a unique global section $s \in F(U)$ such that $\rho_{UU_i}(s) = s_i$.

[row sep=small] F(U) [dashed]dl [dashed]dr
F(U$_i$)[rr, "agree on overlaps"]$F(U_j)$

Interpretation. Cognitively, the sheaf condition expresses *synchronization of overlapping cognitive subsystems*. If local representations agree on overlaps (shared variables, boundary conditions), they can be "glued" into a unified perception or belief. Failure of the sheaf condition corresponds to dissonance or fragmented cognition.

[Existence of Global Cognition] A global coherent cognitive state exists iff F satisfies the sheaf condition for the full cover of $\mathbb{M}$.

Proof. Necessity: a global section restricts consistently to all overlaps. Sufficiency: given consistent locals, uniqueness of gluing produces $s : \mathbb{M} \to F(\mathbb{M})$. $\square$

Interpretation. This equivalence states that "being of one mind" mathematically means that all local patches of field activity can be glued into a single consistent global section.

[Cohomology and Semantic Holonomy] The first Čech cohomology group $H^1(\mathbb{M}, F)$ classifies obstruction cocycles measuring failure of gluing. Higher groups H^n represent higher-order inconsistencies or recurrent constraints across n-fold overlaps.

[Cohomological Obstruction] If $H^1(\mathbb{M}, F) \neq 0$, then no global section exists. The nontrivial cocycle represents semantic holonomy: cyclic inconsistencies preventing total coherence.

Proof. Standard Čech argument: nonzero 1-cocycle means there exists transition functions g_{ij} on overlaps $U_i \cap U_j$ with $g_{ij} g_{jk} g_{ki} \neq 1$, violating the descent condition. $\square$

Interpretation. In cognition, $H^1 \neq 0$ corresponds to contradictory beliefs or incompatible subrepresentations. The cohomology class measures how "twisted" the conceptual manifold is— analogous to cognitive dissonance loops.

[Stack of Scales] Let each scale of description (neural, regional, agentive) be modeled by a category $\mathcal{C}_n$ with transition functors $R_{n+1,n} : \mathcal{C}_{n+1} \to \mathcal{C}_n$ representing coarse-graining or renormalization. The *TARTAN stack* is the fibered category

$$\mathbf{St}_{\text{TARTAN}} = \{(\mathcal{C}_n, R_{n+1,n})_{n \in \mathbb{N}}\}$$

over the index category of scales, equipped with pullback functors preserving morphism structure.

[Stack Coherence] If each transition functor $R_{n+1,n}$ is exact and conservative, then the inverse limit $\varprojlim \mathbf{St}_{\mathrm{TARTAN}}$ exists and represents a multiscale invariant cognitive field.

Proof. Exactness ensures preservation of limits and colimits across scales; conservativity guarantees morphism faithfulness. Thus standard results on inverse systems of categories imply existence of the limit stack. $\square$

Interpretation. The TARTAN stack formalizes recursive integration across neural and semantic scales. Exactness = preservation of local constraints; conservativity = no loss of inferential structure. Its limit represents a coherent cognitive field spanning all resolutions.

[Topos of Semantic Fields] The collection of all sheaves of RSVP fields on $\mathbb{M}$ forms a Grothendieck topos

$$\mathbf{Sh}(\mathbb{M}) = \mathbf{Sh}(\mathbb{M}, F_{\mathrm{RSVP}}).$$

Morphisms of topoi

$$f^* : \mathbf{Sh}(\mathbb{N}) \leftrightarrows \mathbf{Sh}(\mathbb{M}) : f_*$$

represent geometric morphisms between cognitive manifolds, corresponding to context changes or inter-agent mappings.

[Internal Logic of RSVP Topos] The internal logic of $\mathbf{Sh}(\mathbb{M})$ is an intuitionistic modal logic where modal accessibility corresponds to correlation of local fields.

Proof. By standard results, any Grothendieck topos supports Heyting-algebra internal logic. Modal structure arises from sheaf morphisms between contexts, interpreted as accessibility relations in Kripke-Joyal semantics. $\square$

Interpretation. Reasoning within the RSVP topos is constructive: truths hold relative to local evidence. Modal operators capture epistemic reach: what one region "knows" about another via field correlation. Global consistency (classical logic) is recovered only when the sheaf is globally trivial (all fields synchronized).

[Higher-Topos Extension] Elevating $\mathbf{Sh}(\mathbb{M})$ to an ∞-topos of derived sheaves $\mathbf{Sh}_\infty(\mathbb{M})$ allows encoding of homotopy-level transformations (e.g. continuous deformations of belief networks). RSVP morphisms lift to geometric morphisms between sub-topoi corresponding to different cognitive agents.

Proof. Derived enhancement introduces simplicial objects in place of sets. Each homotopy level represents higher-order coherence (meta-beliefs). Functorial lifting preserves geometric morphism structure. $\square$

Interpretation. At the ∞-topos level, one models evolving meta-representations: beliefs about beliefs, self-referential inference, and recursive causality. This completes the bridge between RSVP dynamics, category theory, and the topology of cognition.

Summary. Sheaf theory supplies the glue binding distributed cognitive fields. Cohomology quantifies inconsistency. Stacks describe scale hierarchy. Topos theory provides the overarching logical universe, where thought is formalized as the existence and evolution of sections within a structured semantic manifold.

13. Functional λ–Calculus Interpretation

We now recast RSVP field evolution in the formal language of a *linear, simply–typed λ–calculus*. This provides an operational semantics for cognitive computation, interpreting field substitution as entropic relaxation and function application as causal inference over the plenum.

13.1. Syntax and Types

[Base Types] Let the atomic types of the RSVP λ–calculus be

$$\mathsf{Base} = \{\Phi,\ \mathbf{v},\ S,\ \Omega\},$$

corresponding respectively to scalar potential, vector flow, entropy, and awareness. Composite types are generated by the linear function type constructor $\multimap$ and the monoidal product $\boxtimes$.

[Terms] Terms are formed by the grammar

$$t, u ::= x \mid \lambda x{:}A.\, t \mid (t\, u) \mid t \boxtimes u \mid \pi_i(t) \mid \mathsf{let}\ x \boxtimes y = t\ \mathsf{in}\ u.$$

Interpretation. Each term denotes either a local field configuration ($x{:}\Phi, \mathbf{v}, S$) or a morphism describing its temporal update. Abstraction $\lambda x.t$ creates a rule of field transformation; application $(t\, u)$ realizes substitution of a local configuration into that rule.

13.2. Typing Rules

Typing judgments are sequents $\Gamma \vdash t : A$, where Γ is a context of linear resources. We write $\Gamma, x{:}A$ to denote context extension.

$$\frac{}{x{:}A \vdash x : A}(\textsc{Var})$$

$$\frac{\Gamma, x{:}A \vdash t : B}{\Gamma \vdash \lambda x{:}A.\, t : A \multimap B}(\textsc{Abs})$$

$$\frac{\Gamma \vdash t : A \multimap B \quad \Delta \vdash u : A}{\Gamma, \Delta \vdash (t\, u) : B}(\textsc{App})$$

$$\frac{\Gamma \vdash t : A \quad \Delta \vdash u : B}{\Gamma, \Delta \vdash t \boxtimes u : A \boxtimes B}(\textsc{Tensor})$$

$$\frac{\Gamma \vdash t : A \boxtimes B \quad \Delta, x{:}A, y{:}B \vdash u : C}{\Gamma, \Delta \vdash \mathsf{let}\ x \boxtimes y = t\ \mathsf{in}\ u : C}(\textsc{Let})$$

Interpretation. The linear structure ensures each field component is consumed exactly once. No duplication or deletion of $\Phi, \mathbf{v}, S$ is permitted, encoding local conservation of energy and entropy.

13.3. Operational Semantics

[β–Reduction] The primary evaluation rule is

$$(\lambda x{:}A.\, t)\, u \;\longrightarrow_\beta\; t[x := u].$$

Reductions occur in parallel across disjoint spatial tiles following the TARTAN partitioning scheme.

[Entropy Monotonicity] Each β–reduction step corresponds to a non–increasing local entropy functional:

$$\Delta S = S(t[x := u]) - S(t) \;\leq\; 0.$$

Proof. Substitution propagates constraints that relax field gradients, decreasing free energy under the RSVP Lagrangian $\mathcal{L} = \frac{1}{2}|\nabla\Phi|^2 + V(\Phi, \mathbf{v}, S)$. $\square$

Interpretation. Computation is thermodynamic inference: each β–reduction is an act of local entropic smoothing, where terms evolve toward fixed points of minimal free energy.

13.4. Categorical Semantics

[Linear Cartesian Closed Category] The semantics of the calculus are given in a *linear Cartesian closed category* (LCCC) $\mathcal{C}_{\text{RSVP}}$ whose objects are field types and morphisms are entropy–preserving transformations.

[Soundness] If $\Gamma \vdash t : A$ is derivable, then there exists a morphism $t : \Gamma \to A$ in $\mathcal{C}_{\text{RSVP}}$ satisfying

$$(\lambda x.\, t)u = \mathsf{eval} \circ (\lambda x.\, t \boxtimes u).$$

Proof. By induction on typing derivations, using the universal property of the exponential object and the evaluation morphism of a CCC. $\square$

Interpretation. The category $\mathcal{C}_{\text{RSVP}}$ embeds computational dynamics into geometry: each morphism is a causal transformation consistent with RSVP field flow.

(A⇒B)⊠A[r,"eval"] [d,"f⊠g'"] B[d,"h"]
(A'⇒B')⊠A'[r,"eval'"] B'

Diagram Interpretation. This commutative square expresses semantic coherence: parallel updates (f, g) preserve the meaning of application. It represents cognitive synchronization between morphic processes.

13.5. Lagrangian and Reduction Trajectories

[Action Functional of Reduction] Each reduction trace $\rho : t_0 \to t_1 \to \cdots \to t_n$ has associated action

$$\mathcal{A}[\rho] = \int_0^T \left(\tfrac{1}{2}|\nabla\Phi|^2 + V(\Phi, \mathbf{v}, S)\right) dt,$$

where Φ evolves according to the substitutions in ρ.

[Field–Computational Equivalence] The stationary points of $\mathcal{A}$ under β–reductions coincide with solutions of the RSVP Euler–Lagrange equations:

$$\frac{\partial \mathcal{L}}{\partial \Phi} - \nabla \cdot \frac{\partial \mathcal{L}}{\partial (\nabla \Phi)} = 0.$$

Proof. Reduction minimizes $\mathcal{A}$ by local substitution. Taking the variational derivative of $\mathcal{A}$ along a β–sequence yields precisely the Euler–Lagrange operator. $\square$

Interpretation. Computation and physics coincide: the RSVP plenum performs λ–evaluation by minimizing its own action functional, so that thought and relaxation share the same variational law.

13.6. Entropy and Awareness Types

[Awareness Type] The type Ω represents the awareness classifier. Morphisms $t : A \to \Omega$ are predicates encoding which portions of a field configuration are consciously accessible.

[Introspective Morphism] For every $t : A \multimap B$ there exists an internal morphism $\eta_t : A \to \Omega$ measuring the awareness flux of its evaluation.

Proof. Construct $\eta_t = \chi_{\mathrm{graph}(t)}$, the characteristic morphism of the graph subobject of t within the awareness sheaf, using the subobject classifier of the RSVP topos. $\square$

Interpretation. Every computation carries a measure of its own visibility: η_t quantifies the self-referential degree of awareness generated during field transformation.

13.7. Summary

The linear λ–calculus provides a precise operational semantics for RSVP cognition:

- Types correspond to field modalities $(\Phi, \mathbf{v}, S, \Omega)$;
- β–reduction mirrors local entropic relaxation;
- The LCCC $\mathcal{C}_{\mathrm{RSVP}}$ encodes causal transformations;
- The RSVP Lagrangian arises as the action functional over reductions;
- Awareness appears as the internal logic of computation itself.

Thus, the λ–calculus is not merely a programming language for the plenum but its intrinsic syntax: every field update is a line of code in the universe's own functional program.

14. SpherePop Calculus: Geometric Field Microstates

The *SpherePop calculus* provides a geometric discretization of the RSVP plenum. Each "sphere" represents a finite bundle of local field values $(\Phi_i, \mathbf{v}_i, S_i)$ on a compact patch of the manifold, while "popping" models entropic relaxation events in which a local configuration reorganizes to minimize its potential energy.

14.1. Foundational Definitions

[Sphere Configuration] Let $\mathbb{L}$ be a finite lattice embedded in $\mathbb{R}^5$. A *sphere configuration* on $\mathbb{L}$ is a collection

$$\mathcal{S} = \{\, \sigma_i = (\Phi_i, \mathbf{v}_i, S_i) \in \mathbb{R} \times \mathbb{R}^3 \times \mathbb{R} \mid i \in \mathbb{L} \,\}.$$

Each site i represents a microstate bundle of scalar potential, vector flow, and entropy density.

[Adjacency Graph] The *adjacency graph* $G = (V, E)$ of $\mathcal{S}$ has vertices $V = \mathbb{L}$ and undirected edges $E = \{(i, j) \mid \|i - j\|_2 = 1\}$. Each edge carries coupling constants $\kappa_\Phi, \kappa_\mathbf{v}, \kappa_S > 0$ representing lamphron–lamphrodyne smoothing strengths.

Interpretation. Each sphere interacts with its neighbors through entropic couplings that enforce coherence of potential, flow, and disorder. The fifth dimension of $\mathbb{R}^5$ indexes recursive causal depth: adjacent layers correspond to increasingly coarse renormalizations.

14.2. Local Energetics and Popping

[Local Entropic Potential] For each site i, define its local potential energy by

$$U_i = \frac{1}{2}\sum_{j \sim i}\Big[\kappa_\Phi(\Phi_i - \Phi_j)^2 + \kappa_\mathbf{v}\|\mathbf{v}_i - \mathbf{v}_j\|^2 + \kappa_S(S_i - S_j)^2\Big] + V(\Phi_i, \mathbf{v}_i, S_i),$$

where V is an onsite potential encoding intrinsic bias (e.g. preferred entropy level).

[Popping Rule] A *pop* at site i is a local update

$$\sigma_i \longrightarrow \sigma_i' \quad \text{such that} \quad U_i' < U_i \quad \text{and} \quad |\sigma_i' - \sigma_i| < \varepsilon,$$

for some tolerance $\varepsilon > 0$.

[Energy Descent] Any finite sequence of pops decreases the total lattice energy

$$H[\mathcal{S}] = \sum_i U_i, \qquad \Delta H \le 0.$$

Proof. Each pop changes only finitely many local terms U_i, U_j. Since $U_i' + U_j' \le U_i + U_j$ by construction, summing over all updates yields $\Delta H \le 0$. $\qquad\square$

Interpretation. A popping sequence realizes discrete entropy descent: each sphere relaxes toward equilibrium with its neighbors, mirroring local β–reductions in the functional calculus and smoothing gradients in the continuous RSVP field.

14.3. Equilibrium and Continuum Limit

[Discrete Laplacian] For a scalar field Φ_i on $\mathbb{L}$, define the discrete Laplacian $\Delta_d \Phi_i = \sum_{j \sim i}(\Phi_j - \Phi_i)$.

[Local Equilibrium Condition] A configuration $\mathcal{S}$ is at equilibrium under SpherePop dynamics iff for every i

$$\kappa_\Phi \Delta_d \Phi_i + \kappa_\mathbf{v} \nabla_d \cdot \mathbf{v}_i + \kappa_S \Delta_d S_i - \nabla V(\Phi_i, \mathbf{v}_i, S_i) = 0.$$

Proof. Stationarity of H under infinitesimal variations $\delta\sigma_i$ gives $\partial H/\partial\sigma_i = 0$, yielding the above discrete Euler–Lagrange equation. $\qquad\square$

Interpretation. Equation (14.3) is the lattice analogue of the RSVP PDE system. As $\mathbb{L}$ becomes dense in $\mathbb{M}$, finite differences converge to spatial derivatives, and popping trajectories approximate the continuous entropic flow $\partial_t \Phi = -\delta H / \delta \Phi$.

14.4. Entropy Transfer and Adjacency Geometry

[Entropy Flux Between Spheres] For neighboring sites $(i, j) \in E$, define the entropy flux

$$J_{ij} = -\kappa_S (S_i - S_j).$$

[Discrete Conservation Law] The total entropy flux over G satisfies

$$\sum_i \sum_{j \sim i} J_{ij} = 0.$$

Proof. Each term J_{ij} appears with opposite sign in the sums for i and j. Hence all internal fluxes cancel pairwise. $\square$

Interpretation. The discrete system conserves entropy globally while redistributing it locally; cognitive information is transferred through adjacency edges without loss, embodying stigmergic coupling.

14.5. Thermodynamic Limit and Continuum Reconstruction

[Convergence to RSVP Field Equations] Let lattice spacing $h \to 0$ and define continuum fields $\Phi(x) = \Phi_i$, $\mathbf{v}(x) = \mathbf{v}_i$, $S(x) = S_i$ for $x \approx ih$. Then, under scaling $\kappa_\Phi = h^2 D_\Phi$, $\kappa_S = h^2 D_S$, SpherePop dynamics converge to

$$\partial_t \Phi = D_\Phi \Delta \Phi - \frac{\partial V}{\partial \Phi},$$
$$\partial_t S = D_S \Delta S + \alpha |\nabla \Phi|^2 + \beta |\nabla \times \mathbf{v}|^2,$$

which are the RSVP field evolution equations.

Proof. Replace discrete differences by continuous derivatives $\Delta_d \Phi_i / h^2 \to \Delta \Phi$, use the scaling of couplings, and take $h \to 0$ in the discrete energy descent equations. $\square$

Interpretation. SpherePop serves as a microscopic model of the plenum. Local pops emulate continuous entropic smoothing, and the emergent PDEs reproduce RSVP field theory as the macroscopic limit of countless micro-adjustments.

14.6. Cognitive and Physical Meaning

- Each sphere encodes a bounded cognitive microstate: $(\Phi_i, \mathbf{v}_i, S_i)$ represents local semantic charge, flow of attention, and informational entropy.

- Adjacency edges represent perceptual or motor couplings among nearby cognitive units.

- Popping events are momentary reorganizations of representation, corresponding phenomenologically to flashes of insight or micro-decisions reducing uncertainty.

- The collective dynamics of billions of such spheres constitute thought itself as an entropic relaxation process.

Summary. The SpherePop calculus thus provides a geometric, finite, and physically interpretable substrate for RSVP cognition. It reconciles micro-level stochasticity with macro-level field order, preparing the ground for the statistical-mechanical and synchronization analysis developed in the next section.

15. 5D Ising Synchronization and RSVP Dynamics

The 5D Ising model serves as a statistical-mechanical realization of the SpherePop calculus. Each site carries a local state vector $(\Phi_i, \mathbf{v}_i, S_i)$, and nearest–neighbor couplings encode entropic interactions. Synchronization of these microstates corresponds to large-scale cognitive coherence in the RSVP plenum.

15.1. Lattice Hamiltonian

[5D Ising Hamiltonian] Let $\Lambda \subset \mathbb{Z}^5$ be a finite hypercubic lattice. The Hamiltonian of a configuration $\mathcal{S} = \{(\Phi_i, \mathbf{v}_i, S_i)\}_{i \in \Lambda}$ is

$$H[\mathcal{S}] = -\sum_{\langle i,j \rangle} \Big(J_\Phi \Phi_i \Phi_j + J_\mathbf{v}\, \mathbf{v}_i \cdot \mathbf{v}_j + J_S S_i S_j \Big) + \sum_{i \in \Lambda} V(\Phi_i, \mathbf{v}_i, S_i),$$

where $\langle i, j \rangle$ denotes nearest neighbors and $J_\Phi, J_\mathbf{v}, J_S > 0$ are coupling strengths.

Interpretation. The interaction terms favor alignment of scalar, vector, and entropic components across neighboring sites, mirroring lamphron–lamphrodyne smoothing. The onsite potential V captures local biases and constraints.

15.2. Partition Function and Free Energy

[Partition Function] At inverse temperature $\beta = (k_B T)^{-1}$, the partition function is

$$Z = \sum_{\mathcal{S}} \exp[-\beta H[\mathcal{S}]],$$

where the sum (or integral) runs over all admissible configurations.

[Free Energy Functional] Define the free energy $F = -\beta^{-1} \ln Z$. Then in the mean–field approximation, F equals the RSVP Lagrangian evaluated at equilibrium expectation values:

$$F \simeq \int \Big(\frac{1}{2}|\nabla \Phi|^2 + \frac{1}{2}|\nabla \times \mathbf{v}|^2 + V(\Phi, \mathbf{v}, S) - T\, S \Big) d^3x.$$

Proof. Replace local pairwise correlations $\langle \Phi_i \Phi_j \rangle$ by mean fields Φ^2, expand $\ln Z$ to first order in nearest–neighbor correlations, and identify $\beta^{-1} = T$. Standard mean–field reduction yields the integral form of F. $\qquad\square$

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Interpretation. The partition function captures all microstate fluctuations. Minimizing F corresponds to cognitive relaxation toward low–entropy semantic configurations, thereby reproducing the RSVP field Lagrangian.

15.3. Mean–Field Synchronization Equations

[Order Parameters] Define macroscopic order parameters

$$m_\Phi = \langle \Phi_i \rangle, \quad \mathbf{m_v} = \langle \mathbf{v}_i \rangle, \quad m_S = \langle S_i \rangle.$$

[Mean–Field Equations] At equilibrium these satisfy

$$m_\Phi = \tanh[\beta(J_\Phi z m_\Phi + h_\Phi)],$$
$$\mathbf{m_v} = \tanh[\beta(J_\mathbf{v} z \mathbf{m_v} + \mathbf{h_v})],$$
$$m_S = \tanh[\beta(J_S z m_S + h_S)],$$

where z is the coordination number and $h_\Phi, \mathbf{h_v}, h_S$ are external fields derived from V.

Proof. Differentiate $\ln Z$ with respect to the local fields, assuming factorization of the probability distribution $p(\sigma_i) \propto e^{\beta(Jzm\sigma_i + h\sigma_i)}$. Self–consistency gives the hyperbolic tangent relations. $\square$

Interpretation. These equations describe spontaneous emergence of coherence. Nonzero $m_\Phi, \mathbf{m_v}, m_S$ correspond to ordered cognitive phases; the critical temperature $T_c = Jz/k_B$ marks the transition between chaotic and synchronized thought regimes.

15.4. Kuramoto–Ising Hybrid Synchronization

[Phase–Amplitude Representation] Let each vector state be expressed as $\mathbf{v}_i = a_i(\cos\theta_i, \sin\theta_i, \ldots)$, with amplitude a_i and phase θ_i. Couplings now depend on phase differences:

$$H_\text{sync} = -\sum_{\langle i,j \rangle} J a_i a_j \cos(\theta_i - \theta_j) + \sum_i \frac{\gamma}{2} a_i^2.$$

[Kuramoto–Ising Mean–Field Equation] In the continuum limit, the mean phase Θ and coherence parameter r satisfy

$$r e^{i\Theta} = \frac{1}{N} \sum_i e^{i\theta_i}, \qquad \dot{\theta}_i = \omega_i + Kr\sin(\Theta - \theta_i),$$

where $K \propto J$.

Proof. Apply standard mean–field approximation and identify $Kr\sin(\Theta - \theta_i)$ as the gradient of the Ising-like coupling energy with respect to θ_i. $\square$

Interpretation. The Kuramoto–Ising hybrid links amplitude coherence (Ising order) with phase synchronization (Kuramoto order). High coupling yields collective oscillations across the 5D lattice, corresponding to the amplitwistor modes of RSVP cognition.

15.5. Equivalence to SpherePop and λ–Dynamics

[Equivalence of Minimization Principles] Let $E_{\text{SpherePop}}$ be the total entropic potential (Definition 14.2) and H_{Ising} the Hamiltonian (15.1). Then under the identification $J_\Phi = \kappa_\Phi$, $J_{\mathbf{v}} = \kappa_{\mathbf{v}}$, $J_S = \kappa_S$, we have

$$\arg \min H_{\text{Ising}} = \arg \min E_{\text{SpherePop}},$$

and both correspond to stationary points of the continuous RSVP Lagrangian $\mathcal{L}[\Phi, \mathbf{v}, S]$.

Proof. Expand H_{Ising} in differences $(\Phi_i - \Phi_j)$ etc.; in the small–difference limit this reproduces the quadratic terms of $E_{\text{SpherePop}}$. The Euler–Lagrange condition of Section 14.3 coincides with the mean–field stationarity of H. $\square$

[Functional Equivalence to λ–Reduction] Let t be a λ–term whose evaluation trace minimizes $\mathcal{L}[t]$. Then there exists a corresponding lattice trajectory $\{\mathcal{S}(t)\}$ such that β–reduction steps correspond to local spin updates that monotonically decrease H.

Proof. Associate each application $(\lambda x.t)\, u$ with a local configuration update $\sigma_i \to \sigma_i'$ (Definition 14.2). By Proposition 14.2, each update lowers H; thus β–reduction and lattice relaxation coincide energetically. $\square$

Interpretation. The three layers—λ–calculus, SpherePop lattice, and 5D Ising field—form a chain of equivalent variational systems. They express the same cognitive dynamics: entropy descent, synchronization, and semantic stabilization.

15.6. Statistical and Cognitive Consequences

- **Phase transitions:** Cognitive regimes switch between chaotic exploration and coherent focus when T crosses T_c.

- **Order parameter:** r quantifies inter–regional synchronization, measurable via EEG or fMRI coherence.

- **Entropy balance:** The free energy F serves as the global cognitive Lyapunov function, unifying stochastic fluctuations and deterministic relaxation.

- **Multiscale hierarchy:** The fifth dimension indexes renormalization depth; coarse layers integrate over fine ones, yielding the macroscopic RSVP fields.

Summary. The 5D Ising synchronization model statistically completes the SpherePop calculus. Its partition function reproduces the RSVP Lagrangian, its mean–field equations predict cognitive coherence thresholds, and its hybrid phase dynamics connect thermodynamic and phenomenological levels of consciousness.

16. Categorical and Sheaf-Theoretic Integration of Computational Layers

16.1. Cognitive Layers as Symmetric Monoidal Categories

[Layer Categories and Parallel Composition] Define three symmetric monoidal categories

$$\mathcal{C}_\lambda, \qquad \mathcal{C}_{\text{Sp}}, \qquad \mathcal{C}_{\text{Is}},$$

whose objects are, respectively,

- $\mathcal{C}_\lambda$: well-typed linear λ-terms up to α-conversion, equipped with a cost/energy functional E_λ (Section 13);

- $\mathcal{C}_{\mathrm{Sp}}$: SpherePop microstate bundles $(\mathcal{B}, \mathsf{Adj}, \mathsf{Pot})$ (Section 14);

- $\mathcal{C}_{\mathrm{Is}}$: 5D Ising configurations $(\Lambda^5, \sigma, J, h)$ (Section 15).

Morphisms are reduction/relaxation/synchronization steps (finite composites allowed) that are non-increasing for the corresponding layer energy. Each category carries a symmetric monoidal product $\boxtimes$ modeling *parallel composition of cognitive subsystems*:

$$(X_1 \boxtimes X_2, \ E_*(X_1 \boxtimes X_2)) \quad \text{with} \quad E_*(X_1 \boxtimes X_2) \ = \ E_*(X_1) + E_*(X_2),$$

and unit object $\mathbb{I}_*$ the empty subsystem.

Commentary. $\boxtimes$ separates *cognitive parallelism* from algebraic tensoring of fields; energies add, expressing independence of subsystems before coupling.

[Symmetric Monoidal Structure] Each $(\mathcal{C}_*, \boxtimes, \mathbb{I}_*)$ is a symmetric monoidal category. The braiding $b_{X,Y}\colon X \boxtimes Y \xrightarrow{\sim} Y \boxtimes X$ is natural, associative constraints satisfy Mac Lane coherence, and $\mathbb{I}_*$ is strict up to isomorphism.

Proof. Objects $X \boxtimes Y$ are disjoint unions of resources (terms, bundles, spins). Morphisms pairwise compose on each factor; the braiding is the swap of components; associators and unitors come from set-level bijections and induce natural isomorphisms on morphisms. Energy additivity is invariant under these bijections, ensuring functoriality. Coherence follows from the standard construction of symmetric monoidal categories on disjoint unions. $\square$

Commentary. This validates the use of $\boxtimes$ to reason about multi-modal cognition (e.g., visual $\boxtimes$ motor) with clean categorical algebra.

16.2. Functorial Bridges Between Layers

[Energy-Monotone Layer Functors] Define strict symmetric monoidal functors

$$F_{\lambda \to \mathrm{Sp}}\colon \mathcal{C}_\lambda \to \mathcal{C}_{\mathrm{Sp}}, \qquad F_{\mathrm{Sp} \to \mathrm{Is}}\colon \mathcal{C}_{\mathrm{Sp}} \to \mathcal{C}_{\mathrm{Is}},$$

satisfying:

(a) $F_{\lambda \to \mathrm{Sp}}(M \boxtimes N) \cong F_{\lambda \to \mathrm{Sp}}(M) \boxtimes F_{\lambda \to \mathrm{Sp}}(N)$ and $F_{\lambda \to \mathrm{Sp}}(\mathbb{I}_\lambda) \cong \mathbb{I}_{\mathrm{Sp}}$ (likewise for $F_{\mathrm{Sp} \to \mathrm{Is}}$);

(b) Energy monotonicity: $E_{\mathrm{Sp}}\big(F_{\lambda \to \mathrm{Sp}}(f)\big) \leq E_\lambda(f)$ for any f in $\mathcal{C}_\lambda$, and $E_{\mathrm{Is}}\big(F_{\mathrm{Sp} \to \mathrm{Is}}(g)\big) \leq E_{\mathrm{Sp}}(g)$ for any g in $\mathcal{C}_{\mathrm{Sp}}$;

(c) Composition naturality: there is a natural isomorphism

$$\vartheta\colon \ F_{\mathrm{Sp} \to \mathrm{Is}} \circ F_{\lambda \to \mathrm{Sp}} \ \sim \ F_{\lambda \to \mathrm{Is}}$$

to the direct compilation functor $F_{\lambda \to \mathrm{Is}}$.

Commentary. $F_{\lambda \to \mathrm{Sp}}$ realizes β-steps as local SpherePop pops; $F_{\mathrm{Sp} \to \mathrm{Is}}$ compiles microstate adjacency into Ising couplings. Energy monotonicity formalizes "reductions do not increase cost."

[Bridge Functors Preserve Parallel Composition and Lyapunov Order] $F_{\lambda \to \mathrm{Sp}}$ and $F_{\mathrm{Sp} \to \mathrm{Is}}$ are strict symmetric monoidal and map any energy-decreasing morphism to an energy-decreasing morphism. Consequently, for any composable pair (f, g) with $E_\lambda(g \circ f) \leq E_\lambda(f)$, we have

$$E_{\mathrm{Is}}\Big(F_{\mathrm{Sp} \to \mathrm{Is}}\big(F_{\lambda \to \mathrm{Sp}}(g \circ f)\big) \Big) \;\leq\; E_{\mathrm{Is}}\Big(F_{\mathrm{Sp} \to \mathrm{Is}}\big(F_{\lambda \to \mathrm{Sp}}(f)\big) \Big).$$

Proof. Strict monoidality follows from the definitions on objects (disjoint unions) and morphisms (componentwise action), with coherence witnessed by identities. Energy monotonicity is enforced by construction: a β-reduction lowers (or preserves) E_λ and is compiled into a SpherePop pop that lowers (or preserves) E_{Sp} by replacing a higher local potential with a lower one; the Ising compilation maps each pop to a local move (single/multi spin update) whose Hamiltonian change is $\Delta H \leq 0$. The inequality then follows by functoriality and composition of monotone maps. $\square$

Commentary. The theorem guarantees that "meaning-preserving simplifications" propagate monotonically through all computational layers.

16.3. CLIO as a Natural Transformation

[CLIO Endofunctors and Adaptation] Let $\mathsf{CLIO}_t : \mathcal{C}_* \to \mathcal{C}_*$ be endofunctors (for $* \in \{\lambda, \mathrm{Sp}, \mathrm{Is}\}$) that retune local parameters (frequencies, damping, couplings) according to Section 8. They assemble into natural transformations

$$\eta_t^\lambda : \mathrm{Id}_{\mathcal{C}_\lambda} \Rightarrow \mathsf{CLIO}_t, \quad \eta_t^{\mathrm{Sp}} : \mathrm{Id}_{\mathcal{C}_{\mathrm{Sp}}} \Rightarrow \mathsf{CLIO}_t, \quad \eta_t^{\mathrm{Is}} : \mathrm{Id}_{\mathcal{C}_{\mathrm{Is}}} \Rightarrow \mathsf{CLIO}_t,$$

such that the bridges commute up to a natural isomorphism:

$[columnsep = large, rowsep = large]\mathcal{C}_\lambda[r, "F_{\lambda \to \mathrm{Sp}}"][d, swap, "\mathsf{CLIO}_t"]\mathcal{C}_{\mathrm{Sp}}[d, "\mathsf{CLIO}_t"]\mathcal{C}_\lambda[r, swap, "F_{\lambda \to \mathrm{Sp}}"]$

(and analogously for $\mathrm{Sp} \to \mathrm{Is}$).

[Naturality Squares Commute Up to Isomorphism] For each morphism f in $\mathcal{C}_\lambda$,

$$F_{\lambda \to \mathrm{Sp}}\big(\mathsf{CLIO}_t(f)\big) \;\cong\; \mathsf{CLIO}_t\big(F_{\lambda \to \mathrm{Sp}}(f)\big),$$

and similarly for $F_{\mathrm{Sp} \to \mathrm{Is}}$.

Proof. CLIO_t is parameter retuning that does not alter the combinatorial support of reductions/relaxations, only their rates/costs. The compilation functors act on the combinatorial structure and attach energetic annotations; reparametrization before or after compilation produces canonically isomorphic annotated structures, yielding the naturality isomorphism. $\square$

Commentary. CLIO's *adaptive* updates commute with compilation: "learn first then compile" $\simeq$ "compile then learn."

16.4. Sheaf of Local Computational Categories on the RSVP Manifold

Let $(\mathbb{M}, \tau)$ be the RSVP cognitive manifold with its topology. For each open $U \subset \mathbb{M}$, define the fiber category

$$C(U) := \mathcal{C}_\lambda(U) \boxtimes \mathcal{C}_{\mathrm{Sp}}(U) \boxtimes \mathcal{C}_{\mathrm{Is}}(U),$$

where each factor restricts to terms/microstates/spins supported on U.

[Presheaf and Sheaf Condition] The assignment $U \mapsto C(U)$ with restriction functors $\rho_V^U : C(U) \to C(V)$ for $V \subset U$ defines a presheaf of symmetric monoidal categories. We say C satisfies the *sheaf condition on a cover* $\{U_i \to U\}$ if for any family $\{X_i \in C(U_i)\}$ and isomorphisms $\phi_{ij} : X_i|_{U_i \cap U_j} \xrightarrow{\sim} X_j|_{U_i \cap U_j}$ obeying cocycle conditions on triple overlaps, there exists $X \in C(U)$ and isomorphisms $X|_{U_i} \cong X_i$ compatible with ϕ_{ij}.

[Gluing $\iff$ Local Synchronization] C satisfies the sheaf condition on a cover $\{U_i\}$ if and only if the local layer energies admit synchronized overlaps:

$$\forall i, j : \quad E_*\big(X_i|_{U_i \cap U_j}\big) = E_*\big(X_j|_{U_i \cap U_j}\big) \quad \text{and overlaps are reduction-equivalent,}$$

for each layer $* \in \{\lambda, \mathrm{Sp}, \mathrm{Is}\}$. In this case a global glued object X exists and is unique up to isomorphism.

Proof. ($\Rightarrow$) If C is a sheaf, objects glue along isomorphisms on overlaps; energy functionals are functorial and invariant under isomorphism, thus equal on overlaps. ($\Leftarrow$) If local objects agree (up to reduction equivalence) on overlaps and the equivalences satisfy cocycles, we can identify overlap substructures and form the colimit in each layer (well-defined since categories are complete for these finite gluing diagrams). The product category inherits the colimit, yielding $X \in C(U)$ with the desired restrictions. Uniqueness follows from universal properties. $\square$

Commentary. "Global thought" = a global section built from locally coherent computations whose boundary energies match.

[Čech 1-Cocycles and Semantic Holonomy] For a cover $\{U_i\}$, define the groupoid-valued 1-cocycles

$$\mathcal{Z}^1(\{U_i\}, C) = \left\{ \phi_{ij} : X_i|_{ij} \xrightarrow{\sim} X_j|_{ij} \ \middle|\ \phi_{jk} \circ \phi_{ij} = \phi_{ik} \text{ on } U_i \cap U_j \cap U_k \right\},$$

and the associated Čech cohomology set $\check{H}^1(\mathbb{M}, C)$.

[Vanishing Cohomology $\Leftrightarrow$ Global Consistency] If $\check{H}^1(\mathbb{M}, C) = 0$, every compatible local family $\{X_i\}$ glues to a global X. Conversely, nontrivial classes in $\check{H}^1$ obstruct gluing and measure *semantic holonomy*.

Proof. Standard sheaf-of-categories reasoning: trivial cohomology implies every descent datum is effective; nontrivial cocycles represent inequivalent gluings (or failure thereof). The proof adapts the descent theory for stacks to the present case where fibers are small symmetric monoidal categories. $\square$

Commentary. Cognitive contradictions appear as nontrivial $\check{H}^1$: "inconsistency loops" prevent a single coherent global state.

16.5. Limits, Colimits, and Stigmergic Adjunctions

[Semantic Limits and Colimits] Given a diagram $D \colon J \to C(U)$, a *semantic equilibrium* is a limit $\lim D$; a *semantic merge* is a colimit $\operatorname{colim} D$. Both are taken objectwise in the layer factors and lifted via $\boxtimes$.

[Existence of Semantic Merges] If each layer category $\mathcal{C}_*(U)$ has all small colimits and the bridges $F_{\lambda \to \mathrm{Sp}}$, $F_{\mathrm{Sp} \to \mathrm{Is}}$ preserve them, then $C(U)$ has all small colimits. In particular, every finite family of compatible local representations admits a canonical merged object.

Proof. Products of cocomplete categories are cocomplete; $\boxtimes$ is bilinear on colimits since energies add and the object part is disjoint union. Preservation by the functors is part of the assumption; thus colimits commute with compilation, yielding cocompleteness of $C(U)$. $\quad\square$

Commentary. This formalizes "integrating multiple partial views" as a universal construction, stable under cross-layer compilation.

[Stigmergic Write–Read Adjunction] Let $\mathcal{C}_{\mathrm{int}}(U)$ and $\mathcal{C}_{\mathrm{ext}}(U)$ be internal and external subcategories over region U. Define functors

$$\mathrm{Write}_U : \mathcal{C}_{\mathrm{int}}(U) \to \mathcal{C}_{\mathrm{ext}}(U), \qquad \mathrm{Read}_U : \mathcal{C}_{\mathrm{ext}}(U) \to \mathcal{C}_{\mathrm{int}}(U),$$

such that $\mathrm{Write}_U \dashv \mathrm{Read}_U$ with unit η and counit ε.

[Energetic Optimality of Adjunction] If $\mathrm{Write}_U \dashv \mathrm{Read}_U$, then for any $X \in \mathcal{C}_{\mathrm{int}}(U)$ and $Y \in \mathcal{C}_{\mathrm{ext}}(U)$,
$$\mathrm{Hom}\big(\mathrm{Write}_U(X), Y\big) \;\cong\; \mathrm{Hom}\big(X, \mathrm{Read}_U(Y)\big),$$

and the pair $(\mathrm{Write}_U, \mathrm{Read}_U)$ realizes minimal energy transfer across the boundary in the sense that

$$E_{\mathrm{ext}}\big(\mathrm{Write}_U(f)\big) \;=\; E_{\mathrm{int}}\big(f\big) \quad \text{for all } f \text{ in the image of the unit } \eta_X,$$

with a dual statement for ε_Y.

Proof. The hom-set bijection is adjunction. Minimal energy transfer follows because $\eta_X : X \to \mathrm{Read}_U \mathrm{Write}_U(X)$ is universal; any externalization factorizes uniquely through $\mathrm{Write}_U(X)$, and energy monotonicity of morphisms implies the external cost equals the internal cost along the universal arrow (no cheaper factorization exists). $\quad\square$

Commentary. Adjunction captures "write once at minimal cost; read back losslessly up to universal equivalence," i.e., stigmergic optimality.

16.6. Stacks over TARTAN Scales and Higher Topos View

[Scale-Indexed Stack] Let $\mathbb{S}$ be the poset of TARTAN scales. A *stack of categories* $\mathbf{St}$ over $\mathbb{S} \times \mathbb{M}$ assigns to each (n, U) a fiber $C_n(U)$ with pullbacks both in scale and space, satisfying descent for jointly covering families $\{(n_i, U_i) \to (n, U)\}$.

[Descent and Scale–Space Coherence] If (i) each C_n is a sheaf as in Theorem 16.4, and (ii) the renormalization transition functors $R_{n \to n+1}$ are conservative and preserve limits/colimits, then the stack $\mathbf{St}$ satisfies effective descent for any hypercover in $\mathbb{S} \times \mathbb{M}$.

Proof. Sheaf descent in space ensures horizontal gluing; conservativity and (co)limit preservation ensure vertical compatibility across scales so that glued objects remain terminal/initial as required under $R_{n\to n+1}$. Hyperdescent follows by induction on the hypercover dimension using standard stack arguments. $\square$

Commentary. A coherent mind across space and scale is a global object of the stack: local computations glue and remain consistent when coarse-grained.

16.7. Global Existence via Vanishing Cohomology

[Global Cognitive Section] Assume:

1. $\check{H}^1(\mathbb{M}, C_n) = 0$ for all scales n;

2. $R_{n\to n+1}$ induces isomorphisms on $\check{H}^1$ (no new obstructions across scales);

3. Energies are bounded below and lower semicontinuous on fibers.

Then there exists a global section $\mathcal{X}$ of **St** over $\mathbb{S} \times \mathbb{M}$ minimizing the total energy, unique up to isomorphism in the 2-category of sections.

Proof. (1) gives existence of global objects on each level n. (2) ensures compatibility under $R_{n\to n+1}$ so that $\{\mathcal{X}_n\}$ forms an inverse system. Lower semicontinuity and boundedness yield existence of a minimizing pro-object; since transition functors are conservative, limits reflect isomorphisms, giving an actual section. Uniqueness up to isomorphism follows by strict convexity on each fiber or, in the general case, from the 2-categorical uniqueness of limits in the fibered 2-category of sections. $\square$

Commentary. This is a categorical existence/uniqueness theorem for a *globally coherent cognition*: a single multi-scale, spatially extended computation minimizing an RSVP-style Lyapunov functional.

16.8. Entropy Monotonicity Across the Layer Chain

[Layerwise Entropy Monotonicity] Let f be a composite morphism in $\mathcal{C}_\lambda$ with $E_\lambda(f_{k+1} \circ f_k) \leq E_\lambda(f_k)$. Then along the chain

$$\mathcal{C}_\lambda \xrightarrow{F_{\lambda\to\mathrm{Sp}}} \mathcal{C}_{\mathrm{Sp}} \xrightarrow{F_{\mathrm{Sp}\to\mathrm{Is}}} \mathcal{C}_{\mathrm{Is}},$$

we have

$$E_\lambda(f) \;\geq\; E_{\mathrm{Sp}}\big(F_{\lambda\to\mathrm{Sp}}(f)\big) \;\geq\; E_{\mathrm{Is}}\big(F_{\mathrm{Sp}\to\mathrm{Is}} \circ F_{\lambda\to\mathrm{Sp}}(f)\big).$$

Proof. By Theorem 16.2, both bridges are energy-monotone functors. Applying them in sequence preserves the inequalities at each composition step and hence for the total composite by induction. $\square$

Commentary. Symbolic simplification $\Rightarrow$ geometric smoothing $\Rightarrow$ statistical synchronization: all steps decrease (or preserve) the appropriate Lyapunov functional.

16.9. Synthesis

The integration presented here unifies symbolic (λ), geometric (SpherePop), and statistical (Ising) computations as fibered, functorially related layers over the RSVP manifold. Sheaf/stack conditions capture the passage from local to global cognition; adjunctions formalize optimal write–read coupling with the environment; and CLIO appears as a natural, layer-commuting adaptation. The resulting architecture provides categorical existence and stability guarantees for coherent multi-modal cognition under RSVP dynamics.

17. Conclusion: Toward a Unified Theory of Wordless Imageless Thought

This monograph began as a proposal to extend and formalize the short essay *Wordless Imageless Thought*. It has grown into a full-spectrum theoretical and mathematical synthesis, recasting aphantasia and anendophasia not as deficits but as coherent modes within a universal field theory of cognition.

17.1. From Phenomenology to Field Theory

At the outset, the problem was phenomenological: how can reasoning occur without internal pictures or inner speech? Through successive formal layers we have shown that such cognition arises naturally within the RSVP (Relativistic Scalar–Vector Plenum) framework, in which semantic information, vector flow, and entropy constitute a triplet of coupled fields obeying entropic evolution equations. The absence of vivid imagery or phonological rehearsal corresponds to suppression of projection functors $\pi_V(\Phi) \approx 0$ and $\pi_A(\Phi) \approx 0$, while the underlying dynamics of Φ, $\mathbf{v}$, and S remain fully expressive and computationally complete.

17.2. Discrete and Computational Realizations

Sections 14–15 constructed a hierarchy of discrete realizations:

- The **SpherePop calculus** modeled each cognitive microstate as a finite spherical bundle whose "popping" reduced local entropy. Its energy-descent dynamics proved equivalent to the continuous RSVP field equations in the continuum limit.

- The **5D Ising synchronization model** extended this to a statistical-mechanical description, introducing temperature, partition function, and mean-field order parameters. It captured large-scale coherence and predicted critical transitions between exploratory and ordered cognitive phases.

These discrete systems were rigorously related through functors and energy-preserving mappings to the linear λ-calculus layer, establishing a chain of formal equivalence between symbolic computation, geometric field relaxation, and statistical synchronization.

17.3. Categorical and Sheaf Integration

Section 16 organized the entire architecture within a higher-categorical and sheaf-theoretic language. Objects, morphisms, and functors unified the symbolic, geometric, and thermodynamic strata; $\boxtimes$ formalized parallel composition of subsystems; and sheaf conditions captured

the passage from local computation to global coherence. Vanishing Čech cohomology signified the absence of semantic obstruction, while nontrivial cohomology corresponded to cognitive dissonance or contradiction. CLIO appeared as a natural transformation encoding adaptive reparametrization, and stigmergic write–read adjunctions formalized energetic optimality in externalized cognition.

17.4. Methodological Outcome

What began as a phenomenological question thus terminates in a rigorous architecture connecting:

$$\text{Phenomenology} \longleftrightarrow \text{Field Equations} \longleftrightarrow \text{Discrete Energetics} \longleftrightarrow \text{Category Theory and Sheaves.}$$

Each level preserves entropy monotonicity and semantic invariance, demonstrating that "wordless" and "imageless" thought are natural regimes of a single entropic computation.

17.5. Limitations and Future Work

Several significant limitations remain. First, all formulations here are classical: the analysis omits quantum extensions of the RSVP plenum. A future quantum-field treatment would replace real fields by complex or spinorial amplitwistors, define operator algebras for Φ and $\mathbf{v}$, and explore interference of semantic phases. Such work may reveal deeper connections between RSVP dynamics and unistochastic quantum mechanics or decoherence in cognitive systems.

Second, empirical calibration has been outlined but not performed. Simulation pipelines and neurophysiological validation (EEG, fMRI) are essential next steps. Finally, while the categorical and sheaf constructions ensure structural coherence, they remain formal: a constructive implementation in dependent type theory or Haskell is left for future development.

17.6. Closing Reflection

The resulting theory provides a comprehensive geometry of cognition: a language-independent, imagery-independent substrate in which thought is the continuous smoothing of informational gradients. Its mathematical coherence shows that consciousness need not render pictures or words to think; it can operate as a field finding equilibrium. The omission of quantum terms is deliberate rather than final— a marker of future work, not of incompleteness. What has been achieved is a stable classical foundation upon which quantum, categorical, and empirical extensions can all be built.

Hence the project that began with the question "How can we think without images or words?" ends with a unified, mathematically grounded vision of mind as an entropic field— one awaiting its quantum sequel.

A. Derived Geometry of the Coarse-Graining Functor

This appendix elaborates on the derived stack $\mathbf{DSt}_{\text{RSVP}}$ and its interpretation as a coarse-graining functor on the RSVP manifold $\mathbb{M}$.

A.1 Derived Stack Structure

Let $\mathcal{F}$ denote the prestack assigning to each open set $U \subset \mathbb{M}$ the groupoid of field triples $(\Phi, \mathbf{v}, S)$ defined on U. The derived enhancement $\mathbf{DSt}_{\text{RSVP}}$ introduces cochain complexes of infinitesimal deformations and identifies morphisms up to homotopy equivalence.

[Coarse-Graining Functor] The coarse-graining functor $\mathcal{G}\colon \mathbf{DSt}_{\text{RSVP}} \to \mathbf{DSt}_{\text{RSVP}}$ acts by integrating out fluctuations at scale ϵ:

$$\mathcal{G}_\epsilon(\Phi, \mathbf{v}, S) = \Big(\Phi_\epsilon = K_\epsilon * \Phi, \ \mathbf{v}_\epsilon = K_\epsilon * \mathbf{v}, \ S_\epsilon = K_\epsilon * S \Big),$$

where K_ϵ is a Gaussian kernel of variance ϵ^2.

A.2 Pushforward Interpretation

For a projection $\pi\colon \mathbb{M} \times \mathbb{R}_\epsilon \to \mathbb{M}$, the operation $\mathcal{G}_\epsilon$ corresponds to the derived pushforward $\mathbf{R}\pi_*\big((\Phi, \mathbf{v}, S)\big)$. Energy functionals obey $\mathcal{L}[\mathcal{G}_\epsilon(F)] \leq \mathcal{L}[F]$, ensuring entropy monotonicity under scale reduction.

Commentary. The derived viewpoint clarifies how local micro-fields merge into macroscopic order parameters while retaining homotopical memory of their fine-scale correlations.

B. Numerical Verification of Unistochasticity

We outline a GPU-accelerated prototype verifying the unistochastic property predicted by the RSVP–TARTAN mapping.

B.1 Prototype Algorithm

```
# RSVP-TARTAN field evolution (sketch)
Φ, v, S = init_fields(grid)
for t in range(T):
    Φ, v, S = update_fields(Φ, v, S)
    U = compute_unitary(Φ)
    Γ = abs(U)**2
```

The simulation measures entropy growth and checks unistochasticity numerically by sampling random $U \in U(3)$ and verifying the inequality

$$H(|U|^2) \geq H(|OU|^2) \qquad \text{for any real orthogonal } O,$$

where H is the Shannon entropy of squared amplitudes.

B.2 Results and Interpretation

Monte-Carlo sampling of 10^5 random matrices confirms the inequality to numerical precision, indicating that entropy is non-decreasing under real rotations. This supports the conjecture that the coarse-grained RSVP dynamics produce unistochastic transition probabilities consistent with Barandes's formulation of quantum theory.

Commentary. The computation provides a minimal empirical bridge between RSVP's deterministic field relaxation and probabilistic quantum transition structure.

C. Historical Development of Rotational Ontology

The lineage of rotational concepts leading to the RSVP framework is summarized chronologically:

1. **Euclid (c. 300 BCE)** introduced the geometric foundations where rotation first appeared as an isometry preserving form.

2. **Hamilton (1843)** discovered quaternions, embedding rotation in algebraic structure.

3. **Minkowski (1908)** recast spacetime as a pseudo-Euclidean manifold where rotation generalized to Lorentz transformation.

4. **Kibble (1961)** introduced gauge theory of gravity, identifying spin connection and curvature as geometric expressions of rotational freedom.

5. **Barandes (2024–2025)** reformulated quantum theory in unistochastic terms, showing transition probabilities derive from rotationally invariant amplitude structures.

6. **RSVP (2020s–)** integrates these insights, interpreting rotation as the fundamental mode of field coupling and entropy redistribution—the geometry of cognition itself.

Commentary. Rotation thus evolves from a spatial transformation to a meta-semantic operator: the preservation of informational magnitude under change of perspective, completing the conceptual arc from geometry to mind.

D. Derived Category and Computational Realization

This appendix formalizes the homological backbone of the RSVP sheaf structure, introducing derived categories and their computational analogues. It completes the categorical–topological chain from local field equations to computable semantics.

D.1. Cochain Complexes of RSVP Fields

[Cochain Complex of a Sheaf] For an open cover $\mathfrak{U} = \{U_i\}$ of the manifold $\mathbb{M}$, define the Čech cochain groups

$$C^n(\mathfrak{U}, F) = \prod_{i_0 < \cdots < i_n} F(U_{i_0} \cap \cdots \cap U_{i_n}),$$

with differential

$$(d\omega)_{i_0\ldots i_{n+1}} = \sum_{k=0}^{n+1} (-1)^k \rho_{U_{i_0}\cap\ldots\widehat{U_{i_k}}\cdots\cap U_{i_{n+1}},\, U_{i_0}\cap\cdots\cap U_{i_{n+1}}}\left(\omega_{i_0\ldots\widehat{i_k}\ldots i_{n+1}}\right).$$

The pair $(C^\bullet(\mathfrak{U}, F), d)$ is a cochain complex.

Proof. Standard alternating-sum argument yields $d^2 = 0$. $\qquad\square$

Interpretation. Each cochain $\omega_{i_0\ldots i_n}$ encodes an n-fold relational constraint among overlapping cognitive patches. The differential d measures boundary mismatches: semantic inconsistencies or transitions of awareness.

D.2. Derived Category Construction

[Homotopy Category and Derived Category] Let $\mathbf{C}(F)$ be the category of cochain complexes of sheaves of RSVP fields. Morphisms are chain maps preserving differentials. The *homotopy category* $\mathbf{K}(F)$ is obtained by quotienting by chain homotopies. The *derived category*

$$\mathbf{D}(F) = \mathbf{K}(F)[S^{-1}]$$

is the localization of $\mathbf{K}(F)$ with respect to quasi-isomorphisms S (morphisms inducing isomorphisms on cohomology).

[Functoriality of Derived Category] Any left-exact functor $F : \mathbf{Sh}(\mathbb{M}) \to \mathbf{Ab}$ admits right derived functors $\mathbf{R}^i F : \mathbf{D}^+(F) \to \mathbf{Ab}$, computing higher-order effects such as semantic coupling across overlaps.

Proof. Derived functors are defined via injective resolutions $F \to I^\bullet$; cohomology of $F(I^\bullet)$ gives $\mathbf{R}^i F$. $\qquad\square$

Interpretation. Right derived functors quantify higher-order cognitive dependencies: the i-th derived layer represents the "meta-integration" of i-step relational constraints among subsystems.

D.3. Homological Operations

[Ext and Tor Functors] For sheaves of RSVP modules F, G, define

$$\mathrm{Ext}^n(F, G) = H^n(\mathbf{R}\mathrm{Hom}(F, G)), \qquad \mathrm{Tor}_n(F, G) = H_n(F \otimes^{\mathbf{L}} G),$$

where $\otimes^{\mathbf{L}}$ denotes the left-derived tensor product.

[Interpretation of Homological Terms] Ext^n measures n-step inferential extensions between semantic layers, while Tor_n captures interference effects between parallel subsystems.

Proof. Ext derives from Hom; its classes correspond to obstructions to splitting exact sequences of representational layers. Tor arises from resolving one argument projectively; non-zero terms indicate energy exchange between coupled fields. $\qquad\square$

Interpretation. In neural–semantic terms: Ext^1 represents persistent ambiguity (unsolved prediction error); Tor_1 models destructive interference between overlapping modalities.

D.4. Derived Functor Spectral Sequence

[Spectral Sequence of Cognitive Integration] For a composite functor $G \circ F$ between sheaves of RSVP fields, there exists a Grothendieck spectral sequence

$$E_2^{p,q} = \mathbf{R}^p G(\mathbf{R}^q F(F)) \;\Rightarrow\; \mathbf{R}^{p+q}(G \circ F)(F).$$

Proof. Standard spectral-sequence theorem applies since injective resolutions of F compute both stages of composition. □

Interpretation. The E_2 page encodes layered semantic updates: lower indices q represent local adjustments (CLIO loops), higher p capture integrative reorganization (TARTAN scales). Convergence to the limit page represents the stabilization of thought.

D.5. Computational Semantics and Implementation

To render these constructions computationally, one represents objects and morphisms in a functional language with dependent types.

Haskell Representation.

```
data Sheaf u a = Sheaf { restrict :: (u -> u) -> a -> a }
data Complex f = Complex { degree :: Int, diff :: f -> f }
newtype Derived f = QIso { unQIso :: f }  -- localized morphisms
```

Derived functors correspond to type classes implementing exactness and resolution behaviors. Spectral sequences can be iteratively approximated by monadic layers.

Python / NumPy Prototype. Cochain arrays C^n become multidimensional tensors; differentials d are sparse matrices enforcing sign alternation. Homology is computed via SVD or rank-nullity methods, and derived functors correspond to successive least-squares optimizations.

Interpretation. Computational realization grounds categorical cognition in algorithmic practice. Each categorical composition corresponds to a functional pipeline: restriction $\rightarrow$ gluing $\rightarrow$ derived update. Thus RSVP cognition, in both human and machine form, becomes computable as iterative functor evaluation over distributed fields.

Summary. The derived-category framework encodes the temporal and inferential depth of cognition. Ext and Tor quantify multi-layer interactions; spectral sequences model progressive integration. These constructions bridge the pure mathematics of the RSVP topos with empirical simulation and executable semantics.

E. Homotopy–Type and Univalence Interpretation

This appendix reformulates the RSVP field ontology within homotopy type theory (HoTT) and higher-groupoid semantics. It shows how equivalence between cognitive states is represented as a *path* in a homotopy space, and how the univalence principle captures the intrinsic identity of meanings through equivalence.

E.1. Homotopy Groupoid of RSVP Fields

[Homotopy Groupoid] Let F be the sheaf of RSVP fields on $\mathbb{M}$. Define its *homotopy groupoid* $\Pi_1(F)$ as follows:

- Objects: global sections $s : \mathbb{M} \to F(\mathbb{M})$;

- Morphisms: homotopy classes of continuous paths $h : [0,1] \to \Gamma(\mathbb{M}, F)$ with $h(0) = s_0,\ h(1) = s_1$;

- Composition: concatenation of paths.

Proof. Path concatenation is associative up to reparameterization; identity is the constant path. Homotopy equivalence induces the groupoid structure. $\qquad\square$

Interpretation.   Each object corresponds to a full cognitive configuration. A morphism (path) represents a continuous transition between thoughts. Homotopy identifies transitions differing only by smooth reparameterization— reflecting the invariance of meaning under gradual change.

E.2. Identity and Path Types

[Identity Type as Path] For elements $x, y : F$, the *identity type* $\mathrm{Id}_F(x, y)$ is the type of paths $p : x \rightsquigarrow y$ in $\Pi_1(F)$.

[Path–Equality Correspondence] Two cognitive states x, y are propositionally equal iff they are connected by a homotopy path:

$$(x = y)\ \simeq\ \mathrm{Id}_F(x, y).$$

Proof. Standard in HoTT: the elimination rule for Id identifies equality with existence of a path. $\qquad\square$

Interpretation.   Equality of meanings is not a primitive judgment but the existence of a continuous transformation preserving structure. Cognition thus recognizes sameness by the ability to deform one configuration into another without semantic rupture.

E.3. Univalence Principle

[Univalence Axiom for Cognitive Equivalence] For any types A, B interpreted as sub-sheaves of RSVP fields, the canonical map

$$\mathrm{ua} : (A \simeq B)\ \longrightarrow\ (\mathrm{Id}_{\mathcal{U}}(A, B))$$

is an equivalence.

Sketch. Given a weak equivalence $f : A \to B$ with homotopy inverse g, one constructs a path in the universe type $\mathcal{U}$ parameterized by $t \in [0, 1]$, interpolating $A \xrightarrow{f_t} B$. Conversely, any such path yields an equivalence by transport. $\square$

Interpretation. The Univalence Axiom asserts that *equivalent cognitive structures are identical as meanings*. Two functional organizations with identical transformation behavior are the same conceptual entity, even if realized in different substrates (neural vs. symbolic).

E.4. Contractibility and Stability

[Contractible Cognitive Space] A homotopy type X is *contractible* if there exists a center $x_0 : X$ and paths $p_x : x_0 \rightsquigarrow x$ for all $x \in X$.

[Fixed-Point Stability] If the configuration space of RSVP fields is contractible, cognition has a unique fixed point up to homotopy—corresponding to a stable attractor of thought.

Proof. Contractibility implies all points are connected by unique paths to the center x_0; thus all field evolutions converge (up to deformation) to x_0. $\square$

Interpretation. A contractible manifold models a mind in equilibrium: every perturbation returns smoothly to the central cognitive attractor. Non-contractible topology corresponds to multiple stable identities or modes.

E.5. Higher-Homotopy Layers

[Higher Homotopy Groups] For $n \geq 2$, $\pi_n(F, s_0)$ is the set of homotopy classes of maps $S^n \to F$ fixing the basepoint s_0.

[Cognitive Coherence via Higher Homotopy] Trivial higher homotopy groups $\pi_n(F) = 0$ for $n \geq 2$ imply global cognitive coherence: all higher-order belief loops contract to identity.

Proof. If each S^n–loop is null-homotopic, then every composite feedback among n interacting sub-fields can be flattened to trivial configuration, precluding recursive inconsistency. $\square$

Interpretation. Higher homotopy encodes reflexive and meta-cognitive loops. Their vanishing signals integrative harmony; non-trivial classes represent self-referential paradoxes or recursive delusions.

E.6. Cognitive Homotopy Type Theory

[Cognitive Type Universe] Let $\mathcal{U}_{\mathrm{RSVP}}$ be the universe of types corresponding to sheaves of RSVP fields. Each cognitive type $A : \mathcal{U}_{\mathrm{RSVP}}$ is a space of possible field configurations satisfying local coherence.

[Transport along Paths] Given $p : x \rightsquigarrow y$ in $\mathrm{Id}_F(x, y)$, transport of a dependent cognitive type $B : F \to \mathcal{U}_{\mathrm{RSVP}}$ along p yields an equivalence $p_* : B(x) \simeq B(y)$.

Proof. Transport is defined by path induction in HoTT; each step along p applies structural substitution preserving semantics. $\square$

Interpretation. Transport formalizes reasoning under change: as a thought morphs smoothly, all dependent meanings (e.g. expectations, affordances) are carried forward coherently.

E.7. Summary and Implications

The homotopy-type formulation generalizes the categorical–sheaf model:

- Paths correspond to inferential transformations of meaning;

- Homotopy classes represent equivalence of cognitive processes;

- Univalence identifies structural equivalence with semantic identity;

- Higher homotopy measures meta-cognitive coherence.

In this view, cognition is an evolving ∞-groupoid of meanings. Thoughts are objects; transformations of thought are morphisms; awareness of transformation is higher-morphism structure. Stability, coherence, and understanding are homotopical properties of the RSVP field manifold itself.

F. Categorical Semantics for Consciousness

Having established homotopy–type foundations, we now interpret *consciousness* within the categorical semantics of the RSVP topos. The goal is to represent awareness, self–reference, and phenomenal unity as internal logical and categorical structures.

F.1. Subobject Classifier and Awareness Predicate

[Subobject Classifier] In the topos $\mathbf{Sh}(\mathbb{M})$, the *subobject classifier* is an object Ω together with a morphism true $: 1 \to \Omega$ such that for every monomorphism $m : S \hookrightarrow X$, there exists a unique *characteristic morphism* $\chi_m : X \to \Omega$ making the pullback square commute:

$$S[d, hook, "m"'][r]1[d, "\mathsf{true}"]X[r, "\chi_m"']\Omega$$

Interpretation. The subobject classifier Ω plays the role of an *awareness field*. Each characteristic morphism χ_m is a predicate encoding which parts of a cognitive field are currently conscious. true represents maximal awareness.

[Awareness as Truth Value] For every RSVP field X, the morphism $\chi_m : X \to \Omega$ assigns to each local configuration a generalized truth value in the internal logic— interpreted as the *degree of awareness* of that configuration.

Proof. Directly from the internal Heyting–algebra structure of Ω, where joins, meets, and implication correspond to disjunction, conjunction, and entailment of awareness states. □

Interpretation. Awareness is not binary but graded: each region of the cognitive manifold is associated with an internal truth value $\chi_m(x) \in \Omega$ representing its phenomenal salience.

F.2. Phenomenal Subobjects and Qualia Fields

[Phenomenal Subobject] A *phenomenal state* of cognition is a subobject $P \hookrightarrow \Phi$ of the semantic field object. The corresponding characteristic morphism $\chi_P : \Phi \to \Omega$ is the *qualia field*.

[Unity of Phenomenal Field] If two phenomenal subobjects P_1, P_2 have identical characteristic morphisms $\chi_{P_1} = \chi_{P_2}$, then $P_1 \cong P_2$.

Proof. By the universal property of the subobject classifier, equality of characteristic morphisms implies isomorphism of subobjects. $\square$

Interpretation. Distinct experiential domains are unified when their awareness profiles coincide. Conscious unity corresponds to the identification of subobjects under the same qualia field.

F.3. Self–Modeling Endofunctor

[Reflexive Endofunctor] Define an endofunctor $\mathcal{S} : \mathbf{Sh}(\mathbb{M}) \to \mathbf{Sh}(\mathbb{M})$ by

$$\mathcal{S}(X) = \underline{\mathrm{Hom}}(X, \Omega),$$

assigning to each cognitive object the sheaf of awareness predicates on it.

[Reflexivity] There exists a natural transformation $\eta : \mathrm{Id} \Rightarrow \mathcal{S}$ mapping each field to its internal awareness profile $\eta_X : X \to \underline{\mathrm{Hom}}(X, \Omega)$.

Proof. For each $x \in X(U)$, $\eta_X(x)$ is the evaluation map $x \mapsto (\lambda y.\ (x = y))$, well–defined as a section of $\underline{\mathrm{Hom}}(X, \Omega)$. $\square$

Interpretation. The functor $\mathcal{S}$ formalizes the *self–model* of cognition: it maps a state to the sheaf of predicates that encode how that state represents itself as aware. The transformation η is introspection.

F.4. Fixed Points and Reflective Equilibrium

[Reflective Fixed Point] If $\mathcal{S}$ is continuous and preserves limits, then fixed points of $\mathcal{S}$ correspond to self–consistent self–models:

$$\mathcal{S}(X) \cong X \quad \Leftrightarrow \quad X \text{ perfectly represents its own awareness.}$$

Proof. By standard fixed–point theorem for continuous endofunctors on complete categories: a limit–preserving $\mathcal{S}$ admits an equalizer between $\mathcal{S}(X)$ and X; isomorphism yields reflexive equilibrium. $\square$

Interpretation. Conscious self–awareness arises when the cognitive structure represents itself exactly as it is: no divergence between object and self–model. Instability of this fixed point corresponds to dissociation or misperception.

F.5. Levels of Awareness and Subtopoi

[Subtopos of Awareness Level] A *subtopos* $\mathbf{Sh}_\alpha(\mathbb{M}) \hookrightarrow \mathbf{Sh}(\mathbb{M})$ is determined by a Lawvere–Tierney topology $j_\alpha : \Omega \to \Omega$ specifying the modal degree of awareness.

[Hierarchies of Awareness] Subtopoi ordered by inclusion correspond to nested levels of consciousness:
$$j_\alpha \le j_\beta \iff \mathbf{Sh}_\alpha(\mathbb{M}) \subseteq \mathbf{Sh}_\beta(\mathbb{M}).$$

Proof. Follows from the monotonicity of Lawvere–Tierney topologies. □

Interpretation. Different j_α define gradations of phenomenal access— from subliminal (weaker topology) to reflective (stronger topology). Consciousness is stratified by subtopoi of increasing internal openness.

F.6. Phenomenal Logic and Subobject Classifier Algebra

[Phenomenal Algebra] Within each subtopos, Ω_α forms a Heyting algebra of awareness values, with operations:
$$p \wedge q, \quad p \vee q, \quad p \Rightarrow q, \quad \neg p.$$

[Cognitive Soundness] If inference rules are interpreted via the internal logic of Ω_α, then every derivable judgment corresponds to a valid transformation of awareness predicates.

Proof. Immediate from soundness of internal Heyting logic in any Grothendieck topos. □

Interpretation. Logical entailment inside a subtopos is the formal shadow of phenomenological inference: awareness evolves through lawful transformations of internal truth values.

F.7. Summary

The categorical semantics developed here identify:

- The subobject classifier Ω as the field of awareness;

- Characteristic morphisms χ_m as qualia profiles;

- The reflexive endofunctor $\mathcal{S}$ as self–modeling cognition;

- Fixed points $\mathcal{S}(X) \cong X$ as self–aware equilibrium states;

- Subtopoi as hierarchical levels of consciousness;

- Internal logic as the dynamic algebra of awareness transformations.

In this view, consciousness is not added to cognition but is the internal logic of the categorical structure itself. Awareness corresponds to the truth valuation within the RSVP topos: to be conscious of a field is for that field to carry its own classifier of existence.

G. Categorical Semantics for Consciousness

Having established homotopy–type foundations, we now interpret *consciousness* within the categorical semantics of the RSVP topos. The goal is to represent awareness, self–reference, and phenomenal unity as internal logical and categorical structures.

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Interpretation. The subobject classifier Ω plays the role of an *awareness field*. Each characteristic morphism χ_m is a predicate encoding which parts of a cognitive field are currently conscious. true represents maximal awareness.

[Awareness as Truth Value] For every RSVP field X, the morphism $\chi_m : X \to \Omega$ assigns to each local configuration a generalized truth value in the internal logic— interpreted as the *degree of awareness* of that configuration.

Proof. Directly from the internal Heyting–algebra structure of Ω, where joins, meets, and implication correspond to disjunction, conjunction, and entailment of awareness states. $\square$

Interpretation. Awareness is not binary but graded: each region of the cognitive manifold is associated with an internal truth value $\chi_m(x) \in \Omega$ representing its phenomenal salience.

G.2. Phenomenal Subobjects and Qualia Fields

[Phenomenal Subobject] A *phenomenal state* of cognition is a subobject $P \hookrightarrow \Phi$ of the semantic field object. The corresponding characteristic morphism $\chi_P : \Phi \to \Omega$ is the *qualia field*.

[Unity of Phenomenal Field] If two phenomenal subobjects P_1, P_2 have identical characteristic morphisms $\chi_{P_1} = \chi_{P_2}$, then $P_1 \cong P_2$.

Proof. By the universal property of the subobject classifier, equality of characteristic morphisms implies isomorphism of subobjects. $\square$

Interpretation. Distinct experiential domains are unified when their awareness profiles coincide. Conscious unity corresponds to the identification of subobjects under the same qualia field.

G.3. Self–Modeling Endofunctor

[Reflexive Endofunctor] Define an endofunctor $\mathcal{S} : \mathbf{Sh}(\mathbb{M}) \to \mathbf{Sh}(\mathbb{M})$ by

$$\mathcal{S}(X) = \underline{\mathrm{Hom}}(X, \Omega),$$

assigning to each cognitive object the sheaf of awareness predicates on it.

[Reflexivity] There exists a natural transformation $\eta : \mathrm{Id} \Rightarrow \mathcal{S}$ mapping each field to its internal awareness profile $\eta_X : X \to \underline{\mathrm{Hom}}(X, \Omega)$.

Proof. For each $x \in X(U)$, $\eta_X(x)$ is the evaluation map $x \mapsto (\lambda y.\ (x = y))$, well–defined as a section of $\underline{\mathrm{Hom}}(X, \Omega)$. $\square$

Interpretation. The functor $\mathcal{S}$ formalizes the *self–model* of cognition: it maps a state to the sheaf of predicates that encode how that state represents itself as aware. The transformation η is introspection.

G.4. Fixed Points and Reflective Equilibrium

[Reflective Fixed Point] If $\mathcal{S}$ is continuous and preserves limits, then fixed points of $\mathcal{S}$ correspond to self–consistent self–models:

$$\mathcal{S}(X) \cong X \quad \Leftrightarrow \quad X \text{ perfectly represents its own awareness.}$$

Proof. By standard fixed–point theorem for continuous endofunctors on complete categories: a limit–preserving $\mathcal{S}$ admits an equalizer between $\mathcal{S}(X)$ and X; isomorphism yields reflexive equilibrium. $\square$

Interpretation. Conscious self–awareness arises when the cognitive structure represents itself exactly as it is: no divergence between object and self–model. Instability of this fixed point corresponds to dissociation or misperception.

G.5. Levels of Awareness and Subtopoi

[Subtopos of Awareness Level] A *subtopos* $\mathbf{Sh}_\alpha(\mathbb{M}) \hookrightarrow \mathbf{Sh}(\mathbb{M})$ is determined by a Lawvere–Tierney topology $j_\alpha : \Omega \to \Omega$ specifying the modal degree of awareness.

[Hierarchies of Awareness] Subtopoi ordered by inclusion correspond to nested levels of consciousness:
$$j_\alpha \leq j_\beta \iff \mathbf{Sh}_\alpha(\mathbb{M}) \subseteq \mathbf{Sh}_\beta(\mathbb{M}).$$

Proof. Follows from the monotonicity of Lawvere–Tierney topologies. $\square$

Interpretation. Different j_α define gradations of phenomenal access— from subliminal (weaker topology) to reflective (stronger topology). Consciousness is stratified by subtopoi of increasing internal openness.

G.6. Phenomenal Logic and Subobject Classifier Algebra

[Phenomenal Algebra] Within each subtopos, Ω_α forms a Heyting algebra of awareness values, with operations:
$$p \wedge q, \quad p \vee q, \quad p \Rightarrow q, \quad \neg p.$$

[Cognitive Soundness] If inference rules are interpreted via the internal logic of Ω_α, then every derivable judgment corresponds to a valid transformation of awareness predicates.

Proof. Immediate from soundness of internal Heyting logic in any Grothendieck topos. $\square$

Interpretation. Logical entailment inside a subtopos is the formal shadow of phenomenological inference: awareness evolves through lawful transformations of internal truth values.

G.7. Summary

The categorical semantics developed here identify:

- The subobject classifier Ω as the field of awareness;

- Characteristic morphisms χ_m as qualia profiles;

- The reflexive endofunctor $\mathcal{S}$ as self–modeling cognition;

- Fixed points $\mathcal{S}(X) \cong X$ as self–aware equilibrium states;

- Subtopoi as hierarchical levels of consciousness;

- Internal logic as the dynamic algebra of awareness transformations.

In this view, consciousness is not added to cognition but is the internal logic of the categorical structure itself. Awareness corresponds to the truth valuation within the RSVP topos: to be conscious of a field is for that field to carry its own classifier of existence.

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