Mathematical Supplement: RSVP Field Theory for Neural Representation Dynamics

1 Introduction

This supplement consolidates the mathematical formalisms for the RSVP (Representation, Semantic, Vector, Potential) field theory across three papers: (1) *From Fractured Representations to Modal Coherence*, (2) *Diagnosing Representation Fracture via Scalar-Vector-Entropy Field Dynamics*, and (3) *Beyond Gradient Descent: A Modal-Thermodynamic Paradigm for AI*. The RSVP field triple is defined as:

$$\mathcal{F}(x,t) = \{\Phi(x,t), \vec{v}(x,t), \mathcal{S}(x,t)\}$$

where Φ is the scalar semantic potential, $\vec{v}$ is the vector semantic flow, and $\mathcal{S}$ is the entropy field.

2 Paper 1: From Fractured Representations to Modal Coherence

2.1 RSVP Field Definitions

- **Scalar Semantic Potential Field**:

$$\Phi : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}$$

- **Vector Semantic Flow Field**:

$$\vec{v} : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$$

- **Entropy Field (Semantic Uncertainty)**:

$$\mathcal{S} : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}_{\geq 0}$$

2.2 Field Evolution Equation

Entropy-guided semantic transport:

$$\frac{\partial \Phi}{\partial t} + \nabla \cdot (\Phi \cdot \vec{v}) = -\delta \mathcal{S}$$

2.3 Modal Fixpoint Operator

Modal stability, denoted $\Box A$, is defined as:

$$\Box A := \text{“A is invariant under field evolution”}$$

Löb-stability condition:

$$\Box(\Box A \to A) \to \Box A$$

2.4 Fracture as Torsion and Modal Instability

- **Torsion Entanglement Index**:

$$T_{\text{ent}} = \int_{\Omega} \|\nabla \times \vec{v}\|^2 \, dx$$

- **Modal Fracture**: A representation is fractured if:

$$\neg\Box A \quad \text{or} \quad \lim_{t\to\infty} \Phi^{(t)} \neq \Phi^{(t-1)}$$

3 Paper 2: Diagnosing Representation Fracture via Scalar-Vector-Entropy Field Dynamics

3.1 Field Construction from Model Activations

Given hidden activations $h_i \in \mathbb{R}^d$:

- **Semantic Scalar Field**:

$$\Phi_i := \|h_i\|$$

- **Semantic Flow**:

$$\vec{v}_i := h_{i+1} - h_i$$

- **Entropy Estimate**:

$$\mathcal{S}_i := \mathbb{H}(p(y|x, h_i)) \quad \text{or} \quad \text{Var}(h_i)$$

3.2 Torsion Entanglement Metric

Pointwise torsion:

$$T(x) := \|\nabla \times \vec{v}(x)\|$$

Aggregate:

$$T_{\text{avg}} := \frac{1}{|\Omega|} \int_{\Omega} T(x)^2 dx$$

3.3 Modal Closure Depth

Iterate:

$$\Phi^{(t+1)} = \Phi^{(t)} - \eta \left(\nabla \mathcal{S}(\Phi^{(t)}) - \nabla \cdot (\Phi^{(t)} \cdot \vec{v}^{(t)}) \right)$$

Closure depth:

$$D_{\Box} := \min \left\{ t \in \mathbb{N} \,\middle|\, \|\Phi^{(t+1)} - \Phi^{(t)}\| < \varepsilon \right\}$$

3.4 Redundancy Score

Mutual information between neuron activations:

$$R_{ij} = I(h_i; h_j)$$

Average redundancy:

$$\bar{R} = \frac{1}{n(n-1)} \sum_{i \neq j} R_{ij}$$

4 Paper 3: Beyond Gradient Descent: A Modal-Thermodynamic Paradigm for AI

4.1 Learning as Semantic Convergence

Learning is defined as modal closure:

$$\text{Learn}(A) \iff \Box A$$

4.2 RSVP Learning Update Rule

Thermodynamic descent law:

$$\Phi_{t+1} = \Phi_t - \eta \left(\nabla \mathcal{S}(\Phi_t) - \nabla \cdot (\Phi_t \cdot \vec{v}_t) \right)$$

4.3 Generalized RSVP Field Loss

RSVP energy functional:

$$\mathcal{L}_{\text{RSVP}} = \int_\Omega \left[\frac{1}{2} \|\nabla \Phi\|^2 + \alpha \cdot \|\nabla \times \vec{v}\|^2 + \beta \cdot \mathcal{S} \right] dx$$

Weights α, β adjust penalties for torsion and entropy.

4.4 RSVP vs SGD Comparison

Property	**SGD**	**RSVP Descent**
Objective	Minimize loss	Minimize semantic tension
Update Rule	$\theta \leftarrow \theta - \eta \nabla_\theta \mathcal{L}$	$\Phi \leftarrow \Phi - \eta \nabla \mathcal{S}_{\text{eff}}$
Field Structure	Flat weight space	Recursive modal geometry
Interpretability	Post hoc	Intrinsic
Generalization	Empirical	Emerges from field stability

Table 1: Comparison of SGD and RSVP Descent.