Appendix: CMB Dipole Constraints in RSVP (Entropic Redshift Form)

A.1 Fields, Normalization, and ΛCDM Dictionary

The RSVP framework employs the following plenum fields:

- Scalar capacity Ξ (drives outward expansion),
- Matter density $\delta\rho_m$ (drives inward gravitational attraction),
- Bulk flow $\mathbf{u}$ (peculiar velocity).

The entropic redshift potential is defined as:

$$\boxed{\Upsilon \equiv \delta\Phi - \beta(\eta)\,\varphi_m}$$

$$\boxed{\varphi_m(k,\eta) = \frac{4\pi G\, a^2(\eta)\,\bar{\rho}_m(\eta)}{k^2}\,\mathcal{T}_m(k,\eta)\,\delta_m(k,\eta)}$$

where $\delta\Phi$ is the scalar potential perturbation, φ_m is the dimensionless matter potential, $\beta(\eta)$ is a time-dependent coupling, $a(\eta)$ is the scale factor, $\bar{\rho}_m(\eta)$ is the mean matter density, $\mathcal{T}_m(k,\eta)$ is the matter transfer function, and $\delta_m(k,\eta)$ is the matter density contrast.

Convention. The potential Υ is normalized such that the instantaneous Sachs–Wolfe contribution at decoupling (denoted by subscript $_*$) is:

$$\boxed{\left(\frac{\Delta T}{T}\right)_{\text{SW}} = \frac{1}{3}\Upsilon_*}$$

For consistency with ΛCDM, we set $\beta(\eta_*) = \frac{4\pi G\, a^2(\eta_*)\,\bar{\rho}_m(\eta_*)}{k^2}\mathcal{T}_m(k,\eta_*)$ at last scattering, ensuring $\Upsilon_* = \delta\Phi_* - \alpha_m\delta\rho_m$ with $\alpha_m = \beta(\eta_*)$.

A.2 Large-Angle Anisotropy in RSVP Form

For a sightline $\hat{\mathbf{n}}$, the CMB temperature anisotropy is:

$$\frac{\Delta T}{T}(\hat{\mathbf{n}}) = \underbrace{\hat{\mathbf{n}}\cdot\frac{\mathbf{u}_0}{c}}_{\text{kinematic dipole } \varepsilon_{\text{kin}}\sim 10^{-3}} + \underbrace{\frac{1}{3}\Upsilon_*(\hat{\mathbf{n}})}_{\text{entropic SW}} + \underbrace{2\int_{\eta_*}^{\eta_0}\dot{\Upsilon}\,d\eta}_{\text{entropic ISW}}$$

The intrinsic dipole amplitude, after subtracting the kinematic contribution, is:

$$\left|\left(\frac{\Delta T}{T}\right)^{\text{int}}_{\ell=1}\right| \equiv \varepsilon_{\text{int}}\text{few}\times 10^{-5}$$

A.3 Super-Horizon Gradient Bound

A beyond-horizon inhomogeneity is modeled as a nearly uniform gradient:

$$\Upsilon(\mathbf{x},\eta) \simeq \Upsilon_0(\eta) + \mathbf{G}(\eta)\cdot\mathbf{x}, \quad \|\mathbf{G}\|R_* \ll 1$$

where R_* is the radius of the observable patch at decoupling. The intrinsic dipole contribution is:

$$D_{\text{int}} \approx \frac{1}{3}\|\mathbf{G}_*\|R_* + \mathcal{O}\left(\int\dot{\Upsilon}\,d\eta\right)$$

The gradient bound is:

$$\boxed{\|\nabla\Upsilon_*\|R_* 3\varepsilon_{\text{int}}} \quad\Longleftrightarrow\quad \boxed{\|\mathbf{G}_*\|\frac{3\varepsilon_{\text{int}}}{R_*}}$$

A.4 Effective Potential and “Falling Outward”

The effective potential governing outward and inward accelerations is:

$$\boxed{\mathbf{a}_{\text{eff}} = -\nabla\Phi_{\text{eff}}, \quad \Phi_{\text{eff}} \equiv \Phi - \gamma(\eta)\varphi_m}$$

where $\gamma(\eta)$ is a time-dependent coupling. The dipole bound implies:

$$\boxed{|\delta\Phi|_* \frac{\varepsilon_{\text{int}}}{\alpha_\Phi} = \varepsilon_{\text{int}}, \quad |\delta\rho_m|_* \frac{\varepsilon_{\text{int}}}{\alpha_m}}$$

Under the chosen calibration, $\alpha_\Phi = 1$ at decoupling, ensuring consistency with the normalization.

A.5 Bulk-Flow Convergence (Vector Test)

The RSVP bulk-flow estimator over a sphere of radius R is:

$$\mathbf{u}_0^{\text{RSVP}}(R) := \arg\min_{\mathbf{u}} \sum_{i:r_i<R} w_i \left(z_i^{\text{obs}} - z_i^{\text{RSVP}}(\mathbf{u})\right)^2, \quad w_i \propto \frac{1}{\sigma_{S,i}}$$

where z_i^{obs} is the observed redshift, z_i^{RSVP} is the model-predicted redshift, and $\sigma_{S,i}$ is the uncertainty. The estimator converges as:

$$\angle\left(\mathbf{u}_0^{\text{RSVP}}(R), \mathbf{d}_{\text{CMB}}\right) \to 0, \quad \|\mathbf{u}_0^{\text{RSVP}}(R)\| \to c\varepsilon_{\text{kin}}$$

A.6 Long-Mode Consistency (Semantic-Slicing Gauge)

Super-horizon adiabatic modes in RSVP correspond to a semantic-slicing gauge redefinition. The leading dipole cancels, and any residual dipole is linked to the quadrupole via Υ evolution at horizon entry, analogous to the Grishchuk–Zel’dovich relation. The observed quadrupole ($\sim 10^{-5}$) further constrains the bound in A.3.

A.7 Bottom Line

The potential Υ encodes both outward expansion (Ξ) and inward gravitational pull ($\delta\rho_m$) in the CMB. The residual dipole limit is:

$$\Delta\Upsilon_* \equiv \|\nabla\Upsilon_*\| R_* \text{few} \times 10^{-5}$$

Bulk-flow alignment and the small quadrupole amplitude indicate that homogeneity and isotropy extend at least one horizon length beyond the observable universe, disfavoring adjacent “bubble” domains with radically different large-scale parameters.