1 The Relativistic Scalar Vector Plenum (RSVP) Framework

The Relativistic Scalar Vector Plenum (RSVP) framework reinterprets modal logic within a field-theoretic context, modeling recursive phenomena as dynamic interactions of scalar, vector, and entropy fields. Below, we formalize key components mathematically.

1.1 Field Definitions

Let $\mathcal{G}$ be a 64×64 grid. The RSVP system is defined by:

- Scalar field: $\Phi : \mathcal{G} \to \mathbb{R}$, representing the primary state variable.
- Vector field: $\sqsubseteq : \mathcal{G} \to \mathbb{R}^2$, guiding recursive transport.
- Entropy field: $\mathcal{S} : \mathcal{G} \to \mathbb{R}$, enforcing thermodynamic relaxation.

The field configuration at time t is denoted $A_t = (\Phi_t, \sqsubseteq_t, \mathcal{S}_t)$.

1.2 Recursive Dynamics

The evolution of Φ_t is governed by:

- **Vector Transport**: $\Phi_{t+1}(x) = \Phi_t(x - \sqsubseteq_t(x) \cdot \Delta t)$, where Δt is the time step.
- **Entropy Smoothing**: $\Phi_{t+1} = \Phi_t + \kappa \nabla^2 \mathcal{S}_t$, where $\kappa > 0$ is a diffusion constant and ∇^2 is the Laplacian on $\mathcal{G}$.

1.3 Modal Operator

The modal operator $\Box : \mathcal{C}_{RSVP} \to \mathcal{C}_{RSVP}$ is defined as:

$$\Box A = \lim_{t \to \infty} A_t,$$

where convergence is measured by thermodynamic closure:

$$\|\Phi_{t+1} - \Phi_t\| < \epsilon,$$

and $\|\cdot\|$ is the L^2-norm on $\mathcal{G}$.

For Gödel-incomplete fields, $\Box A$ does not converge, satisfying:

$$G \leftrightarrow \neg \Box G,$$

modeled as persistent oscillation.

1.4 Categorical Structure

Define the category $\mathcal{C}_{RSVP}$:

- **Objects**: Field configurations $A = (\Phi, \sqsubseteq, \mathcal{S})$.
- **Morphisms**: Recursive updates $f : A \to A'$, parameterized by time steps.
- **Functor** $\Box$: Maps $A \to \Box A$, preserving stability properties.

Löb-stable fields satisfy the endomorphism condition:

$$f(f(X)) \cong f(X),$$

while Gödel-incomplete fields lack a global section to $\Box$.

1.5 Topos-Theoretic Extension

The category $\mathcal{T}_{RSVP}$ is posited as a topos with:

- **Subobject Classifier**: Ω, representing stability states.
- **Forcing Condition**: For $X \in \mathcal{T}_{RSVP}$, $X \Vdash \Box A \Rightarrow A$ if for all $f : Y \to X$, $Y \Vdash \Box A$ implies $Y \Vdash A$.

If $\mathcal{T}_{RSVP}$ is a Grothendieck topos, sheaf theory models field dynamics over a spacetime base $\mathcal{S}$, with sheaves representing Φ, $\sqsubseteq$, and $\mathcal{S}$.

1.6 Commutative Diagram

The functorial action of $\Box$ is illustrated by:

$$\begin{tikzcd}
A \arrow[r, "f"] \arrow[d, "\Box"] & B \arrow[d, "\Box"] \\
\Box A \arrow[r, "\Box f"] & \Box B
\end{tikzcd}$$