From RSVP Field Dynamics to TAG Multi-Agent Hierarchies

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Abstract

Hierarchical organization is fundamental to intelligent behavior in both biological and artificial systems, yet current approaches to multi-agent reinforcement learning (MARL) struggle with scalability and stability. Recent work on TAG introduces a decentralized framework for constructing arbitrarily deep agent hierarchies via the LevelEnv abstraction, but lacks a unifying theoretical foundation. In this paper we embed TAG within the Relativistic Scalar–Vector Plenum (RSVP), a field-theoretic framework where scalar density (Φ), vector flow (v), and entropy flux (S) jointly govern system dynamics. We show that LevelEnv corresponds to a boundary compression of RSVP fields, establishing TAG as a concrete instantiation of RSVP dynamics. This embedding yields new predictive laws: (i) conservation principles under symmetry, (ii) entropy production as a bound on stability, (iii) a depth–compression scaling law for hierarchy efficiency, and (iv) interface tightness as a transfer criterion. Each law translates into empirical protocols for MARL benchmarks, enabling falsifiable tests that go beyond notational generalization. By linking MARL to thermodynamic and categorical perspectives, our work advances both the design of scalable multi-agent systems and the development of RSVP as a predictive unifying theory.

1 Introduction

Modern research confronts a dual scaling problem. On one hand, **interdisciplinary scaling** arises because progress on complex questions—such as intelligence, coordination, and emergence—requires integrating insights from physics, computer science, neuroscience, mathematics, and philosophy. Each field develops its own models, languages, and benchmarks, and the combinatorial explosion of cross-disciplinary dependencies makes synthesis increasingly intractable. On the other hand, **intra-subject scaling** manifests within each discipline itself: specialized subfields proliferate, producing exponentially growing parameter spaces, technical vocabularies, and internal models that resist unification. As a result, researchers often master narrow fragments of their domain while losing sight of the broader structural connections.

In reinforcement learning and multi-agent systems, these scaling issues appear in microcosm. Hierarchical reinforcement learning (HRL) is motivated precisely by the difficulty of learning in high-dimensional state and action spaces, yet most existing frameworks are limited to two-level structures or centralized training regimes. Multi-agent reinforcement learning (MARL) further compounds the challenge: as the number of agents grows, joint state–action spaces expand exponentially, and coordination failures become increasingly likely. Attempts to remedy this by communication protocols, parameter sharing, or specialized abstractions alleviate symptoms but do not address the fundamental scaling barrier.

This paper argues that a field-theoretic perspective, provided by the Relativistic Scalar–Vector Plenum (RSVP), offers a principled way to tame both interdisciplinary and intra-subject scaling. By embedding TAG—a framework for decentralized hierarchical MARL—into RSVP, we show that multi-agent hierarchies can be treated as structured entropy–flow systems. This mapping transforms scaling problems into conservation laws, stability criteria, and transfer diagnostics that are mathematically well-defined and empirically testable.

The TAG framework was developed specifically to address the intractability in MARL and HRL by enabling arbitrary-depth hierarchies with decentralized coordination. Motivated by the limitations of shallow structures in handling non-stationarity and scalability, TAG introduces the LevelEnv abstraction, where higher-level agents shape the environments of lower-level ones through observation modifications and message passing. This bottom-up and top-down flow allows for heterogeneous agents and deeper hierarchies, outperforming traditional methods in benchmarks. However, without a unifying theoretical foundation, TAG risks being seen as an ad hoc engineering solution rather than a manifestation of deeper principles. The RSVP embedding provides this foundation, linking TAG's mechanisms to field dynamics and enabling novel predictions.

Contributions:

1. Formal derivation of TAG from RSVP dynamics.
2. Sheaf-theoretic interpretation of coordination feasibility.
3. Predictive laws linking entropy flux to stability and sample efficiency.
4. Critical discussion of what counts as meaningful theoretical progress.

2 Background

2.1 Multi-Agent Reinforcement Learning (MARL)

Independent learners, parameter sharing, communication-based approaches.

Challenges: non-stationarity, scalability, coordination.

2.2 Hierarchical Reinforcement Learning (HRL)

Options framework, Feudal RL, symbolic and model-based approaches.

Limitations of two-level structures and value estimation under non-stationarity.

2.3 TAG Framework (Paolo et al. 2025)

LevelEnv abstraction.

Arbitrary depth, heterogeneous agents, bottom-up/top-down message flow.

2.4 RSVP Theory (Guimond, 2024–25)

Field triple = scalar density, vector flow, entropy flux.

Recursive causality and boundary-based compression.

Prior applications in cosmology, cognition, and semantic computation.

3 Deriving TAG from RSVP

3.1 RSVP recursion

$\Phi^l(t+1) = f(S^{l-1}, v^{l+1}, \Phi^l), \quad v^l = g(\Phi^l, v^{l+1}), \quad S^l = h(\Phi^{l-1}, v^l)$

3.2 Boundary compression

Define observation/action/reward triples as , , .

3.3 Emergence of LevelEnv

Show each level treats the one below as its environment.

Derive TAG update cycle directly from RSVP recursion.

3.4 Theorem: TAG = boundary-compressed RSVP

Statement and sketch proof.

Implication: TAG is not an ad hoc construct but a realizable projection of a more general field law.

3.5 RSVP Field Dynamics

[RSVP System] An RSVP system on levels $L = \{0, \ldots, D\}$ is a family

$$\mathcal{E}^l(t) = (\Phi^l(t),\, v^l(t),\, S^l(t)) \quad l \in L,$$

where Φ^l is the scalar density, v^l the vector flow, and S^l the entropy flux associated with level l.

[Locality] Each level updates only from its immediate neighbors:

$$\mathcal{E}^l(t+1) = F^l\big(\mathcal{E}^{l-1}(t),\, \mathcal{E}^l(t),\, \mathcal{E}^{l+1}(t)\big).$$

For suitable coordinates, this decomposes as

$$\Phi^l(t+1) = f\big(S^{l-1}(t),\, v^{l+1}(t),\, \Phi^l(t)\big), \tag{1}$$
$$v^l(t+1) = g\big(\Phi^l(t),\, v^{l+1}(t)\big), \tag{2}$$
$$S^l(t+1) = h\big(\Phi^{l-1}(t),\, v^l(t)\big). \tag{3}$$

3.6 Boundary Compression and RL Interface

To align RSVP with reinforcement learning, define compression maps

$$o^l = T_\Phi(\Phi^l), \quad a^l = T_v(v^l), \quad (m^l, r^l) = T_S(S^l),$$

corresponding to observations, actions, and messages/rewards.

3.7 LevelEnv as Boundary Object

[Level Environment] The boundary object at level l is

$$\mathsf{Env}^l = (o^l, a^l, r^l) = \big(T_\Phi(\Phi^l), T_v(v^l), T_S(S^l)\big).$$

Each level $l+1$ interacts with Env^l as its environment, while level l itself interacts with Env^{l-1}.

3.8 Derivation of TAG Recursions

Applying the compression maps to RSVP dynamics yields:

$$(m^l, r^l) = T_S\big(h(\Phi^{l-1}, v^l)\big) = \varphi^l(o^{l-1}, r^{l-1}), \tag{4}$$
$$o^l = T_\Phi\big(f(S^{l-1}, v^{l+1}, \Phi^l)\big) = A^l(m^l, r^l), \tag{5}$$
$$a^l = T_v\big(g(\Phi^l, v^{l+1})\big) = \pi^l(a^{l+1}, o^{l-1}). \tag{6}$$

[TAG as Boundary-Compressed RSVP] Given an RSVP system $\{\mathcal{E}^l\}$ with compression maps (T_Φ, T_v, T_S), the induced boundary processes $\{\mathsf{Env}^l\}$ evolve by

$$(m^l, r^l) = \varphi^l(o^{l-1}, r^{l-1}), \quad o^l = A^l(m^l, r^l), \quad a^l = \pi^l(a^{l+1}, o^{l-1}),$$

which are precisely the TAG update equations. Conversely, any TAG hierarchy with sufficient statistics (o, a, r) lifts to an RSVP system under suitable inverse charts.

[Sketch] Apply T_Φ, T_v, T_S to the RSVP update laws. Since cross-level interactions occur only through (Φ, v, S) boundaries, the compressed dynamics close over (o, a, r). This reproduces the TAG cycle of bottom-up messages, observation aggregation, and top-down action shaping.

3.9 Interpretation and Implications

The derivation shows that TAG is not an ad hoc construction but a quotient of RSVP dynamics under boundary compression. This yields three important consequences:

1. Stability through entropy flux. The RSVP update laws guarantee that bottom-up flux S^l influences upper-level scalar states Φ^{l+1}. After compression, this manifests in TAG as the reward/message channel (m^l, r^l). Fluctuations in S^l therefore bound the stability of higher-level learning. Empirically, this implies that monitoring information-theoretic measures of message entropy provides an early-warning signal for policy instability.

2. Depth–compression tradeoff. In RSVP, scalar density Φ^l accumulates structure by compressing flux from below while being shaped by vector influences from above. After boundary compression, this corresponds to the quality of observation summaries o^l. The benefit of deeper hierarchies in TAG depends on the net compression ratio χ achieved at each interface. There exists an optimal depth D^* maximizing efficiency $\chi^D/(\lambda D)$, where λ represents per-level coordination cost. This predicts that sample efficiency improves with depth only up to D^*.

3. Coordination feasibility via gluing. Because each LevelEnv is a boundary object, global coordination reduces to compatibility of local sections across overlaps. In sheaf-theoretic terms, the existence of a global policy corresponds to trivial Čech cohomology of the policy sheaf. Nontrivial classes represent obstructions: no globally consistent policy exists without architectural change. Practically, this means persistent coordination failures are structural and can be resolved only by adding a mediator level or widening communication bandwidth.

These consequences move the TAG–RSVP connection beyond notational unification. They yield testable predictions: entropy flux as a stability indicator, compression laws governing depth, and sheaf obstructions diagnosing coordination. Each will be explored in Section 5 on predictive laws.

4 Categorical & Sheaf-Theoretic Embedding

4.1 RSVP as a Category

Objects: -systems.

Morphisms: maps preserving entropy–vector–scalar invariants.

4.2 TAG as a Subcategory

Objects: LevelEnvs .

Morphisms: policy update operators.

Functor .

4.3 Sheaf Interpretation of Coordination

Base site: communication hypergraph.

Sheaf of local stochastic policies.

Čech 1-cohomology = obstruction to global consistency.

4.4 Practical Computation

Linearization via log-probs.

Sparse least-squares coboundary fit.

Nontrivial cohomology need for new mediator level.

5 Predictive Laws from RSVP-to-TAG Mapping

The boundary-compressed derivation establishes TAG as a special case of RSVP. This provides additional explanatory power in the form of predictive laws that can be empirically tested. We highlight four such laws.

5.1 Conservation under Symmetry

[Entropy–Reward Conservation] If a TAG hierarchy admits a symmetry under permutation of agents at level l that preserves the LevelEnv interface, then the RSVP entropy flux S^l is conserved in expectation. Consequently, the aggregate return across symmetric agents is invariant up to statistical fluctuations.

Prediction: In symmetric cooperative tasks, the variance of per-episode rewards across agent permutations decays proportionally to $1/L$ with hierarchy depth L, provided communication functions are learned rather than fixed.

5.2 Entropy Production as a Stability Bound

[Flux–Drift Bound] Let $\dot{S}^l$ denote entropy production at level l as measured by inter-level message divergence. Then the expected Bellman error drift at level $l+1$ is bounded by

$$\|\Delta V^{l+1}\| \ \leq \ C \cdot \mathbb{E}[\dot{S}^l],$$

for some constant C depending on the Lipschitz properties of the compression maps.

Prediction: Episodes with large spikes in entropy production at level l precede instability in value estimation at level $l+1$. Reducing $\dot{S}^l$ via learned communication improves stability.

5.3 Depth–Compression Scaling Law

[Optimal Hierarchy Depth] Let χ denote the average compression ratio at each interface, and λ the effective penalty per level (computation and coordination cost). Then the sample efficiency of a hierarchy of depth D scales as

$$\eta(D) \;\propto\; \frac{\chi^D}{\lambda D}.$$

There exists an optimal depth D^* that maximizes efficiency.
Prediction: Empirically, adding levels improves sample efficiency only up to D^*, after which additional depth degrades performance. Increasing interface compression (e.g., via learned communication) shifts D^* upward.

5.4 Interface Tightness and Transferability

[Interface Tightness] The tightness of interface l with respect to task goal g is defined as

$$\tau^l = \frac{I(o^l; g)}{H(o^l)},$$

the ratio of mutual information between observation summaries o^l and goal variables g to their entropy.

[Transfer Criterion] Upper-level policies trained on interface l transfer across tasks with related goals if and only if τ^l exceeds a threshold τ^*.
Prediction: When $\tau^l > \tau^*$, pre-trained upper-level policies can be reused with new lower-level implementations. When $\tau^l < \tau^*$, transfer fails regardless of optimization method.

Together, these four laws demonstrate that the RSVP perspective contributes more than notational generality: it yields conservation principles, stability bounds, scaling laws, and transfer criteria that are testable within multi-agent benchmarks. They also provide design rules for hierarchy depth, communication protocols, and interface evaluation in TAG systems.

6 Empirical Program

To demonstrate that the RSVP-to-TAG mapping yields more than notational generalization, we propose four empirical protocols that correspond directly to the predictive laws of Section 5. Each experiment is designed to be feasible in standard multi-agent benchmarks such as PettingZoo [Terry et al., 2021], MPE-Spread, or cooperative navigation tasks.

6.1 Symmetry and Conservation

Setup. Construct a symmetric task (e.g., cooperative navigation with N identical agents). Instantiate TAG hierarchies of varying depth L with either identity communication functions or learned communication modules.

Measurement. Compute variance of per-episode cumulative reward across agent permutations.

Prediction. Variance decays as $1/L$ when learned communication is enabled, confirming the conservation principle. Flat baselines and identity comms do not show this decay.

6.2 Entropy Production and Stability

Setup. Instrument entropy production $\dot{S}^l$ at each level by measuring KL divergence between successive message distributions:

$$\dot{S}_t^l = \mathbb{E}\big[D_{\mathrm{KL}}(m_t^l \,\|\, m_{t-1}^l)\big].$$

Measurement. Track $\dot{S}^l$ alongside Bellman error drift in upper levels.

Prediction. Episodes with spikes in $\dot{S}^l$ precede instability in value estimation at level $l{+}1$. Learned communication that reduces $\dot{S}^l$ improves stability.

6.3 Depth–Compression Scaling

Setup. Train TAG hierarchies with depth $D \in \{1, \ldots, 5\}$, with both identity and learned compression functions T_Φ, T_S.

Measurement. Estimate interface compression ratio χ as the ratio of entropy in observations before and after aggregation. Measure sample efficiency as the number of steps to achieve a fixed return threshold.

Prediction. Sample efficiency increases with D up to an optimal D^* consistent with

$$\eta(D) \propto \frac{\chi^D}{\lambda D}.$$

Learned compression increases χ and shifts D^* upward.

6.4 Interface Tightness and Transferability

Setup. Pre-train upper levels of a TAG hierarchy on a source task. Swap in new lower-level agents for a target task with related but not identical goals.

Measurement. Compute interface tightness

$$\tau^l = \frac{I(o^l; g)}{H(o^l)}$$

where g encodes task goals. Evaluate transfer success as performance after limited fine-tuning.

Prediction. When $\tau^l > \tau^*$, pre-trained upper-level policies transfer; when $\tau^l < \tau^*$, transfer fails regardless of optimization. This criterion provides an actionable design rule for re-use of high-level policies.

Together, these four experimental protocols provide a direct test of whether the RSVP embedding generates *new predictions, stability diagnostics, and design rules* beyond those offered by TAG alone. Their success would establish the RSVP framework as more than a notational generalization: as a predictive theory of hierarchical multi-agent learning.

Experiments in PettingZoo benchmarks.

Metrics: entropy flux, overlap KLs, cohomology residual, stability curves.

Ablations: identity vs. learned comms; varying depth; mediator-level insertion.

Predictions: nontrivial cohomology persistent failures; mediator levels resolve obstructions.

7 Philosophical and Methodological Reflection

7.1 Notation vs. Progress

Comparison with physics (Maxwell's equations in differential forms).

When generalization is meaningful: prediction, unification, simplification, connection.

7.2 The danger of theoretical ornamentation

How sophisticated frameworks (category theory, sheaf theory) can obscure vacuity.

The need for empirical and algorithmic consequences.

7.3 Satirical dimension

Commentary on the allure of mathematical jargon in AI theory.

How to distinguish genuine explanatory power from "category-theoretic wallpaper."

8 Related Work

Our approach situates TAG [Paolo et al., 2025] within a broader theoretical landscape that spans multi-agent reinforcement learning, hierarchical reinforcement learning, physics-based entropy theories, and sheaf-theoretic formalisms. We highlight relevant prior work across these domains and clarify how RSVP provides a unifying framework.

8.1 Multi-Agent Reinforcement Learning (MARL)

The study of multi-agent systems has accelerated in recent years, motivated by the need to coordinate multiple adaptive units in complex environments [Nguyen et al., 2020, Oroojlooyjadid et al., 2023]. Leibo et al. [Leibo et al., 2019] emphasized the role of autocurricula—emergent challenges driven by agent interaction—in driving innovation. To support this growing field, standardized benchmarks such as PyMARL [Samvelyan et al., 2019], PettingZoo [Terry et al., 2021], and BenchMARL [Bettini et al., 2024] have been developed.

Existing MARL methods fall broadly into independent learners [Thorpe et al., 1997, de Oliveira et al., 2020], parameter-sharing methods [Yu et al., 2021], and communication-based agents [Foerster et al., 2016, Jorge et al., 2016]. The dominant paradigm of centralized training with decentralized execution addresses non-stationarity [Oroojlooyjadid et al., 2023], but remains brittle in lifelong learning settings. TAG extends this literature by showing that fully decentralized hierarchies with LevelEnv abstraction can outperform flat and two-level baselines, suggesting that hierarchical structuring itself confers scalability.

8.2 Hierarchical Reinforcement Learning (HRL)

Hierarchical methods have long been viewed as critical for abstraction and credit assignment. The Options framework [Sutton et al., 1999] formalized temporal abstraction through semi-MDPs, while Feudal RL [Dayan and Hinton, 1992] introduced manager–worker decomposition. Later work advanced these approaches with end-to-end training [Bacon et al., 2016], feudal networks [Vezhnevets et al., 2017], and multi-agent hierarchical extensions [Nachum et al., 2019, Yang and Nachum, 2021].

TAG builds most directly on these traditions but replaces explicit goal-passing with environmental shaping. Rather than specifying intrinsic rewards or goals, higher-level agents in TAG modify the observation spaces of their subordinates. This mechanism resonates with biological theories of modular control [Levin, 2022], where environmental constraints induce emergent coordination.

8.3 Entropy and Physics-Inspired Perspectives

Beyond AI, our framework is inspired by thermodynamic and entropic accounts of coordination. Jacobson [Jacobson, 1995] famously derived Einstein's field equations from local thermodynamic arguments, while Verlinde [Verlinde, 2011] proposed that gravity itself emerges from entropic gradients. More recently, Carney [Carney, 2022] reviewed insights from quantum information that frame gravity as an entropic phenomenon.

The RSVP framework adopts a similar stance: it treats information flow, constraint relaxation, and entropy flux as the fundamental invariants across domains. Embedding TAG into RSVP thus situates multi-agent coordination in the same lineage as thermodynamic accounts of emergent laws. Conservation, stability bounds, and scaling relations in TAG can be interpreted as specific cases of RSVP's entropic field dynamics.

8.4 Information Geometry and Variational Principles

RSVP also connects to broader mathematical traditions. Information geometry [Amari, 2016] provides a natural setting for interpreting policies as points on curved statistical manifolds. Variational principles in control and RL [Gallego et al., 2020] echo RSVP's formulation of learning as entropy minimization under vector flow constraints. The free-energy principle in neuroscience [Friston, 2010] offers another parallel, framing cognition itself as entropy reduction under predictive models.

These traditions highlight that abstraction, learning, and control can all be cast as thermodynamic processes—aligning precisely with RSVP's scalar–vector– entropy decomposition.

8.5 Sheaves and Category-Theoretic Approaches

Finally, RSVP's categorical interpretation draws on sheaf theory [Mac Lane and Moerdijk, 1992, Bredon, 1997]. In this view, local policies are sections of a sheaf over a base space defined by system configurations, with gluing conditions enforcing global consistency. Recent work has applied sheaves to machine learning [Curry et al., 2021], signal processing [Robinson and Hansen, 2021], and distributed computation, demonstrating the fruitfulness of this perspective.

This provides RSVP with diagnostic tools: coordination failures in TAG can be viewed as non-trivial cohomology classes obstructing the existence of global sections. Such an interpretation is not merely metaphorical—it suggests concrete diagnostics for when hierarchical structures fail to integrate.

8.6 Cross-Domain Hybrid Systems

Recent work explores hybrids between MARL and language models, using LLMs as zero-shot coordinators [Wang et al., 2023]. These efforts demonstrate the utility of leveraging heterogeneous agents across abstraction levels. TAG's support for heterogeneous policies across layers, interpreted through RSVP, offers a principled way to study such mixed architectures.

8.7 Summary

Taken together, these literatures show three gaps that our work addresses: (1) MARL lacks a unifying theoretical framework; (2) HRL approaches often stop at two levels and rely on hand-designed goals; (3) categorical and entropic perspectives have yet to be integrated with scalable multi-agent benchmarks. The RSVP embedding of TAG contributes to filling these gaps by offering a field-theoretic account that unifies conservation, scaling, and coordination within a single mathematical framework.

Existing research on multi-agent reinforcement learning (MARL) has produced a range of independent, parameter-sharing, and communication-based approaches [Samvelyan et al., 2019, Terry et al., 2021, Bettini et al., 2024], while hierarchical reinforcement learning (HRL) has emphasized temporal abstraction through Options [Sutton et al., 1999] and Feudal methods [Dayan and Hinton, 1992, Vezhnevets et al., 2017]. These methods remain limited in scalability and often stop at shallow hierarchies. Recent frameworks such as TAG [Paolo et al., 2025] demonstrate that decentralized hierarchies with LevelEnv abstraction can outperform flat baselines, yet lack a unifying theory for conservation, scaling, and coordination. Physics-based accounts of entropy [Jacobson, 1995, Verlinde, 2011, Carney, 2022] and mathematical tools from information geometry [Amari, 2016] and sheaf theory [Mac Lane and Moerdijk, 1992, Curry et al., 2021] suggest deeper connections between learning, thermodynamics, and category-theoretic gluing. Our contribution embeds TAG into the Relativistic Scalar-Vector Plenum (RSVP), demonstrating that multi-agent hierarchies are special cases of a more general field-theoretic framework. This embedding yields not only notational unity but new predictive laws—conservation principles, stability bounds, and depth– compression tradeoffs—that can be empirically tested in MARL benchmarks.

9 Conclusion

This work has shown that TAG, a decentralized framework for hierarchical multi-agent reinforcement learning [Paolo et al., 2025], can be formally embedded as a special case of the Relativistic Scalar–Vector Plenum (RSVP) framework. The LevelEnv abstraction corresponds directly to RSVP's scalar, vector, and entropy fields, with observations as scalar densities, actions as vector flows, and rewards as entropy flux. This embedding demonstrates that TAG's empirical effectiveness is not an isolated engineering success but a manifestation of deeper field-theoretic dynamics.

From this correspondence we derived predictive laws: conservation principles under symmetry, bounds on stability through entropy production, scaling laws for optimal hierarchy depth, and interface tightness as a transfer criterion. Each law yields empirical protocols that can be tested within standard MARL benchmarks. These predictions move the TAG–RSVP connection beyond notational generalization and toward falsifiable science.

The broader implication is twofold. For MARL, RSVP provides a principled lens through which to analyze stability, scalability, and transfer in hierarchical systems. For RSVP, TAG offers a concrete instantiation that grounds its abstract thermodynamic and categorical claims in implementable benchmarks. This mutual reinforcement illustrates the value of cross-domain synthesis: mathematical generality can illuminate empirical design rules, and empirical results can validate field-theoretic abstractions.

Future work will extend this framework along two directions: (1) integrating learned communication functions to reduce entropy production and test deeper hierarchies, and (2) developing categorical diagnostics for coordination failures as sheaf obstructions, with experimental validation. By embedding TAG into RSVP, we hope to advance not only the scalability of multi-agent systems but also the credibility of RSVP as a predictive, unifying theory.

TAG as a special case of RSVP field theory.

Implications for multi-agent AI, distributed robotics, and human–AI coordination.

Open problems: dynamic hierarchy growth, adversarial agents, integration with model-based planning.

Broader vision: RSVP as a unifying field framework across physics, cognition, and multi-agent intelligence.

10 Appendices

A Full Proof of Theorem (TAG as Boundary-Compressed RSVP)

We provide the full derivation showing that TAG dynamics arise from boundary-compressed RSVP fields.

A.1 Setup

Recall that an RSVP system is given by

$$\mathcal{E}^l(t) = (\Phi^l(t), v^l(t), S^l(t)), \quad l \in L,$$

with local update rule

$$\mathcal{E}^l(t+1) = F^l(\mathcal{E}^{l-1}(t), \mathcal{E}^l(t), \mathcal{E}^{l+1}(t)).$$

A.2 Compression Maps

Define $o^l = T_\Phi(\Phi^l)$, $a^l = T_v(v^l)$, and $(m^l, r^l) = T_S(S^l)$. These are surjective maps that reduce high-dimensional fields to the observation–action–reward triple of RL.

A.3 Proof

Applying the compression maps to the RSVP updates:

$$o^l(t+1) = T_\Phi\big(f(S^{l-1}, v^{l+1}, \Phi^l)\big), \tag{7}$$

$$a^l(t+1) = T_v\big(g(\Phi^l, v^{l+1})\big), \tag{8}$$

$$(m^l, r^l)(t+1) = T_S\big(h(\Phi^{l-1}, v^l)\big). \tag{9}$$

Because all cross-level dependencies in RSVP are local and boundary-mediated, the induced processes over (o^l, a^l, m^l, r^l) close under these equations. This is precisely the TAG update cycle. Conversely, any TAG hierarchy can be lifted by embedding its (o, a, r) variables into latent RSVP fields via inverse charts.

B Sheaf-Theoretic Formalism

B.1 Base Site

Let $\mathcal{U} = \{U_i\}$ be an open cover of the agent–communication hypergraph. Each U_i corresponds to a neighborhood of agents sharing information.

B.2 Sheaf of Local Policies

Define a sheaf $\mathcal{F}$ on $\mathcal{U}$ such that:

- $\mathcal{F}(U_i)$ is the set of stochastic policies over U_i.
- Restriction maps $\rho_{ij} : \mathcal{F}(U_i) \to \mathcal{F}(U_i \cap U_j)$ enforce consistency on overlaps.

B.3 Nerve Construction

The nerve $N(\mathcal{U})$ is the simplicial complex with vertices $= U_i$, edges = non-empty intersections, etc. Cohomology $H^k(N(\mathcal{U}), \mathcal{F})$ encodes obstructions.

B.4 Interpretation

Non-trivial H^1 corresponds to persistent coordination failures: no global section exists. Adding mediator levels is equivalent to refining the cover until Čech cohomology vanishes.

C Experimental Details and Pseudocode

C.1 Entropy Production Measurement

Entropy production is estimated as

$$\dot{S}_t^l = \mathbb{E}[D_{\mathrm{KL}}(m_t^l \,\|\, m_{t-1}^l)].$$

C.2 Sample Efficiency Estimation

Interface compression ratio χ is measured as:

$$\chi = \frac{H(o^{l-1}) - H(o^l)}{H(o^{l-1})}.$$

C.3 Pseudocode: Depth–Compression Scaling

```
for depth D in {1,...,5}:
    init TAG hierarchy(D)
    while not converged:
        run_episode()
        measure  ,  (D)
record optimal depth D*
```

C.4 Benchmarks

All experiments can be implemented in PettingZoo and MPE. Parameters: 3–6 agents, 10k training episodes, entropy regularization coefficient $\beta = 0.1$.

D Critical Discussion

D.1 Limitations

- **Compression maps not unique.** Multiple T_Φ, T_v, T_S may yield the same TAG interface, raising identifiability issues.

- **Finite sample artifacts.** Entropy flux estimates are sensitive to small-batch KL divergences.
- **Sheaf formalism.** While elegant, computing Čech cohomology for large hypergraphs may be infeasible in practice.

D.2 Failed Generalizations

- Extending the depth–compression law to adversarial settings failed: entropy production may increase without clear stability breakdown.
- Attempts to generalize symmetry conservation to heterogeneous agents showed counterexamples.

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