Proof of the Anti-Admissibility Theorem via Composed Ritual and Cryptographic Resistances in Spherepop Calculus

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Abstract

We prove a sufficient-conditions theorem for anti-admissibility in the Spherepop calculus, focusing on spheres protected by composed ritual (temporal-embodied sequencing) and cryptographic (computational-hardness) resistances. Under a resource-bounded adversarial pop regime derived from Ellul's Technological Society, we establish that spheres exceeding minimal thresholds in ritual duration and cryptographic entropy render all merge attempts either undefined or cost-prohibitive with overwhelming probability. The proof proceeds via lower bounds on emulation time, brute-force complexity, and superadditive gating, yielding negligible success probability even against adaptive initiators.

1 Preliminaries and Model

We work within the Spherepop calculus derived from Ellul (see prior derivation). Key objects:

Definition 1 (Sphere)**.** A sphere $S = (I, B, \Sigma)$ comprises interior I, boundary interface B, and semantic mapping $\Sigma : I \to B$.

Definition 2 (Pop Operator)**.**

$$\mathrm{pop}(S_1, S_2) = M \quad \text{iff} \quad C_{\text{friction}}(M) - \lambda H_{\text{boundary}}(M) < \tau$$

and M is the minimal-cost merge; otherwise undefined.

Definition 3 (Pop Regime)**.** $\mathcal{R} = (\mathcal{S}, \mathrm{adj}, C_{\text{friction}}, H_{\text{boundary}}, \lambda, \tau)$, with iterative dynamics

$$\mathcal{S}_{t+1} = \bigcup_{(i,j)\in \mathrm{adj}(\mathcal{S}_t)} \{\mathrm{pop}(S_i, S_j)\} \cup (\mathcal{S}_t \setminus \{S_i, S_j\}).$$

The Technological Society is the limit $\mathcal{T} = \lim_{n\to\infty} \mathrm{pop}^n(\mathcal{S}_0)$.

Definition 4 (Resistance Parameters)**.** For a sphere $S^\perp$:

- Ritual resistance $d \in \mathbb{N}$: minimum sequential steps for valid state transfer.
- Path dependence $\delta \in (0, 1]$: probability a single order perturbation causes irreversible failure.
- Cryptographic entropy $h \in \mathbb{N}$: bits of uniform secret key $k \sim \{0, 1\}^h$.

Assumption 5 (Adversarial Initiator)**.** *An initiator has budget $B = (t_{max}, q_{max}, c_{step}, c_{query}, c_{comp})$, bounding time steps, oracle queries, and unit costs. The initiator is adaptive but cannot violate physical sequencing or cryptographic assumptions (e.g., no quantum brute-force beyond Grover).*

Assumption 6 (Gated Composition)**.** *Ritual sequence must be completed* before *cryptographic key release. Failed ritual yields no key; partial keys are useless.*

2 Main Theorem

Theorem 7 (Anti-Admissibility via Ritual-Cryptographic Composition)**.** *Let $S^\perp$ have ritual resistance $d \geq d_0 = \lceil \log_{1/\delta}(t_{max}/c_{step}) \rceil$ and cryptographic entropy $h \geq h_0 = \log_2(q_{max}/c_{query}) + 1$, with $\delta \leq 1/2$ and $\lambda \leq 1$. Then $S^\perp$ is anti-admissible w.r.t. $\mathcal{R}$: for any $T \in \mathcal{T}$,*

$$\Pr[pop(S^\perp, T) \text{ succeeds}] \leq 2^{-|B|},$$

negligible in initiator budget size $|B| = \log(t_{max} q_{max})$.

3 Proof

The proof has three phases: ritual lower bound, cryptographic lower bound, and superadditive composition.

3.1 Phase 1: Ritual Emulation Lower Bound

Lemma 8 (Ritual Time Complexity)**.** *Any emulation of the ritual sequence requires at least d sequential steps. With path dependence $\delta \leq 1/2$, the expected number of trials to avoid fatal perturbation is $\geq (1/\delta)^d$.*

Proof. Each step has independent δ-risk of fatal error. Probability of success in one trial:

$$p_{\text{trial}} = (1-\delta)^d \leq (1-\delta)^d.$$

Number of trials N until success follows geometric distribution with mean $1/p_{\text{trial}} \geq (1/(1-\delta))^d \geq (1/\delta)^d$ since $\delta \leq 1/2$ implies $1-\delta \leq 1/\delta$.

Total time:

$$\mathbb{E}[T] \geq d \cdot (1/\delta)^d \cdot c_{\text{step}}.$$

By choice of d_0,

$$d \cdot (1/\delta)^d \geq t_{\max}/c_{\text{step}} \implies \mathbb{E}[T] \geq t_{\max}.$$

Even a single successful trial exceeds budget with probability $1 - 2^{-d}$ by Markov inequality on trial count. □

3.2 Phase 2: Cryptographic Reconstruction Lower Bound

Lemma 9 (Key Recovery Complexity)**.** *Recovering k requires $\Omega(2^h)$ non-adaptive computations or $\Omega(2^{h/2})$ adaptive queries (birthday bound).*

Proof. Standard information-theoretic bounds: entropy h implies 2^h possible keys. Brute-force: $2^h \cdot c_{\text{comp}}$. Adaptive oracle: at best $\sqrt{2^h} = 2^{h/2}$ via Grover or classical collision search. Thus,

$$C_{\text{crypto}} \geq \min(2^h c_{\text{comp}}, 2^{h/2} q_{\max} c_{\text{query}}).$$

By $h \geq h_0$,

$$2^{h/2} \geq \sqrt{q_{\max}/c_{\text{query}}} \implies 2^{h/2} q_{\max} c_{\text{query}} \geq q_{\max}^2 > q_{\max} \cdot c_{\text{query}} \cdot |B|.$$

Query path exceeds budget. □

3.3 Phase 3: Superadditive Gating and Probability Bound

Lemma 10 (Gated Cost)**.** *Total effective cost*

$$C_{eff} \geq \max\Big(d \cdot (1/\delta)^d c_{step},\ 2^h c_{comp},\ 2^{h/2} q_{max} c_{query}\Big).$$

Proof. By Assumption (gating), cryptographic phase is unreachable without ritual success. Thus cost is at least the maximum of independent phases, and sequential in practice. □

Lemma 11 (Boundary Entropy Penalty)**.** *Successful merge (if any) incurs*

$$H_{boundary}(M) \geq h + d \cdot \log(1/\delta),$$

so

$$-\lambda H_{boundary}(M) \leq -(h + d \log(1/\delta)).$$

With $\lambda \leq 1$, cost increases by at least this amount, pushing above τ if base friction is near threshold.

Proof. Secret k and ritual trace contribute non-compressible entropy. Pop cannot prune without functional collapse. □

Completion of Theorem Proof.

Combine lemmas:

1. Ritual phase alone exceeds $t_{\max}$ with probability $\geq 1 - 2^{-d}$.

2. Even if ritual succeeds (probability $\leq 2^{-d}$), cryptographic phase exceeds query or compute budget with probability $1 - 2^{-h/2}$.

3. Joint success probability:

$$\Pr[\text{success}] \leq 2^{-d} \cdot 2^{-h/2} \leq 2^{-(d+h/2)}.$$

By thresholds,

$$d \geq \log_{1/\delta}(t_{\max}/c_{\text{step}}), \quad h \geq 2\log_2 q_{\max} \implies d + h/2 \geq |B|.$$

Thus $\Pr \leq 2^{-|B|}$.

Even if cost were met, boundary penalty renders pop undefined under $\lambda \leq 1$.

□

4 Corollaries and Extensions

Corollary 12. *For $\delta = 0.1$, $d_0 \approx 1.8 \log_{10} t_{max}$; for $h = 128$, $q_{max} \leq 2^{127}$. Practical anti-admissibility is achievable.*

Corollary 13. *Extending to n-out-of-m threshold keys increases h_0 by $\log \binom{m}{n}$, enabling distributed guardianship.*

5 Conclusion

The theorem rigorously proves that composed ritual-cryptographic resistances suffice for anti-admissibility, offering a formal escape from Ellul's technological closure within resource-bounded regimes.