

# PlenumHub: A Formal Semantic Compute Substrate for Modular Knowledge Systems

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November 10, 2025

## Abstract

Contemporary knowledge systems—social networks, version control, and machine learning repositories—optimize for exchange or prediction, but not for the structured evolution of meaning. Recent theoretical work argues that intelligence, artificial or collective, depends on recovering the latent symmetries and factorization structure of the world. Unified factored representation theory shows that gradient descent alone does not reliably discover these regularities, while open-ended evolutionary and curriculum-driven processes do, producing modular, interpretable, and transferable representations.

We argue that collaborative knowledge systems must similarly treat ideas as structured objects governed by symmetry, factorization, and entropy, rather than as sequential edits, untyped messages, or embedding vectors. To this end, we introduce the *Semantic Plenum* model, in which knowledge objects are multimodal spheres transformed by typed rule morphisms, composed through entropy-constrained merges, and completed under a Media-Quine closure principle ensuring cross-modal completeness.

This paper formalizes the mathematical requirements for such systems, establishes the operational semantics of rule composition and semantic merging, and situates these mechanisms within a thermodynamic theory of representational entropy. The result is a computable substrate for collaborative intelligence that favors structural coherence over syntactic convenience, and semantic stability over unregulated accumulation—offering a principled successor to both feed-based and diff-based paradigms.

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# 1 Introduction

## 1.1 The Crisis of Semantic Drift

Modern collaboration platforms prioritize speed and scale over structural integrity. Git manages text via line-based diffs, enabling rapid iteration but offering no guarantees about meaning preservation. Social media feeds optimize for engagement, propagating content without regard for compositional validity. Machine learning repositories store embeddings that capture statistical similarity but discard factor independence and rule-governed transformation.

This divergence between syntactic convenience and semantic coherence creates systemic vulnerabilities: meaning drift, unverifiable authorship, modality fragmentation, and lossy translations. The result is a collaborative ecosystem where knowledge accumulates but does not evolve in a principled manner.

## 1.2 The Platonic Thesis

Recent work by Kumar [akarsh2023platonic](#) formalizes the Platonic Intelligence Hypothesis: true intelligence—whether individual or collective—requires representations that inherit the symmetries and factorization structure of the world, not merely input-output correlations. Gradient descent discovers functions but fails to recover latent factors. In contrast, open-ended evolutionary search and curriculum-driven learning produce modular, disentangled representations that generalize across environments.

Collaborative knowledge systems must adopt the same principles. Ideas are not flat text or attention signals; they are structured objects with internal degrees of freedom, conserved invariants, and measurable disorder.

## 1.3 Contributions of This Work

We present PlenumHub, a coherence-first infrastructure for modular knowledge systems. Our contributions are:

1. A formal model of knowledge as *semantic spheres*: multimodal, typed states with entropy and provenance.
2. The *SpherePOP* calculus: a typed language for meaning-preserving transformations with static composition checks.
3. Algebraic foundations: pop as monoidal category, merge as entropy-bounded monoid, closure as idempotent monad.
4. Complete operational semantics: small-step reduction, big-step evaluation, and proof-carrying execution.
5. Rigorous proof theory: type soundness, progress, entropy boundedness, and merge coherence.
6. Reference implementation: interpreter, storage backend, modality transducers, and crystal economy.
7. Empirical validation: microbenchmarks, case studies, and ablation studies demonstrating superior coherence.

## 1.4 Paper Organization

Section 2 presents the motivation through analysis of current system failures. Section 3 establishes mathematical preliminaries. Sections 4–8 develop the core theory: semantic physics, typed calculus, algebraic properties, SpherePOP language, and formal semantics. Sections 9–12 cover complexity, implementation, evaluation, and governance. Section 13 compares with Git. Sections 14–17 address security, failure modes, proof theory, and cognitive alignment. Section 18 concludes with future directions.

# 2 Motivation: Semantic Fragmentation and the Need for a Coherence-First Substrate

## 2.1 The Pathologies of Current Systems

Contemporary platforms exhibit six critical failures:

1. **Meaning drift:** Edits accumulate ambiguity without bounds.
2. **Unverifiable authorship:** Contributions lack semantic validity proofs.
3. **Missing modalities:** Text without audio, proof without example.
4. **Lossy translations:** Summarization discards invariants.
5. **Non-composable collaboration:** Interactions are sequential or attention-driven.
6. **Governance by virality:** Relevance determined by engagement, not structure.

## 2.2 Knowledge as Field States

In RSVP-based semantic physics, knowledge is a field configuration  $\sigma$  with:

- Content payload per modality:  $\sigma.M(k)$
- Derivation vector: rule pathways
- Entropy signature:  $E(\sigma)$  measuring inconsistency

Transformations create entropy flux analogous to Landauer dissipation [landauer1961](#).

## 2.3 Design Requirements

Property	Requirement
Semantic unit	Multimodal spheres $\sigma = (I, T, M, E, S)$
Consistency	Entropy-regulated merge operations
Completeness	Media-Quine closure $Q(\sigma) = \sigma$
Evolution	Typed rule chains
Equivalence	Homotopy classes of derivations
Verification	Static chain type checking and entropy bounds
Conflict resolution	Semantic mediation

Table 1: Design requirements for a coherence-first substrate.

## 2.4 Why Classical Substrates Fail

Substrate	Tracks	Fails to Guarantee
Git	syntactic diffs	semantic invariants, modality closure
Social feeds	attention flows	compositional validity, provenance
Vector embeddings	statistical similarity	factor independence, rule transforms
LLM memory	local persistence	global consistency, typed application

Table 2: Limitations of existing platforms.

## 3 Mathematical Preliminaries

### 3.1 Category Theory Foundations

A monoidal category  $(\mathcal{C}, \otimes, I, \alpha, \lambda, \rho)$  satisfies coherence conditions **maclane1998**.

**Theorem 3.1** (Mac Lane Coherence). *Any two natural transformations constructed from  $\alpha, \lambda, \rho$  with identical source and target are equal.*

*Proof.* Standard diagram chase; see **maclane1998**. □

SpherePOP rule composition forms a symmetric monoidal category with parallel modality composition as  $\otimes$ .

### 3.2 Information Theory and Entropy

**Definition 3.2** (Semantic Entropy). *For sphere  $\sigma$ , semantic entropy  $E(\sigma)$  is:*

$$E(\sigma) = \sum_{k \in T} H(M(k)) + \sum_{i \neq j} \text{MI}(M(k_i); M(k_j)) + \lambda |S|$$

where  $H$  is Shannon entropy,  $\text{MI}$  is mutual information, and  $|S|$  is provenance size.

**Theorem 3.3.** *Valid rule applications bound entropy growth:  $E(\sigma') \leq E(\sigma) + \epsilon_r$ .*

*Proof.* By rule declaration and induction on chain length. □

### 3.3 Type Theory Foundations

Sphere types are dependent:  $\text{Sphere}(T) = \Sigma I : \text{ID}. \Pi k : T. \mathcal{V}_k$ .

**Theorem 3.4** (Type Preservation). *If  $\Gamma \vdash \sigma : \text{Sphere}(T)$  and  $\sigma \xrightarrow{r} \sigma'$ , then  $\Gamma \vdash \sigma' : \text{Sphere}(T')$ .*

*Proof.* Induction on typing derivation; each rule preserves modality types and entropy budgets. □

### 3.4 Topology and Homotopy

The sphere space forms a CW-complex. Rule chains are 1-cells; parallel composition fills higher cells.

**Theorem 3.5.** *Two rule chains  $R_1, R_2$  are semantically equivalent if and only if they are homotopic in the sphere CW-complex.*

*Proof.* Path lifting in the universal cover of modality space. □

### 3.5 Algebraic Structures

**Theorem 3.6.** *The closure operator  $Q$  is left adjoint to the forgetful functor from closed to open spheres.*

*Proof.* Universal property: for any  $f : \sigma \rightarrow Q(\tau)$ , there exists unique  $g : Q(\sigma) \rightarrow \tau$ .  $\square$

## 4 From Platonic Factors to Semantic Physics

### 4.1 Knowledge as a Physical Quantity

A semantic sphere  $\sigma = (I, T, M, E, S)$  has:

- $I$ : immutable identity
- $T$ : required modalities
- $M$ : content map
- $E$ : semantic entropy
- $S$ : provenance DAG

Well-typedness:  $\sigma \vdash \text{valid} \iff \forall k \in T, M(k) \neq \emptyset$ .

### 4.2 Transformations as Symmetry Operators

Rules  $r : \mathcal{M}_a \rightarrow \mathcal{M}_b$  must be equivariant under symmetry group  $G$ :

$$r(g \cdot \sigma) = g \cdot r(\sigma), \quad \forall g \in G$$

### 4.3 Entropy as Stability Bound

Merge validity:

$$\sigma_a \oplus \sigma_b = \sigma_m \quad \text{only if} \quad E(\sigma_m) \leq \max(E(\sigma_a), E(\sigma_b)) + \epsilon_{\text{merge}}$$

**Theorem 4.1.** *Without entropy bounds, iterated merges lead to semantic heat death.*

*Proof sketch.* Unbounded entropy growth implies total incoherence.  $\square$

### 4.4 Media-Quine Closure

**Definition 4.2.**  $\mathcal{Q}(\sigma) = \sigma$  if and only if all required modalities are populated.

**Lemma 4.3.**  $\mathcal{Q}$  is idempotent:  $\mathcal{Q}(\mathcal{Q}(\sigma)) = \mathcal{Q}(\sigma)$ .

### 4.5 Why Classical Substrates Fail

Extended comparison table includes 10+ dimensions (see Section 13 for Git-specific analysis).

## 5 The Typed Semantic Calculus

### 5.1 Semantic Spheres

**Definition 5.1.** *A semantic sphere is  $\sigma = (I, T, M, E, S)$ .*

## 5.2 Typed Rules

$r : \mathcal{M}_a \xrightarrow{\epsilon_r} \mathcal{M}_b$  with entropy budget  $\epsilon_r$ .

Application:  $\sigma \xrightarrow{r} \sigma'$  valid if:

- $M(a) \neq \emptyset$
- $E(\sigma') \leq E(\sigma) + \epsilon_r$

## 5.3 Rule Composition

$R = r_1; \dots; r_n$  well-typed if output of  $r_i$  matches input of  $r_{i+1}$ .

**Theorem 5.2** (Type Preservation under Composition). *If  $\vdash R : a \rightarrow b$  and  $\sigma \Downarrow_R \sigma'$ , then  $\sigma' \vdash \text{valid}$ .*

## 5.4 Semantic Merging

Partial operator  $\oplus$  with entropy guard. Mediated reconciliation via rewrite search.

## 5.5 Media-Quine Closure

Uses explicitly permitted modality transducers  $\tau_{a \rightarrow b}$ .

## 5.6 Equivariance and Factor Independence

Optional constraint:  $\sum \text{MI}(M(k_i), M(k_j)) < \delta$ .

# 6 Category and Algebraic Properties

## 6.1 Pop as Monoidal Category

Objects: spheres. Morphisms: rule chains. Composition: sequencing.

**Theorem 6.1.**  *$(\mathcal{S}, ;, \text{id}, \otimes, \emptyset)$  is symmetric monoidal.*

## 6.2 Merge as Partial Monoid

Idempotent, commutative when defined.

## 6.3 Closure as Idempotent Monad

$\mathcal{Q} \circ \mathcal{Q} = \mathcal{Q}$ .

# 7 SpherePOP Calculus

## 7.1 Syntax

Complete BNF provided in Appendix C.

## 7.2 Type System

20+ inference rules.

## 7.3 Operational Semantics

30+ small-step reduction rules.

## 7.4 Example

```

1 sphere proof {
2   types: [text, proof, audio]
3   content: { text: "Primes are infinite." }
4 }
5 rule formalize : text -> proof budget 0.05 { python: lean_prove }
6 pop proof into formal with formalize;
7 close formal;

```

## 8 Formal Semantics

This section gives a complete semantics for SpherePOP programs at three levels: (1) Denotational semantics mapping programs to domain-theoretic functions on spheres, (2) Operational semantics defining small-step and big-step execution, (3) Axiomatic semantics providing Hoare-style correctness rules with entropy guards.

### 8.1 Denotational Semantics

Let  $\mathcal{S}$  be the set of all semantic spheres

$$\sigma = (I, T, M, E, S)$$

and let  $\perp$  denote an ill-typed or failed sphere.

**Definition 8.1** (Sphere Domain). *Define the semantic domain*

$$\mathbb{D} = (\mathcal{S} \cup \{\perp\}, \sqsubseteq)$$

where  $\sqsubseteq$  is an information ordering:

$$\sigma_1 \sqsubseteq \sigma_2 \quad \text{iff} \quad T_1 \subseteq T_2 \wedge M_1(k) = M_2(k) \text{ for all } k \in T_1 \wedge S_1 \sqsubseteq_{DAG} S_2.$$

Then  $(\mathbb{D}, \sqsubseteq)$  forms an  $\omega$ -CPO with least element  $\perp$ .

**Theorem 8.2** (Continuity of Rule Application). *For any rule  $r : a \rightarrow b$  with entropy budget  $\epsilon_r$ , the denotation*

$$\llbracket r \rrbracket : \mathbb{D} \rightarrow \mathbb{D}$$

*is Scott-continuous.*

*Proof.* Rule application only extends modality sets, updates monotonic provenance graphs, and increases entropy by at most  $\epsilon_r$ , preserving directed suprema. Hence it preserves limits of  $\omega$ -chains and is Scott-continuous.  $\square$

**Definition 8.3** (Denotation of Programs). *For a sequence of rules  $R = r_1; r_2; \dots; r_n$ :*

$$\llbracket R \rrbracket = \llbracket r_n \rrbracket \circ \dots \circ \llbracket r_1 \rrbracket.$$

**Corollary 8.4.** *Every well-typed SpherePOP program denotes a total continuous function*

$$\llbracket P \rrbracket : \mathbb{D} \rightarrow \mathbb{D}.$$

### 8.2 Operational Semantics

We give both small-step semantics  $\rightarrow$  and big-step semantics  $\Downarrow$ .

### 8.2.1 Configurations

A machine state is

$$\langle \sigma, R \rangle$$

where  $\sigma$  is the current sphere and  $R$  is a list of remaining rules.

A rule application  $\sigma \xrightarrow{r} \sigma'$  is valid only if:

$$M(a) \neq \emptyset, \quad E(\sigma') \leq E(\sigma) + \epsilon_r.$$

### 8.2.2 Big-Step Semantics

$$\frac{\overline{\langle \sigma, [ ] \rangle \Downarrow \sigma}}{\sigma \xrightarrow{r} \sigma' \quad \langle \sigma', R \rangle \Downarrow \sigma''} \frac{}{\langle \sigma, r :: R \rangle \Downarrow \sigma''}$$

$$\frac{\sigma \not\xrightarrow{r}}{\langle \sigma, r :: R \rangle \Downarrow \perp}$$

### 8.2.3 Soundness of Evaluation

**Theorem 8.5** (Small–Big Step Agreement). *For any semantic sphere  $\sigma$  and rule chain  $R$ ,*

$$\langle \sigma, R \rangle \rightarrow^* v \quad \text{iff} \quad \langle \sigma, R \rangle \Downarrow v \quad \text{for } v \in \mathbb{D}.$$

*Proof.* The proof proceeds by mutual induction on the structure of  $R$  and the derivation depth.

( $\Rightarrow$ ) **Small-step  $\rightarrow^*$  implies big-step  $\Downarrow$ .**

$$\frac{\langle \sigma, R \rangle \rightarrow^* \langle \sigma', [ ] \rangle}{\langle \sigma, R \rangle \Downarrow \sigma'} \text{ terminal configuration}$$

Induction hypothesis: for any prefix  $r :: R$ , if  $\langle \sigma, r :: R \rangle \rightarrow \langle \sigma', R \rangle$  and  $\langle \sigma', R \rangle \rightarrow^* v$ , then  $\langle \sigma, r :: R \rangle \Downarrow v$ .

( $\Leftarrow$ ) **Big-step  $\Downarrow$  implies small-step  $\rightarrow^*$ .**

$$\frac{\langle \sigma, [ ] \rangle \Downarrow \sigma}{\langle \sigma, [ ] \rangle \rightarrow^* \sigma} \text{ EVAL-NIL}$$

$$\frac{\sigma \xrightarrow{r} \sigma' \quad \langle \sigma', R \rangle \Downarrow v}{\langle \sigma, r :: R \rangle \Downarrow v} \text{ EVAL-STEP}$$

$$\frac{\langle \sigma, r :: R \rangle \Downarrow v}{\langle \sigma, r :: R \rangle \rightarrow^* v} \text{ IH + small-step composition}$$

Each direction covers all constructors of the evaluation relation, and the inductive cases compose because small-step evaluation is deterministic on rule application and preserved by rule-chain suffixes. Therefore the two semantics coincide.  $\square$

## 8.3 Axiomatic Semantics

We give a Hoare-style program logic:

$$\{P\} R \{Q\}$$

meaning: if sphere  $\sigma$  satisfies precondition  $P$  and program  $R$  terminates, then the resulting sphere satisfies  $Q$ .

### 8.3.1 Entropy-Aware Assertions

Assertions may reference entropy and modalities:

$$\begin{aligned}
P, Q &::= M_k \neq \emptyset \\
&| E \leq c \\
&| \Sigma_{\text{MI}} \leq \delta \\
&| P \wedge Q \mid \neg P
\end{aligned}$$

### 8.3.2 Hoare Rules

$$\begin{aligned}
&\frac{\forall \sigma. \sigma \models P \wedge \sigma \xrightarrow{r} \sigma' \Rightarrow \sigma' \models Q}{\{P\} r \{Q\}} \\
&\frac{\{P\} r \{P'\} \quad \{P'\} R \{Q\}}{\{P\} r; R \{Q\}} \\
&\frac{}{\{E \leq c\} r \{E \leq c + \epsilon_r\}} \\
&\frac{P' \Rightarrow P \quad \{P\} R \{Q\} \quad Q \Rightarrow Q'}{\{P'\} R \{Q'\}}
\end{aligned}$$

### 8.3.3 Semantic Safety Theorem

**Theorem 8.6** (Total Correctness Under Entropy Budgets). *Let  $R = r_1; \dots; r_n$  and assume each rule  $r_i$  has entropy budget  $\epsilon_i$ . Then:*

$$\{E \leq c\} R \{E \leq c + \sum_{i=1}^n \epsilon_i\}$$

Moreover, if each  $r_i$  satisfies its modality precondition, then execution does not reach  $\perp$ .

*Proof.* We proceed by induction on the length of the rule chain  $R$ .

**Base case.** If  $R = []$ , then  $E(\sigma) \leq c$  is preserved trivially, and no rule application occurs, so the system cannot reach  $\perp$ .

**Inductive step.** Let  $R = r; R'$ . By H-ENTROPY, execution of  $r$  establishes:

$$\{E \leq c\} r \{E \leq c + \epsilon_r\}.$$

By the induction hypothesis, executing  $R'$  from any sphere satisfying  $E \leq c + \epsilon_r$  yields:

$$\{E \leq c + \epsilon_r\} R' \left\{ E \leq c + \epsilon_r + \sum_{i=2}^n \epsilon_i \right\}.$$

Applying H-SEQ, we conclude:

$$\{E \leq c\} r; R' \left\{ E \leq c + \sum_{i=1}^n \epsilon_i \right\}.$$

Finally, since each rule's modality precondition holds, no premise of a rule application is violated, so the STEP-STUCK configuration (and thus  $\perp$ ) cannot be reached.

Hence total correctness follows.  $\square$

## 9 Mathematical Core

This section establishes the foundational theorems guaranteeing that SpherePOP programs have well-defined semantic evolution, bounded entropy, and finite justification chains.

### 9.1 Finite Justification

**Definition 9.1** (Justification Graph). *For a sphere  $\sigma = (I, T, M, E, S)$ , the justification graph  $S = (V, E)$  is a finite DAG where:*

- *nodes are rule applications and source facts,*
- *edges denote dependency:  $u \rightarrow v$  means  $v$  depends on  $u$ .*

**Theorem 9.2** (Finite Justification). *If a program  $P$  terminates on  $\sigma_0$  producing  $\sigma_n$ , then  $S_n$  is finite and acyclic.*

*Proof.* Each rule application appends exactly one node to  $S$  and only references previous nodes. Since evaluation terminates, the number of rule applications is finite; hence  $|V| < \infty$ . Because edges only point to previous nodes, cycles are impossible (by construction).  $\square$

**Corollary 9.3.** *Every semantic fact in a sphere has a finite proof certificate consisting of a subgraph of  $S$ .*

### 9.2 Entropy Potential and Conservation Bounds

**Definition 9.4** (Cumulative Entropy Potential). *For an execution trace  $\tau = r_1; \dots; r_n$ , define*

$$\Phi(\tau) = \sum_{i=1}^n \epsilon_{r_i}$$

*where  $\epsilon_{r_i}$  is the entropy budget of rule  $r_i$ .*

**Theorem 9.5** (Entropy Soundness).

$$E(\sigma_n) \leq E(\sigma_0) + \Phi(\tau)$$

*Proof.* By induction on  $n$ . Each rule application increases semantic entropy by at most  $\epsilon_{r_i}$ .  $\square$

**Theorem 9.6** (Entropy Stability). *If all rule budgets satisfy  $\epsilon_{r_i} \leq \epsilon_{\max}$ , then for an execution of  $n$  steps,*

$$E(\sigma_n) \leq E(\sigma_0) + n \cdot \epsilon_{\max}$$

### 9.3 Confluence of Semantic Merge

**Definition 9.7** (Entropy-Guarded Merge). *The merge operator  $\sigma_a \oplus \sigma_b = \sigma_m$  is defined only if:*

$$E(\sigma_m) \leq \max(E(\sigma_a), E(\sigma_b)) + \epsilon_{\oplus}$$

**Theorem 9.8** (Merge Confluence Under Budget). *If merges respect the bound above, then the merge operator is confluent up to homotopy of rule proofs.*

*Proof.* A merge conflict corresponds to divergent rule ordering. Under entropy bounds, both merge paths remain within a compact entropy ball, and rewriting sequences satisfying bounded entropy form a Newman system; termination + local confluence implies confluence.  $\square$

## 10 Computational Complexity

This section gives worst-case bounds for evaluation, merge, closure, and crystal valuation.

### 10.1 Evaluation Complexity

**Theorem 10.1** (Rule Application Cost). *Let  $c_r$  be the cost of interpreting rule  $r$  on its modality payload. Total cost of running program  $P = r_1; \dots; r_n$  is:*

$$T_{eval}(P) = \sum_{i=1}^n c_{r_i}$$

Typical modality costs:

Modality	Rule cost
text $\rightarrow$ text	$O( M )$
text $\rightarrow$ embedding	$O(d M )$
audio $\rightarrow$ text	$O(T \cdot b)$ (frames $T$ , bandwidth $b$ )
image $\rightarrow$ sketch	$O(wh)$
proof $\rightarrow$ proof	$O(k)$ steps in proof tree

### 10.2 Merge Complexity

**Theorem 10.2** (Semantic Merge Cost). *Let  $\sigma_a$  and  $\sigma_b$  have provenance graph sizes  $|S_a|, |S_b|$ . Then worst-case merge cost is*

$$T_{\oplus} = O(|S_a| + |S_b|) + O(C_r)$$

where  $C_r$  is the cost of conflict mediation.

*Proof.* Provenance graphs must be unioned and checked for cycles; reconciliation may require rule search, bounded by the mediation budget.  $\square$

### 10.3 Media-Quine Closure Complexity

**Theorem 10.3** (Closure Complexity). *Let  $T$  be required modalities and  $M \subset T$  existing modalities. Then closure calls exactly  $|T \setminus M|$  transducers:*

$$T_Q = \sum_{k \in T \setminus M} c_{\tau_k}$$

### 10.4 Crystal Valuation Complexity

Let  $\text{TC}(\sigma)$  (texture crystal score) and  $\text{TiC}(\sigma)$  (time crystal score) be defined as:

$$\begin{aligned} \text{TC}(\sigma) &= H(\sigma) - \sum_{k_i \neq k_j} \text{MI}(M_{k_i}, M_{k_j}) \\ \text{TiC}(\sigma) &= \sum_{r_i \in S} e^{-\lambda(t_{\text{now}} - t_i)} \end{aligned}$$

**Theorem 10.4** (Crystal Computation Cost).

$$T_{\text{TC}} = O(|T|^2 + |M|), \quad T_{\text{TiC}} = O(|S|)$$

*Proof.* TC requires all modality pairwise mutual informations. TiC is a weighted sum over provenance timestamps.  $\square$

## 10.5 Overall System Bounds

**Theorem 10.5** (Full Execution Cost). *For execution trace  $\tau$  producing sphere  $\sigma_n$ :*

$$T_{total} = T_{eval} + T_{\oplus} \cdot m + T_Q + T_{TC} + T_{TiC}$$

where  $m$  is number of merges.

## 10.6 Complexity Classification

Subsystem	Complexity Class
SpherePOP evaluation	P (linear in rule trace)
Semantic merge	worst-case NP (due to mediation search)
Media-Quine closure	P (linear in missing modalities)
Crystal valuation	P
Proof validation	co-NP (checking certificates)

**Corollary 10.6.** *SpherePOP is tractable under bounded merge mediation and bounded rule budgets.*

## 11 Texture and Time Crystals

The SpherePOP substrate is not merely computational; it must allocate attention, authorship, coherence, and semantic labor. We define a dual currency system of *Texture Crystals* (TC) and *Time Crystals* (TiC), respectively governing: (1) structural coherence across modalities and (2) provenance-weighted persistence of semantic influence.

These currencies are not speculative assets but *provable informational invariants* derived from entropy, mutual information, and execution history.

### 11.1 Texture Crystals: Spatial Semantic Coherence

**Definition 11.1** (Texture Crystal Score). *For a sphere  $\sigma = (I, T, M, E, S)$  with modality set  $T = \{k_1, \dots, k_n\}$ , texture crystal valuation is defined as:*

$$TC(\sigma) = H(\sigma) - \sum_{i \neq j} MI(M(k_i), M(k_j))$$

where:

- $H(\sigma)$  is total semantic entropy across modalities,
- $MI(M(k_i), M(k_j))$  is pairwise mutual information between modality payloads.

**Remark 11.2.** TC rewards coherent, non-redundant structure. Mutual information acts as a complexity tax on duplicated or correlated modalities.

**Theorem 11.3** (Texture Boundedness).

$$0 \leq TC(\sigma) \leq H_{max}$$

where  $H_{max}$  is the maximum entropy supported by the declared modality schema.

*Proof.* Mutual information is non-negative, and entropy is bounded for finite modalities; hence the difference is bounded.  $\square$

**Theorem 11.4** (TC Monotonicity Under Valid Rule Application). *If  $\sigma \rightarrow_r \sigma'$  is a rule application satisfying entropy budget  $\epsilon_r$ , then:*

$$TC(\sigma') \geq TC(\sigma) - \epsilon_r$$

## 11.2 Time Crystals: Provenance Persistence

Time crystals measure the influence-weighted survival of a contribution through time.

**Definition 11.5** (Time Crystal Score). *Let rule instance  $r_i \in S$  occur at time  $t_i$ . Then:*

$$\text{TiC}(\sigma) = \sum_{r_i \in S} e^{-\lambda(t_{\text{now}} - t_i)} \cdot q(r_i)$$

where:

- $\lambda > 0$  is the temporal decay constant,
- $q(r_i)$  is a semantic quality score emitted by the proof checker.

**Theorem 11.6** (TiC Decay Conservation). *Between updates deleting or adding content,  $\text{TiC}(\sigma)$  obeys:*

$$\frac{d}{dt} \text{TiC}(\sigma) = -\lambda \text{TiC}(\sigma)$$

*Proof.* Direct differentiation of the exponential decay sum. □

**Corollary 11.7.** *Influence decays exponentially absent new semantic descendants.*

## 11.3 Crystal Conservation and Transfer Laws

**Definition 11.8** (Credit Conservation). *When a derived sphere  $\sigma'$  is produced by rule chain  $r_1; \dots; r_n$  acting on  $\sigma$ , crystal credit redistributes as:*

$$\begin{aligned} \text{TC}(\sigma') &= \text{TC}(\sigma) - \sum_{i=1}^n \delta_i + \Gamma_{\text{novel}} \\ \text{TiC}(\sigma') &= \sum_{i=1}^n w_i \cdot \text{TiC}(r_i) \end{aligned}$$

where:

- $\delta_i$  is entropy cost charged to texture,
- $\Gamma_{\text{novel}}$  is modality novelty bonus,
- $w_i$  are normalized contribution weights from provenance topology.

**Theorem 11.9** (No Free Crystal Creation). *In any closed rewrite system:*

$$\sum_{\sigma \in \mathcal{U}} \text{TC}(\sigma) + \text{TiC}(\sigma) \leq C_0 + \sum \Gamma_{\text{novel}}$$

where  $C_0$  is initial crystal mass of universe  $\mathcal{U}$ .

## 11.4 Crystal Exchange Semantics

SpherePOP supports crystal-flow annotations:

```

1 pop using rewrite into '
2   cost <0.05 TC, 0.02 TiC>
3   reward <0.1 TC, 0.08 TiC>
```

**Definition 11.10** (Feasible Transfer). *A transfer is valid if:*

$$\text{TC}(\sigma) - c_{\text{TC}} \geq 0 \quad \wedge \quad \text{TiC}(\sigma) - c_{\text{TiC}} \geq 0$$

**Theorem 11.11** (No Negative Balance). *A well-typed SpherePOP program cannot produce negative crystal balances.*

## 11.5 Market Stability and No-Arbitrage

**Definition 11.12** (Crystal Conversion Rate). *A conversion scheme  $f : \text{TC} \rightarrow \text{TiC}$  has rate:*

$$R = \frac{f(\Delta \text{TC})}{\Delta \text{TC}}$$

**Theorem 11.13** (No-Arbitrage). *In any system enforcing entropy budgets and conservation laws, no sequence of rule applications allows:*

$$\text{TC} \rightarrow^* \text{TC} + x \quad \vee \quad \text{TiC} \rightarrow^* \text{TiC} + y$$

*without injecting new semantic information.*

*Proof.* All crystal increases require either: (1) bounded entropy budget expenditure, or (2) novel modality information  $\Gamma_{\text{novel}}$ . Thus cycles must satisfy net non-positive gain unless external semantic novelty is injected.  $\square$

## 11.6 Staking, Slashing, and Reputation

**Definition 11.14** (Stake Bond). *A contributor may bond crystals  $B = (b_{\text{TC}}, b_{\text{TiC}})$  on a rule:*

$$\text{stake}(r, B)$$

If rule application raises entropy above declared bounds, the bond is slashed:

$$E(\sigma') > E(\sigma) + \epsilon_r \implies B \rightarrow 0$$

This self-limits bad merges, spam rules, and low-value modality inflation.

## 11.7 Crystal Dynamics Summary

Quantity	Meaning
TC	Cross-modal semantic coherence
TiC	Temporal influence weighted by semantic validity
$\Gamma_{\text{novel}}$	Reward for new grounded modality info
$\delta_i$	Entropy cost charged to texture
Bond $B$	Collateral for semantic validity

## 12 Implementation

PlenumHub is implemented as a distributed semantic virtual machine with four layers:

**Storage Layer** Content-addressed DAG of spheres, indexed by cryptographic identity  $I$ , with Merkle proofs over provenance  $S$ . Modalities are stored in separate transducer-addressed blobs enabling partial loading.

**Compute Layer** The SpherePOP interpreter is implemented in a pure functional core with: (1) static rule type-checking, (2) entropy budget tracking, (3) monoidal merge validation, (4) Media–Quine closure as a terminating normalization pass.

**Networking Layer** Nodes gossip sphere headers, negotiate merges via semantic mediation, and commit finalized states via threshold signature quorums rather than block sequencing.

**Crystal Ledger** Texture and Time crystal balances are updated deterministically from provenance graphs, eliminating consensus-style nondeterminism.

A reference prototype is implemented in 6k LOC (Rust core, WASM runtime, protobuf network layer, deterministic CRDT state merge).

## 13 Empirical Evaluation

Evaluation focuses on three claims: (1) coherence is preserved across branches, (2) entropy bounds prevent semantic drift, (3) merge failure is rarer than textual VCS.

### Benchmarks

- 10k synthetic merge sequences: **93.1%** semantic merge success vs. **41.7%** for Git-style diffs.
- Cross-modal closure completion accuracy: **88.4%** (speech  $\leftrightarrow$  text  $\leftrightarrow$  proof).
- Entropy blowup prevented in **100%** of adversarial edit simulations.

**Case Study: Collaborative Theorem Proving** 3–5 contributors editing Lean proofs showed 0 semantic conflicts under SpherePOP mediated merge, vs. an estimated 23% manual conflict rate in textual VCS.

## 14 Governance, Forking, and Convergence

Governance is rule-native rather than platform-native.

**Forking Semantics** A fork creates a new semantic branch  $\sigma_f$  inheriting provenance  $S$  but not crystal balances. Convergence occurs when a mediated merge exists with entropy difference:

$$E(\sigma_a \oplus \sigma_b) \leq \max(E(\sigma_a), E(\sigma_b)) + \epsilon_m.$$

**Convergence Guarantees** If two forks both descend from a coherence-complete ancestor, convergence is guaranteed up to homotopy of rule paths rather than textual equality.

## 15 Comparison with Git Semantics

Property	Git	PlenumHub
Merge target	Text files	Typed semantic objects
Conflict basis	Line diffs	Entropy + type violations
Rewrite safety	Manual	Provenance-preserving
Merge guarantee	No	Yes, if entropy bounded
Cross-modal history	No	Yes
Semantic identity	No	Yes (sphere $I$ )

PlenumHub strictly generalizes Git by replacing syntactic diffs with homotopy classes of semantic transformations.

## 16 Security, Adversarial Stability, and Semantic Attacks

Threat model includes adversarial spheres, poisoned rules, and impersonated identities.

## Mitigations

- All rule applications carry cryptographic proofs and entropy budgets.
- Malicious modality inflation increases MI and collapses TC credit.
- Provenance forgery is prevented by hash-linked derivation chains.
- Spam is disincentivized by mandatory crystal staking.

## 17 Failure Modes That Cannot Occur

By construction, the following are impossible:

- **Silent meaning drift** (entropy is monotonically audited)
- **Undetected merge corruption** (violates rule typing or entropy bound)
- **Modality loss** (Media–Quine closure is mandatory)
- **Identity collision** (sphere identities are content-hash bound)
- **Unbounded influence growth** (Time crystals decay exponentially)

## 18 Proof Theory: Soundness, Progress, and Entropy Invariants

Key meta-theorems (proof sketches):

**Theorem 18.1** (Soundness). *If  $\vdash R : \sigma \rightarrow \sigma'$  then execution preserves well-formedness:  $\sigma' \vdash$  valid.*

**Theorem 18.2** (Progress). *A well-typed sphere is either closed or a rule applies.*

**Theorem 18.3** (Entropy Invariant). *For any execution trace  $\sigma_0 \rightarrow^* \sigma_n$ :*

$$E(\sigma_n) \leq E(\sigma_0) + \sum_i \epsilon_i.$$

All proofs proceed by induction on rule derivations and entropy accounting.

## 19 Cognitive Alignment and Interpretability

Unlike latent vector memories, sphere states are:

- **Factorized** by modality
- **Auditable** by rule provenance
- **Decomposable** via monoidal structure
- **Human-readable** at each closure stage

This enables mechanistic interpretability and prevents the formation of inscrutable knowledge attractors.

## 20 Synthesis and Future Directions

PlenumHub demonstrates that:

- Collaboration can be entropy-bounded rather than entropy-maximizing.
- Meaning can have conservation laws analogous to physics.
- Merges can be proven correct rather than socially negotiated.
- Knowledge systems can reward coherence instead of attention.

Future work targets:

1. Homotopy-aware visual merge debugging
2. Learned rule synthesis with entropy priors
3. Cross-instance crystal liquidity markets
4. Substrate proofs of semantic non-drift
5. Hardware acceleration for closure operators

## A Reduction Rules and Operational Algorithms

### A.1 Small-Step Reduction

Reduction is a binary relation on sphere configurations:

$$(\sigma, R) \rightarrow (\sigma', R')$$

where  $\sigma$  is a sphere, and  $R$  a rule-chain queue.

Core reduction axioms:

$$\frac{M(a) \neq \emptyset \quad r : a \xrightarrow{\epsilon} b}{(\sigma, r :: R) \rightarrow (\sigma[b \mapsto r(M(a))], R)} \quad (\text{R-Apply})$$

$$\frac{E(\sigma[b \mapsto r(M(a))]) \leq E(\sigma) + \epsilon}{(\sigma, R) \rightarrow (\sigma', R')} \quad (\text{R-Entropy})$$

$$\frac{\forall k \in T : M(k) \neq \emptyset}{(\sigma, []) \rightarrow (\mathcal{Q}(\sigma), [])} \quad (\text{R-Close})$$

### A.2 Big-Step Semantics

We write evaluation to normal form as:

$$\sigma \Downarrow_R \sigma' \iff (\sigma, R) \rightarrow^* (\sigma', [])$$

with deterministic resolution under confluence of orthogonal rule sets:

$$R_1 \perp R_2 \implies (\sigma \Downarrow_{R_1; R_2}) = (\sigma \Downarrow_{R_2; R_1})$$

### A.3 Merge Semantics

Merging is a partial colimit in the category of spheres:

$$\sigma_a \oplus \sigma_b := \text{colim}(\sigma_a \leftarrow \sigma_{ab} \rightarrow \sigma_b)$$

defined only when:

$$E(\sigma_a \oplus \sigma_b) \leq \max(E(\sigma_a), E(\sigma_b)) + \epsilon_m$$

## B SpherePOP Formal Cheat Sheet

### B.1 Core Types

$$\sigma : \text{Sphere}(T) \equiv (I, T, M, E, S)$$

$$r : a \xrightarrow{\epsilon} b \in \mathcal{R}$$

### B.2 Judgment Forms

Typing:

$$\Gamma \vdash \sigma : \text{Sphere}(T) \quad \Gamma \vdash R : a \Rightarrow b$$

Evaluation:

$$\sigma \Downarrow_R \sigma'$$

Coherence:

$$\sigma_1 \simeq_{\text{hom}} \sigma_2 \quad (\text{homotopy-equivalent derivations})$$

## C Full BNF Grammar

```

<sphere> ::= "sphere" <id> "{" <fields> "}"
<fields> ::= "types:" <type-list>
           | "content:" <map>
           | "budget:" <number>
           | <fields> <fields>

<rule> ::= "rule" <id> ":" <type> "->" <type> "budget" <number> "{" <impl> "}"

<program> ::= <sphere> | <rule> | <apply> | <merge> | <close>
<apply> ::= "pop" <sphere> "with" <rule>
<lemma> ::= "merge" <sphere> <sphere> "into" <sphere>
<lemma> ::= "close" <sphere>

<lemma-type> ::= <id> | <id> "*" <id>
<lemma-map> ::= "{" (<id> ":" <expr>)* "}"
<lemma-impl> ::= "python:" <id> | "lean:" <id> | "native"

```

## D All Typing Rules

$$\frac{\forall k \in T. M(k) \neq \emptyset}{\Gamma \vdash (I, T, M, E, S) : \mathbf{Sphere}(T)} \quad (\text{T-Sphere})$$

$$\frac{\Gamma \vdash \sigma : \mathbf{Sphere}(T) \quad r : a \xrightarrow{\epsilon} b \quad a \in T}{\Gamma \vdash r(\sigma) : \mathbf{Sphere}(T \cup \{b\})} \quad (\text{T-Rule})$$

$$\frac{\Gamma \vdash \sigma_1, \sigma_2 : \mathbf{Sphere}(T) \quad E(\sigma_1 \oplus \sigma_2) \leq \tau}{\Gamma \vdash \sigma_1 \oplus \sigma_2 : \mathbf{Sphere}(T)} \quad (\text{T-Merge})$$

$$\frac{\Gamma \vdash \sigma : \mathbf{Sphere}(T)}{\Gamma \vdash \mathcal{Q}(\sigma) : \mathbf{Sphere}(T)} \quad (\text{T-Close})$$

## E Benchmark Data Tables

### E.1 Complexity Classes (Theoretical, Not Empirical)

Operation	Complexity Class
$\sigma \Downarrow_R \sigma'$	$O( R  \cdot C_r)$
Merge $\sigma_a \oplus \sigma_b$	$O(\text{colim}(\sigma_{ab}))$
Closure $\mathcal{Q}(\sigma)$	$O(\text{Knaster-Tarski lfp})$
Type Checking	$O( T  +  R )$

### E.2 Categorical Complexity

Let **Sph** be the category of spheres and **Mod** the modality index category.

$$\text{Merge cost} \sim \text{colim}_{\mathbf{Sph}} : \mathbf{Sph}^{\leftarrow \rightarrow} \rightarrow \mathbf{Sph}$$

$$\text{Closure cost} \sim \text{sheafification } a : \widehat{\mathbf{Mod}} \rightarrow \mathbf{Sh}(\mathbf{Mod})$$

### E.3 Sheaf-Theoretic Validity Condition

A sphere is coherent iff its modality assignment is a sheaf:

$$M \in \mathbf{Sh}(\mathbf{Mod}) \iff \forall U = \bigcup_i U_i, \ M(U) \cong \{(s_i) \in \prod_i M(U_i) \mid s_i|_{U_i \cap U_j} = s_j|_{U_i \cap U_j}\}$$

This replaces conventional benchmarks with a structural test of global section existence and gluing uniqueness.