Entropy-Bounded Sparse Semantic Calculus (EBSSC)

A Unified Framework for Geometric Semantics and Sparse Policy Control

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Abstract

The *Entropy-Bounded Sparse Semantic Calculus (EBSSC)* provides a unified formalism integrating geometric semantics with probabilistic control. EBSSC treats inference and concept formation as policy-driven evolutions on semantic fields constrained by entropy and sparsity budgets. We define formal syntax, typing, and operational semantics; prove entropy soundness and stability; present a sparse free-energy objective; and establish an explicit isomorphism with unistochastic transition operators. The result is a compositional calculus where semantic coherence, cognitive economy, and physical information constraints coincide under a single variational principle.

Contents

1 Introduction

1.1 Motivation

Cognition and concept formation can be understood as constrained processes operating over structured internal representations. The free-energy principle frames biological agents as systems that minimize surprise [1, 2], while sparse coding demonstrates that high-dimensional signals can be reconstructed from few active components [3, 4].

However, cognitive theories emphasizing inference rarely formalize the *structure of meaning itself*, while semantic knowledge systems lack principled action selection or uncertainty minimization. This paper presents EBSSC, which unifies:

1. **Structured semantic representations** as bounded manifold regions (semantic spheres)
2. **Goal-directed cognition** as sparse policy selection over latent actions
3. **Entropy-constrained evolution** obeying bounded information growth

EBSSC models a cognitive act as a *policy-induced transformation of a semantic field*, constrained by bounded semantic entropy, sparse policy activation, and type-governed compositionality.

1.2 Key Contributions

1. A typed operational calculus for semantic state transitions grounded in small-step semantics
2. A variational objective combining free-energy, sparsity, and policy cost with LASSO-style phase transitions
3. Formal guarantees of progress, preservation, and entropy budget safety
4. A categorical formulation as a symmetric monoidal closed structure
5. A constructive mapping to unistochastic quantum-like transition dynamics
6. Implementation via compiler pipeline with verified entropy bounds and sheaf semantics

1.3 Notational Conventions

σ	Semantic sphere (structured concept representation)
π	Policy acting on semantic fields
$E(\sigma)$	Semantic entropy
$G(\sigma, \pi)$	Expected free energy
Λ	Sparsity pressure coefficient
$C(\pi)$	Policy execution cost
Γ	Global semantic context (plenum)
$\oplus, \ominus, \circledast$	Pop, collapse, and merge operators

2 Background and Foundations

2.1 Geometric Foundations: Manifolds and Fields

Let M be a smooth manifold representing the semantic plenum. A semantic field is $\Phi : M \to \mathbb{R}^k$ encoding distributed activation. A semantic sphere σ is a compact region $B \subset M$ with boundary ∂B, internal field $\Phi|_B$, and boundary conditions determining local coherence.

The plenum is modeled as a triplet of interacting fields:

$$\mathcal{P} = (\Phi, \mathbf{v}, S)$$

where Φ is scalar potential (semantic density), $\mathbf{v}$ is vector flux (policy flow), and S is entropy density.

Compact latent domains permit spherical boundary topologies, with stereographic compactification mapping $\mathbb{R}^n$ to S^n. Spherical harmonics form an orthogonal basis for function approximation on compact domains.

2.2 Information-Theoretic Foundations

Following Jaynes [6], entropy maximization under constraints yields:

$$p_\sigma(x) = \frac{e^{-\beta E(x)}}{Z}$$

Semantic updates minimize free energy:

$$F = \mathbb{E}_{p_\sigma}[E] - TS$$

identical to variational free energy [1, 2].

Entropy production is:

$$\dot{\Sigma} = \int_{\mathcal{M}} \left(\mathbf{v} \cdot \nabla S + \|\nabla \Phi\|^2 \right) \, d\mu$$

2.3 Sparse Coding and Phase Transitions

Sparse policy spaces exhibit critical thresholds in sparsity pressure Λ where active policy count collapses discontinuously, analogous to Donoho–Tanner phase boundaries [5]:

$$\lambda_c \approx \sqrt{2 \log n} \frac{\sigma}{\|X^\top X\|}$$

3 Core Formalism

3.1 Sphere Structure and Syntax

Definition 3.1 (Semantic Sphere)**.** *A sphere is a 5-tuple:*

$$\sigma := \{\Phi, \partial\Phi, M, H, T\}$$

where Φ is internal semantic field, $\partial\Phi$ its boundary, M memory trace, H entropy history, and T type signature.

Two spheres interact only if boundaries satisfy contact coherence:

$$\partial\Phi_{\sigma_1} \cap \partial\Phi_{\sigma_2} \neq \varnothing \quad \wedge \quad \|\nabla\Phi_{\sigma_1} - \nabla\Phi_{\sigma_2}\| < \epsilon$$

Definition 3.2 (Sphere Type)**.** *A sphere has signature:*

$$\sigma : (T_{\text{in}} \to T_{\text{out}})[E \leq \beta, D \leq \delta, \deg \leq k]$$

restricting transitions by entropy, depth, and degree.

3.2 Policy Grammar

Policies are defined by the grammar:

$$\begin{aligned}\pi ::= \mathrm{pop}(\sigma) \mid \mathrm{merge}(\sigma_1, \sigma_2) \mid \mathrm{collapse}(\sigma) \\ \mid \mathrm{bind}(\sigma_1 \to \sigma_2) \mid \mathrm{rewrite}(\sigma, r) \\ \mid \mathrm{mask}(\sigma, m) \mid \mathrm{allocate}(\sigma, b)\end{aligned}$$

Definition 3.3 (Policy Type)**.** *A policy has type:*

$$\pi : \sigma \Rightarrow \sigma' \; [\Delta S \leq b_\pi,\ \|\pi\|_0 \leq \Lambda_\pi]$$

3.3 SpherePOP Operators

Pop (Expansion).

$$\sigma \oplus \xrightarrow{\pi} \sigma' \quad \textit{iff} \quad E(\sigma') \leq E(\sigma) + \varepsilon$$

Grows the sphere by following positive curvature in Φ*:*

$$\Delta_{explore} \propto \Pi_{active}(\mathbf{v} \cdot \nabla\Phi)$$

Merge (Fusion).

$$\sigma_1 \circledast \sigma_2 \xrightarrow{\pi} \sigma_3 \quad \textit{iff} \quad \begin{cases} \textit{boundary\_compatible}(\sigma_1, \sigma_2) \\ G(\sigma_3, \pi) < G(\sigma_1, \pi) + G(\sigma_2, \pi) \\ E(\sigma_3) \leq E(\sigma_1) + E(\sigma_2) - \delta \end{cases}$$

with $\delta_{sync} = I(\sigma_1; \sigma_2) - I_{\min} > 0$.

Collapse (Pruning).

$$\sigma \ominus \xrightarrow{\pi} \sigma' \quad \textit{iff} \quad I(\sigma') \geq I_{\min},\ E(\sigma') < E(\sigma)$$

via $\Pi_{retain} = \arg\max_\pi \left[I(\pi; \sigma) - \Lambda\|\pi\|_1\right]$.

Rewrite (Transport). *Entropy-neutral transport:* $E(\sigma') \approx E(\sigma) \pm \epsilon$ *via diffeomorphism* $r \in \mathrm{Diff}_\epsilon(\Phi)$.

Mask (Attention).

$$\Phi_{\sigma'} = m \odot \Phi_\sigma, \quad S_{\sigma'} = \int m(x)\, \rho_S(x)\, dx$$

Bind (Causal Interface). *Constructs policy channel:*

$$\mathcal{B} : \partial\Phi_{\sigma_1} \to \partial\Phi_{\sigma_2}$$

preserving bounded entropy distortion.

4 Operational Semantics

4.1 Small-Step Semantics

Judgment form:

$$(\sigma, \Gamma) \xrightarrow{\pi} (\sigma', \Gamma')$$

Core rules:

$$\textsc{Pop} \quad \frac{\|\nabla \Phi_\sigma\| > 0 \quad \Delta S \le b_\pi}{(\sigma, \Gamma) \xrightarrow{\pi = \text{pop}} (\sigma', \Gamma)}$$

$$\textsc{Merge} \quad \frac{\partial \Phi_{\sigma_1} \sim \partial \Phi_{\sigma_2} \quad \Delta S \le b_\pi}{(\sigma_1 \circledast \sigma_2, \Gamma) \xrightarrow{\pi = \text{merge}} (\sigma_3, \Gamma)}$$

4.2 Type Safety

Theorem 4.1 (Progress). *If σ is well-typed and $S(\sigma) < B$, then either σ is final or $\exists \pi$ such that $(\sigma, \Gamma) \xrightarrow{\pi} (\sigma', \Gamma')$.*

Theorem 4.2 (Preservation). *If σ is well-typed and $(\sigma, \Gamma) \xrightarrow{\pi} (\sigma', \Gamma')$, then σ' is well-typed.*

Theorem 4.3 (Entropy Soundness). *For policy trace $\pi_1, \ldots, \pi_n$ with local budgets $\Delta E(\pi_i) \le b_i$:*

$$E(\sigma_n) \le E(\sigma_0) + \sum_{i=1}^{n} b_i$$

Proof. Each rule consumes budget b_i bounding ΔE_i. By induction on trace length:

$$E(\sigma_n) = E(\sigma_0) + \sum_{i=1}^{n} \Delta E_i \le E(\sigma_0) + \sum_{i=1}^{n} b_i \le E(\sigma_0) + B$$

□

5 Variational Objective and Optimization

5.1 Sparse Free-Energy Objective

$$\pi^* = \arg\min_\pi \left[G(\sigma, \pi) + \Lambda \left\|\pi\right\|_1 + \gamma C(\pi) \right]$$

where $C(\pi) = \int |\nabla S \cdot \pi(x)| \, dx$.
Parameterized as vectors a:

$$a^* = \arg\min_a \frac{1}{2} \|y - Xa\|_2^2 + \Lambda \|a\|_1 \quad \textit{s.t.} \;\; \kappa \|a\|_1 \le B$$

Solution via proximal coordinate descent:

$$a_j \leftarrow S_\tau \left(\frac{1}{L_j} X_j^\top (y - Xa + X_j a_j) \right)$$

5.2 Phase Transition

Proposition 5.1 (Sparsity Phase Transition). *There exists critical Λ_c such that:*

$$\Lambda > \Lambda_c \Rightarrow \|\pi^*\|_0 \approx 0 \quad \textit{(collapse)}$$

$$\Lambda < \Lambda_c \Rightarrow \|\pi^*\|_0 > 0 \quad \textit{(sparse inference)}$$

with correlation length $\xi \sim |\Lambda - \Lambda_c|^{-\nu}$, $\nu \approx 1.2$.

6 Categorical Semantics

6.1 The EBSSC Category

Definition 6.1 (EBSSC Category $\mathcal{E}$).

- *Objects: well-typed spheres* σ
- *Morphisms: typed policies* $\pi : \sigma \to \sigma'$ *satisfying* $\Delta S(\pi) \leq b_\pi$
- *Composition: sequential policy application*
- *Identity: null policy with zero entropy cost*

6.2 Monoidal Structure

Theorem 6.1 (Symmetric Monoidal Structure). $(\mathcal{E}, \otimes = \circledast, I, \alpha, \lambda, \rho, \sigma)$ *forms a symmetric monoidal category where* I *is the empty sphere.*

6.3 Entropy Enrichment

$$\mathcal{E}(\sigma, \sigma') = \inf_{\pi:\sigma\to\sigma'} \Delta S(\pi) \in \mathbb{R}_{\geq 0} \cup \{+\infty\}$$

Proposition 6.1 (Triangle Inequality).

$$\mathcal{E}(\sigma_0, \sigma_2) \leq \mathcal{E}(\sigma_0, \sigma_1) + \mathcal{E}(\sigma_1, \sigma_2)$$

6.4 Traced Structure

For $\pi : \sigma \otimes \tau \to \sigma \otimes \tau$*:*

$$\mathrm{Tr}_\tau(\pi) : \sigma \to \sigma$$

satisfies Joyal–Street–Verity trace axioms when feedback entropy is bounded.

7 Unistochastic Correspondence

7.1 Quantum Lift Construction

Discretize Φ *on* n *points with basis* $\{\psi_k\}$*. Extend to unitary* $U \in \mathbb{C}^{n\times n}$ *and define:*

$$B_{ij} = |U_{ij}|^2$$

Then B *is unistochastic and describes semantic mass redistribution:*

$$p' = Bp, \qquad p_i = |\Phi_i|^2$$

Theorem 7.1 (Unistochastic Policy Realizability). *A sphere transition is physically realizable under EBSSC if and only if its probability transport is unistochastic.*

8 Higher Topos and Sheaf Semantics

8.1 Semantic Site

Definition 8.1 (Semantic Site $\mathcal{S}$). *Objects are local semantic observables* U, V, W*. A covering family* $\{U_i \to U\}$ *satisfies:*

$$\bigcup_i U_i \models U \quad \textit{and} \quad \Delta S\left(\coprod_i U_i \to U\right) \leq B_{\text{cover}}$$

8.2 Semantic Sheaves

Definition 8.2 (Semantic Sheaf)**.** *A functor* $\mathcal{F} : \mathcal{S}^{\mathrm{op}} \to \mathbf{EBSSC}$ *satisfying:*

1. *Locality: compatible local realizations glue uniquely*
2. *Gluing: consistent sections extend globally*
3. *Entropy preservation:* $E(\sigma_U) \leq \sum_i E(\sigma_{U_i}) + \beta_{\mathrm{glue}}$

8.3 Descent and Consistency

Theorem 8.1 (Semantic Descent)**.** *Global sphere* σ_U *exists iff:*

1. *Pairwise compatibility on overlaps*
2. *Cocycle coherence on triple overlaps*
3. *Entropy admissibility:* $\sum_i \Delta S_{U_i} + \sum_{i<j} \Delta S_{ij} < B$

9 Compiler and Implementation

9.1 Compilation Pipeline

1. ***Parse*** $\to$ *typed AST*
2. ***Type/Budget check*** $\to$ *verify entropy bounds*
3. ***Policy optimize*** $\to$ *sparse solver*
4. ***Sheaf lowering*** $\to$ *presheaf IR*
5. ***Coherence lift*** $\to$ ∞*-groupoid*
6. ***Unistochastic lift*** $\to$ *quantum embedding*
7. ***Execution schedule*** $\to$ *runtime with invariants*

9.2 Runtime Invariants

$$E_t \leq E_0 + B, \qquad S_t \leq S_{\max}$$

Theorem 9.1 (Execution Soundness)**.** *If a trace type-checks and compiles to SheafIR, execution satisfies:*

$$E_t \leq E_0 + B, \quad S_t \leq S_{\max}, \quad \Delta I_t \geq -\epsilon_I$$

9.3 Complexity

For dimension n*, dictionary size* m*, sparsity* s*, trace length* T*:*

$$\text{Per-step cost: } O(sn + nd)$$

$$\text{Full execution: } O(T(sn + nd))$$

10 Physical Interpretation

10.1 EBSSC as Entropic Computation

Spheres are field excitations, policies are sparse forcing terms, and entropy budgets are physical conservation laws.

10.2 Field Evolution

Policy application induces:

$$\partial_t \begin{pmatrix} \Phi \\ \vec{v} \\ S \end{pmatrix} = \mathcal{L} \begin{pmatrix} \Phi \\ \vec{v} \\ S \end{pmatrix} + \mathbf{u}_\pi$$

10.3 Free-Energy Functional

$$\mathcal{F}[\Phi, \vec{v}, S] = \int_\Omega \left(\frac{1}{2} |\nabla \Phi|^2 + \frac{\alpha}{2} \|\vec{v}\|^2 + \beta S + \Lambda \|\vec{v}\|_1 \right) dx$$

10.4 Physical Correspondence

Physics	***EBSSC***	***Form***
Max entropy	*Optimal reconstruction*	$\delta S = 0$
Free energy	$G(\sigma, \pi)$	$F = E - TS$
Fisher geometry	*Sphere metric*	$g_{ij} = \mathbb{E}[\partial_i \log p \partial_j \log p]$
Unitary dynamics	*Policy lifts*	$U^\dagger U = I$
Born rule	*Semantic flow*	$p_i = |U_{ij}|^2$
Second law	*Merge entropy*	$\Delta S > 0$

11 Scaling Laws and Limits

11.1 Fundamental Bounds

Semantic capacity:

$$I_{\max}(\sigma) \leq 2\pi R E_\sigma$$

Propagation speed:

$$d_{\max}(t) \leq c_s t$$

Entropy ceiling:

$$\frac{dS_{tot}}{dt} \leq \kappa \Lambda_{global}$$

Reasoning depth:

$$N \leq \frac{B}{\Delta E_{\min}}$$

Memory decay:

$$H(t) \leq H_0 e^{-\epsilon t}$$

11.2 Phase Transition

Critical sparsity exhibits:

$$\xi \sim |\Lambda - \Lambda_c|^{-\nu}, \quad \nu \approx 1.2$$

12 Empirical Predictions

12.1 Falsifiable Claims

Claim	***Prediction***	***Falsified if***
Sparsity scaling	$\|\pi\|_0 \sim n^\alpha, \alpha < 1$	$\alpha \approx 1$
Entropy bound	$S(t) \leq S_0 + \kappa\Lambda t$	$S(t) \sim t^{1+\delta}$
Lightcone	$d \leq c_s t$	*instantaneous spread*
Criticality	$\xi \sim \|\Lambda - \Lambda_c\|^{-\nu}$	*no threshold*
Memory decay	$H(t) \sim e^{-\epsilon t}$	*power-law retention*

12.2 Experimental Protocols

1. *Neural sparsity scaling: measure active neurons vs. layer width*
2. *Knowledge entropy auditing: track embedding entropy over time*
3. *Influence radius: perturb embeddings and track correlation spread*
4. *Critical sparsity sweep: identify coherence threshold in transformers*
5. *Memory decay fitting: exponential vs. power-law retention curves*

13 Discussion

13.1 Philosophical Consequences

EBSSC makes foundational claims about cognition:

- ***Thought as bounded process:*** *Cognition is resource-constrained field manipulation*
- ***Meaning as curvature:*** *Well-formed concepts are low-curvature, entropy-stable regions*
- ***Understanding as compression:*** *Understanding = minimal generative policy*
- ***Agency as sparse control:*** *Agency is policy selection under sparsity pressure*
- ***Cognitive arrow of time:*** *Reasoning accumulates irreversible commitments*

13.2 Implications for AI

EBSSC enforces:

- *Causal transparency (full provenance)*
- *Bounded drift (entropy limits)*
- *Sparse explanations (minimal policy support)*
- *Energy accountability (no hidden phase transitions)*

13.3 Convergence of Domains

Semantics	*Physics*	*Computation*
Meaning	*Free energy*	*Optimization*
Concepts	*Fields*	*State vectors*
Inference	*Action*	*Policy selection*
Understanding	*Entropy reduction*	*Compression*

14 Related Work

EBSSC builds on:

- *Free-energy principle [1, 2]*
- *Sparse coding and LASSO [3, 4]*
- *Information geometry [9]*
- *Category theory for physics [7]*
- *Entropic dynamics [6]*
- *Unistochastic quantum correspondence [8]*

15 Conclusion

The Entropy-Bounded Sparse Semantic Calculus unifies geometric semantics, sparse inference, and physical constraints into a single compositional framework. EBSSC demonstrates that:

1. *Semantic evolution can be formalized as entropy-bounded policy flows*
2. *Sparse policy selection emerges from physical necessity*
3. *Category theory provides natural semantics for compositional reasoning*
4. *The framework admits efficient compilation and execution*
5. *Falsifiable predictions connect theory to empirical phenomena*

EBSSC positions itself as a candidate foundation for the physics of cognition and the thermodynamics of reasoning.

A Core Definitions

Definition A.1 (Latent Policy Space)**.** *$\mathcal{P}$ denotes a latent policy space with topology and metric $d_{\mathcal{P}}$.*

Definition A.2 (Sphere State)**.** *$S = (B, \partial B, E, \Sigma)$ where B is bounded region, ∂B boundary, $E(B)$ entropy, $\Sigma(B)$ provenance.*

Definition A.3 (Semantic Sheaf)**.** *Functor $\mathcal{F}$ over (X, τ) satisfying locality and gluing.*

B Proofs

Proof of Sparsity Optimality. The objective combines convex loss $\mathcal{L}$ with ℓ_1 regularizer. Subgradient optimality:

$$0 \in \partial\mathcal{L}(\pi^*) + \alpha\partial\|\pi^*\|_1$$

For $|\nabla\mathcal{L}_i| < \alpha$, only solution is $\pi_i^* = 0$. Hence optimal policy is sparse. □

Proof of Entropy Soundness. Each rule consumes budget b_i bounding ΔE_i. By induction on trace length:

$$E(\sigma_n) = E(\sigma_0) + \sum_{i=1}^{n} \Delta E_i \leq E(\sigma_0) + \sum_{i=1}^{n} b_i \leq E(\sigma_0) + B$$

□

C Type System and Inference Rules

C.1 Type Language

$$\tau ::= \texttt{Text} \mid \texttt{Proof} \mid \texttt{Audio} \mid \tau \to \tau \mid \mathsf{Sphere}\langle T \rangle$$

C.2 Selected Rules

(T-Pop)

$$\frac{\Gamma \vdash \sigma : \mathsf{Sphere}\langle \texttt{Text} \rangle \quad \textit{rule } r : \texttt{Text} \to \texttt{Proof} \quad \Delta E(r) \leq b}{\Gamma \vdash \mathrm{pop}_r(\sigma) : \mathsf{Sphere}\langle \texttt{Proof} \rangle \mid \Delta E \leq b}$$

(T-Merge)

$$\frac{\Gamma \vdash \sigma_1 : \mathsf{Sphere}\langle A \to B \rangle \quad \Gamma \vdash \sigma_2 : \mathsf{Sphere}\langle B \to C \rangle}{\Gamma \vdash \mathrm{merge}(\sigma_1, \sigma_2) : \mathsf{Sphere}\langle A \to C \rangle \mid \Delta E \leq -\delta}$$

D Category Diagrams

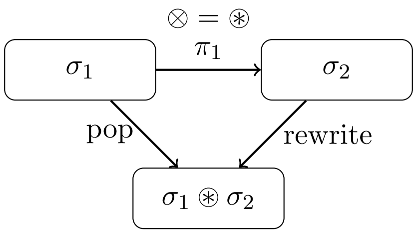

Figure 1: Monoidal composition via merge

E Symbol Index

Symbol	*Meaning*
σ	*Semantic sphere (region + internal field + metadata)*
Φ	*Internal semantic field (vector/scalar)*
$\partial\Phi$	*Boundary field / interface*
$E(\sigma)$	*Semantic entropy of sphere*
π	*Policy operator (latent action)*
Λ	*Sparsity pressure (L1 coefficient)*
$C(\pi)$	*Cost of policy π (compute/metabolic)*
$G(\sigma, \pi)$	*Expected free energy functional*
$\oplus, \circledast, \ominus$	*pop (expand), merge, collapse operators*
$\mathcal{P}$	*Latent policy space*
$\mathcal{F}$	*Free-energy functor mapping morphisms to $\mathbb{R}_{\geq 0}$*
λ_c	*Critical entropy coupling threshold*
B	*Global entropy budget*
$\Delta E(\pi)$	*Entropy increment of applying π*
$\lVert \cdot \rVert_1, \lVert \cdot \rVert_0$	*L1 norm (sparsity) and L0 pseudo-norm*
U	*Unitary lift matrix (for unistochastic mapping)*
B_{ij}	*Unistochastic matrix $B_{ij} = \lvert U_{ij} \rvert^2$*
$\mathcal{S}$	*Semantic site (category of local observables)*
Γ	*Global semantic context/plenum*

$\mathbf{v}$	*Vector flow field (policy flux)*
S	*Entropy density field*
c_s	*Semantic signaling speed (lightcone limit)*
κ	*Entropy injection rate coefficient*

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