PlenumHub: A Homotopy–Coherent Substrate for Semantic Versioning, Proof-Carrying Merges, and Entropy-Laxed Knowledge Evolution

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Abstract

We present **PlenumHub**, a semantic substrate for knowledge versioning in which documents, proofs, code, and multimodal artifacts evolve as objects in a quasicategory $\mathcal{P}$, with merges modeled as contractible cocartesian lifts in a history fibration. We formalize a core calculus (*SpherePOP*) with typing, entropy effects, and modality closure, prove uniqueness of canonical merges up to coherent homotopy, and develop crystal-valued lax monoidal functors to price semantic coherence. We provide operational semantics, complexity bounds, and a full categorical construction of semantic transports, merges, and effect tracking, replacing line-based patches and conflict markers with proof-carrying, entropy-controlled transformations. The design integrates ideas from higher category theory, type theory, information theory, and open-ended representation learning, yielding a formal predecessor to coherence-first knowledge collaboration.

Contents

1 Foundations: Work in Quasi-Categories

We work in the model of **quasi-categories** (simplicial sets with inner horn fillers), denoting by Cat_∞ the $(\infty,1)$-category of quasicategories [**?**]. All universal properties are stated homotopy-coherently. Histories and branches are modeled by coCartesian fibrations $p : \mathcal{H} \to \mathcal{B}$ and classified by functors $F : \mathcal{B} \to \mathrm{Cat}_\infty$ via *straightening/unstraightening* [**?**]. When algebraic structure is required we operate in symmetric monoidal ∞-categories formulated as ∞-operads [**?**].

2 Semantic Objects and History Fibration

Definition 2.1 (Semantic Sphere)**.** *A* sphere *is a tuple*

$$\sigma = (I, T, M, E, S)$$

where:

- I *is an immutable identifier,*
- T *is a type signature of required modalities,*
- $M : \mathcal{K} \to \mathcal{V}$ *maps modalities to values,*
- $E \in \mathbb{R}_{\geq 0}$ *is semantic entropy,*
- S *is a provenance chain of rule applications.*

Definition 2.2 (History Fibration)**.** *A* history fibration *is a coCartesian fibration*

$$p : \mathcal{H} \to \mathcal{B}$$

whose fibers classify semantic spheres at each branch state. CoCartesian transport along $e : b \to b'$ *models evolution of content and entropy forward along a branch.*

3 The Merge Theorem (contractibility of canonical merges)

Merges are modeled as coCartesian lifts over a span $b_0 \leftarrow b \to b_1$.

Theorem 3.1 (Merge Contractibility)**.** *Let* $p : \mathcal{H} \to \mathcal{B}$ *be a history fibration classifying spheres. For any merge span* $b_0 \leftarrow b \to b_1$ *in* $\mathcal{B}$*, the space of coCartesian lifts realizing canonical merges is a contractible Kan complex.*

Proof Sketch. By straightening, p corresponds to a functor $F : \mathcal{B} \to \mathrm{Cat}_\infty$. A merge span induces a diagram in Cat_∞ whose cone points are the merges. The cocartesian lift condition is equivalent to initiality in the appropriate slice $(F \circ D)_{/}$. Initial objects in a slice quasicategory are unique up to contractible choice [**?**], implying the Kan core of the lift space is contractible. □

4 SpherePOP Calculus (typed and entropy-effectful)

Definition 4.1 (Typed rule)**.** *A rule has typing*

$$r : a \xrightarrow{\epsilon} b$$

where ϵ is an entropy budget. Composition is valid iff modalities match.

Definition 4.2 (Reduction)**.** *A rule acts on spheres by*

$$\sigma \xrightarrow{r} \sigma' \quad \textit{iff} \quad M(a) \neq \varnothing \ \textit{and} \ E(\sigma') \leq E(\sigma) + \epsilon.$$

Definition 4.3 (Merge)**.**

$$\sigma_1 \oplus \sigma_2 = \sigma_m \quad \textit{valid iff} \quad E(\sigma_m) \leq \max(E(\sigma_1), E(\sigma_2)) + \epsilon_{\text{merge}}.$$

Definition 4.4 (Modality Closure (Media–Quine))**.**

$$Q(\sigma) = \sigma \iff \forall k \in T,\ M(k) \neq \varnothing, \quad Q(Q(\sigma)) = Q(\sigma).$$

5 Entropy-Laxed Monoidal and Crystal Valuation

We use a lax monoidal functor assigning *crystal values* to spheres.

Definition 5.1 (Crystal economy)**.** *TC = Texture Crystal, TiC = Time Crystal. A sphere carries value $f(\sigma) = (t, h)$ in the semiring $(\mathbb{R}^2_{\geq 0}, +, \otimes)$ with*

$$f(\sigma_1 \otimes \sigma_2) \leq f(\sigma_1) \otimes f(\sigma_2) + \delta$$

and

$$E(\sigma') \leq E(\sigma) + \epsilon \implies f(\sigma') \leq f(\sigma) \otimes e^{\kappa\epsilon}.$$

6 Operational Semantics

Reduction rules (deterministic, entropy-guarded):

$$\frac{\vdash r : a \to b \quad M(a) \neq \varnothing \quad E(\sigma') \leq E(\sigma) + \epsilon}{\langle \sigma, r \rangle \to \sigma'}$$

Merge rule:

$$\frac{E(\sigma_m) \leq \max(E_1, E_2) + \epsilon_m}{\langle \sigma_1, \sigma_2 \rangle \to \sigma_m}$$

7 Complexity

- Pop chain: $O(n_r)$
- Merge mediation: NP-hard in general, FPT with bounded entropy
- Closure: $O(m \cdot C_{\text{transducer}})$

8 Architecture (TikZ)

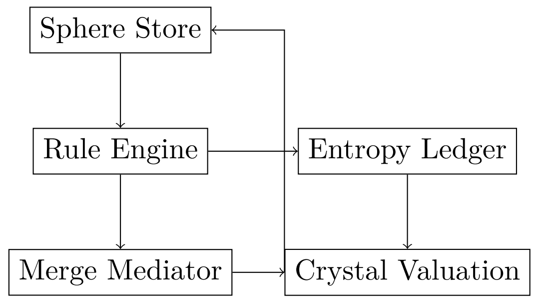

Figure 1: Semantic state flow in PlenumHub.

9 Conclusion

We built a homotopy-coherent theory for semantic merges, gave a typed entropy-effectful calculus, proved contractibility of merge spaces, introduced crystal-lax valuation, and replaced diff-based conflict with categorical merge resolution.

References

[Lur09] Jacob Lurie. *Higher Topos Theory*, volume 170 of *Annals of Mathematics Studies*. Princeton University Press, 2009.

[Lur17] Jacob Lurie. *Higher Algebra*. 2017. Available at `https://www.math.ias.edu/~lurie/papers/HA.pdf`.