RSVP Semantic Framework: Formal Proofs and Structural Foundations

A. Well-Posedness of RSVP Field Equations

Theorem A.1 (Well-Posedness of RSVP SPDE System) Let Φ_t, v■_t, S_t evolve on a Minkowski manifold M = ■ × ■³ via the SPDE system: $d\Phi_t = [\nabla\cdot(D\nabla\Phi_t)$ - v■_t$\cdot\nabla\Phi_t + \lambda S_t]dt + \sigma_\Phi\, dW_t$, dv■_t = $[-\nabla S_t + \gamma\Phi_t$ v■_t]dt + $\sigma_v\, dW'_t$, $dS_t = [\delta\nabla\cdot$v■_t $- \eta S_t^2]dt + \sigma_S\, dW''_t$, with compact support and smooth initial conditions. Then under standard Lipschitz and linear growth assumptions, the system admits a unique global strong solution in $L^2([0,T];$ H■(M)), and the energy functional $E(t) = \int_M (\frac{1}{2}|\nabla\Phi_t|^2 + \frac{1}{2}|$v■_t$|^2 + \frac{1}{2}S_t^2)\, dx$ is conserved in expectation.

B. Sheaf Gluing and Semantic Coherence

Theorem B.1 (Semantic Coherence via Sheaf Gluing) Let F be the RSVP sheaf assigning field triples (Φ, v■, S) to open sets $U \subset X$. Suppose for an open cover $\{U_i\}$, the local fields agree on overlaps: $\Phi_i|_{U_i \cap U_j} = \Phi_j|_{U_i \cap U_j}$, etc. Then there exists a unique global field triple (Φ, v■, S) over X such that: $F(X) \cong \lim F(U_i)$.

C. Merge Obstruction and Homotopy Colimit Coherence

Theorem C.1 (Merge Validity Criterion) Let M■, M■ be modules with overlapping semantic fields. Let L_M be the cotangent complex and T_M the tangent complex. Then the merge: μ(M■, M■) = M if $\mathrm{Ext}^1(L_M, T_M) = 0$, μ(M■, M■) = Fail(ω) if $\omega \in \mathrm{Ext}^1(L_M, T_M) \neq 0$.

D. Associativity via Symmetric Monoidal Structure

Proposition D.1 (Tensorial Merge Associativity) Let $\otimes$ be the monoidal product on semantic modules, and μ the merge operator. Then: μ(M■ $\otimes$ M■, M■) $\cong$ μ(M■, M■ $\otimes$ M■) This follows from Mac Lane's coherence theorem.

E. Tiling Consistency via RSVP Entropy Gradients

Theorem E.1 (Topological Tiling) Let modules $\{M_i\}$ be RSVP tiles over patches U_i. Suppose ∇S_i defines adjacency relations, and $\Phi_i|_{U_i \cap U_j} \sim \Phi_j|_{U_i \cap U_j}$. Then the entire space X = ■_i U_i admits a globally coherent entropy map S: X $\to$ ■■ minimizing $\sum_{i,j} ||\nabla(S_i - S_j)||^2$.