



Delving deep into rectifiers: Surpassing human-level performance on imagenet classification



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Abstract

we study rectifier neural networks for image classification from two aspects.

1. we propose a **Parametric Rectified Linear Unit (PReLU)** that generalizes the traditional rectified unit.

-> improves model fitting with nearly zero extra computational cost and little overfitting risk.

2. we derive a robust initialization method that particularly considers the rectifier nonlinearities.

-> enables us to train extremely deep rectified models directly from scratch and to investigate deeper or wider network architectures

Our result is the first¹ to surpass the reported **human-level performance** (5.1%, [26]) on this dataset.

Introduction

Rectified Linear Unit (ReLU), is one of several keys to the recent success of deep networks.

- > It expedites convergence of the training procedure and leads to better solutions than conventional sigmoid like units

Unlike traditional sigmoid-like units, ReLU is not a symmetric function.

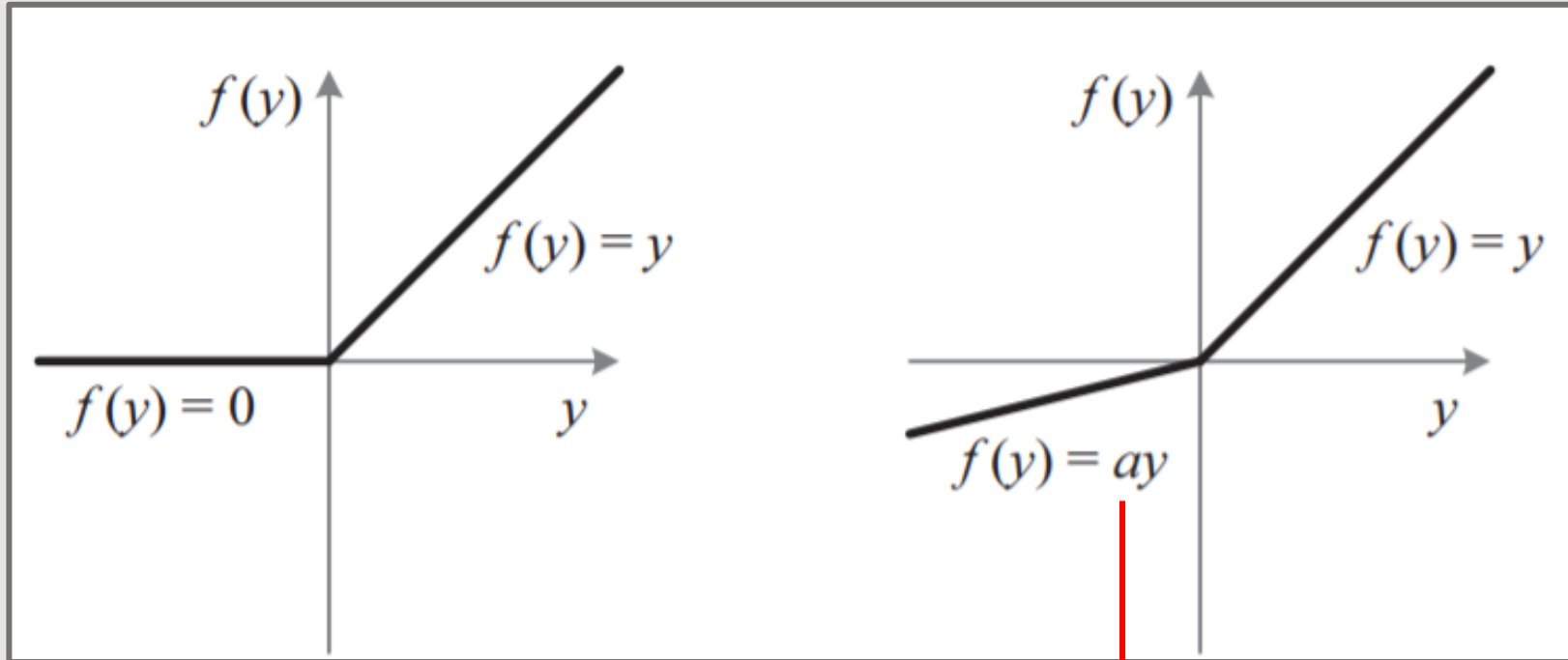
- > the mean response of ReLU is always no smaller than zero
- Even assuming the inputs/weights are subject to symmetric distributions, the distributions of responses can still be asymmetric because of the behavior of ReLU.

Introduction

We investigate neural networks from two aspects

1. we propose a new extension of ReLU, which we call **Parametric Rectified Linear Unit (PReLU)**
2. we study the difficulty of training rectified models that are very deep. By explicitly modeling the nonlinearity of rectifiers (ReLU/PReLU), we derive a theoretically sound initialization method, which helps with convergence of very deep models

Introduction



ReLU vs PReLU

If $a = 0.01$:
Leaky ReLU

Parametric Rectifiers

We show that replacing the parameter-free ReLU by a learned activation unit improves classification accuracy.


PReLU introduces a very small number of extra parameters.

- The number of extra parameters is equal to the total number of channels, which is negligible when considering the total number of weights.
- > we expect no extra risk of overfitting.

Parametric Rectifiers

The gradient of a_i for one layer is:

$$\frac{\partial \mathcal{E}}{\partial a_i} = \sum_{y_i} \frac{\partial \mathcal{E}}{\partial f(y_i)} \frac{\partial f(y_i)}{\partial a_i},$$


$$\frac{\partial f(y_i)}{\partial a_i} = \begin{cases} 0, & \text{if } y_i > 0 \\ y_i, & \text{if } y_i \leq 0 \end{cases}.$$

The time complexity due to PReLU is negligible for both forward and backward propagation

Parametric Rectifiers

We adopt the momentum method when updating a_i :

$$\Delta a_i := \mu \Delta a_i + \epsilon \frac{\partial \mathcal{E}}{\partial a_i}.$$

Momentum

Learning
rate

Initialization : $a_i=0.25$

we do not use weight decay when updating a_i .

A weight decay tends to push a_i to zero, and thus biases PReLU toward ReLU.

Initialization of Filter Weights for Rectifiers

Rectifier networks are easier to train,
But a bad initialization can still hamper the learning of a highly non-linear system

-> we propose a robust initialization method that removes an obstacle of training extremely deep rectifier networks.

Xavier initialization

Glorot and Bengio [8] proposed to adopt a properly scaled uniform distribution for initialization.

-> Its derivation is based on the assumption that the activations are linear.

→ This assumption is **invalid for ReLU and PReLU**.

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$$Var(W_i) = \frac{2}{n_{in} + n_{out}}$$

→ "He" initialization

Initialization of Filter Weights for Rectifiers

Forward Propagation Case

The central idea is to investigate the variance of the responses in each layer. For a conv layer, a response is:

$$\mathbf{y}_l = \mathbf{W}_l \mathbf{x}_l + \mathbf{b}_l.$$

$\mathbf{x} : k^2 c$ by 1vector (k by k pixels in c input channel)

$n = k^2 c$

$\mathbf{W} : d$ by n (d : the number of filter)

$\mathbf{b} : \text{bias}$

$\mathbf{y} : \text{response at a pixel of the output map}$

Initialization of Filter Weights for Rectifiers

We let the initialized elements in W_l be independent and identically distributed (i.i.d.)

- x_l are also i.i.d
- x_l and W_l are independent of each other

$$\text{Var}[y_l] = n_l \text{Var}[w_l x_l],$$

the variance of the product of independent variables gives us

$$\text{Var}[y_l] = n_l \text{Var}[w_l] E[x_l^2].$$

Initialization of Filter Weights for Rectifiers

let W_{l-1} have a symmetric distribution around zero and $b_{l-1} = 0$,
then y_{l-1} has zero mean and has a symmetric distribution around zero

$$\longrightarrow E[x_l^2] = \frac{1}{2} \text{Var}[y_{l-1}] \quad \text{when } f \text{ is ReLU}$$

$$\text{Var}[y_l] = \frac{1}{2} n_l \text{Var}[w_l] \text{Var}[y_{l-1}].$$

$$\text{Var}[y_L] = \text{Var}[y_1] \left(\prod_{l=2}^L \frac{1}{2} n_l \text{Var}[w_l] \right). \quad \frac{1}{2} n_l \text{Var}[w_l] = 1, \quad \forall l.$$

Key to the initialization design

Sufficient condition

zero-mean Gaussian distribution whose standard deviation (std) is $\sqrt{2/n_l}$

Initialization of Filter Weights for Rectifiers

Backward Propagation Case

The gradient of a conv layer is computed by:

$$\Delta \mathbf{x}_l = \hat{W}_l \Delta \mathbf{y}_l.$$

Note that $\hat{n} \neq n = k^2 c$.

W_l and Δy_l are independent of each other, then Δx_l has zero mean for all l , when W_l is initialized by a symmetric distribution around zero

Initialization of Filter Weights for Rectifiers

$$\Delta y_l = f'(y_l) \Delta x_{l+1}$$

We assume that $f'(y_l)$ and Δx_{l+1} are independent of each other

$$E[\Delta y_l] = E[\Delta x_{l+1}]/2 = 0,$$

$$E[(\Delta y_l)^2] = \text{Var}[\Delta y_l] = \frac{1}{2} \text{Var}[\Delta x_{l+1}].$$

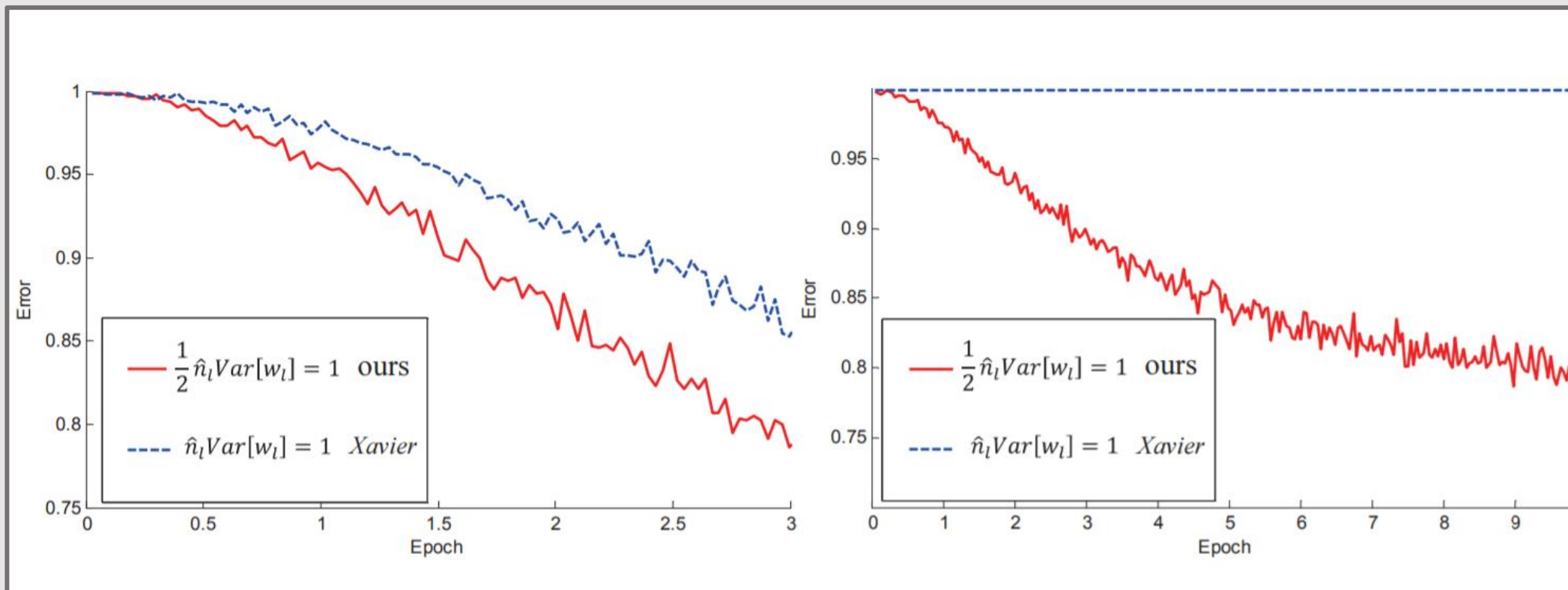
$$\begin{aligned} \text{Var}[\Delta x_l] &= \hat{n}_l \text{Var}[w_l] \text{Var}[\Delta y_l] \\ &= \frac{1}{2} \hat{n}_l \text{Var}[w_l] \text{Var}[\Delta x_{l+1}]. \end{aligned}$$

$$\frac{1}{2} \hat{n}_l \text{Var}[w_l] = 1, \quad \forall l.$$

Sufficient condition

$$\text{Var}[\Delta x_2] = \text{Var}[\Delta x_{L+1}] \left(\prod_{l=2}^L \frac{1}{2} \hat{n}_l \text{Var}[w_l] \right)$$

Initialization of Filter Weights for Rectifiers



We use ReLU in both figures

He initialization is able to make the extremely deep model converge. On the contrary, the **Xavier method** completely stalls the learning, and the gradients are diminishing as monitored in the experiments.

Initialization of Filter Weights for Rectifiers

We found that this degradation is because of the increase of training error when the model is deeper. Such a degradation is still an open problem

Though our attempts of extremely deep models have **not** shown benefits on accuracy, our initialization paves a foundation for further study on increasing depth. We hope this will be helpful in understanding deep networks.

Experiments on ImageNet

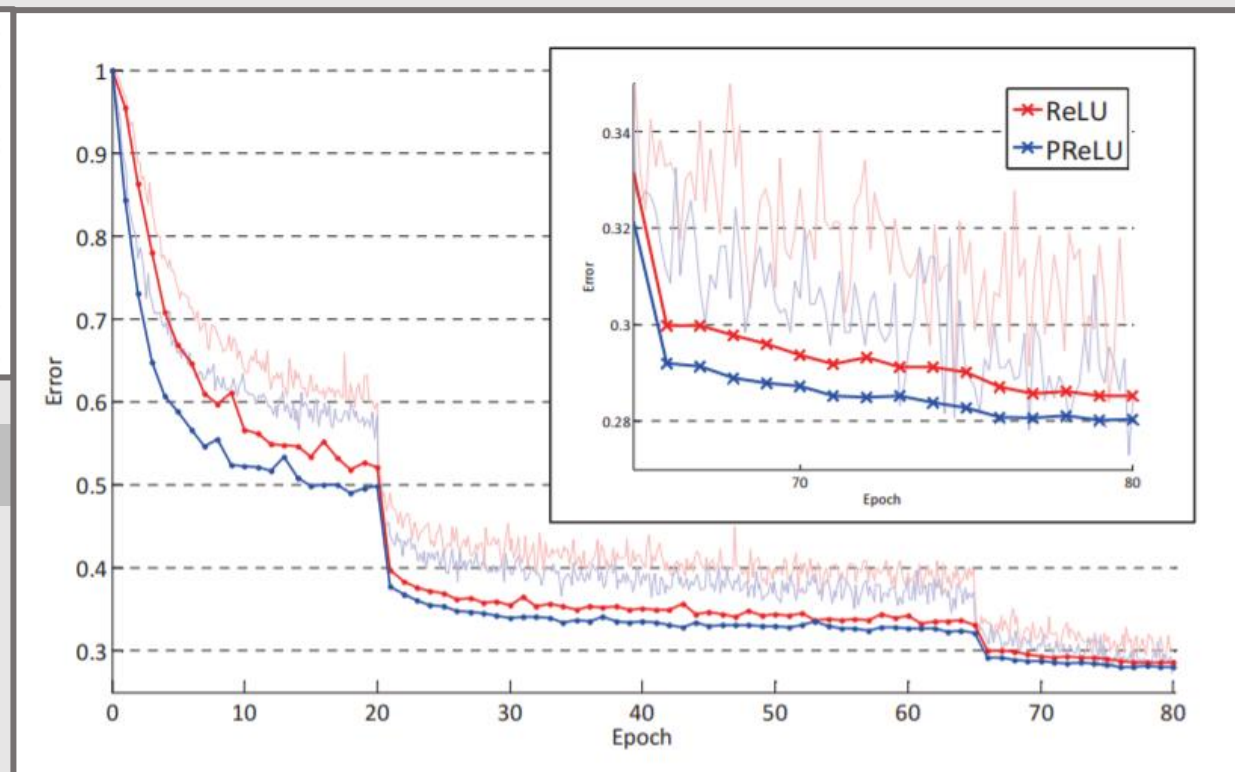
We perform the experiments on the 1000-class **ImageNet 2012 dataset**

- 1.2 million training images
- > 50,000 validation images, and 100,000 test images
- top1/top-5 error rates

Experiments on ImageNet

| model A | ReLU | | PReLU | |
|-------------|-------|-------|--------------|-------------|
| | top-1 | top-5 | top-1 | top-5 |
| 256 | 26.25 | 8.25 | 25.81 | 8.08 |
| 384 | 24.77 | 7.26 | 24.20 | 7.03 |
| 480 | 25.46 | 7.63 | 24.83 | 7.39 |
| multi-scale | 24.02 | 6.51 | 22.97 | 6.28 |

Comparisons between ReLU and PReLU



PReLU converges faster than ReLU.

Moreover, PReLU has lower train error and val error than ReLU throughout the training procedure.

Experiments on ImageNet

| | method | top-1 | top-5 |
|-------------------|---------------------|--------------|-------------------|
| in ILSVRC 14 | SPP [12] | 27.86 | 9.08 [†] |
| | VGG [29] | - | 8.43 [†] |
| | GoogLeNet [33] | - | 7.89 |
| post ILSVRC 14 | VGG [29] (arXiv v2) | 24.8 | 7.5 |
| | VGG [29] (arXiv v5) | 24.4 | 7.1 |
| | ours (A, ReLU) | 24.02 | 6.51 |
| | ours (A, PReLU) | 22.97 | 6.28 |
| | ours (B, PReLU) | 22.85 | 6.27 |
| | ours (C, PReLU) | 21.59 | 5.71 |

Single-model results evaluated from test set

| | method | top-5 (test) |
|-------------------|---------------------|--------------|
| in ILSVRC 14 | SPP [12] | 8.06 |
| | VGG [29] | 7.32 |
| | GoogLeNet [33] | 6.66 |
| post ILSVRC 14 | VGG [29] (arXiv v5) | 6.8 |
| | ours | 4.94 |

Multi-model results for ImageNet test set

Combine six models

Experiments on ImageNet



GT: horse cart

1: horse cart

- 2: minibus
- 3: oxcart
- 4: stretcher
- 5: half track



GT: birdhouse

1: birdhouse

- 2: sliding door
- 3: window screen
- 4: mailbox
- 5: pot



GT: forklift

1: forklift

- 2: garbage truck
- 3: tow truck
- 4: trailer truck
- 5: go-kart



GT: coucal

1: coucal

- 2: indigo bunting
- 3: lorikeet
- 4: walking stick
- 5: custard apple



GT: komondor

1: komondor

- 2: patio
- 3: llama
- 4: mobile home
- 5: Old English sheepdog



GT: yellow lady's slipper

1: yellow lady's slipper

- 2: slug
- 3: hen-of-the-woods
- 4: stinkhorn
- 5: coral fungus

Human performance yields a **5.1%** top-5 error on the ImageNet dataset.

-> Our result (**4.94%**) exceeds the reported human-level performance

Comparisons with Human Performance