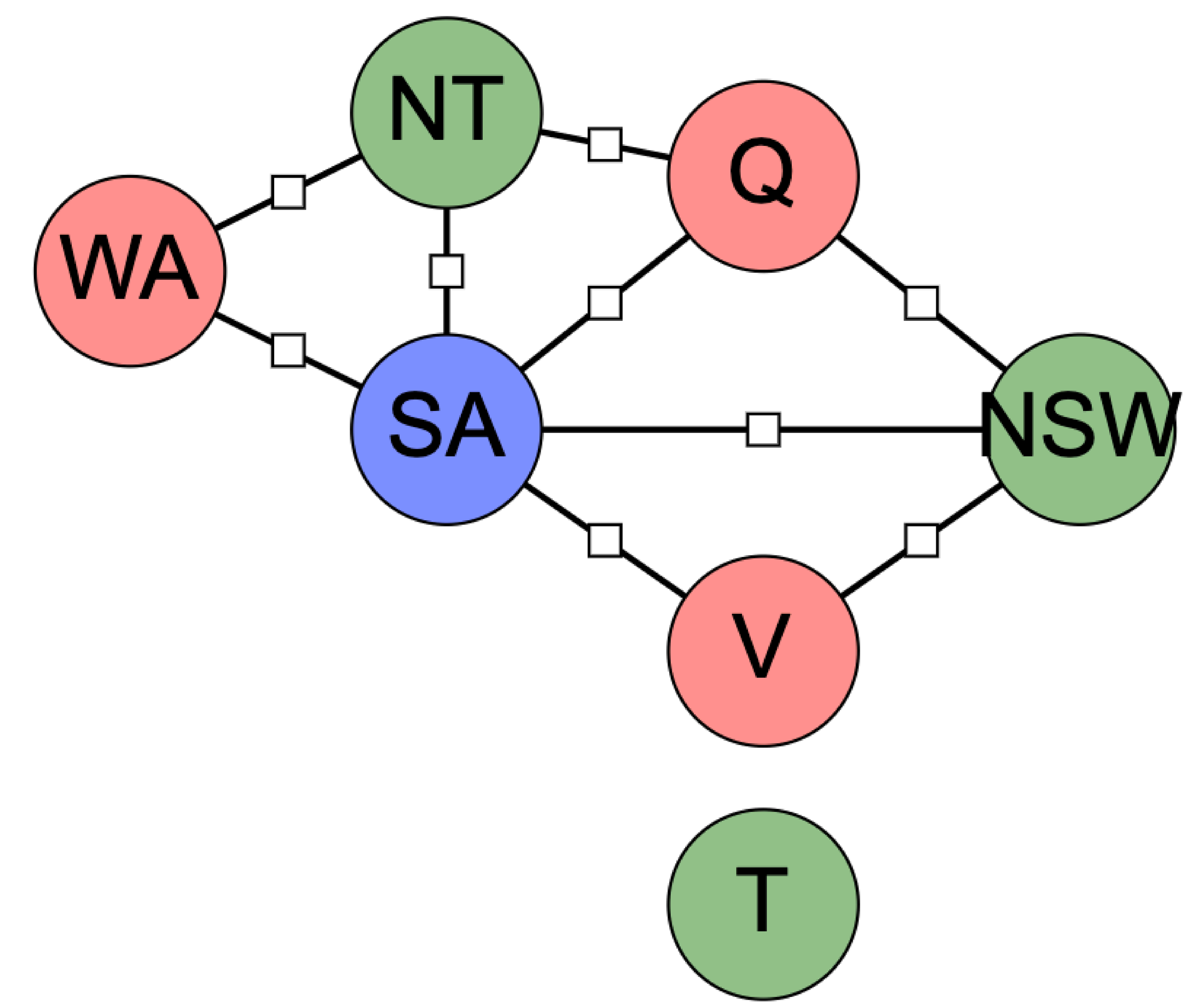
Example: map coloring

CSP: $f_j \in \{0,1\}$



Assignment:

$$x = \{\mathrm{WA}:\mathrm{R}, \mathrm{NT}:\mathrm{G}, \mathrm{SA}:\mathrm{B}, \mathrm{Q}:\mathrm{R}, \mathrm{NSW}:\mathrm{G}, \mathrm{V}:\mathrm{R}, \mathrm{T}:\mathrm{G}\}$$

Weight:

$$\mathrm{Weight}(x) = 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 1$$

Assignment:

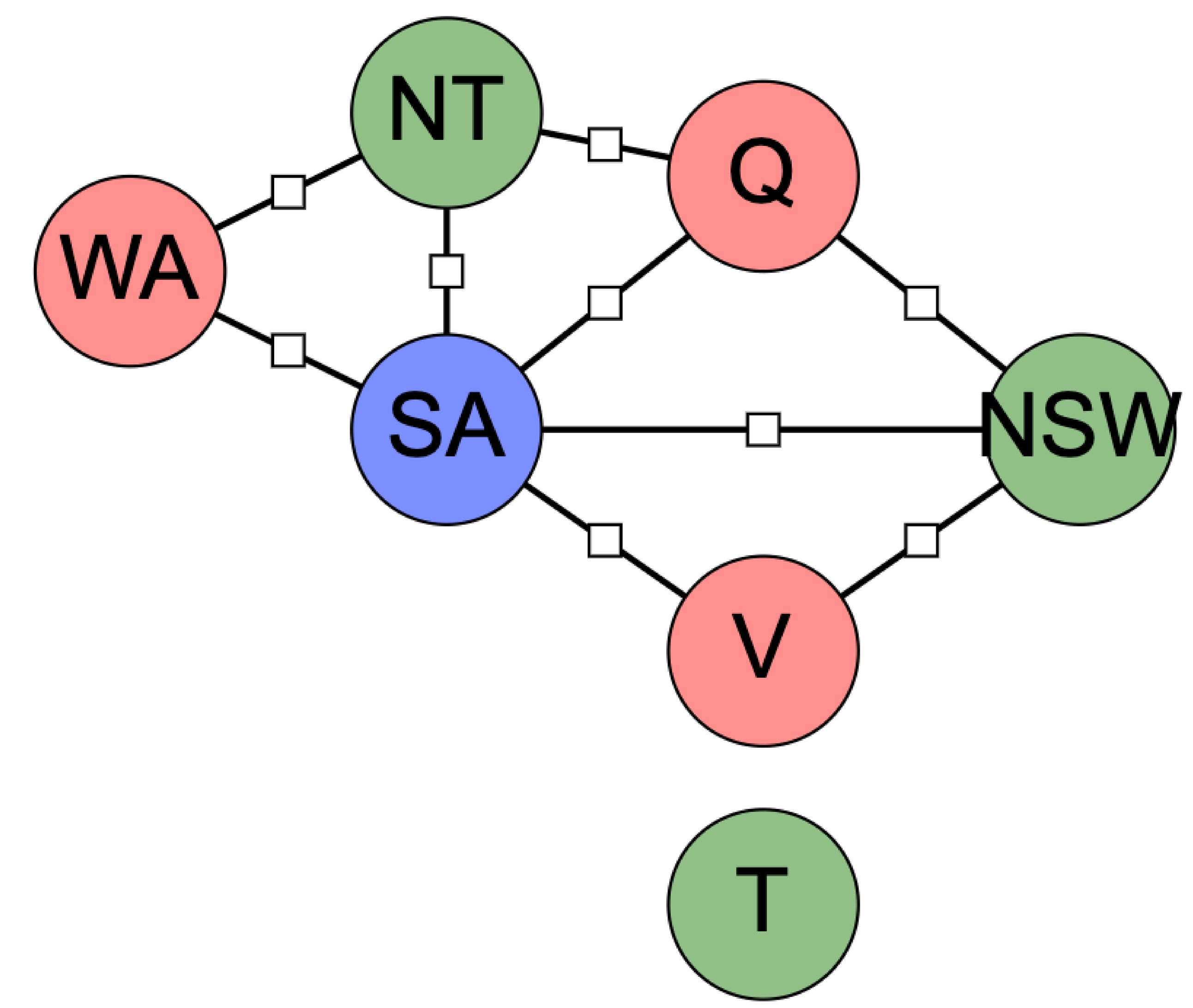
$$x' = \{\mathrm{WA}:\mathrm{R}, \mathrm{NT}:\mathrm{R}, \mathrm{SA}:\mathrm{B}, \mathrm{Q}:\mathrm{R}, \mathrm{NSW}:\mathrm{G}, \mathrm{V}:\mathrm{R}, \mathrm{T}:\mathrm{G}\}$$

Weight:

$$\mathrm{Weight}(x') = 0 \cdot 0 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 0$$

Example: map coloring

$$\lg(f_1 \cdots f_m)$$
$$= \lg f_1 + \lg f_2 + \cdots + \lg f_m$$



Assignment:

$$x = \{\mathrm{WA} : \mathrm{R}, \mathrm{NT} : \mathrm{G}, \mathrm{SA} : \mathrm{B}, \mathrm{Q} : \mathrm{R}, \mathrm{NSW} : \mathrm{G}, \mathrm{V} : \mathrm{R}, \mathrm{T} : \mathrm{G}\}$$

Weight:

$$\mathrm{Weight}(x) = 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 1$$

Assignment:

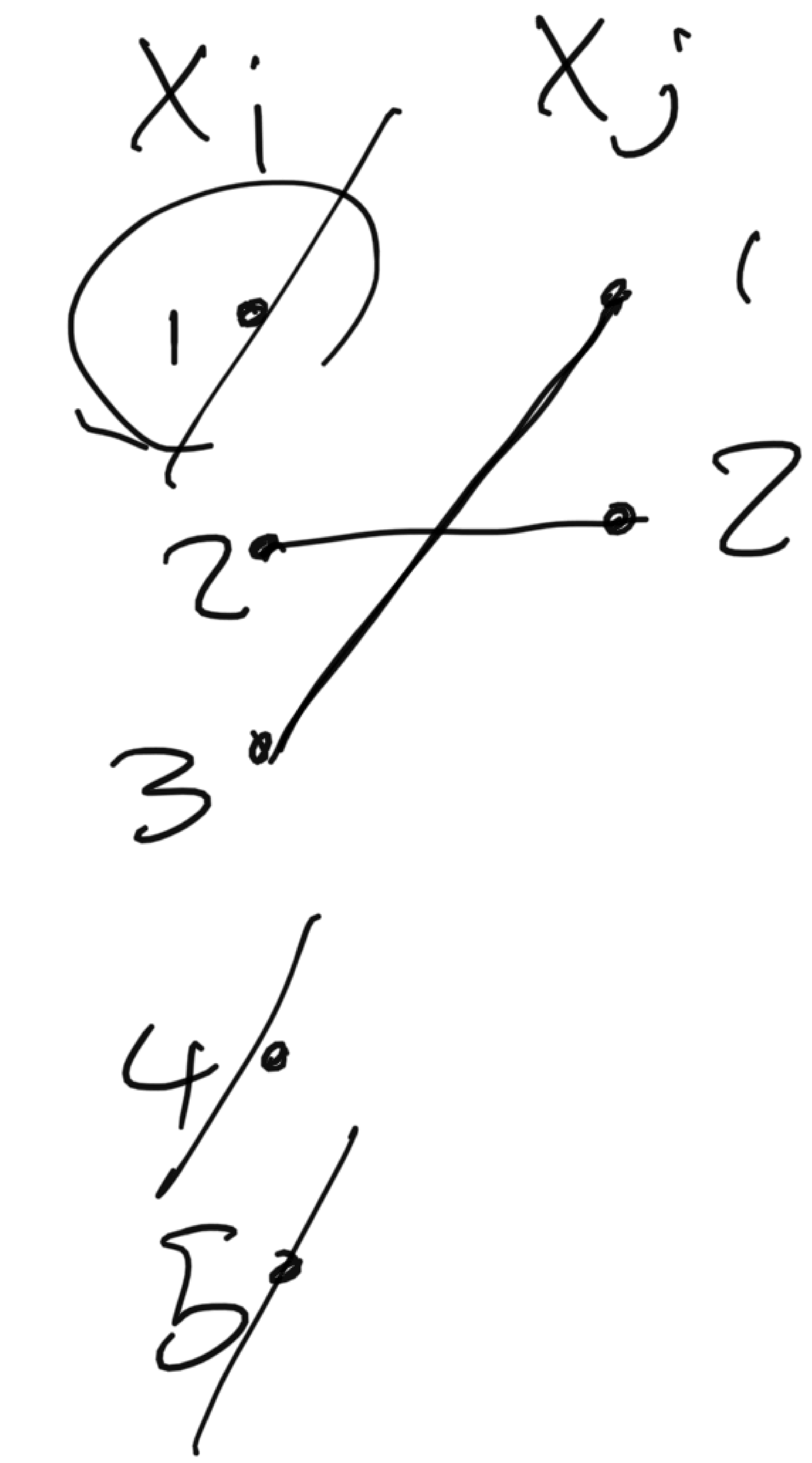
$$x' = \{\mathrm{WA} : \mathrm{R}, \mathrm{NT} : \mathrm{R}, \mathrm{SA} : \mathrm{B}, \mathrm{Q} : \mathrm{R}, \mathrm{NSW} : \mathrm{G}, \mathrm{V} : \mathrm{R}, \mathrm{T} : \mathrm{G}\}$$

Weight:

$$\mathrm{Weight}(x') = 0 \cdot 0 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 0$$

Arc consistency

Idea: eliminate values from domains $\Rightarrow$ reduce branching



Example: numbers

Before enforcing arc consistency on X_i:

$$X_i \in \text{Domain}_i = \{1, 2, 3, 4, 5\}$$
$$X_j \in \text{Domain}_j = \{1, 2\}$$
$$f_1(X) = [X_i + X_j = 4]$$

[whiteboard]

AC-3

Forward checking: when assign $X_j : x_j$, set $\text{Domain}_j = \{x_j\}$ and enforce arc consistency on all neighbors X_i with respect to X_j

AC-3: repeatedly enforce arc consistency on all variables

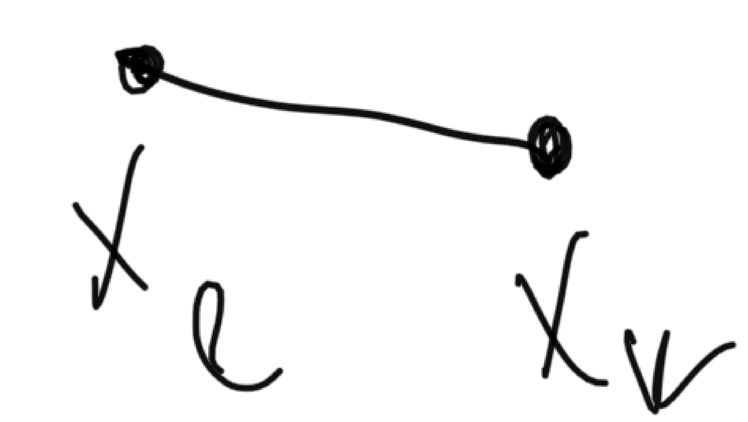
$e \times d \times d^2 + d = e \times d^3$

Algorithm: AC-3

Add X_j to set.

While set is non-empty:

- Remove any X_k from set.
- For all neighbors X_l of X_k:
  - Enforce arc consistency on X_l w.r.t. X_k.
  - If Domain_l changed, add X_l to set.



AC-3 (example)