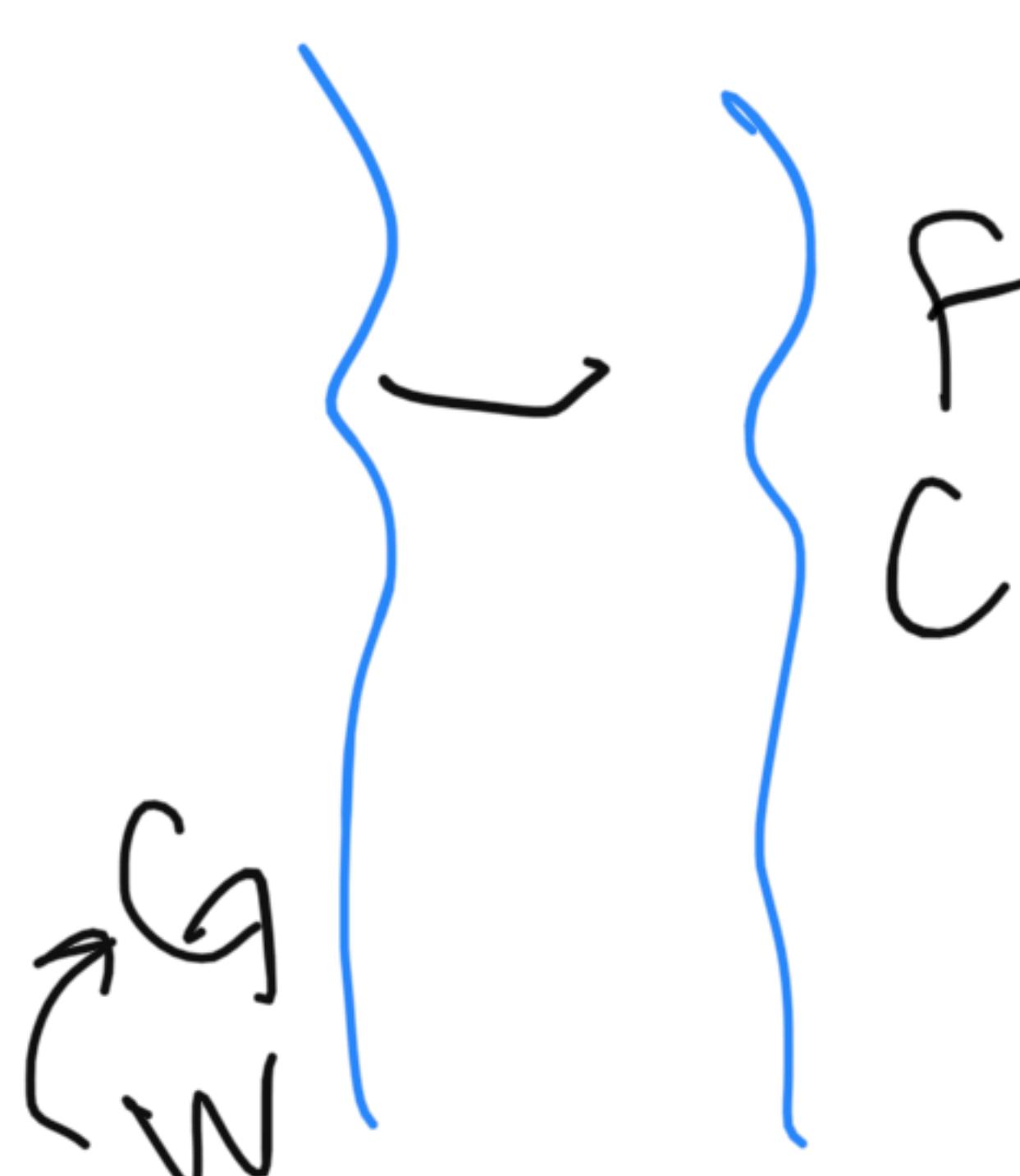




answer in chat

Question

A **farmer** wants to get his **cabbage**, **goat**, and **wolf** across a river. He has a boat that only holds two. He cannot leave the cabbage and goat alone or the goat and wolf alone. How many river crossings does he need?



4

5

6

7

no solution

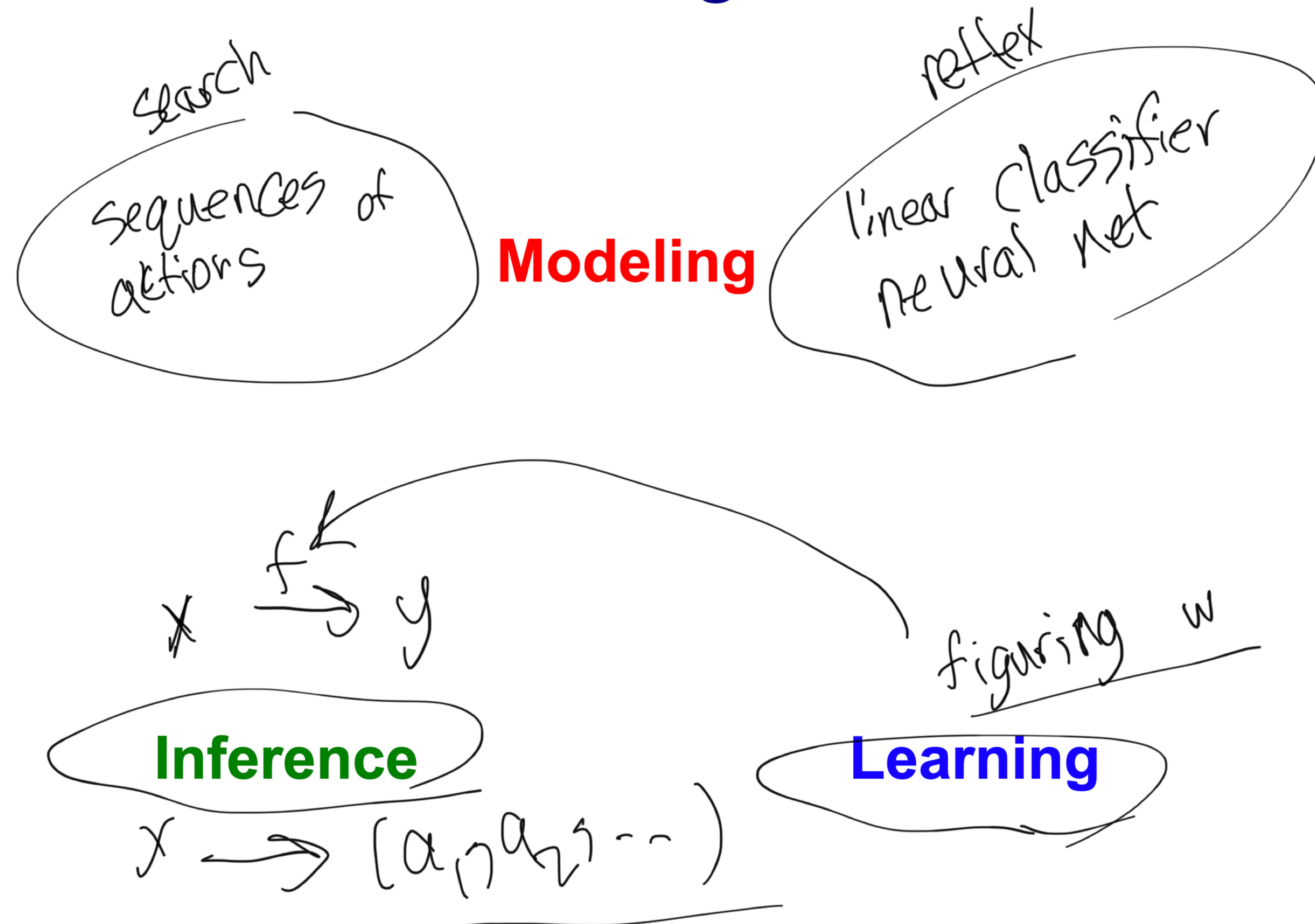
activate

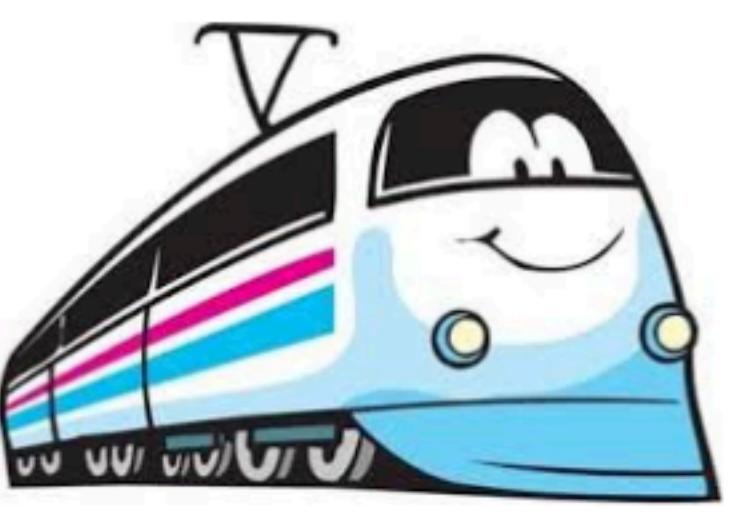
deactivate

reset

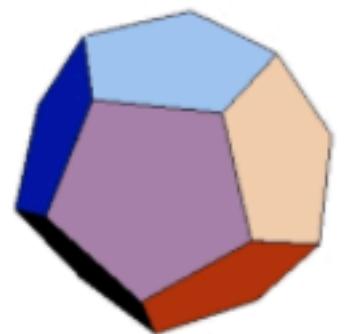
report

Paradigm





Transportation example



Example: transportation

Street with blocks numbered 1 to n .

Walking from s to $s + 1$ takes 1 minute.

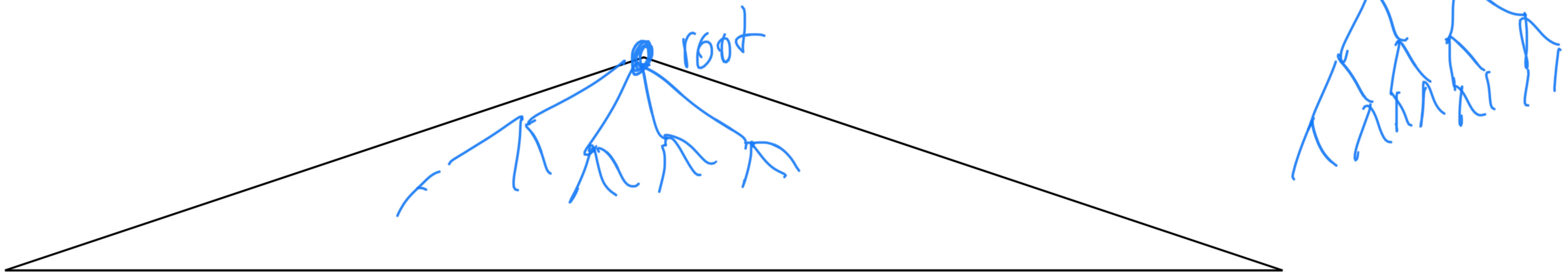
Taking a magic tram from s to $2s$ takes 2 minutes.

How to travel from 1 to n in the least time?



[semi-live solution: `TransportationProblem`]

Backtracking search



[whiteboard: search tree]

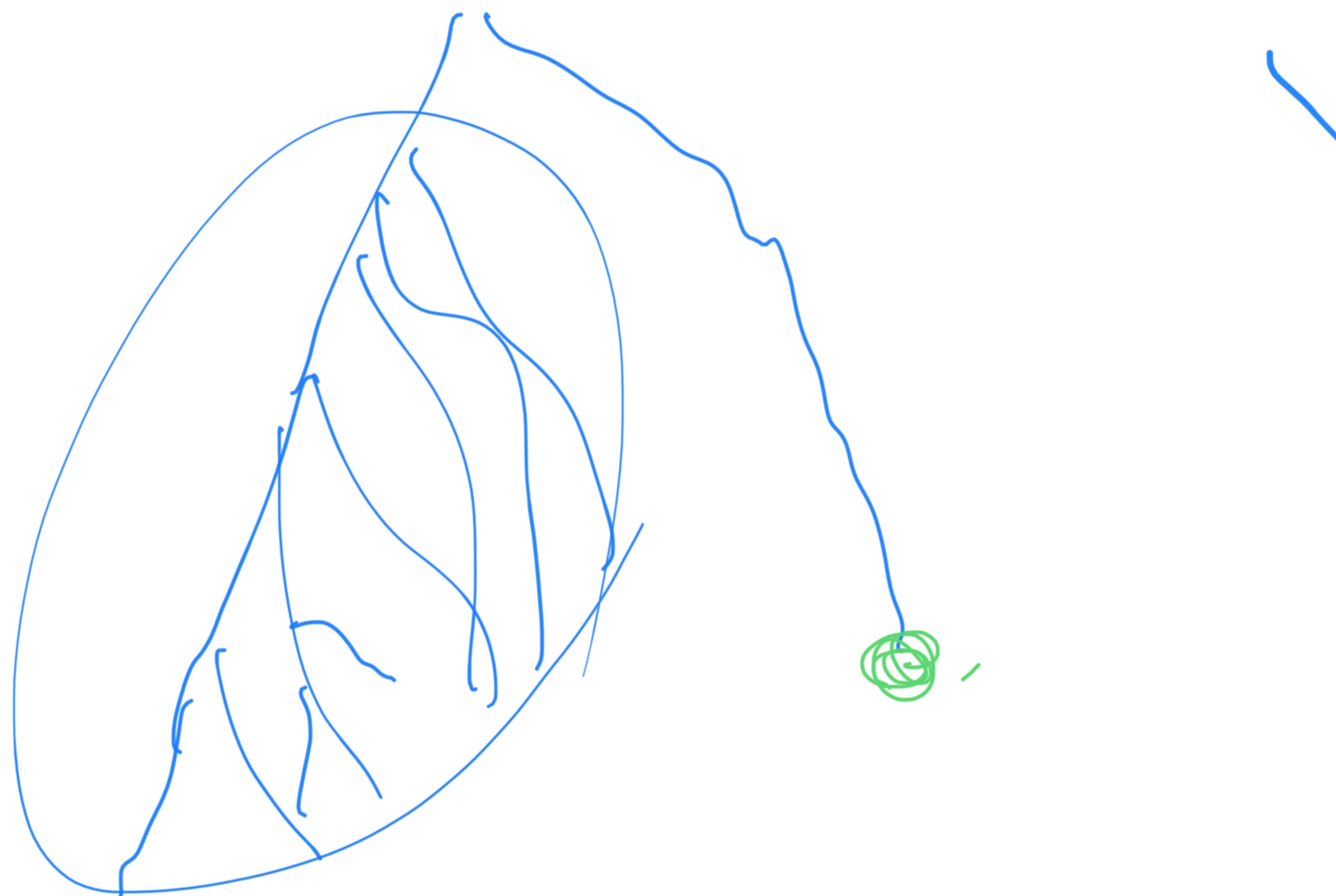
If b actions per state, maximum depth is D actions:

Breadth-first search



Assumption: constant action costs

Assume action costs $\text{Cost}(s, a) = c$ for some $c \geq 0$.



Breadth-first search



Assumption: constant action costs

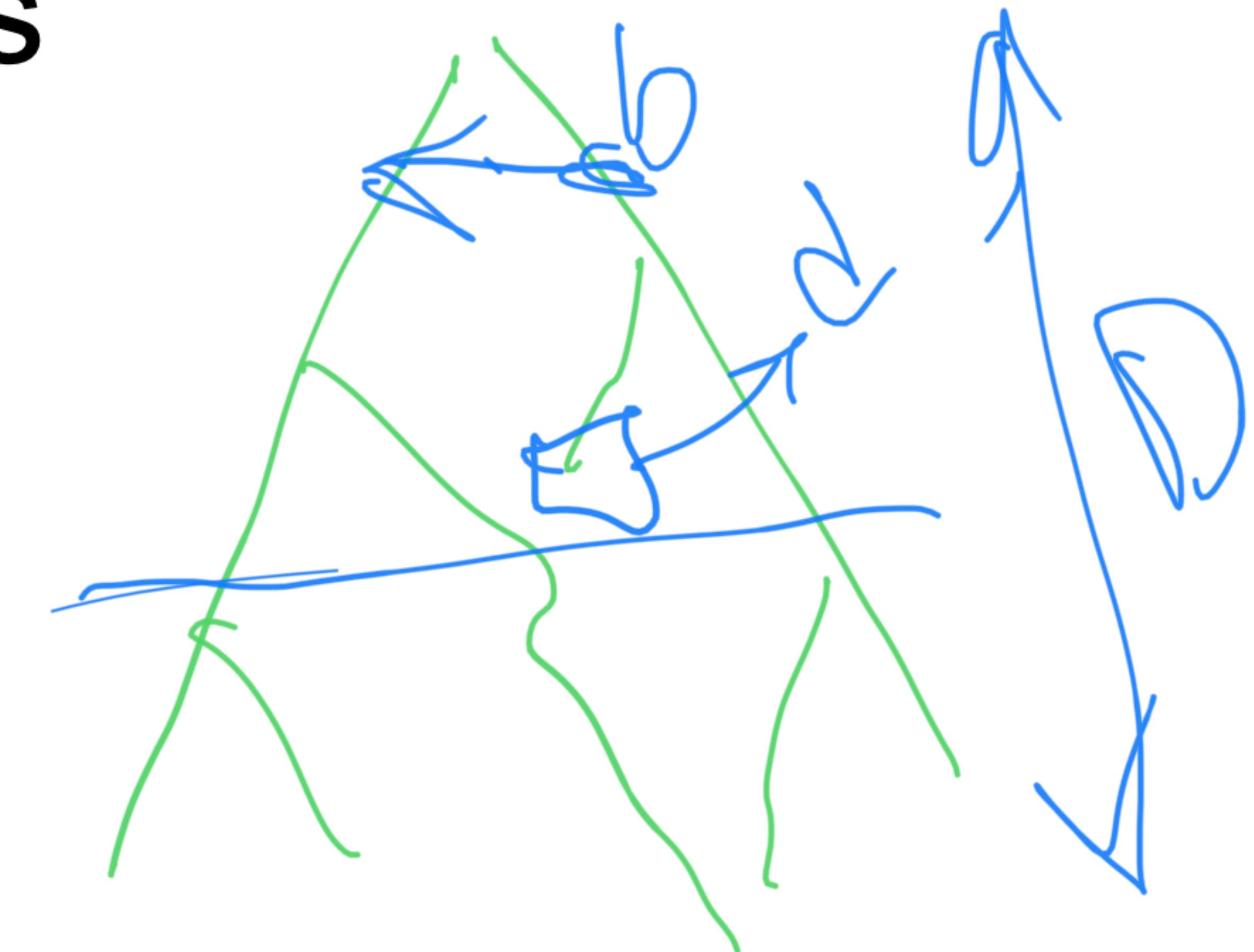
Assume action costs $\text{Cost}(s, a) = c$ for some $c \geq 0$.

Idea: explore all nodes in order of increasing depth.

b^d $2 \leq b$

Legend: b actions per state, solution has d actions

- Space: now $O(b^d)$ (a lot worse!)



Depth-first search



$$b + b^2 + b^3 + \dots + b^D$$



Assumption: zero action costs

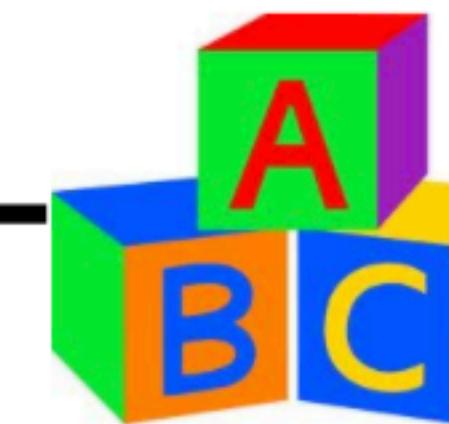
Assume action costs $\text{Cost}(s, a) = 0$.

Idea: Backtracking search + stop when find the first end state.

If b actions per state, maximum depth is D actions:

- **Space:** still $O(D)$
- **Time:** still $O(b^D)$ worst case, but could be much better if solutions are easy to find

DFS with iterative deepening



Assumption: constant action costs

Assume action costs $\text{Cost}(s, a) = c$ for some $c \geq 0$.

Idea:

- Modify DFS to stop at a maximum depth.
- Call DFS for maximum depths 1, 2, ...

DFS on d asks: is there a solution with d actions?

Legend: b actions per state, solution size d

- Space: $O(d)$ (saved!)
- Time: $O(b^d)$ (same as BFS)

$$\begin{aligned} b &= 2 \\ 2^1 + 2^2 + 2^3 + 2^4 + \dots + 2^{d-1} &= O(2^d) \\ 2^0 + 1 - 2 &= 1 \end{aligned}$$

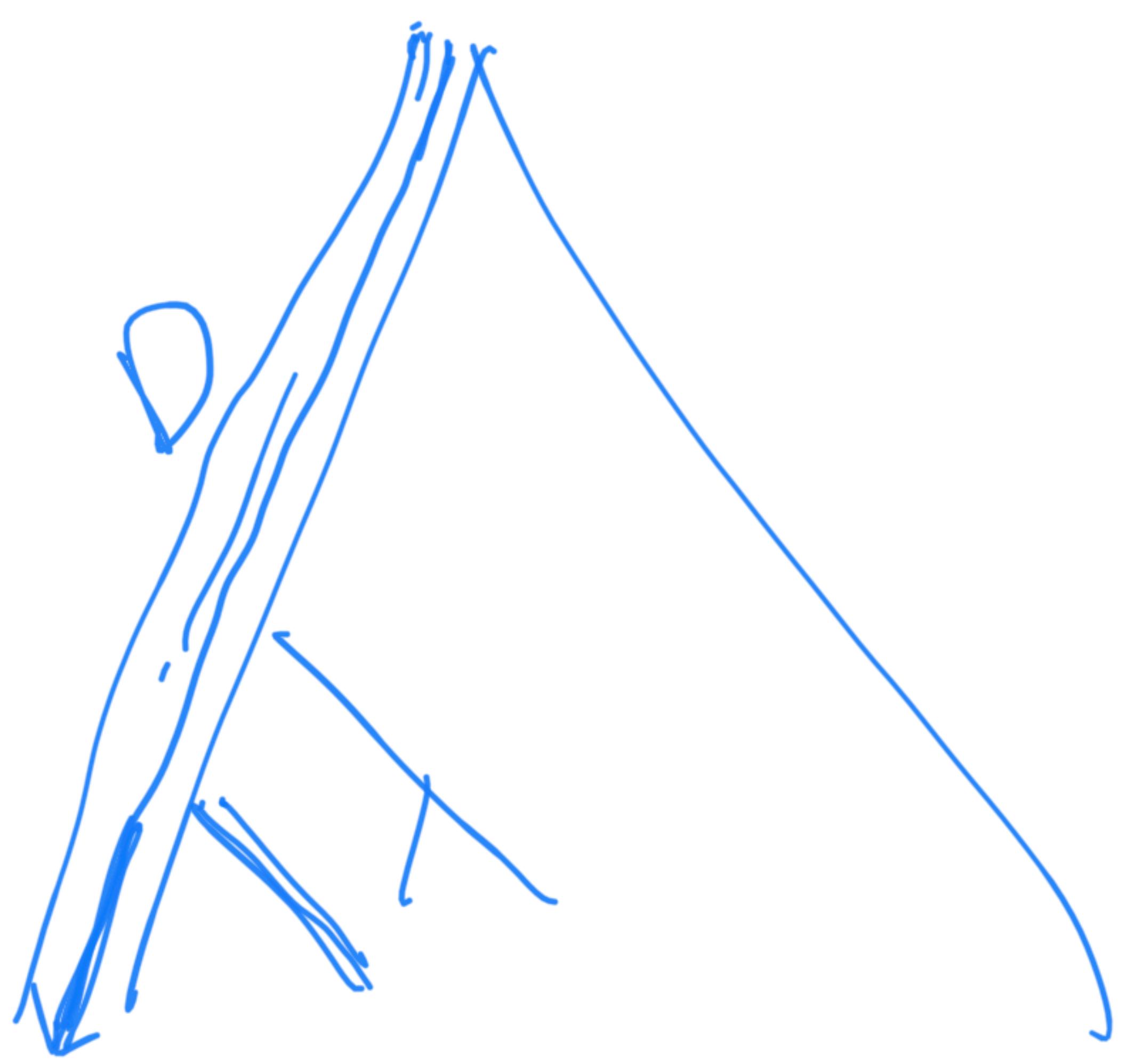
b^2

b^3

b^4

b^d

b^{d-1}

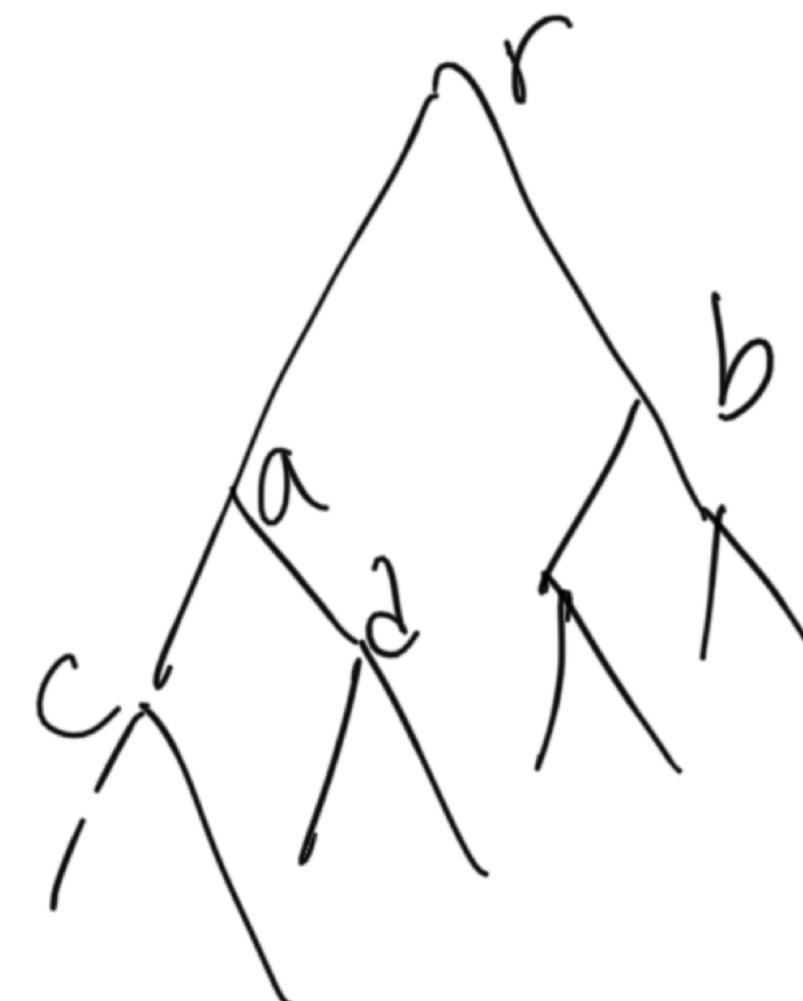


DFS with iterative deepening



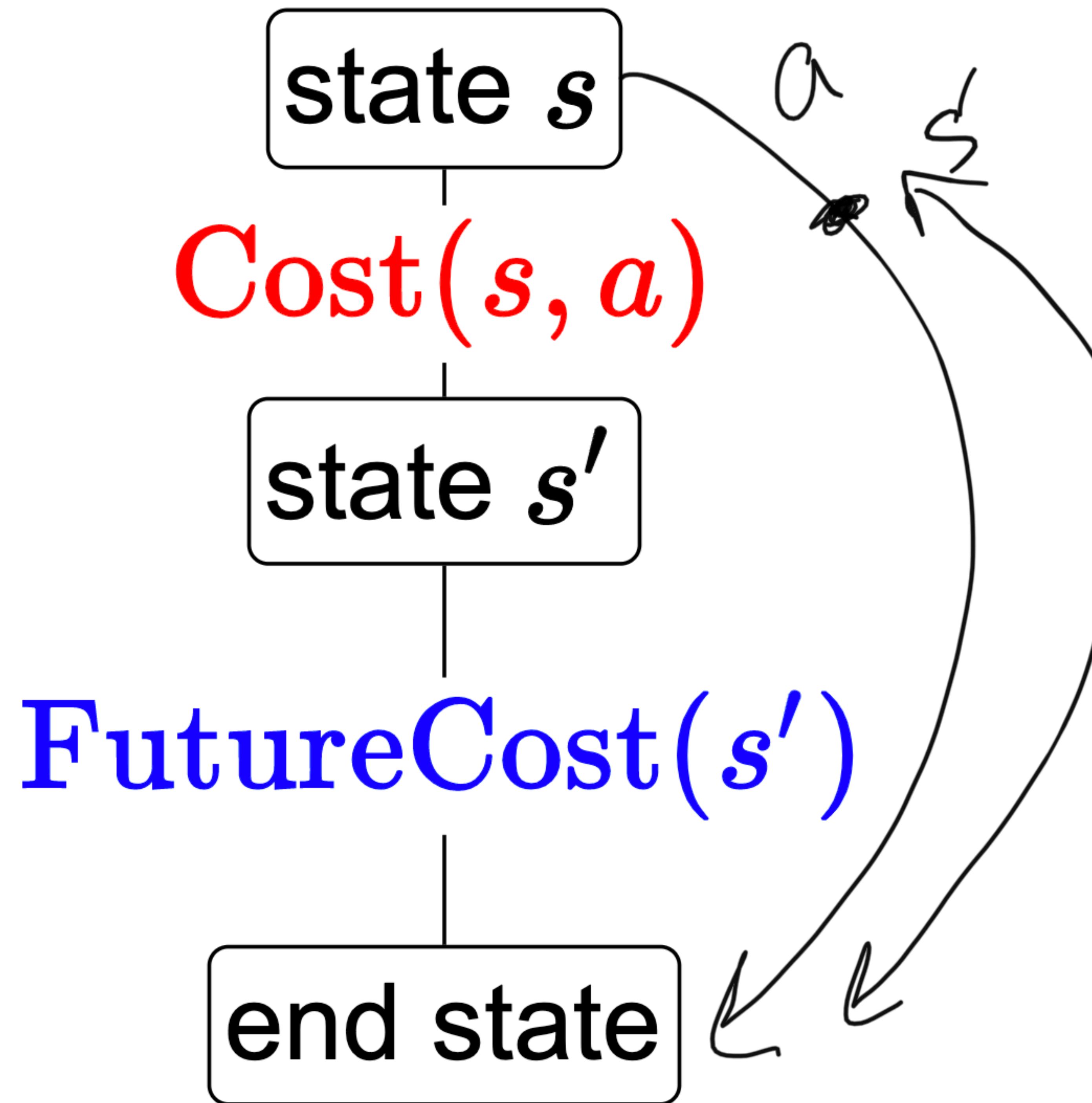
Assumption: constant action costs

Assume action costs $\text{Cost}(s, a) = c$ for some $c \geq 0$.



$$Q = \begin{bmatrix} r & ab & cd \\ f & & \end{bmatrix}$$
A large, hand-drawn arrow originates from the bottom right of the search tree and points towards the matrix Q , indicating a correspondence between the two concepts.

Dynamic programming



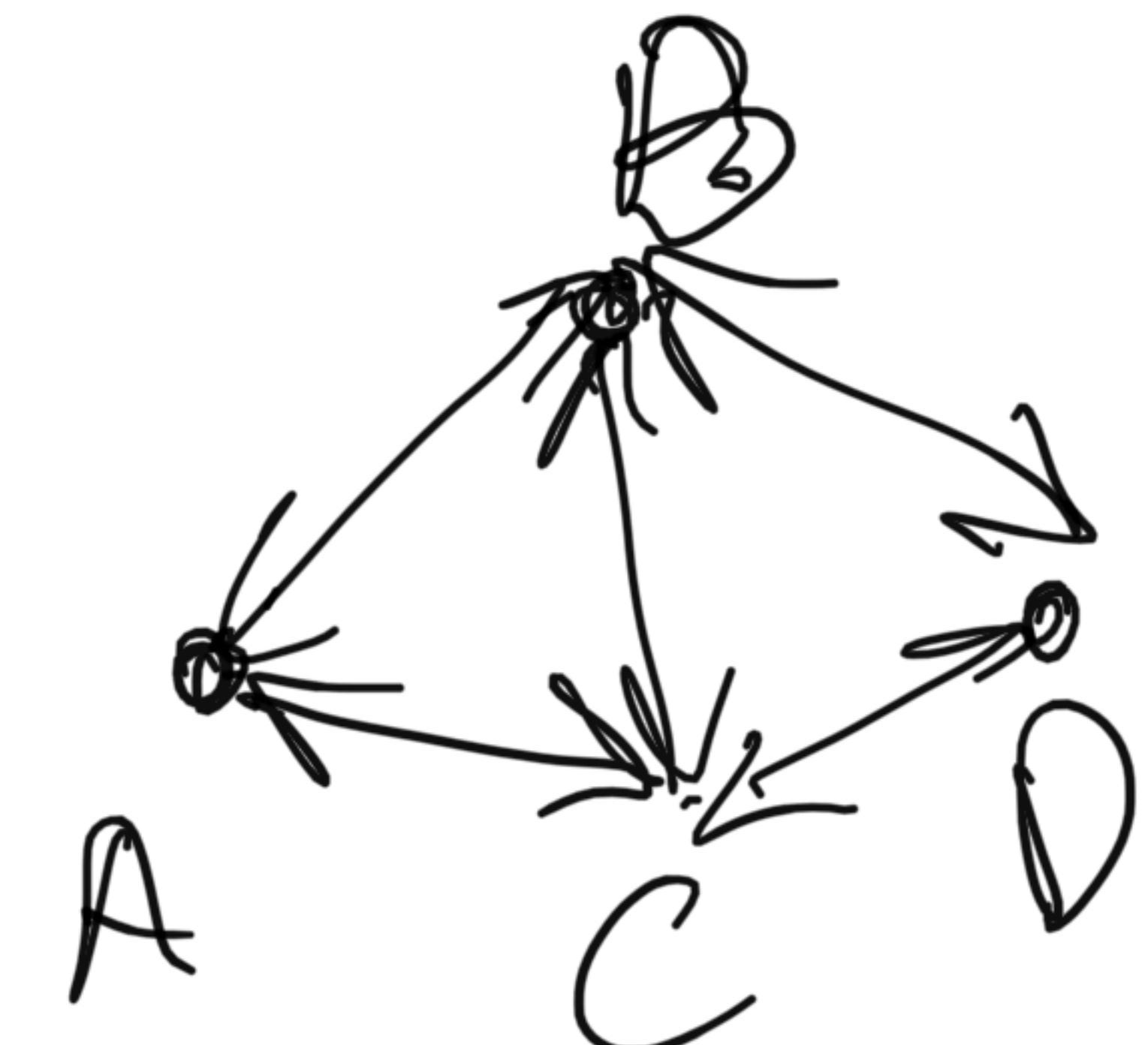
subproblems:
find the best
path from s'
to end.

Dynamic programming



Algorithm: dynamic programming

```
def DynamicProgramming(s):  
    If already computed for s, return cached answer.  
    If IsEnd(s): return solution  
    For each action  $a \in \text{Actions}(s)$ : ...
```



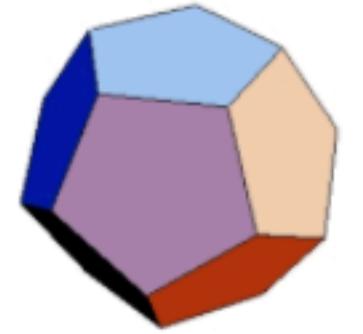
[semi-live solution: Dynamic Programming]



Assumption: acyclicity

The state graph defined by $\text{Actions}(s)$ and $\text{Succ}(s, a)$ is acyclic.

Handling additional constraints



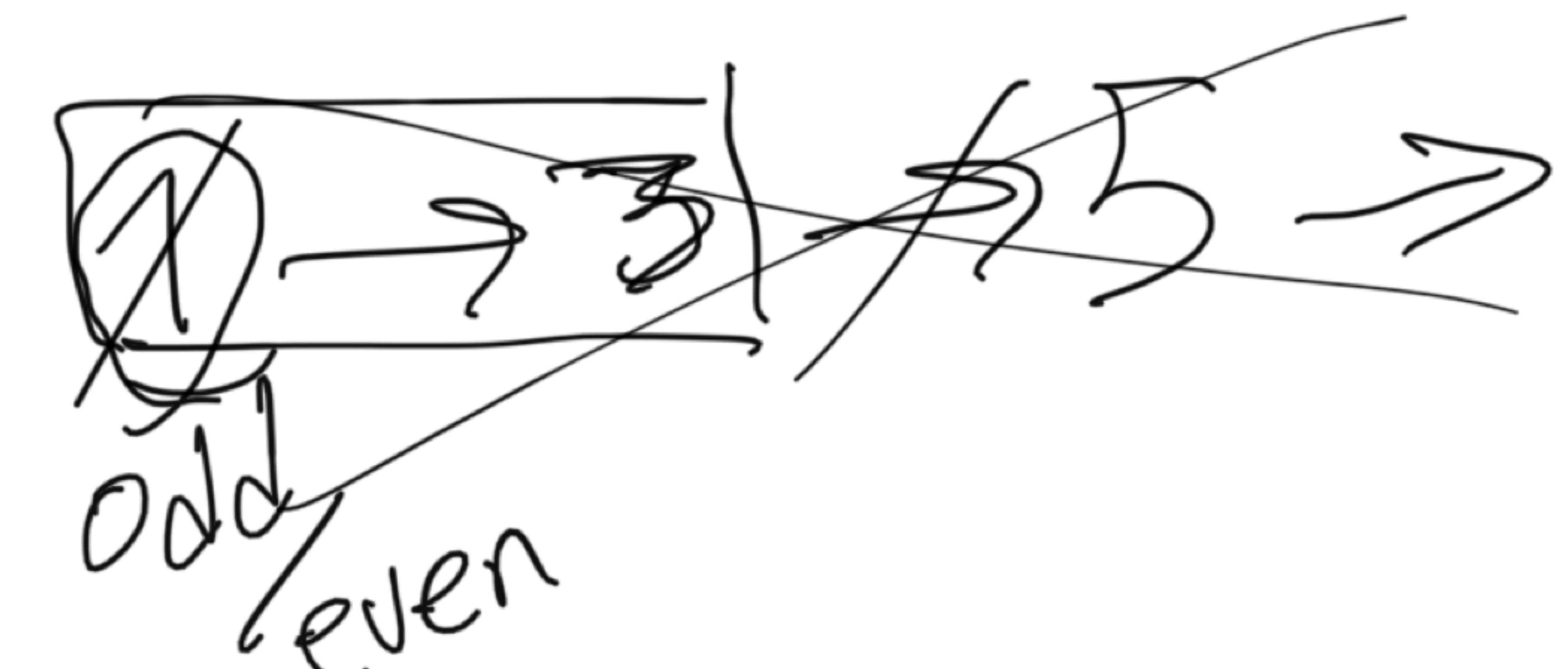
Example: route finding

Find the minimum cost path from city 1 to city n , only moving forward. It costs c_{ij} to go from i to j .

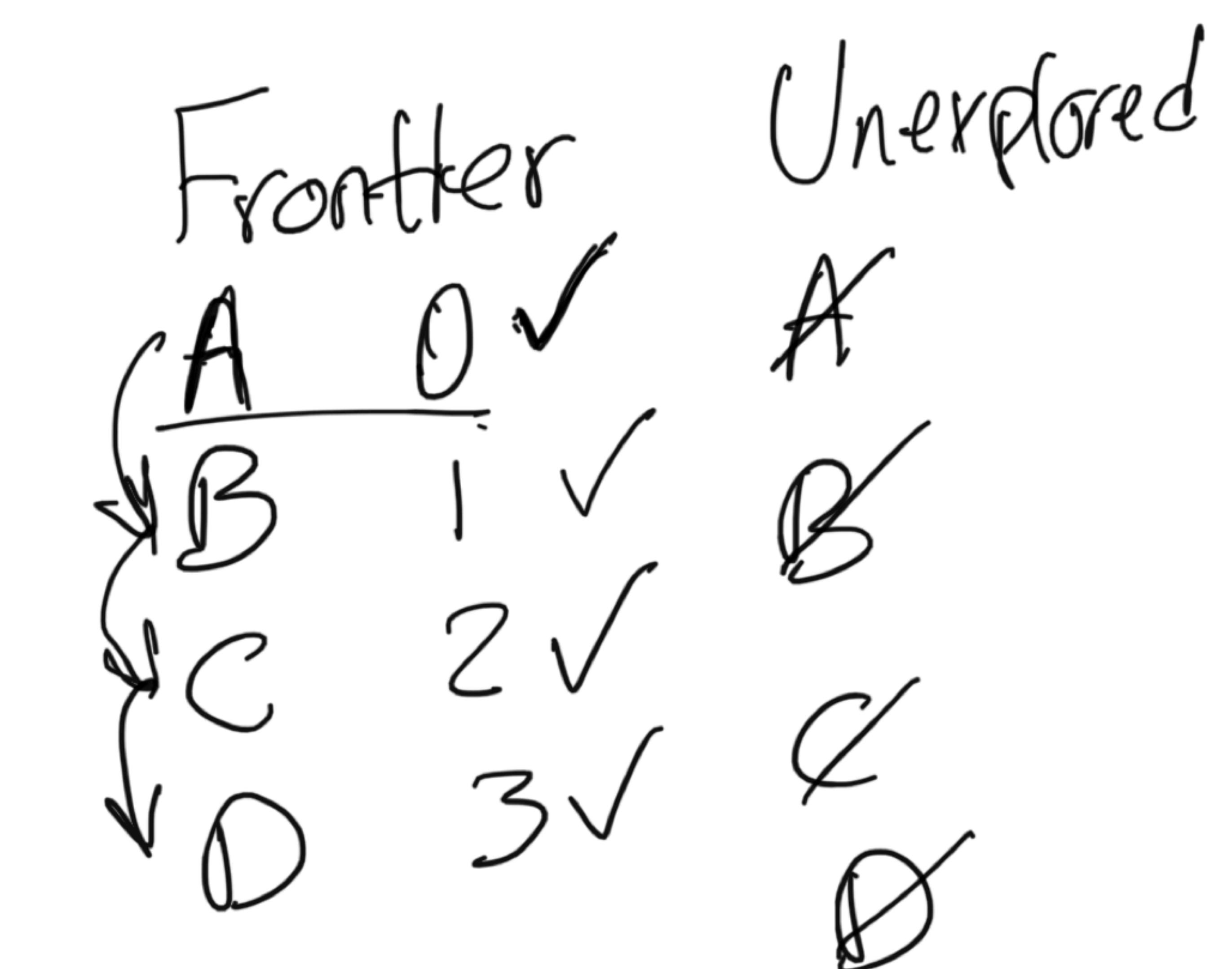
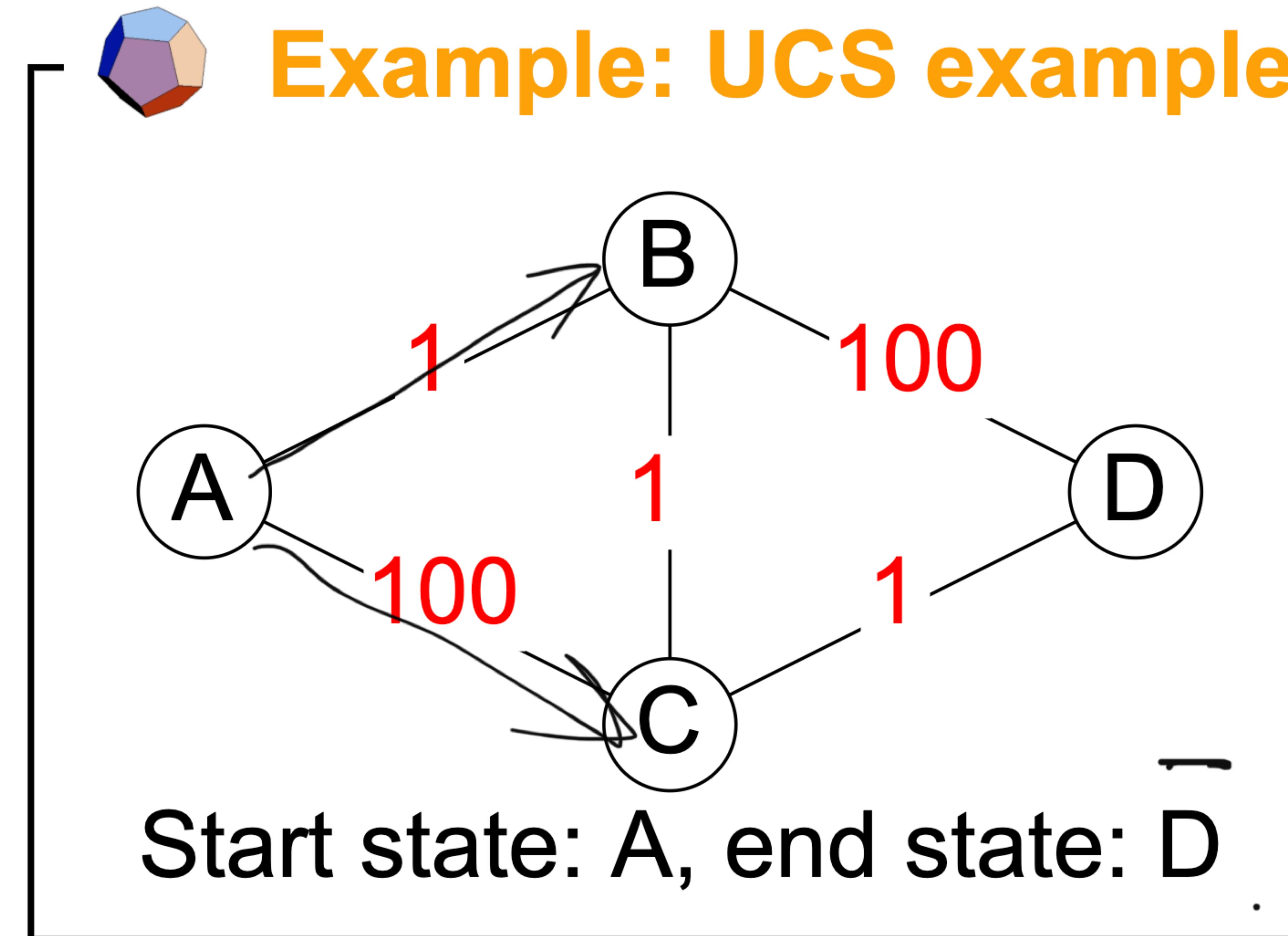
Constraint: Can't visit three odd cities in a row.

State: (previous city, current city)

$O(n^2)$



Uniform cost search example



[whiteboard]

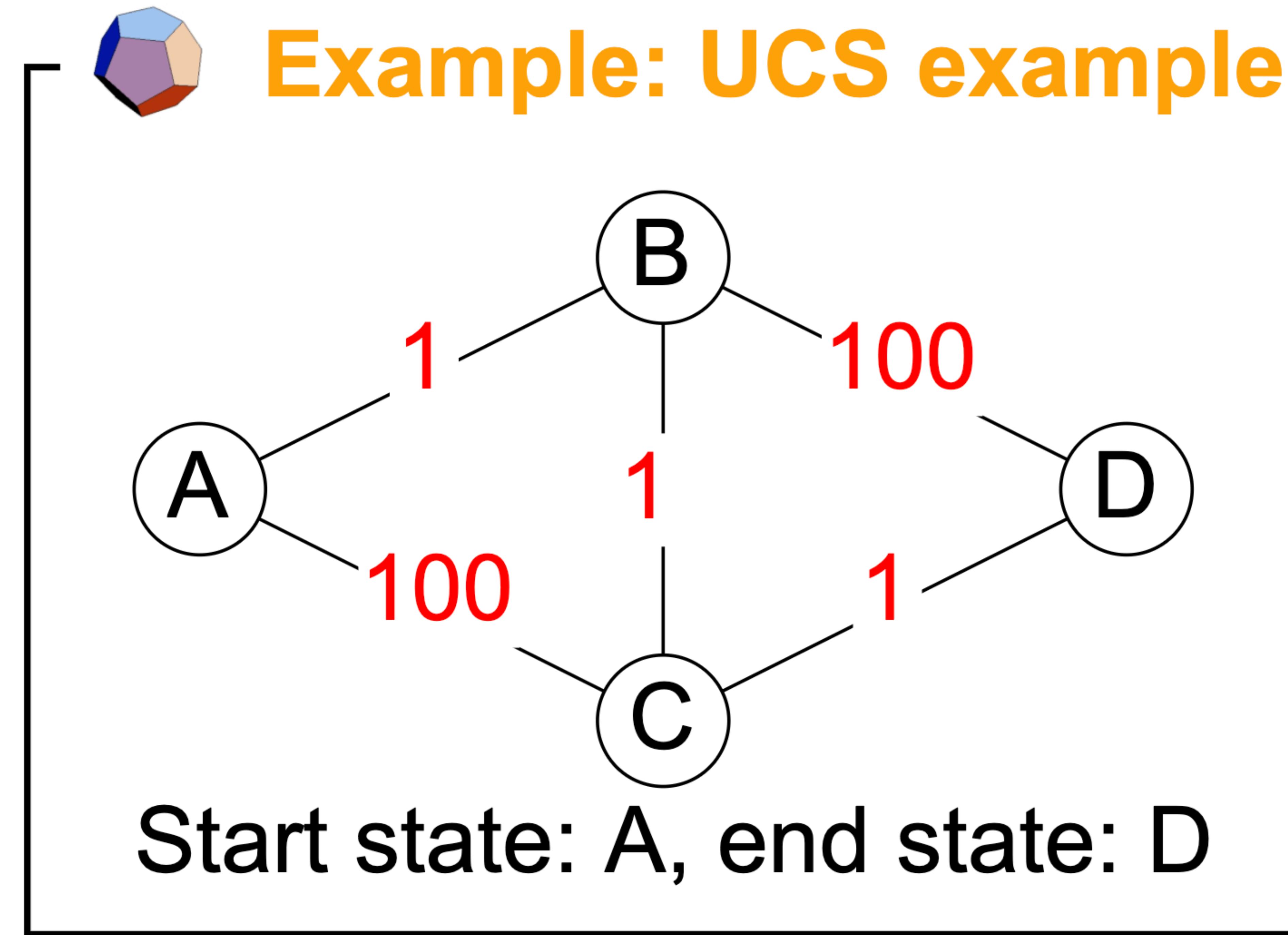
tree search : $O(b^d)$

dynamic programming $O(N \cdot A)$

uniform cast search $O(N \cdot A + N \cdot N)$

A: actions per state
N: states

Uniform cost search example



[whiteboard]

Minimum cost path:

$A \rightarrow B \rightarrow C \rightarrow D$ with cost 3

