An assessment of the use of the Kozeny-Carman relationship to estimate permeability in anisotropic materials from NMR data

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Summary

Calculating permeability from NMR data has been accomplished through the use of empirical relationships based on the Kozeny-Carman (K-C) relationship. The parameters in the K-C relationship are surface area to volume ratio (S/V), porosity and tortuosity. Due to the link between NMR measured parameters and S/V, there is potential for using NMR data to obtain estimates of permeability, where the NMR parameter T_{2ML} replaces S/V. To examine the validity of the K-C relationship in an anisotropic media, two systems were examined, 1) anisotropic grain packs and 2) layered sand/clay systems.

In the anisotropic grain pack, permeability was calculated numerically in the vertical (k_v) and horizontal (k_h) directions using lattice-Boltzmann and compared with K-C derived permeability ($k_{K\text{-}C}$). Results show that K-C was a good predictor of both k_v and k_h. In the sand/clay layered system, values of permeability and porosity were assigned to the sand and clay, and the volume fraction of clay was increased from 0 to 1.0. The k_v and k_h were calculated analytically. Predicted permeability values were calculated with the K-C relationship using an average S/V, an amplitude-weighted average S/V, and an amplitude weighted mean log S/V (which corresponds to T_{2ML}). Results show strong directional dependence in the agreement between the K-C-predicted permeability and true permeability. When S/V is calculated in a way so that it corresponds to T_{2ML}, K-C is a good predictor of k_h up to a critical volume fraction clay content; beyond this, K-C underestimates k_h. When S/V is calculated in a way so that it corresponds to the average T_2, the K-C relationship was a good predictor of k_v only after a critical volume fraction clay content. Further research is needed to quantify the errors associated with the permeability predicted from NMR data using the K-C relationship when averaging over an anisotropic material occurs.

Introduction

One of the critical needs of the 21st century is the evaluation and management of our groundwater resources. This requires improved methods for characterizing the properties of groundwater aquifers that control the storage and movement of groundwater. One such property is permeability. In groundwater applications, hydrologists most commonly measure permeability by conducting an aquifer test. There is, however, growing interest in the use of borehole and surface-based geophysical methods for estimating permeability. Borehole methods can provide high spatial resolution; surface-based methods have the appeal of being a non-invasive form of measurement. The challenge in the use of any geophysical method, is that permeability is not measured directly, but must be estimated using a relationship between the measured geophysical parameter and permeability. Of interest in our research is the estimation of permeability from the proton nuclear magnetic resonance (NMR) response of water-saturated geological materials.

There are two methods available which can be used in the field to measure the NMR response of subsurface materials: surface nuclear magnetic resonance (SNMR) and nuclear magnetic resonance logging (NML). SNMR is a non-invasive surface-based NMR method. Data are collected using a loop on the surface, about 100 m diameter, to measure the NMR response over a series of depth intervals, on the order of meters in thickness, to a total depth of ~100 m. NML uses a tool lowered in a borehole to measure the NMR response of the materials directly surrounding the borehole, with a spatial resolution on the order of ~10 cm. Both methods can be used to quantify subsurface properties such as water-filled porosity and permeability. Calculating permeability from the NMR response has been done through the use of empirical relationships based on the Kozeny-Carman (K-C) relationship (Seevers, 1966). The parameters in the K-C relationship are the ratio of the surface area (S) of the pore space to the total volume of the pore space (V), porosity and tortuosity. In an NMR measurement, S/V is one of the main parameters that controls the NMR response; this is the basis for the estimation of permeability from NMR data.

The use of the K-C relationship as the basis for estimating permeability from NMR data inherently assumes that the material is isotropic; both porosity and S/V are scalar variables and as such have no directional dependence. But it is very likely that both logging and surface NMR measurements sample over volumes in which there could exist anisotropy in permeability. The vertical resolution of NML is about 10's of centimeters whereas in SNMR, it can be meters to 10's of meters. In NML, measurement averaging may occur at the pore-scale where it is possible to have anisotropy due to the shapes of pores/grains. Both NML and SNMR measurements may average over geologic layering where permeability anisotropy, at the measurement scale, arises due to differences in layer permeability. If the NMR measurement averages over a volume of material, where permeability is anisotropic, how valid is a permeability estimate based on the K-C relationship? This study examines the robustness of the K-

C relationship as it is applied in NMR permeability calculations. Two scenarios are examined: 1) systems composed of anisotropic grains and 2) layered sand/clay systems.

The Link Between the NMR Measurement and Permeability

NMR relaxation measurements can provide information about the pore size distribution, porosity, fluid content and permeability of a water-saturated geologic material. The NMR technique is sensitive to atoms that have an odd number of protons or neutrons, and therefore, possesses an angular momentum and nuclear spin. When the objective is to measure the response of a water-saturated material, the atom of interest is hydrogen. The NMR measurement starts with the protons in a water-saturated material polarized and aligned with a static magnetic field. An external magnetic field is then applied, which rotates the magnetization into the plane perpendicular to the static field. After perturbation, the magnetization returns, or "relaxes", to its original state. The relaxation of the transverse magnetization is described in terms of a distribution of relaxation times, where each relaxation time T_2, corresponds to the response of water occupying a single pore environment. T_2 is determined by three mechanisms: relaxation in the bulk fluid, relaxation at the surface of the pore space, and relaxation due to diffusion. Because the total magnetization is proportional to the number of hydrogen protons, regions in the sampled material with higher water-filled porosity will have a larger total magnetization and therefore higher amplitude in the NMR received signal.

NMR data are typically represented by the arithmetic mean of log T_2 (T_{2ML}). In many geologic applications, where the objective is to obtain permeability estimates, the surface relaxation, with corresponding relaxation time T_{2S}, is assumed to be the dominant mechanism ($T_{2ML} \cong T_{2S}$). The surface relaxation rate is given by (Brownstein and Tarr, 1979; Kenyon et al., 1988)

$$\frac{1}{T_{2S}} = \rho \frac{S}{V}, \tag{1}$$

where ρ is the surface relaxivity. The K-C relationship that relates permeability to S/V can be written in the following form (as given by Seevers (1966); Kenyon et al. (1988)):

$$k_{K-C} = \frac{\phi}{\tau (S/V)^2}, \tag{2}$$

where ϕ is porosity and τ is tortuosity. It is the link between T_{2ML} and S/V that is the basis for estimating permeability from NMR data using the above K-C relationship.

It is important to consider the way in which the parameter S/V is calculated for use in the K-C relationship. In applications where the measured geophysical parameter is presumed to correspond directly to S/V, an average S/V can be used and is given by:

$$\left(\frac{S}{V}\right)_{avg} = \sum_i \theta_i \left(S/V\right)_i, \tag{3}$$

where θ_i is the volume fraction of the ith layer within the system with corresponding $(S/V)_i$. The amplitude weighted S/V, which is analogous to an average T_2, is given by:

$$\left(\frac{S}{V}\right)_{aw} = \frac{\sum_i \phi_i \theta_i \left(S/V\right)_i}{\sum_i \phi_i \theta_i}, \tag{4}$$

where ϕ_i is the porosity. The amplitude weighted mean log S/V, which is analogous to T_{2ml}, is given by:

$$\left(\frac{S}{V}\right)_{awML} = \exp\left(\frac{\sum_i \phi_i \theta_i \ln\left(S/V\right)_i}{\sum_i \phi_i \theta_i}\right). \tag{5}$$

In this study, we will estimate permeability from K-C using each type of S/V calculation.

Method

The relationship between permeability and S/V was examined for two scenarios, 1) anisotropic grain packs and 2) layered sand/clay systems.

<u>Anisotropic Grain Packs</u>
Anisotropic digital grain packs were created using a modified Finney pack, a random packing of uniform spheres that provides the center coordinate for each sphere (Finney, 1970). We created an anisotropic system by increasing the particle aspect ratio, which is defined as the ratio of the major axis (dx = dy) to the minor axis (dz). The center coordinates of the Finney pack were modified from a spherical grain supported pack to an oblate grain supported pack by shifting the z- axis coordinates proportionally to the minor axis. Eleven anisotropic grain packs were created in which the major axis varied from 2 to 5 mm (30 to 75 voxels), and the minor axis varied from 0.5 mm to 1.5 mm (8 to 25 voxels). The grain packs were cuboids with xy-dimensions of 10 by 10 mm (150 by 150 voxels) and y-dimensions ranging from 2 to 4.5 mm (30 to 70 voxels). There were approximately 5 to 30 grains in each pack. Vertical permeability (k_v) and horizontal permeability (k_h) were calculated numerically using the Navier-Stokes based lattice-Boltzmann (LB) method. The code used here was developed and described in detail in Keehm (2003).

Kozeny-Carman permeability ($k_{K\text{-}C}$) was calculated using numerically derived values of S/V and porosity.

Layered Sand/Clay Systems

Each layered system consisted of one horizontal layer of sand and one of clay with isotropic permeability and porosity values for each layer assigned, using values from the literature. Tortusity was set equal to 2. S/V was then calculated using the K-C relationship. For the clay layer, the permeability was 0.001 mD, the porosity was 0.4, and the S/V was 4.47×10^6 (cm^{-1}). The permeability of sand layer was 500 mD, the porosity was 0.2, and the S/V was 3.54×10^3 (cm^{-1}). The total dimensions of the system remained constant but the volume fraction of clay was varied from 0 to 1.0. Permeability for the total layered system was calculated with the K-C relationship using an average S/V (K-C_{avg}), an amplitude weighted average S/V (K-C_{aw}) (which would correspond to an average T_2), and an amplitude weighted mean log S/V (K-C_{awML}) (which would correspond to T_{2ml}). The vertical permeability of the layered system was calculated using the weighted harmonic mean given by:

$$k_v = \left(\sum_i \theta_i k_i^{-1}\right)^{-1}, \tag{6}$$

where k_i is the defined permeability of the layer and θ_i is the fraction content of that layer in the system. The horizontal permeability was calculated using the weighted arithmetic mean given by:

$$k_h = \sum_i \theta_i k_i . \tag{7}$$

Discussion

Anisotropic Grain Packs

For the scope of this study, LB derived k_v and k_h are considered the "true" permeability. $k_{K\text{-}C}$ was compared to k_v and k_h to determine the direction of permeability that better corresponds to the permeability predicted using K-C. Results are shown in Figure 1. A weak trend of permeability decreasing with increasing S/V is shown. For most of the grain packs, k_v was larger than k_h, even though the grains were elongated in the horizontal direction. Stewart, et al. (2006), also found higher k_v in body-centered anisotropic grain packs; but higher k_h in face-centered grain packs. The values of k_v and k_h are plotted against $k_{K\text{-}C}$ in Figure 2. Results show scattering along a 1:1 trend line, which indicates fairly good agreement between K-C and LB permeabilities. It is not clear from these data if permeability predicted by K-C better matches the vertical or horizonal permeability,

Layered Sand/Clay Systems

In the layered sand and clay systems, k_v and k_h were calculated using the weighted harmonic and arithmetic means and $k_{K\text{-}C}$ was estimated using the three types of S/V calculations. In this scenario, the calculated means are considered the "true" permeabilities. Results of permeability using the three different calculations of S/V in K-C (K-C_{avg}, K-C_{aw}, K-C_{awML}) are compared to the true k_v and k_h in Figure 3. Figure 3a shows on a logarithmic scale the values of permeability plotted against the volume fraction of clay on the lower x- axis and average S/V on the upper x- axis. This figure reveals the effect of a small amount of clay (< 0.1) in the system. Figure 3b shows logarithmic permeability values plotted against linear volume fraction clay on the lower –x axis and linear S/V on the upper x- axis. This figure better displays the overall effects of variations in clay content. Figure 3a shows that K-C_{waML} predicts k_v up to a clay content of 0.01. After this, K-C_{waML} begins to decrease more rapidly than k_v with increasing clay content but stays between the true k_v and k_h. At small increases in clay content, K-C_{avg} and K-C_{aw} follow a trend qualitatively similar to k_v, but do not predict either k_v or k_h well. At volume fractions of clay above 0.1, K-C_{avg} and K-C_{aw} closely follow the k_v trend, as seen in Figure 3b. For clay content above 0.2, K-C_{aw} closely matches k_v, but slightly underestimates k_v when the volume fraction of clay increases above ~0.25. In contrast, at this level of clay content K-C_{awML}, does not accurately predict k_v or k_h.

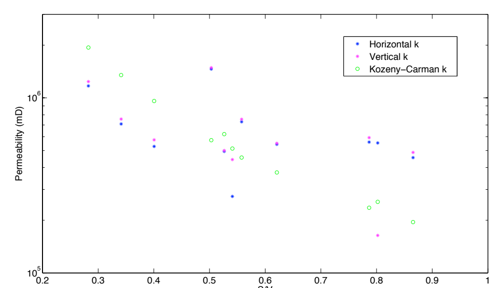


Figure 1: Results of the LB and K-C permeability from the anisotropic grain packs. Horizontal permeability is shown with a blue asterisk, vertical permeability is shown with a pink asterisk and K-C permeability is shown in green circles. In most cases, vertical permeabiliy is larger than horizontal permeabilty.

The effect of the difference between the porosity of the sand and the clay was also examined as this controls the contribution of each in the weighted calculations of S/V. If the sand porosity is held constant, a lower clay porosity results in a K-C_{awML} that is a good predictor of k_h for larger volume fractions of clay. For a clay layer with porosity of 0.2 K-C_{awML} will follow the k_h trend for volume fractions of clay up to 0.05. At lower clay porosity, K-C_{aw} becomes a poor predictor of k_v and approaches K-C_{avg}. When the clay porosity is increased and sand porosity held constant, the match between K-C_{aw} and k_v increases, and the match between K-C_{awML} and $k_{h.}$ decreases.

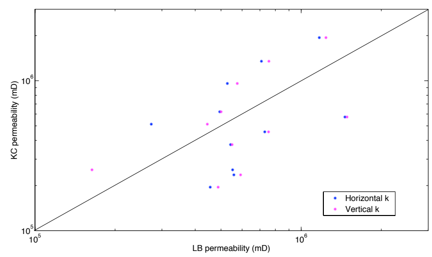
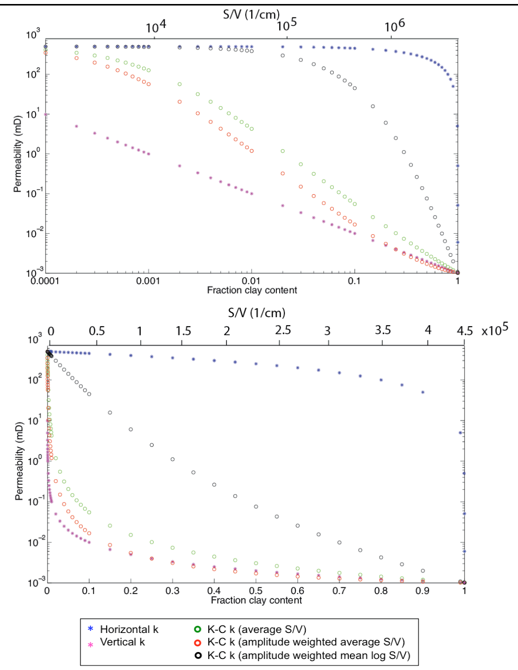


Figure 2: K-C permeability plotted against LB vertical (pink asterisk) and horizontal (blue asterisk) permeability. The black line is a 1:1 trend line. K-C is a good predictor of both vertical and horizontal permeability, with no strong directional dependence.



a)

b)

Figure 3: Permeability vs.fraction clay content (lower x- axis) and S/V (upper x- axis) for sand and clay system. True permeability values are in asterisks: horizontal in blue and vertical in pink. K-C permeability values are in circles: K-C with average S/V in green (K-Cavg), K-C with amplitude weighted average S/V in red (K-C-aw), and amplitude weighted mean log S/V in black (K-CawML). Figure 3a. A logarithmic scale on the x- axis is used to better show the effects of adding a small amount of clay into the system. For small amounts of clay content (<0.01), K-CawML is a good predictor of kh while kv is not predicted by either K-Cavg or K-Caw. Figure 3b. A linear scale is used on the x- axis to better show the effects of larger amounts of clay in the system. For large amounts of clay content (>0.01), K-Caw is a good predictor of kv, while K-CawML becomes a poor predictor of kh.

Conclusions

Due to the link between NMR measured parameters and S/V, there is potential for using NMR data to obtain estimates of permeability. This is typically done using the K-C relationship, where the NMR parameter T_{2ML} replaces S/V. This study examined the robustness of the K-C relationship as it is applied in obtaining estimates of permeability in anisotropic materials from NMR data. Two scenarios were examined, 1) systems composed of anisotropic grains and 2) layered sand/clay systems.

The results from the anisotropic grains revealed that the K-C relationship was a good predictor of both vertical and horizontal permeability, with no strong directional dependence in the quality of the agreement between K-C predictions and the true permeability. The results from the layered systems, composed of horizontal layers of sand and clay, showed strong directional dependence in the agreement between K-C predicted permeability and true permeability. When S/V was calculated in a way that corresponds to T_{2ML} we found good agreement between K-C_{awML} and the horizontal permeability up to a certain volume fraction of clay; above this level K-C_{awML} increasingly under-estimates the true permeability. This critical level of clay was found to depend upon the relative porosity of the sand and the clay, increasing as the difference between porosity values decreased. These results suggest that NMR data, from an anisotropic material, can provide good estimates of horizontal permeability, over a range of clay content.

This study highlights the need for further research to allow us to quantify errors likely to result from using NMR data to predict permeability in anisotropic materials. Given the large averaging volume of the surface NMR measurement this could be an important consideration in the further development of this method for characterization of groundwater aquifers.

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EDITED REFERENCES

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