MS&E 228: Unobserved Confounding

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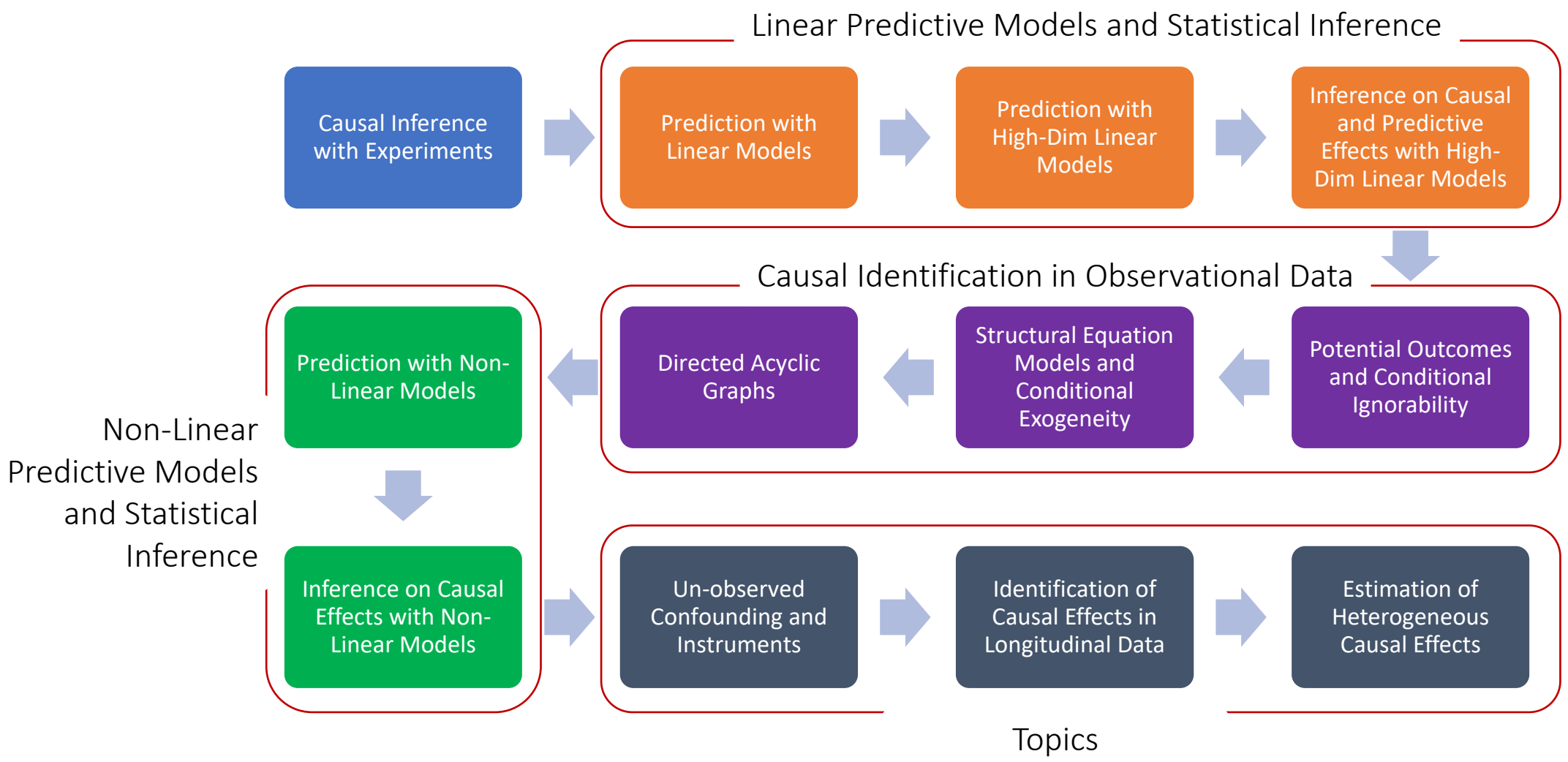
MS&E, Stanford


Linear Predictive Models and Statistical Inference
Causal Inference with Experiments
Prediction with Linear Models
Prediction with High-Dim Linear Models
Inference on Causal and Predictive Effects with High-Dim Linear Models
Causal Identification in Observational Data
Potential Outcomes and Conditional Ignorability
Structural Equation Models and Conditional Exogeneity
Directed Acyclic Graphs
Non-Linear Predictive Models and Statistical Inference
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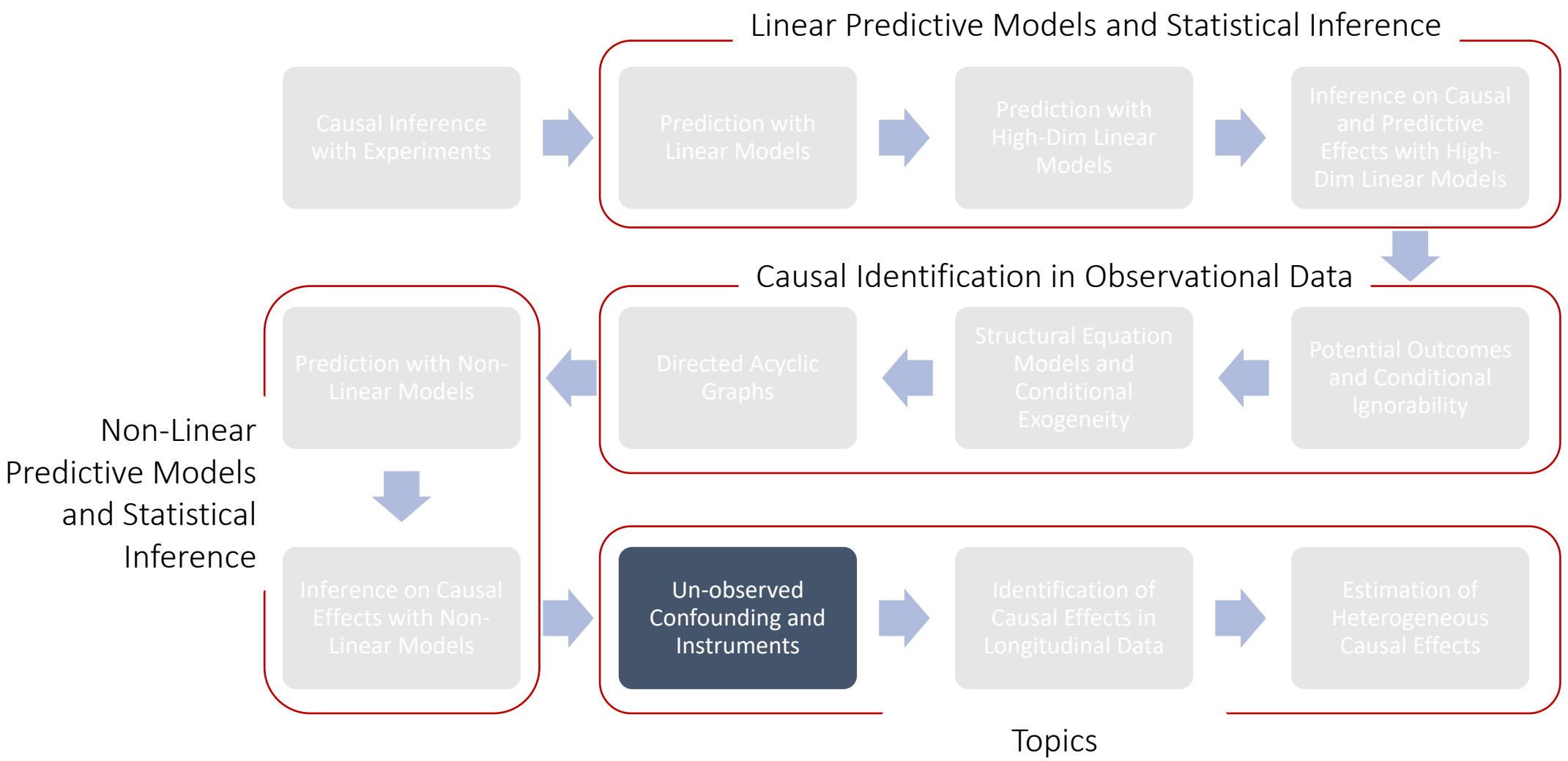
Inference on Causal Effects with Non-Linear Models
Un-observed Confounding and Instruments
Identification of Causal Effects in Longitudinal Data
Estimation of Heterogeneous Causal Effects
Topics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Topics

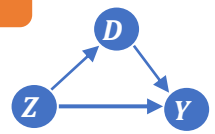
Goals for Today

- What can we do when we have un-observed confounding
- Omitted variable bias bounds
- Introduction to “Instruments”

Causal Inference Pipeline

Theory

DAGs/ASEMs



$$a_d = E[E[Y|D=d,Z]]$$

Construct estimate $\hat{g}(d,z)$ of predictive model

$$E[Y \mid D=d, Z=z]$$

Estimate mean counterfactual

$$\hat{a}_d = E_n[\hat{g}(d,Z)]$$

$$a_d \in \hat{a}_d \pm 2\hat{\sigma}$$

Practice

Data Collection → Domain Assumption Elicitation → Identification → Estimation (training) → Validation (testing) → Inference (Confidence Intervals) → Sensitivity Analysis → Decision Policy → Adaptive Experiments → Data Collection

What if my conditional ignorability assumption was violated? How much would the estimate change?

$$a_d \in \hat{a}_d \pm (2\hat{\sigma} + \epsilon)$$

Bias Bounds

Reduction in unexplained variance of Y when adding A in the model that predicts Y from treatment and controls

- The analyst provides bounds on the partial R^2

$$R^2_{Y\sim A|D,X} \leq C^2_Y, \qquad R^2_{D\sim A|X} \leq C^2_D$$

Reduction in unexplained variance of D when adding A in the model that predicts D from controls

- Based on these bounds we can conclude that

$$\theta_0 \in \theta_s \pm \sqrt{C^2_Y \frac{C^2_D}{1-C_D} \frac{E\left[\left(\tilde{Y}-\theta_s\tilde{D}\right)^2\right]}{E\left[\tilde{D}^2\right]}}$$

Measurable from the data

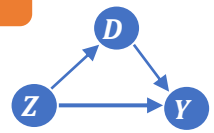
For more details:
[Making Sense of Sensitivity: Extending Omitted Variable Bias]

For more general analysis see:
[Long Story Short: Omitted Variable Bias in Causal Machine Learning]

Causal Inference Pipeline

Theory

DAGs/ASEMs



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Practice

Data Collection → Domain Assumption Elicitation → Identification → Estimation (training) → Validation (testing) → Inference (Confidence Intervals) → Sensitivity Analysis → Decision Policy → Adaptive Experiments → Data Collection

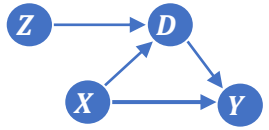
Are there other ways of identifying the treatment effect I care about (other than identification by conditioning) when I have un-observed confounding?

$$a_d \in \hat{a}_d \pm (2\hat{\sigma} + \epsilon)$$

Causal Inference Pipeline

Theory

DAGs/ASEMs



$$\alpha = \frac{E[\tilde{Y}\tilde{Z}]}{E[\tilde{D}\tilde{Z}]}$$

Estimate approximate residuals $\check{Y}, \check{Z}, \check{D}$ using cross-fitting

Solve moment restriction

$$E_n[(\check{Y} - \hat{a}\check{D})\check{Z}] = 0$$

$$a \in \hat{a} \pm 2\hat{\sigma}$$

Practice

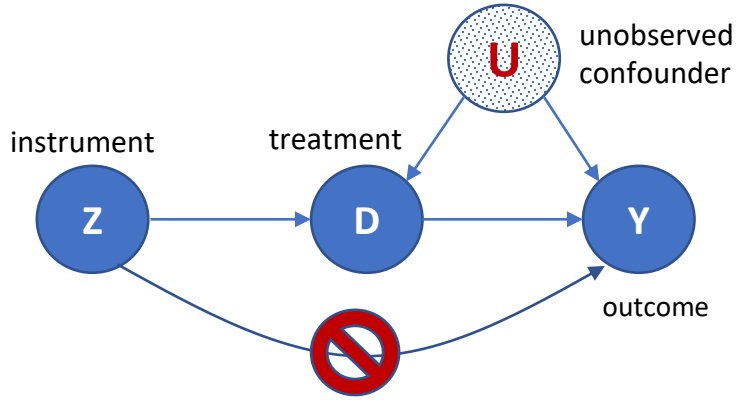
Data Collection → Domain Assumption Elicitation → Identification → Estimation (training) → Validation (testing) → Inference (Confidence Intervals) → Sensitivity Analysis → Decision Policy → Adaptive Experiments → Data Collection

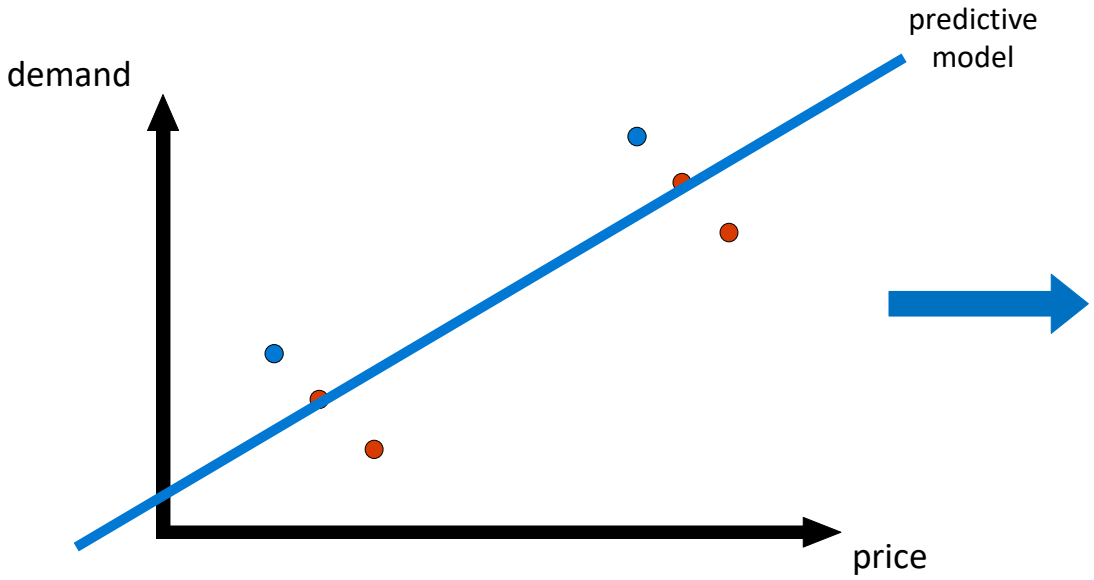
Instrumental Variable: any random variable Z that affects the treatment (log-price) D but does not affect the outcome (log-demand) Y other than through the treatment

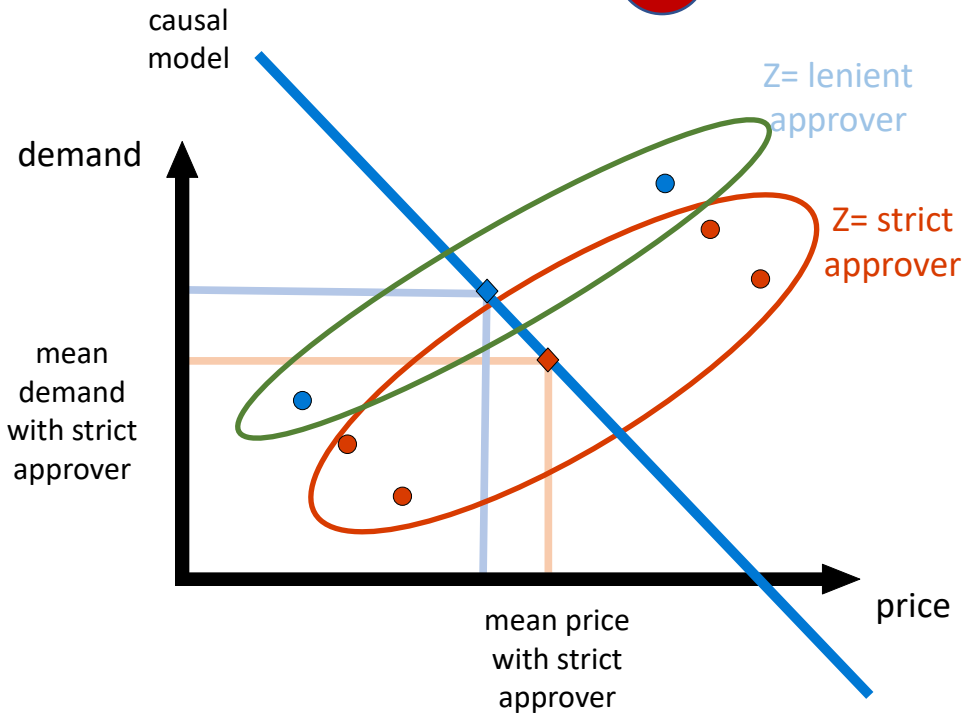
$$a \in \hat{a} \pm (2\hat{\sigma} + \epsilon)$$

Instrumental Variables and 2SLS

Instrumental Variable: any random variable Z that affects the treatment (log-price) D but does not affect the outcome (log-demand) Y other than through the treatment [Wright'28, Bowden-Turkington'90, Angrist-Krueger'91, Imbens-Angrist'94]







Partially Linear Instrumental Variable Model

- After partialling out the observed controls X

$$\tilde{Y} \coloneqq \theta_0 \tilde{D} + \tilde{A} + \epsilon_Y$$
$$\tilde{D} \coloneqq \beta \tilde{Z} + \gamma \tilde{A} + \epsilon_D$$
$$\tilde{Z} \coloneqq \epsilon_Z$$
$$\tilde{A} \coloneqq \epsilon_A$$

- Setting falls into the general moment estimation framework

$$M(\theta, h, p, m) = E\left[\Big(Y - h(X) - \theta\left(D - p(X)\right)\Big)\ \left(Z - m(X)\right)\right] = 0$$

- Where $h(X) = E[Y|X]$, $p(X) = E[D|X]$, $m(Z) = E[Z|X]$

Orthogonal Method: Double ML for IV

Double ML. Split samples in half

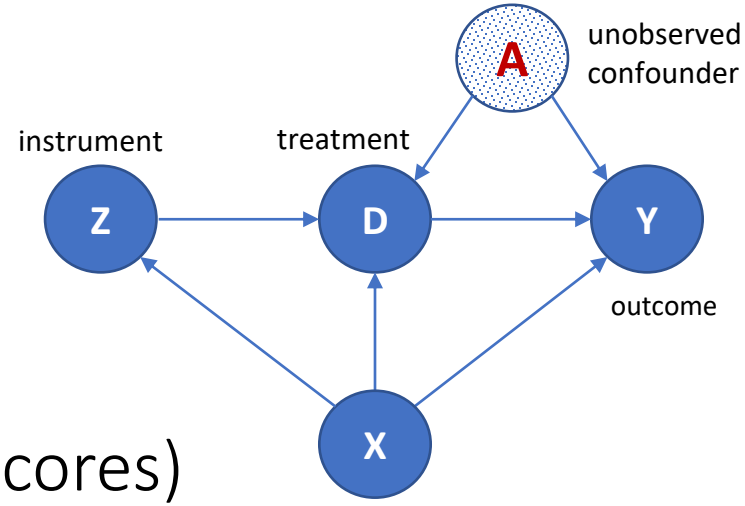
- Regress $Y \sim X$ with ML on first half, to get estimate $\hat{h}(S)$ of $E[Y|X]$
- Regress $D \sim X$ with ML on first half, to get estimate $\hat{p}(S)$ of $E[D|X]$
- Regress $Z \sim X$ with ML on first half, to get estimate $\hat{m}(S)$ of $E[Z|X]$
- Construct residuals on other half, $\hat{Z} = Z - \hat{m}(X), \hat{D} \coloneqq D - \hat{p}(X)$ and $\hat{Y} \coloneqq Y - \hat{h}(X)$
- Solve moment condition:

$$E_n\left[\left(\hat{Y} - \theta\hat{D}\right)\hat{D}\right] = 0$$

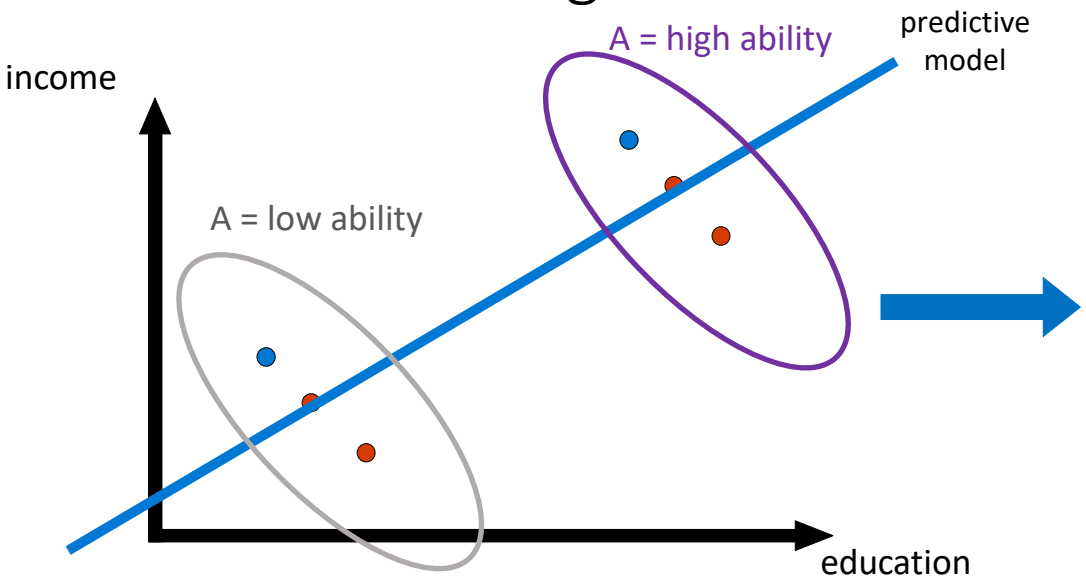
```
from econml.iv.dml import OrthoIV
orthoiv = OrthoIV()
orthoiv.fit(y, D, Z, W=X).effect_inference()
```

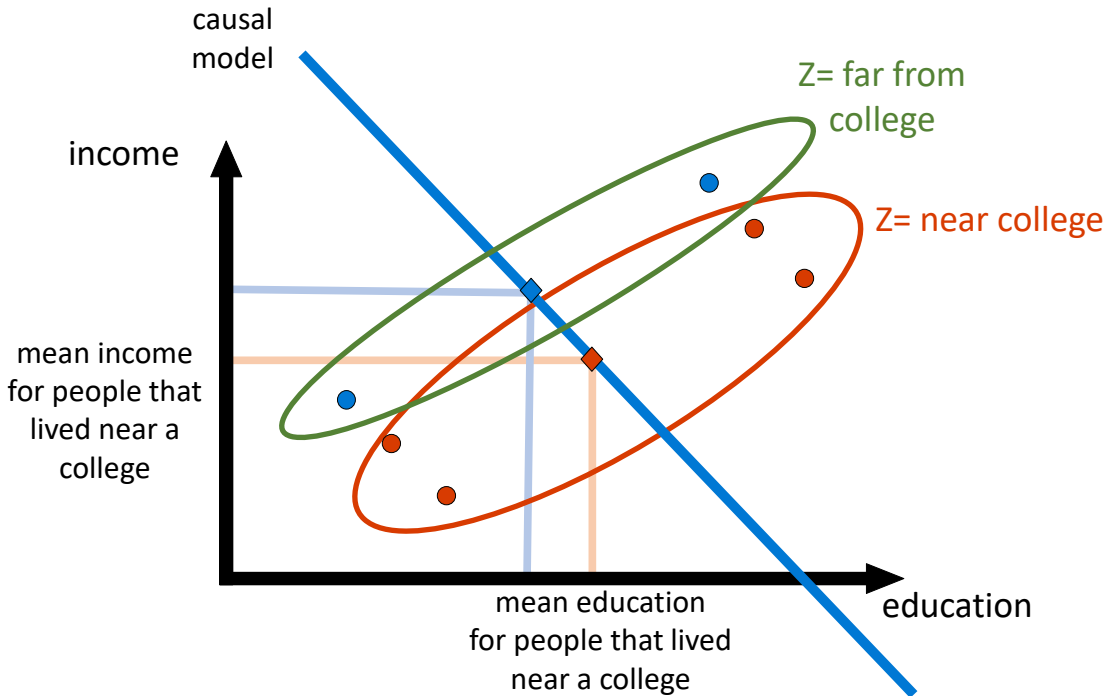
Examples

Returns to Education

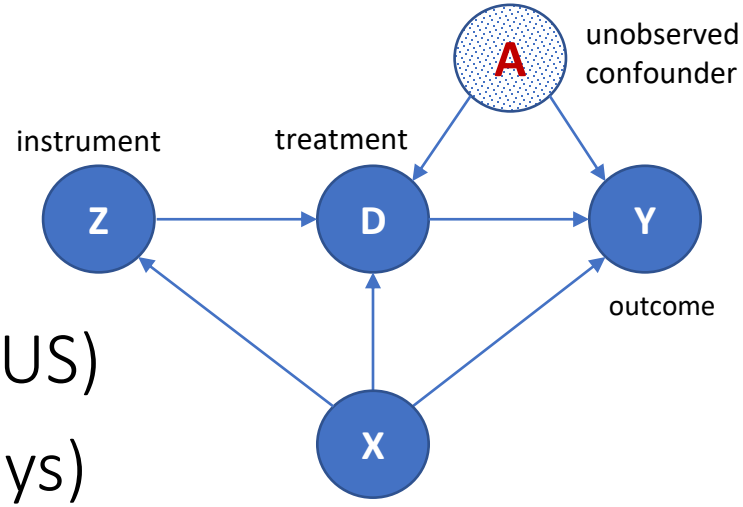


- D: years of college, Y: income
- X: observable characteristics of a student (e.g. test scores)
- A: unobserved “ability”
- Z: distance to college

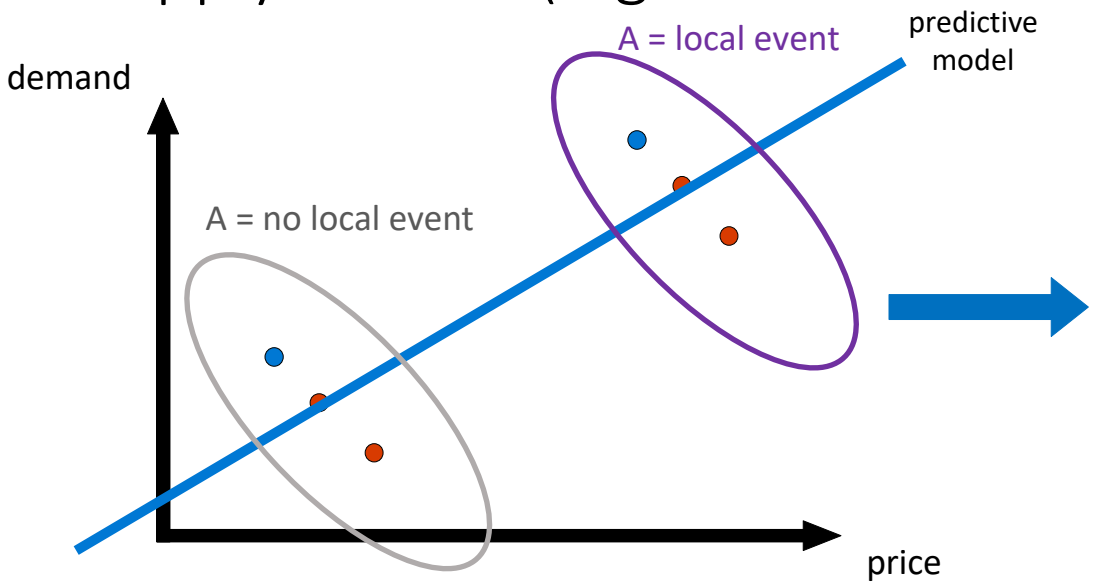


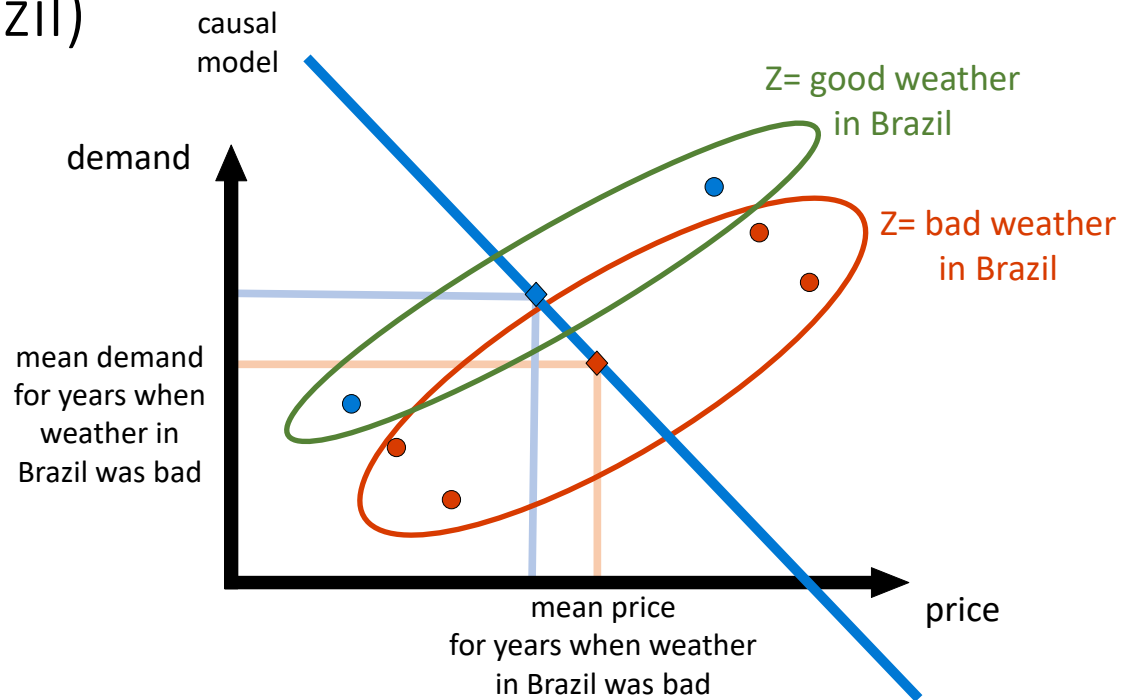


Demand Estimation

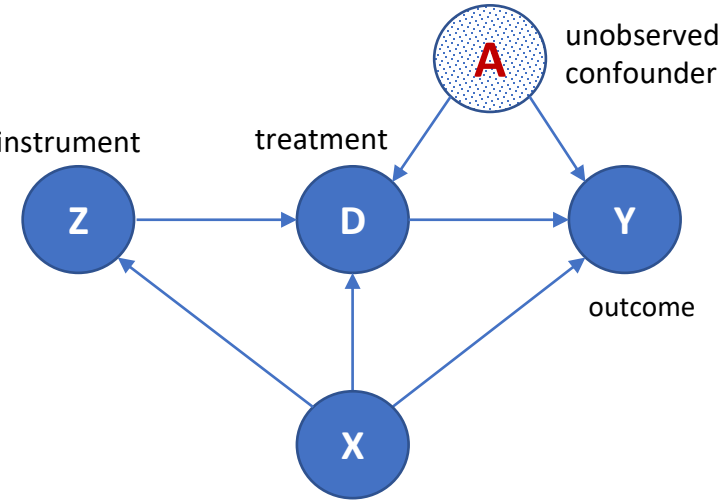


- D: price (e.g. of coffee), Y: demand (e.g. of coffee in US)
- X: observable characteristics of a market (e.g. holidays)
- A: unobserved “demand shocks” (e.g. local event)
- Z: supply shifters (e.g. weather in brazil)

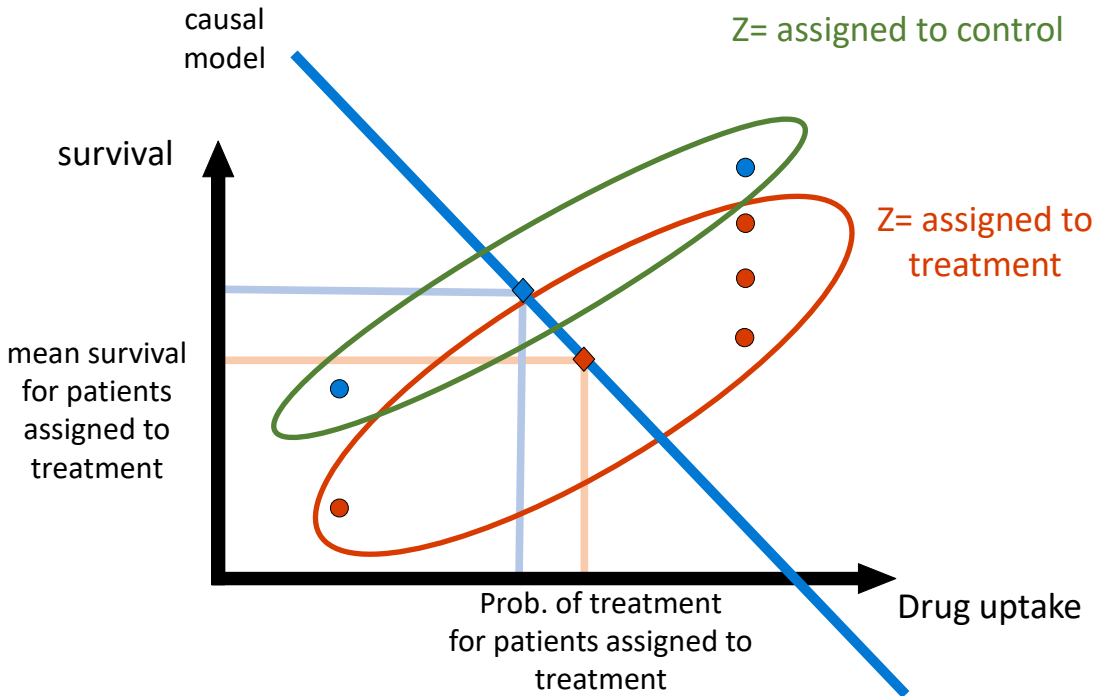




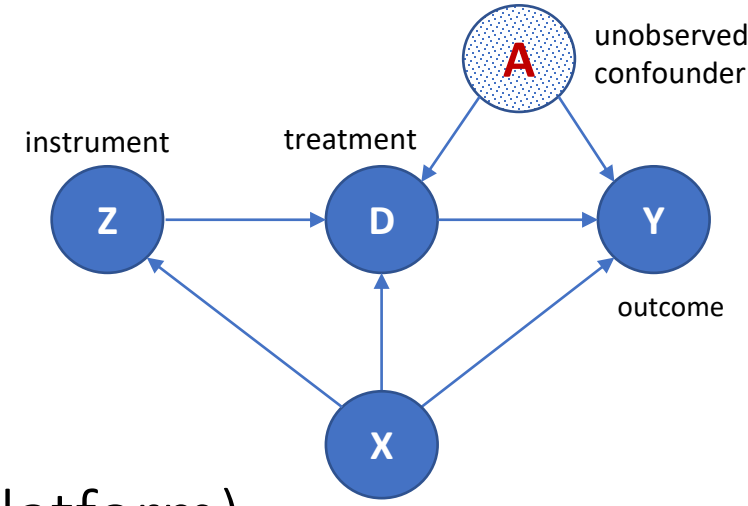
Clinical Trials with Non-Compliance



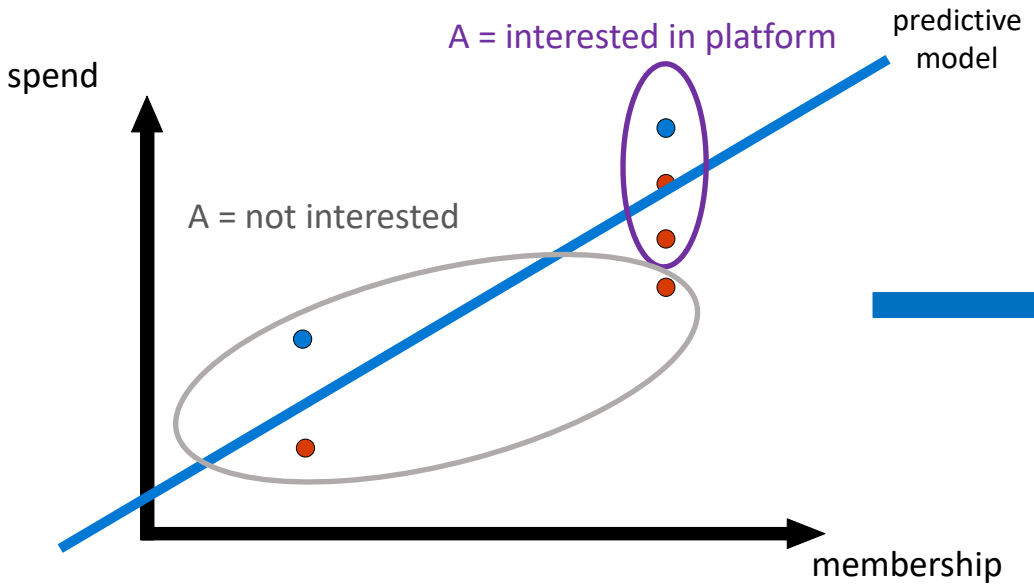
- D: drug treatment, Y: survival
- X: observable characteristics of a patient
- A: unobserved “compliance factors” (e.g. health habits)
- Z: randomized cohort assignment

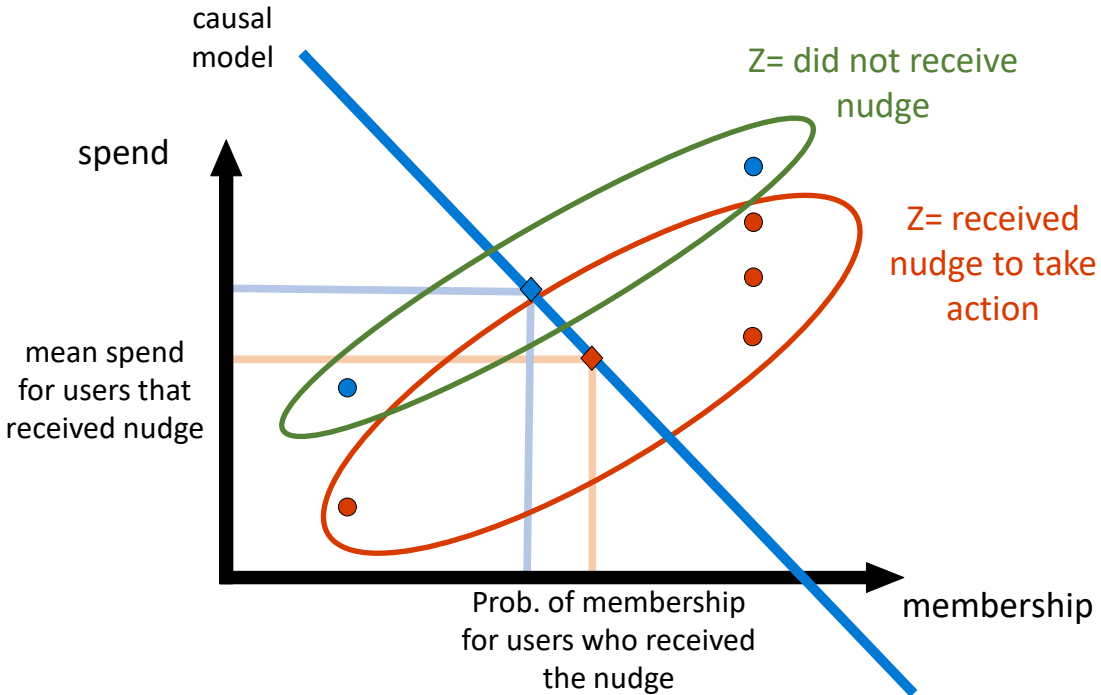


Digital Recommendation A/B tests



- D: action taken by user (e.g. membership), Y: spend
- X: observable characteristics of a user
- A: unobserved confounding factors (e.g. interest in platform)
- Z: randomized nudge to take action (e.g. one-click sign-up pop-up)





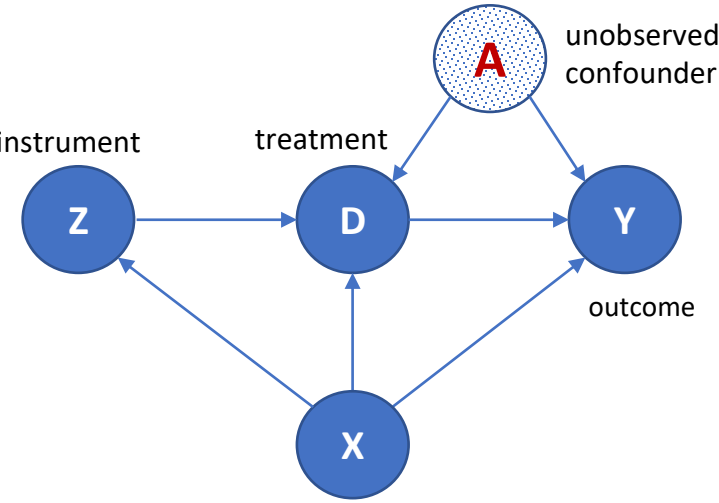
Limits of Identification via Instruments

- ATE identification via Instruments not based solely on DAG restrictions
- Requires further restrictions on structural equation models

Example

- Binary treatment D (drug) and binary instrument Z (drug recommendation)
- Consider an unobserved confounder A = “smoking”
- Suppose that smokers (A=1) never take the drug (never comply) and non-smokers (A=0) always follow the recommendation (comply)
- Suppose that drug has positive effects for non-smokers but has severe side-effects for smokers

Clinical Trials with Non-Compliance



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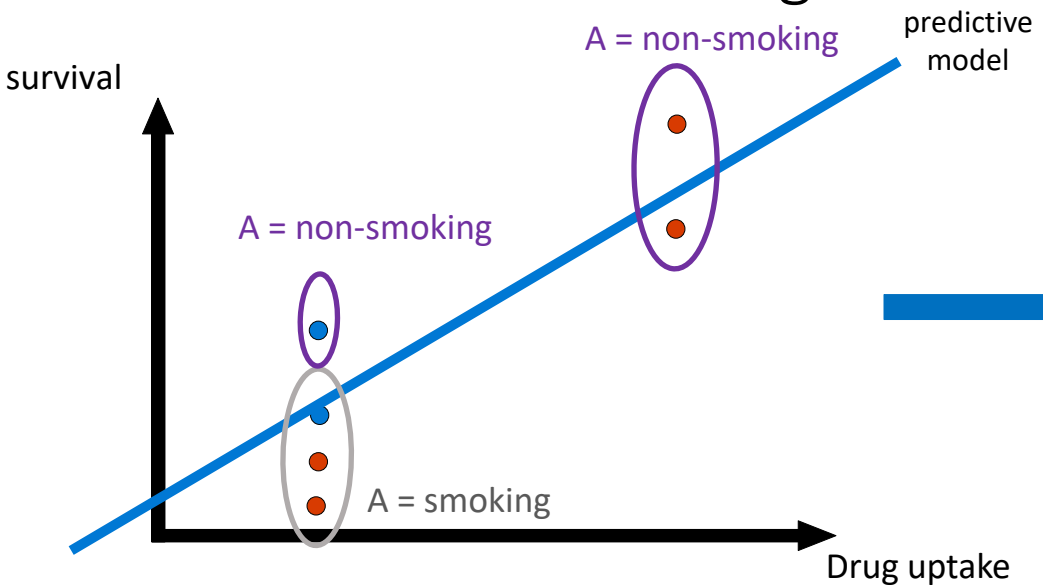

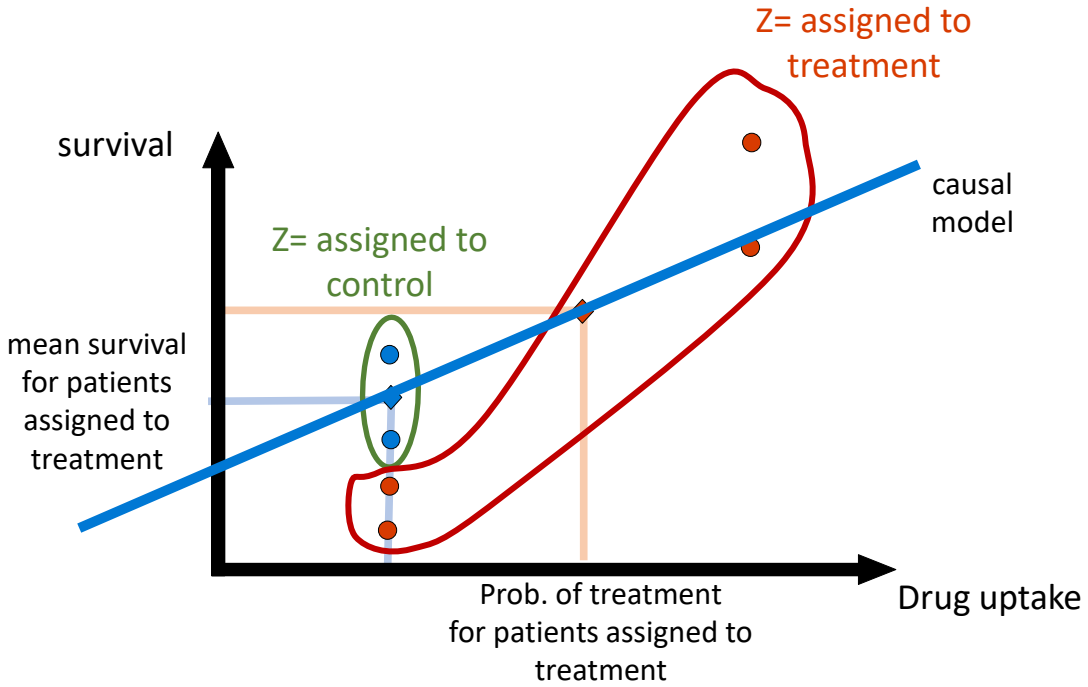

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Example

- IV regression will never be able to uncover the side effects of drug treatment on smokers
- Nothing in the data is informative of that
- Effect will be biased as compared to average effect in whole population

What do we need for ATE

- Either the compliance behavior (effect of instrument on treatment) does not vary with A (or X)
- Or the treatment effect (effect of treatment on outcome) does not vary with A (or X)

$$
\begin{aligned}
Y &:= g_Y(\epsilon_Y)\, D + f_Y(X, A, \epsilon_Y) \\
D &:= f_D(Z, X, A, \epsilon_D) \\
Z &= f_Z(X, \epsilon_Z) \\
A &:= f_A(X, \epsilon_A)
\end{aligned}
$$

$$
\begin{aligned}
Y &:= g_Y(X, A, \epsilon_Y)\, D + f(X, A, \epsilon_Y) \\
D &:= g_D(\epsilon_D) Z + f_D(X, A, \epsilon_D) \\
Z &= f_Z(X) + \epsilon_Z \\
A &:= f_A(X, \epsilon_A)
\end{aligned}
$$

Joint Variation on Observables

- If joint variation is captured through observables then ATE is feasible

$$Y \coloneqq g_Y(X, \epsilon_Y)\, D + f_Y(X, A, \epsilon_Y)$$
$$D \coloneqq f_D(Z, X, A, \epsilon_D)$$
$$Z = f_Z(X, \epsilon_Z)$$
$$A \coloneqq f_A(X, \epsilon_A)$$

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$$D \coloneqq g_D(X, \epsilon_D)Z + f_D(X, A, \epsilon_D)$$
$$Z = f_Z(X, \epsilon_Z)$$
$$A \coloneqq f_A(X, \epsilon_A)$$

- We just need to do our identification analysis conditional on X and then average

$$\beta(X) = \frac{E[\tilde{Y}\tilde{Z} \mid X]}{E[\tilde{D}\tilde{Z} \mid X]}, \qquad a = E[\beta(X)]$$

- Roughly: reweighting data based on compliance level $E[\tilde{D}\tilde{Z} \mid X]$

What if joint variation happens through unobservables?

$$
\begin{aligned}
Y &\coloneqq f_Y(D, X, A, \epsilon_Y) \\
D &\coloneqq f_D(Z, X, A, \epsilon_D) \\
Z &= f_Z(X, \epsilon_Z) \\
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\end{aligned}
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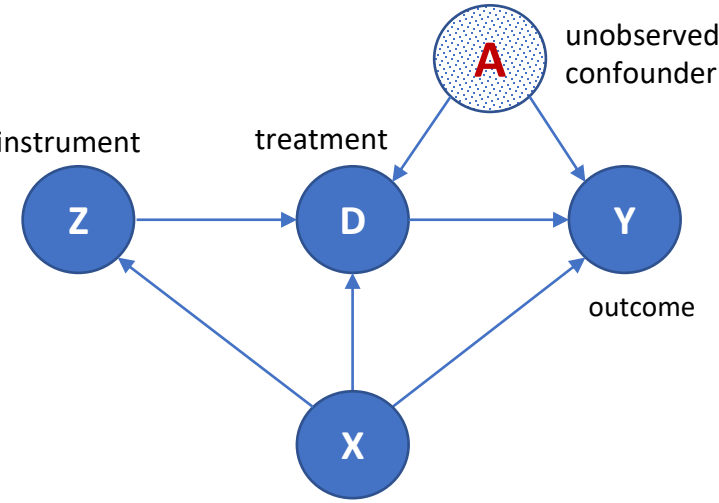
Clinical Trials with Non-Compliance



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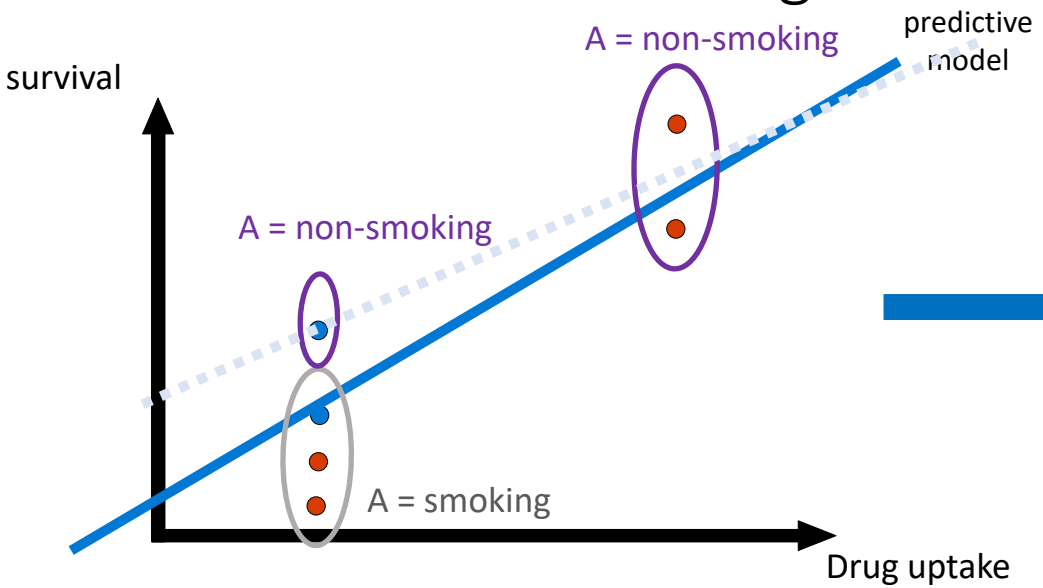
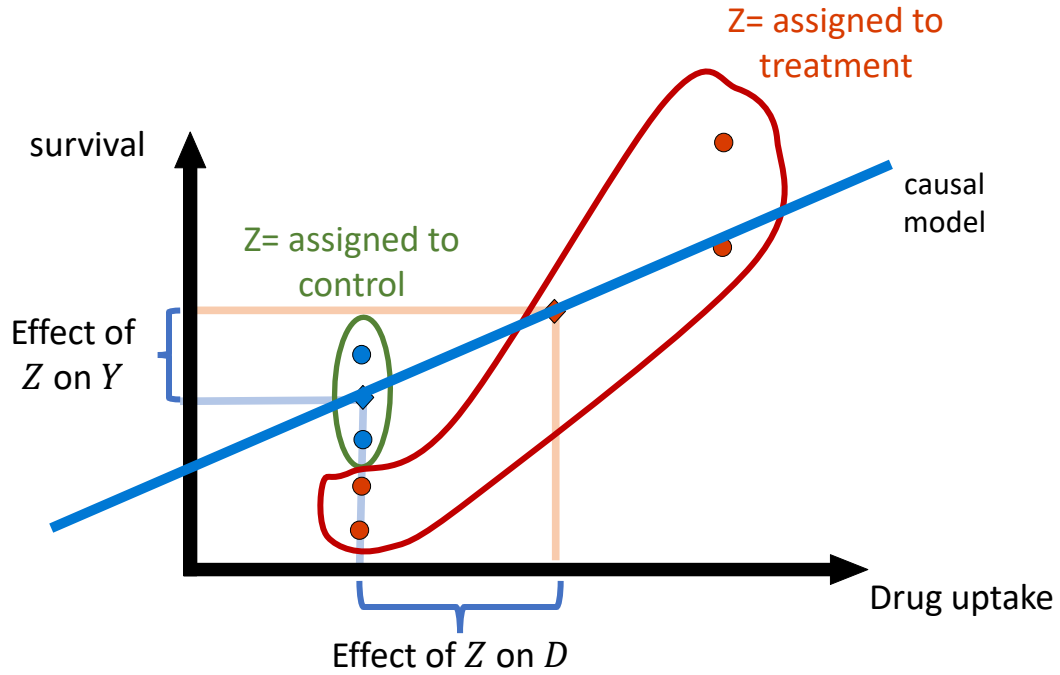
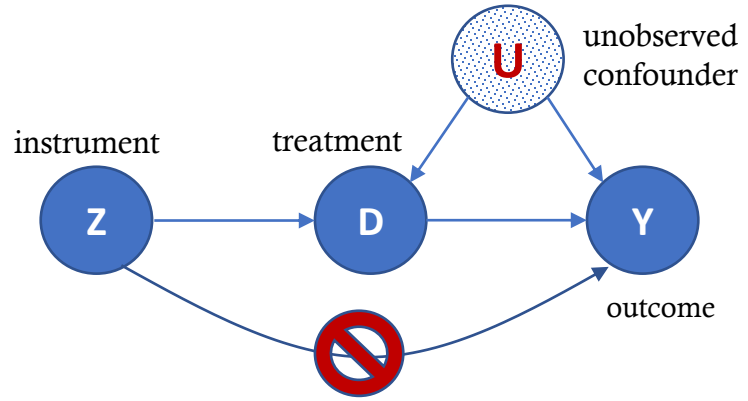




Does the IV estimate coincide with the average effect for some sub-population?

The Binary Case



Imbens-Angrist (1994): core contribution of Nobel 2022 award

- Instrument/Treatment are binary (instrument=recommended treatment)
- Assume monotonicity: $D^{(1)} \geq D^{(0)}$
- Recommended treatment cannot reverse taken treatment
- Object of interest: Local Average Treatment Effect (ATE among compliers)

$$\theta_0 = E\left[\mathrm{Y}^{(1)} - \mathrm{Y}^{(0)} \middle| \mathrm{D}^{(1)} > \mathrm{D}^{(0)}\right]$$

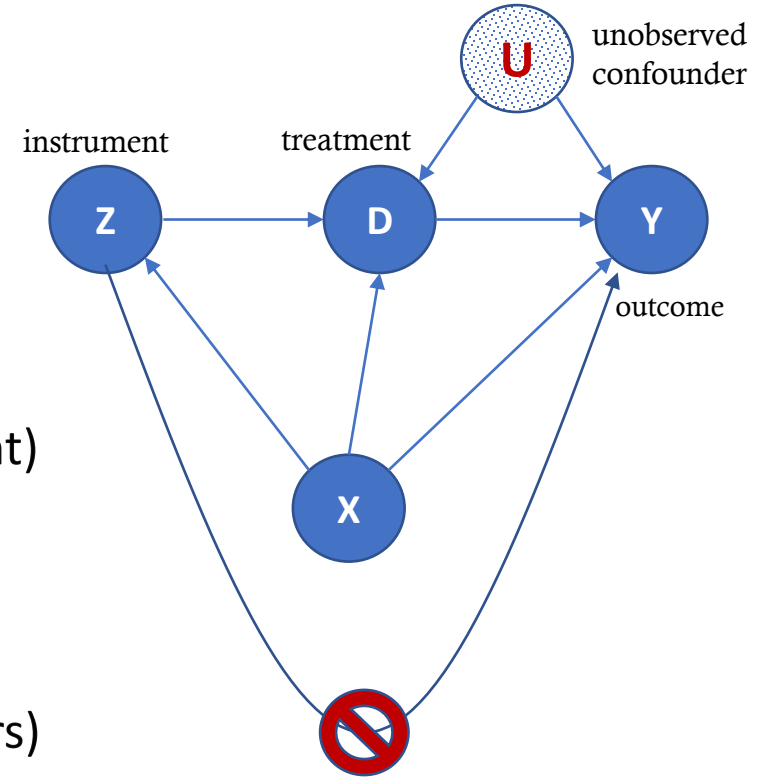
- Proof [Angrist-Imbens'94]:

$$\theta_0 = \frac{E\left[\left(Y^{(1)} - Y^{(0)}\right)1\left\{D^{(1)} > D^{(0)}\right\}\right]}{E\left[1\left\{D^{(1)} > D^{(0)}\right\}\right]} = \frac{E\left[Y^{(D(1))} - Y^{(D(0))}\right]}{E\left[D^{(1)} - D^{(0)}\right]} = \frac{ATE(Z \to Y)}{ATE(Z \to D)}$$

$$\gamma = \frac{\mathbb{E}\left[\tilde{Z}\tilde{y}\right]}{\mathbb{E}\left[\tilde{Z}^2\right]}$$

$$\delta = \frac{\mathbb{E}\left[\tilde{\mathrm{Z}}\tilde{\mathrm{D}}\right]}{\mathbb{E}\left[\tilde{\mathrm{Z}}^2\right]}$$

The Binary Case



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$$E\left[E[Y|Z=1,X] - E[Y|Z=0,X]\right]$$

$$E\left[E[D|Z=1,X] - E[D|Z=0,X]\right]$$

LATE in the Binary Case

- Under monotonicity

$$\theta_0 = \frac{E\big[E[\,Y \mid Z=1,X\,] - E[Y|Z=0,X]\big]}{E\big[E[D|Z=1,X] - E[D|Z=0,X]\big]}$$

- Moment formulation

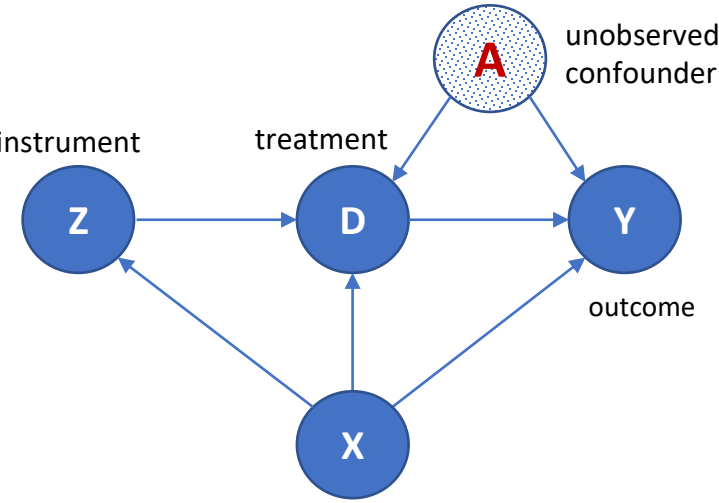
$$E[E[Y|Z=1,X] - E[Y|Z=0,X] - \theta_0(E[D|Z=1,X] - E[D|Z=0,X])] = 0$$

$$+ \boldsymbol{H(Z,X)(Y - E[Y|Z,X])} \qquad + \boldsymbol{H(Z,X)(D - E[D|Z,X])}$$

$$H(Z,X) = \frac{Z}{P(Z=1|X)} - \frac{1-Z}{1-P(Z=1|X)}$$

- Orthogonal moment formulation: apply ATE debiasing twice

Clinical Trials with Non-Compliance



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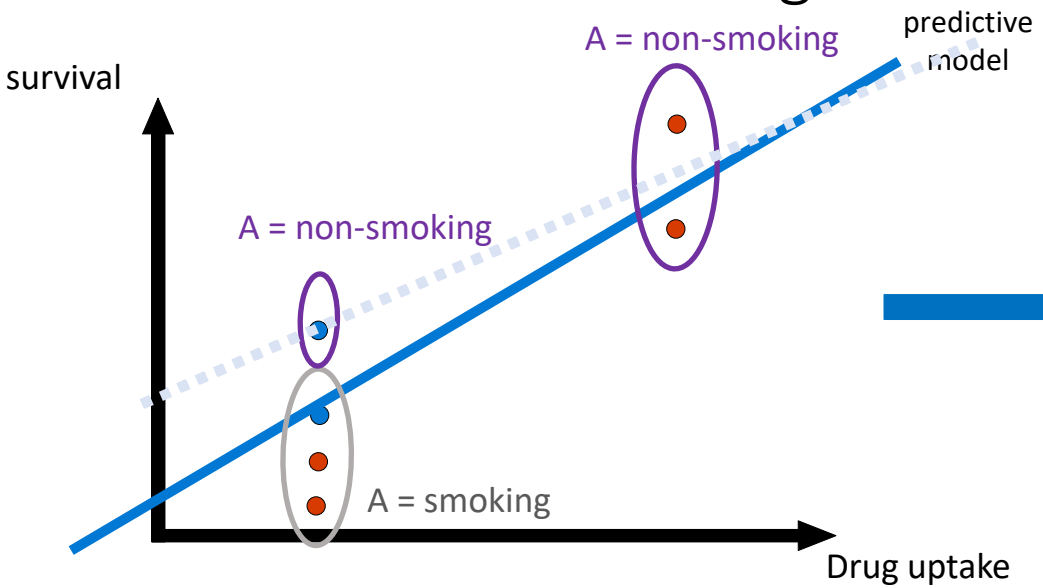
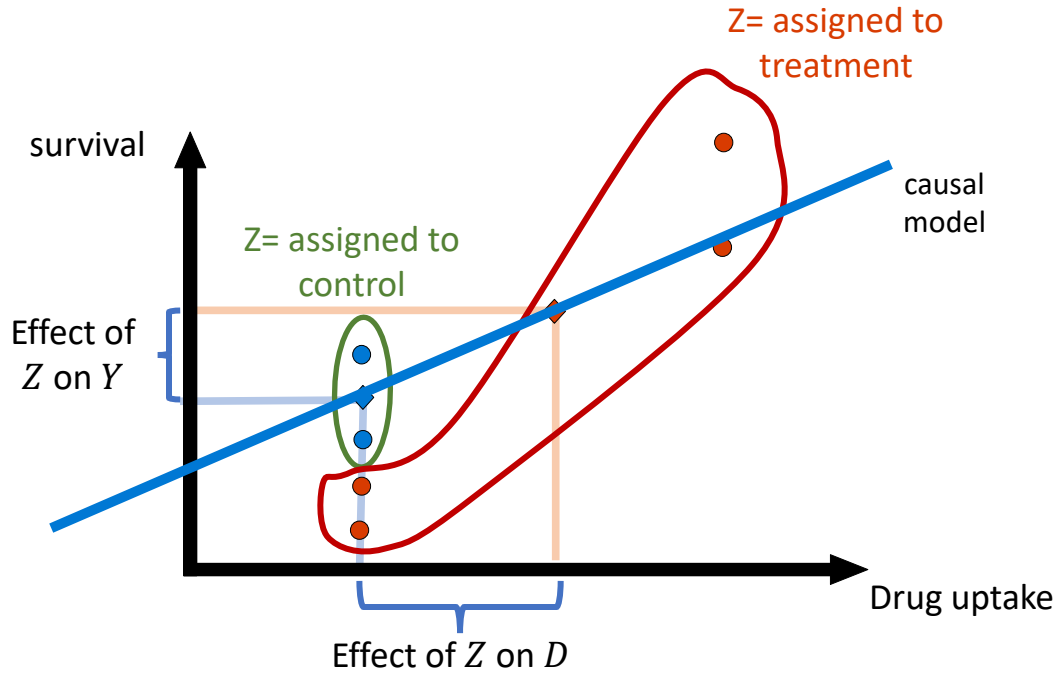
- A: unobserved “compliance factors” (e.g. health habits)
- Z: randomized cohort assignment





Confidence Intervals

Main Theorem (expanded) Define RMSE: $\|h\|_{L^2} = \sqrt{E[h(X)^2]}$

- If moment is Neyman orthogonal and RMSE of $\hat{g}$ goes down at rate $n^{1/4}$, plus regularity conditions
$$n^{1/4}\|\hat{g} - g_0\|_{L^2} \approx 0$$

- Then the estimate $\hat{\theta}$ is *asymptotically linear*
$$\sqrt{n}\left(\hat{\theta} - \theta_0\right) \approx \sqrt{n}\, E_n[\phi_0(Z)], \qquad \phi_0(Z) = -J_0^{-1}\, m(Z; \theta_0, g_0), \qquad J_0 \coloneqq \partial_\theta E[m(Z; \theta_0, g_0)]$$

- Consequently, it is *asymptotically normal*
$$\sqrt{n}\left(\hat{\theta} - \theta_0\right) \sim_a N(0, V), \qquad V \coloneqq E[\phi_0(Z)\phi_0(Z)']$$

- *Confidence intervals* for any projection based on estimate of variance are asymptotically valid

$$\ell'\theta \in \left[\ell'\hat{\theta} \pm c\sqrt{\frac{\ell'\hat{\mathbb{V}}\ell}{n}}\right], \qquad \hat{\mathbb{V}} = \mathrm{Var_n}\left(\hat{\phi}(Z)\right), \qquad \hat{\phi}(Z) \coloneqq -\hat{J}^{-1}m\left(Z; \hat{\theta}, \hat{g}\right), \qquad \hat{J} = \partial_\theta E_n\left[m\left(Z; \hat{\theta}, \hat{g}\right)\right]$$

Partially Linear Instrumental Variable Model

- Suppose that we can identify parameter of interest via moment

$$E\left[\left(\tilde{Y} - \theta_0 \tilde{D}\right)\tilde{Z}\right] = 0$$

- Setting falls into the general moment estimation framework

$$M(\theta, h, p, m) = E\left[\left(Y - h(X) - \theta\left(D - p(X)\right)\right)\ \left(Z - m(X)\right)\right] = 0$$

Where $h(X) = E[Y|X]$, $p(X) = E[D|X]$, $m(Z) = E[Z|X]$

Inference with DML in PLIV Setting

- If RMSE of propensity model and outcome model goes down at rate $n^{1/4}$, plus regularity conditions
- Then the estimate $\hat{\theta}$ is *asymptotically linear*

$$\sqrt{n}\left(\hat{\theta} - \theta_0\right) \approx \sqrt{n}\, E_n[\phi_0(Z)], \qquad \phi_0(Z) = -J_0^{-1}\left(\tilde{Y} - \theta_0 \tilde{D}\right)\tilde{Z}, \qquad J_0 := E\left[\tilde{D}\tilde{Z}\right]$$

Jacobian of moments with respect to parameter; relates to strength of instrument

- Consequently, it is *asymptotically normal*

$$\sqrt{n}\left(\hat{\theta} - \theta_0\right) \sim_a N(0, V), \qquad V := \frac{E\left[\left(\tilde{Y} - \theta_0 \tilde{D}\right)^2 \tilde{Z}^2\right]}{E\left[\tilde{D}\tilde{Z}\right]^2}$$

- *Confidence intervals* for any projection based on estimate of variance are asymptotically valid

$$\ell'\theta \in \left[\ell'\hat{\theta} \pm c\sqrt{\frac{\ell'\hat{V}\ell}{n}}\right], \qquad \hat{\mathrm{V}} = \mathrm{Var_n}\left(\hat{\phi}(Z)\right), \qquad \hat{\phi}(Z) := -\hat{J}^{-1}\left(\hat{Y} - \hat{\theta}\hat{D}\right)\hat{Z}, \qquad \hat{J} = E_n\left[\hat{D}\hat{Z}\right]$$

LATE in the Binary Case

- Under monotonicity

$$\theta_0 = \frac{E\big[E[\,Y \mid Z=1,X\,] - E[Y|Z=0,X]\big]}{E\big[E[D|Z=1,X] - E[D|Z=0,X]\big]}$$

- Moment formulation

$$E[E[Y|Z=1,X] - E[Y|Z=0,X] - \theta_0(E[D|Z=1,X] - E[D|Z=0,X])] = 0$$

$$+ \boldsymbol{H(Z,X)(Y - E[Y|Z,X])} \qquad + \boldsymbol{H(Z,X)(D - E[D|Z,X])}$$

$$H(Z,X) = \frac{Z}{P(Z=1|X)} - \frac{1-Z}{1-P(Z=1|X)}$$

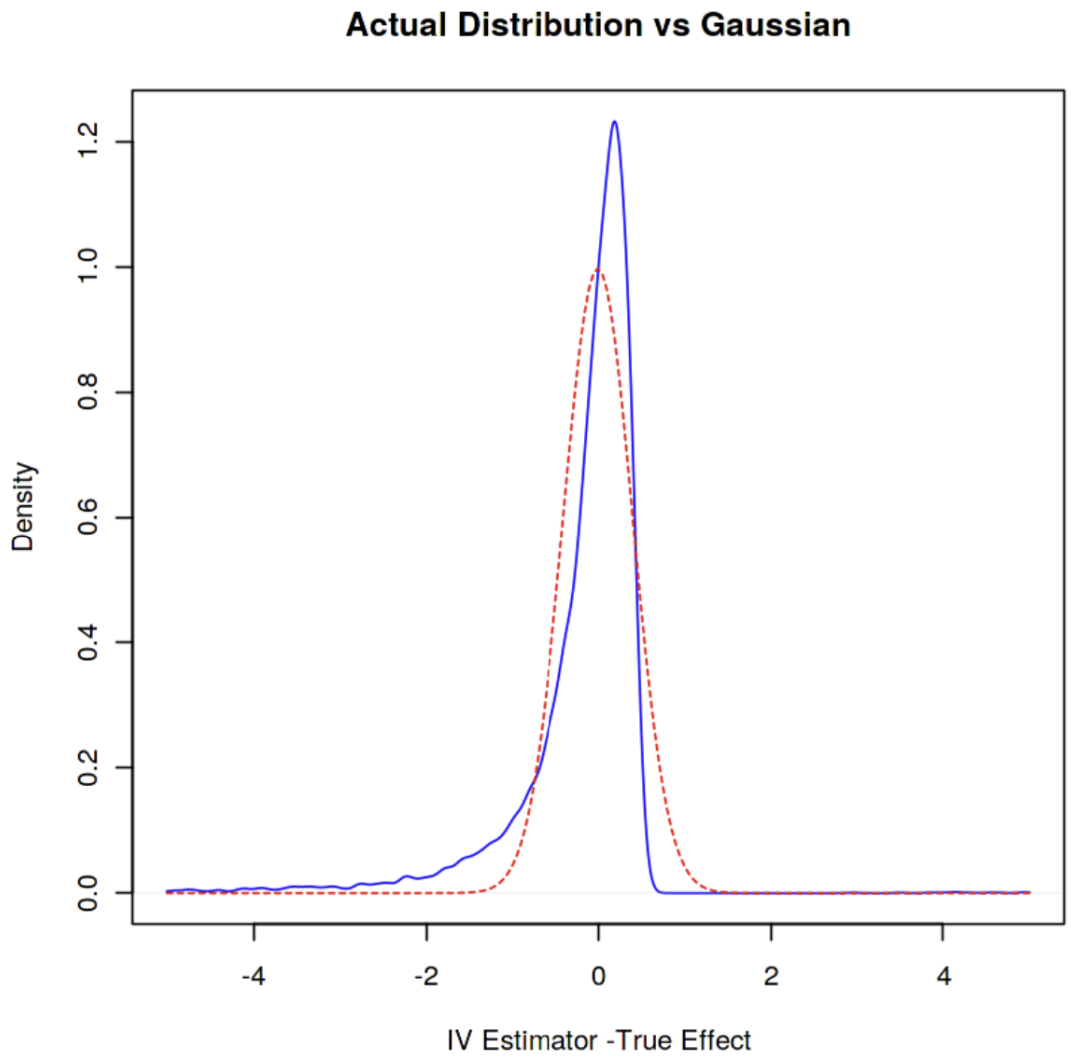
- Orthogonal moment formulation: apply ATE debiasing twice

Weak Identification

- If $\boldsymbol{E}\left[\widetilde{\boldsymbol{D}}\widetilde{\boldsymbol{Z}}\right]$ is small and comparable with the sample size, then approximation

$$E_n\left[\widetilde{D}\widetilde{Z}\right]^{-1} \approx E\left[\widetilde{D}\widetilde{Z}\right]$$

- Can be inaccurate in finite samples and normal based approximation will yield in-correct confidence intervals



A More Robust Inference Approach

- Even in the weak regime the moment constraint is still well-behaved

$$E\left[\left(\tilde{Y} - \theta\tilde{D}\right)\tilde{Z}\right]$$

- At the true parameter θ_0 we know that:

$$\mathrm{C}(\theta) \coloneqq \frac{\left(\sqrt{n}\, E_n\left[\left(\tilde{Y} - \theta\tilde{D}\right)\tilde{Z}\right]\right)^2}{Var_n\left(\left(\tilde{Y} - \theta\tilde{D}\right)\tilde{Z}\right)} \sim_a \left(N(0,1)\right)^2 = \chi^2(1)$$

- This statistic does not hinge on inversion of the Jacobian; approximation remains valid even with cross-fitted approximate residuals due to Neyman orthogonality
- We can perform a grid search over candidate parameters θ and for every such parameter test whether (for confidence interval with confidence α)

$$C(\theta) \leq (1-\alpha)\ \text{quantile of}\ \chi^2(1)$$

- Then by construction: $\Pr\left(\theta_0 \in C(\theta)\right) \approx 1 - \alpha$

General Moments and Weak Identification

- For a general Neyman orthogonal moment

$$E[m(Z;\theta_0, g_0)] = 0$$

- We can construct a statistic that is robust to weak identification (i.e. Jacobian $\partial_\theta E[m(Z;\theta_0, g_0)]$ very small)

$$C(\theta) = \frac{\left(\sqrt{n}E_n[m(Z;\theta,\hat{g})]\right)^2}{Var_n\left(m(Z;\theta,\hat{g})\right)} \sim_a \chi^2(1)$$

- Construct a α-confidence region by including all parameter values θ s.t.

$$C(\theta) \leq (1-\alpha) \text{ quantile of } \chi^2(1)$$

- Then by construction: $\Pr\left(\theta_0 \in C(\theta)\right) \approx 1 - \alpha$