MS&E 228: Unobserved Confounding and Instruments

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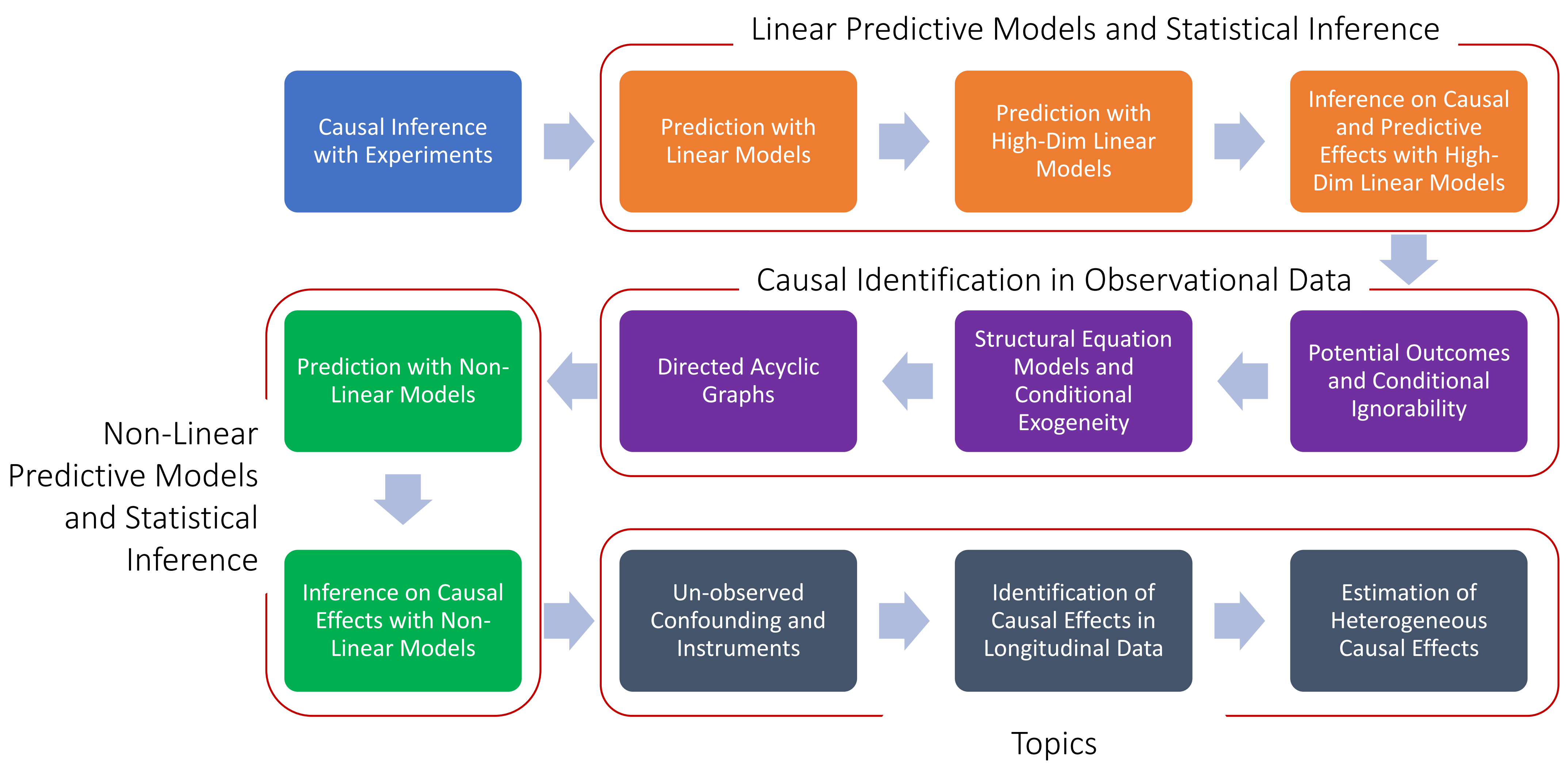

Linear Predictive Models and Statistical Inference
Causal Inference with Experiments
Prediction with Linear Models
Prediction with High-Dim Linear Models
Inference on Causal and Predictive Effects with High-Dim Linear Models
Causal Identification in Observational Data
Potential Outcomes and Conditional Ignorability
Structural Equation Models and Conditional Exogeneity
Directed Acyclic Graphs
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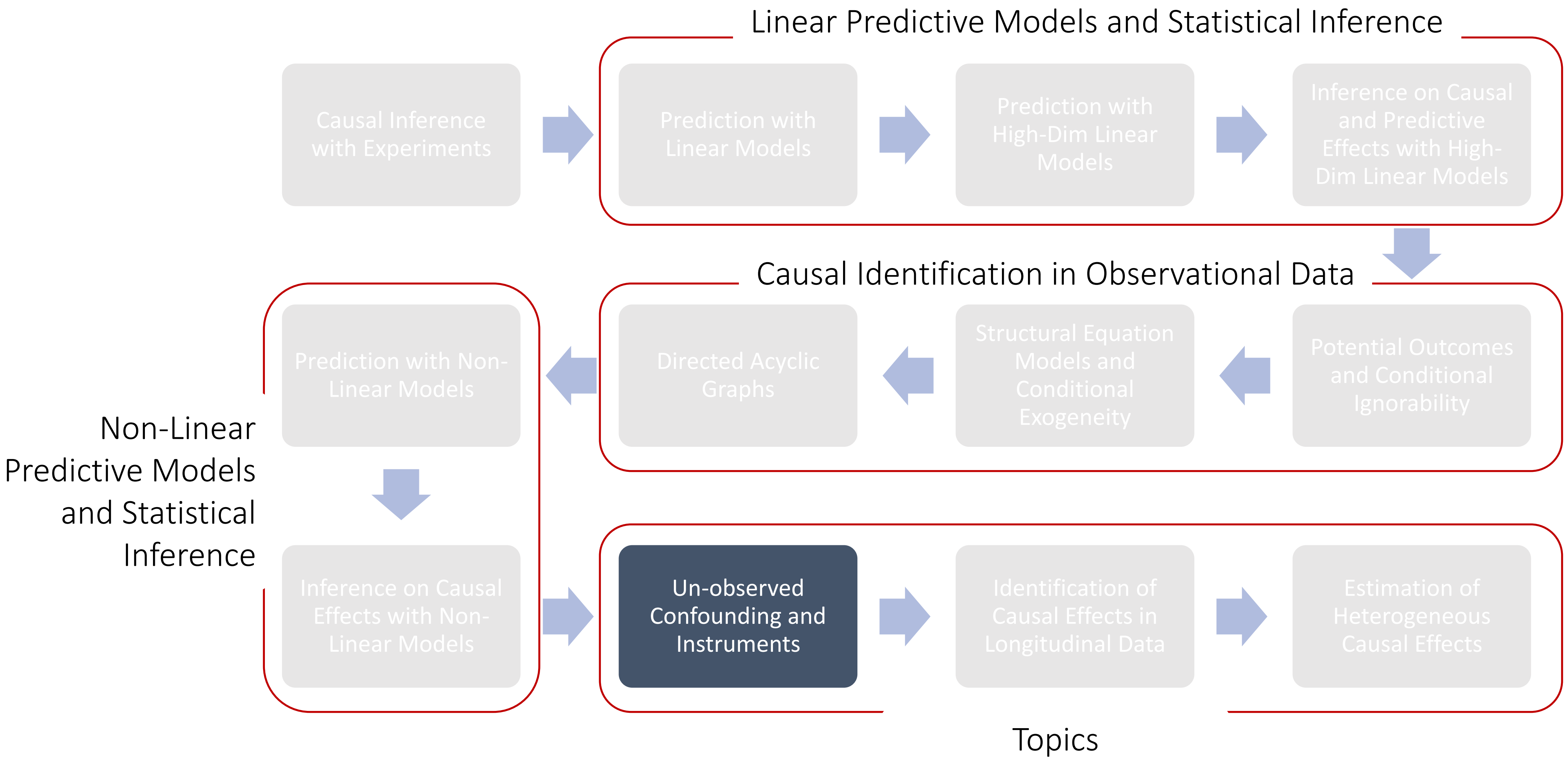
Identification of Causal Effects in Longitudinal Data
Estimation of Heterogeneous Causal Effects
Topics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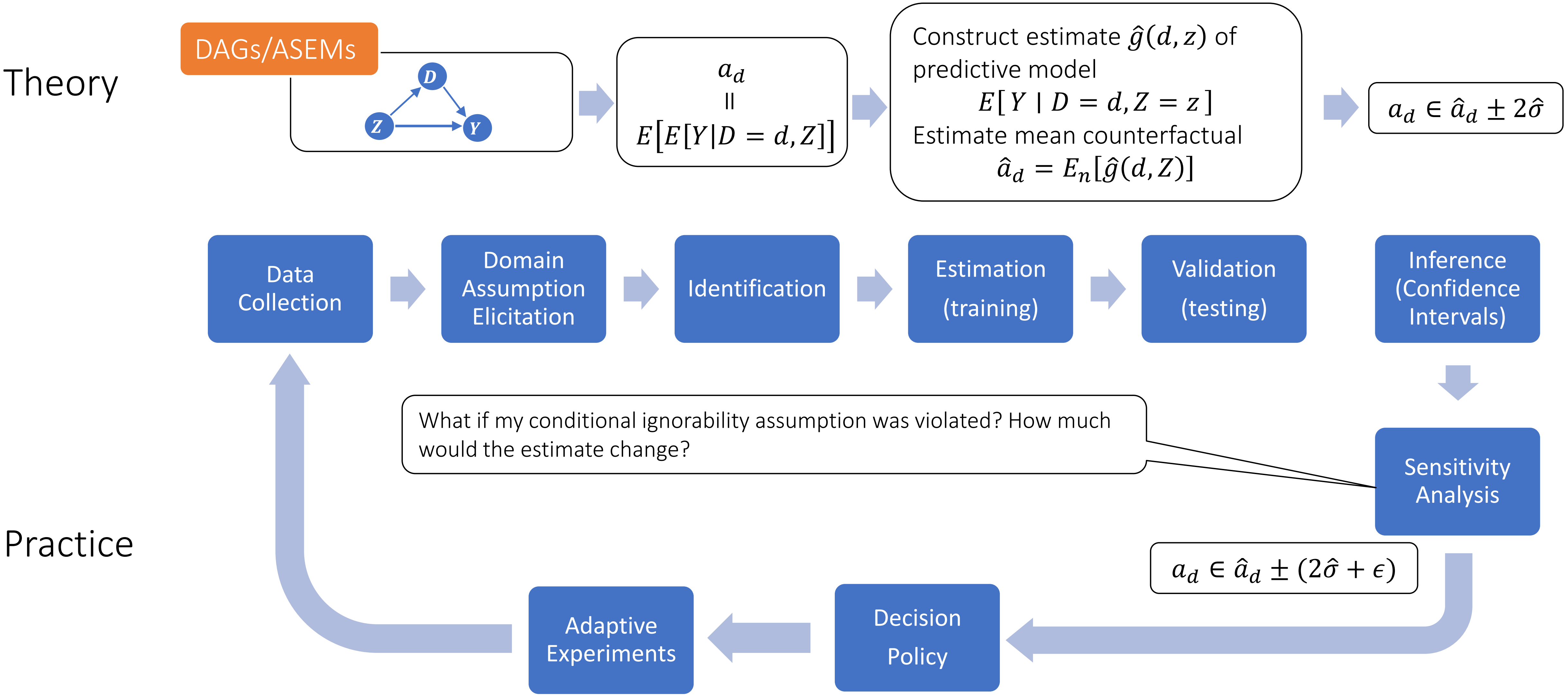
Goals for Today

- What can we do when we have un-observed confounding
- Omitted variable bias bounds
- Introduction to “Instruments”

Causal Inference Pipeline



Bias Bounds

Reduction in unexplained variance of Y when adding A in the model that predicts Y from treatment and controls

- The analyst provides bounds on the partial R^2

$$\boxed{R^2_{Y\sim A|D,X}} \leq C_Y^2, \qquad \boxed{R^2_{D\sim A|X}} \leq C_D^2$$

Reduction in unexplained variance of D when adding A in the model that predicts D from controls

- Based on these bounds we can conclude that

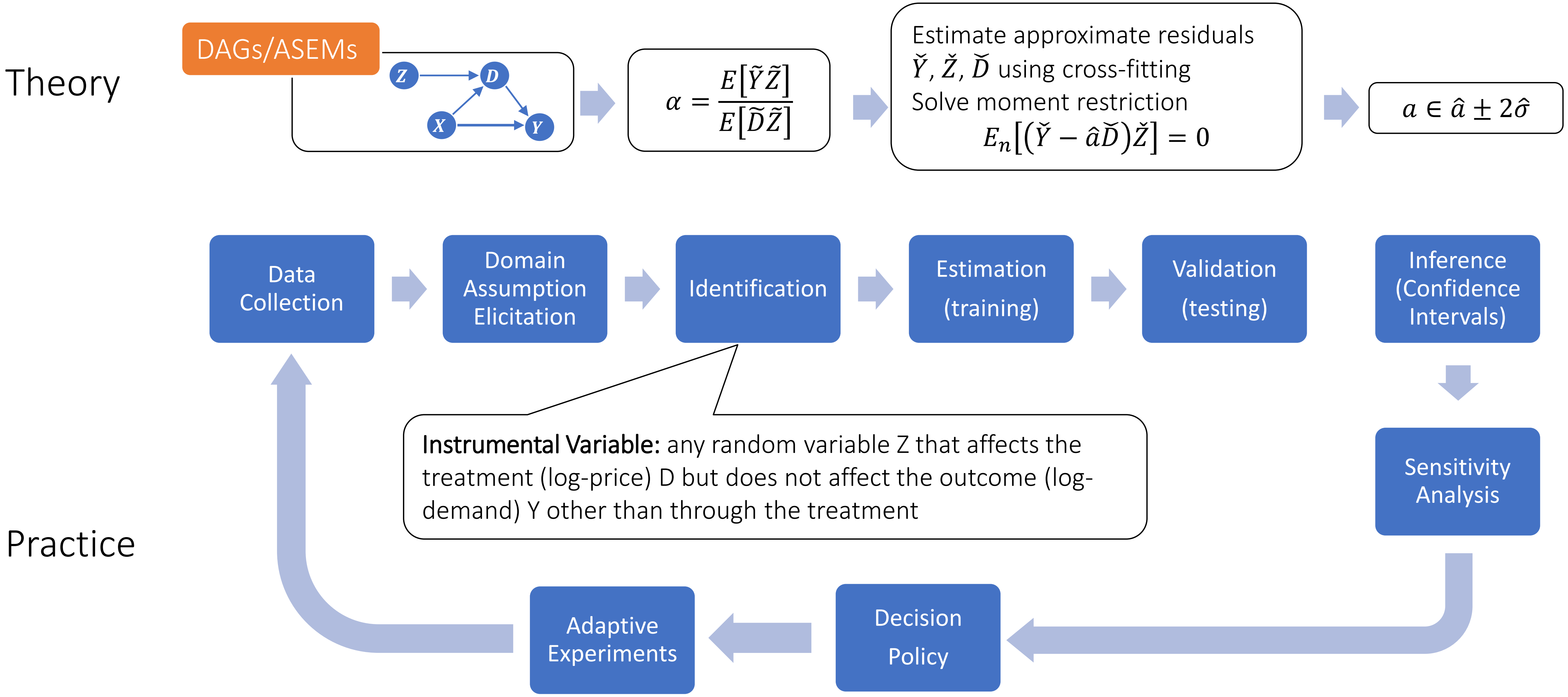
$$\theta_0 \in \theta_s \pm \sqrt{C_Y^2 \frac{C_D^2}{1 - C_D} \boxed{\frac{E\left[\left(\tilde{Y} - \theta_s \tilde{D}\right)^2\right]}{E\left[\tilde{D}^2\right]}}}$$

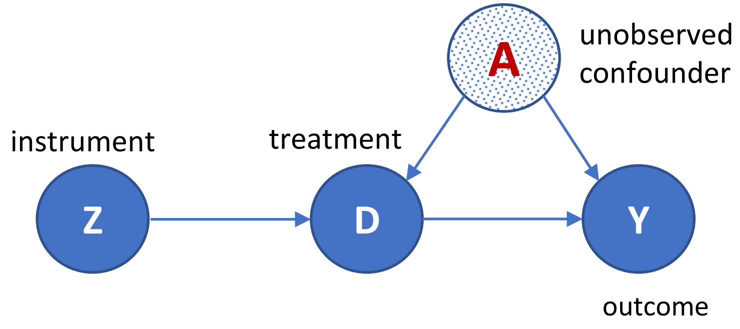
Measurable from the data

For more details:
Making Sense of Sensitivity: Extending Omitted Variable Bias

For more general analysis see:
Long Story Short: Omitted Variable Bias in Causal Machine Learning

Causal Inference Pipeline

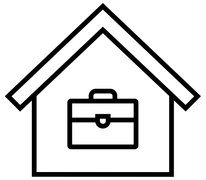




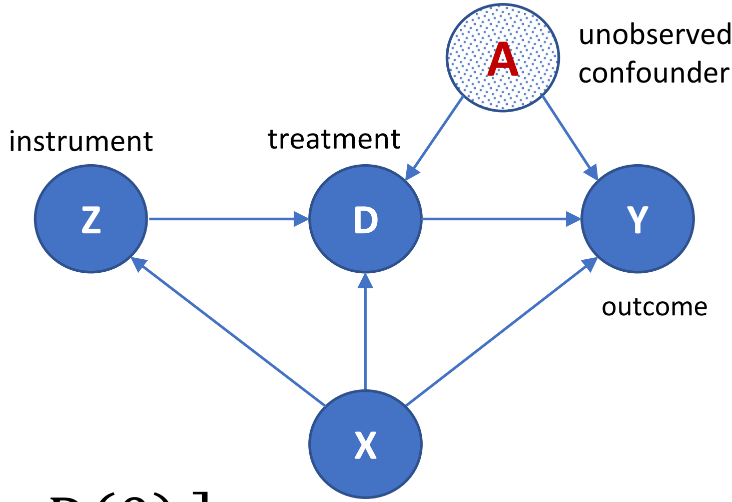
If we have access to a valid instrument Z, i.e. $Y(d), D(z) \perp\!\!\!\perp Z$ then typical approach is to estimate an effect via 2SLS ($\tilde{V} = V - E[V]$):

$$\theta = \frac{ATE(Z \rightarrow Y)}{ATE(Z \rightarrow D)} = \frac{E\left[\tilde{Y}\tilde{Z}\right]}{E\left[\tilde{D}\tilde{Z}\right]} = \frac{Cov(Y,Z)}{Cov(D,Z)}$$

If treatment and instrument are binary + monotonicity (instrument cannot “reverse” treatment choice), then this estimates the **Local Average Treatment Effect** (average effect among compliers):



$$\theta = \frac{ATE(Z \rightarrow Y)}{ATE(Z \rightarrow D)} = E[\,Y(1) - Y(0) \mid D(1) > D(0)\,]$$



If we only have a conditionally valid instrument, i.e.

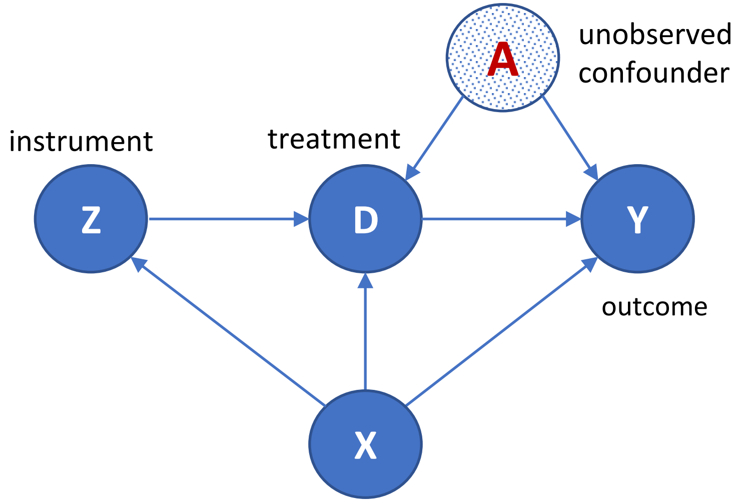
$$Y(d), D(z) \perp\!\!\!\perp Z \mid X$$

then still we have that:

$$\theta = \frac{ATE(Z \to Y)}{ATE(Z \to D)} = E[\, Y(1) - Y(0) \mid D(1) > D(0) \,]$$

But now this ratio is identified by a ratio of g-formulas

$$\theta = \frac{E\big[E[\, Y \mid Z = 1, X \,] - E[\, Y \mid Z = 0, X \,]\big]}{E\big[E[\, D \mid Z = 1, X \,] - E[\, D \mid Z = 0, X \,]\big]}$$



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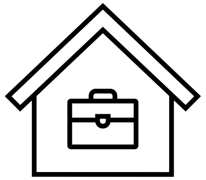
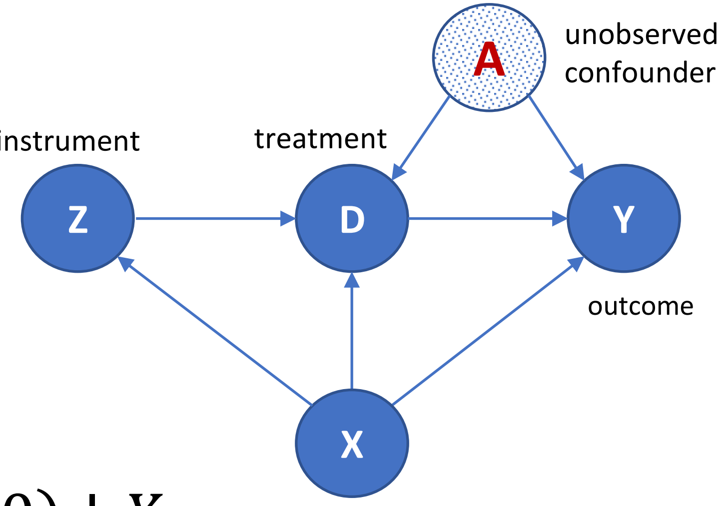


$$\theta = \frac{E\big[E[\,Y \mid Z = 1, X\,] - E[\,Y \mid Z = 0, X\,]\big]}{E\big[E[\,D \mid Z = 1, X\,] - E[\,D \mid Z = 0, X\,]\big]}$$

If we further assume that

$$Y(d = 1) - Y(d = 0) \perp\!\!\!\perp \mathrm{D(z = 1) - D(z = 0)}$$

Then this estimates the ATE



If we only have a conditionally valid instrument, i.e.

$$Y(d), D(z) \perp\!\!\!\perp Z \mid X$$

If we just assume that

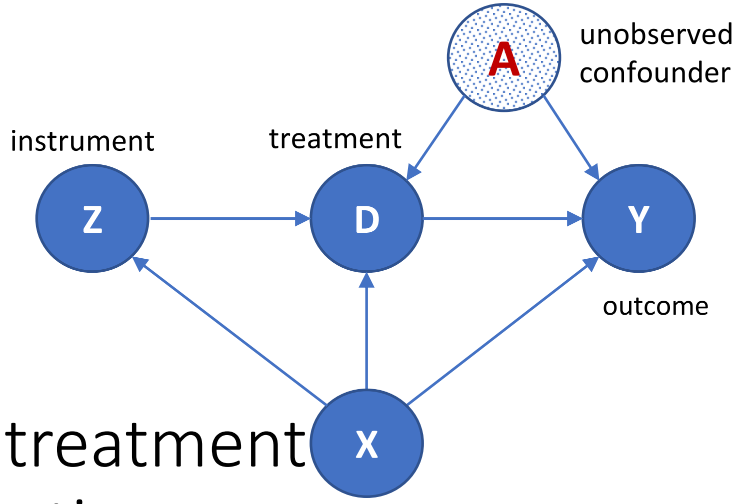
$$Y(d=1) - Y(d=0) \perp\!\!\!\perp \mathrm{D(z=1) - D(z=0)} \mid X$$

Then the ATE can be identified as:

$$\theta = E_X\left[\frac{ATE(Z \to Y \mid X)}{ATE(Z \to D \mid X)}\right] = E[Y(1) - Y(0)]$$

This is identified by the expected ratio of g-formulas

$$\theta = E_X\left[\frac{E[\,Y \mid Z = 1, X\,] - E[\,Y \mid Z = 0, X\,]}{E[\,Y \mid Z = 1, X\,] - E[\,Y \mid Z = 0, X\,]}\right]$$



If we have a possibly continuous instrument or treatment and assume the semi-parametric structural equation:

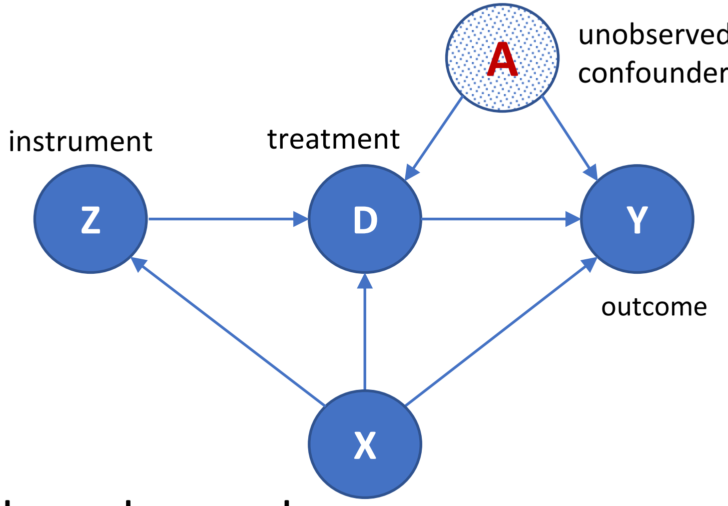
$$Y(d, x, a) \coloneqq \theta \cdot D + f_Y\left(x, a, \epsilon_y\right)$$

then we have the identifying moment condition:

$$E\left[\left(\tilde{Y} - \theta_0 \tilde{D}\right) \tilde{Z}\right] = E\left[f_Y(X, A, \epsilon_Y)\, \tilde{Z}\right] = 0$$

where $\tilde{V} = V - E[V|X]$

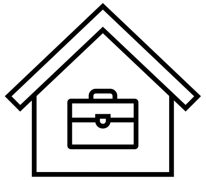
This moment is also Neyman orthogonal



If we have a possibly continuous instrument or treatment and assume the semi-parametric structural equation:

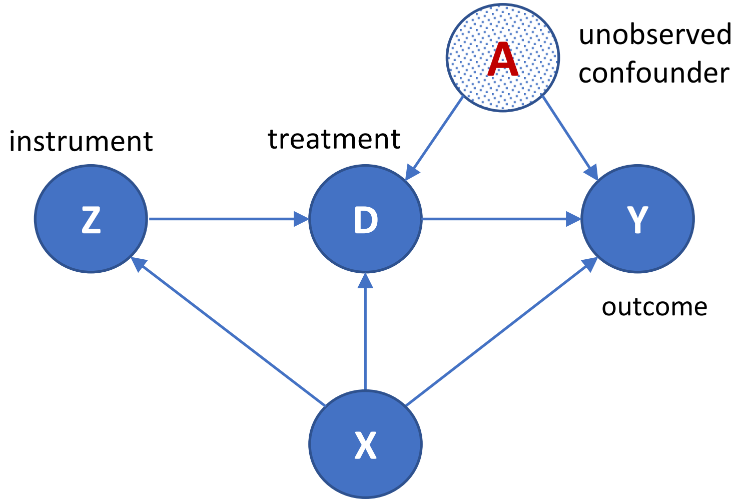
$$Y(d,x,a) \coloneqq g_Y(\epsilon_Y)\cdot D + f_Y\left(x,a,\epsilon_y\right)$$

then we have the identifying moment condition:



$$\begin{aligned} E\left[\left(\tilde{Y} - \theta\tilde{D}\right)\tilde{Z}\right] &= E\left[\left((g_Y(\epsilon_Y)-\theta)D + f_Y(X,A,\epsilon_Y)\right)\tilde{Z}\right] \\ &= E\left[(g_Y(\epsilon_Y)-\theta)\,D\tilde{Z}\right] \\ &= E[g_Y(\epsilon_Y)-\theta]\cdot E\left[D\tilde{Z}\right] \end{aligned}$$

$$E\left[\left(\tilde{Y} - \theta\tilde{D}\right)\tilde{Z}\right] = 0 \Rightarrow \theta = E[g(\epsilon_Y)] = \text{Average Marginal Effect}$$



If we have a possibly continuous instrument or treatment and assume the semi-parametric structural equation:

$$Y(d,x,a) \coloneqq g_Y(x,a,\epsilon_Y)\cdot D + f_Y\left(x,a,\epsilon_y\right)$$
$$D(z,x,a) \coloneqq g_D(\epsilon_D)\cdot Z + f_D(x,a,\epsilon_D)$$
$$Z(x) \coloneqq f_Z(x) + \epsilon_Z$$

then we can also argue that:

$$E\left[\left(\tilde{Y} - \theta\tilde{D}\right)\tilde{Z}\right] = 0 \Rightarrow \theta = E[g(X,A,\epsilon_Y)] = \text{Average Marginal Effect}$$

Orthogonal Method: Double ML for IV

Double ML. Split samples in half

- Regress $Y \sim X$ with ML on first half, to get estimate $\hat{h}(S)$ of $E[Y|X]$
- Regress $D \sim X$ with ML on first half, to get estimate $\hat{p}(S)$ of $E[D|X]$
- Regress $\mathrm{Z} \sim X$ with ML on first half, to get estimate $\widehat{m}(S)$ of $E[Z|X]$
- Construct residuals on other half, $\hat{Z} = Z - \widehat{m}(X), \widehat{D} \coloneqq D - \hat{p}(X)$ and $\hat{Y} \coloneqq Y - \hat{h}(X)$
- Solve moment condition:

$$E_n[(\hat{Y} - \theta\widehat{D})\widehat{D}] = 0$$

Inference with DML in PLIV Setting

- The estimate can be written as:

$$\hat{\theta} = \frac{E_n\big[\hat{Y}\hat{Z}\big]}{E_n\big[\hat{Z}\widehat{D}\big]}$$

- If RMSE of propensity models and outcome model goes down at rate $n^{1/4}$, plus regularity conditions

$$\sqrt{n}\,\big(\hat{\theta} - \theta_0\big) \sim_a N(0, V), \qquad V := \frac{E\Big[\big(\tilde{Y} - \theta_0\widetilde{D}\big)^2\tilde{Z}^2\Big]}{E\big[\widetilde{D}\tilde{Z}\big]^2}$$

- *Confidence intervals* for any projection based on estimate of variance are asymptotically valid

$$\ell'\theta \in \left[\ell'\hat{\theta} \pm c\sqrt{\frac{\ell'\hat{V}\ell}{n}}\right], \qquad \widehat{\mathrm{V}} = \frac{E_n\Big[\big(\hat{Y} - \hat{\theta}\,\widehat{D}\big)^2\hat{Z}^2\Big]}{E_n\big[\hat{Z}\widehat{D}\big]^2}$$

LATE in the Binary Case

- Under monotonicity

$$\theta_0 = \frac{E\left[E[Y \mid Z = 1, X] - E[Y|Z = 0, X]\right]}{E\left[E[D|Z = 1, X] - E[D|Z = 0, X]\right]}$$

- Moment formulation

$$E[E[Y|Z = 1, X] - E[Y|Z = 0, X] - \theta_0(E[D|Z = 1, X] - E[D|Z = 0, X])] = 0$$

$$+ \qquad\qquad\qquad\qquad +$$

$$\boldsymbol{a(Z,X)(Y - E[Y|Z,X])} \qquad\qquad \boldsymbol{a\,(Z,X)(D - E[D|Z,X])}$$

$$a(Z, X) = \frac{Z}{P(Z = 1|X)} - \frac{1 - Z}{1 - P(Z = 1|X)}$$

- Orthogonal moment formulation: apply ATE debiasing twice

Inference on LATE in the Binary Case

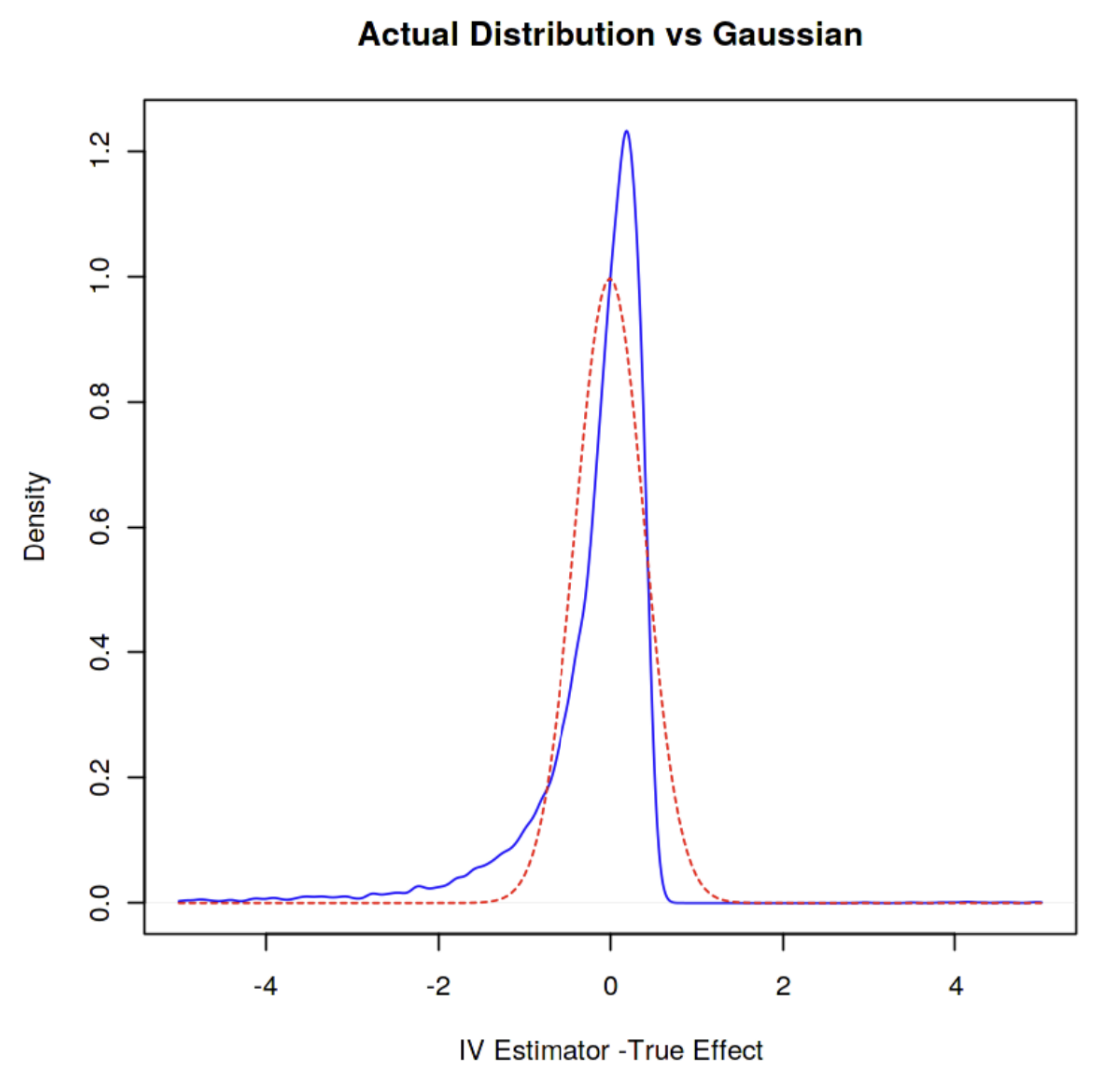
$$\hat{\theta} = \frac{E_n\left[\hat{h}_{Z\to Y}(1,X) - \hat{h}_{Z\to Y}(0,X) + a(Z,X)\left(Y - \hat{h}_{Z\to Y}(Z,X)\right)\right]}{E_n\left[\hat{h}_{Z\to D}(1,X) - \hat{h}_{Z\to D}(0,X) + a(Z,X)\left(D - \hat{h}_{Z\to D}(Z,X)\right)\right]}$$

- If RMSE of propensity models and outcome model goes down at rate $n^{1/4}$, plus regularity conditions

$$\sqrt{n}\left(\hat{\theta} - \theta_0\right) \sim_a N(0,V)$$

Weak Identification

- If $\boldsymbol{E}[\widetilde{\boldsymbol{D}}\widetilde{\boldsymbol{Z}}]$ is small and comparable with the sample size, then approximation
$$E_n[\widetilde{D}\widetilde{Z}]^{-1} \approx E[\widetilde{D}\widetilde{Z}]^{-1}$$
- Can be inaccurate in finite samples and normal based approximation will yield in-correct confidence intervals



A More Robust Inference Approach

- Even in the weak regime the moment constraint is still well-behaved
$$E\left[\left(\tilde{Y} - \theta\widetilde{D}\right)\tilde{Z}\right]$$
- At the true parameter θ_0 we know that:
$$\mathrm{C}(\theta) := \frac{\left(\sqrt{n}\, E_n\left[\left(\tilde{Y} - \theta\widetilde{D}\right)\tilde{Z}\right]\right)^2}{Var_n\left(\left(\tilde{Y} - \theta\widetilde{D}\right)\tilde{Z}\right)} \sim_a \left(N(0,1)\right)^2 = \chi^2(1)$$
- This statistic does not hinge on inversion of $E\left[\widetilde{D}\tilde{Z}\right]$; approximation remains valid even with cross-fitted approximate residuals due to Neyman orthogonality
- We can perform a grid search over candidate parameters θ and for every such parameter test whether (for confidence interval with confidence α)
$$C(\theta) \leq (1-\alpha) \text{ quantile of } \chi^2(1)$$
- Then by construction: $\Pr\left(\theta_0 \in C(\theta)\right) \approx 1 - \alpha$

General Moments and Weak Identification

- For a general Neyman orthogonal moment
$$E[m(Z;\theta_0,g_0)]=0$$
- We can construct a statistic that is robust to weak identification (i.e. Jacobian $\partial_\theta E[m(Z;\theta_0,g_0)]$ very small)
$$C(\theta)=\frac{\left(\sqrt{n}E_n[m(Z;\theta,\hat{g})]\right)^2}{Var_n\left(m(Z;\theta,\hat{g})\right)}\sim_a\chi^2(1)$$
- Construct a α-confidence region by including all parameter values θ s.t.
$$C(\theta)\leq(1-\alpha)\text{ quantile of }\chi^2(1)$$
- Then by construction: $\Pr\left(\theta_0\in C(\theta)\right)\approx 1-\alpha$