MS&E 228: Unobserved Confounding

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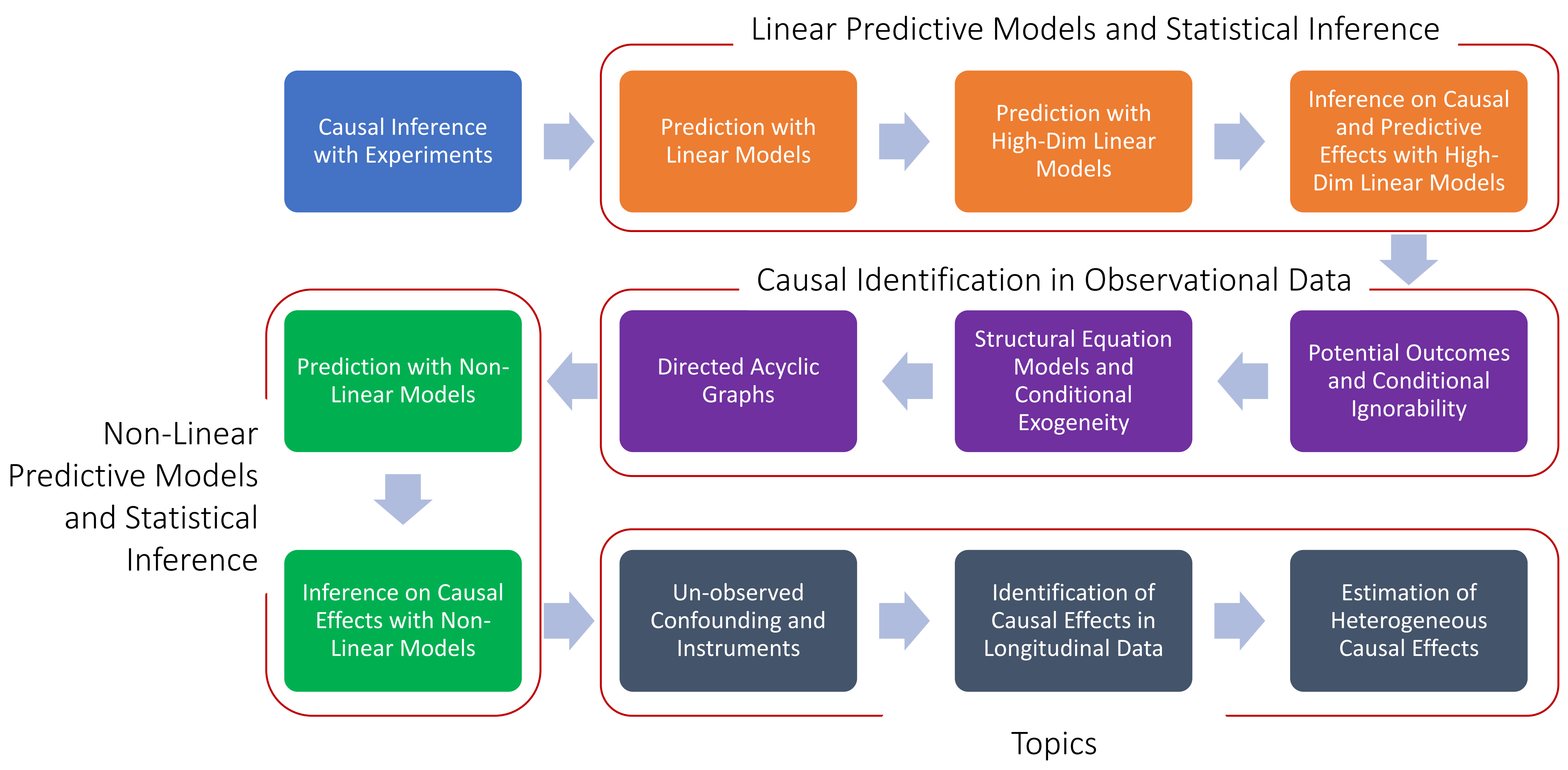
Linear Predictive Models and Statistical Inference
Causal Inference with Experiments
Prediction with Linear Models
Prediction with High-Dim Linear Models
Inference on Causal and Predictive Effects with High-Dim Linear Models
Causal Identification in Observational Data
Potential Outcomes and Conditional Ignorability
Structural Equation Models and Conditional Exogeneity
Directed Acyclic Graphs
Non-Linear Predictive Models and Statistical Inference
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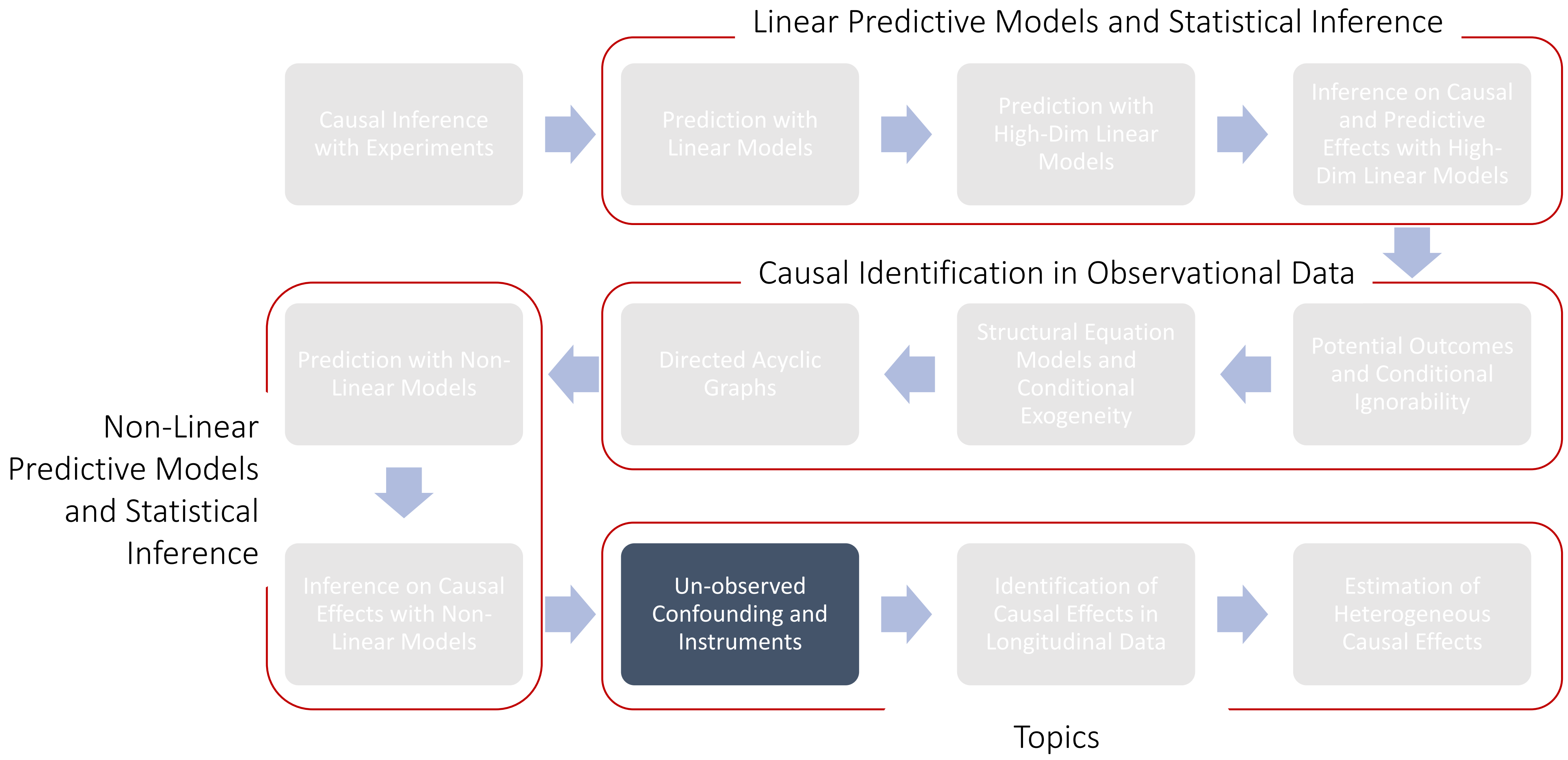
Un-observed Confounding and Instruments
Identification of Causal Effects in Longitudinal Data
Estimation of Heterogeneous Causal Effects
Topics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Topics

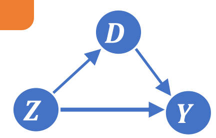
Goals for Today

- What can we do when we have un-observed confounding
- Omitted variable bias bounds
- Introduction to “Instruments”

Causal Inference Pipeline

Theory

DAGs/ASEMs



$$a_d = E[E[Y|D=d,Z]]$$

Construct estimate $\hat{g}(d,z)$ of predictive model

$$E[\,Y \mid D=d, Z=z\,]$$

Estimate mean counterfactual

$$\hat{a}_d = E_n[\hat{g}(d,Z)]$$

$$a_d \in \hat{a}_d \pm 2\hat{\sigma}$$

Practice

Data Collection

Domain Assumption Elicitation

Identification

Estimation (training)

Validation (testing)

Inference (Confidence Intervals)

There is no way to validate my causal estimate $\hat{\alpha}_d$, given the data that I have! I cannot observe counterfactuals. The best alternative is to **quantify uncertainty of my estimate** to see how confident I should be in the result.

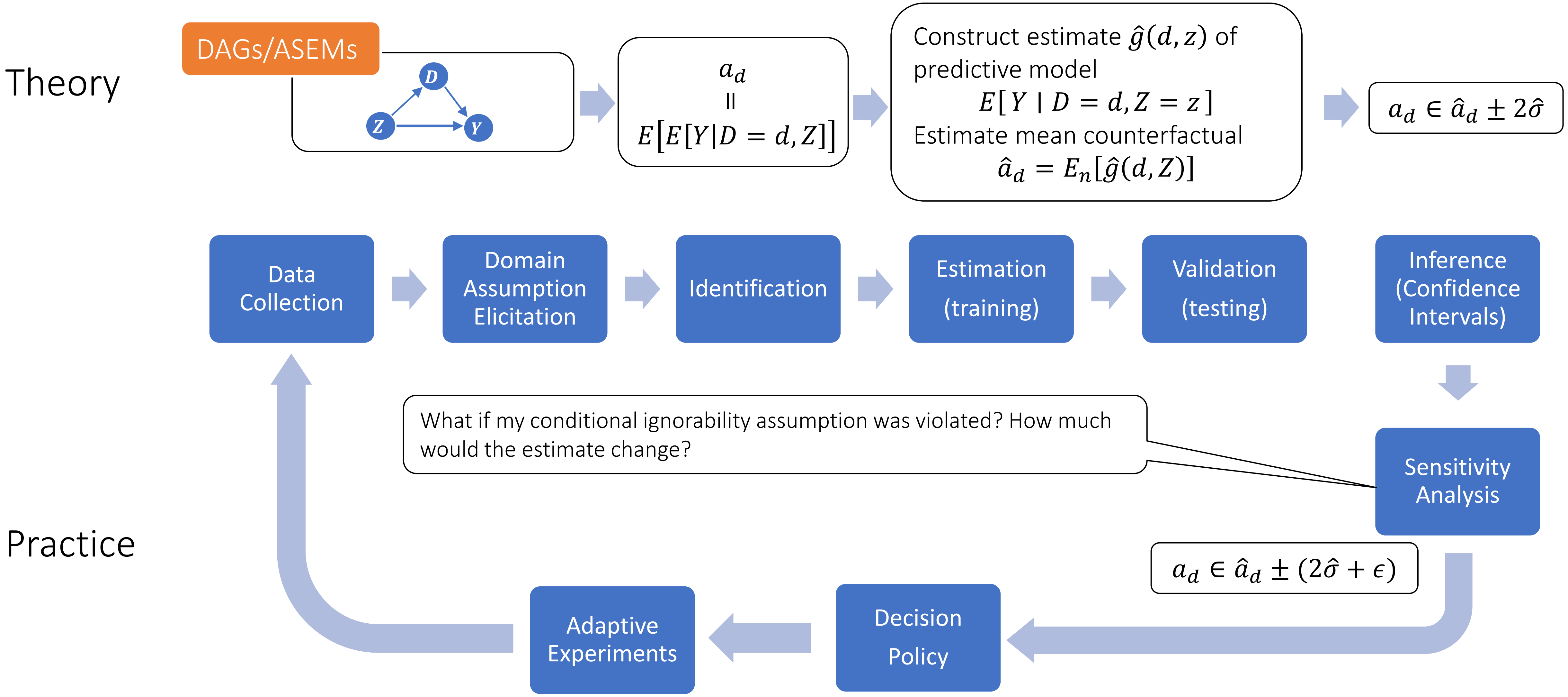
Sensitivity Analysis

$$a_d \in \hat{a}_d \pm (2\hat{\sigma} + \epsilon)$$

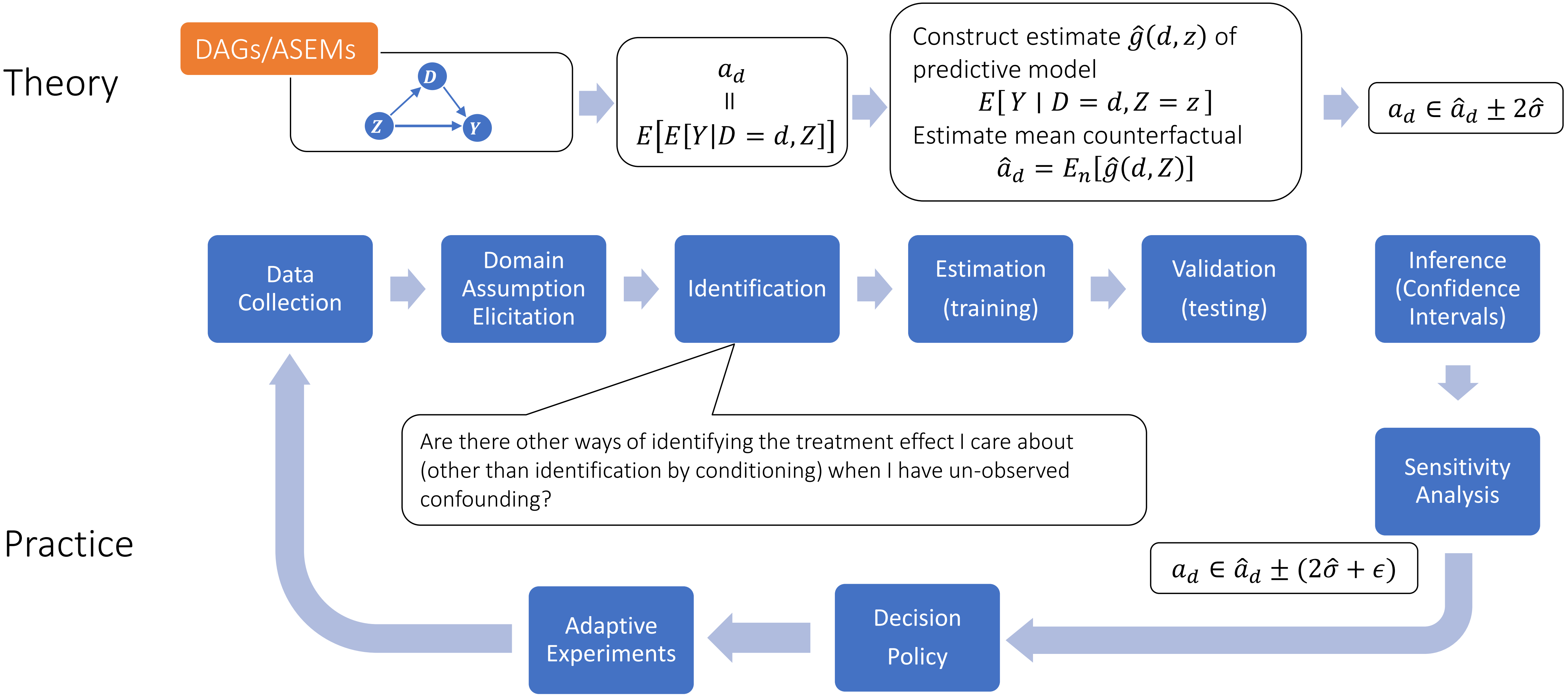
Decision Policy

Adaptive Experiments

Causal Inference Pipeline

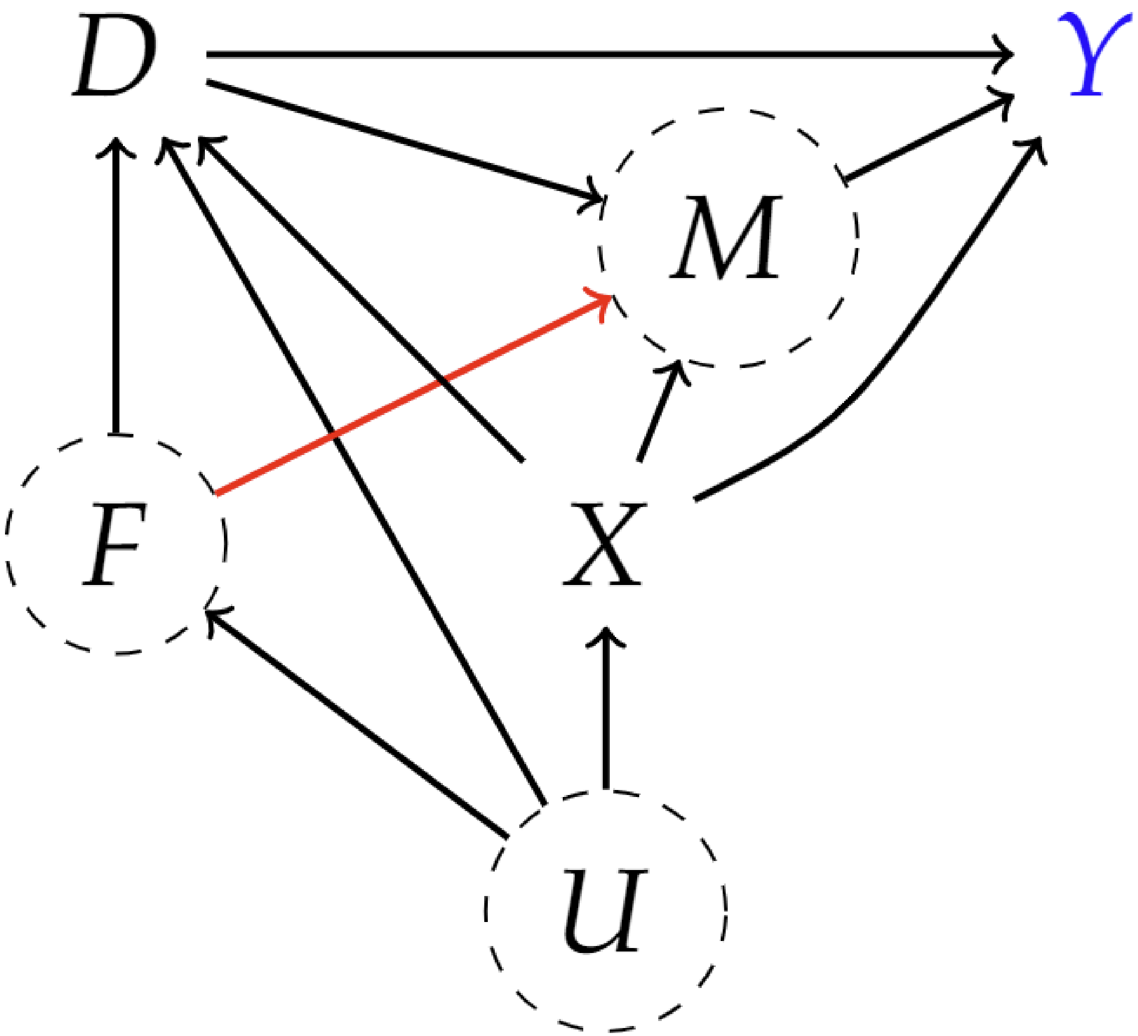


Causal Inference Pipeline



Possible Violations

- F: firm characteristics
- D: eligibility for 401k
- Y: net financial assets
- X: age, income, family size, years of education, a married indicator, a two-earner status indicator, a defined benefit pension status indicator, an IRA participation indicator, and a home ownership indicator
- M: match amount



Bias Bounds

Very Frequently Unobserved Confounders

- Average treatment effect not "identifiable"

$$\theta_0 \neq E\big[E[Y|T=1,X] - E[\,Y \mid T=0,X\,]\big]$$

- Realistic conditional exogeneity

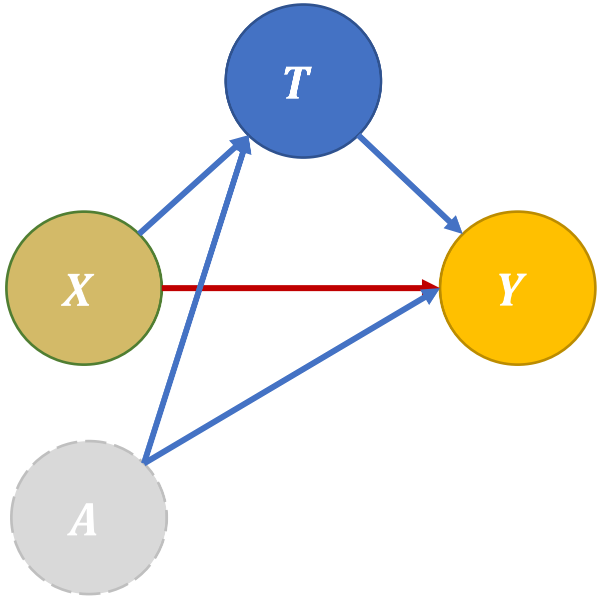
$$Y^{(d)} \perp T \mid X, A \qquad \text{(conditional exogeneity)}$$

- Ideal quantity: hypothetical g-formula

$$\theta_0 = E\big[E[Y|T=1,X,A] - E[\,Y \mid T=0,X,A\,]\big]$$

- Identifiable quantity:

$$\theta_s = E\big[E[Y|T=1,X] - E[\,Y \mid T=0,X\,]\big]$$



Omitted Variable Bias

- We want to estimate:

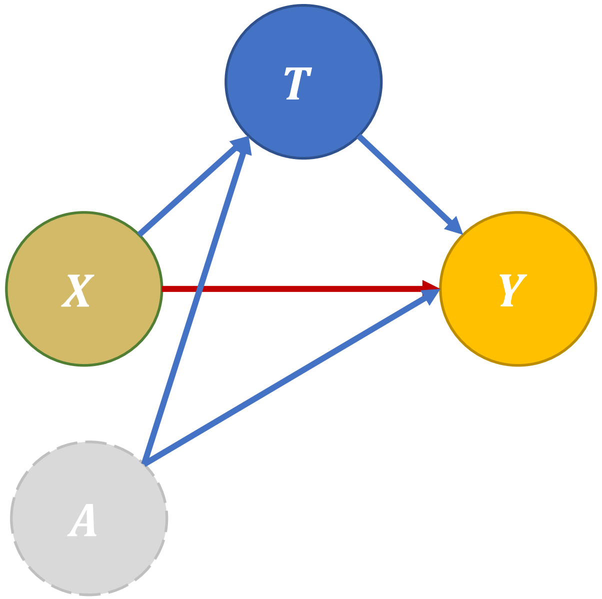
$$\theta_0 := E[g(1,X,A) - g(0,X,A)]$$
$$g(T,X,A) := E[\,Y \mid T,X,A\,]$$

- Which depends on an un-attainable “long regression”
- We can only estimate a “short regression”

$$g_s(T,X) := E[Y|T,X] = E[g(T,X,A)|T,X]$$

- And compute “short estimate”

$$\theta_s = E[g_s(1,X) - g_s(0,X)]$$



Omitted Variable Bias Bounds

- Provide expression and construct bounds on Omitted Variable Bias (OMVB)
$$\theta_s - \theta_0$$
- Under interpretable assumptions that limit the strength of unobserved confounding
- Perform statistical inference on θ_0 allowing for ML regressions

- Sensitivity analysis has a long history:
  - Rosenbaum-Rubin'83: non-parametric bounds [non-sharp]
  - A lot of follow-up work making parametric assumptions

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Omitted Variable Bias: Partially Linear Models

- Let's consider a simpler structural equation model

$$
\begin{aligned}
Y &:= aD + \delta A + f_Y(X) + \epsilon_Y \\
D &:= \gamma A + f_D(X) + \epsilon_D \\
A &:= f_A(X) + \epsilon_A \\
X &:= \epsilon_X
\end{aligned}
$$

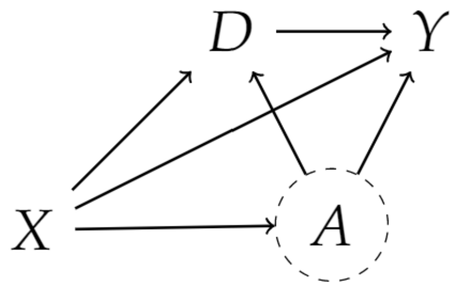


Figure 4.5: X are observed confounders, and A are unobserved confounders.

Omitted Variable Bias: Partially Linear Models

- Suppose we run the residual-on-residual process and first partial out X
- For convenience $E\left[\epsilon_A^2\right] = 1$

$$\tilde{Y} := a\widetilde{D} + \delta\tilde{A} + \epsilon_Y$$
$$\widetilde{D} := \gamma\tilde{A} + \epsilon_D$$
$$\tilde{A} := \epsilon_A$$

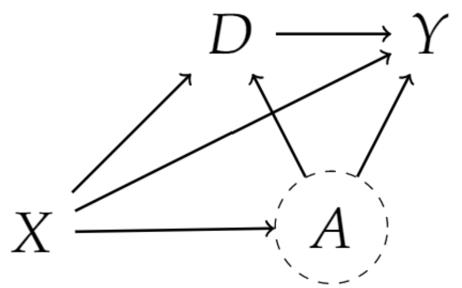


Figure 4.5: X are observed confounders, and A are unobserved confounders.

- Then the double ML method would run OLS of $\tilde{Y}$ on $\widetilde{D}$

$$\theta_s = \frac{E\left[\tilde{Y}\widetilde{D}\right]}{E\left[\widetilde{D}^2\right]} = \frac{E\left[a\,\widetilde{D}^2 + \delta\tilde{A}\widetilde{D}\right]}{E\left[\widetilde{D}^2\right]} = a + \delta\frac{E\left[\gamma\tilde{A}^2\right]}{E\left[\widetilde{D}^2\right]} = a + \frac{\delta\gamma}{\gamma^2 + E\left[\epsilon_D^2\right]}$$

Bias Bounds

- If the analyst can provide bounds on the strength of each relationship (e.g. $\delta, \gamma, E\left[\epsilon_D^2\right]$) then we can provide bounds on the true effect

$$\theta_0 = \theta_s \pm \left|\frac{\delta\gamma}{\gamma^2 + E\left[\epsilon_D^2\right]}\right|$$

- More interpretable bounds:

$$\left(\frac{\delta\gamma}{\gamma^2 + E\left[\epsilon_D^2\right]}\right)^2 = \boxed{R^2_{\tilde{Y}\sim\tilde{A}|\tilde{D}}}\,\frac{R^2_{\tilde{D}\sim\tilde{A}}}{1 - \boxed{R^2_{\tilde{D}\sim\tilde{A}}}}\,\boxed{\frac{\mathrm{E}\left[\left(\tilde{Y} - \theta_s\tilde{D}\right)^2\right]}{E\left[\tilde{D}^2\right]}}$$

Measurable from the data

R^2 in linear regression of $\tilde{Y}$ on $\tilde{A}$ after linearly partialling out $\tilde{D}$

R^2 in linear regression of $\tilde{D}$ on $\tilde{A}$

Bias Bounds

- If the analyst can provide bounds on the strength of each relationship (e.g. $\delta, \gamma, E\left[\epsilon_D^2\right]$) then we can provide bounds on the true effect

$$\theta_0 = \theta_s \pm \left|\frac{\delta\gamma}{\gamma^2 + E\left[\epsilon_D^2\right]}\right|$$

Measurable from the data

- More interpretable bounds:

$$\left(\frac{\delta\gamma}{\gamma^2 + E\left[\epsilon_D^2\right]}\right)^2 = \boxed{R^2_{Y\sim A|D,X}} \frac{R^2_{D\sim A|X}}{1 - \boxed{R^2_{D\sim A|X}}} \boxed{\frac{\mathrm{E}\left[\left(\tilde{Y} - \theta_s \tilde{D}\right)^2\right]}{E\left[\tilde{D}^2\right]}}$$

Reduction in unexplained variance of Y when adding A in the model that predicts Y from treatment and controls

Reduction in unexplained variance of D when adding A in the model that predicts D from controls

For more details:
Making Sense of Sensitivity: Extending Omitted Variable Bias

For more general analysis see:
Long Story Short: Omitted Variable Bias in Causal Machine Learning

Bias Bounds

- The analyst provides bounds on the partial R^2

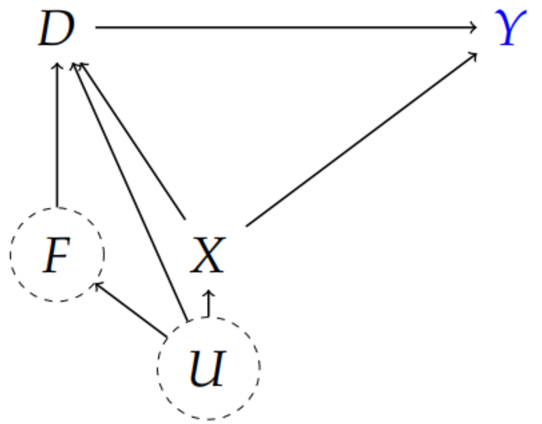
$$R^2_{Y\sim A|D,X} \leq C_Y^2, \qquad R^2_{D\sim A|X} \leq C_D^2$$

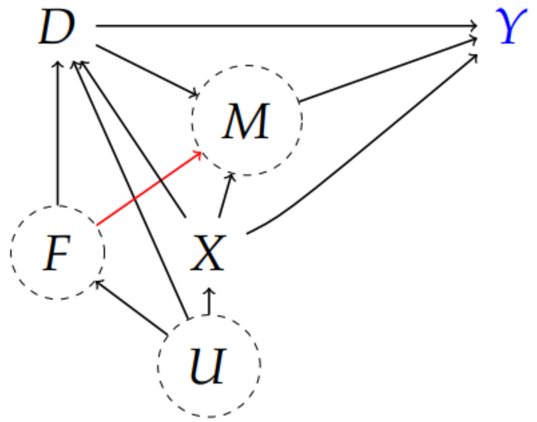
- Based on these bounds we can conclude that

$$\theta_0 \in \theta_s \pm \sqrt{C_Y^2 \frac{C_D^2}{1 - C_D} \frac{E\left[\left(\tilde{Y} - \theta_s \widetilde{D}\right)^2\right]}{E\left[\widetilde{D}^2\right]}}$$

Application: 401k eligibility

- Y=net financial assets
- D=eligibility to enroll in 401(k) program
- X=pre-treatment worker-level covariates (observed)
- F=pre-treatment firm-level covariates (unobserved)
- M=amount of contribution matched by employer
- U=general latent factors





Controlling for X is sufficient in top figure but not bottor

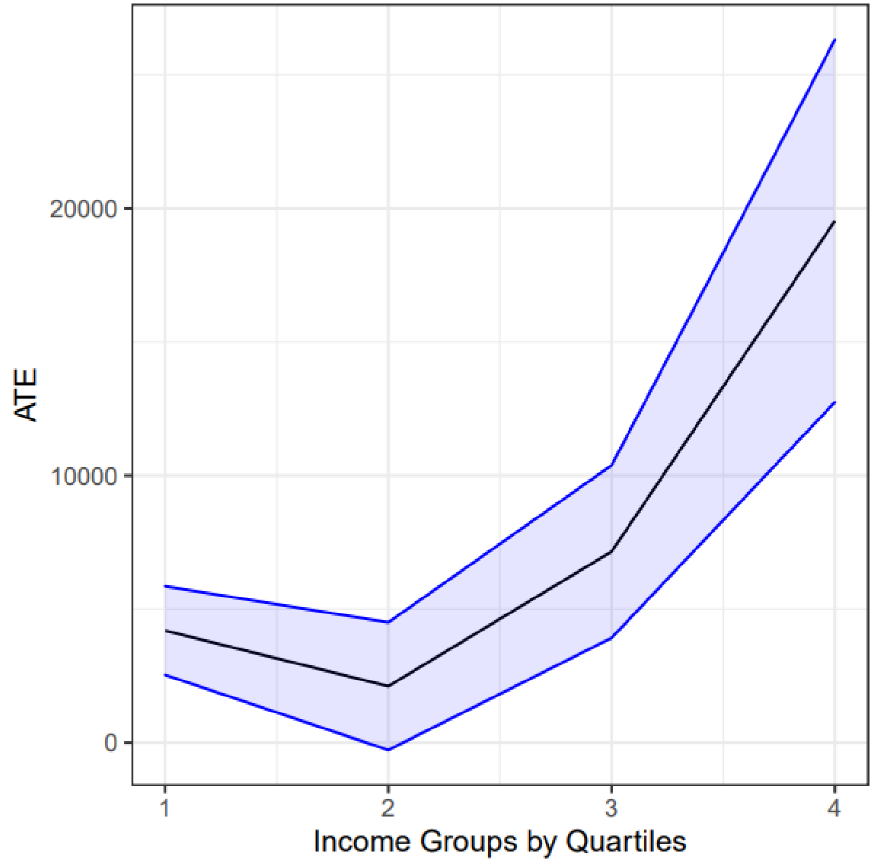
Confounding Scenario

- Posit that F explains as much variation in net financial assets as the total variation of maximal matched percentage (5%) of income over period of three years
- Posit that F explains an additional 2.5% of the variation in 401k eligibility, a 20% relative increase in the baseline R^2 of the treatment of 13%

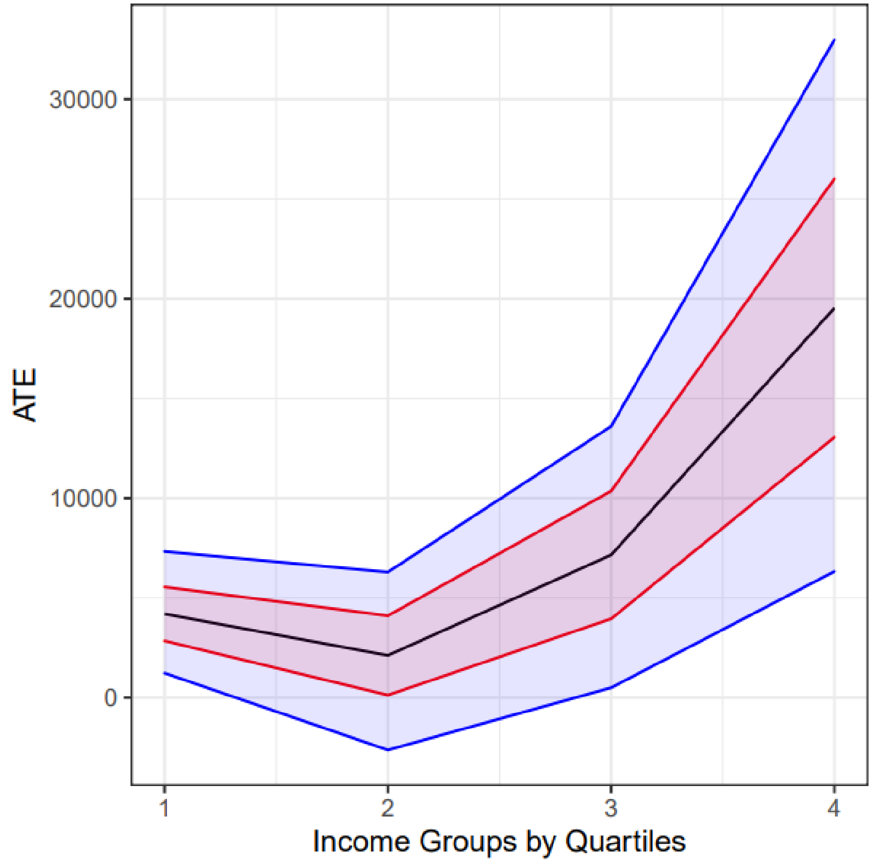
- In PLR: translates to $\mathrm{C}_{\mathrm{Y}}^2 \coloneqq R_{Y\sim A|D,X}^2 \approx 4\%, C_D^2 \coloneqq R_{D\sim A|X}^2 \approx 3\%$

Robustness value (RV) = minimal equal strength of C_Y^2 and C_D^2 s.t. bound includes zero

- RV=5.5% (at 95% significance level) > 4%,3%
- Finding that 401k eligibility has positive effect is robust to this confounding scenario

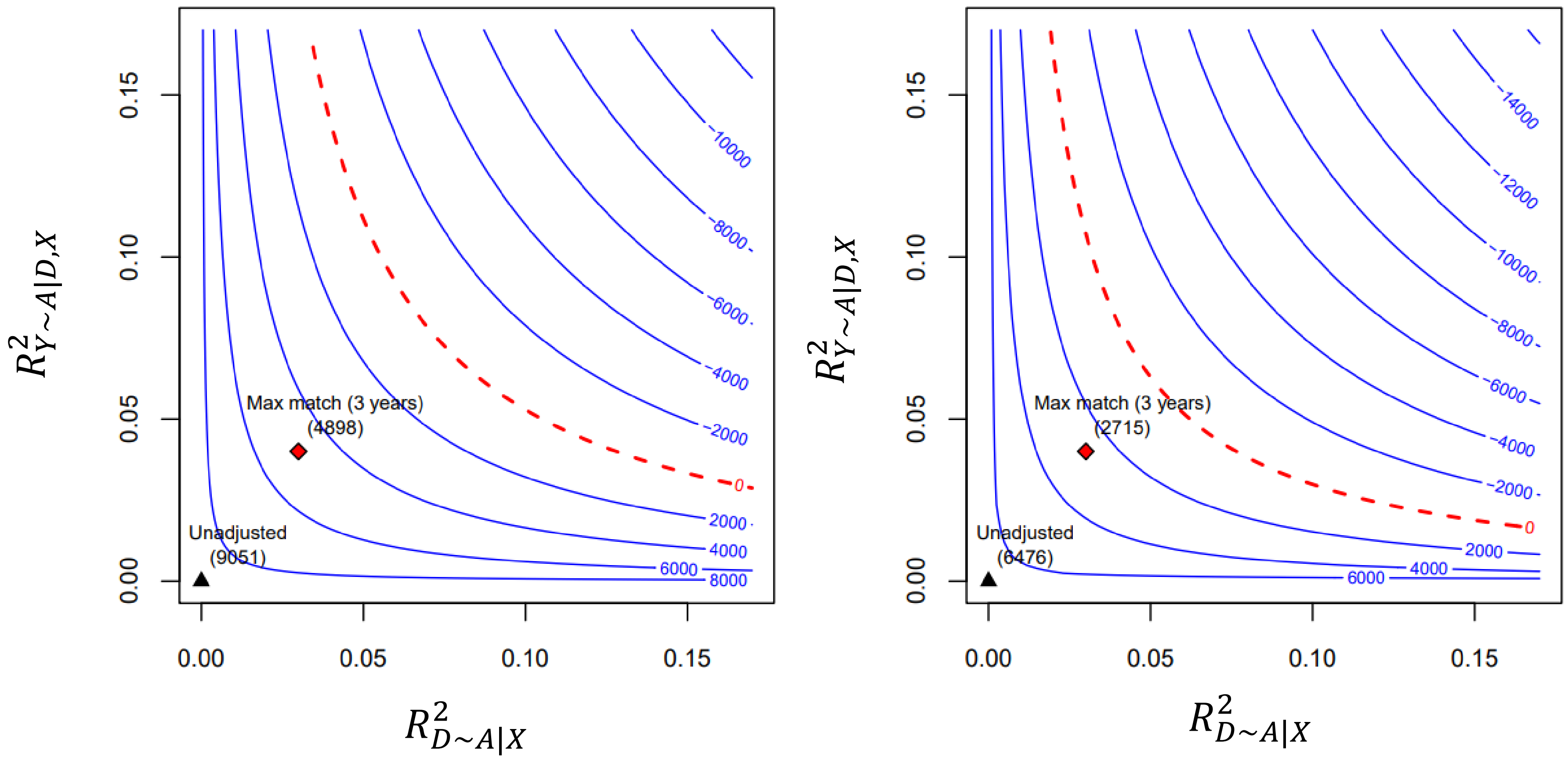


(a) Estimates under no confounding.



(b) Bounds under posited confounding.

Note: Estimate (black), bounds (red), and confidence bounds (blue) for the ATE. Confounding scenario: $\rho^2 = 1$; $C_Y^2 \approx 0.04$; $C_D^2 \approx 0.031$. Significance level of 5%.



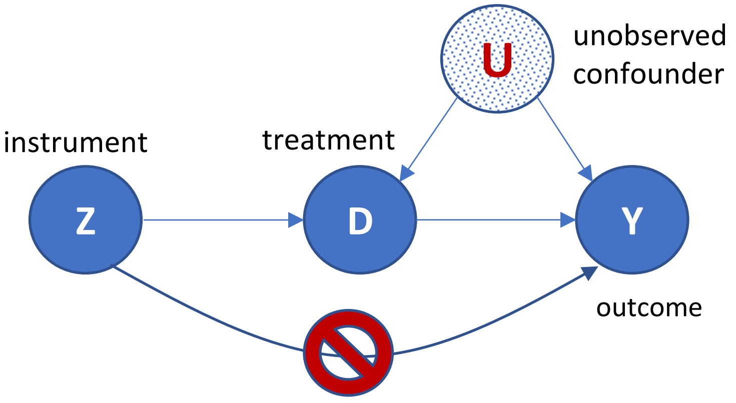
(a) Contours for $\theta_{-} = \theta_s - |B|$.

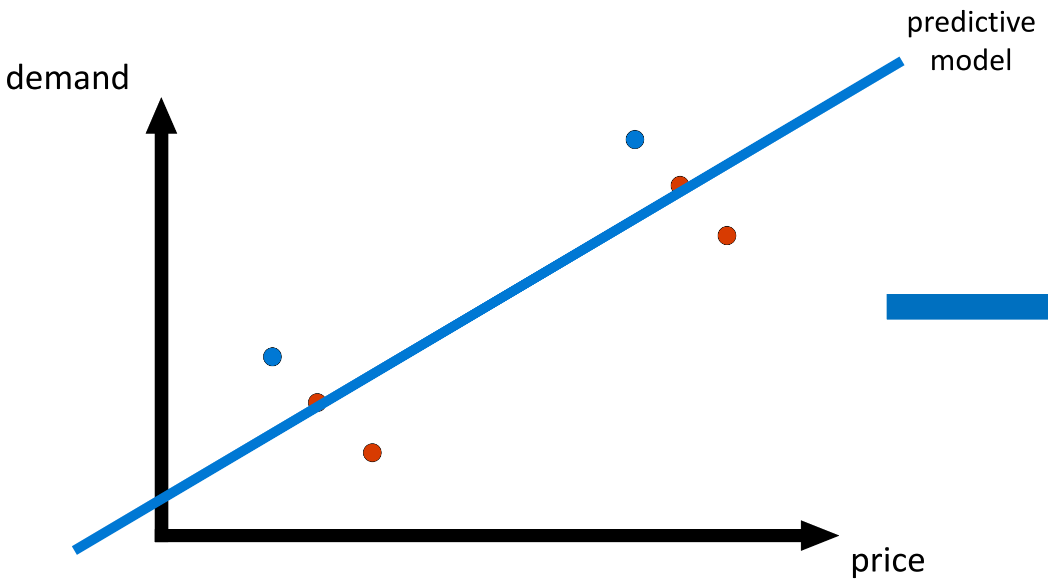
(b) Contours lower limit confidence bound.

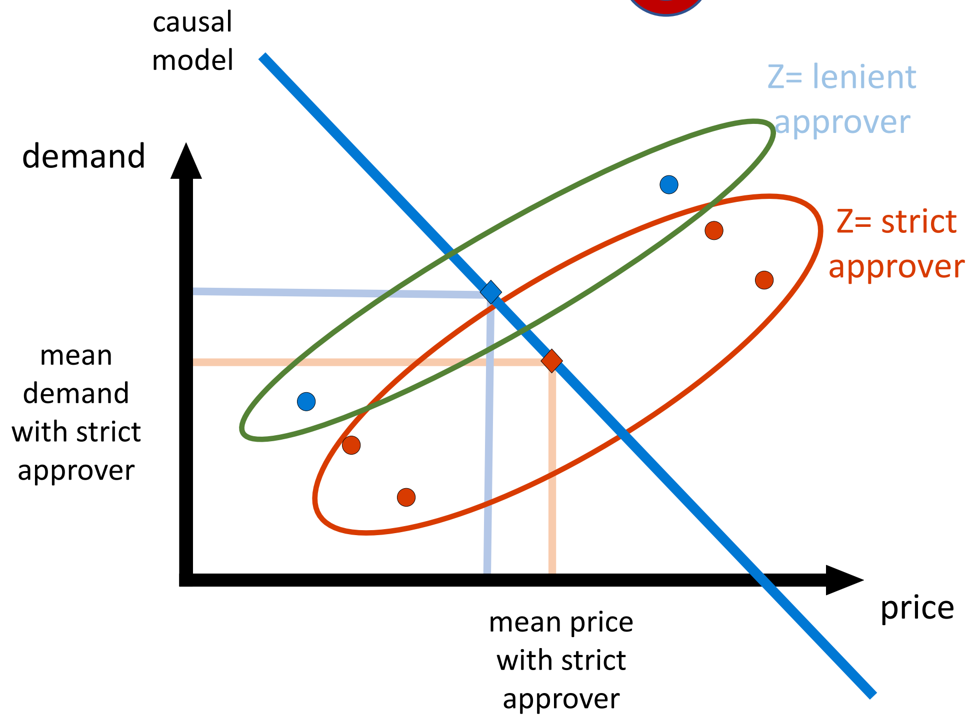
Can we recover the true effect?
Instrumental Variables

Instrumental Variables and 2SLS

Instrumental Variable: any random variable Z that affects the treatment (log-price) D but does not affect the outcome (log-demand) Y other than through the treatment [Wright'28, Bowden-Turkington'90, Angrist-Krueger'91, Imbens-Angrist'94]



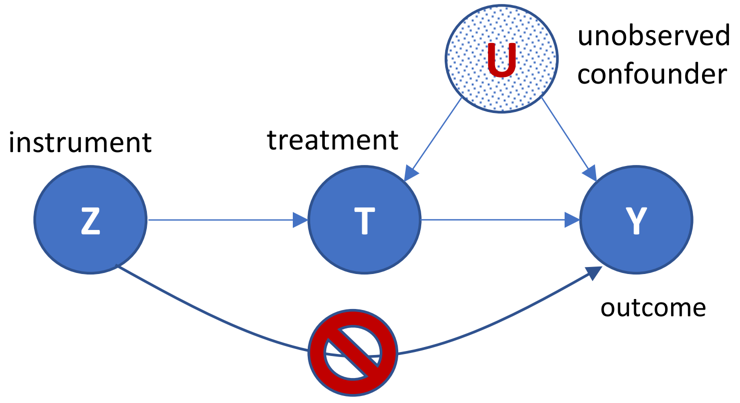




Instrumental Variables and 2SLS

Instruments are widely used

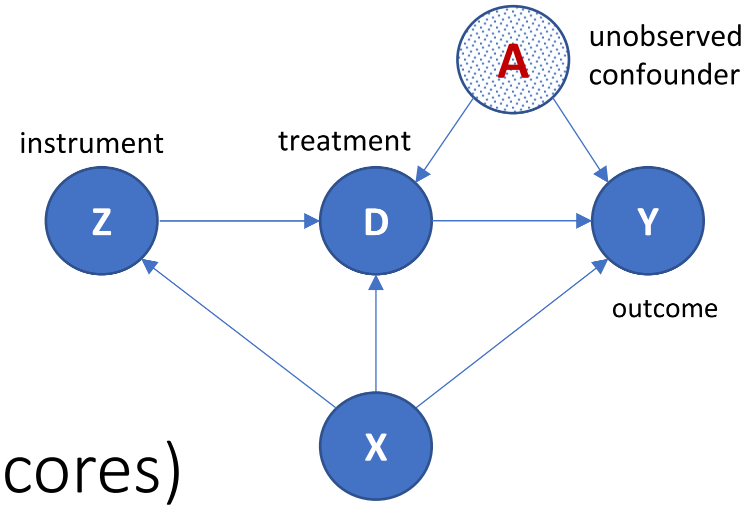
- In the discount example (see also [Kling AER06] for effects of incarceration)
    - Discounts are sent to an approver desk
    - Approver assignment is random and different approvers are more or less “lenient”
    - Approver leniency is an instrument



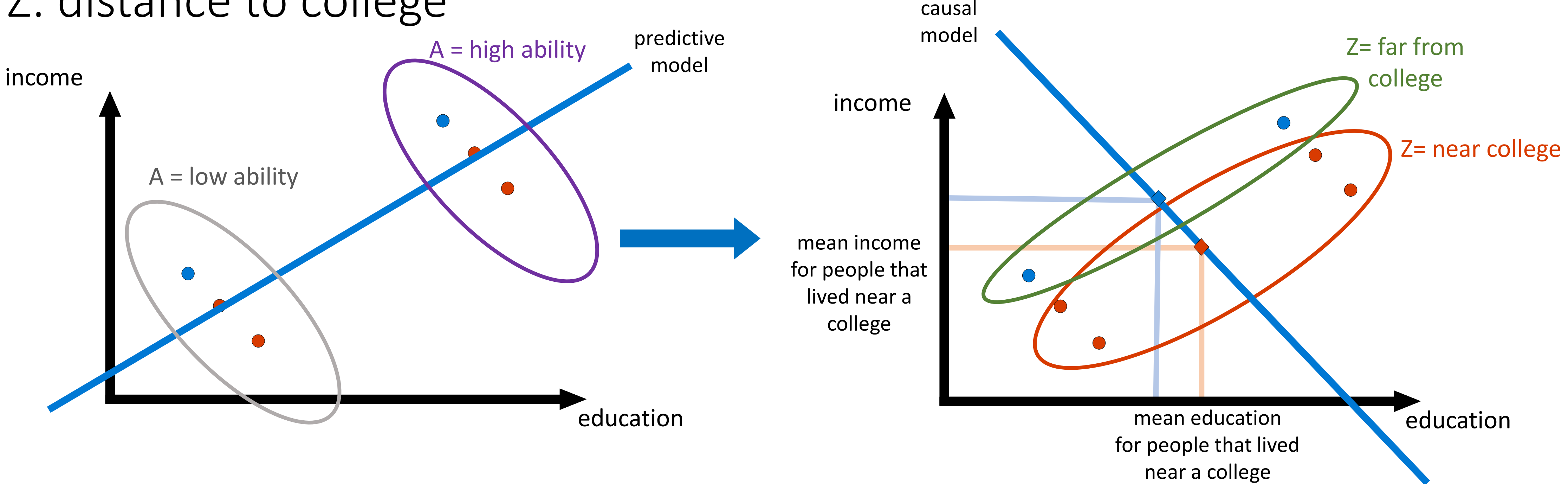
- In healthcare [Doyle et al., JPE15]
    - Random assignment to ambulance companies of nearby patients is an instrument for measuring hospital quality

- In Tech [S., NeurIPS19]
    - Recommendation A/B tests as instruments for the effects of downstream actions

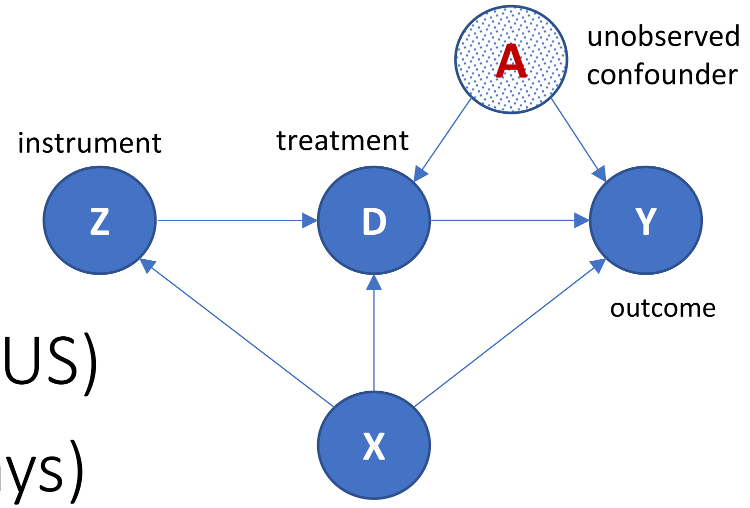
Returns to Education



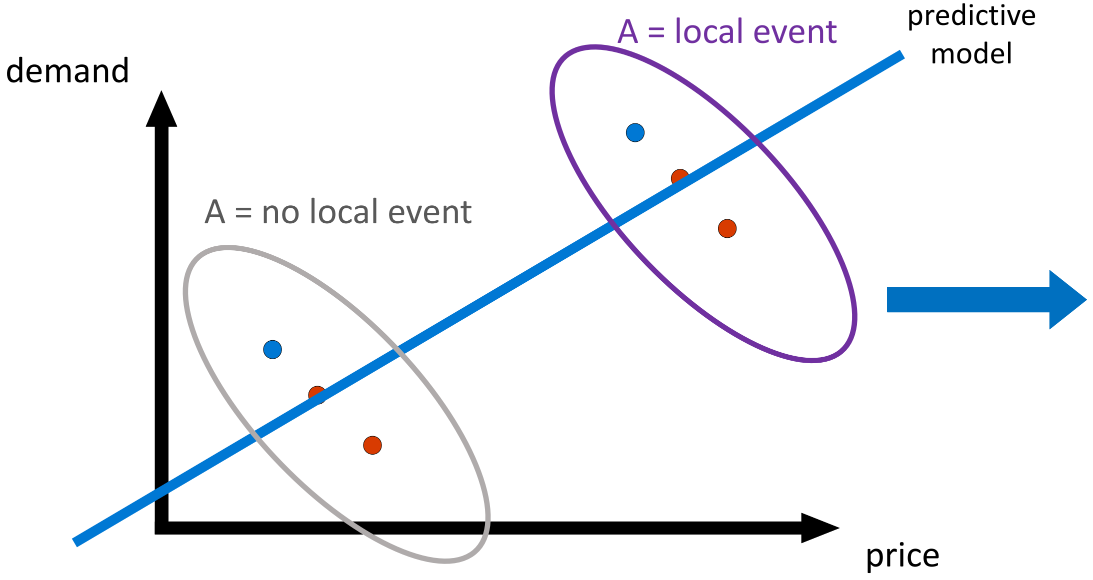
- D: years of college, Y: income
- X: observable characteristics of a student (e.g. test scores)
- A: unobserved "ability"
- Z: distance to college

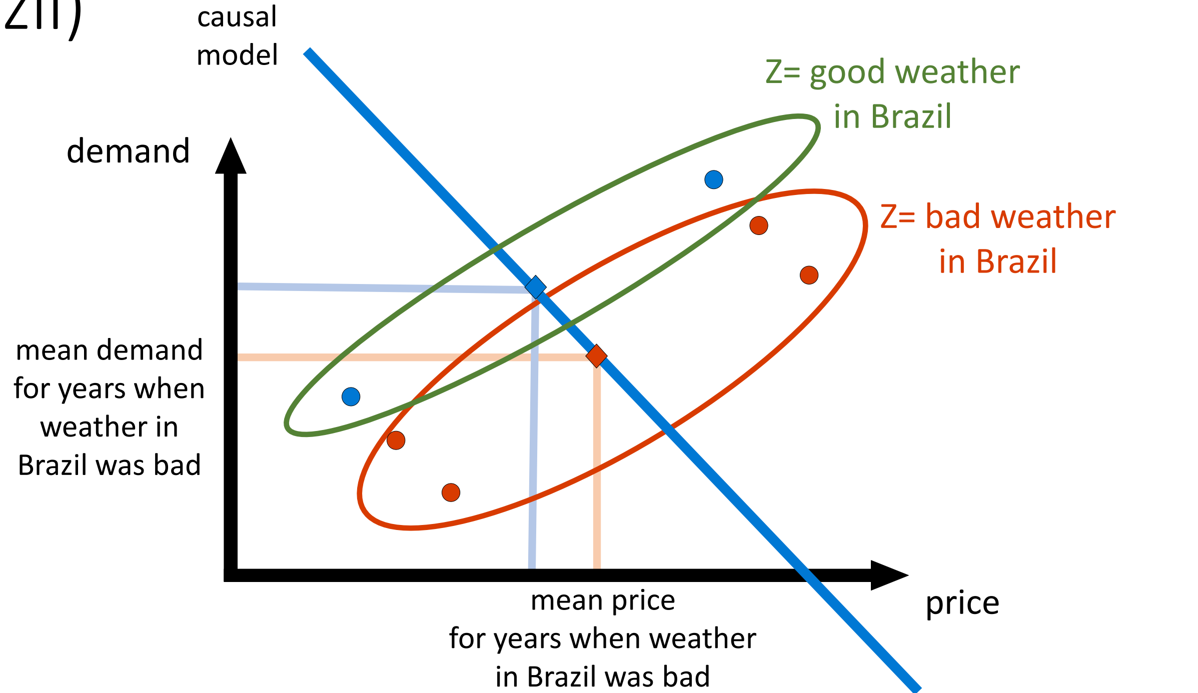


Demand Estimation

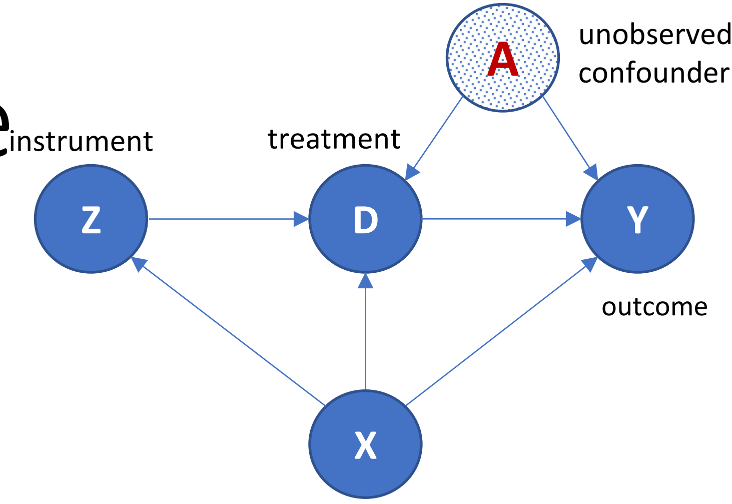


- D: price (e.g. of coffee), Y: demand (e.g. of coffee in US)
- X: observable characteristics of a market (e.g. holidays)
- A: unobserved "demand shocks" (e.g. local event)
- Z: supply shifters (e.g. weather in brazil)

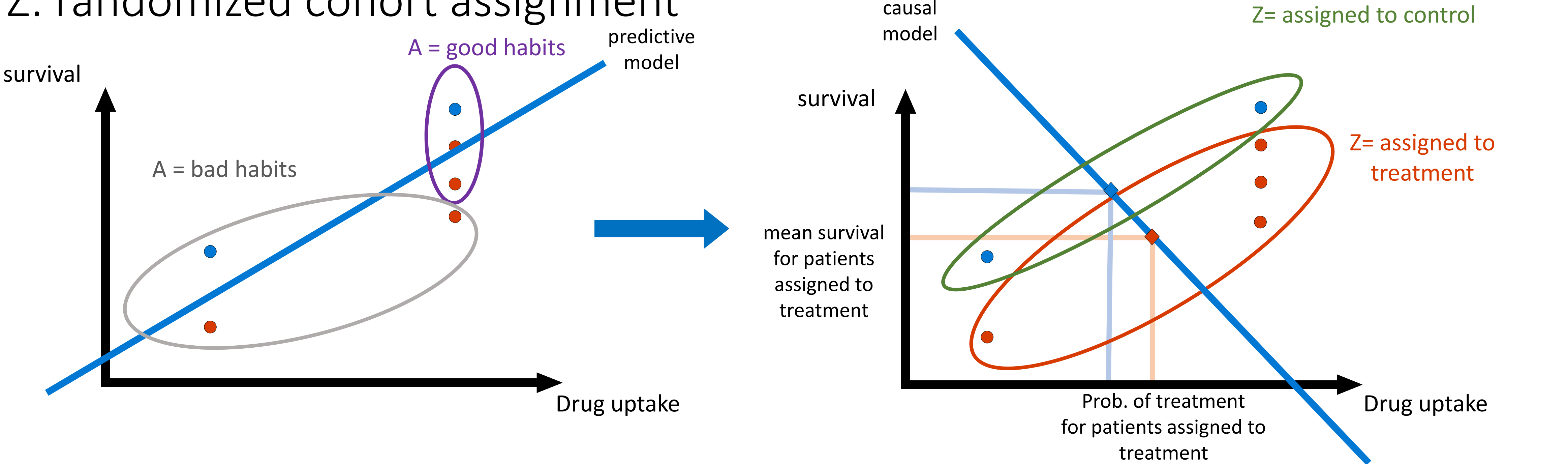




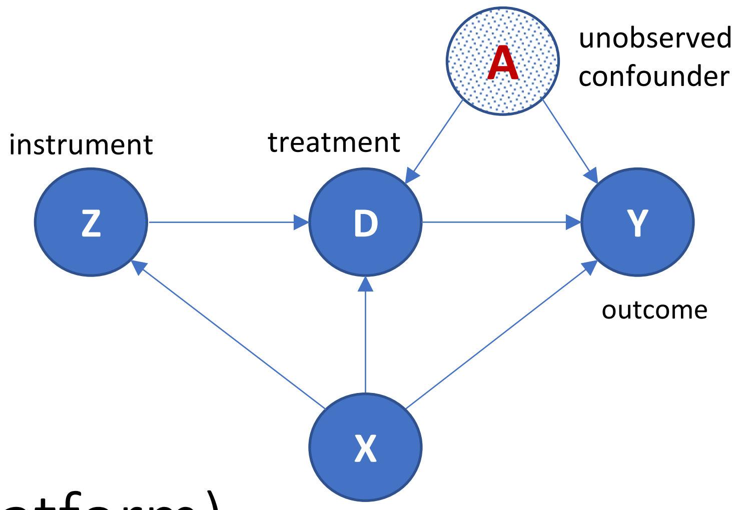
Clinical Trials with Non-Compliance



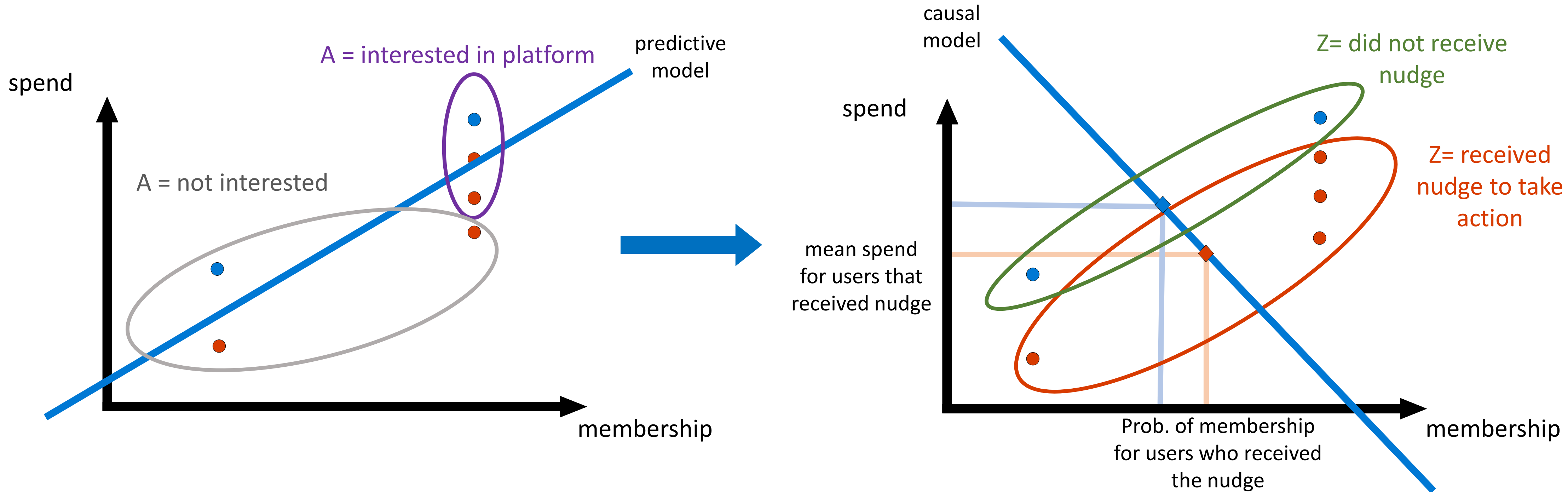
- D: drug treatment, Y: survival
- X: observable characteristics of a patient
- A: unobserved “compliance factors” (e.g. health habits)
- Z: randomized cohort assignment



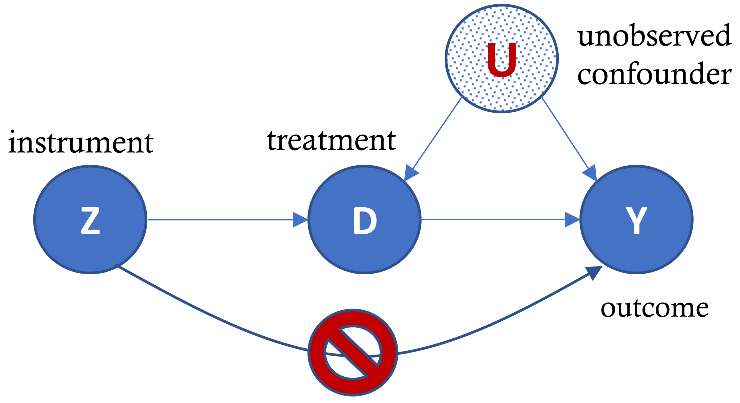
Digital Recommendation A/B tests



- D: action taken by user (e.g. membership), Y: spend
- X: observable characteristics of a user
- A: unobserved confounding factors (e.g. interest in platform)
- Z: randomized nudge to take action (e.g. one-click sign-up pop-up)



Identification of Causal Effects via Instruments



Phillip Wright's idea (1928): the first causal path diagram analysis

- We can estimate effect of Z on y via a regression

$$\gamma = \frac{\mathbb{E}[\tilde{Z}\tilde{y}]}{\mathbb{E}[\tilde{Z}^2]}$$

- We can estimate the effect of Z on D via a regression

$$\delta = \frac{\mathbb{E}[\tilde{\mathrm{Z}}\tilde{\mathrm{D}}]}{\mathbb{E}[\tilde{\mathrm{Z}}^2]}$$

- The effect of Z on Y (γ) is the product of the effect of Z on T (δ) multiplied by the effect of T on y (θ)

$$\theta = \frac{\gamma}{\delta} = \frac{\mathbb{E}[\tilde{Z}\tilde{y}]}{\mathbb{E}[\tilde{Z}\tilde{D}]}$$

Partially Linear Instrumental Variable Model

- Typically for continuous treatment/instrument a partially linear structural equation assumed

$$
\begin{aligned}
Y &\coloneqq \theta_0 D + f_Y(X) + \delta A + \epsilon_Y \\
D &\coloneqq \beta Z + f_D(X) + \gamma A + \epsilon_D \\
Z &\coloneqq f_Z(X) + \epsilon_Z \\
A &\coloneqq f_A(X) + \epsilon_A
\end{aligned}
$$

All errors are exogenous and un-correlated

Partially Linear Instrumental Variable Model

- After partialling out the observed controls X

$$\tilde{Y} := \theta_0 \widetilde{D} + \delta \tilde{A} + \epsilon_Y$$
$$\widetilde{D} := \beta \tilde{Z} + \gamma \tilde{A} + \epsilon_D$$
$$\tilde{Z} := \epsilon_Z$$
$$\tilde{A} := \epsilon_A$$

- We see immediately that:

$$\tilde{Y} := \theta_0 \widetilde{D} + U, \qquad U := \delta \tilde{A} + \epsilon_Y \perp \tilde{Z}$$

- Since $\epsilon_A, \epsilon_Y, \epsilon_Z$ are un-correlated: $E\left[\left(\delta \tilde{A} + \epsilon_Y\right)\tilde{Z}\right] = 0$
- Thus we have the moment restriction: $E\left[\left(\tilde{Y} - \theta_0 \widetilde{D}\right)\tilde{Z}\right] = 0$

Partially Linear Instrumental Variable Model

- After partialling out the observed controls X

$$
\begin{gathered}
\tilde{Y} := \theta_0 \widetilde{D} + \delta \tilde{A} + \epsilon_Y \\
\widetilde{D} := \beta \tilde{Z} + \gamma \tilde{A} + \epsilon_D \\
\tilde{Z} := \epsilon_Z \\
\tilde{A} := \epsilon_A
\end{gathered}
$$

- Thus we have the moment restriction: $E\left[\left(\tilde{Y} - \theta_0 \widetilde{D}\right)\tilde{Z}\right] = 0$
- We re-derive a generalization of Wright's formula

$$
\theta_0 = \frac{E\left[\tilde{Y}\tilde{Z}\right]}{E\left[\widetilde{D}\tilde{Z}\right]}
$$

Partially Linear Instrumental Variable Model

- After partialling out the observed controls X

$$\tilde{Y} \coloneqq \theta_0 \tilde{D} + \tilde{A} + \epsilon_Y$$
$$\tilde{D} \coloneqq \beta \tilde{Z} + \gamma \tilde{A} + \epsilon_D$$
$$\tilde{Z} \coloneqq \epsilon_Z$$
$$\tilde{A} \coloneqq \epsilon_A$$

- Setting falls into the general moment estimation framework

$$M(\theta, h, p, m) = E\left[\left(Y - h(X) - \theta\left(D - p(X)\right)\right)\ \left(Z - m(X)\right)\right] = 0$$

- Where $h(X) = E[Y|X], p(X) = E[D|X], m(Z) = E[Z|X]$

Orthogonal Method: Double ML for IV

Double ML. Split samples in half

- Regress $Y \sim X$ with ML on first half, to get estimate $\hat{h}(S)$ of $E[Y|X]$
- Regress $D \sim X$ with ML on first half, to get estimate $\hat{p}(S)$ of $E[D|X]$
- Regress Z $\sim X$ with ML on first half, to get estimate $\widehat{m}(S)$ of $E[Z|X]$
- Construct residuals on other half, $\hat{Z} = Z - \widehat{m}(X), \widehat{D} \coloneqq D - \hat{p}(X)$ and $\hat{Y} \coloneqq Y - \hat{h}(X)$
- Solve moment condition:

$$E_n\left[\left(\hat{Y} - \theta\widehat{D}\right)\hat{Z}\right] = 0$$

```
from econml.iv.dml import OrthoIV
orthoiv = OrthoIV()
orthoiv.fit(y, D, Z, W=X).effect_inference()
```

Asymptotic Normality of DoubleML Estimate

$$E_n\left[\left(\hat{Y} - \theta\hat{D}\right)\hat{Z}\right] = 0 \Leftrightarrow \hat{\theta} = \frac{E_n\left[\hat{Y}\hat{Z}\right]}{E_n\left[\hat{D}\hat{Z}\right]}$$

- *Assume* that $E\left[\tilde{D}\tilde{Z}\right] = E[Cov(D, Z \mid X)] > 0$ (average overlap)
- *Assume* $\hat{h}, \hat{p}, \hat{m}$ estimated on separate sample (or cross-fitting), are consistent and:

$$\sqrt{n}\ \left(\mathrm{RMSE}(\hat{h}) \cdot \mathrm{RMSE}(\hat{m}) + \mathrm{RMSE}(\hat{p}) \cdot \mathrm{RMSE}(\hat{m})\right) \to_p 0$$

- *Assume* random variables Y, D, X, Z have bounded fourth moments

$$\sqrt{n}(\hat{\theta} - \theta_0) \to_d N(0, \sigma^2),$$

Estimate asymptotically normal

$$\sigma^2 := \frac{E\left[\left(\tilde{Y} - \theta_0\tilde{D}\right)^2\tilde{Z}^2\right]}{E\left[\tilde{D}\tilde{Z}\right]^2},$$

Asymptotic variance

$$\hat{\sigma}^2 = \frac{E_n\left[\left(\hat{Y} - \hat{\theta}\ \hat{D}\right)^2\hat{Z}^2\right]}{E_n\left[\hat{D}\hat{Z}\right]^2}$$

Estimate of variance ⇒ 95% CI $\left[\theta \pm 1.96\frac{\hat{\sigma}}{\sqrt{n}}\right]$

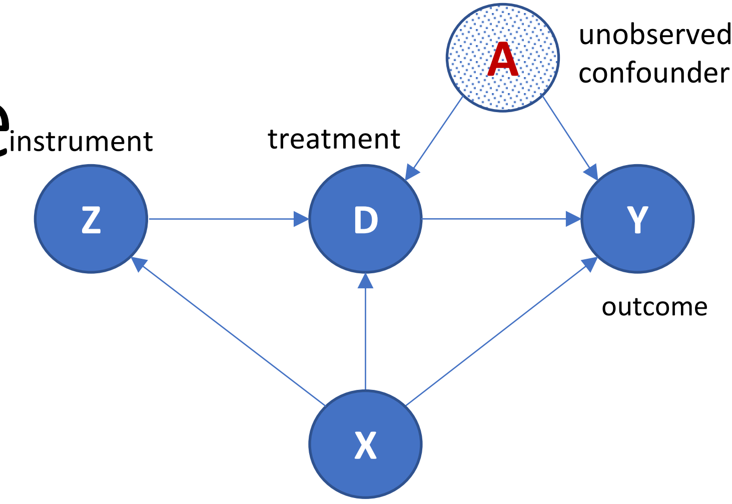
Limits of Identification via Instruments

- ATE identification via Instruments not based solely on DAG restrictions
- Requires further restrictions on structural equation models (e.g. additive error, partial linearity)

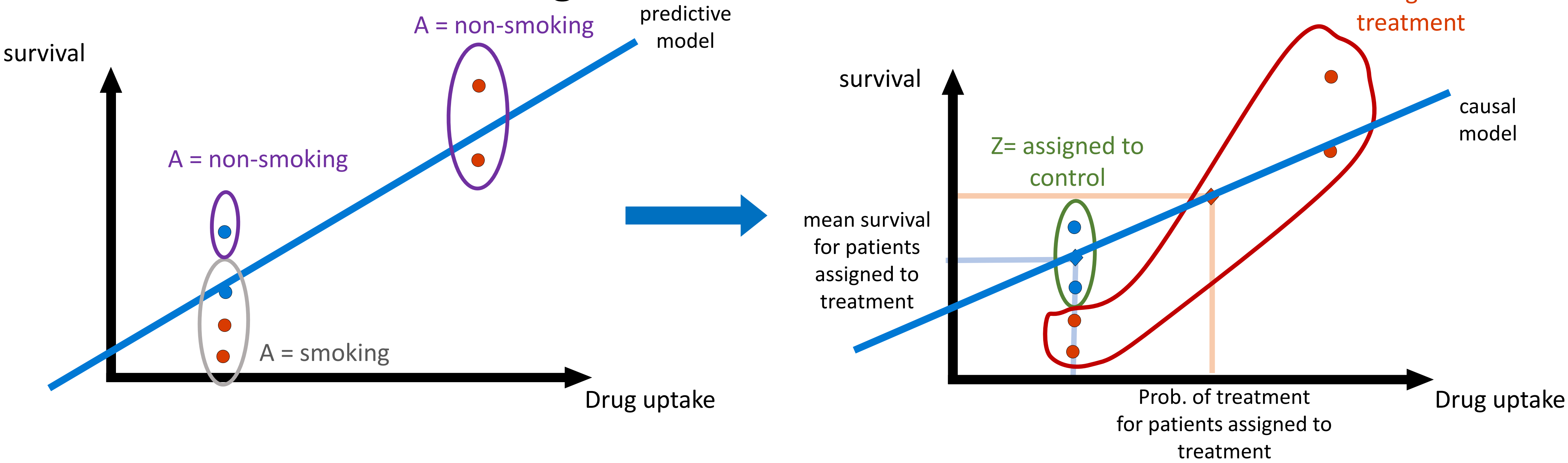
Example

- Binary treatment D (drug) and binary instrument Z (drug recommendation)
- Consider an unobserved confounder A = “smoking”
- Suppose that smokers (A=1) never take the drug (never comply) and non-smokers (A=0) always follow the recommendation (comply)
- Suppose that drug has positive effects for non-smokers but has severe side-effects for smokers

Clinical Trials with Non-Compliance



- D: drug treatment, Y: survival
- X: observable characteristics of a patient
- A: unobserved “compliance factors” (e.g. health habits)
- Z: randomized cohort assignment



Limits of Identification via Instruments

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Example

- IV regression will never be able to uncover the side effects of drug treatment on smokers
- Nothing in the data is informative of that
- Effect will be biased as compared to average effect in whole population

What do we need for ATE

- Either the compliance behavior (effect of instrument on treatment) does not vary with A (or X)
- Or the treatment effect (effect of treatment on outcome) does not vary with A (or X)

$$Y \coloneqq g_Y(\epsilon_Y)\, D + f_Y(X, A, \epsilon_Y)$$
$$D \coloneqq f_D(Z, X, A, \epsilon_D)$$
$$Z = f_Z(X, \epsilon_Z)$$
$$A \coloneqq f_A(X, \epsilon_A)$$

$$Y \coloneqq g_Y(X, A, \epsilon_Y)\, D + f(X, A, \epsilon_Y)$$
$$D \coloneqq g_D(\epsilon_D) Z + f_D(X, A, \epsilon_D)$$
$$Z = f_Z(X) + \epsilon_Z$$
$$A \coloneqq f_A(X, \epsilon_A)$$

Joint Variation on Observables

- If joint variation is captured through observables then ATE is feasible

$$
\begin{aligned}
Y &:= g_Y(X, \epsilon_Y)\, D + f_Y(X, A, \epsilon_Y) \\
D &:= f_D(Z, X, A, \epsilon_D) \\
Z &= f_Z(X, \epsilon_Z) \\
A &:= f_A(X, \epsilon_A)
\end{aligned}
$$

$$
\begin{aligned}
Y &:= g_Y(X, A, \epsilon_Y)\, D + f(X, A, \epsilon_Y) \\
D &:= g_D(X, \epsilon_D) Z + f_D(X, A, \epsilon_D) \\
Z &= f_Z(X, \epsilon_Z) \\
A &:= f_A(X, \epsilon_A)
\end{aligned}
$$

- We just need to do our identification analysis conditional on X and then average

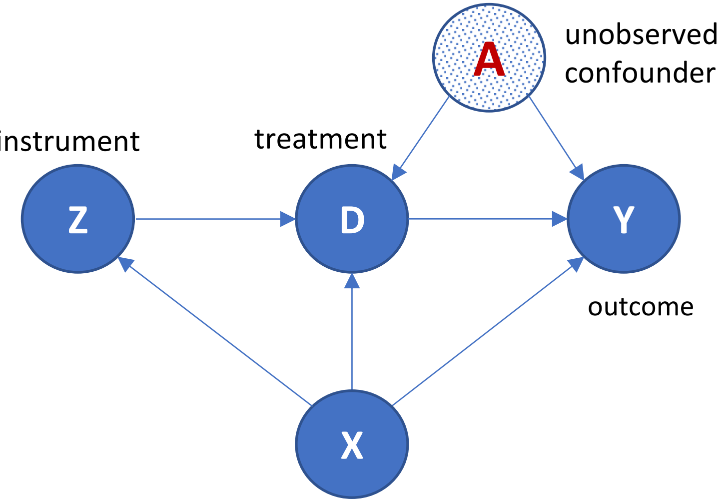
$$\beta(X) = \frac{E[\tilde{Y}\tilde{Z} \mid X]}{E[\tilde{D}\tilde{Z} \mid X]}, \qquad a = E[\beta(X)]$$

- Roughly: reweighting data based on compliance level $E[\tilde{D}\tilde{Z} \mid X]$

What if joint variation happens through unobservables?

$$Y := f_Y(D, X, A, \epsilon_Y)$$
$$D := f_D(Z, X, A, \epsilon_D)$$
$$Z = f_Z(X, \epsilon_Z)$$
$$A := f_A(X, \epsilon_A)$$

Clinical Trials with Non-Compliance



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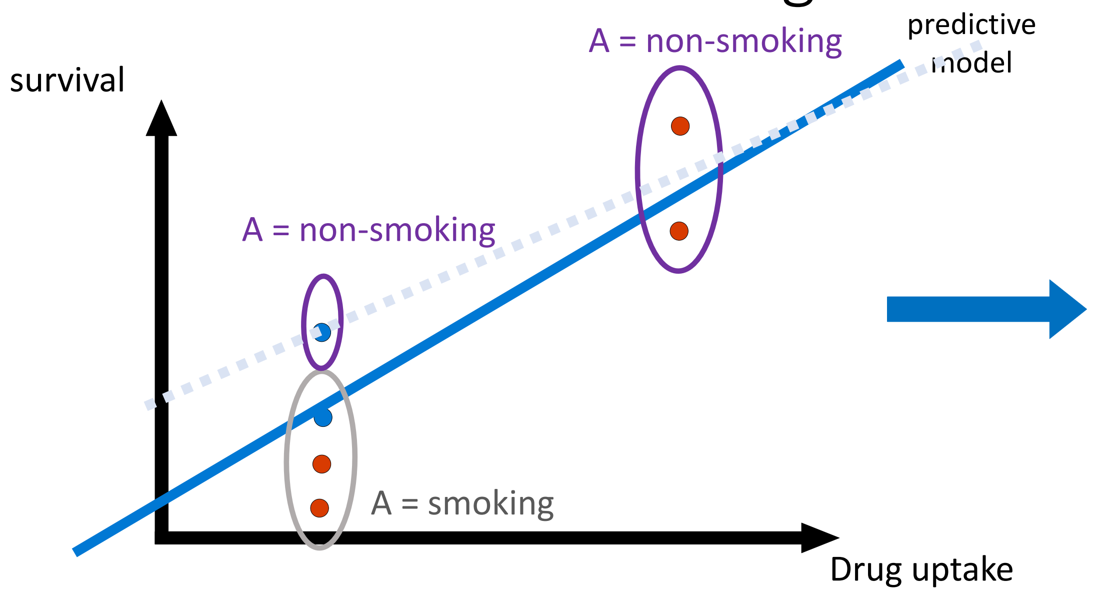
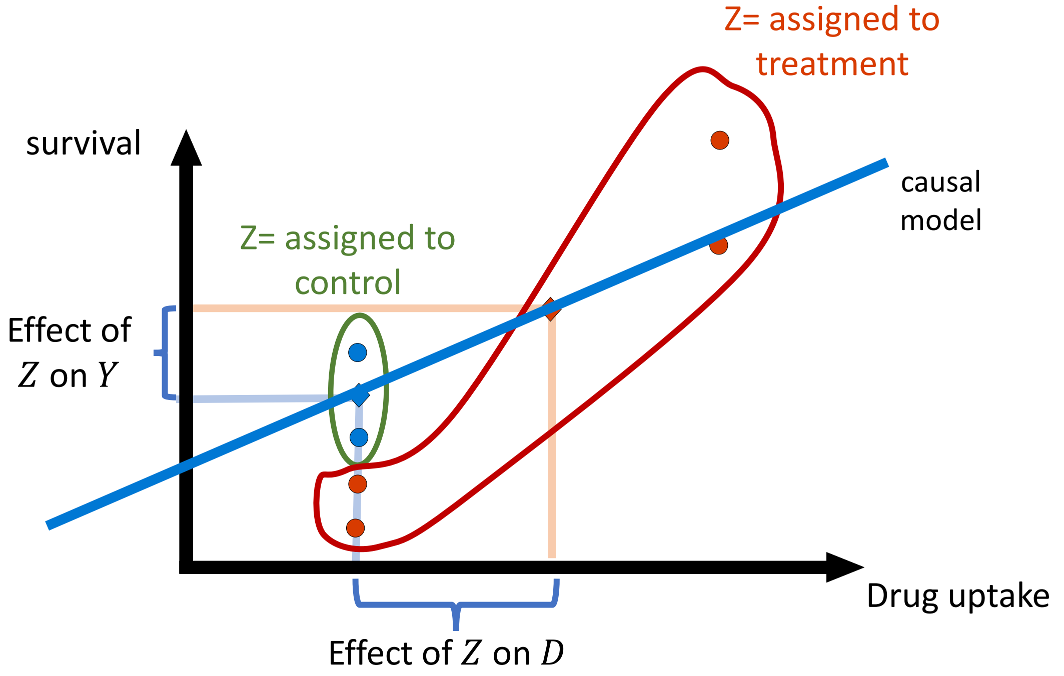
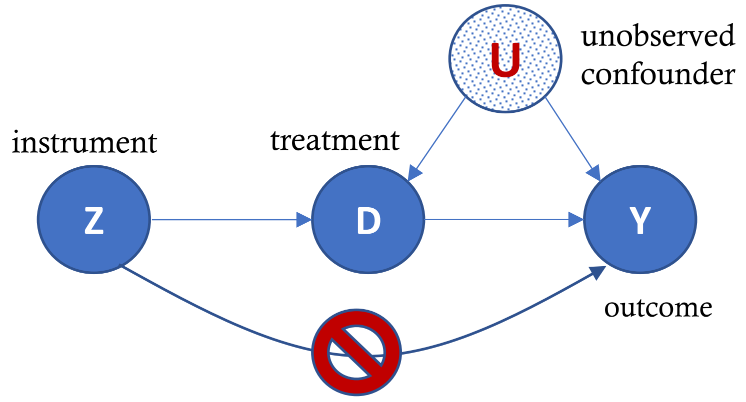




Does the IV estimate coincide with the average effect for some sub-population?

The Binary Case



Imbens-Angrist (1994): core contribution of Nobel 2022 award

- Instrument/Treatment are binary (instrument=recommended treatment)
- Assume monotonicity: $D^{(1)} \geq D^{(0)}$
- Recommended treatment cannot reverse taken treatment
- Object of interest: Local Average Treatment Effect (ATE among compliers)

$$\theta_0 = E\left[\mathrm{Y}^{(1)} - \mathrm{Y}^{(0)} \middle| \mathrm{D}^{(1)} > \mathrm{D}^{(0)}\right]$$

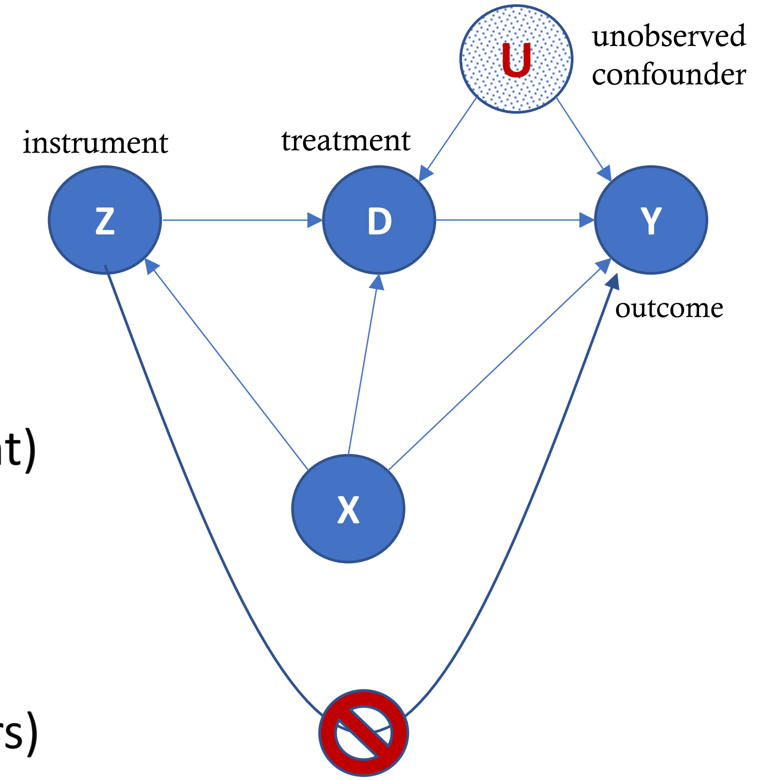
- Proof [Angrist-Imbens'94]:

$$\theta_0 = \frac{E\left[\left(Y^{(1)} - Y^{(0)}\right)1\left\{D^{(1)} > D^{(0)}\right\}\right]}{E\left[1\left\{D^{(1)} > D^{(0)}\right\}\right]} = \frac{E\left[Y^{(D(1))} - Y^{(D(0))}\right]}{E\left[D^{(1)} - D^{(0)}\right]} = \frac{\boxed{ATE(Z \to Y)}}{\boxed{ATE(Z \to D)}}$$

$$\boxed{\gamma = \frac{\mathbb{E}[\tilde{Z}\tilde{y}]}{\mathbb{E}[\tilde{Z}^2]}}$$

$$\boxed{\delta = \frac{\mathbb{E}[\tilde{Z}\tilde{D}]}{\mathbb{E}[\tilde{Z}^2]}}$$

The Binary Case



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$$\boxed{E\left[E[Y|Z=1,X] - E[Y|Z=0,X]\right]}$$

$$\boxed{E\left[E[D|Z=1,X] - E[D|Z=0,X]\right]}$$

LATE in the Binary Case

- Under monotonicity

$$\theta_0 = \frac{E\left[E[Y \mid Z = 1, X] - E[Y|Z = 0, X]\right]}{E\left[E[D|Z = 1, X] - E[D|Z = 0, X]\right]}$$

- Moment formulation

$$E[E[Y|Z = 1, X] - E[Y|Z = 0, X] - \theta_0(E[D|Z = 1, X] - E[D|Z = 0, X])] = 0$$

$$+ \qquad\qquad\qquad\qquad +$$

$$\boldsymbol{H(Z, X)(Y - E[Y|Z, X])} \qquad\qquad \boldsymbol{H(Z, X)(D - E[D|Z, X])}$$

$$H(Z, X) = \frac{Z}{P(Z = 1|X)} - \frac{1 - Z}{1 - P(Z = 1|X)}$$

- Orthogonal moment formulation: apply ATE debiasing twice

Weak-IV Robust Confidence Intervals

Asymptotic Normality of DoubleML Estimate

$$E_n\left[\left(\hat{Y}-\theta\hat{D}\right)\hat{Z}\right]=0 \Leftrightarrow \hat{\theta}=\frac{E_n\left[\hat{Y}\hat{Z}\right]}{E_n\left[\hat{D}\hat{Z}\right]}$$

- *Assume* that $E\left[\tilde{D}\tilde{Z}\right]=E[Cov(D,Z \mid X)]>0$ (average overlap)
- *Assume* $\hat{h},\hat{p},\hat{m}$ estimated on separate sample (or cross-fitting), are consistent and:

$$\sqrt{n}\,\left(\mathrm{RMSE}(\hat{h})\cdot\mathrm{RMSE}(\hat{m})+\mathrm{RMSE}(\hat{p})\cdot\mathrm{RMSE}(\hat{m})\right)\to_p 0$$

- *Assume* random variables Y,D,X,Z have bounded fourth moments

$$\sqrt{n}\left(\hat{\theta}-\theta_0\right)\to_d N\left(0,\sigma^2\right),$$

Estimate asymptotically normal

$$\sigma^2 := \frac{E\left[\left(\tilde{Y}-\theta_0\tilde{D}\right)^2\tilde{Z}^2\right]}{E\left[\tilde{D}\tilde{Z}\right]^2},$$

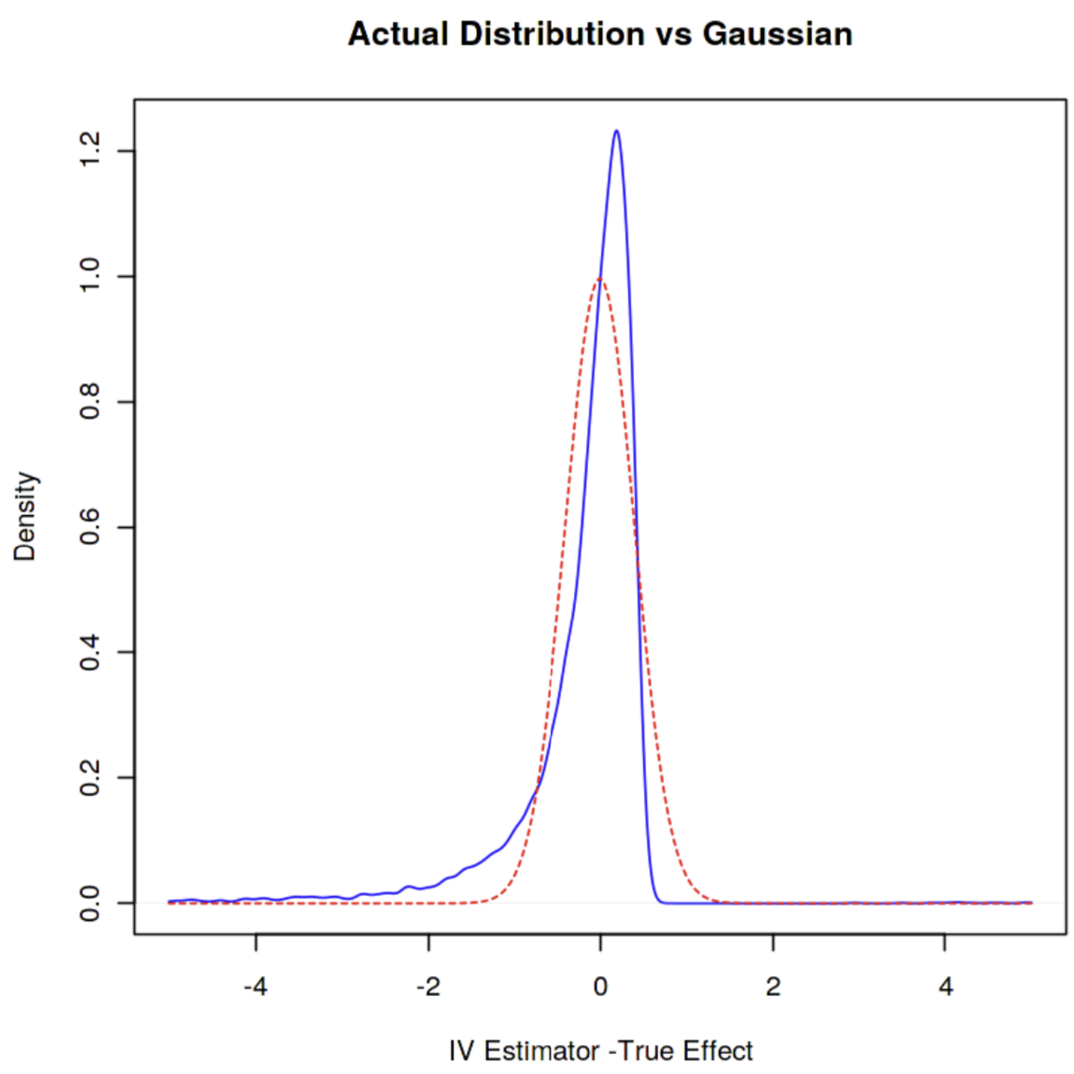
Asymptotic variance

$$\hat{\sigma}^2=\frac{E_n\left[\left(\hat{Y}-\hat{\theta}\,\hat{D}\right)^2\hat{Z}^2\right]}{E_n\left[\hat{D}\hat{Z}\right]^2}$$

Estimate of variance ⇒ 95% CI $\left[\theta \pm 1.96\frac{\hat{\sigma}}{\sqrt{n}}\right]$

Weak Identification

- If $\boldsymbol{E}[\widetilde{\boldsymbol{D}}\widetilde{\boldsymbol{Z}}]$ is small and comparable with the sample size, then approximation
$$E_n[\tilde{D}\tilde{Z}]^{-1} \approx E[\tilde{D}\tilde{Z}]^{-1}$$
- Can be inaccurate in finite samples and normal based approximation will yield in-correct confidence intervals



A More Robust Inference Approach

- Even in the weak regime the moment constraint is still well-behaved
$$E\left[\left(\tilde{Y} - \theta\widetilde{D}\right)\tilde{Z}\right]$$
- At the true parameter θ_0 we know that:
$$\mathrm{C}(\theta) := \frac{\left(\sqrt{n}\, E_n\left[\left(\tilde{Y} - \theta\widetilde{D}\right)\tilde{Z}\right]\right)^2}{Var_n\left(\left(\tilde{Y} - \theta\widetilde{D}\right)\tilde{Z}\right)} \sim_a \left(N(0,1)\right)^2 = \chi^2(1)$$
- This statistic does not hinge on inversion of $E\left[\widetilde{D}\tilde{Z}\right]$; approximation remains valid even with cross-fitted approximate residuals due to Neyman orthogonality
- We can perform a grid search over candidate parameters θ and for every such parameter test whether (for confidence interval with confidence α)
$$C(\theta) \leq (1-\alpha) \text{ quantile of } \chi^2(1)$$
- Then by construction: $\Pr\left(\theta_0 \in C(\theta)\right) \approx 1 - \alpha$

General Moments and Weak Identification

- For a general Neyman orthogonal moment
$$E[m(Z;\theta_0,g_0)]=0$$
- We can construct a statistic that is robust to weak identification (i.e. Jacobian $\partial_\theta E[m(Z;\theta_0,g_0)]$ very small)
$$C(\theta)=\frac{\left(\sqrt{n}E_n[m(Z;\theta,\hat{g})]\right)^2}{Var_n\left(m(Z;\theta,\hat{g})\right)}\sim_a\chi^2(1)$$
- Construct a α-confidence region by including all parameter values θ s.t.
$$C(\theta)\leq(1-\alpha)\text{ quantile of }\chi^2(1)$$
- Then by construction: $\Pr\left(\theta_0\in C(\theta)\right)\approx 1-\alpha$