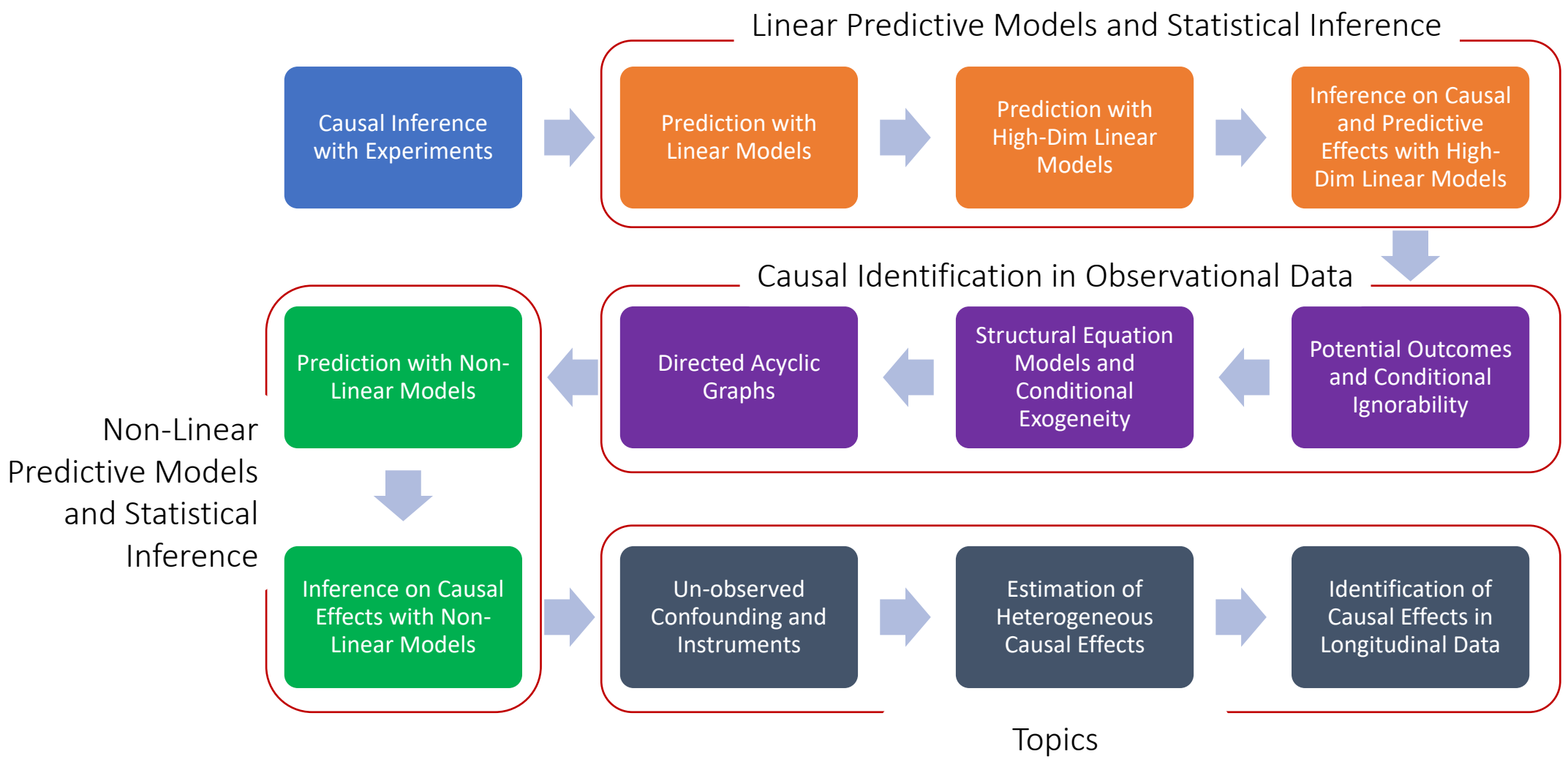
MS&E 228: Topics on Longitudinal Data and Causal Machine Learning

Vasilis Syrgkanis

MS&E, Stanford


Linear Predictive Models and Statistical Inference
Causal Inference with Experiments
Prediction with Linear Models
Prediction with High-Dim Linear Models
Inference on Causal and Predictive Effects with High-Dim Linear Models
Causal Identification in Observational Data
Potential Outcomes and Conditional Ignorability
Structural Equation Models and Conditional Exogeneity
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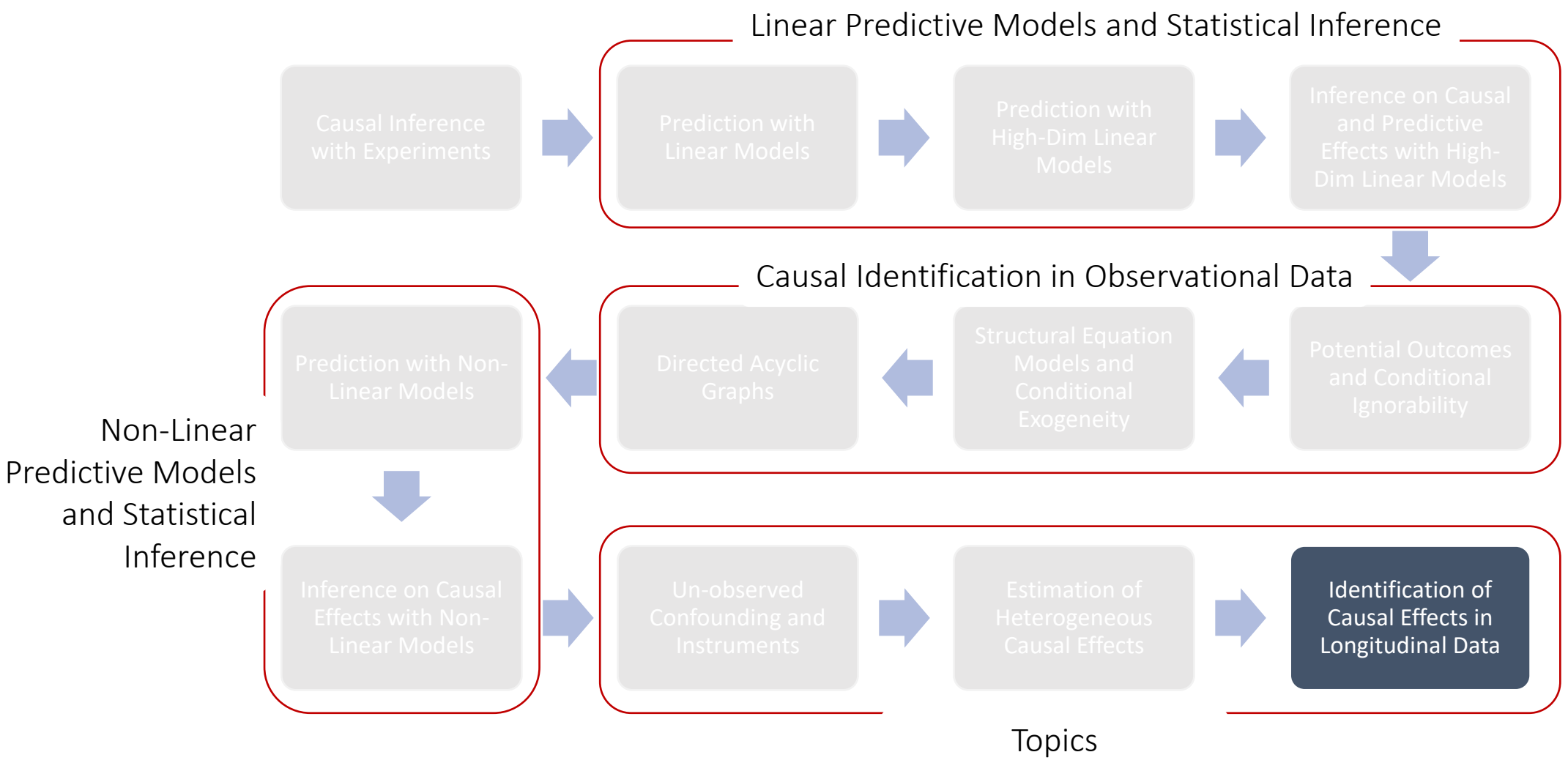
Un-observed Confounding and Instruments
Estimation of Heterogeneous Causal Effects
Identification of Causal Effects in Longitudinal Data
Topics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Topics

Longitudinal Data

- Longitudinal data refer to datasets where we have multiple observations of the same unit over time

They present a separate set of new problems

- **Correlation:** samples stemming from each unit (or sometimes cluster of units from the same “site”) are correlated
- **Censoring/Missingness:** units typically drop out at (potentially not) random times before the end of the study
- **Dynamic treatments:** units are treated with multiple treatments over time in a manner that is adaptive and auto-correlated

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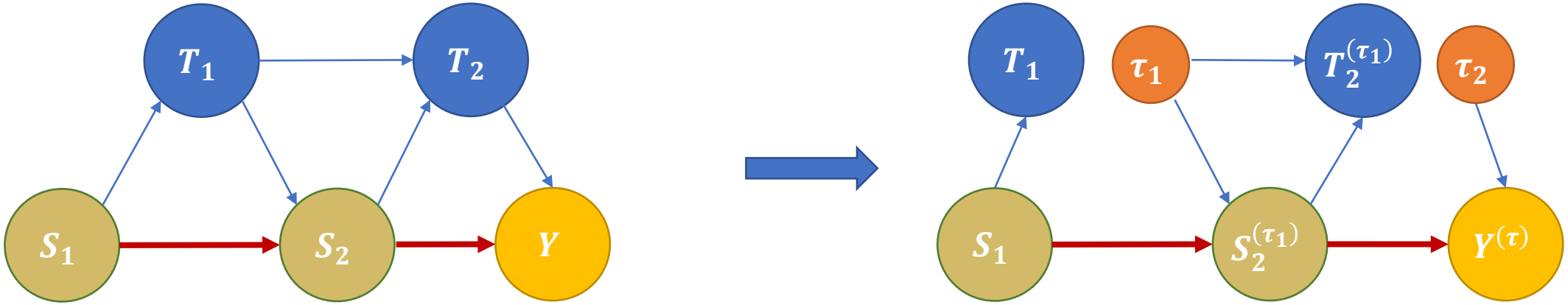
Dynamic Treatment Regime

Dynamic Treatment Regime

observed data (panel)

target quantity: average outcome under a static treatment sequence (regime) $\tau = (\tau_1, \tau_2)$

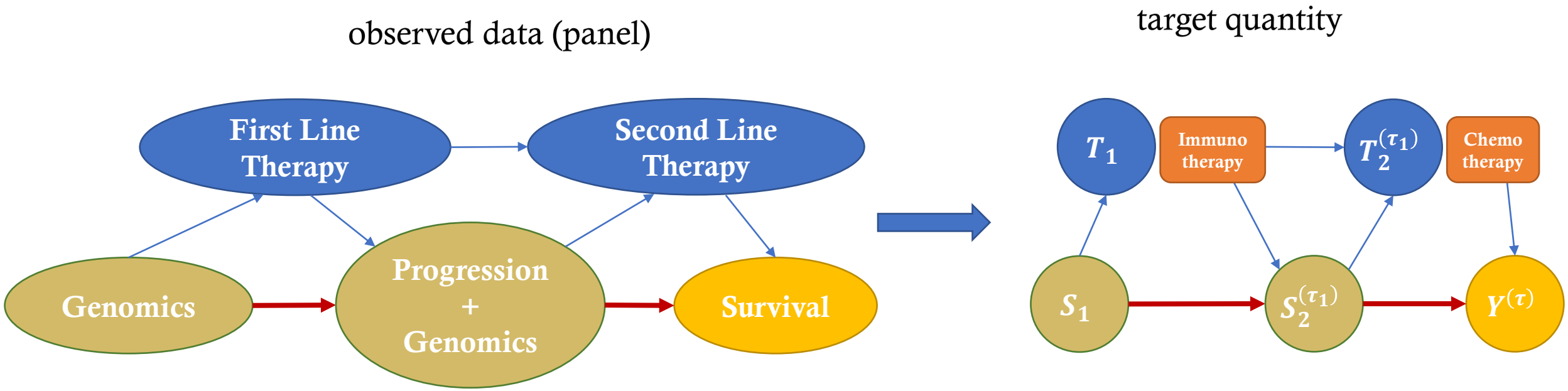
$$\theta := E\left[Y^{(\tau)}\right]$$



- Treatments are offered in an adaptive manner, in response to previous period controls
- The surrogate – control feedback precludes viewing this as a one-shot treatment problem
- Setting is known as the dynamic treatment regime [Robins'94,'04, Chakraborty-Murphy'14]

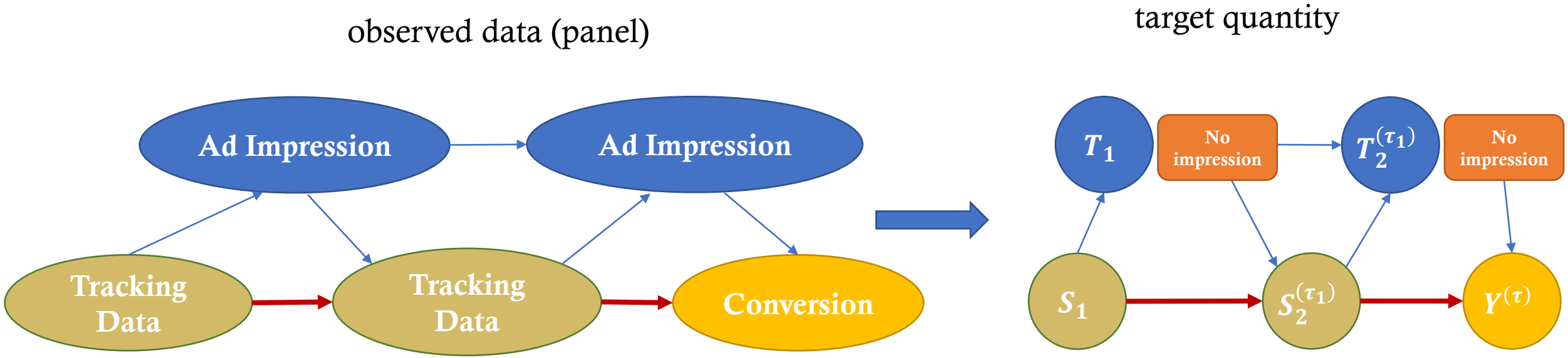
Examples

- Healthcare: Patients treated over time and adaptively



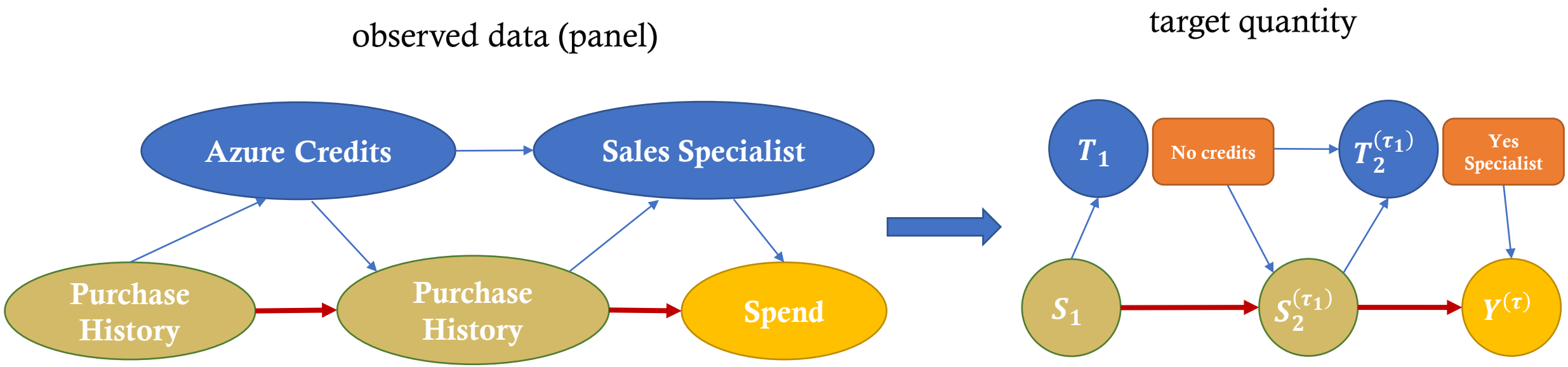
Examples

- Digital Marketing: Web users shown ads multiple times



Examples

- Business-to-Business (B2B) Operations: Customers offered multiple discount/support interventions over time



Identification in the Dynamic Treatment Regime

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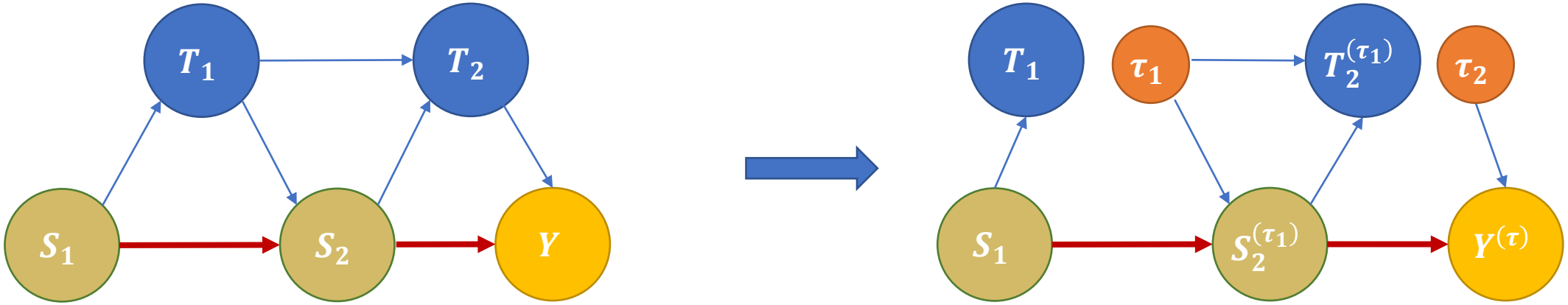

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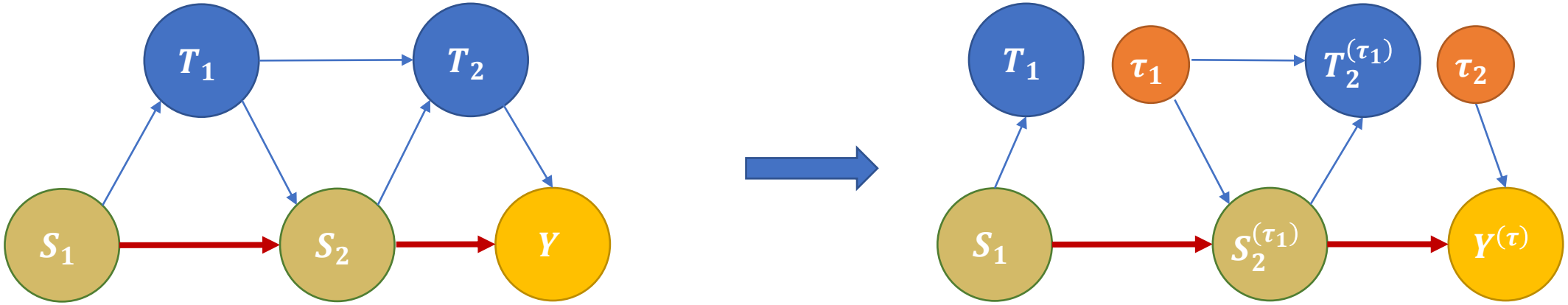

- Since there is no unobserved confounding, why not just estimate the effect of $\mathrm{T} = (T_1, T_2)$ by conditioning, conditioning on $S = (S_1, S_2)$
- Wrong: conditioning on S_2 blocks all the effect from T_1!

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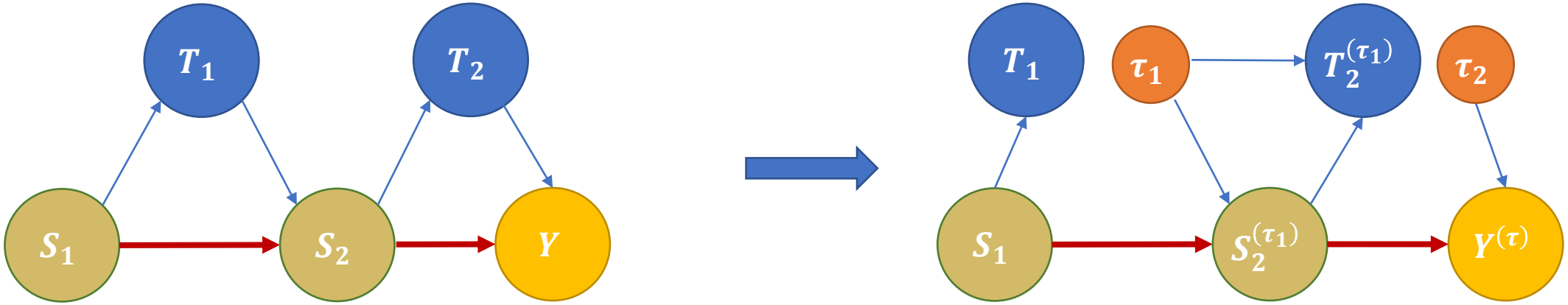

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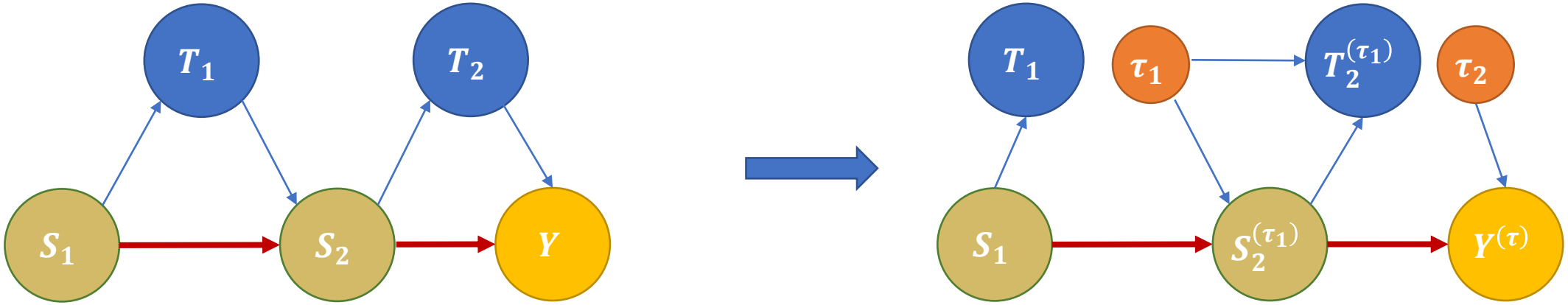

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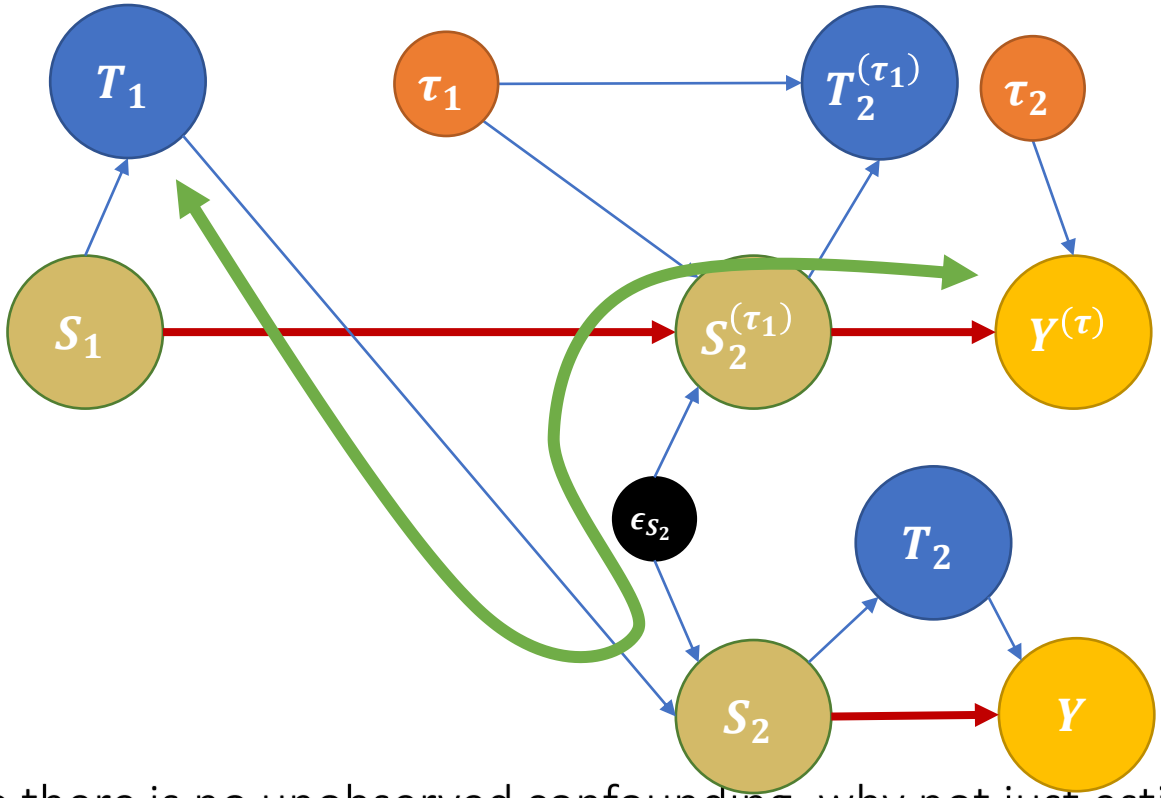

Intervention World

Two-World Graphs

Observed World

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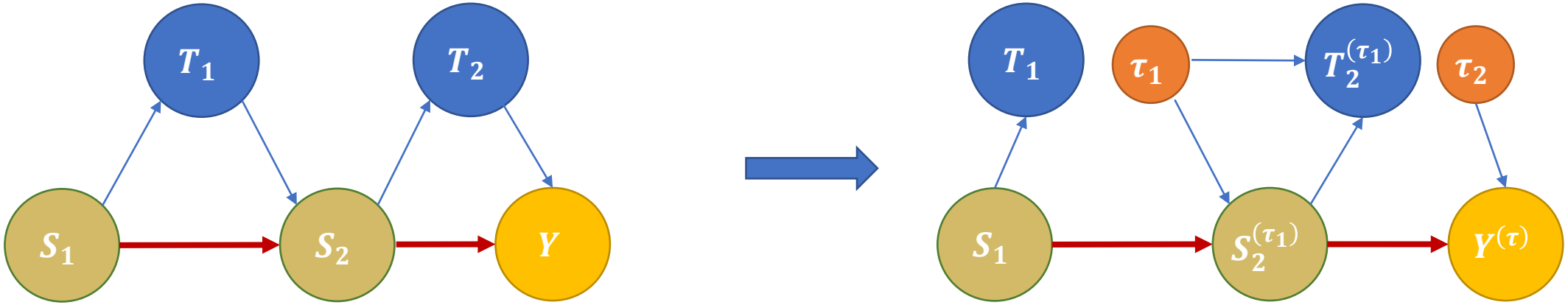

- Then let's condition only on S_1, i.e. estimate the effect of $\mathrm{T} = (T_1, T_2)$ by conditioning, conditioning on S_1
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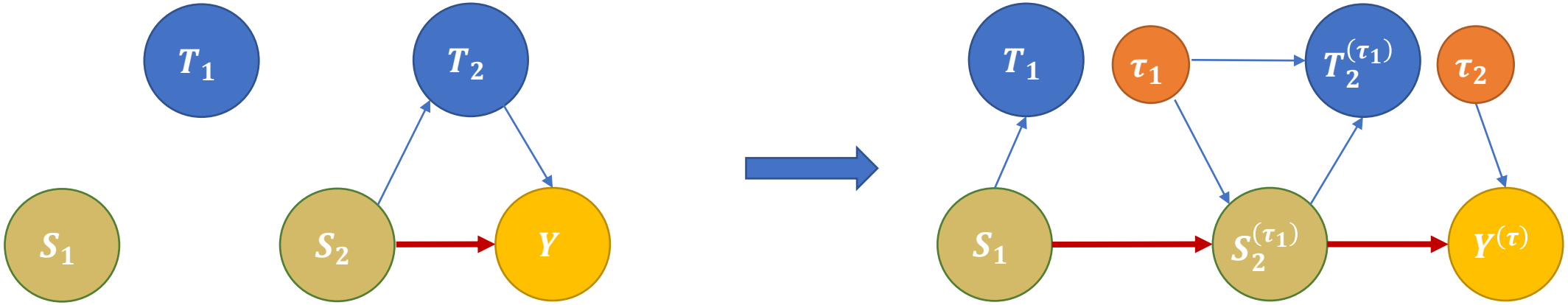

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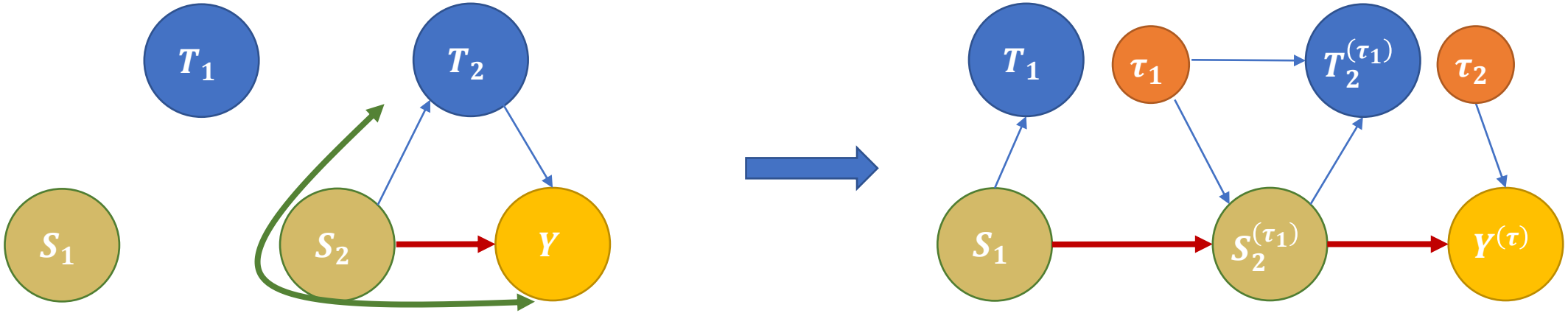

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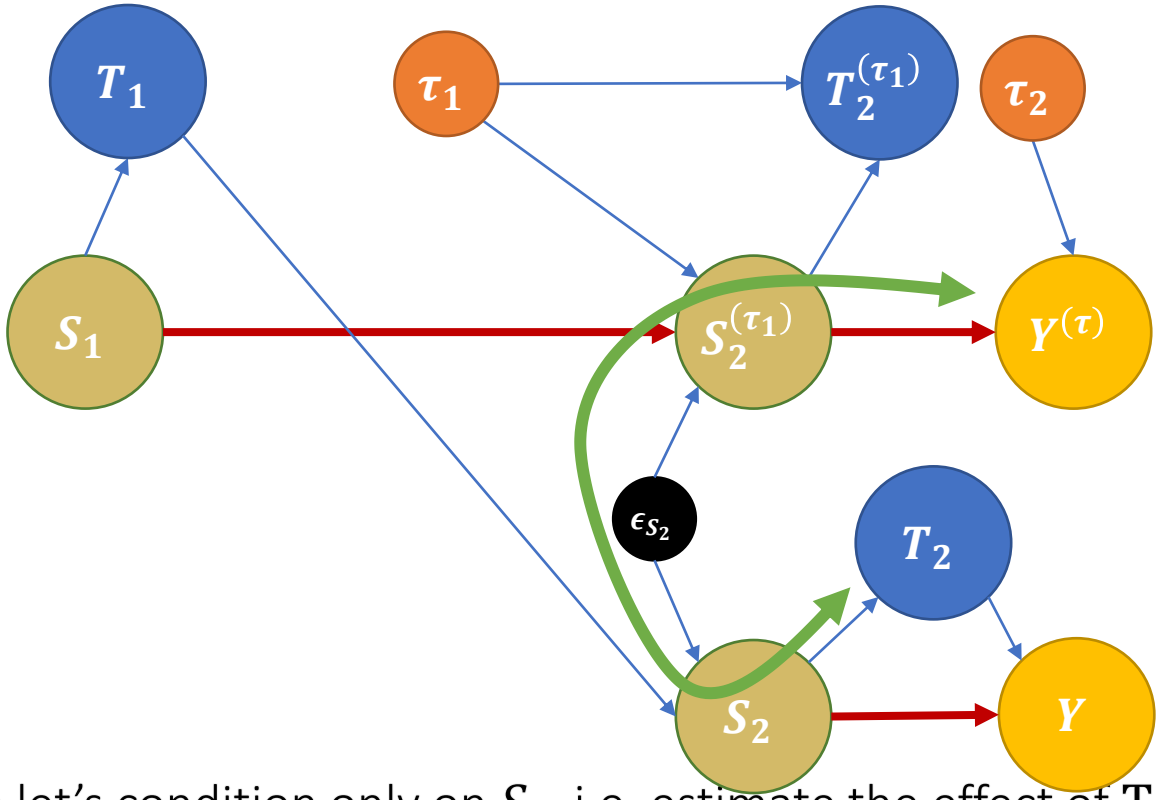

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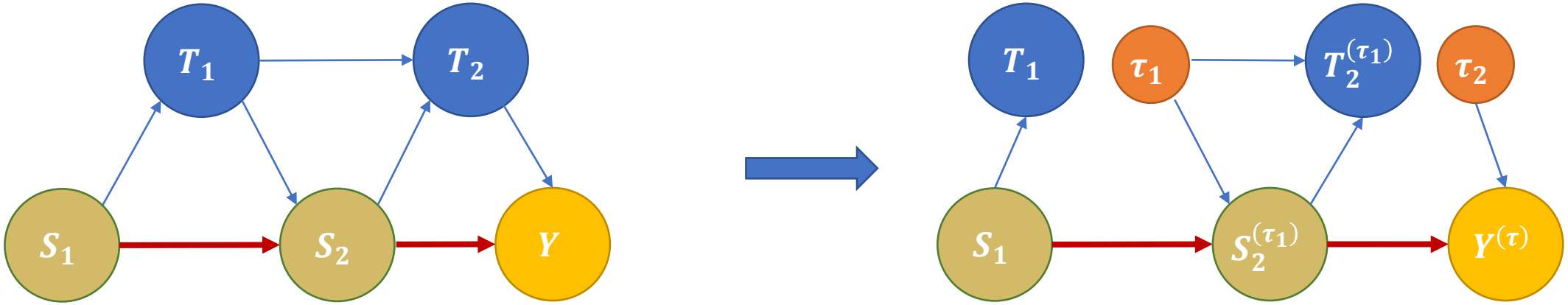
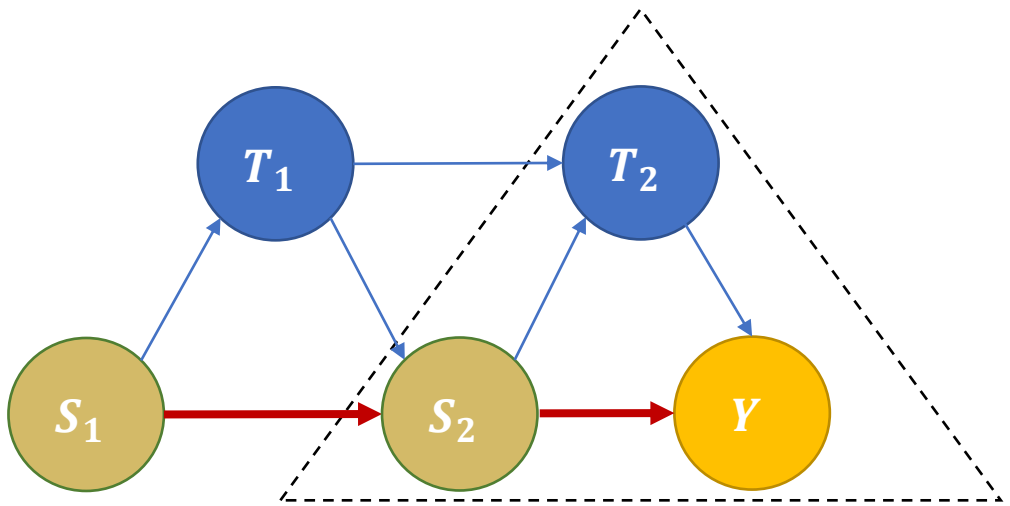
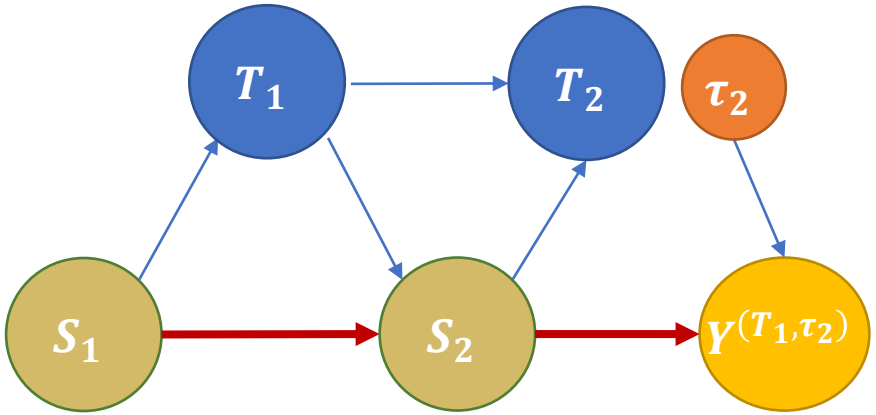


- We cannot identify this expected potential outcome simply by conditioning!
- Let's take it one step at a time
- What can we identify by conditioning?

Identification of Last Period Intervention



target quantity: $\theta_2(S_2) \coloneqq E\left[Y^{(T_1,\tau_2)}|S_2\right]$



We have conditional ignorability $Y^{(T_1,\tau_2)} \perp\!\!\!\perp T_2 \mid S_2$

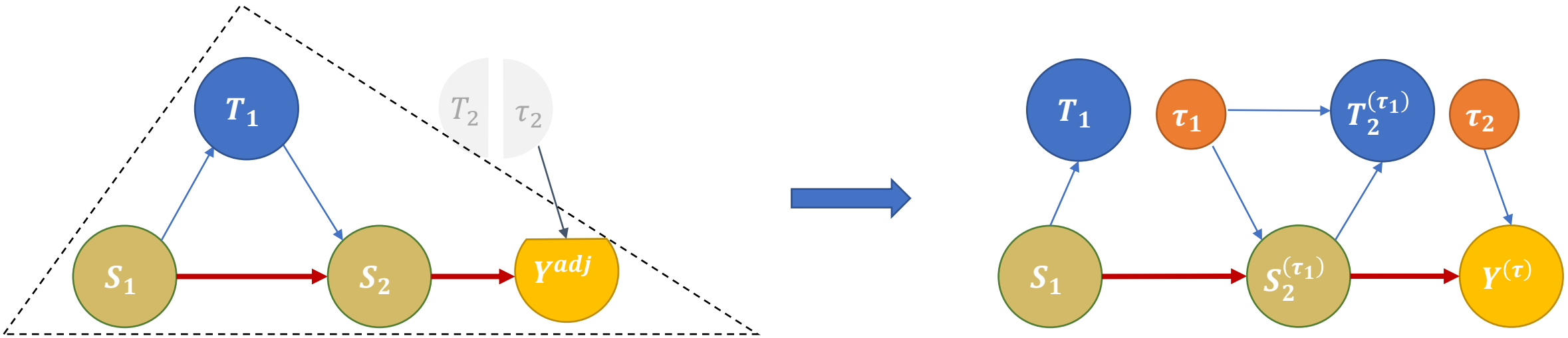
Target quantity $\theta_2(S_2)$ can be identified by conditioning

$$\theta_2(S_2) = E\left[Y^{(T_1,\tau_2)}\middle|S_2\right]$$
$$= E\left[Y^{(T_1,\tau_2)}\middle|T_2 = \tau_2, S_2\right]$$
$$= E[Y|T_2 = \tau_2, S_2]$$

Identification by conditioning:

- Train a predictive model $f_2(T_2, S_2)$
  $Y \sim T_2, S_2$
- Evaluate the model at $T_2 = \tau_2$
  $\theta_2(S_2) = f_2(\tau_2, S_2)$

Identification via Backwards Recursion

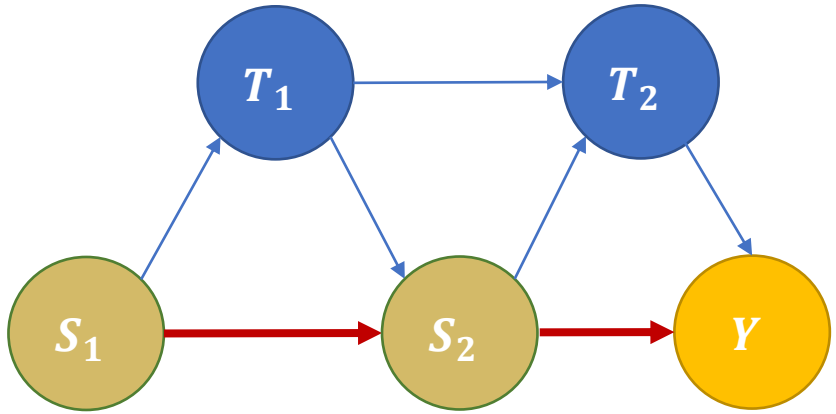


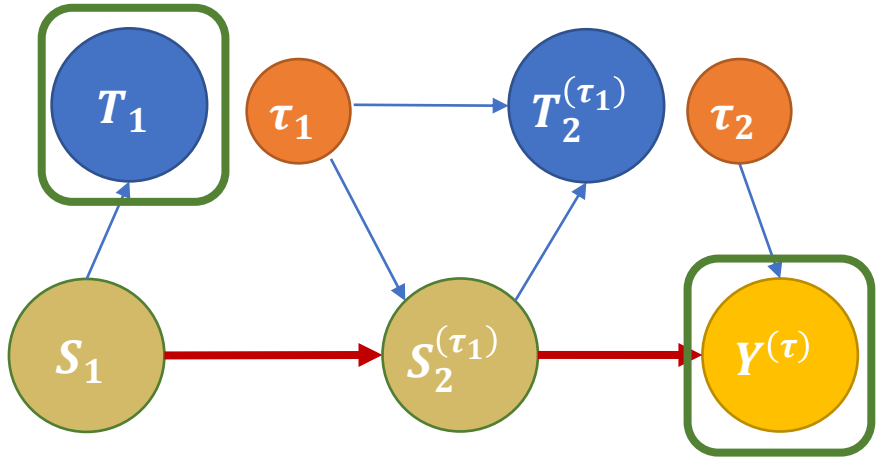
- But now we can "adjust the observed outcome" using this counterfactual model to remove the second period treatment!

- Replace Y with $Y_{\text{adj}} \coloneqq f_2(\tau_2, S_2)$

- Estimating the target quantity $E\left[Y^{(\tau_1,\tau_2)}\right]$ is the same as estimating the effect of T_1 on Y_{adj}
- We can estimate this by conditioning on S_1

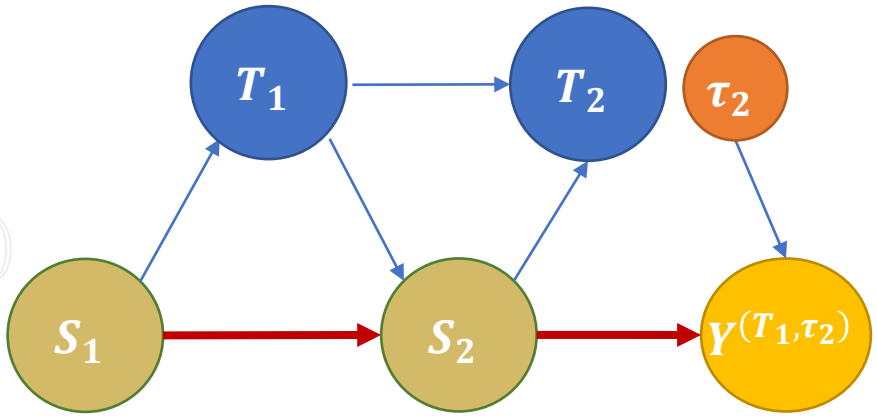
Identification via Backwards Recursion





$$E\left[Y^{(\tau_1,\tau_2)}\right] = E\left[E\left[Y^{(\tau_1,\tau_2)}\middle|S_1\right]\right]$$
$$= E\left[E\left[Y^{(T_1,\tau_2)}\middle|T_1=\tau_1,S_1\right]\right]$$
$$= E\left[E\left[E\left[Y^{(T_1,\tau_2)}\middle|S_2,T_1,S_1\right]\middle|T_1=\tau_1,S_1\right]\right] \quad \text{(tower law)}$$
$$= E\left[E\left[E\left[Y^{(T_1,\tau_2)}\middle|S_2\right]\middle|T_1=\tau_1,S_1\right]\right] \quad \left(Y^{(T_1,\tau_2)} \perp\!\!\!\perp S_1,T_1|S_2\right)$$
$$= E\left[E\left[\theta_2(S_2)\middle|T_1=\tau_1,S_1\right]\right]$$

$$\left(Y^{(\tau)} \perp\!\!\!\perp T_1|S_1\right)$$



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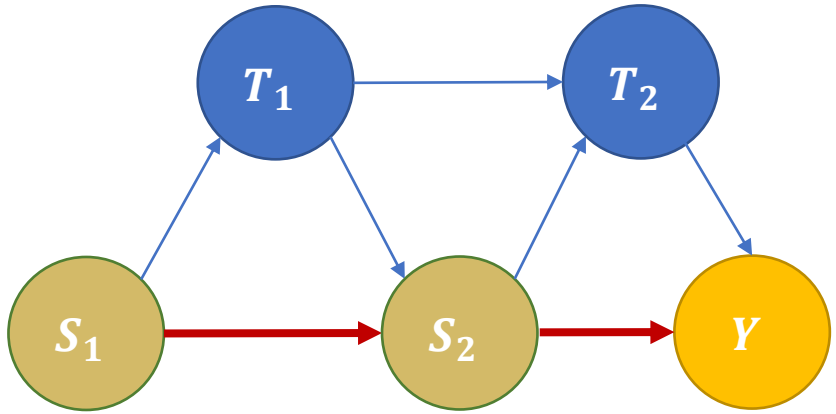

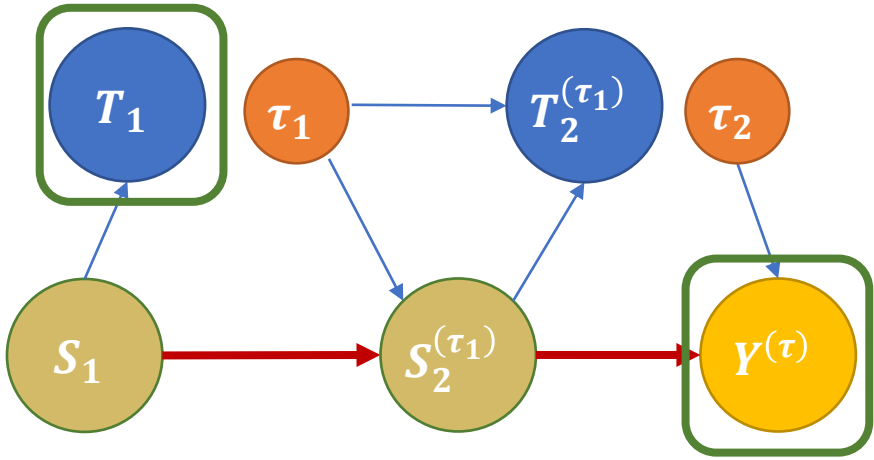

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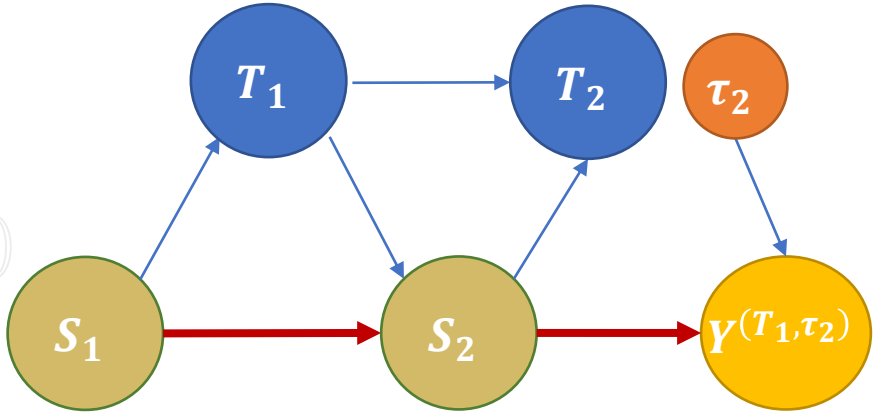
$$= E\left[E\left[\theta_2(S_2)\middle|T_1=\tau_1,S_1\right]\right]$$

$\left(Y^{(\tau)} \perp\!\!\!\perp T_1\middle|S_1\right)$

(tower law)



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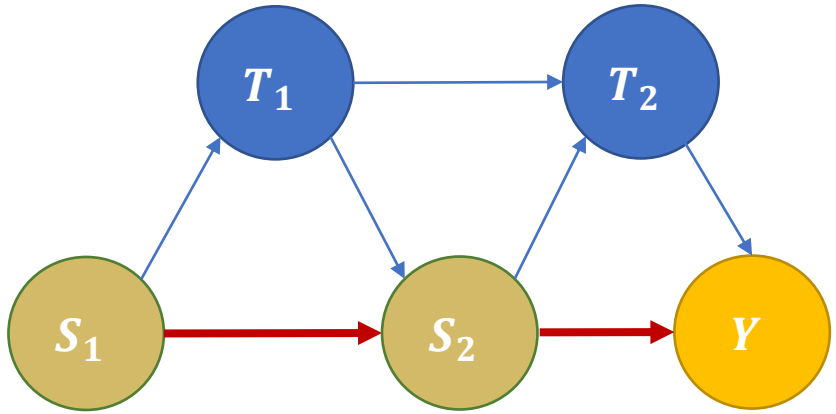

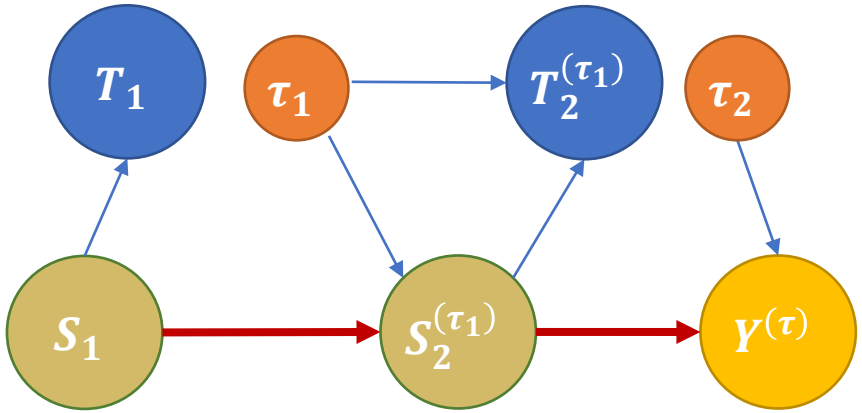

$$
\begin{aligned}
E\left[Y^{(\tau_1,\tau_2)}\right] &= E\left[E\left[Y^{(\tau_1,\tau_2)}\middle|S_1\right]\right] \\
&= E\left[E\left[Y^{(T_1,\tau_2)}\middle|T_1=\tau_1,S_1\right]\right] && \left(Y^{(\tau)} \perp\!\!\!\perp T_1 \middle| S_1\right) \\
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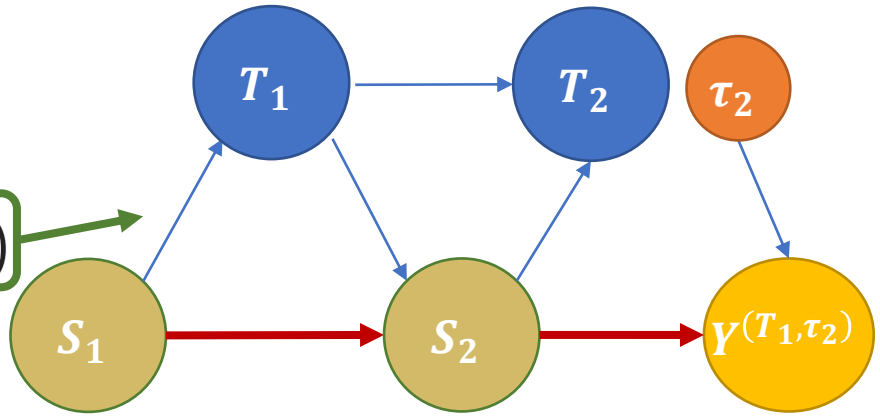
\end{aligned}
$$



Identification Process

- Estimate predictive model $f_2: Y \sim T_2, S_2$
- Adjust outcome for second period treatment

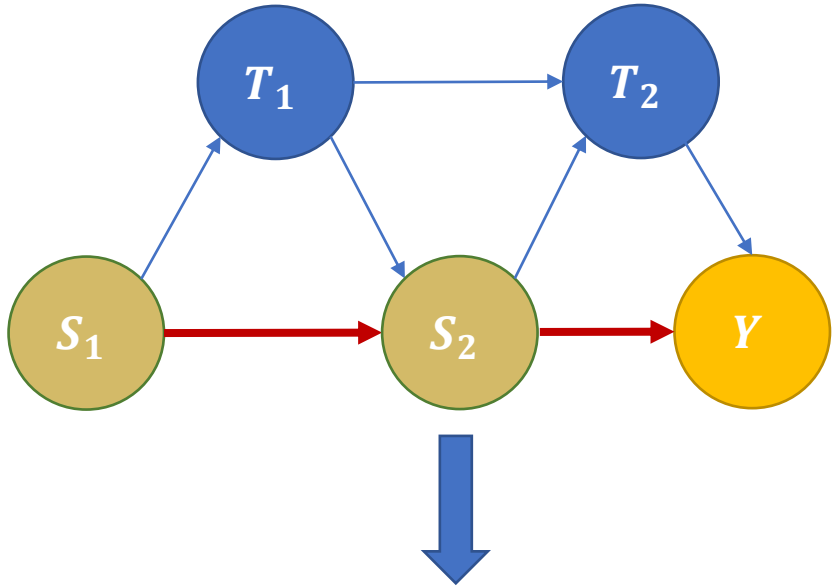
$$Y_{adj} \leftarrow f_2(\tau_2, S_2)$$

- Estimate predictive model $f_1: Y_{adj} \sim T_1, S_1$
- Adjust outcome for first period treatment

$$Y_{adj} \leftarrow f_1(\tau_1, S_1)$$

- Return target quantity: $\theta = E[Y_{adj}]$

observed data (panel)



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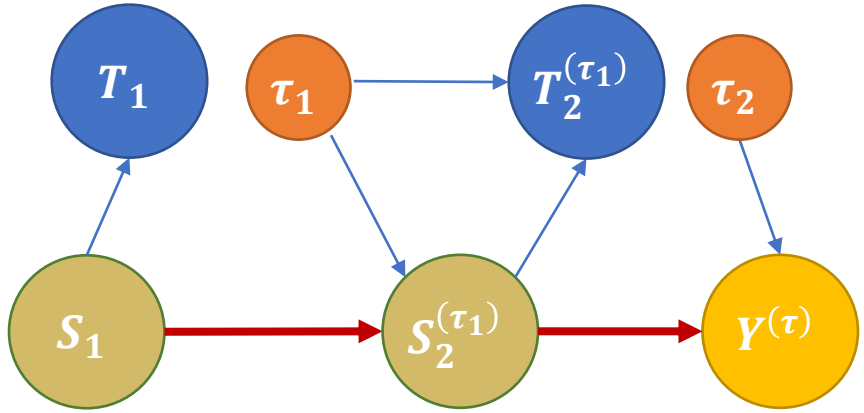
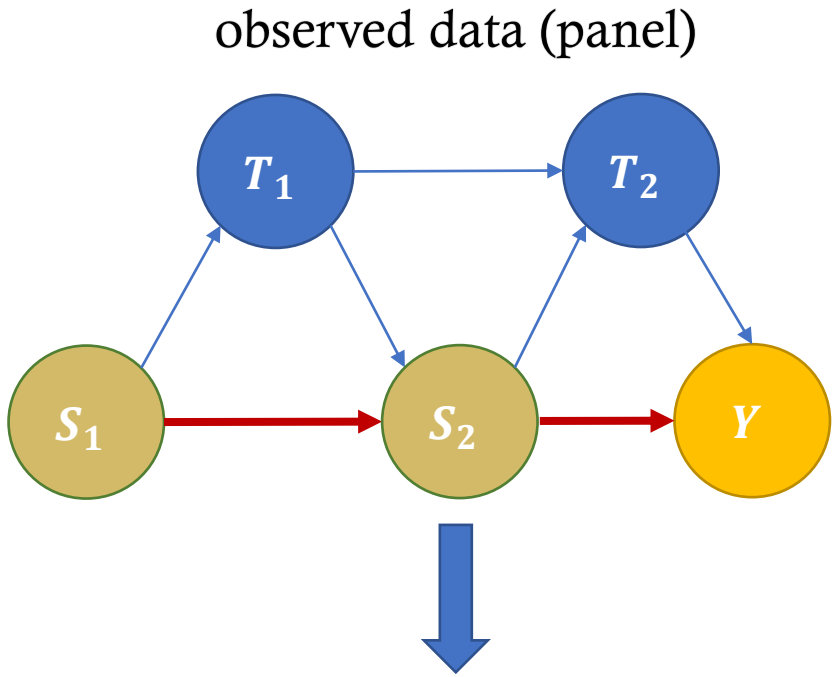


Moment Based Framework

$$\theta = E[f_1(\tau_1, S_1)]$$

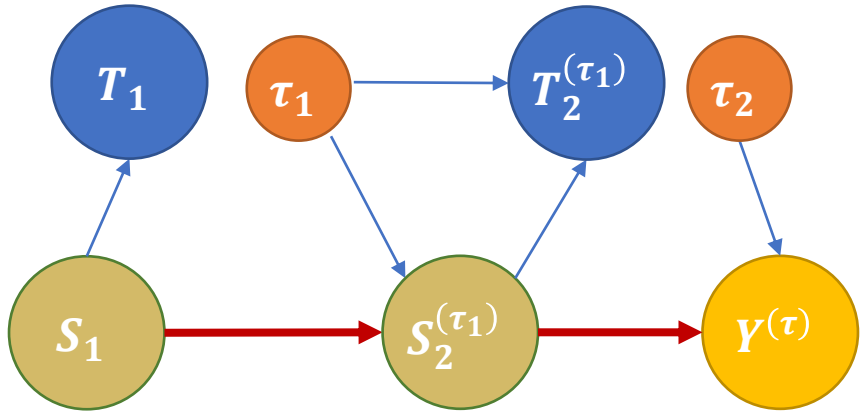
$$f_1(T_1, S_1) := E[f_2(\tau_2, S_2)|T_1, S_1]$$
$$f_2(T_2, S_2) := E[Y|T_2, S_2]$$

These set of equations (and their generalization to many periods) are known as the g-formula



target quantity: average outcome under a static treatment sequence (regime) $\tau = (\tau_1, \tau_2)$

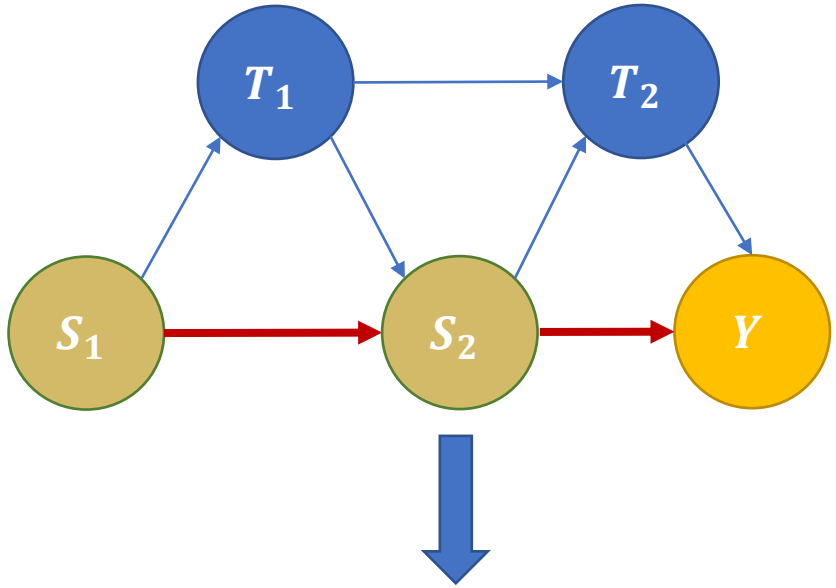
$$\theta := E[Y^{(\tau)}]$$



Estimation in the Dynamic Treatment Regime

G-computation

observed data (panel)



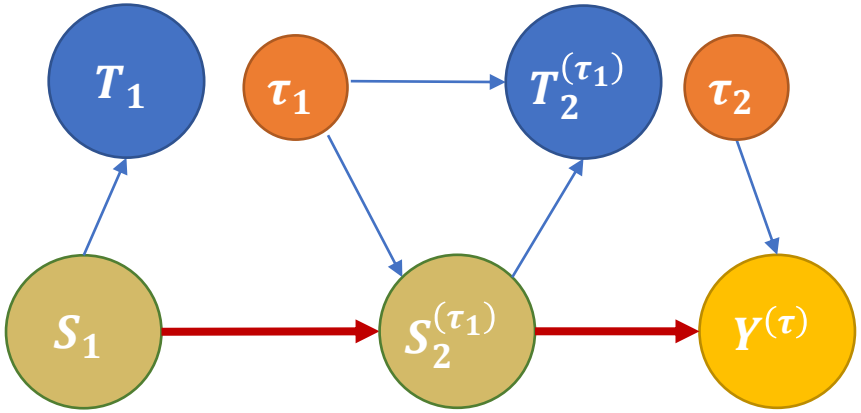
Moment function: $m(Z;\theta,f)$

$$E[\theta - f_1(\tau_1, S_1)] = 0$$

$$f_1(T_1, S_1) := E[f_2(\tau_2, S_2)|T_1, S_1]$$
$$f_2(T_2, S_2) := E[Y|T_2, S_2]$$

target quantity: average outcome under a static treatment sequence (regime) $\tau = (\tau_1, \tau_2)$

$$\theta := E[Y^{(\tau)}]$$



Nuisance functions

- Using parametric models to estimate these functions (e.g. linear or logistic regression) and plug them in the formula is known as the “parametric g computation”

ML Based Estimation

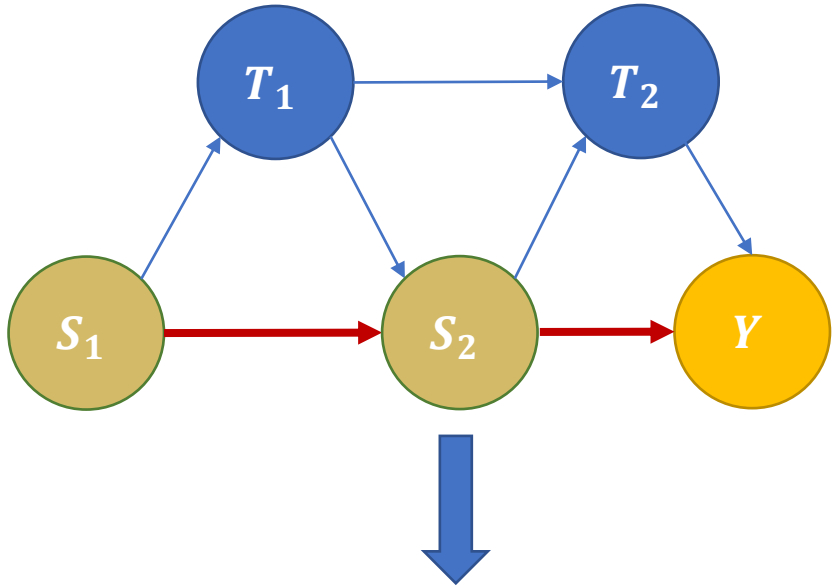
Moment function: $m(Z;\theta,f)$

$$E[\theta - f_1(\tau_1, S_1)] = 0$$

$$f_1(T_1, S_1) := E[f_2(\tau_2, S_2)|T_1, S_1]$$
$$f_2(T_2, S_2) := E[Y|T_2, S_2]$$

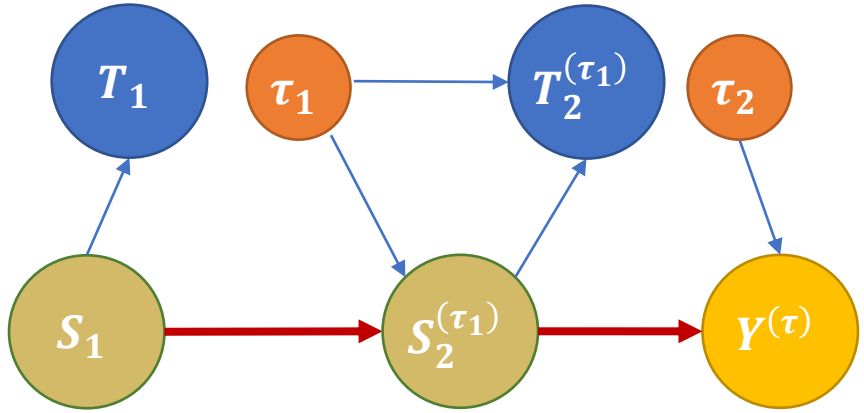
- If we use ML for these predictive problems, then we wont be able to construct confidence intervals for θ because moment is not Neyman orthogonal
- We can apply the Inverse Propensity based debiasing idea

observed data (panel)



target quantity: average outcome under a static treatment sequence (regime) $\tau = (\tau_1, \tau_2)$

$$\theta := E[Y^{(\tau)}]$$



Debiasing Moments and Neyman Orthogonality

Original moment

Inverse Propensity

Residual of the Regression Problem that defines nuisance function f_1

$$E\left[\theta - f_1(\tau_1, S_1) + \frac{1\{T_1 = \tau_1\}}{\Pr(T_1 = \tau_1 | S_1)}\left(f_2(\tau_2, S_2) - f_1(T_1, S_1)\right)\right] = 0$$

Debiasing correction for nuisance function f_1

$$f_1(T_1, S_1) := E[f_2(\tau_2, S_2) | T_1, S_1]$$
$$f_2(T_2, S_2) := E[Y | T_2, S_2]$$

Nuisance functions

Debiasing Moments and Neyman Orthogonality

Original moment

We are still left with this nuisance, that we have not "debiased"

$$E\left[\theta - f_1(\tau_1, S_1) + \frac{1\{T_1 = \tau_1\}}{\Pr(T_1 = \tau_1 | S_1)}\left(f_2(\tau_2, S_2) - f_1(T_1, S_1)\right)\right] = 0$$

Debiasing correction for nuisance function f_1

$$f_1(T_1, S_1) := E[f_2(\tau_2, S_2) | T_1, S_1]$$
$$f_2(T_2, S_2) := E[Y | T_2, S_2]$$

Nuisance functions

Debiasing Moments and Neyman Orthogonality

We are still left with this nuisance, that we have not "debiased"

Original moment

$$E\left[\theta - f_1(\tau_1, S_1) + \frac{1\{T_1 = \tau_1\}}{\Pr(T_1 = \tau_1 | S_1)}\left(f_2(\tau_2, S_2) - f_1(T_1, S_1)\right)\right] = 0$$

$$+ \frac{1\{T_1 = \tau_1\}}{\Pr(T_1 = \tau_1 | S_1)} \frac{1\{T_2 = \tau_2\}}{\Pr(T_2 = \tau_2 | S_2)}\left(Y - f_2(T_2, S_2)\right)$$

Debiasing correction for nuisance function f_2

$$f_1(T_1, S_1) := E[f_2(\tau_2, S_2) | T_1, S_1]$$
$$f_2(T_2, S_2) := E[Y | T_2, S_2]$$

Nuisance functions

Debiasing Moments and Neyman Orthogonality

We are still left with this nuisance, that we have not "debiased"

Original moment

$$E\left[\theta - f_1(\tau_1, S_1) + \frac{1\{T_1 = \tau_1\}}{\Pr(T_1 = \tau_1 | S_1)}\left(f_2(\tau_2, S_2) - f_1(T_1, S_1)\right)\right] = 0$$

IPS term already multiplying f_2

$$+ \frac{1\{T_1 = \tau_1\}}{\Pr(T_1 = \tau_1 | S_1)} \frac{1\{T_2 = \tau_2\}}{\Pr(T_2 = \tau_2 | S_2)}\left(Y - f_2(T_2, S_2)\right)$$

New IPS term for second period treatment introduced to transform $f_2(T_2, S_2) \to f_2(\tau_2, S_2)$

Residual of the regression problem that defines f_2

$$f_1(T_1, S_1) := E[f_2(\tau_2, S_2) | T_1, S_1]$$
$$f_2(T_2, S_2) := E[Y | T_2, S_2]$$

Nuisance functions

Debiasing Moments and Neyman Orthogonality

Original moment

$$E\left[\theta - f_1(\tau_1, S_1) + \frac{1\{T_1 = \tau_1\}}{\Pr(T_1 = \tau_1 | S_1)}\left(f_2(\tau_2, S_2) - f_1(T_1, S_1)\right)\right] = 0$$

$$+ \frac{1\{T_1 = \tau_1\}}{\Pr(T_1 = \tau_1 | S_1)} \frac{1\{T_2 = \tau_2\}}{\Pr(T_2 = \tau_2 | S_2)}\left(Y - f_2(T_2, S_2)\right)$$

$$f_1(T_1, S_1) := E[f_2(\tau_2, S_2) | T_1, S_1]$$
$$f_2(T_2, S_2) := E[Y | T_2, S_2]$$

Nuisance functions

- This moment now satisfies Neyman orthogonality with respect to all the nuisance functions
- We need to also estimate the propensity functions via generic ML classification

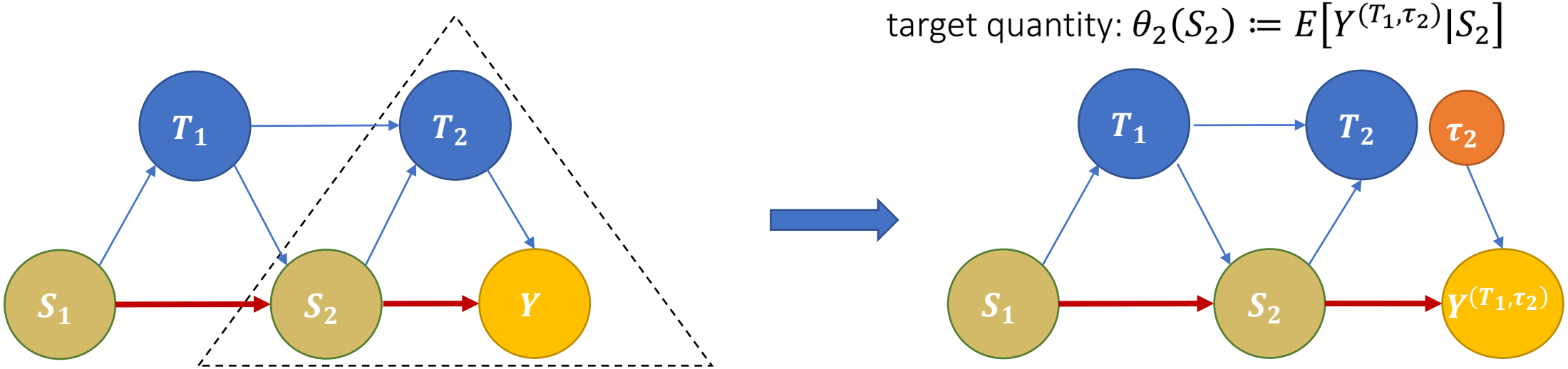
Continuous Treatments and Alternative Approach to Identification and Estimation

What if we have continuous treatments

- The propensity based approach to de-biasing does not apply
- What is the analogue of the “Residual-on-Residual” or “Partialling-Out” (FWL) approach for the dynamic treatment regime, which is also applicable to continuous treatments?

Identification via “Instantaneous” or “Blip” Effects

target quantity: $\theta_2(S_2) \coloneqq E\left[Y^{(T_1,\tau_2)}|S_2\right]$



- Instead of estimating the outcome $Y^{(T_1,\tau_2)}$, we will estimate the “effect” of T_2

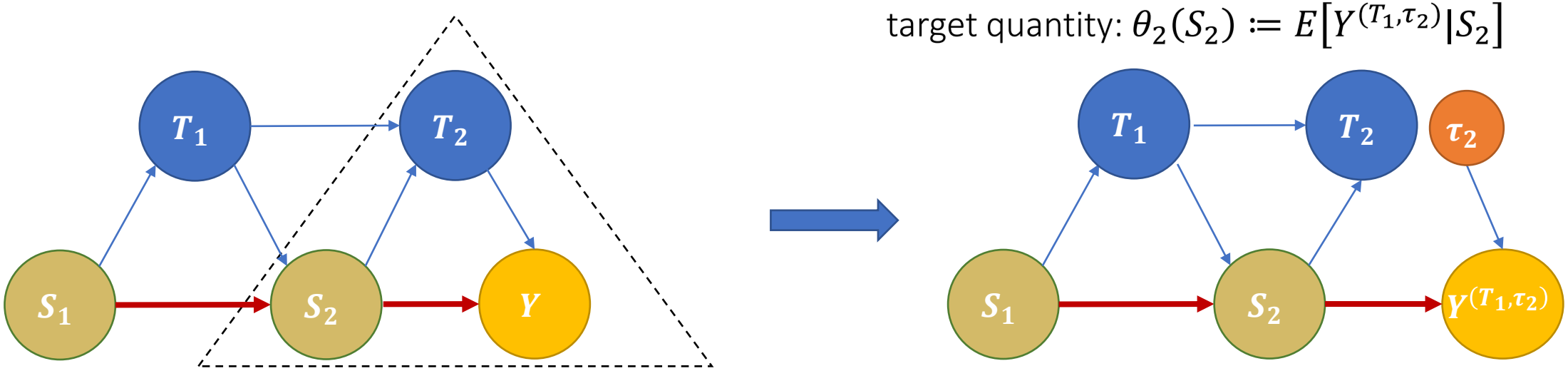
$$\alpha_2(\tau_2, S_2) \coloneqq E\left[Y^{(T_1,\tau_2)} - Y^{(T_1,0)}\middle|S_2\right]$$

- Adjust outcome by subtracting effect of observed treatment and adding effect of target treatment

$$Y_{\text{adj}} \coloneqq Y - \alpha_2(T_2, S_2) + \alpha_2(\tau_2, S_2)$$

Identification via “Instantaneous” or “Blip” Effects

target quantity: $\theta_2(S_2) \coloneqq E\left[Y^{(T_1,\tau_2)} \middle| S_2\right]$



- Instead of estimating the outcome $Y^{(T_1,\tau_2)}$, we will estimate the “effect” of T_2

$$\alpha_2(\tau_2, S_2) \coloneqq E\left[Y^{(T_1,\tau_2)} - Y^{(T_1,0)} \middle| S_2\right]$$

- Adjust outcome by subtracting effect of observed treatment and adding effect of target treatment

$$Y_{\text{adj}} \coloneqq Y - \alpha_2(T_2, S_2) + \alpha_2(\tau_2, S_2)$$

- If “effect” is assumed to have a simple parametric form, then this approach leverages this simplicity!

Repeat in a backwards manner

Identification Process

Estimate the "effect" $\alpha_2(\tau_2, S_2)$

$$\alpha_2(\tau_2, S_2) := E\left[Y^{(T_1,\tau_2)} - Y^{(T_1,0)} \middle| S_2\right]$$

Adjust outcome for second period treatment

$$Y_{\text{adj}} \leftarrow Y - \alpha_2(T_2, S_2) + \alpha_2(\tau_2, S_2)$$

Estimate the effect $\alpha_1(\tau_1, S_1)$

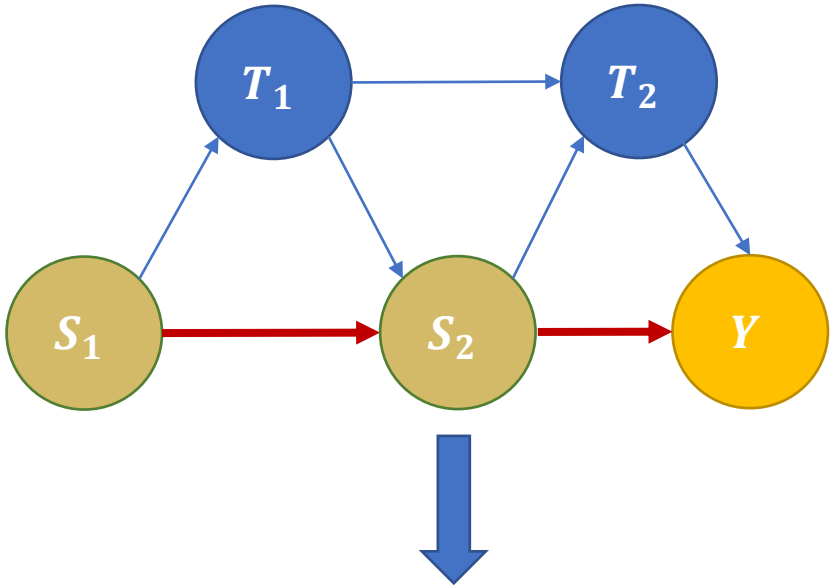
$$E\left[Y^{(\tau_1,\tau_2)} - Y^{(0,\tau_2)} \middle| S_1\right] \approx E\left[Y_{\text{adj}}^{(\tau_1)} - Y_{\text{adj}}^{(0)} \,\middle|\, S_1\right]$$

Adjust outcome for first period treatment

$$Y_{\text{adj}} \leftarrow Y_{\text{adj}} - \alpha_1(T_1, S_1) + \alpha_1(\tau_1, S_1)$$

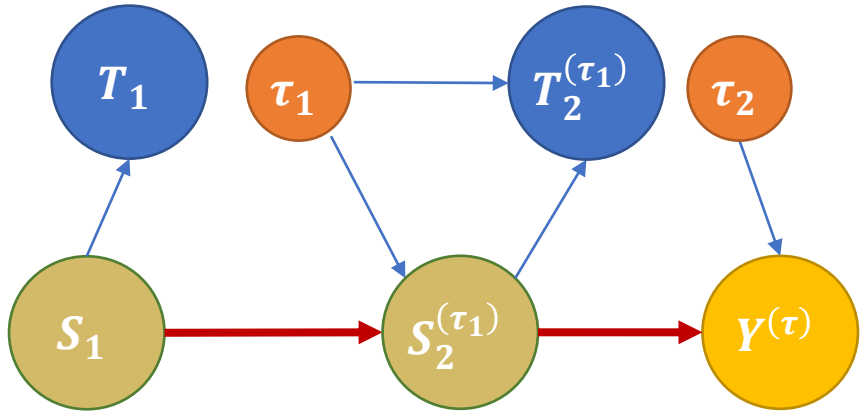
Return target quantity: $\theta = E[Y_{\text{adj}}]$

observed data (panel)



target quantity: average outcome under a static treatment sequence (regime) $\tau = (\tau_1, \tau_2)$

$$\theta := E\left[Y^{(\tau)}\right]$$

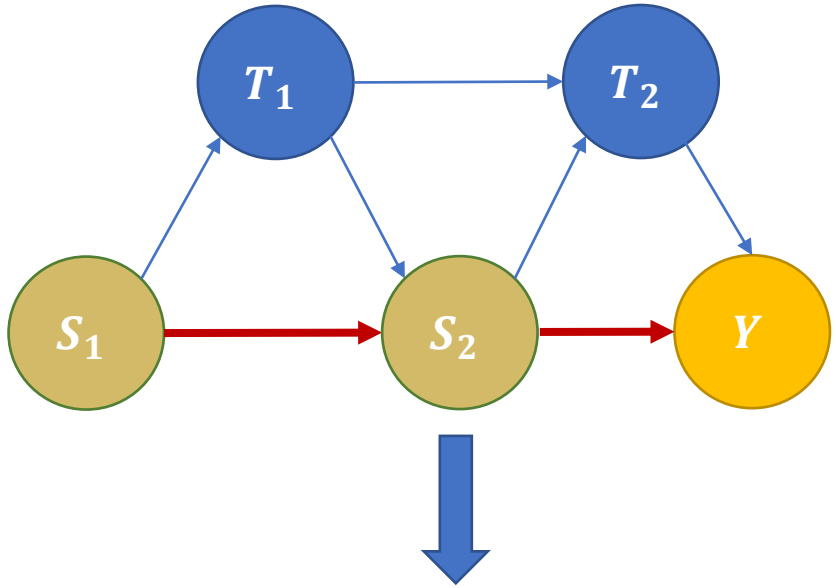


Partial Linearity: Linear Effects

- If we assume that these effects are partially linear:

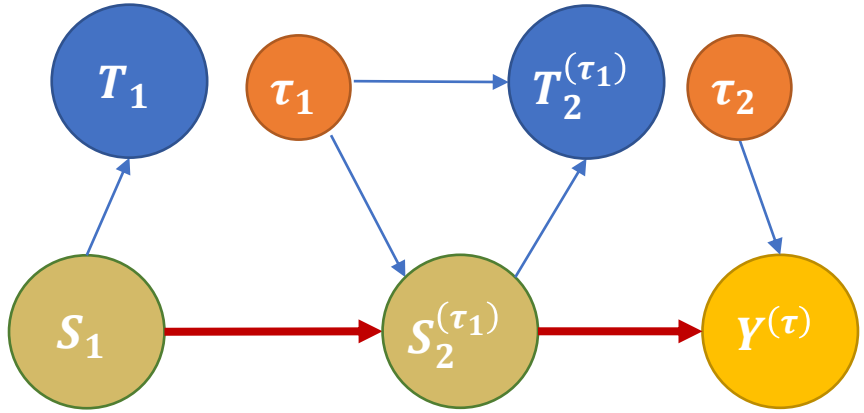
$$a_2(\tau_2, S_2) = \delta_2 \tau_2$$
$$a_1(\tau_1, S_1) := \delta_1 \tau_1$$

observed data (panel)



target quantity: average outcome under a static treatment sequence (regime) $\tau = (\tau_1, \tau_2)$

$$\theta := E\left[Y^{(\tau)}\right]$$

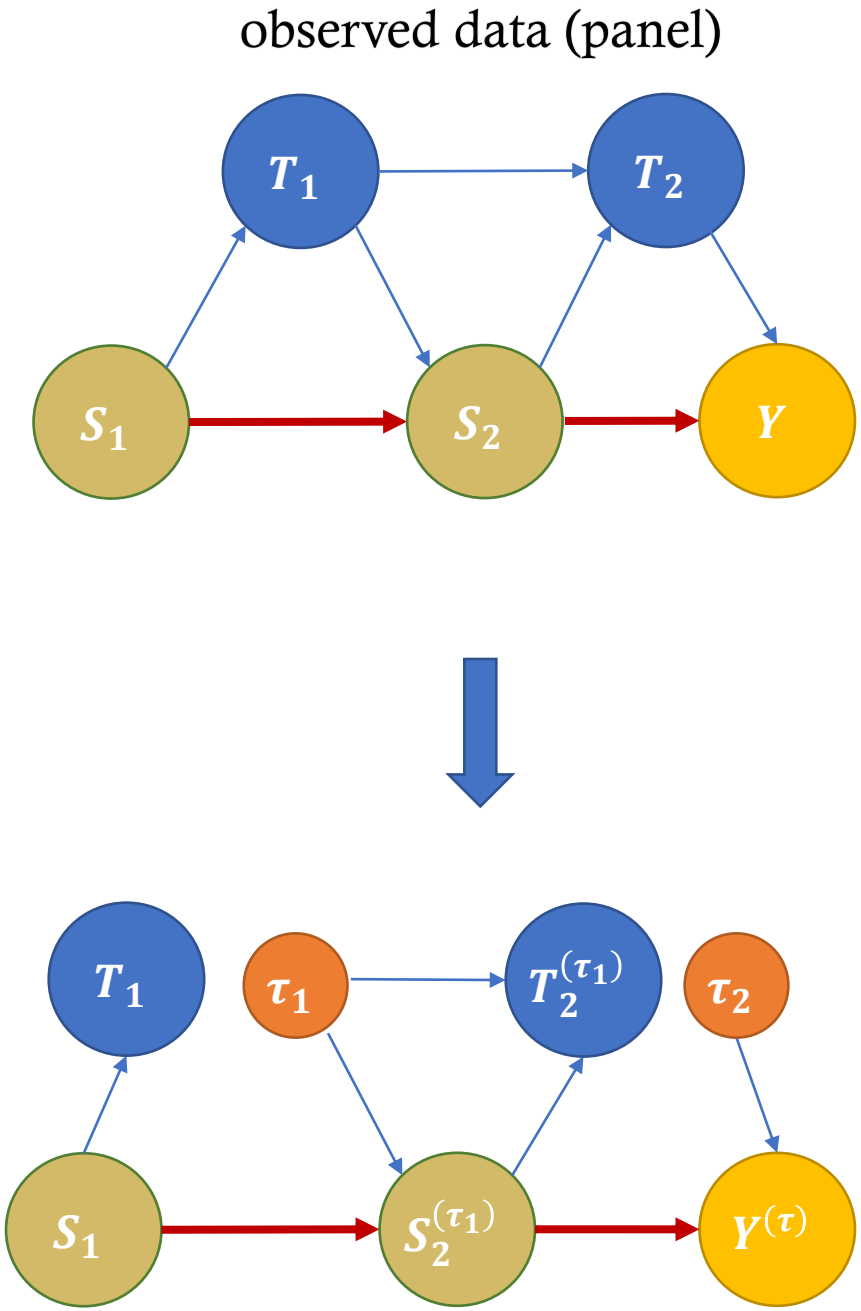


Identification Process

- If we assume that these effects are partially linear:

$$a_2(\tau_2, S_2) = \delta_2 \tau_2$$
$$a_1(\tau_1, S_1) \coloneqq \delta_1 \tau_1$$

- We can estimate these effects via partialling out!



Dynamic DML (Partialling Out)

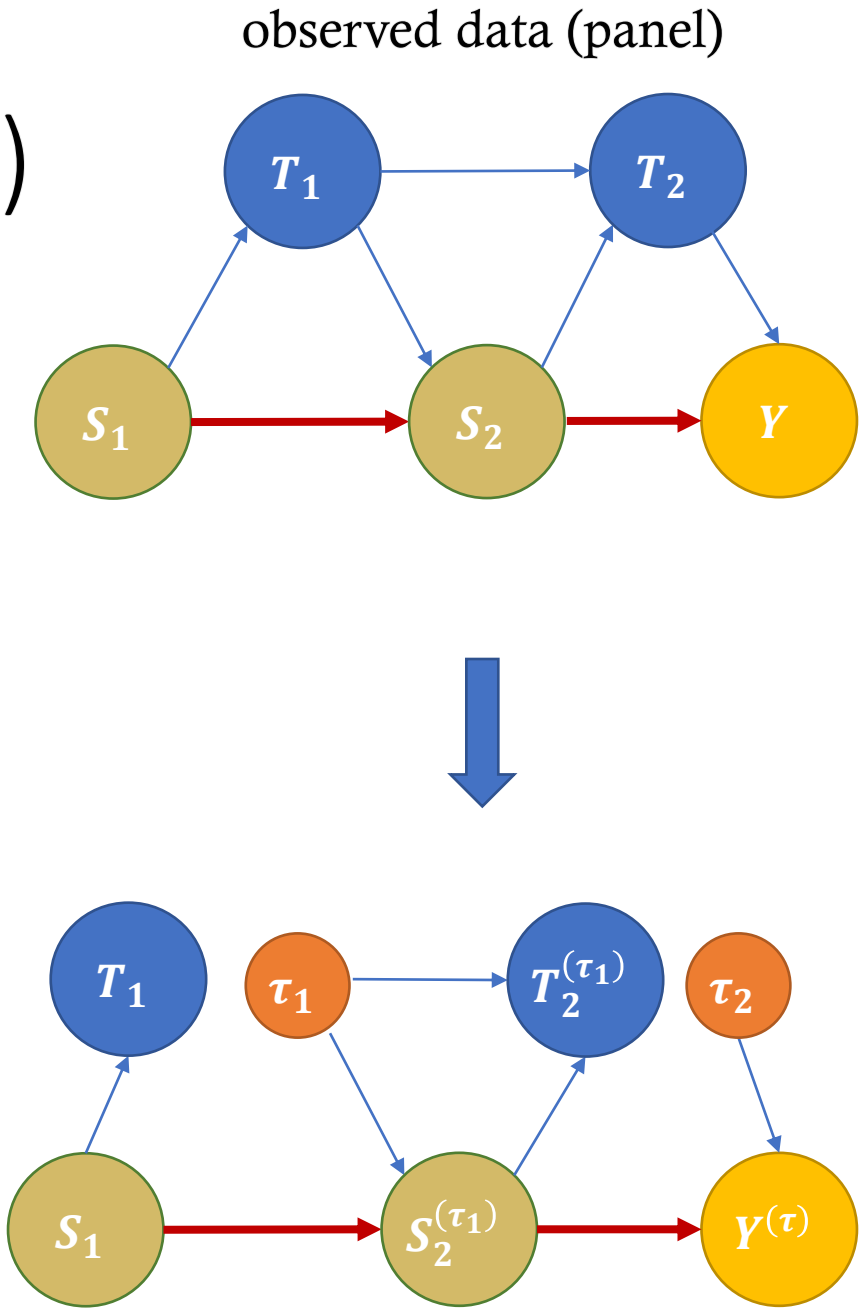
- Construct residuals
  $\tilde{Y}_2 := Y - E[Y|S_2], \qquad \tilde{T}_2 = T_2 - E[T_2 \mid S_2]$
- Run OLS: $\tilde{Y}_2 \sim \tilde{T}_2$ to estimate δ_2

- Adjust outcome $Y_{adj} = Y - \delta_2 T_2 + \delta_2 \tau_2$

- Construct residuals:
  $\tilde{Y}_1 := Y_{adj} - E[Y_{adj}|S_1], \qquad \tilde{T}_1 := T_1 - E[T_1|S_1]$
- Run OLS: $\tilde{Y}_1 \sim \tilde{T}_1$ to estimate δ_1

- Adjust outcome $Y_{adj} \leftarrow Y_{adj} - \delta_1 T_1 + \delta_1 \tau_1$

- Return: $\theta := E[Y_{adj}]$



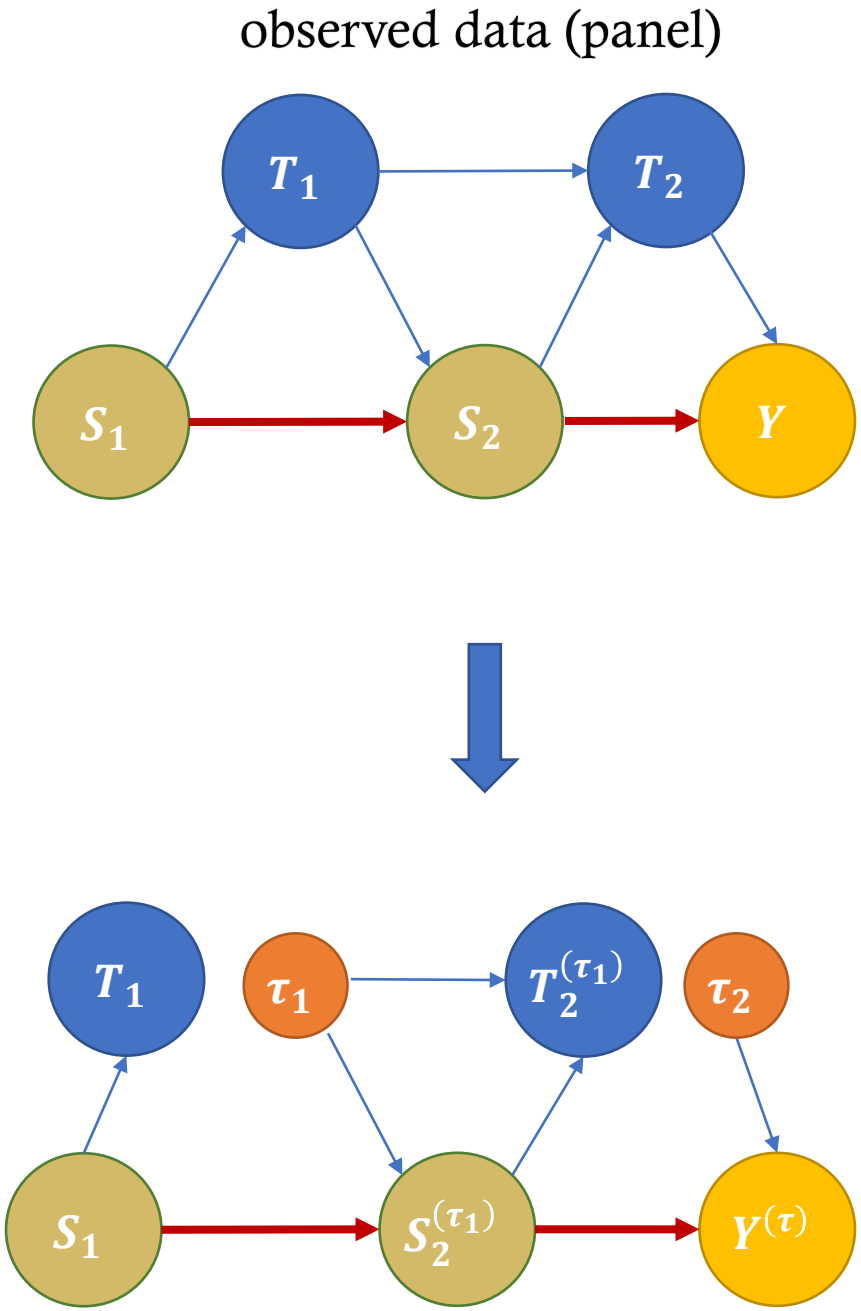
Moment Based Framework

$$\theta := E[Y - \delta_2 T_2 + \delta_2 \tau_2 - \delta_1 T_1 + \delta_1 \tau_1] = 0$$

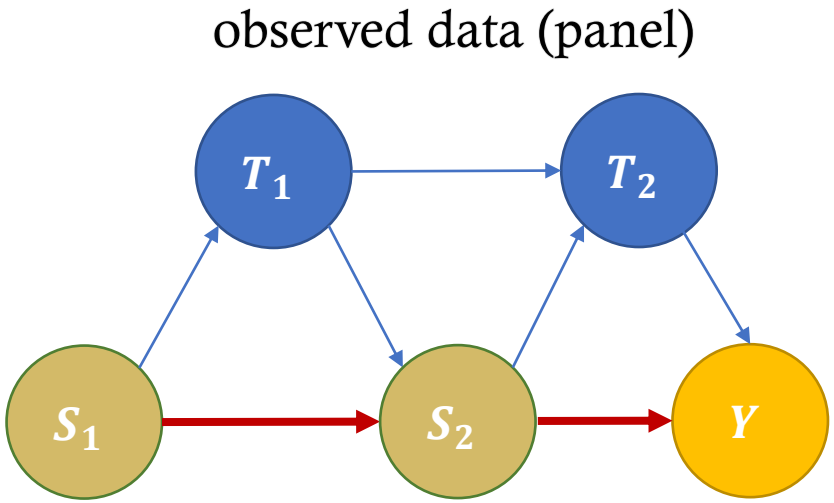
$$E\left[\left(\tilde{Y}_2 - \delta_2 \tilde{T}_2\right)\tilde{T}_2\right] = 0$$
$$E\left[\left(\tilde{Y}_1 - \delta_1 \tilde{T}_1\right)\tilde{T}_1\right] = 0$$

$E[Y|S_2]$, $\mathrm{E}[\mathrm{T}_2|\mathrm{S}_2]$
$E[Y - \delta_2 T_2 + \delta_2 \tau_2|S_1]$, $\mathrm{E}[\mathrm{T}_1|\mathrm{S}_1]$

Nuisance functions



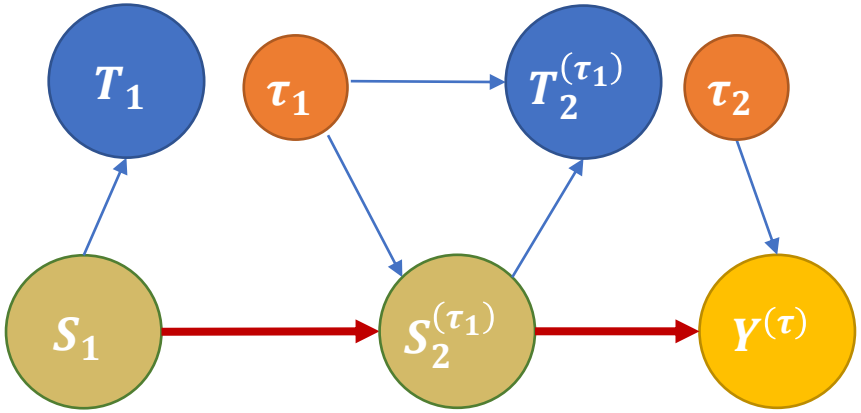
Moment Based Framework



$$\theta := E[Y - \delta_2 T_2 + \delta_2 \tau_2 - \delta_1 T_1 + \delta_1 \tau_1] = 0$$

$$E\left[\left(\tilde{Y}_2 - \delta_2 \tilde{T}_2\right)\tilde{T}_2\right] = 0$$
$$E\left[\left(\tilde{Y}_1 - \delta_1 \tilde{T}_1\right)\tilde{T}_1\right] = 0$$

$$E[Y|S_2], \quad \mathrm{E}[\mathrm{T}_2|\mathrm{S}_2]$$
$$E[Y - \delta_2 T_2 + \delta_2 \tau_2|S_1], \quad \mathrm{E}[\mathrm{T}_1|\mathrm{S}_1]$$



For the same reason why the Residual-on-Residual moment was Neyman orthogonal, this estimation process is also orthogonal and we can use ML for these problems

Long-Term Causal Inference from Short-Term Data with Surrogates

Estimating Long-Term Returns on Investment

- Companies frequently deploys new discount or customer support programs
- Which of these programs ("investments") are more successful than others?
- Success is a long-term objective: what is the effect of the program on the two-year customer journey (e.g., effect on two-year revenue)
- We cannot wait two years to evaluate a program
- Main Question. Can we construct estimates of the values of these programs with short-term data, e.g. after 6 months?

Long-Term Effects from Short-Term Surrogates

- Suppose that there are many short-term signals S that are indicative of a customer's long-term reward Y (e.g. the next 6-month purchase patterns of a customer could be indicative of their long-term spend)
- Suppose that investment program T affects long-term rewards if and only if it affects these short-term signals

- We will call these short-term signals S surrogates

Causal Inference with Surrogates 101

- Since long-term effect goes only through surrogates:

  expected effect on long-term reward = effect on projected long-term reward based on surrogates

$$E\left[Y^{(\tau)}\right] = E\left[Y^{(\tau)} \middle| T = \tau\right] = E[Y|T = \tau] = E[E[Y|T = \tau, S]|T = \tau] = E[E[Y|S]|T = \tau]$$

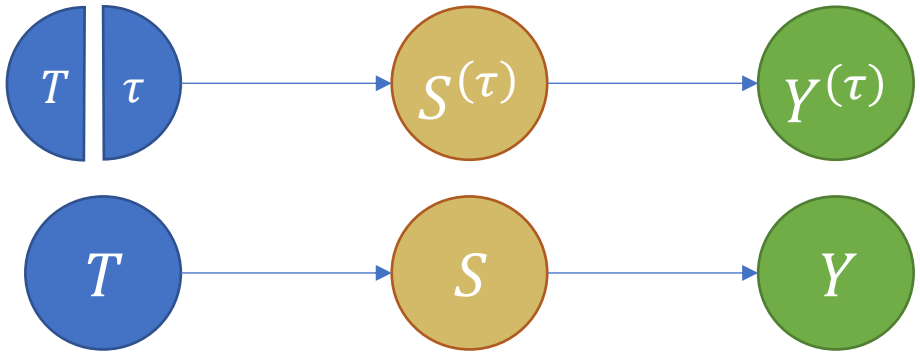
Average reward if intervene and set investment= τ

Independence in counterfactual graph

Average reward of samples that received investment=τ in data

Tower Law of Expectations

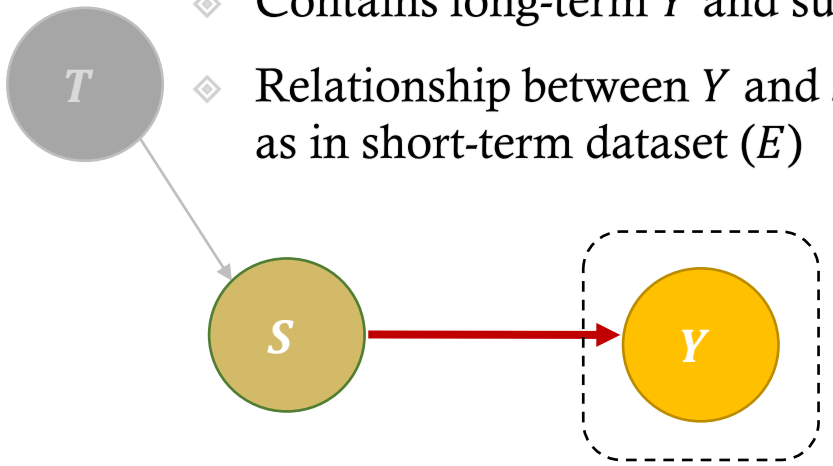
Forecasted reward from surrogates



Causal Inference with Surrogates 101

historical/long-term (O)

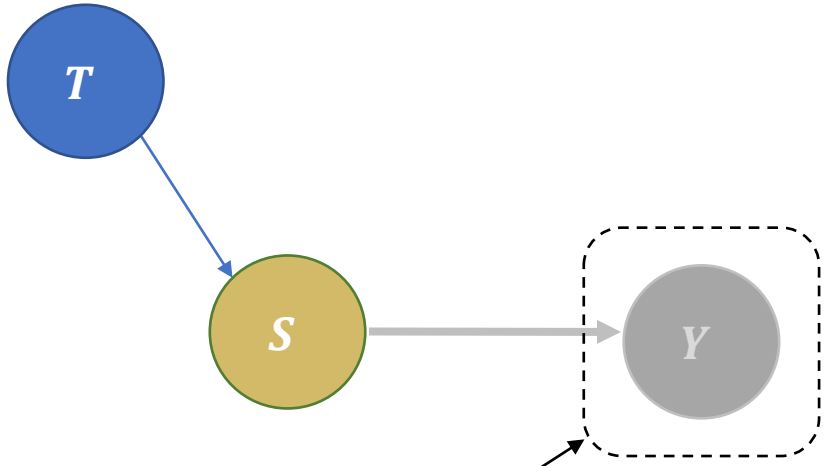
- Contains long-term Y and surrogates S
- Relationship between Y and S the same as in short-term dataset (E)



$g(S) = E_o[Y|S]$

1. Estimate $g(S) := E[Y|S]$ (surrogate index) from (O) by regressing $Y \sim S$

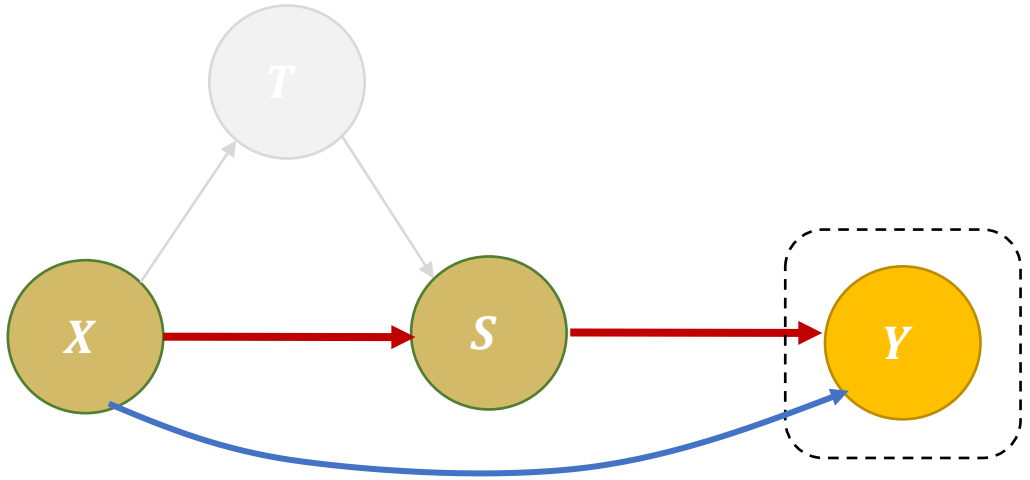
recent/short-term (E)



2. Impute expected long-term outcomes in (E)
3. Regress $g(S) \sim T$ to estimate effect of T on Y from (E)

Causal Inference with Surrogates

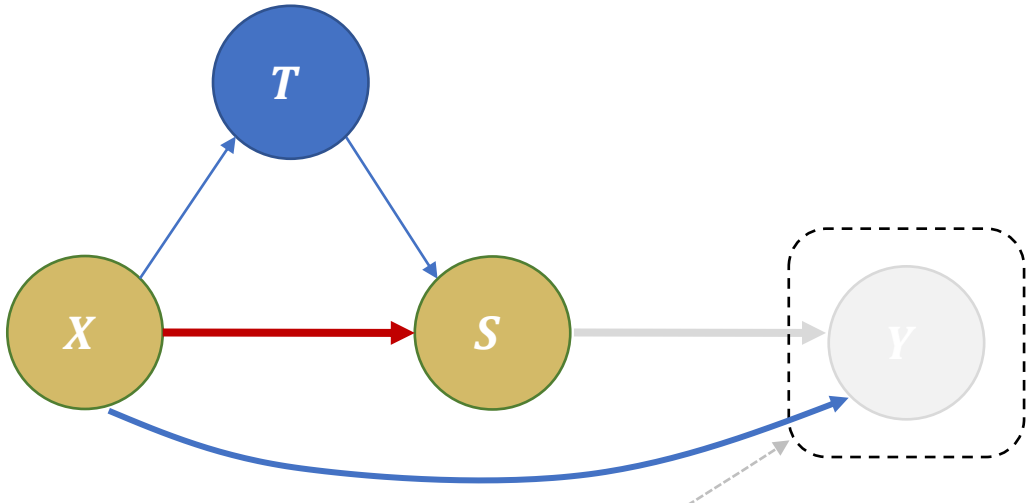
long-term data (ℓ)



$h(S, X) = E_{\ell}[Y|S, X]$

1. Estimate $h(S, X) \coloneqq E[Y|S, X]$ (surrogate index) by regressing $Y \sim S, X$

short-term data (s)



2. Impute expected long-term outcomes
3. Estimate effect of T on imputed outcomes

(Athey et al., 2020)

Causal Inference with Surrogates 102

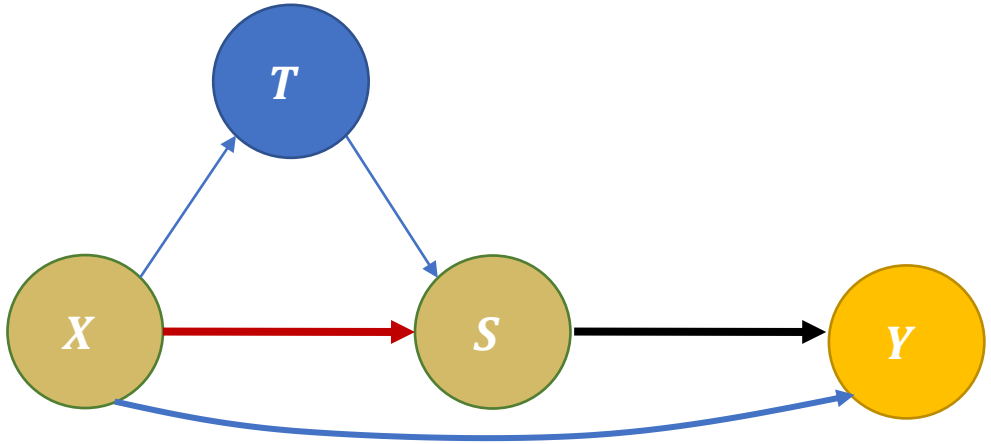
$$\textit{Reminder: } \theta_0 := E\left[Y^{(\tau)}\right] = E\left[E[Y|T=\tau, X]\right], \text{ (g–formula)}$$

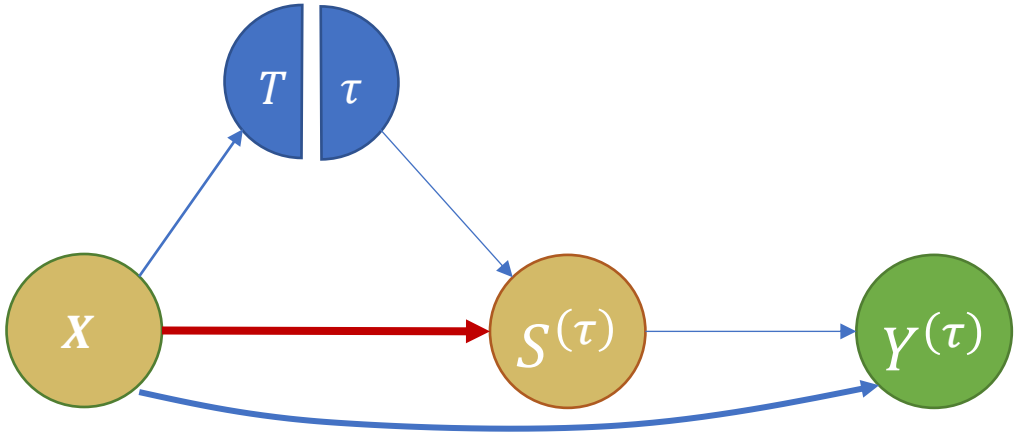
- Surrogacy assumption implies:

$$E[Y|T=\tau, X] = E[\, E[Y|S, X] \mid T=\tau, X\,]$$

- Surrogacy based g-formula

$$\theta_0 = E\left[E[h(S,X)|T=\tau, X]\right]$$





Nested Regression Estimand

- Surrogacy based g-formula

$$\theta_0 = E_s[g(\tau, X)]$$
$$g(T, X) = E_s[h(S, X)|T, X]$$
$$h(S, X) = E_\ell[Y|S, X]$$

- g regression function is trained on the outputs of the h regression function
- Surrogate Index Model h is trained on long-term data distribution
- Target parameter is averaged over short-term data distribution

Automatic Debiased Machine Learning

- Surrogacy based g-formula

$$\theta_0 = E_s[g(\tau, X)]$$
$$g(T, X) = E_s[h(S, X)|T, X]$$
$$h(S, X) = E_\ell[Y|S, X]$$

- Automatic debiased formula

$$\theta_0 = E_s\left[g(\tau, X) + a_1(T, X)\left(h(S, X) - g(T, X)\right)\right]$$

Recursive Automatic Debiasing

- Surrogacy based g-formula

$$\theta_0 = E_s[g(\tau, X)]$$
$$g(T,X) = E_s[h(S,X)|T,X]$$
$$h(S,X) = E_\ell[Y|S,X]$$

- Automatic debiased formula

$$\theta_0 = E_s\left[g(\tau,X) + a_1(T,X)\left(h(S,X) - g(D,X)\right)\right] + E_\ell\left[a_2(S,X)\left(Y - h(S,X)\right)\right]$$

Inverse Propensity ratio

$$\frac{1\{T=\tau\}}{\Pr(T=\tau \mid X)}$$

Surrogate Score(S,X) * $E[\,a_1(T,X) \mid S,X\,]$

Surrogate score: (density ratio of S,X between ℓ and s "environment")

$$\frac{p(S,X \mid E=s)}{p(S,X \mid E=\ell)} = \frac{\Pr(E=s \mid S,X)\Pr(E=\ell)}{\Pr(E=\ell \mid S,X)\Pr(E=s)}$$

Multiplied by

$$E[\,a_1(T,X) \mid S,X\,] = \frac{\Pr(T=t \mid S,X)}{\Pr(T=t \mid X)}$$

An Application from Operations Management

Return on Investment at Microsoft using Surrogates and Dynamic Treatment Effects

Estimating Long-Term Returns on Investment

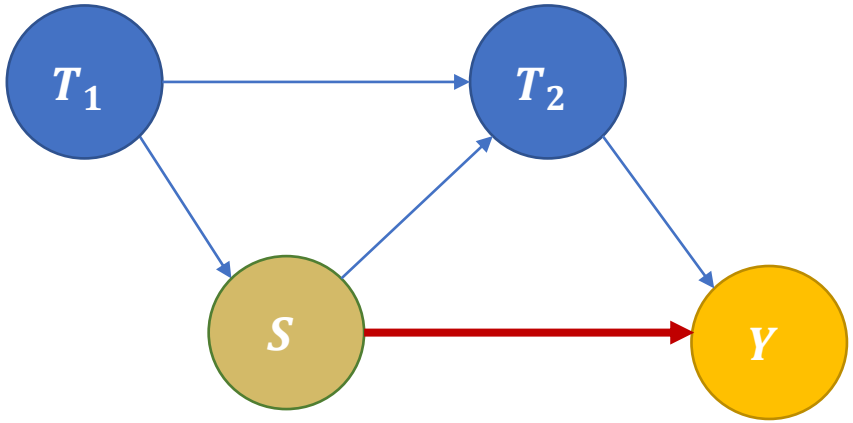
- Companies frequently deploys new discount or customer support programs
- Which of these programs ("investments") are more successful than others?
- Success is a long-term objective: what is the effect of the program on the two-year customer journey (e.g., effect on two-year revenue)
- We cannot wait two years to evaluate a program
- Main Question. Can we construct estimates of the values of these programs with short-term data, e.g. after 6 months?

Key Assumptions

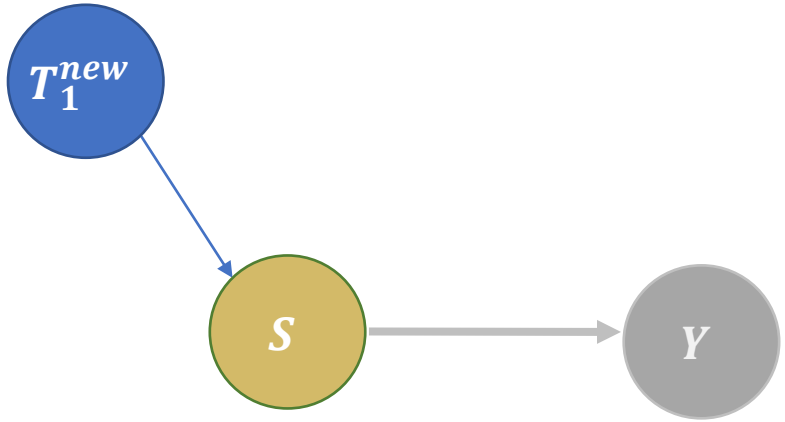
- Long-term effect only goes through surrogates
- Expected relationship between surrogates and long-term reward is the same long-term setting (O) and in short-term setting (E)

Key Assumptions can be Easily Violated
Investment policies are dynamic and change

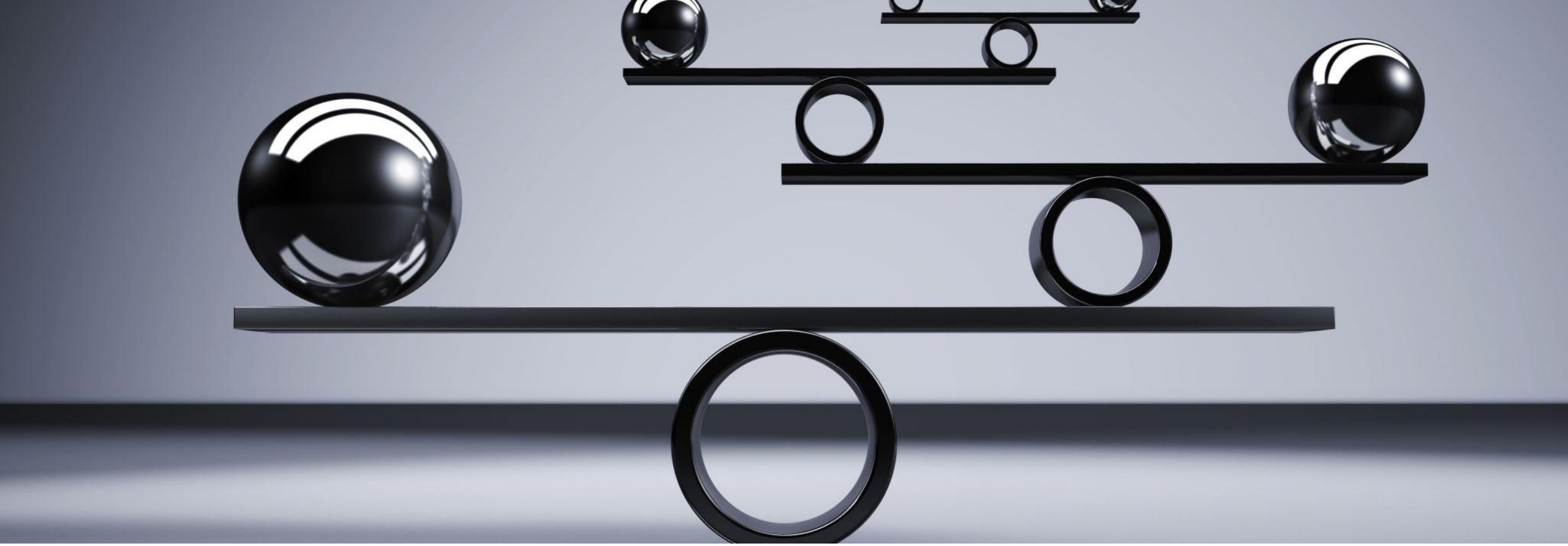
historical/long-term (O)



recent/short-term (E)



- We deployed older/deprecated investments
- In a potentially long-term highly auto-correlated manner
- Investments are potentially adaptive
- Investment policies change

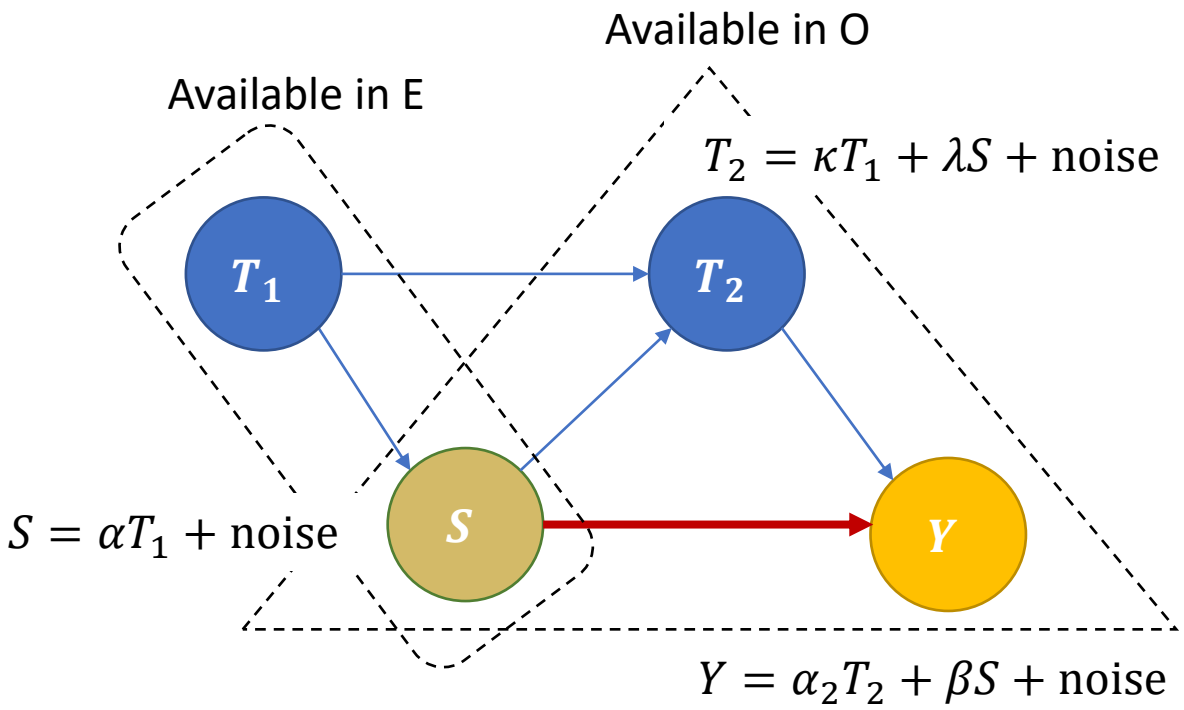


Bias of Vanilla Surrogate

Illustrative Example

Illustrative Example

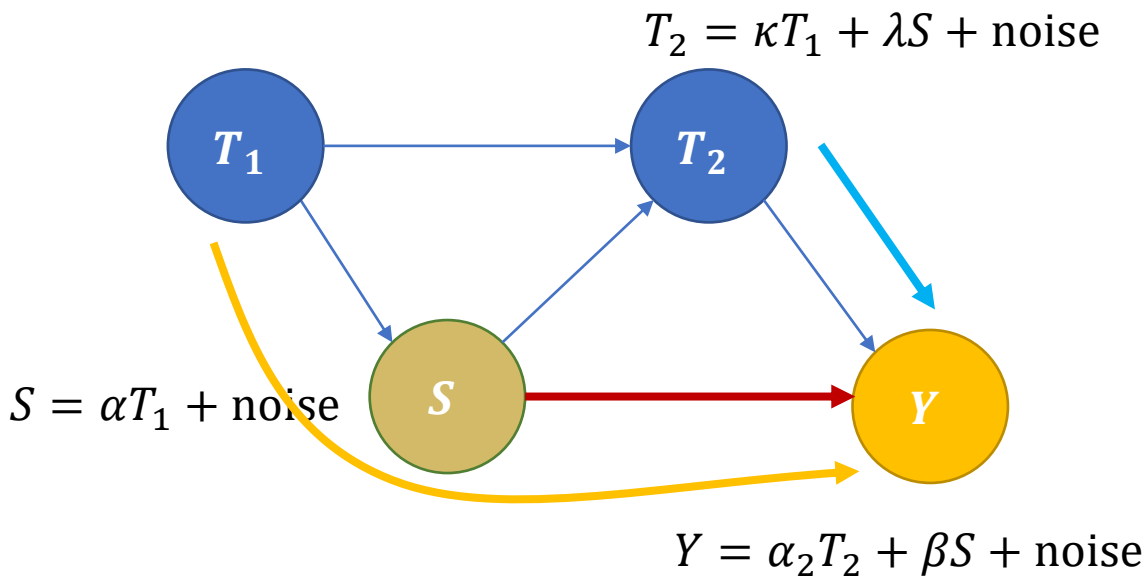
- Want the effect of T_1 on Y, with no future treatments



Bias of Vanilla Surrogate

- What do we want to estimate?

$$Y = \boldsymbol{\beta}\ \boldsymbol{\alpha}\ T_1 + \alpha_2\ T_2 + noise$$
$$\boldsymbol{\theta_0}$$



Bias of Vanilla Surrogate

- What do we want to estimate?

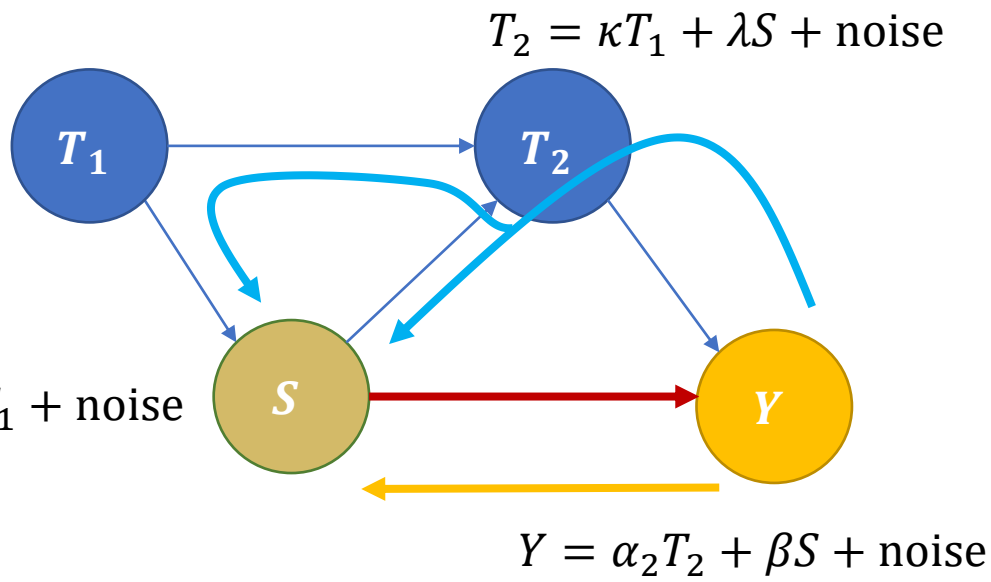
$$Y = \boldsymbol{\beta}\,\boldsymbol{\alpha}\,T_1 + \alpha_2\,T_2 + noise$$

$$\boldsymbol{\beta}\,\boldsymbol{\alpha} \;\;!!\;\; \boldsymbol{\theta_0}$$

- What does the surrogate approach estimate?

$$g_0(S) \coloneqq E[Y|S] = \boldsymbol{\beta}\,S + \boldsymbol{\alpha_2}\,E[T_2|S]$$

$$\boldsymbol{\kappa}E[T_1|S] + \boldsymbol{\lambda}\boldsymbol{S}$$



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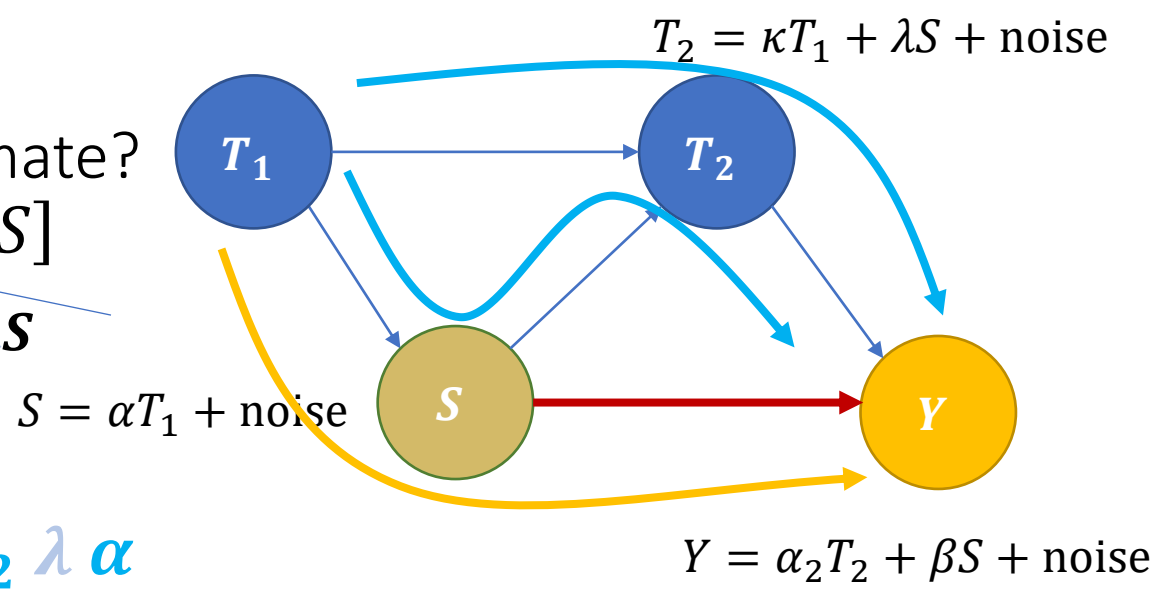
- The effect of T_1 on $g_0(S)$ is:

$$\theta_* \coloneqq \boldsymbol{\beta}\,\boldsymbol{\alpha} + \boldsymbol{\alpha_2}\,\boldsymbol{\kappa}\,\frac{E[E[T_1|S]\cdot T_1]}{E[T_1^2]} + \boldsymbol{\alpha_2}\,\boldsymbol{\lambda}\,\boldsymbol{\alpha}$$



Bias of Vanilla Surrogate

- What do we want to estimate?

$$Y = \boldsymbol{\beta}\,\boldsymbol{\alpha}\,T_1 + \alpha_2\,T_2 + noise$$

$$\boldsymbol{\beta}\,\boldsymbol{\alpha} \;\;\text{!!}\;\; \boldsymbol{\theta_0}$$

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$$g_0(S) := E[Y|S] = \boldsymbol{\beta}\,S + \boldsymbol{\alpha_2}\,E[T_2|S]$$

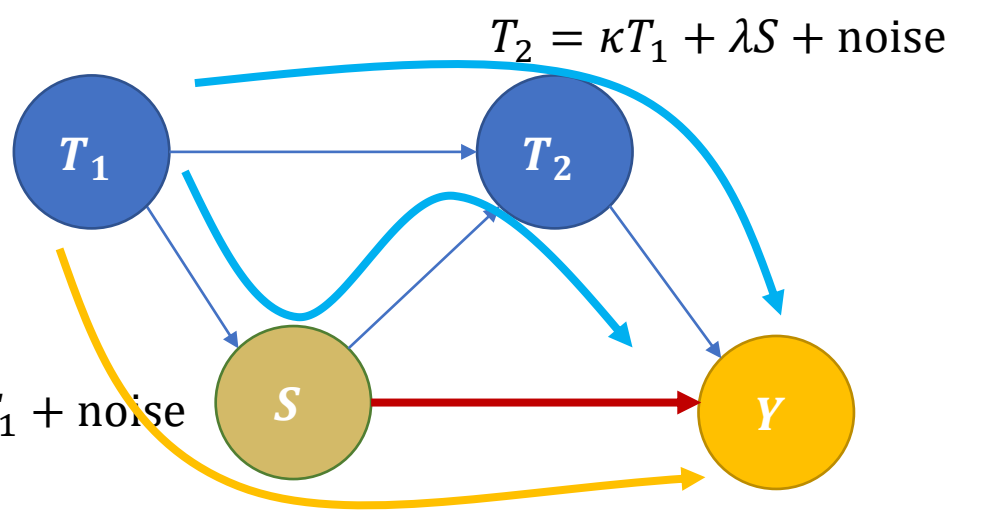
$$\boldsymbol{\alpha_2}\,E[T_2|S] = \boldsymbol{\alpha_2}\,\boldsymbol{\kappa}E[T_1|S] + \boldsymbol{\alpha_2}\,\boldsymbol{\lambda}\boldsymbol{S}$$

- The effect of T_1 on $g_0(S)$ is:

$$\theta_* := \boldsymbol{\beta}\,\boldsymbol{\alpha} + \boldsymbol{\alpha_2}\,\boldsymbol{\kappa}\frac{E[E[T_1|S]\cdot T_1]}{E[T_1^2]} + \boldsymbol{\alpha_2}\,\boldsymbol{\lambda}\,\boldsymbol{\alpha}$$

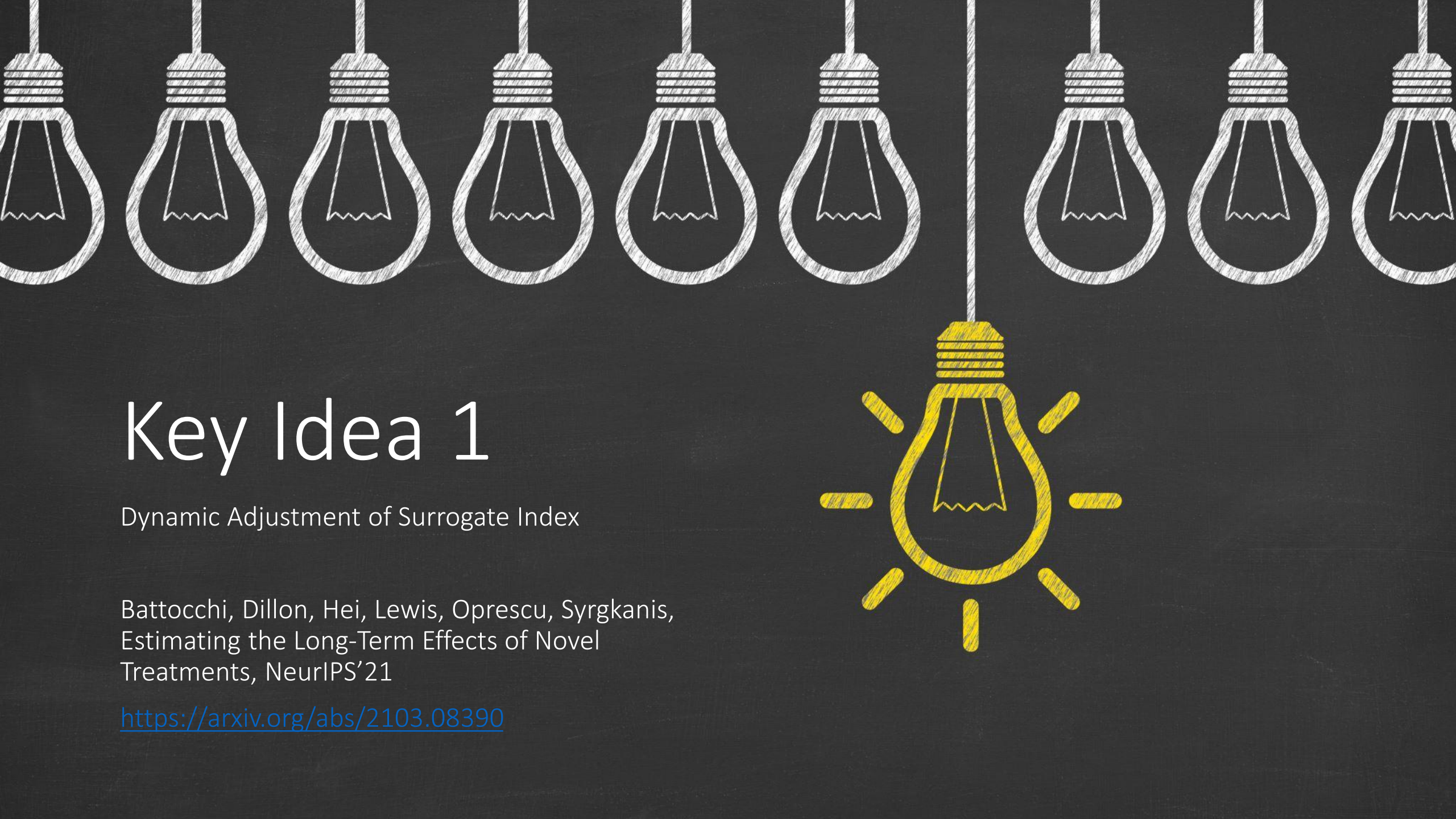
Bias from the fact that policy is auto-correlated violating the "effect only through surrogates" assumption

Bias from the fact that policy is adaptive on past surrogates violating "relationship of S and Y unchanged" assumption



Prevalence of Such Violation

- Marketing departments frequently deploy new marketing campaigns
  - In historical data, older campaigns were deployed
  - Due to targeting, if a customer received an ad, most probably will receive one in the future
  - Ad display is adaptive to signals of customer behavior
- Pharmaceutical companies frequently deploy new drugs
  - Can we get estimates of long-term effect from short-term trials
  - Use short-term signals of patient response that correlated with long-term survival
  - In historical long-term clinical data, patients are treated with multiple treatments over time
  - Treatments are typically adaptive to signals of patient trajectory



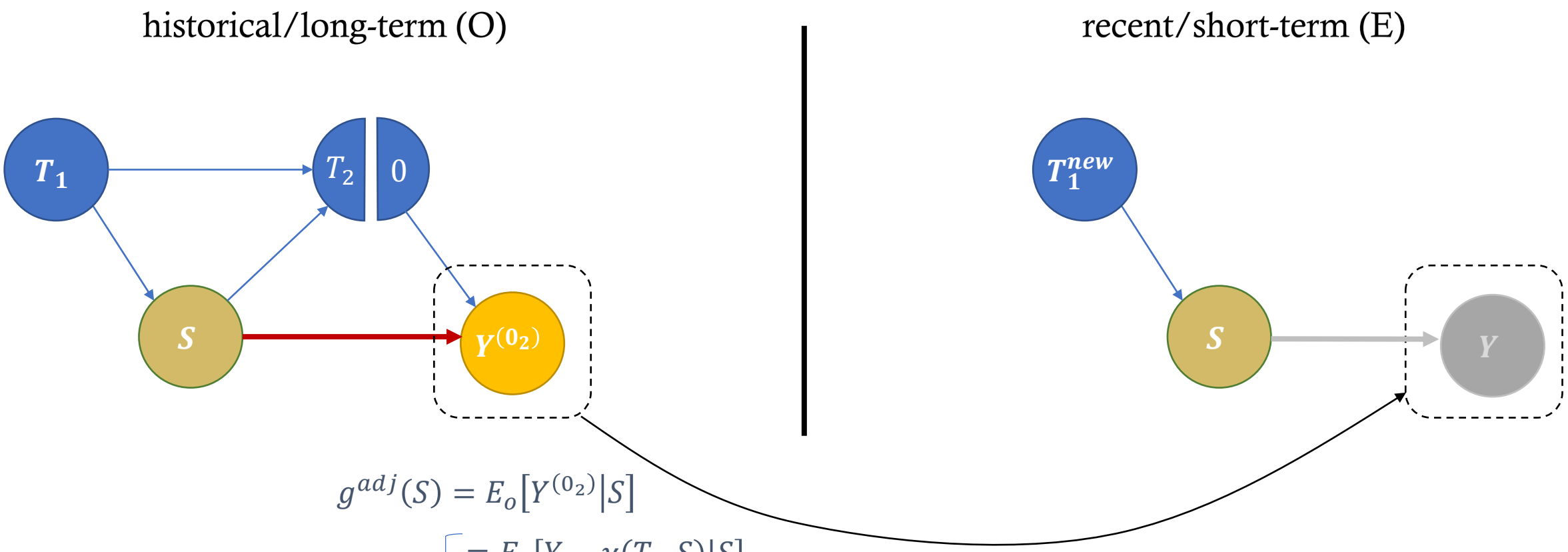
Key Idea 1

Dynamic Adjustment of Surrogate Index

Battocchi, Dillon, Hei, Lewis, Oprescu, Syrgkanis, Estimating the Long-Term Effects of Novel Treatments, NeurIPS'21

https://arxiv.org/abs/2103.08390

Dynamically Adjusted Surrogate Index



$$g^{adj}(S) = E_o\left[Y^{(0_2)}|S\right]$$

$$= E_o[Y - \gamma(T_2, S)|S]$$

Identification argument

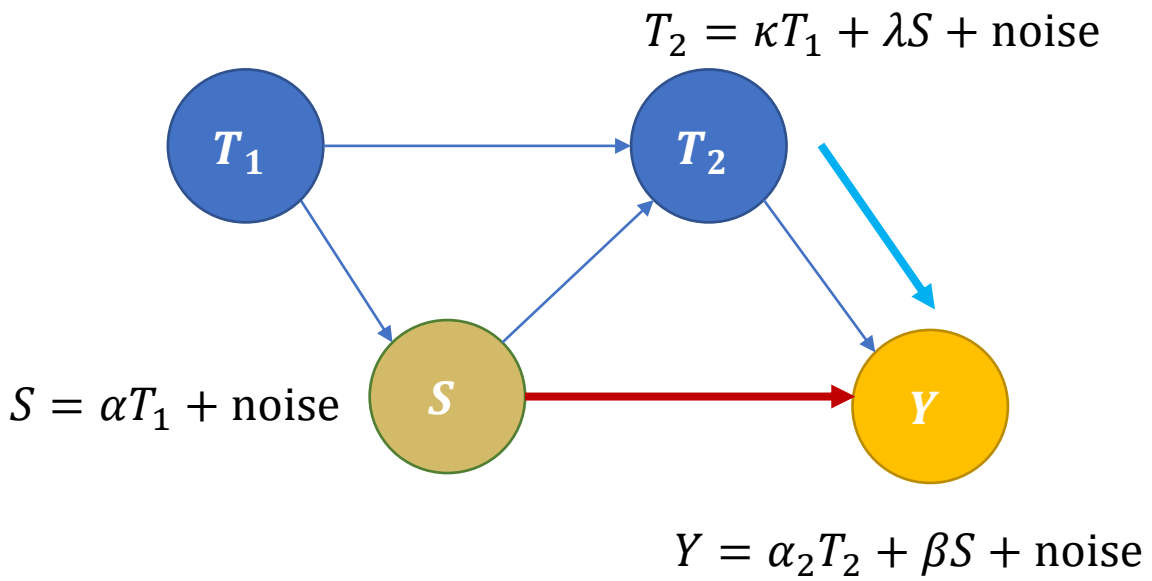
"blip" effect of second treatment conditional on surrogate

⟺ effect of T_2 controlling for S

Illustrative Example: Dynamically Adjusted Index

- Estimate effect α_2 of T_2 on Y, controlling for S

$$E[Y|T_2, S] = \alpha_2 T_2 + \beta S$$



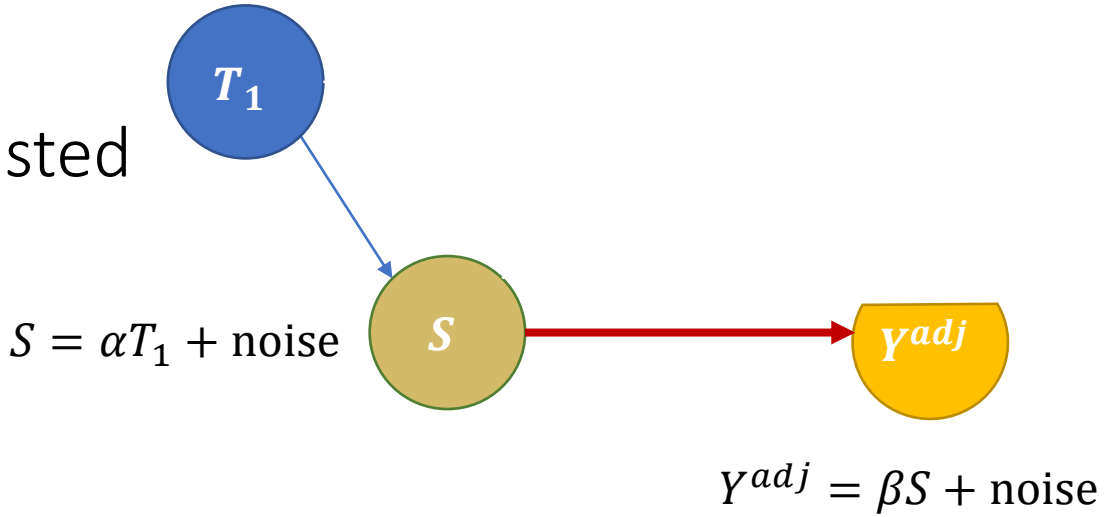
Illustrative Example: Dynamically Adjusted Index

- Estimate effect α_2 of T_2 on Y, controlling for S

$$E[Y|T_2, S] = \alpha_2 T_2 + \beta S$$

- Subtract that effect to create the adjusted outcome

$$Y^{adj} := Y - \alpha_2 T_2$$



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- Estimate effect α_2 of T_2 on Y, controlling for S

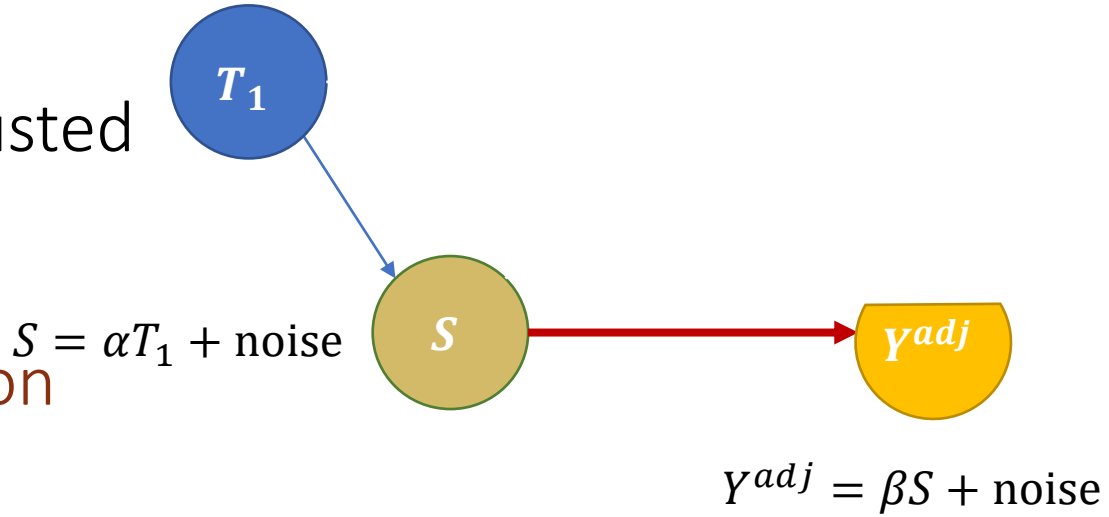
$$E[Y|T_2, S] = \alpha_2 T_2 + \beta S$$

- Subtract that effect to create the adjusted outcome

$$Y^{adj} := Y - \alpha_2 T_2$$

- Estimate dynamically adjusted index on long-term data

$$g_0^{adj}(S) := E\left[Y^{adj}\middle|S\right] = \beta\, S$$



Illustrative Example: Dynamically Adjusted Index

- Estimate effect α_2 of T_2 on Y, controlling for S

$$E[Y|T_2, S] = \alpha_2 T_2 + \beta S$$

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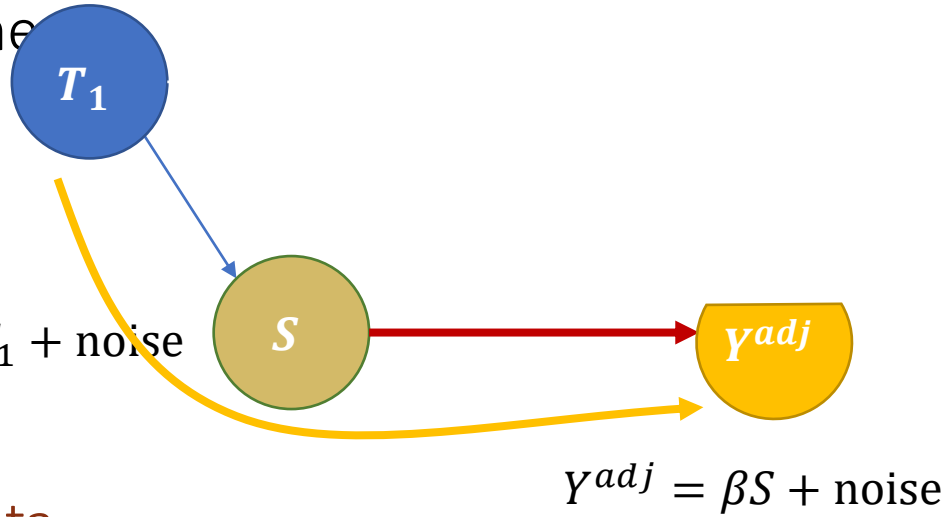
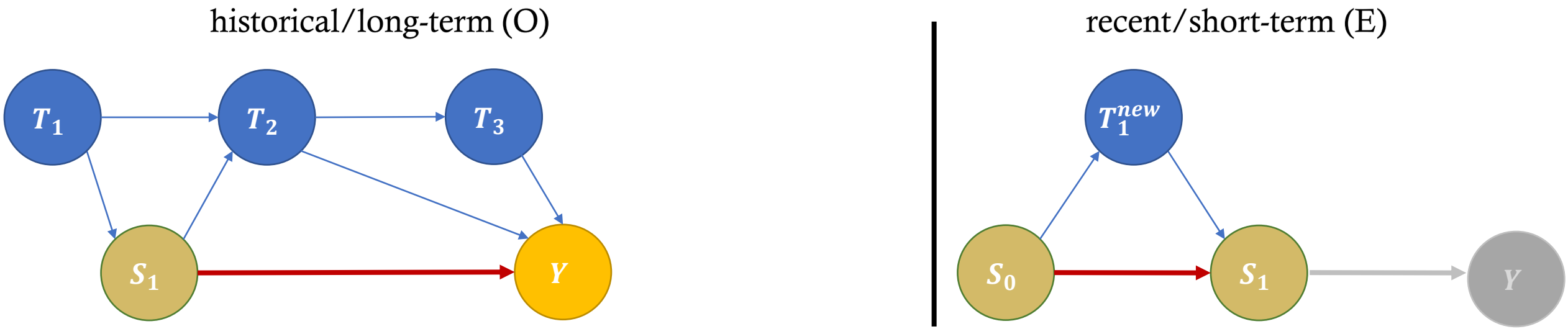


- Estimate effect of T_1 on $g_0^{adj}(S)$ on short-term data

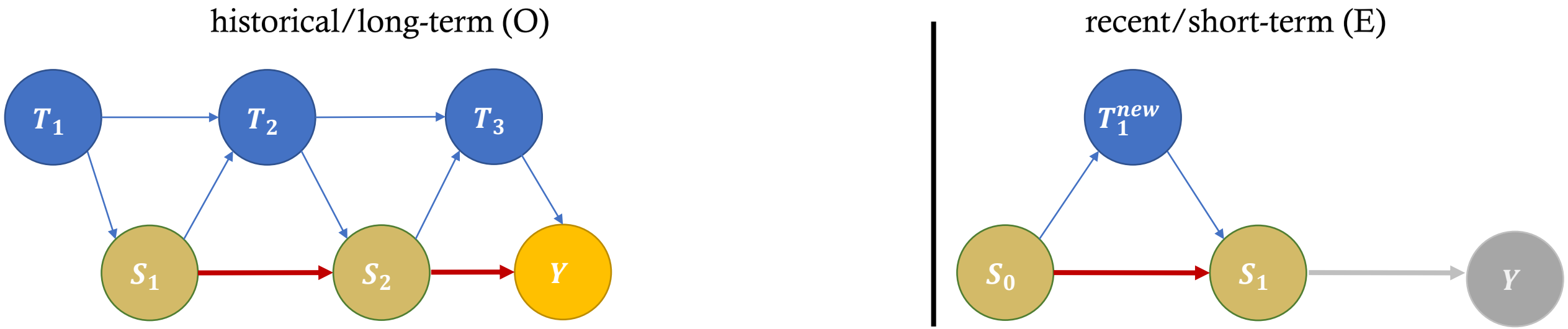
$$\theta_0 := \frac{E\left[g_0^{adj}(S) \cdot T_1\right]}{E\left[T_1^2\right]} = \beta \cdot \alpha$$

Beyond Two Periods



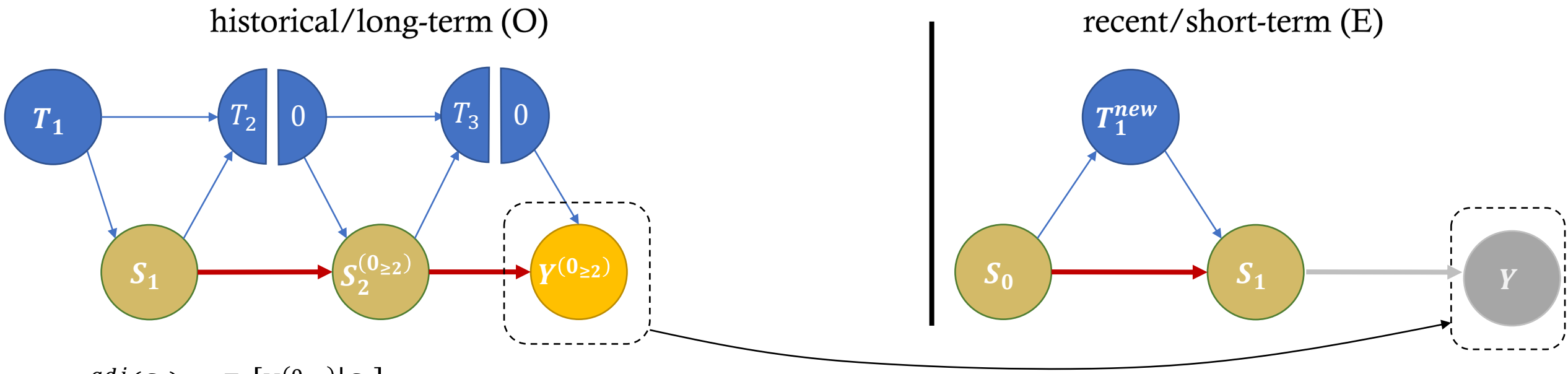
- Multiple treatments are offered after the surrogate variable
- Easy: estimate their effect controlling for S_1
- Wrong!

Beyond Two Periods



- Treatments are offered in an adaptive manner, in response to previous period surrogate/state
- The surrogate – treatment feedback precludes viewing this as a one-shot treatment problem
- Setting is known as the dynamic treatment regime [Robins'94,'04, Chakraborty-Murphy'14]

Beyond Two Periods: Target Quantity



$$g^{adj}(S_1) = E_o\left[Y^{(0_{\geq 2})} \middle| S_1\right]$$

$$= E_o\left[Y - \sum_{t \geq 2} \gamma_t(T_t, S_{t-1}) \middle| S_1\right]$$

"blip" effect of treatment at period t

- What effect do we subtract from Y?
- Do we estimate effect of T_2, T_3 controlling for S_1, S_2? Wrong
- Do we estimate effect of T_2, T_3 controlling for S_1? Wrong