MS&E 233
Game Theory, Data Science and AI
Lecture 8

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(by courtesy) Computer Science and Electrical Engineering

Institute for Computational and Mathematical Engineering

Computational Game Theory for Complex Games

1
- Basics of game theory and zero-sum games (T)
- Basics of online learning theory (T)
- Solving zero-sum games via online learning (T)
- *HW1: implement simple algorithms to solve zero-sum games*
- Applications to ML and AI (T+A)
- *HW2: implement boosting as solving a zero-sum game*

2
- Basics of extensive-form games
- Solving extensive-form games via online learning (T)
- *HW3: implement agents to solve very simple variants of poker*

3
- General games, equilibria and online learning (T)
- **Online learning in general games**
- *HW4: implement no-regret algorithms that converge to correlated equilibria in general games*

Data Science for Auctions and Mechanisms

4
- Basics and applications of auction theory (T+A)
- Learning to bid in auctions via online learning (T)
- *HW5: implement bandit algorithms to bid in ad auctions*

5
- Optimal auctions and mechanisms (T)
- Simple vs optimal mechanisms (T)
- *HW6: calculate equilibria in simple auctions, implement simple and optimal auctions, analyze revenue empirically*

6
- Optimizing mechanisms from samples (T)
- Online optimization of auctions and mechanisms (T)
- *HW7: implement procedures to learn approximately optimal auctions from historical samples and in an online manner*

Further Topics

7
- Econometrics in games and auctions (T+A)
- A/B testing in markets (T+A)
- *HW8: implement procedure to estimate values from bids in an auction, empirically analyze inaccuracy of A/B tests in markets*

Guest Lectures

- Mechanism Design for LLMs, Renato Paes Leme, Google Research
- Auto-bidding in Sponsored Search Auctions, Kshipra Bhawalkar, Google Research

Recap: Regret vs Correlated Equilibrium

Distributions that satisfy this are called **Coarse Correlated Equilibria**

- No-regret property, implies

$$\forall s_i': \sum_{s} \pi^T(s)\left(u_i(s) - u_i(s_i', s_{-i})\right) \geq -\tilde{\epsilon}(T,\delta) \to 0$$

- Correlated equilibrium requires conditioning on recommendation

$$\forall s_i^*, s_i': \sum_{s:s_i=s_i^*} \pi^T(s)\left(u_i(s) - u_i(s_i', s_{-i})\right) \geq 0$$

At subset of periods when **played s_i^***

You don’t regret **switching to s_i'**

Recap: Swaps and Correlated Equilibrium

- Correlated equilibrium requires conditioning on recommendation

$$\forall s_i^*, s_i' : \sum_{s: s_i = s_i^*} \pi^T(s) \left(u_i(s) - u_i\left(s_i', s_{-i}\right) \right) \geq 0$$

- Equivalently: for any **swap** function ϕ that maps original actions s_i to deviating actions s_i' (potentially different for each original s_i)

$$\sum_s \pi^T(s) \left(u_i(s) - u_i(\phi(s_i), s_{-i}) \right) \geq 0$$

At all periods

You don't regret **swapping your original action based on the mapping ϕ**

Recap: No-Swap Regret!

- No-regret property requires

$$\frac{1}{T}\sum_{t=1}^{T} u_i\left(s^t\right) \geq \max_{s_i' \in S_i} \frac{1}{T}\sum_{t=1}^{T} u_i\left(s_i', s_{-i}^t\right) - \tilde{\epsilon}(T, \delta)$$

- No-swap regret property requires

$$\forall \phi\colon \frac{1}{T}\sum_{t=1}^{T} u_i\left(s^t\right) \geq \frac{1}{T}\sum_{t=1}^{T} u_i\left(\phi\left(s_i^t\right), s_{-i}^t\right) - \tilde{\epsilon}(T, \delta)$$

Theorem. If all players use no-swap regret algorithms, then the empirical joint distribution converges to a Correlated Equilibrium

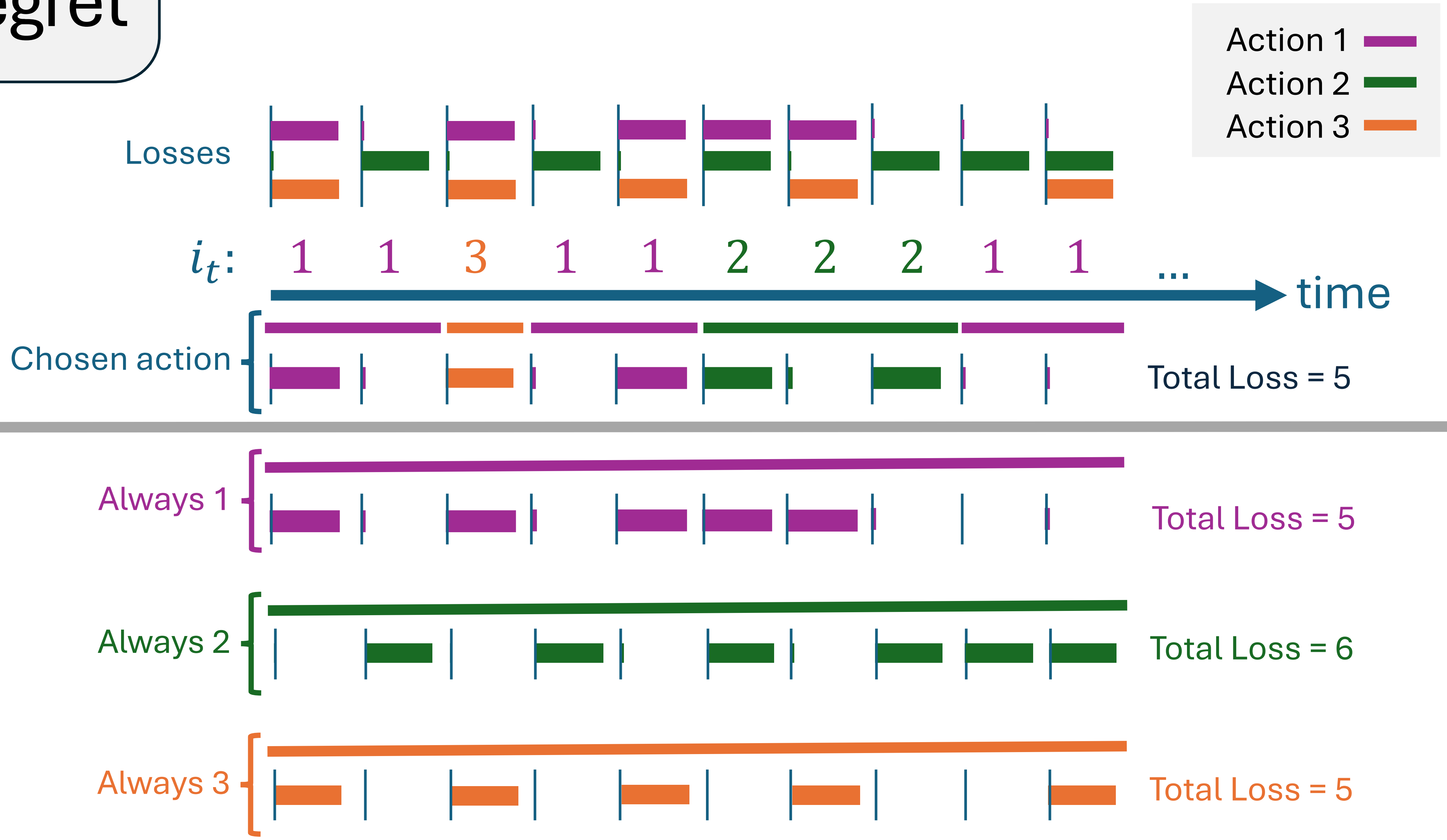
Can we construct algorithms with vanishing no-swap regret?

No Swap Regret vs No Regret

- At period t you choose action i_t from distribution x_t over n actions
- Observe vector $\ell_t = \left(\ell_t^1, \ldots, \ell_t^n\right)$ containing loss of each action
- You incur the loss of the action you chose $\ell_t^{i_t}$
- No-regret: for any action i, you do not regret always taking action i

$$\frac{1}{T}\sum_t \ell_t^{i_t} \le \frac{1}{T}\sum_t \ell_t^i + \tilde{\epsilon}(T,\delta), \qquad \text{w.p. } 1-\delta$$

No-Regret


Action 1
Action 2
Action 3
Losses
i_t: 1 1 3 1 1 2 2 2 1 1 ...
time
Chosen action
Total Loss = 5
Alternatives
Always 1
Total Loss = 5
Always 2
Total Loss = 6
Always 3
Total Loss = 5

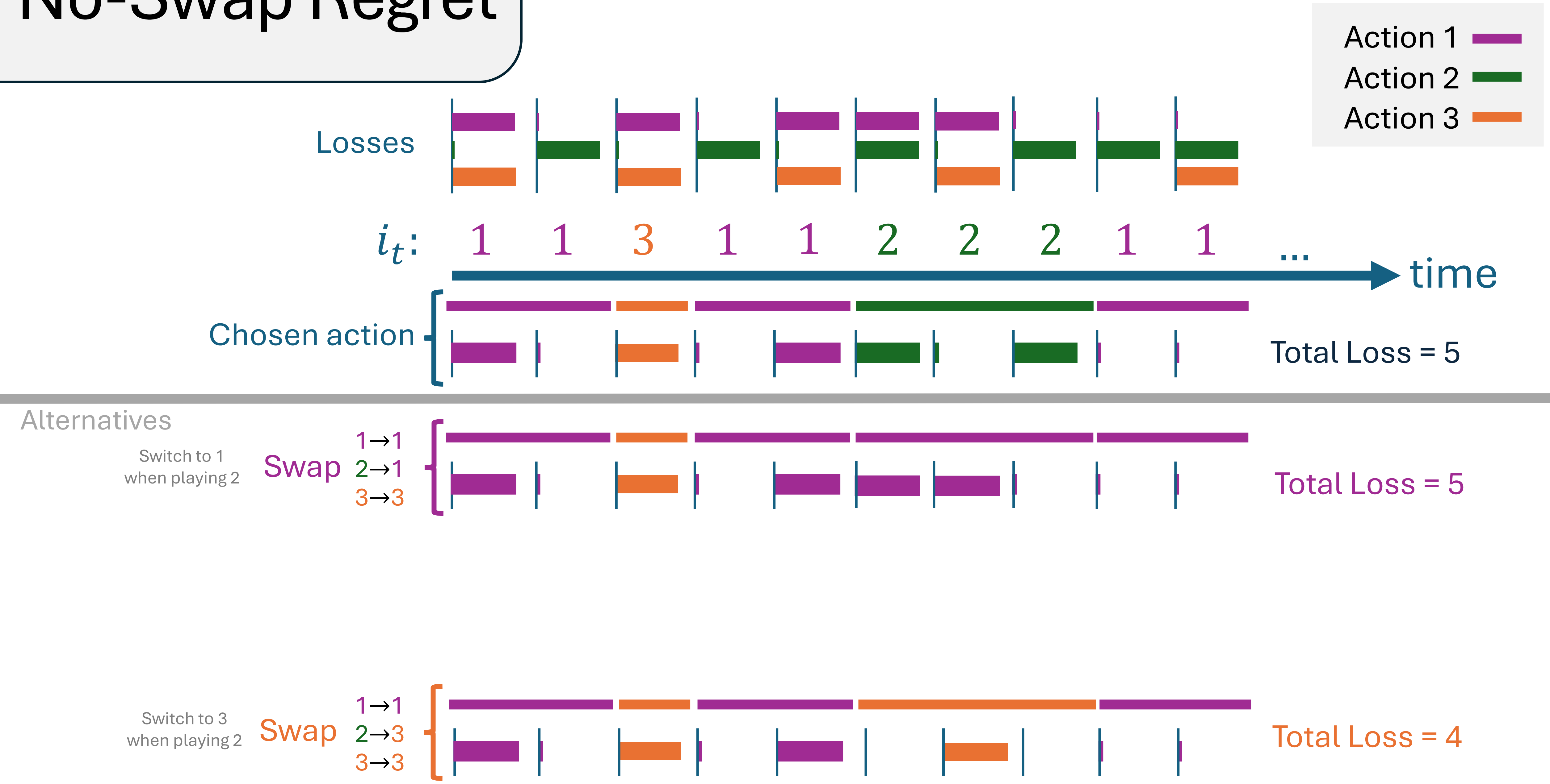
No Swap Regret vs No Regret

- At period t you choose action i_t from distribution x_t over n actions
- Observe vector $\ell_t = \left(\ell_t^1, \ldots, \ell_t^n\right)$ containing loss of each action
- You incur the loss of the action you chose $\ell_t^{i_t}$
- No-swap regret: for any swap function ϕ mapping original actions i to alternatives $i' = \phi(i)$, you do not regret making that swap

$$\frac{1}{T}\sum_t \ell_t^{i_t} \leq \frac{1}{T}\sum_t \ell_t^{\phi(i_t)} + \tilde{\epsilon}(T, \delta), \qquad \text{w.p. } 1 - \delta$$

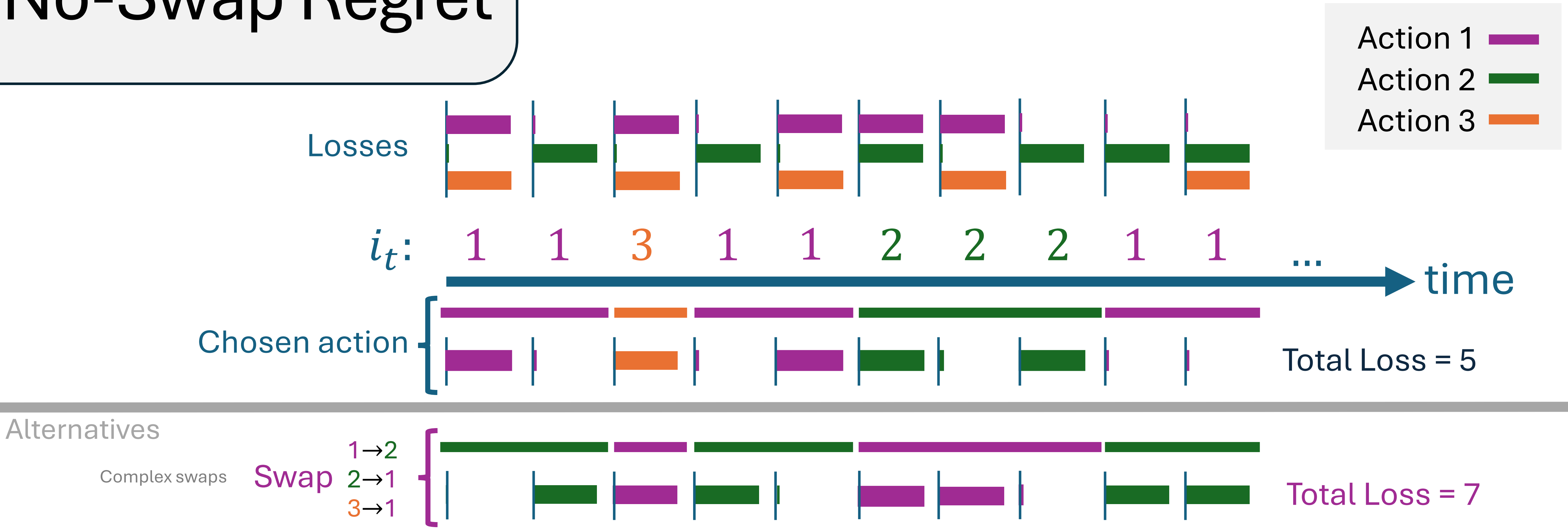
No-Swap Regret

No-Swap Regret



No-Swap Regret

No-Swap Regret



Vanishing regret for complex swaps is implied by vanishing regret of simple swaps:
switch to j' whenever you had played j and leave everything else as is

No Swap Regret vs No Regret

- **No-swap regret:** for any swap function ϕ mapping original actions i to alternatives $i' = \phi(i)$, you do not regret making that swap

$$\frac{1}{T}\sum_{t} \ell_t^{i_t} \le \frac{1}{T}\sum_{t} \ell_t^{\phi(i_t)} + \tilde{\epsilon}(T,\delta), \qquad \text{w.p. } 1-\delta$$

- **Equivalently:** for **subset of periods** when you played i you don't regret any other action i'

$$\frac{1}{T}\sum_{t:i_t=i} \ell_t^{i_t} \le \max_{i'} \frac{1}{T}\sum_{t:i_t=i} \ell_t^{i'} + \tilde{\epsilon}(T,\delta), \qquad \text{w.p. } 1-\delta$$

You have an online learning problem, for simplicity, with 2 actions. Is any no-swap regret sequence a no-regret sequence?

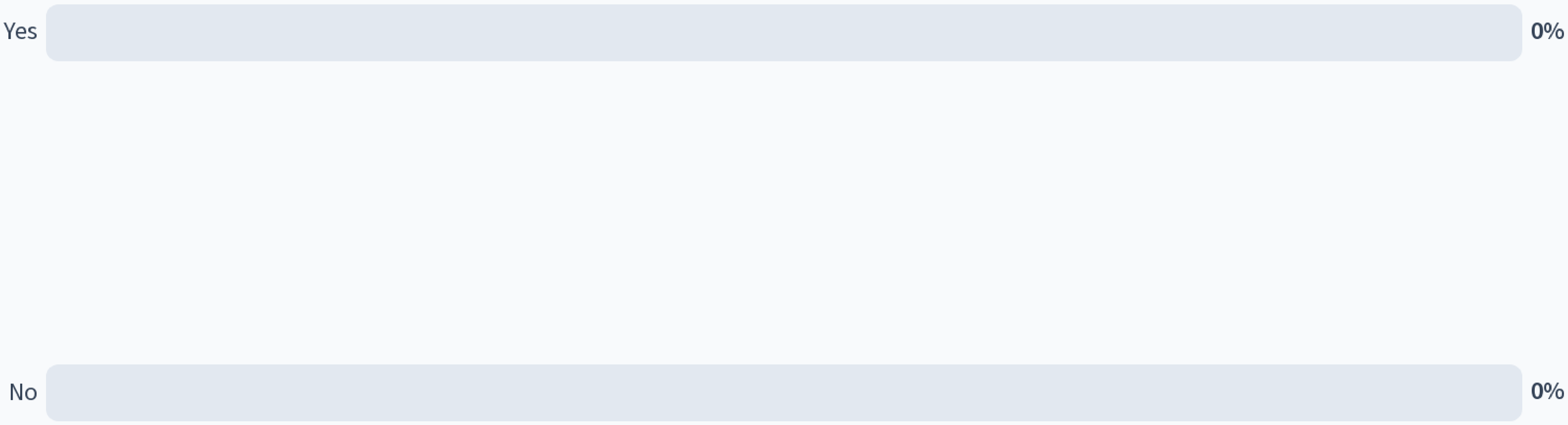
Yes	0%
No	0%

You have an online learning problem, for simplicity, with 2 actions. Is any no-regret sequence a no-swap regret sequence?

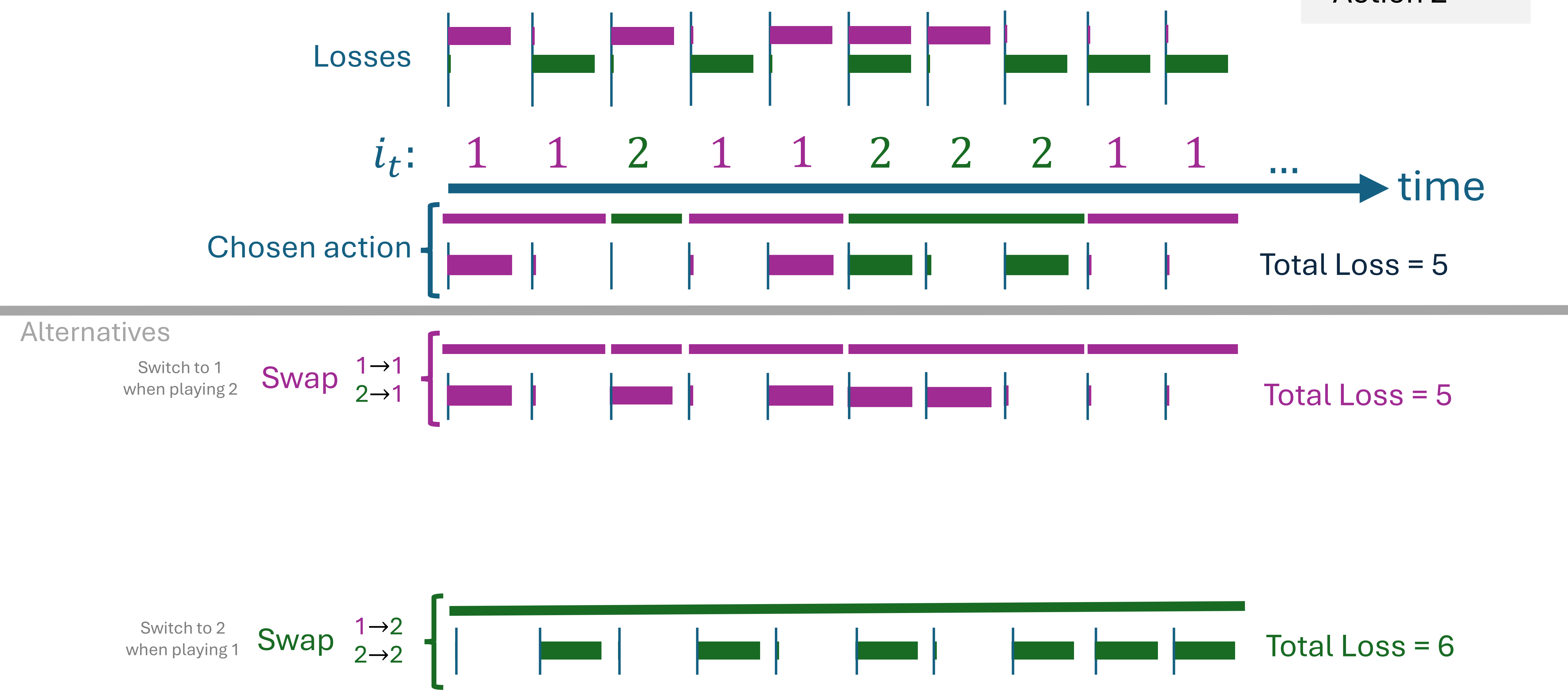
Yes 0%

No 0%

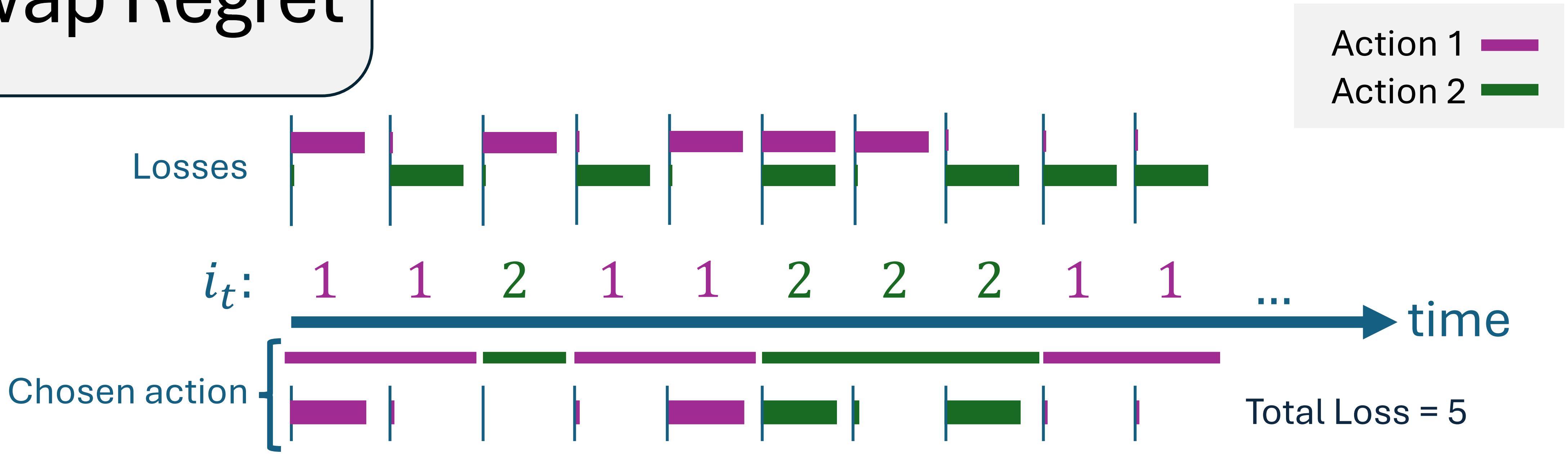
You have an online learning problem, for simplicity, with 3 actions. Is any no-regret sequence a no-swap regret sequence?

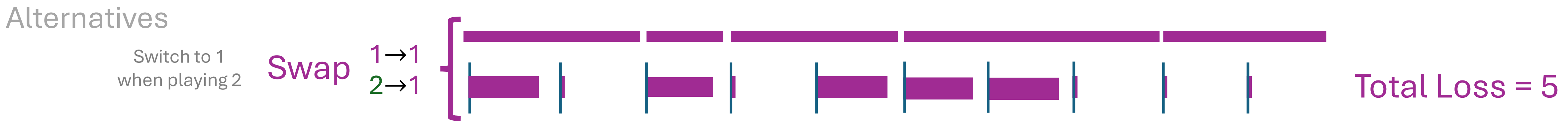


No-Swap Regret

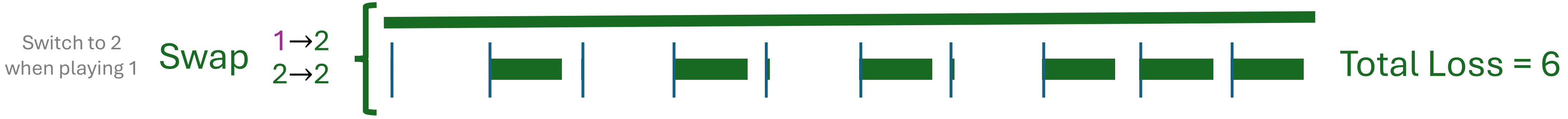


No-Swap Regret





No-swap regret is weirdly implied by no-regret when you only have two actions.
Intuition: no-regret towards action j is the same as no-regret on the subset of periods when you did not play j. With two actions, these are exactly the periods when you played j'



Can we reduce no-swap regret to no-regret?

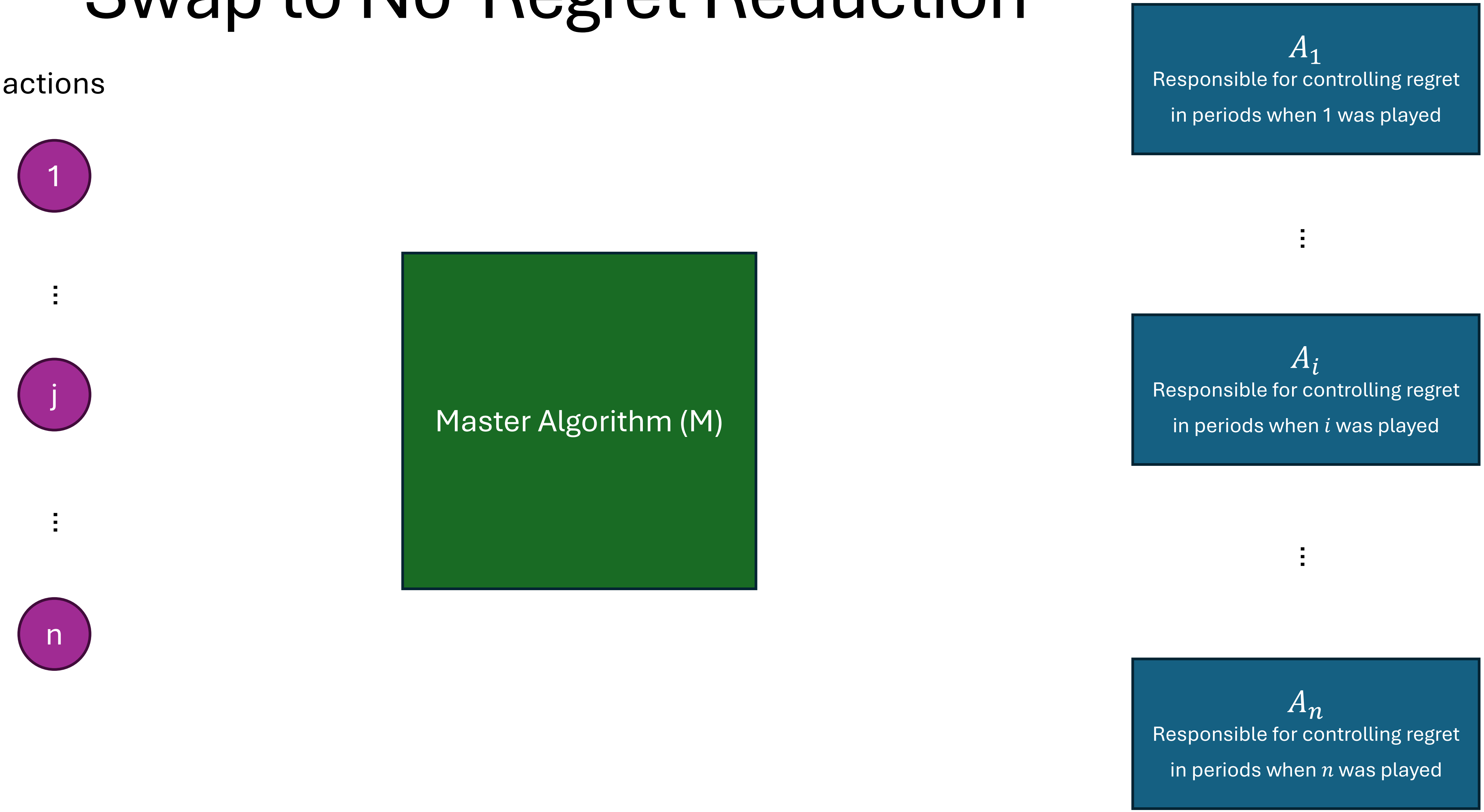
No Swap Regret vs No Regret

- **For subset of periods** when played i don't regret any other i'

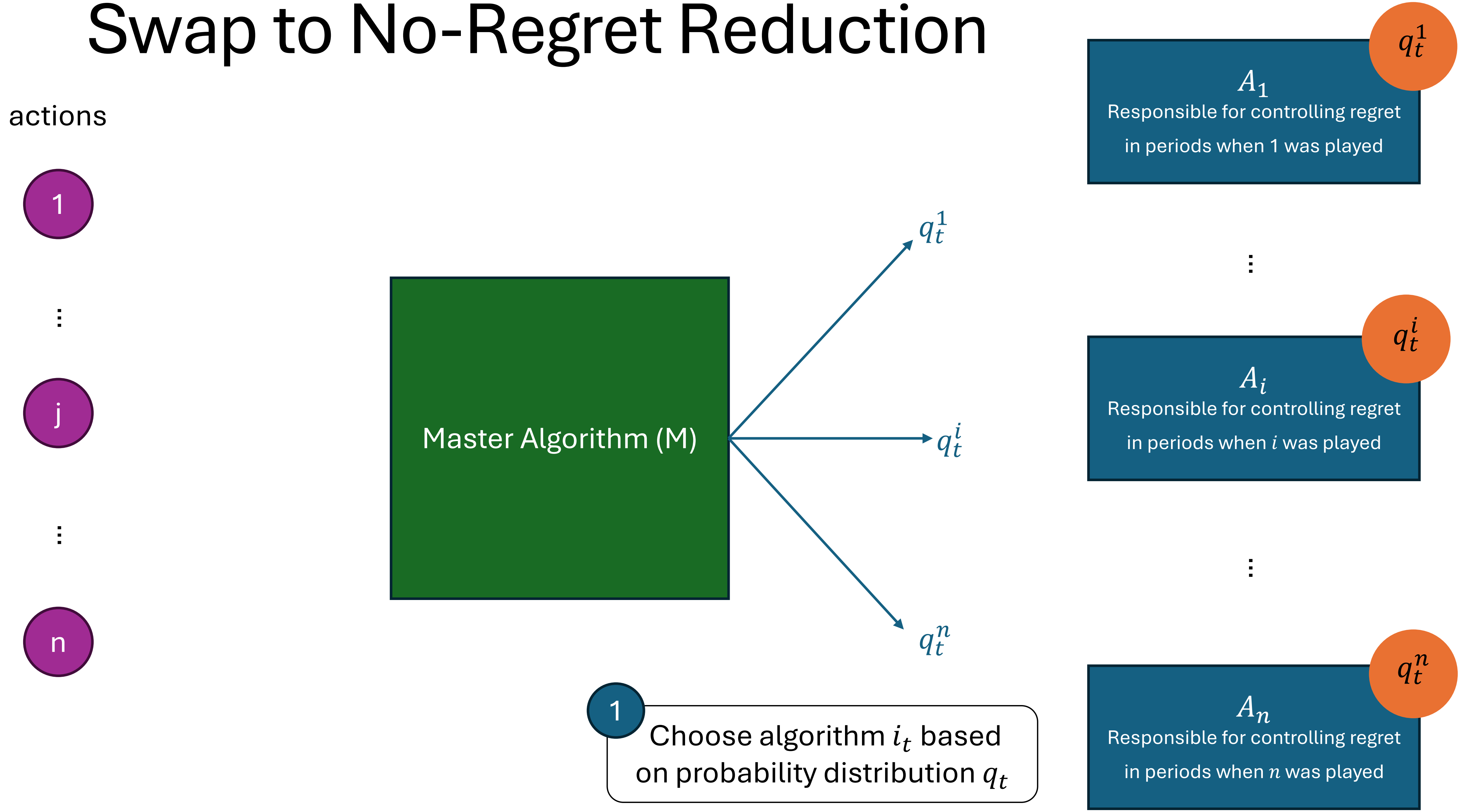
$$\frac{1}{T}\sum_{t:i_t=i} \ell_t^{i_t} \leq \max_{i'} \frac{1}{T}\sum_{t:i_t=i} \ell_t^{i'} + \tilde{\epsilon}(T,\delta), \qquad \text{w.p. } 1-\delta$$

- This looks like the no-regret property, but on a subset of periods
- If ahead of time we knew on which subset of periods we'd play i
- We could spawn a separate no-regret algorithm A_i
- When it was time to play i we would call A_i and report back loss

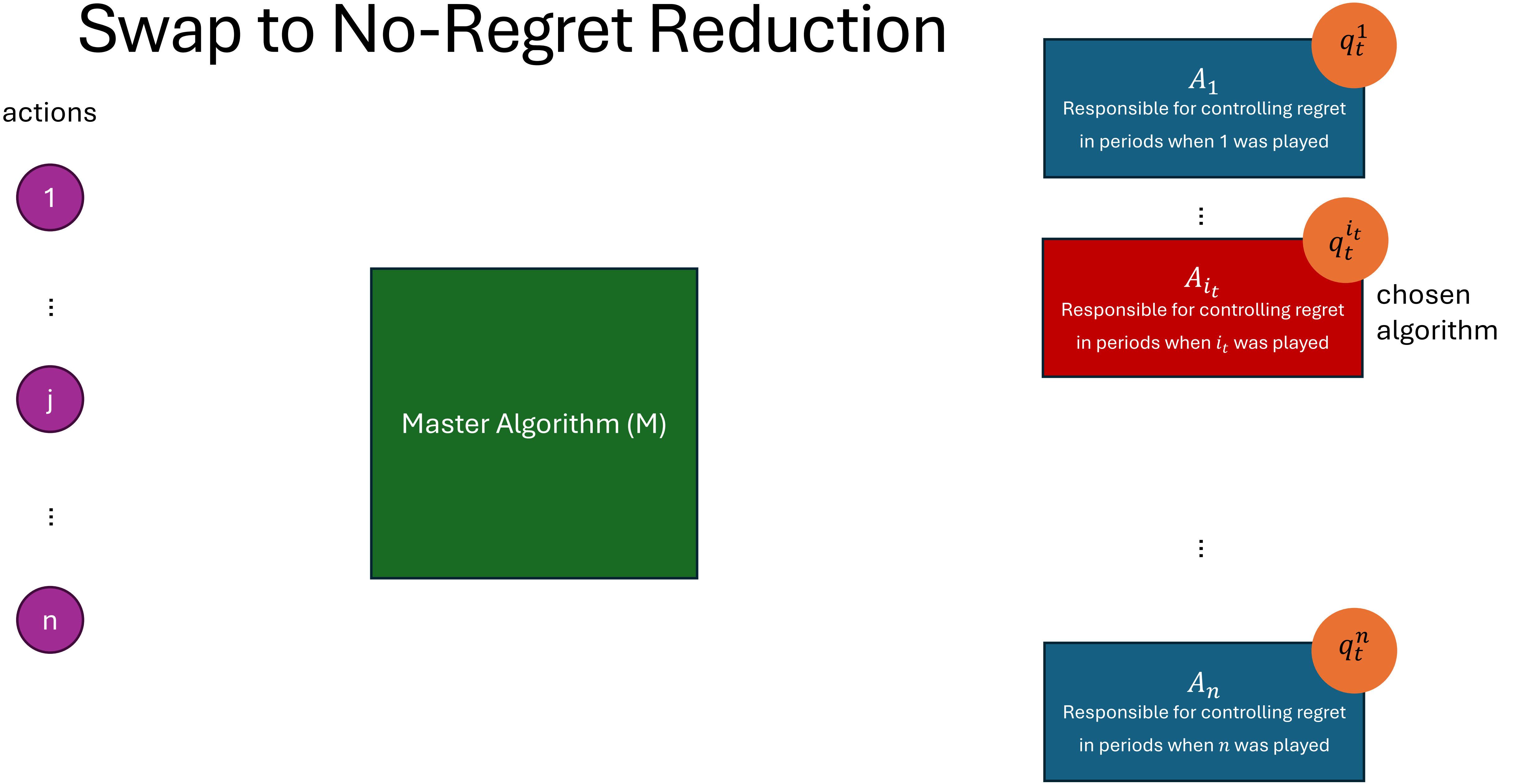
Swap to No-Regret Reduction



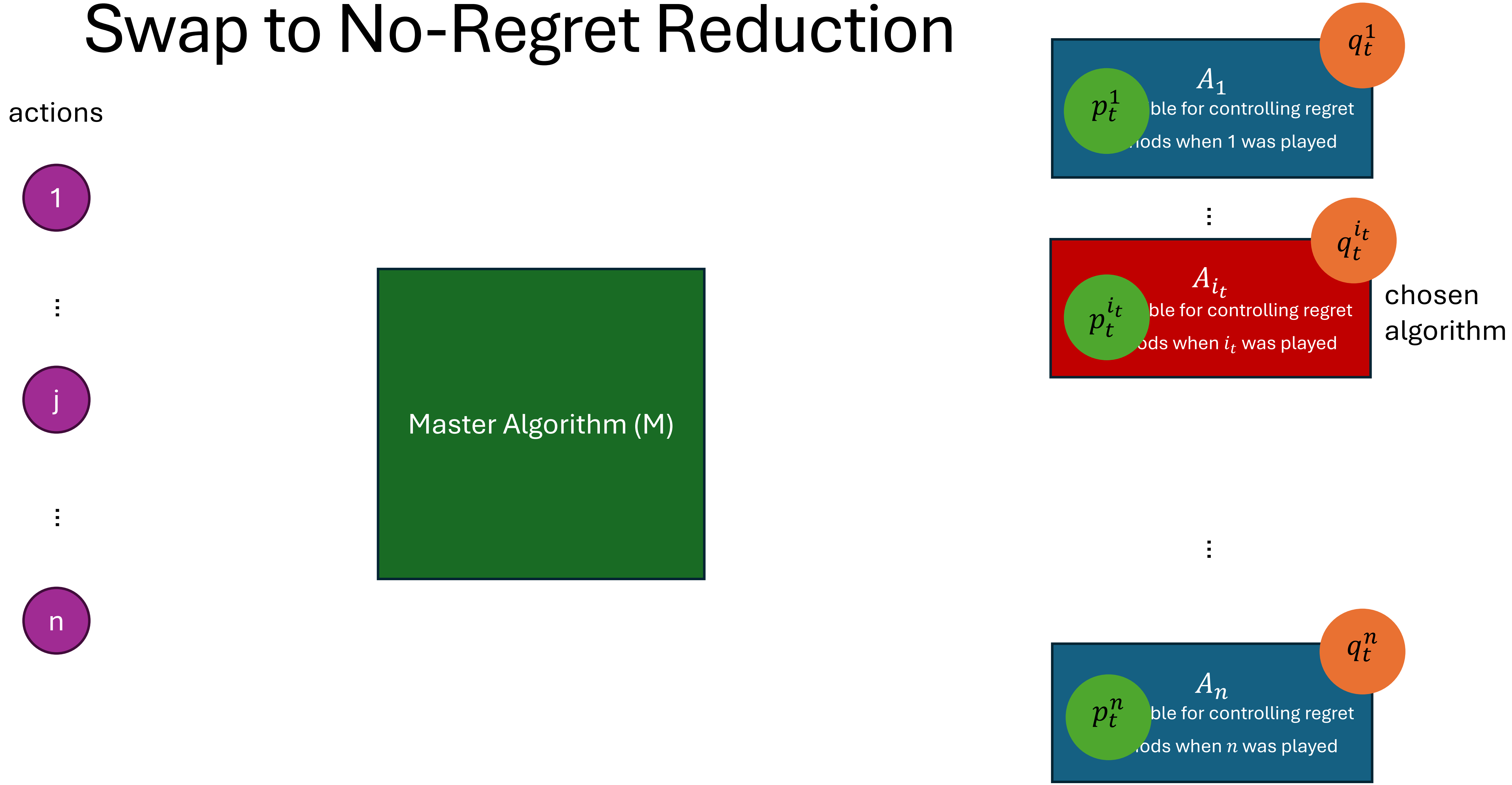
Swap to No-Regret Reduction



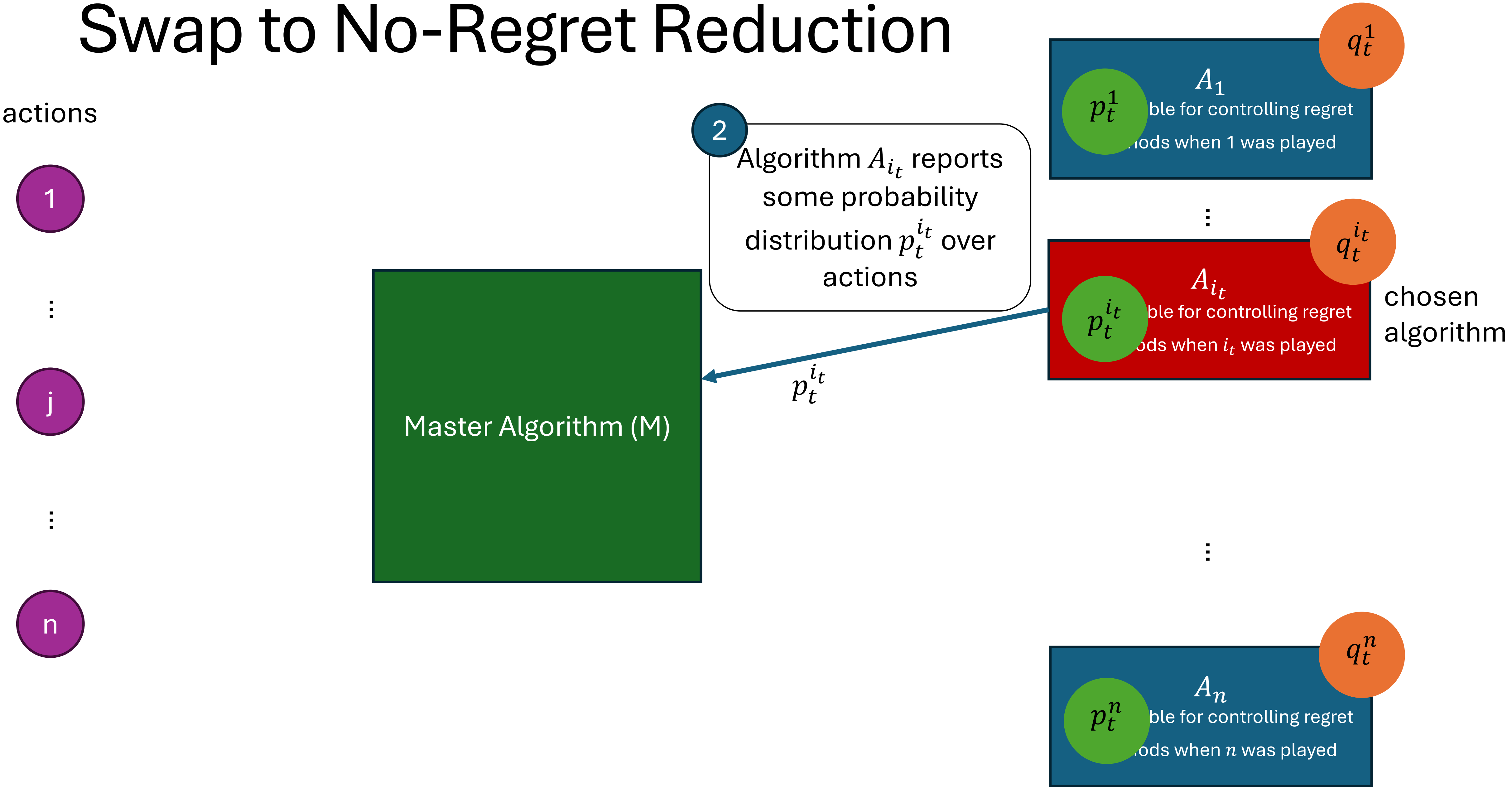
Swap to No-Regret Reduction



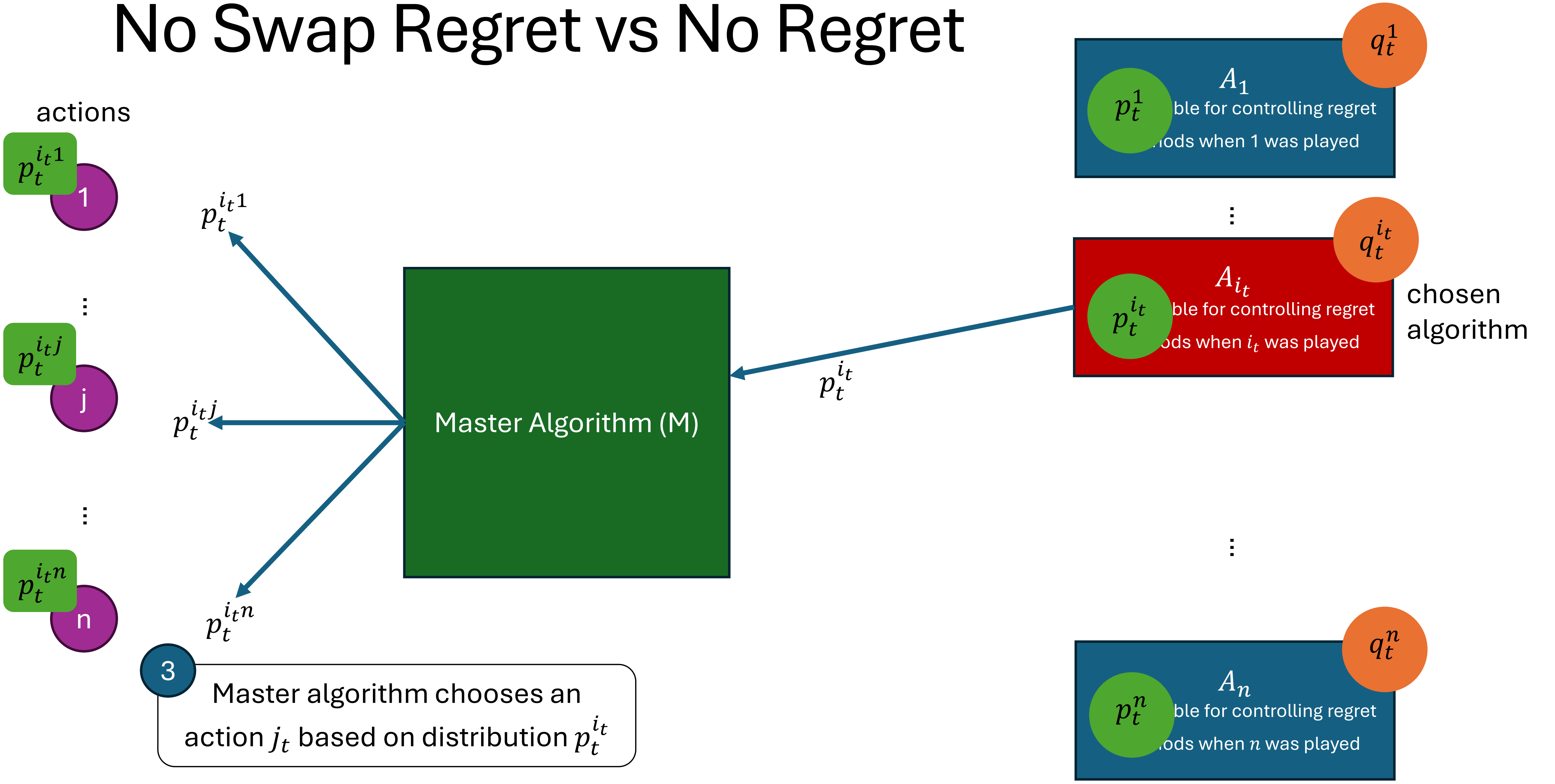
Swap to No-Regret Reduction



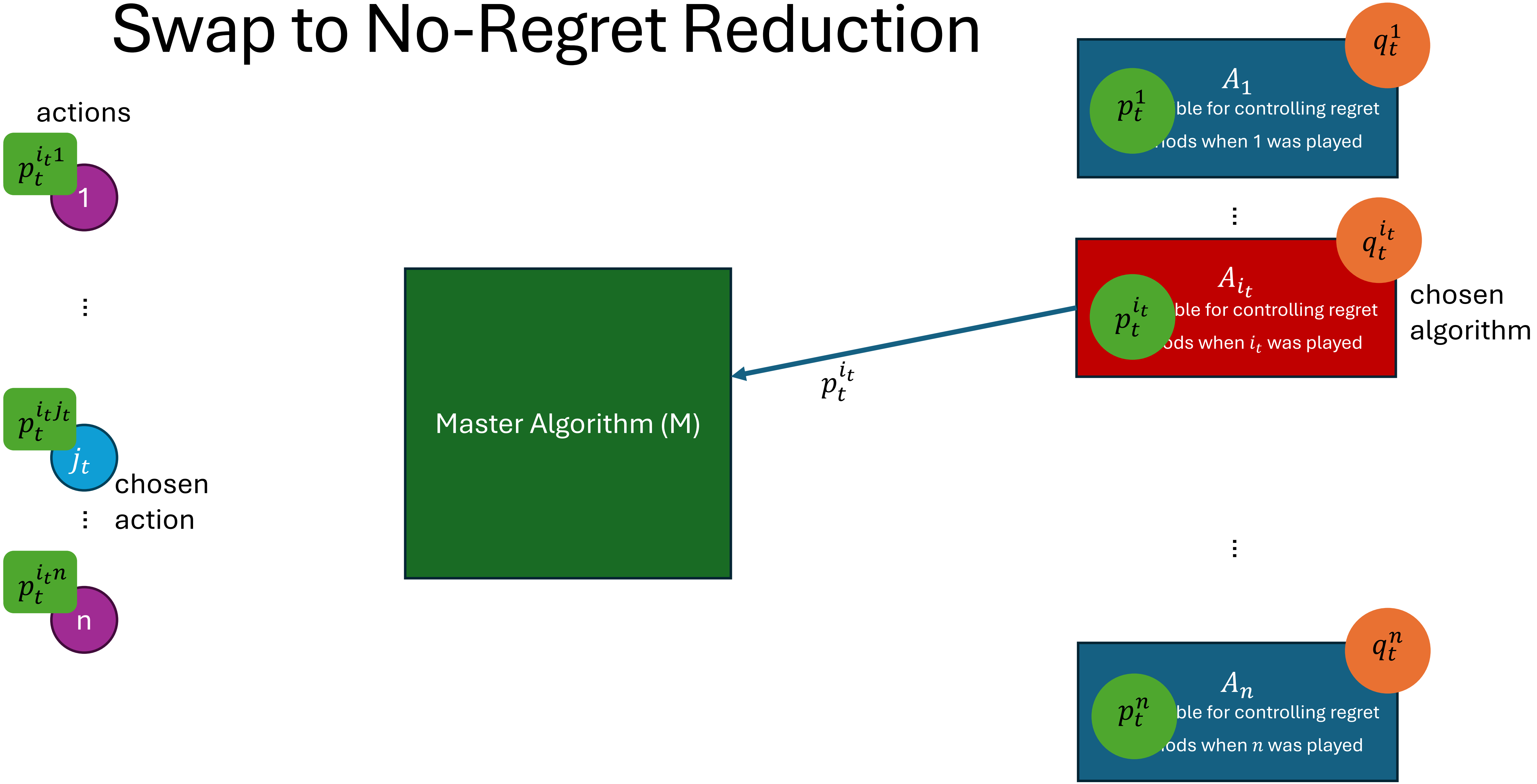
Swap to No-Regret Reduction



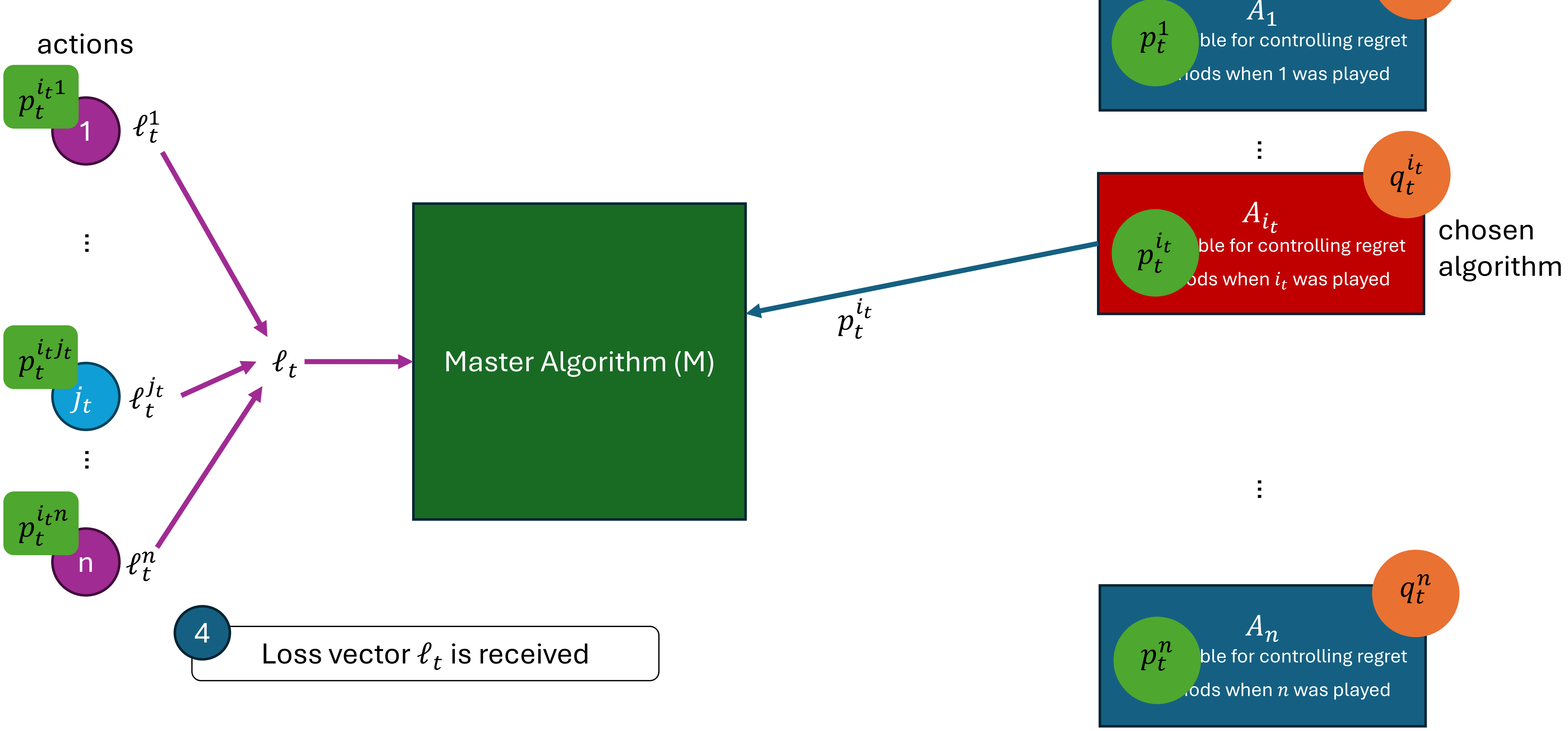
No Swap Regret vs No Regret



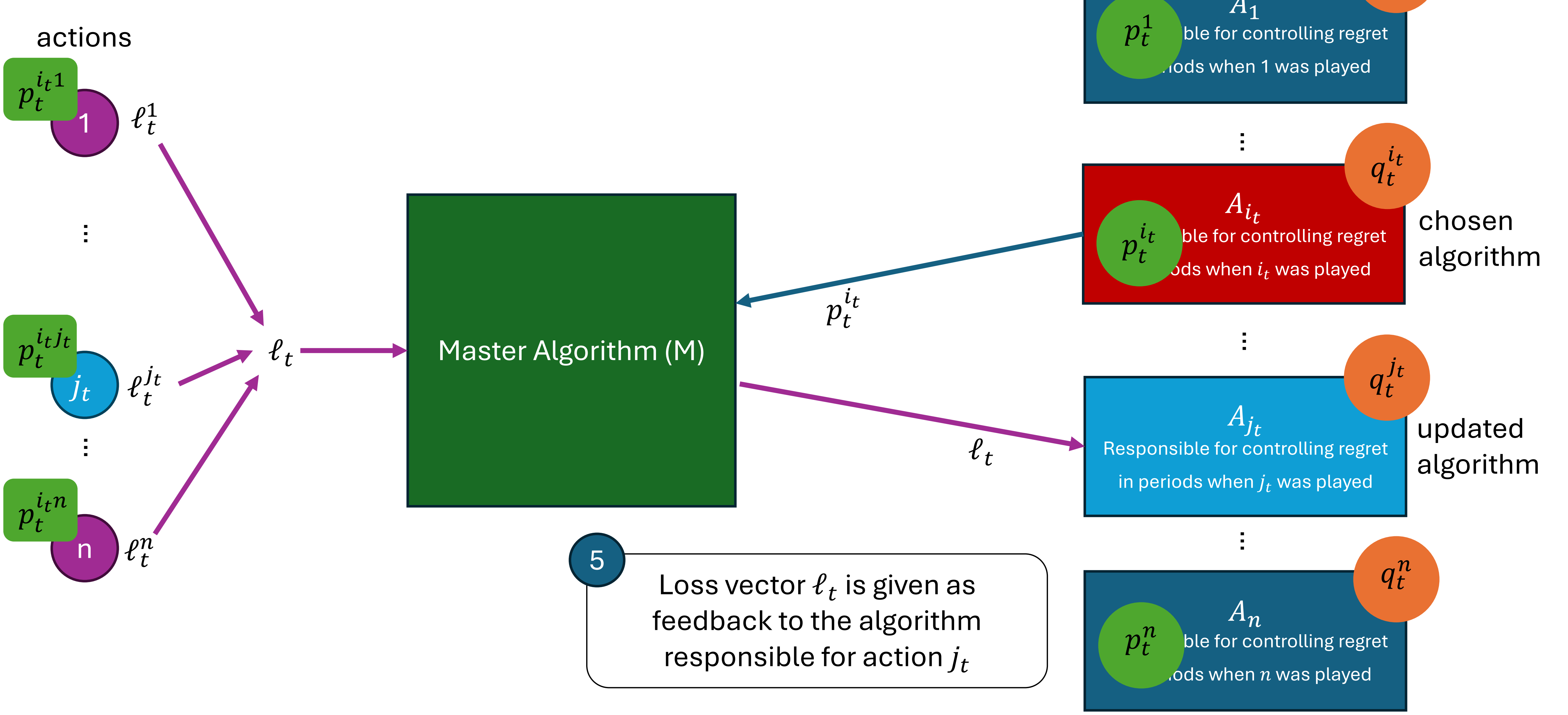
Swap to No-Regret Reduction



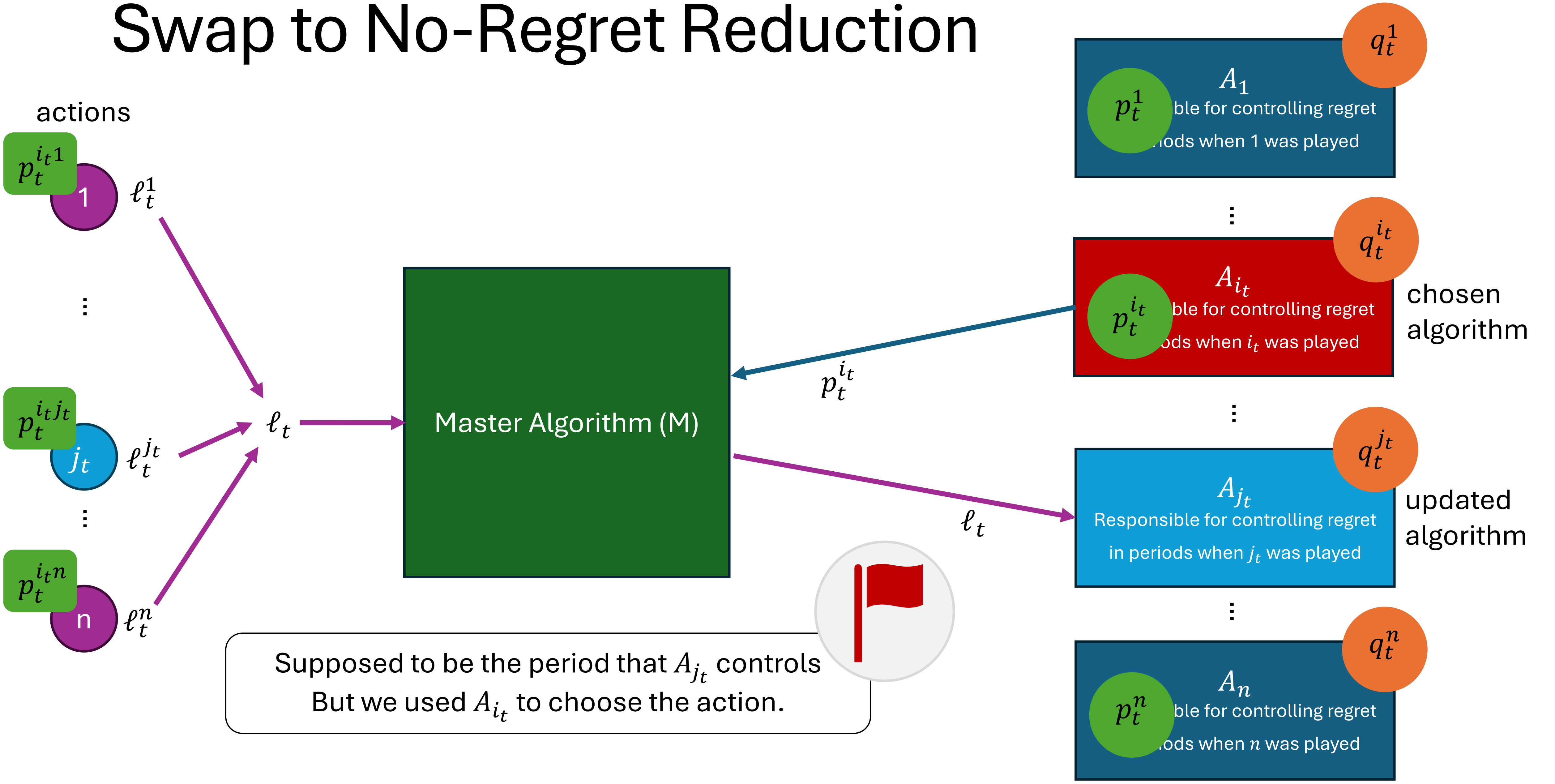
Swap to No-Regret Reduction



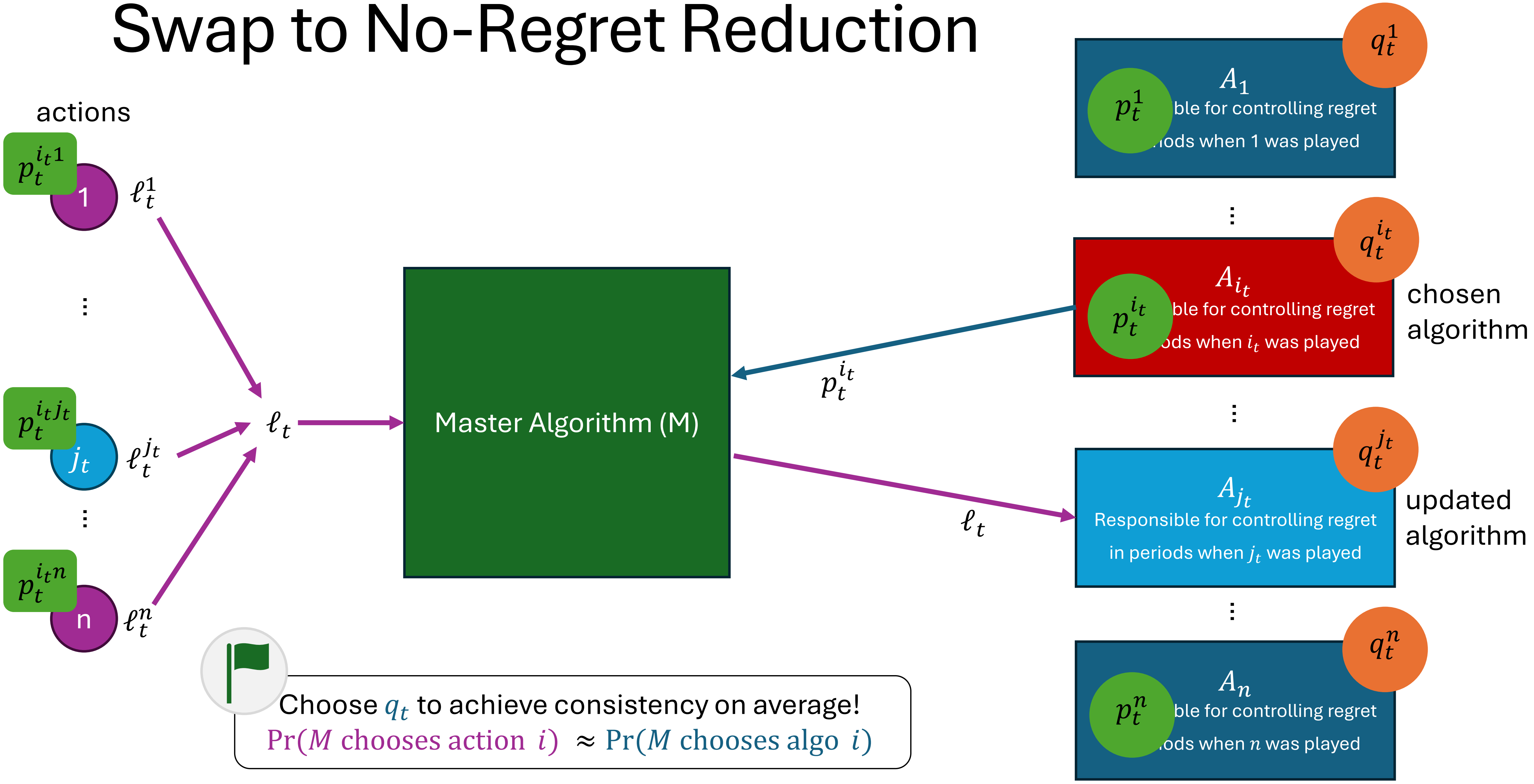
Swap to No-Regret Reduction



Swap to No-Regret Reduction



Swap to No-Regret Reduction

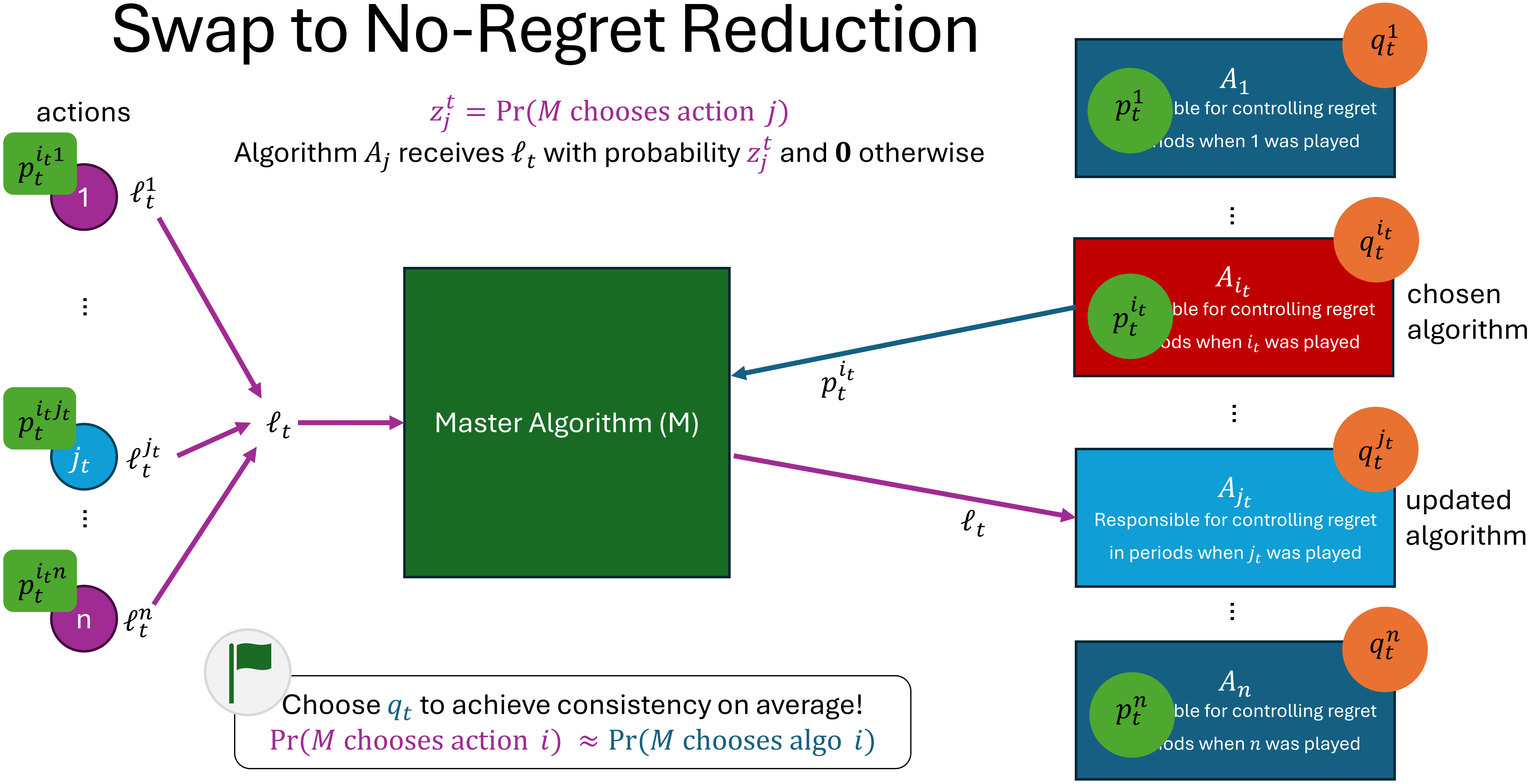


Choose q_t to achieve consistency on average!
$\Pr(M \text{ chooses action } i) \approx \Pr(M \text{ chooses algo } i)$

Swap to No-Regret Reduction

$z_j^t = \Pr(M \text{ chooses action } j)$

Algorithm A_j receives ℓ_t with probability z_j^t and $\mathbf{0}$ otherwise



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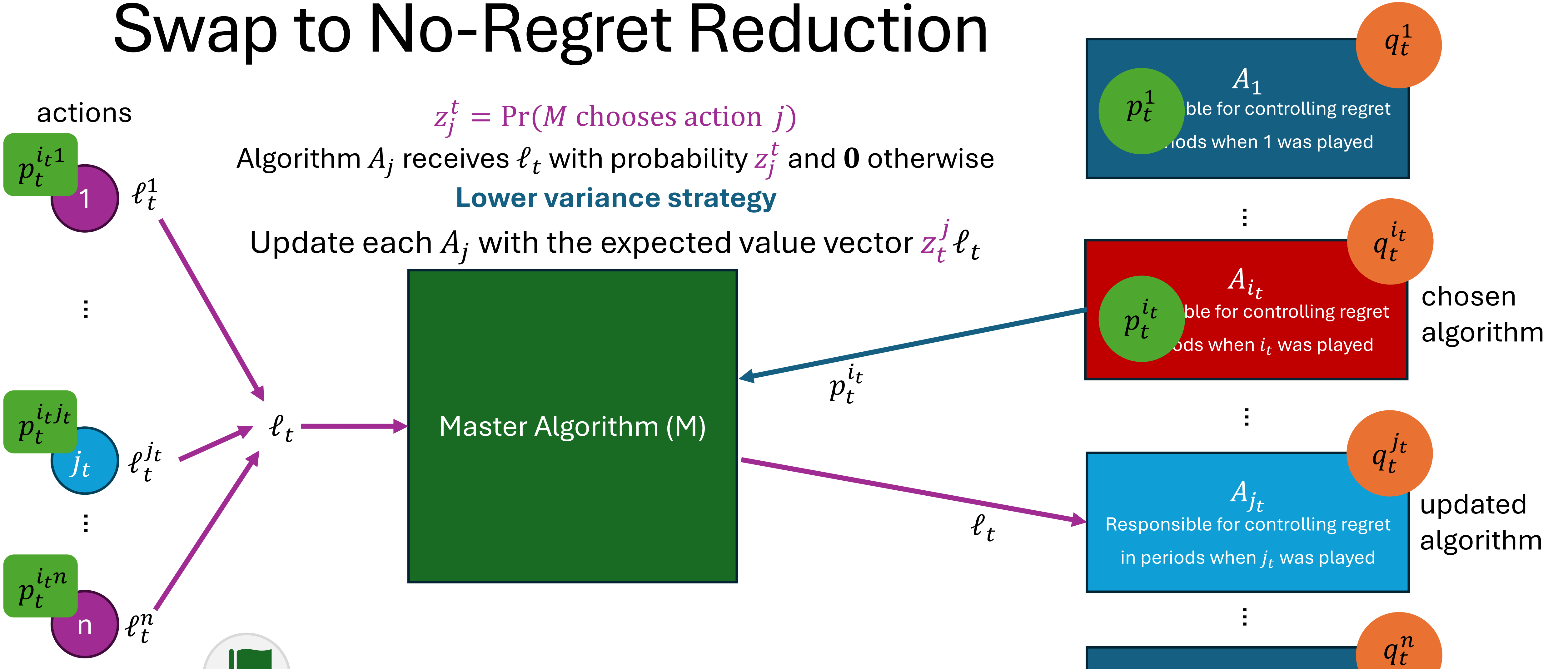
Swap to No-Regret Reduction

$z_j^t = \Pr(M \text{ chooses action } j)$

Algorithm A_j receives ℓ_t with probability z_j^t and $\mathbf{0}$ otherwise

Lower variance strategy

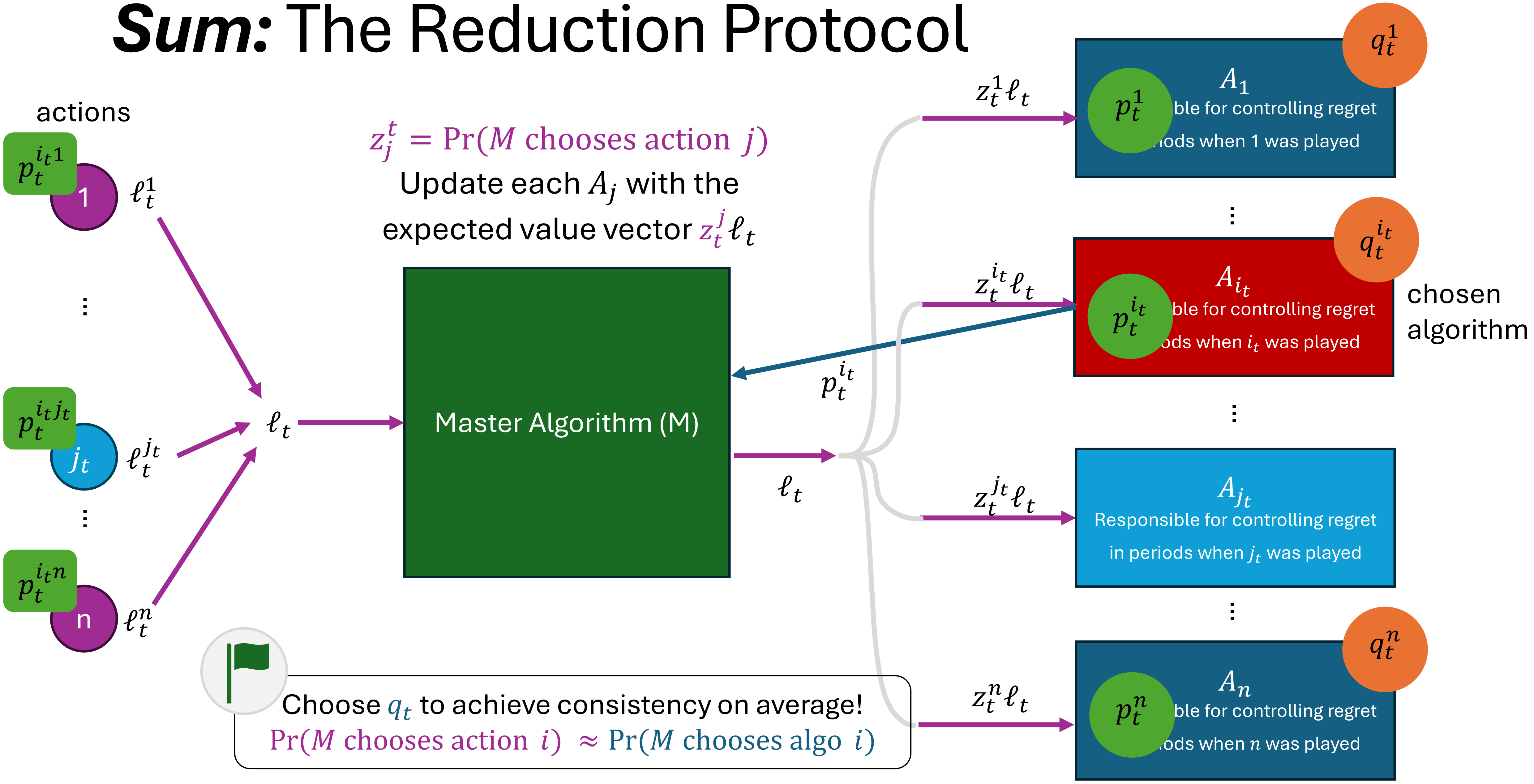
Update each A_j with the expected value vector $z_t^j \ell_t$



Choose q_t to achieve consistency on average!

$\Pr(M \text{ chooses action } i) \approx \Pr(M \text{ chooses algo } i)$

Sum: The Reduction Protocol



Sum: The reduction protocol

- At each period we choose each action with probability

$$
\begin{aligned}
z_t^j &= \Pr(M \text{ choose action } j) \\
&= \sum_i \underbrace{\Pr(M \text{ choose algo } A_i)}_{q_t^i} \cdot \underbrace{\Pr(A_i \text{ choose action } j)}_{p_t^{ij}}
\end{aligned}
$$

- We update each algorithm A_j with loss vector

$$z_t^j \ell_t = \Pr(M \text{ choose action } j) \cdot (\text{loss vector})$$

- The distribution over algorithms q_t is chosen such that

$$\Pr(M \text{ choose action } j) \approx \Pr(M \text{ choose algo } A_j)$$

From No-Regret of Algos
to No-Swap Regret of Master

Regret = Loss – Benchmark Loss

Loss Analysis at Each Step

- How much loss does algorithm A_i perceive?

$$\underbrace{\Pr(M \text{ choose action } i)}_{\text{The fraction of the loss vector that M attributed and reported back to } A_i} \sum_j \Pr(A_i \text{ choose action } j) \cdot \text{loss}(j)$$

- How much total loss do all the algorithms perceive?

$$\sum_i \Pr(M \text{ choose action } i) \sum_j \Pr(A_i \text{ choose action } j) \cdot \text{loss}(j)$$

- How much loss does the master algorithm incur?

$$\sum_i \Pr(M \text{ choose algo } A_i) \sum_j \Pr(A_i \text{ choose action } j) \cdot \text{loss}(j)$$

Loss Analysis at Each Step

- How much loss does algorithm A_i perceive?

$$\underbrace{\Pr(M \text{ choose action } i)}_{\text{The fraction of the loss vector that M attributed and reported back to } A_i} \sum_j \Pr(A_i \text{ choose action } j) \cdot \mathrm{loss}(j)$$

- How much total loss do all the algorithms perceive?

$$\sum_i \Pr(M \text{ choose action } i) \sum_j \Pr(A_i \text{ choose action } j) \cdot \mathrm{loss}(j)$$

- How much loss does the master algorithm incur?

$$\sum_i \Pr(M \text{ choose algo } A_i) \sum_j \Pr(A_i \text{ choose action } j) \cdot \mathrm{loss}(j)$$

Recap: Loss Analysis at Each Step

Corollary. If we can guarantee that

$$\underbrace{\Pr(M \text{ choose action } i)}_{z_t^i} \approx \underbrace{\Pr(M \text{ choose algo } A_i)}_{q_t^i}$$

Then the total loss perceived by the separate algorithms is approximately the same as the total loss experienced by the master

$$\text{total loss perceived by algos} \approx \text{total loss of master}$$

Competing Benchmark Analysis at Each Step

- What can each algorithm A_i compete with based on **no-regret**?

$$\underbrace{\Pr(M \text{ choose action } i)}_{\substack{\text{The fraction of the loss vector that M} \\ \text{attributed and reported back to } A_i}} \cdot \text{loss}\big(\underbrace{\phi}_{\substack{\text{For each algo } A_i \text{ this is a constant} \\ \text{action comparison with } i' = \phi(i)}}(i)\big)$$

- What can in total all algorithms compete with based on **no-regret**?

$$\sum_i \Pr(M \text{ choose action } i) \cdot \text{loss}\big(\phi(i)\big)$$

- What does the master want to compete with for **no-swap regret**?

$$\sum_j \Pr(M \text{ choose action } j) \cdot \text{loss}\big(\phi(j)\big)$$

Competing Benchmark Analysis at Each Step

- What can each algorithm A_i compete with based on **no-regret**?

$$\underbrace{\Pr(M \text{ choose action } i)} \cdot \text{loss}\big(\underbrace{\phi(i)}\big)$$

The fraction of the loss vector that M attributed and reported back to A_i

For each algo A_i this is a constant action comparison with $i' = \phi(i)$

- What can in total all algorithms compete with based on **no-regret**?

$$\underbrace{\sum_i \Pr(M \text{ choose action } i) \cdot \text{loss}\big(\phi(i)\big)} = \overbrace{\sum_j \Pr(M \text{ choose action } j) \cdot \text{loss}\big(\phi(j)\big)}$$

- What does the master want to compete with for **no-swap regret**?

Recap: Benchmark Analysis at Each Step

Corollary. The total *perceived* benchmark loss that algorithms compete with, *where each algorithm i considers the no-regret benchmark of always playing action $i' = \phi(i)$*, is equal to the *true* swap benchmark loss that the master wants to compete with, *associated with the swap function ϕ.*

Regret = Loss – Benchmark Loss

Regret Analysis at Each Step

Corollary. If we can guarantee that

$$\Pr(M \text{ choose action } i) \approx \Pr(M \text{ choose algo } A_i)$$

then swap regret of master is upper bounded by sum of plain regrets of algos

$$\begin{aligned}
\text{Swap Regret of Master } &= \text{Total Loss of Master} - \text{Swap Benchmark} \\
&\approx \text{Total Perceived Loss by Algos} - \text{Total Algo Fixed Action Benchmark} \\
&= \text{Total Perceived Regret of Algos}
\end{aligned}$$

Regret Analysis at Each Step

Corollary. If we can guarantee that

$$\Pr(M \text{ choose action } i) \approx \Pr(M \text{ choose algo } A_i)$$

then swap regret of master is upper bounded by sum of plain regrets of algos

$$\underbrace{\sum_t \sum_j z_t^j \ell_t^j - z_t^j \ell_t^{\phi(j)}}_{\text{Swap Regret of Master}} = \underbrace{\sum_t \sum_i \boldsymbol{q_t^i} \sum_j p_t^{ij} \ell_t^j}_{\text{Total Loss of Master}} - \underbrace{\sum_t \sum_j z_t^j \ell_t^{\phi(j)}}_{\text{Swap Benchmark}}$$

$$\approx \underbrace{\sum_t \sum_i \boldsymbol{z_t^i} \sum_j p_t^{ij} \ell_t^j}_{\text{Total Loss Perceived by Algos}} - \underbrace{\sum_t \sum_i z_t^i \ell_t^{\phi(i)}}_{\text{Total Algo Fixed Action Benchmark}}$$

$$= \underbrace{\sum_i \sum_t \langle p_t^i, z_t^i \ell_t \rangle - z_t^j \ell_t^{\phi(i)}}_{\text{Total Perceived Regret of Algos}}$$

Can we pick q_t such that:

$$\Pr(M \text{ choose action } j) \approx \Pr\left(M \text{ choose algo } A_j\right)$$

Choosing distribution over algos

- Choose q_t such that

$$\Pr(M \text{ choose action } j) \approx \Pr\big(M \text{ choose algo } A_j\big)$$

- Remember that

$$\Pr(M \text{ choose action } j) = \sum_j \Pr(M \text{ choose algo } A_i) \cdot \Pr(A_i \text{ choose action } j)$$

- We need the distribution over algos q_t to satisfy the self-consistency property

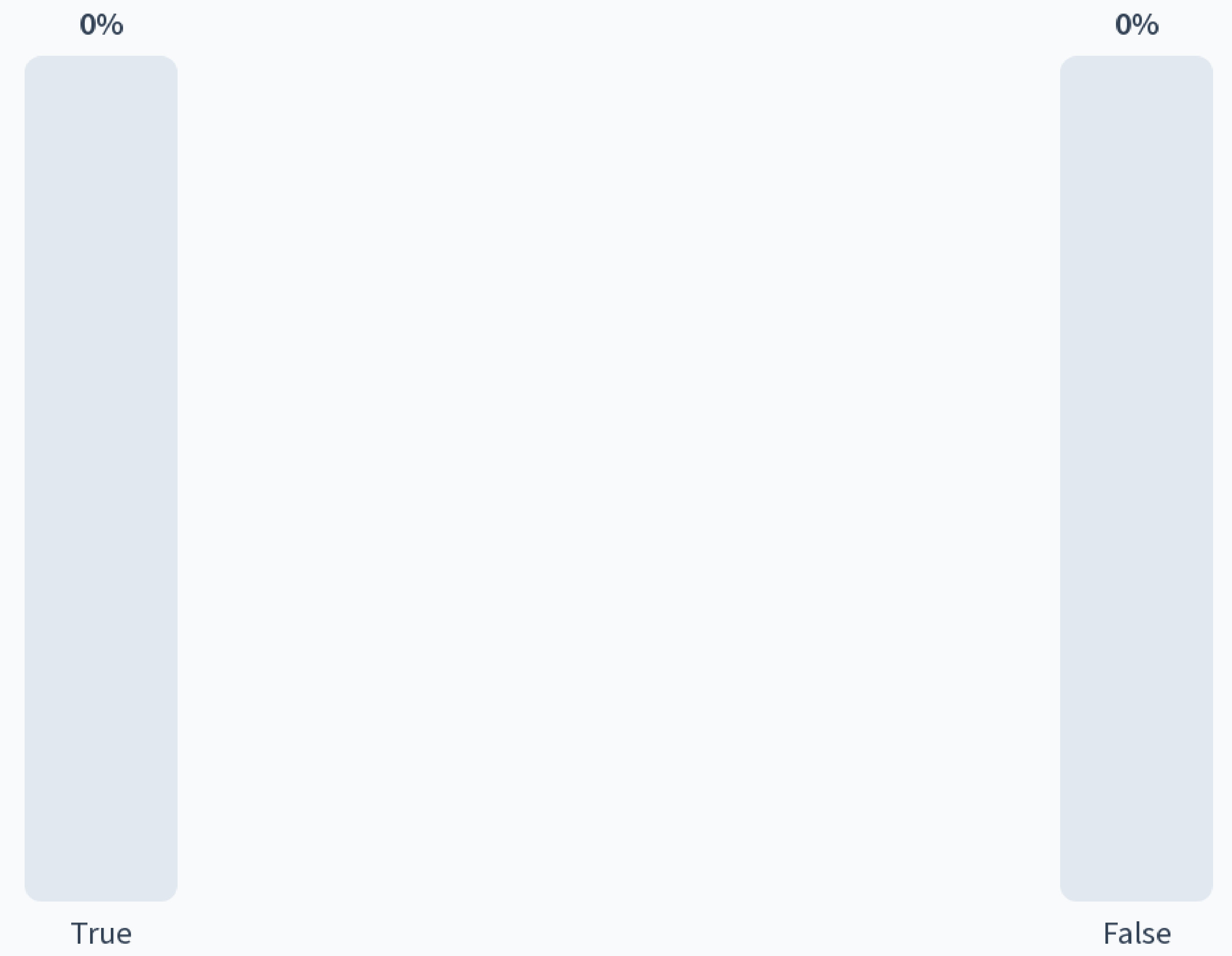
$$\sum_i \underbrace{\Pr(M \text{ choose algo } A_i)}_{q_t^i} \cdot \underbrace{\Pr(A_i \text{ choose action } j)}_{p_t^{ij}} = \underbrace{\Pr\big(M \text{ choose algo } A_j\big)}_{q_t^j}$$

Does there exist a distribution q_t such that:

$$\sum_i \Pr(M \text{ choose algo } A_i) \cdot \Pr(A_i \text{ choose action } j) = \Pr\left(M \text{ choose algo } A_j\right)$$

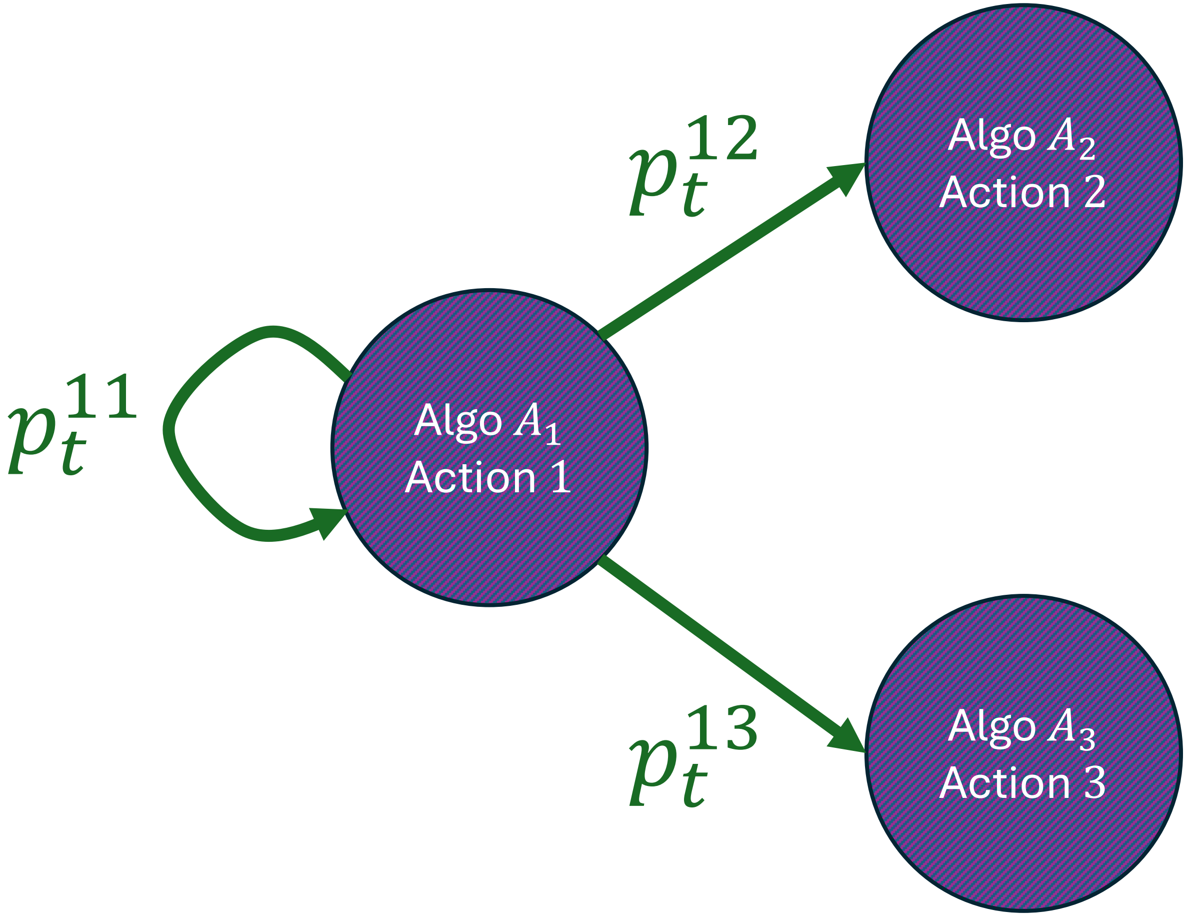
$$\sum_{i=1}^{n} q_t^i \cdot p_t^{ij} = q_t^j$$

There always exists a distribution q that satisfies this property



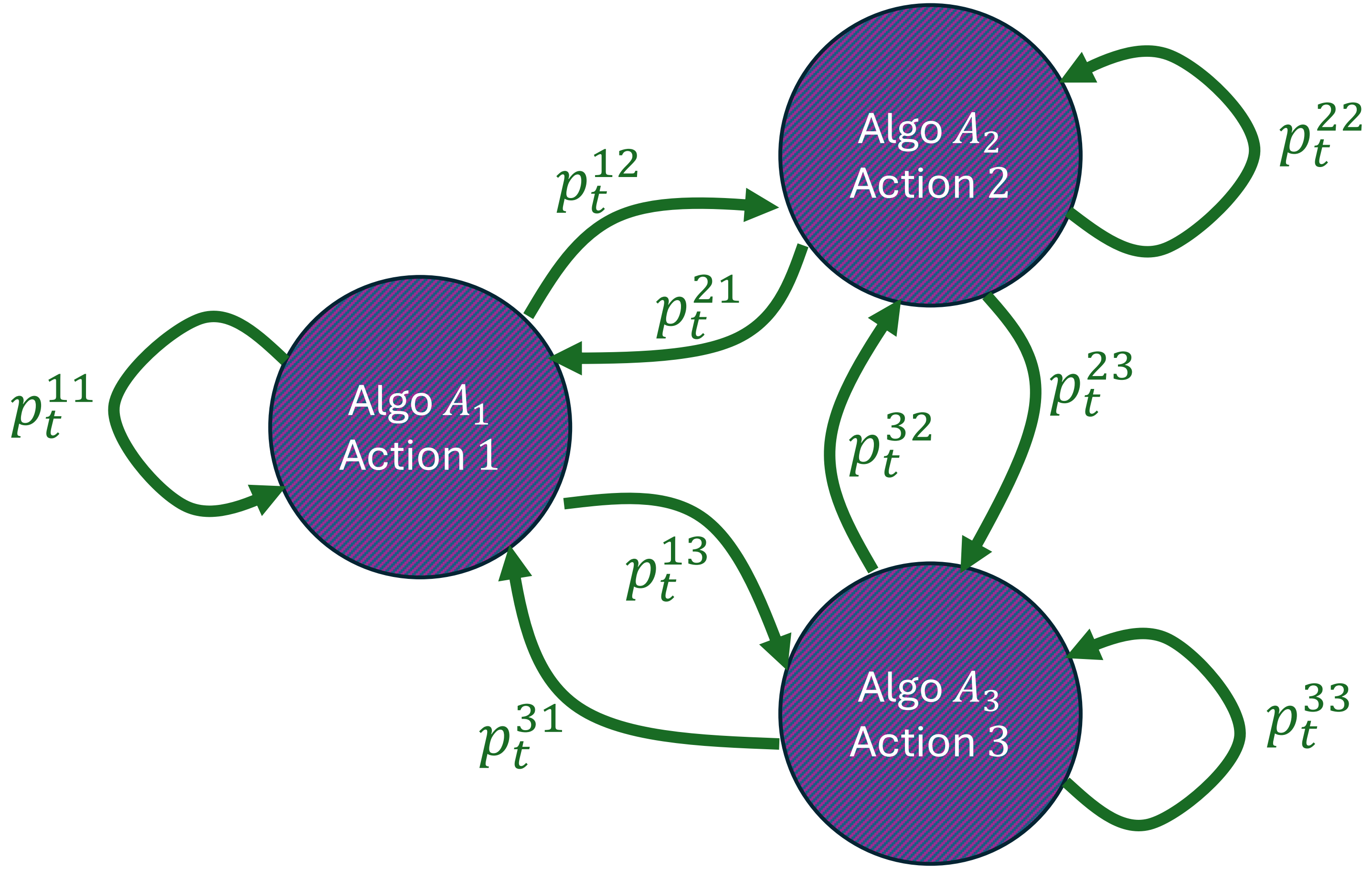
Choosing distribution over algos

$$\sum_i \Pr(M \text{ choose algo } A_i) \cdot \Pr(A_i \text{ choose action } j) = \Pr\big(M \text{ choose algo } A_j\big)$$



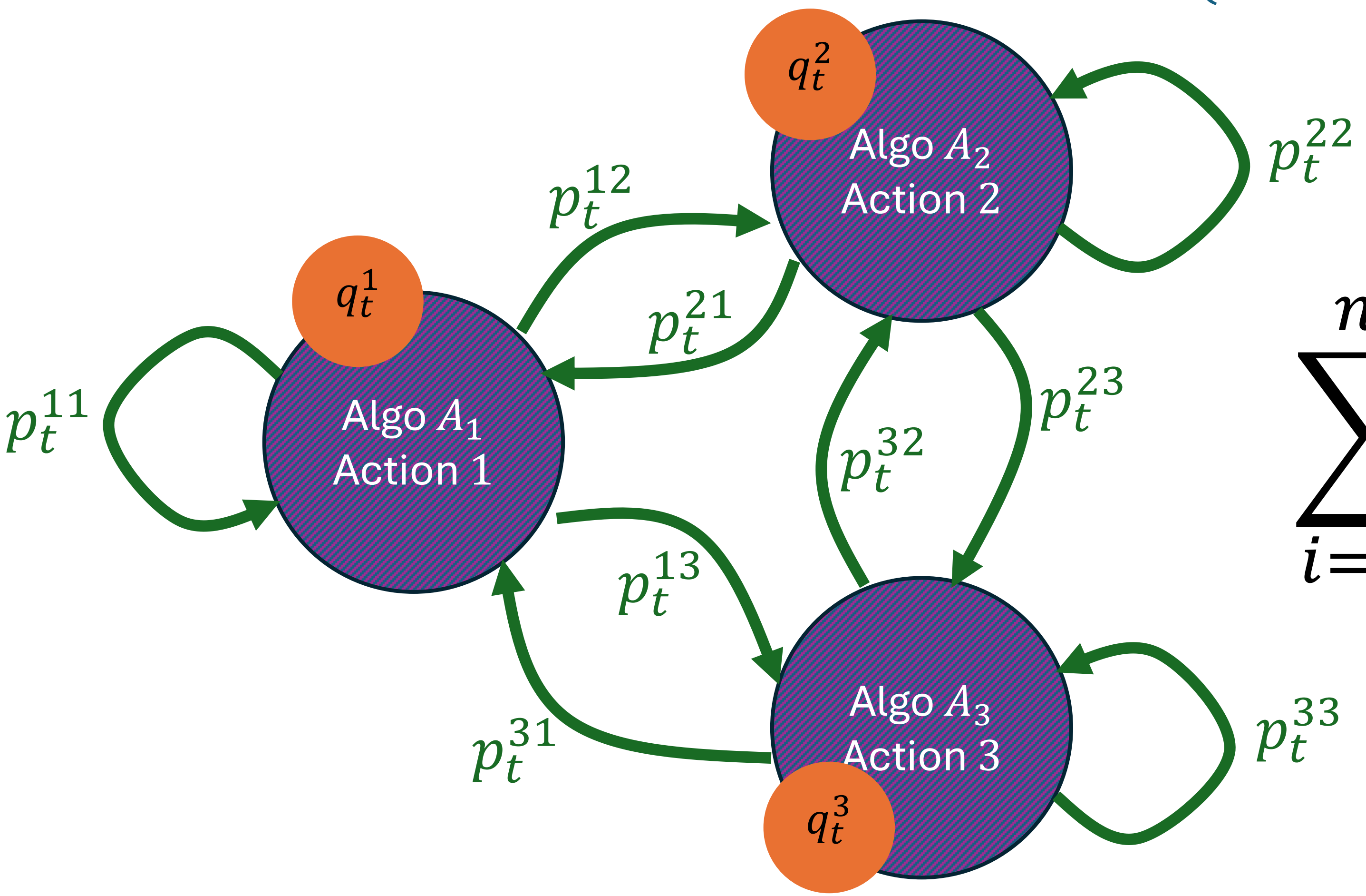
Choosing distribution over algos

$$\sum_i \Pr(M \text{ choose algo } A_i) \cdot \Pr(A_i \text{ choose action } j) = \Pr\big(M \text{ choose algo } A_j\big)$$



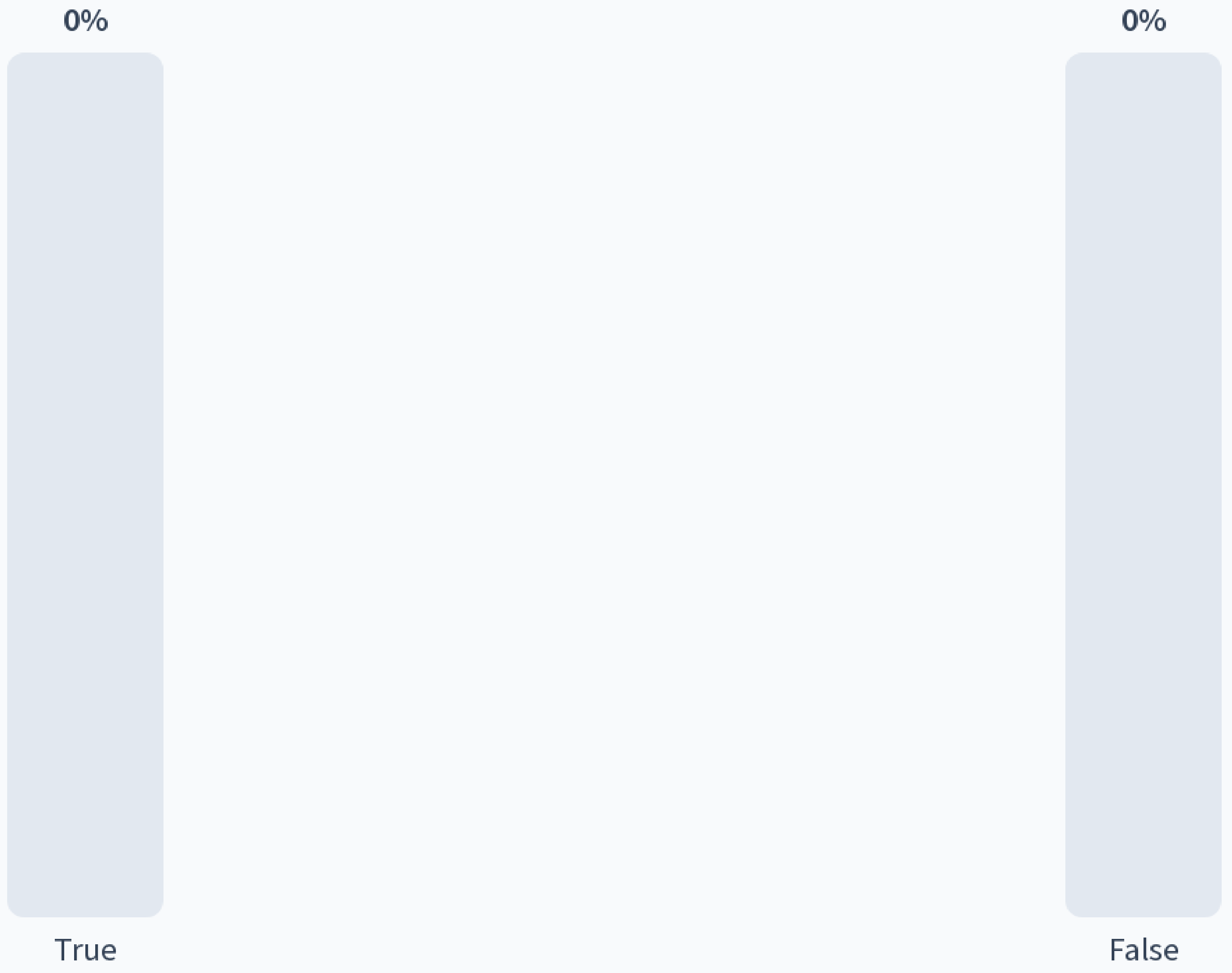
Choosing distribution over algos

$$\sum_i \Pr(M \text{ choose algo } A_i) \cdot \Pr(A_i \text{ choose action } j) = \Pr\big(M \text{ choose algo } A_j\big)$$



$$\sum_{i=1}^{n} q_t^i \cdot p_t^{ij} = q_t^j$$

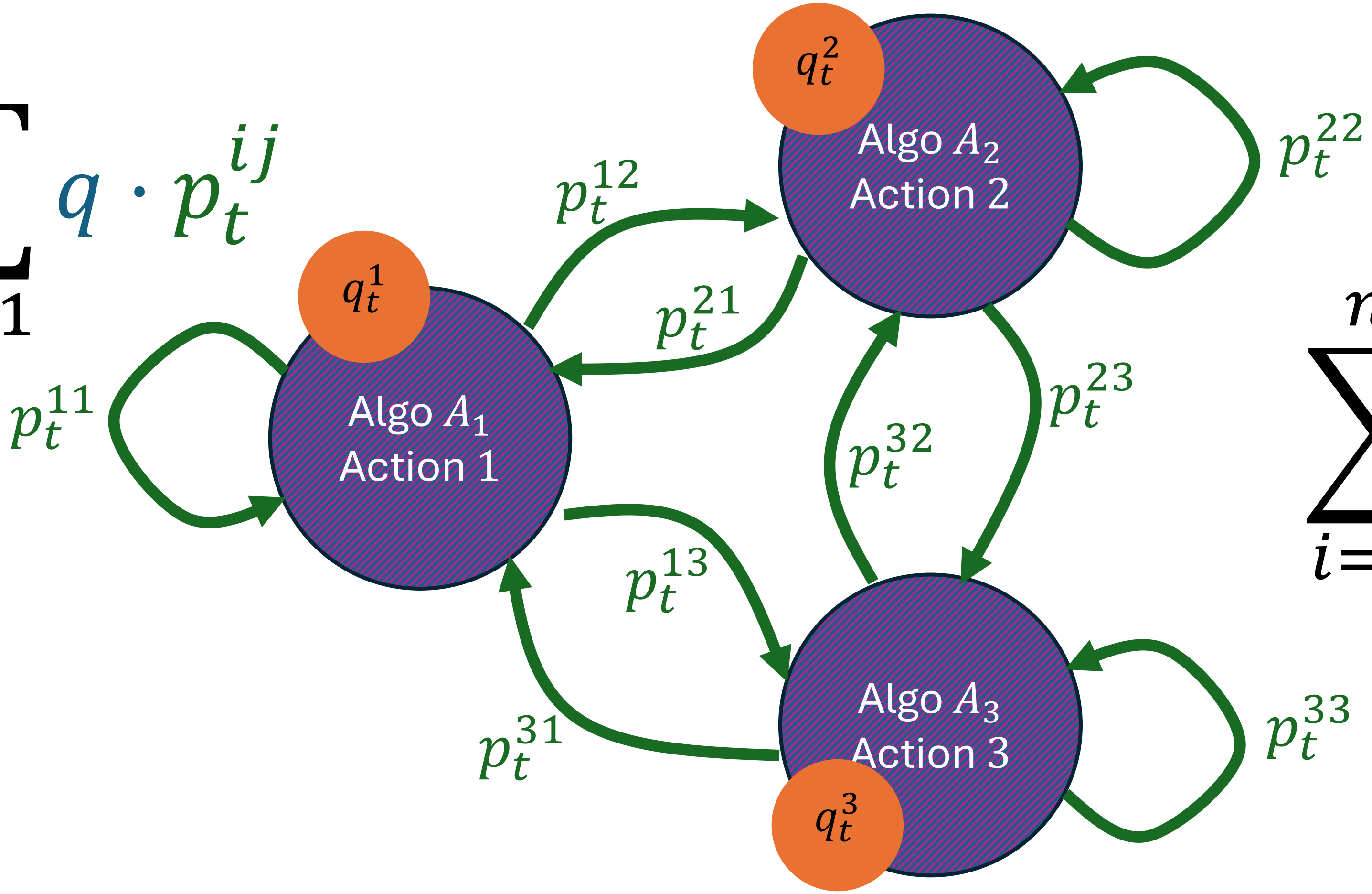
There always exists a distribution q that satisfies this property



A Markov Chain over the Algos/Actions

Starting from a distribution q over nodes and applying one step of the random transitions, brings us to a new distribution over states

$$q' = \sum_{i=1}^{n} q \cdot p_t^{ij}$$

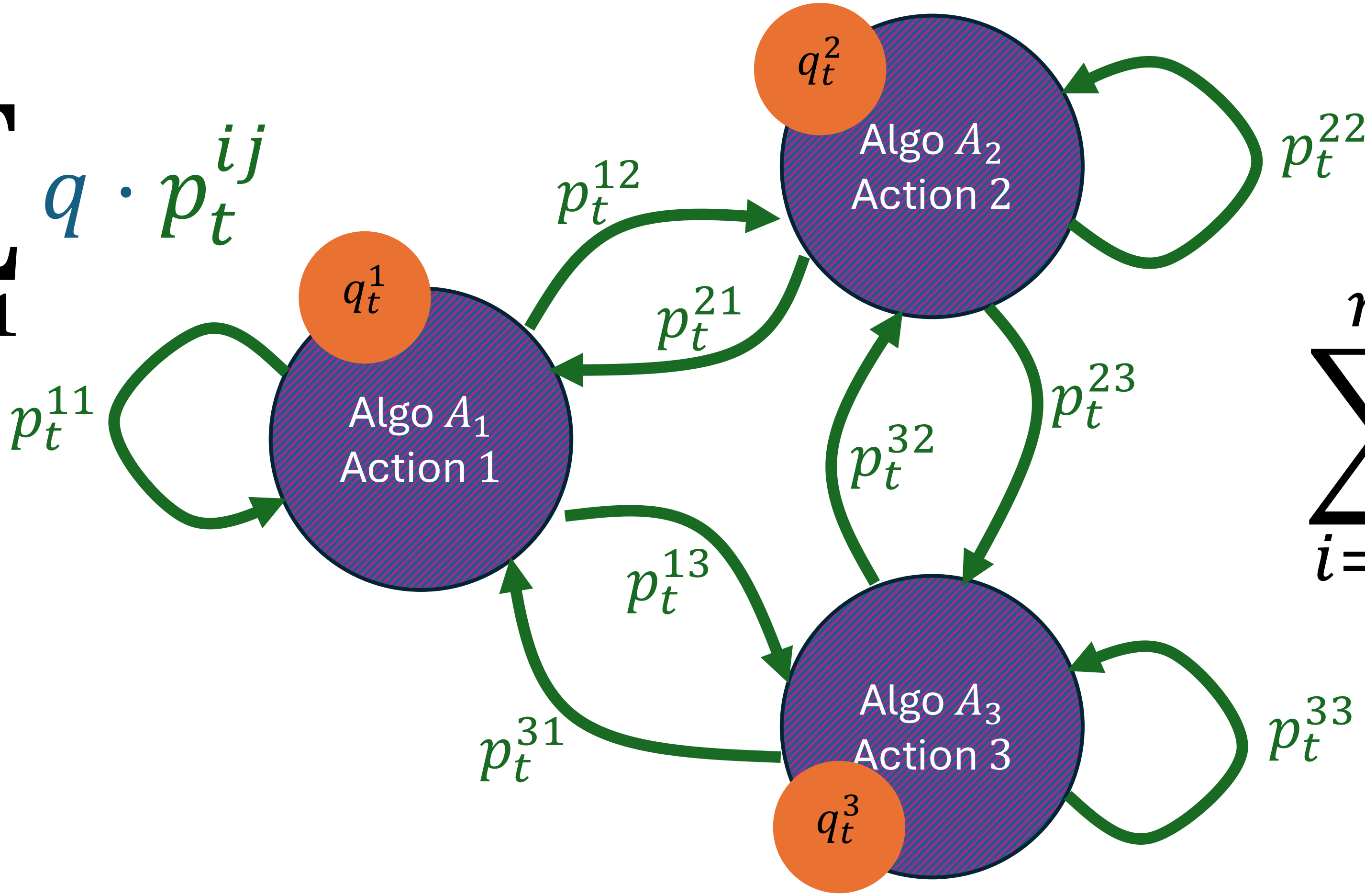


$$\sum_{i=1}^{n} q_t^i \cdot p_t^{ij} = q_t^j$$

Stationary Distributions of Markov Chains

If new distribution is the same as the original distribution, then this distribution is called a Stationary Distribution of the Markov Chain

$$q = \sum_{i=1}^{n} q \cdot p_t^{ij}$$



$$\sum_{i=1}^{n} q_t^i \cdot p_t^{ij} = q_t^j$$

Stationary Distributions of Markov Chains

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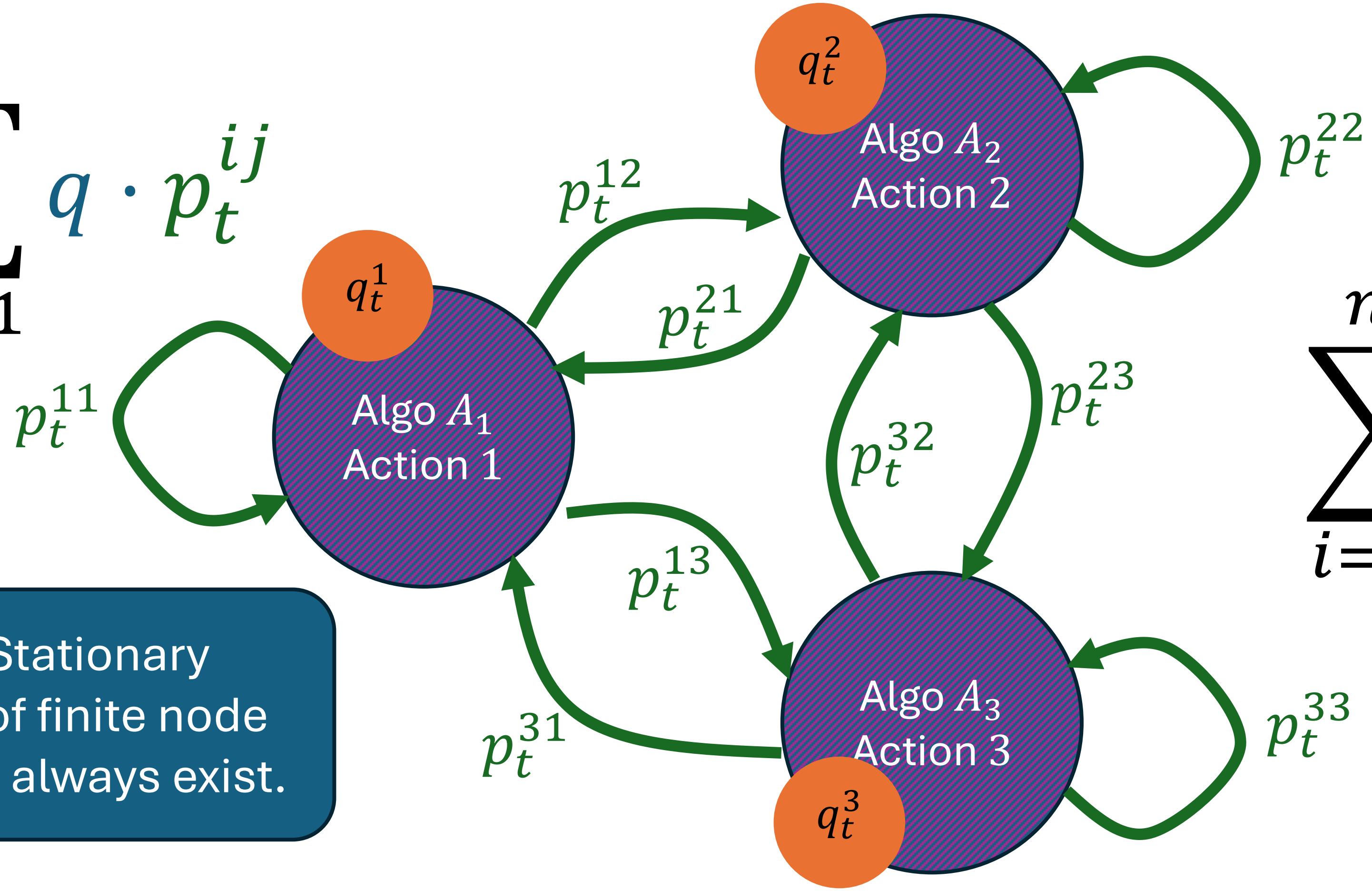

$$\sum_{i=1}^{n} q_t^i \cdot p_t^{ij} = q_t^j$$

Theorem. Stationary distributions of finite node Markov Chains always exist.

Recap: Choosing Distribution over Algos

Corollary. If we choose q_t as stationary distribution of the Markov Chain defined by transition probabilities $\Pr(\mathrm{i} \to \mathrm{j}) = p_t^{ij}$ then

$$\Pr(M \text{ choose action } j) = \Pr\left(M \text{ choose algo } A_j\right)$$

Therefore

$$\text{Swap Regret of Master} = \text{Total Fixed Action Regret of Algos} \to 0$$

Sum: The reduction protocol

- At each period calculate stationary distribution q_t of the Markov Chain defined by the transition probabilities $\Pr(i \to j) = p_t^{ij}$
- Choose each action with probability

$$z_t^j = \Pr(M \text{ choose action } j) = \Pr(M \text{ choose algo } j) = q_t^j$$

- Update each algorithm A_j with loss vector

$$z_t^j \ell_t = \Pr(M \text{ choose action } j) \cdot (\text{loss vector})$$

Finding Stationary Distributions

- Define the matrix P_t, whose (i, j) entry is p_t^{ij}

- Then the stationary distribution satisfies
$$q^\top = q^\top P_t$$
- q is a left eigenvector of P_t associated with eigenvalue 1

- We can calculate q via eigen-decomposition of P_t and identifying the eigenvector associated with eigenvalue 1

Overall Algorithm using EXP for each Algo

```
Initialize Pt with each row being the uniform distribution
For t in 1..T
    # Calculate choice probability q of master based on
    # choice probabilities Pt of algos
    Calculate stationary distribution q of matrix Pt
    Draw action jt based on distribution q
    Observe loss vector lt

    # update each algorithms choice probabilities
    For i in 1..n
        Calculate perceived loss plt[i] = q[i] * lt
        Pt[i] = EXP-Update(Pt[i], plt[i])
```

Recap: Final Theorem

Theorem. If we choose q_t as stationary distribution of the Markov Chain defined by transition probabilities $\Pr(\mathrm{i} \to \mathrm{j}) = p_t^{ij}$ and each algorithm updates their choice probabilities using the EXP rule then

$$\text{Average Swap Regret of Master} \leq 2n\sqrt{\frac{2\log(n)}{T}} \to 0$$

Convergence to Correlated Equilibrium

Theorem. If all players use such an algorithm, then the empirical joint distribution of actions converges to the set of correlated equilibria.

At every T the empirical joint distribution of strategies π^T is an $\epsilon(T)$ approximate correlated equilibrium, in the sense that:

$$\mathrm{SwapRegret}_i(s_i, s_i', T) = \sum_{s_{-i}} \pi^T(s_i, s_{-i}) \cdot \left(u_i(s_i, s_{-i}) - u_i(s_i', s_{-i})\right) \leq \epsilon(T)$$

with $\epsilon(T) = 2n\sqrt{\frac{2\log(n)}{T}}$, where n is number of actions of player i

Recent example research in multi-agent RL using Correlated Equilibrium Techniques

Multi-Agent Training beyond Zero-Sum with Correlated Equilibrium Meta-Solvers

Luke Marris[1 2] Paul Muller[1 3] Marc Lanctot[1] Karl Tuyls[1] Thore Graepel[1 2]

Abstract

Two-player, constant-sum games are well studied in the literature, but there has been limited progress outside of this setting. We propose Joint Policy-Space Response Oracles (JPSRO), an algorithm for training agents in n-player, general-sum extensive form games, which provably converges to an equilibrium. We further suggest correlated equilibria (CE) as promising meta-solvers, and propose a novel solution concept Maximum Gini Correlated Equilibrium (MGCE), a principled and computationally efficient family of solutions for solving the correlated equilibrium selection problem. We conduct several experiments using CE meta-solvers for JPSRO and demonstrate convergence on n-player, general-sum games.

1. Introduction

Recent success in tackling two-player, constant-sum games (Silver et al., 2016; Vinyals et al., 2019) has outpaced progress in n-player, general-sum games despite a lot of interest (Jaderberg et al., 2019; OpenAI et al., 2019; Brown & Sandholm, 2019; Lockhart et al., 2020; Gray et al., 2020; Anthony et al., 2020). One reason is because Nash equilibrium (NE) (Nash, 1951) is tractable and interchangeable in the two-player, constant-sum setting but becomes intractable (Daskalakis et al., 2009) and potentially non-interchangeable[1] in n-player and general-sum settings. The problem of selecting from multiple solutions is known as the equilibrium selection problem (Goldberg et al., 2013; Avis et al., 2010; Harsanyi & Selten, 1988).[2]

Outside of normal form (NF) games, this problem setting arises in multi-agent training when dealing with empirical games (also called meta-games), where a game payoff tensor is populated with expected outcomes between agents playing an extensive form (EF) game, for example the StarCraft League (Vinyals et al., 2019) and Policy-Space Response Oracles (PSRO) (Lanctot et al., 2017), a recent variant of which reached state-of-the-art results in Stratego Barrage (McAleer et al., 2020).

In this work we propose using correlated equilibrium (CE) (Aumann, 1974) and coarse correlated equilibrium (CCE) as a suitable target equilibrium space for n-player, general-sum games[3]. The (C)CE solution concept has two main benefits over NE; firstly, it provides a mechanism for players to correlate their actions to arrive at mutually higher payoffs and secondly, it is computationally tractable to compute solutions for n-player, general-sum games (Daskalakis et al., 2009). We provide a tractable approach to select from the space of (C)CEs (MG), and a novel training framework that converges to this solution (JPSRO). The result is a set of tools for theoretically solving any complete information[4] multi-agent problem. These tools are amenable to scaling approaches; including utilizing reinforcement learning, function approximation, and online solution solvers, however we leave this to future work.

In Section 2 we provide background on a) correlated equilibrium (CE), an important generalization of NE, b) coarse correlated equilibrium (CCE) (Moulin & Vial, 1978), a similar solution concept, and c) PSRO, a powerful multi-agent training algorithm. In Section 3 we propose novel solution concepts called Maximum Gini (Coarse) Correlated Equilibrium (MG(C)CE) and in Section 4 we thoroughly explore its properties including tractability, scalability, invariance, and

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Proceedings of the 38[th] International Conference on Machine

[2]The equilibrium selection problem is subtle and can have various interpretations. We describe it fully in Section 4.1 based