MS&E 125: Intro to Applied Statistics

Hypothesis Testing

Professor Udell

Management Science and Engineering
Stanford

April 26, 2023

Announcements

- Thursday 11:59pm (4/27): HW 3
- Friday (4/28): Project proposal
  (Required project meetings happen this week)
- Next Monday (5/1): Quiz 1 (in class)

Outline

Hypothesis testing

Comparing two samples

Choosing a cutoff

Multiple hypotheses

Summary

Jury selection

Amendment VI of the United States Constitution states,

> *In all criminal prosecutions, the accused shall enjoy the right to a speedy and public trial, by an impartial jury of the State and district wherein the crime shall have been committed.*

Swain vs. Alabama (1965)

- Robert Swain, a Black man, was convicted in Talladega County, Alabama, in 1962
- 26% of eligible jurors were Black
- jurors were selected from among 100 panelists
- only 8 of the 100 panelists were Black

Poll: was the jury rigged?

source: `https://inferentialthinking.com/chapters/11/1/Assessing_a_Model.html`

Hypothesis testing

how likely is this outcome, if the jury were selected at random?

- **null hypothesis:** the jury was selected at random from the eligible population
- **alternative hypothesis:** the jury was not selected at random from the eligible population
- **test statistic:** the number of Black jurors

Demo

approach:

- simulate the jury selection process many times
- visualize the **sampling distribution** of the test statistic using simulation
- compute the **p-value**: the proportion of simulations where the test statistic is at least as extreme as the observed value
- if the p-value is small (often, $< .05$), we **reject the null hypothesis**

`https://colab.research.google.com/github/stanford-mse-125/demos/blob/main/testing.ipynb`

Statistics on the supreme court

Swain vs. Alabama (1965): "the overall percentage disparity has been small"

- how was the supreme court measuring the disparity?
- how would you suggest measuring it?

Outline

Hypothesis testing

Comparing two samples

Choosing a cutoff

Multiple hypotheses

Summary

Deflategate

the New England Patriots were accused of deflating footballs in the 2015 AFC Championship game

- NFL rules require footballs to be inflated to 12.5–13.5 psi
- each team must ensure their footballs are properly inflated
- Colts intercepted a ball and measured < 12.5 psi
- Patriots were accused of deflating the footballs to make them easier to grip

Comparing two samples

- **null hypothesis:** the two samples are drawn from the same population
- **alternative hypothesis:** the two samples are drawn from different populations
- **test statistic:** the difference between the two sample means

Demo

approach:

- simulate the process of assigning footballs to teams many times
- visualize the **sampling distribution** of the test statistic using simulation
- compute the **p-value**: the proportion of simulations where the test statistic is at least as extreme as the observed value
- if the p-value is small (often, $< .05$), we **reject the null hypothesis**

`https://colab.research.google.com/github/stanford-mse-125/demos/blob/main/testing.ipynb`

Outline

Hypothesis testing

Comparing two samples

Choosing a cutoff

Multiple hypotheses

Summary

Choosing a cutoff

the p-value is the probability of observing a test statistic at least as extreme as the one observed, under the null hypothesis

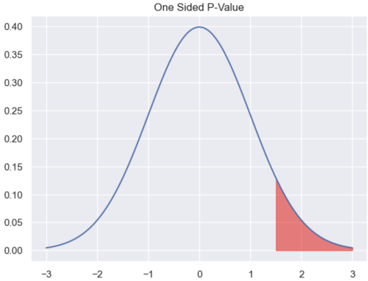
- example: if the p-value is 0.05, then there is a 5% chance of observing a test statistic at least as extreme as the one observed, under the null hypothesis

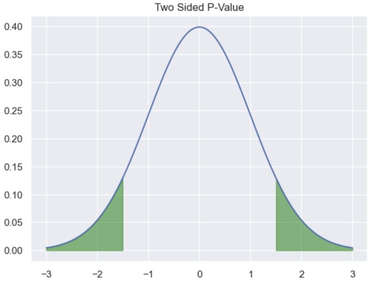
in typical parlance, we say

- a p-value > 0.05 is **not statistically significant**
- a p-value < 0.05 is **statistically significant**
- a p-value < 0.01 is **highly statistically significant**

One-sided vs two-sided tests

- one-sided test: what is the probability under the null that the test statistic Y is at least as extreme as the observed value?
- two-sided test: what is the probability under the null that the absolute value of the test statistic is at least as extreme as the observed value?





Statistical vs practical significance

- an effect is statistically significant if it is unlikely to be due to chance
- an effect is practically significant if the observed effect is large enough to be considered important in a clinical or practical sense

a statistically significant result may not be practically significant if the effect size is small.

Statistical vs practical significance

- an effect is statistically significant if it is unlikely to be due to chance
- an effect is practically significant if the observed effect is large enough to be considered important in a clinical or practical sense

a statistically significant result may not be practically significant if the effect size is small.

example:

- medical study tests if a new drug reduces blood pressure
- after data analysis, effect has $p = 0.0126 < .01$
- but estimated effect size is small: 2 mmHg
  (n.b., a cup of coffee can raise blood pressure by 5 mmHg)
- drug side effects: nausea, dizziness, fatigue
- would you recommend the drug?

False positives vs false negatives

- **false positive:** we reject the null hypothesis when it is true
- **false negative:** we fail to reject the null hypothesis when it is false

example: cancer screening based on blood test

- cost of false positive: unnecessary treatment
- cost of false negative: cancer goes undetected

cost is different for different patients, so cutoff should also be different!

Example: prostate cancer screening

PSA (prostate-specific antigen) is a protein produced by the prostate gland

- most men w/o prostate cancer have PSA $<$ 4 ng/ml
- men with PSA between 4 and 10 ng/ml have a 25% chance of having prostate cancer
- men with PSA $>$ 10 ng/ml have a 50% chance of having prostate cancer

Poll: who has a higher cost for a false positive? for a false negative? what cutoff would you use for follow-up testing in

- 80yo patient
- 40yo patient

Outline

Hypothesis testing

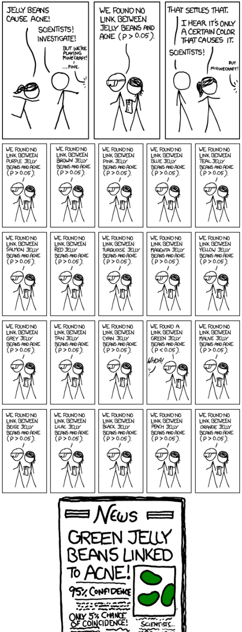
Comparing two samples

Choosing a cutoff

Multiple hypotheses

Summary

Multiple hypothesis testing


JELLY BEANS CAUSE ACNE!
SCIENTISTS! INVESTIGATE!
BUT WE'RE PLAYING MINECRAFT!
...FINE.
WE FOUND NO LINK BETWEEN JELLY BEANS AND ACNE (P > 0.05).
THAT SETTLES THAT.
I HEAR IT'S ONLY A CERTAIN COLOR THAT CAUSES IT.
SCIENTISTS!
BUT MIIIINECRAFT!
WE FOUND NO LINK BETWEEN PURPLE JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN BROWN JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN PINK JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN BLUE JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN TEAL JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN SALMON JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN RED JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN TURQUOISE JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN MAGENTA JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN YELLOW JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN GREY JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN TAN JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN CYAN JELLY BEANS AND ACNE (P > 0.05).
WE FOUND A LINK BETWEEN GREEN JELLY BEANS AND ACNE (P < 0.05).
WHOA!
WE FOUND NO LINK BETWEEN MAUVE JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN BEIGE JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN LILAC JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN BLACK JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN PEACH JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN ORANGE JELLY BEANS AND ACNE (P > 0.05).
NEWS
GREEN JELLY BEANS LINKED TO ACNE!
95% CONFIDENCE
ONLY 5% CHANCE OF COINCIDENCE!
SCIENTISTS...

Multiple hypothesis testing

imagine the experiment:

- scientists divide students into **test** and **control** population

Multiple hypothesis testing

imagine the experiment:

- scientists divide students into **test** and **control** population
- scientists give the test group jellybeans

Multiple hypothesis testing

imagine the experiment:

- scientists divide students into **test** and **control** population
- scientists give the test group jellybeans
- (what do they give the control group?)

Multiple hypothesis testing

imagine the experiment:

- scientists divide students into **test** and **control** population
- scientists give the test group jellybeans
- (what do they give the control group?)
- scientists compare the students' acne levels

Multiple hypothesis testing

imagine the experiment:

- scientists divide students into **test** and **control** population
- scientists give the test group jellybeans
- (what do they give the control group?)
- scientists compare the students' acne levels

- null hypothesis: jellybeans have no effect on acne

Multiple hypothesis testing

imagine the experiment:

- scientists divide students into **test** and **control** population
- scientists give the test group jellybeans
- (what do they give the control group?)
- scientists compare the students' acne levels

- null hypothesis: jellybeans have no effect on acne
- alternative hypothesis: jellybeans have an effect on acne

Multiple hypothesis testing

imagine the experiment:

- scientists divide students into **test** and **control** population
- scientists give the test group jellybeans
- (what do they give the control group?)
- scientists compare the students' acne levels

- null hypothesis: jellybeans have no effect on acne
- alternative hypothesis: jellybeans have an effect on acne
- test statistic: proportion of patients with acne in test vs control group

Demo

`https://colab.research.google.com/github/stanford-mse-125/demos/blob/main/testing.ipynb`

What to do?

if you read a scientific finding, consider

- how many hypotheses do you think they tested to find this result?
- how many similar hypotheses did other research groups test?

What to do?

if you read a scientific finding, consider

- how many hypotheses do you think they tested to find this result?
- how many similar hypotheses did other research groups test?

in your own work, consider methods to control the **false discovery rate** (FDR)

- **Bonferonni correction**: divide the cutoff significance level by the number of hypotheses tested
- . . . many more!

Outline

Hypothesis testing

Comparing two samples

Choosing a cutoff

Multiple hypotheses

Summary

Demo

https://colab.research.google.com/github/stanford-mse-125/demos/blob/main/bootstrap_cheatsheet.ipynb