2. Find the first 2 iterations of

a) Jacobi for

$$3x_1 - x_2 + x_3 = 4$$
$$3x_1 + 6x_2 + 2x_3 = 0$$
$$3x_1 + 3x_2 + 7x_3 = 4$$

$$\Rightarrow \quad x_1 = \frac{1 + x_2 - x_3}{3}$$
$$x_2 = \frac{-x_1}{2} - \frac{x_3}{3}$$
$$x_3 = \frac{4 - 3x_1 - 3x_2}{7}$$

$$\vec{x}^{(0)} = 0$$

$$x_1^{(1)} = \frac{1}{3} \qquad x_2^{(1)} = 0 \qquad x_3^{(1)} = \frac{4}{7}$$

$$x_1^{(2)} = \frac{1 + 0 - \frac{4}{7}}{3} = \frac{1}{7}$$

$$x_2^{(2)} = -\frac{\frac{1}{3}}{2} - \frac{\frac{4}{7}}{3} = -\frac{5}{14}$$

$$x_3^{(2)} = \frac{4 - 1 - 0}{7} = \frac{3}{7}$$

$$2x_1 - 2x_2 + x_3 + x_4 = 0.8 \Rightarrow \quad x_1 = \frac{0.8 + 2x_2 - x_3 - x_4}{2}$$

$$-3x_2 + 0.5x_3 + x_4 = -6.6 \Rightarrow \quad x_2 = \frac{-6.6 - 0.5x_3 - x_4}{-3}$$

$$5x_3 - x_4 = 4.5 \Rightarrow \quad x_3 = \frac{4.5 + x_4}{5}$$

$$2x_4 = 2 \qquad x_4 = 1$$

$$x_1^{(1)} = 0.4$$
$$x_2^{(1)} = -2.2$$
$$x_3^{(1)} = 0.9$$
$$x_4^{(1)} = 1$$

$$x_1^{(2)} = \frac{0.5 + 2(-2.2) - 0.9 - 1}{2} = -5.5$$

$$x_2^{(2)} = \frac{-6.6 - (0.5)(0.9) - 1}{3} = \frac{-8.05}{3} = -2.68333$$

$$x_3^{(2)} = \frac{4.5+1}{5} = 1.1$$

$$x_4^{(2)} = 1$$

b) Gauss - Seidel:

$$x_1 = \frac{1 + x_2 - x_3}{3} \qquad x_2 = \frac{-3x_1 - 2x_3}{6} \qquad x_3 = \frac{4 - 3x_1 - 3x_2}{7}$$

$$x_1^{(1)} = \frac{1}{3}$$

$$x_2^{(1)} = \frac{(-3)(\frac{1}{3}) - 2(0)}{6} = -\frac{1}{6}$$

$$x_3^{(1)} = \frac{4 - 3(\frac{1}{3}) - 3(\frac{-1}{6})}{7} = \frac{1}{2}$$

$$x_1^{(2)} = \frac{1 + \frac{-1}{6} + \frac{1}{2}}{3} = \frac{1}{9}$$

$$x_2^{(2)} = \frac{(-3)(\frac{1}{9}) - 2(\frac{1}{2})}{6} = -2/9$$

$$x_3^{(2)} = \frac{4 - (3)(\frac{1}{9}) - 3(\frac{-2}{9})}{7} = \frac{13}{21}$$

$$x_1 = \frac{0.8 + 2x_2 - x_3 - x_4}{2} \qquad x_3 = \frac{4.5 + x_4}{5}$$

$$x_2 = \frac{-6.6 - 0.5x_3 - x_4}{-3} \qquad x_4 = 1$$

$x_1^{(1)} = 0.4$

$x_2^{(1)} = \frac{-6.6}{3} = -2.2$

$x_3^{(1)} = \frac{4.5}{5} = 0.9$

$x_4^{(1)} = 1$

$x_1^{(2)} = \frac{0.8 + 2(-2.2) - 0.9 - 1}{2} = -5.5$

$x_2^{(2)} = \frac{-6.6 - (0.5)(0.9) - 1}{-3} = 2.6833$

$x_3^{(2)} = \frac{5.5}{5} = 1.1$

$x_4^{(2)} = 1$

c) SOR w/ $w = 1.1$

$$x_1 = \frac{1}{3}(1 + x_2 - x_3) \qquad x_2 = \frac{1}{6}(-3x_1 - 2x_3)$$

$$x_3 = \frac{1}{7}(4 - 3x_1 - 3x_2)$$

$$x_1^{(1)} = (1.1)\left(\frac{1}{3}\right) - (0.1)(0) = 0.36667$$

$$x_2^{(1)} = (1.1)\left(\frac{1}{6}(-3 \cdot 0.36667 - 2(0))\right) + (1 - 1.1)(0) = -0.201667$$

$$x_3^{(1)} = (1.1)\left(\frac{1}{7}(4 - 3(0.36667) - 3(-.201667))\right) + 0 = 0.550784$$

$$x_1^{(2)} = (1.1)\left(\frac{1}{3}(1 + -0.201667 - 0.550784)\right) + (1 - 1.1)(0.36667)$$

$$= 0.054101$$

$$x_2^{(2)} = (1.1)\left(\frac{1}{6}((-3 \cdot 0.054101) - 2 \cdot 0.550784)\right) + (1.0 - 1.1)(-0.201667)$$

$$x_2^{(2)} = -0.2115$$

$$x_3^{(2)} = \left(\frac{1.1}{7}\right)(4 - 3 \cdot 0.054101 - 3 \cdot (-0.2115)) + (1.0 - 1.0)(0.550784)$$

$$= 0.6477$$

$$x_1 = \frac{1}{2}(0.8 + 2x_2 - x_3 - x_4)$$

$$x_2 = -\frac{1}{3}(-6.6 - 0.5x_3 - x_4)$$

$$x_3 = \frac{1}{5}(4.5 + x_4)$$

$$x_4 = 1$$

$$x_1^{(1)} = (1.1)(0.4) + (1.0-1.1)(0) = 0.44$$

$$x_2^{(1)} = (1.1)\left(-\frac{1}{3}\right)(-6.6) = 2.42$$

$$x_3^{(1)} = (1.1)\left(\frac{1}{5}\right)(4.5) = 0.99$$

$$x_4^{(1)} = (1.1)(1) + 0 = 1.1$$

$$x_1^{(2)} = (1.1)\left(\frac{1}{2}\right)(0.8 + 2(2.4) - 0.99 - 1.1) + (1.0-1.1)(0.44) = 1.8865$$

$$x_2^{(2)} = (1.1)\left(-\frac{1}{3}\right)(-6.6 - 0.5 \cdot 0.99 - 1.1) + (1.0-1.1)(2.42) = 2.40167$$

$$x_3^{(2)} = (1.1)\left(\frac{1}{5}\right)(4.5 + 1.1) + (1.0-1.1)(0.99) \approx 1.133$$

$$x_4^{(3)} = (1.1)(1) + (1.0-1.1)(1.1) = 0.99.$$