Thermal Physics Homework #2

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1) A nice way to summarize it is that Microstates are the permutation (ordered) tuples while Macrostates are the combination (unordered) tuples.

2) $N=60,\ p=1/2$

a) $2^N = 2^{60} = (1024)^6 \sim 10^{18}$ Permutations

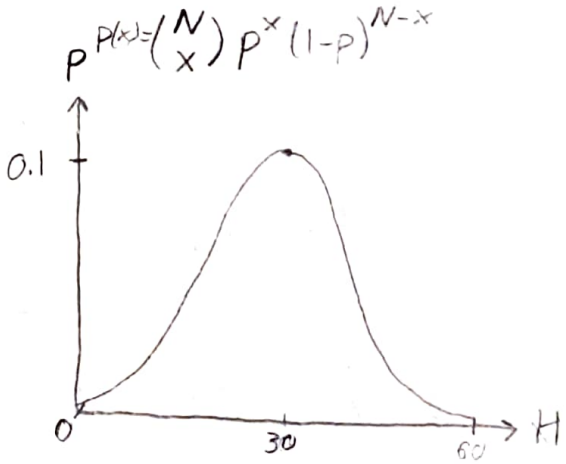
b) $\binom{60}{30} = \frac{60!}{30!\,(60-30)!} \sim 1.183\cdot 10^{17}$ Combinations

c) $p(H=30) = \frac{60!}{(30!)^2} p^{60} = 10.26\%$

d) $p(H=40) = \frac{60!}{40!\,20!} p^{60} = 0.364\%$

e) $p(H=50) = \frac{60!}{50!\,10!} p^{60} \sim 6.5\cdot 10^{-6}\%$

f) $p(H=60) = \frac{60!}{60!\,0!} p^{60} \sim 10^{-16}\%$



Binomial Distribution

3) $N=2,\ E=4$: $S=\{(4,0),(3,1),(2,2),(1,3),(0,4)\}$, $|S|=5$ ✓

$N=2,\ E=3$: $S=\{(3,0),(2,1),(1,2),(0,3)\}$, $|S|=4$ ✓

$N=2,\ E=2$: $S=\{(2,0),(1,1),(0,2)\}$, $|S|=3$ ✓

$N=2,\ E=1$: $S=\{(1,0),(0,1)\}$, $|S|=2$ ✓

$N=1,\ E=E$: $|S| = \frac{(1+E-1)!}{E!\,0!} = \frac{E!}{E!} = \boxed{1}$

$N=N,\ E=1$: $|S| = \frac{(N+1-1)!}{1!\,(N-1)!} = \frac{N!}{(N-1)!} = \boxed{N}$

4) $\Omega(N,E) = \frac{(N+E-1)!}{E!\,(N-1)!}$, $\Omega(25,25) = \frac{49!}{25!\cdot 24!} = \boxed{6.321\cdot 10^{13} \text{ States}}$

5) The probability of microstates is homogenous.
It seems to make sense since there shouldn't be any preferred state for a system to be in.

6) The most probable macrostate depends on the cardinality of its microstates. $P(\text{Macro}) \propto (\text{Micro states} \Rightarrow \text{Macro state})$.

7) Whether addition or multiplication: $N+a \sim N$ & $N\cdot a \sim N$

8) $\Omega = \left(\frac{e}{N}\right)^{2N} \cdot q_A^N q_B^N$, $z = \frac{q_A}{q}$, $1-z = \frac{q_B}{q}$

$= [4z(1-z)]^N$, $0 \le z \le 1$.

Binomial Distribution $\rightarrow$ Gaussian Distribution

$\bar{p} = 1/2$, $\sigma = \sqrt{\frac{1}{8N}} = \frac{1}{2\sqrt{2}}\frac{1}{\sqrt{N}}$

N	1	10	100	1000	10,000
$(2\sqrt{2})s$	1	$\sqrt{1/10}$	$1/10$	$\frac{1}{10}\sqrt{\frac{1}{10}}$	$1/100$

HWM: $\Delta x = \frac{\sqrt{\ln(2)}}{2}\cdot N^{-1/2}$ about $x = 1/2$ (Half-Width Maximum)

1

$N = 22$

0 — 100

2

$(4x(1-x))^N\{0 \le x \le 1\}$

3

$e^{-\frac{(x-\frac{1}{2})^2}{2s^2}}\{0 \le x \le 1\}$

4

$s = \sqrt{\frac{1}{8N}}$

$s = 0.0753778361444$

5

6

$x = \frac{1}{2} + \frac{\sqrt{\ln(2)}}{2}\frac{1}{\sqrt{N}}$

$x = 0.588750619752$

7

$x = \frac{1}{2} - \frac{\sqrt{\ln(2)}}{2}\frac{1}{\sqrt{N}}$

$x = 0.411249380248$