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(9/7/2024)

Electromagnetism Homework #2

Question 1:

b) For $|x| \geq a$, $|\vec{E}| = \frac{kQ}{x^2}$, looking like a point charge.

For $-a < x \leq 0$ & $2b \leq x < b$, $\vec{E} = 0$, which is the inside of Conductor.

For $0 < x < 2b$, $|\vec{E}| = \frac{kQ}{(x-b)^2}$, being a point charge at $x = b$.

a) For the inner surface, $\sigma_{inner} = \frac{Q}{4\pi b^2}$, since the point charge is centered in the internal sphere.

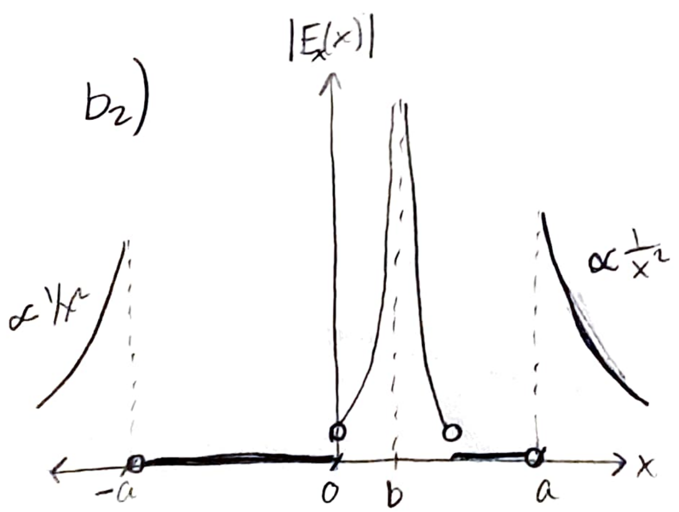
For the outer surface, $\sigma_{outer} = \frac{Q}{4\pi a^2}$, and will always be uniform for any internal charge distribution.

c) For $|x| \geq a$, $V = \frac{kQ}{|x|}$, like a point charge

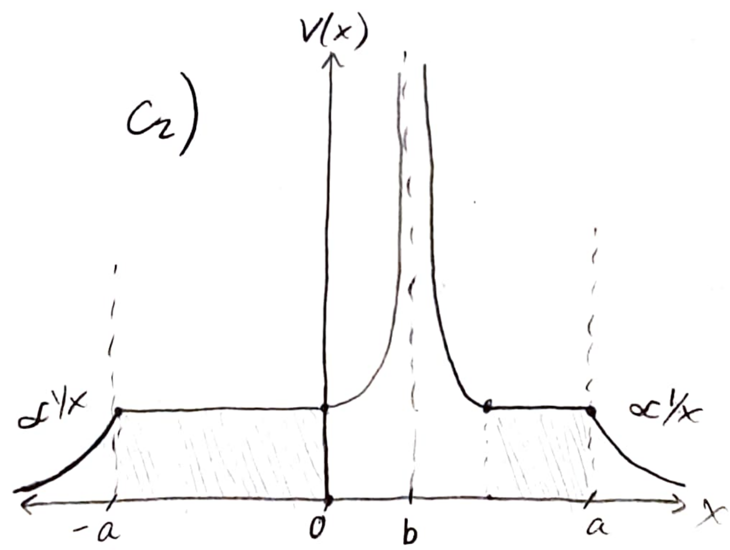
For $-a < x \leq 0$ & $2b \leq x < b$, $V = \frac{kQ}{a}$, uniform voltage inside conductor.

For $0 < x < 2b$, $V = \frac{kQ}{|x-b|}$, a voltage from the point charge inside.

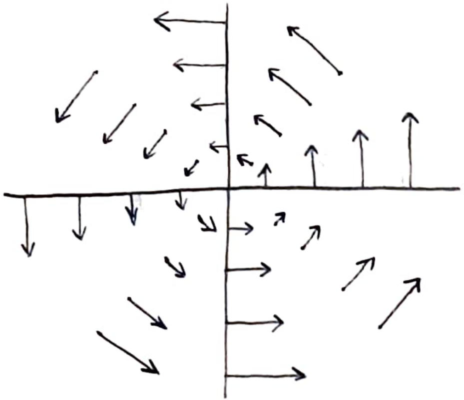
b₂)



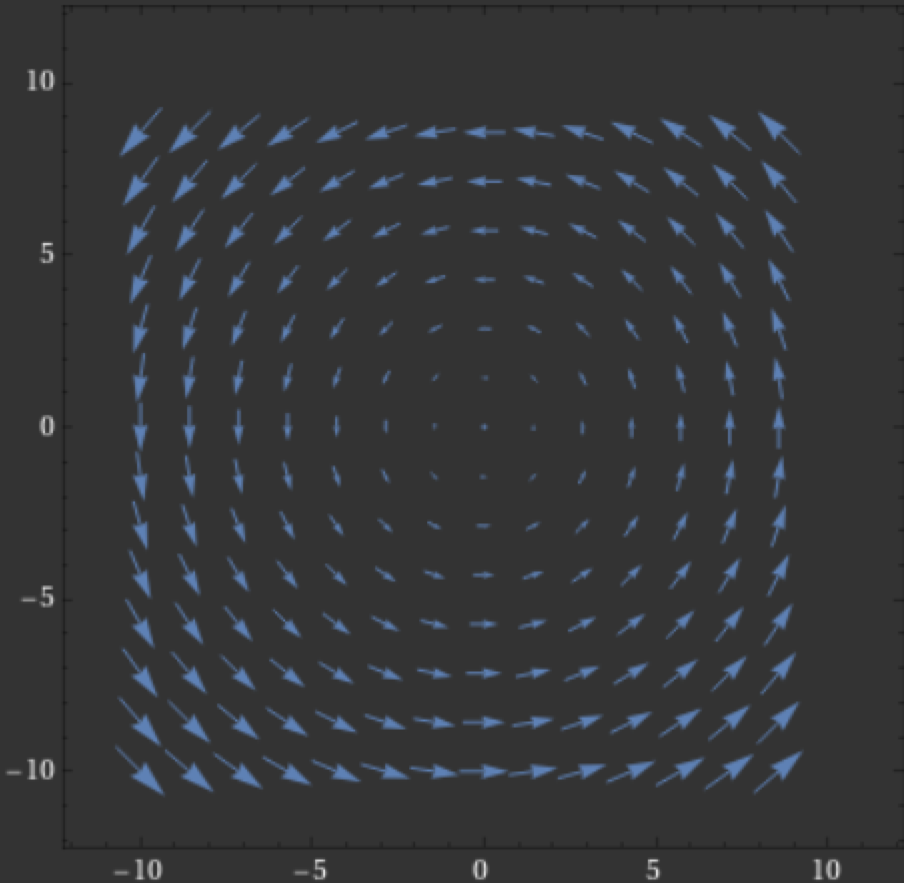
c₂)



$$f(x,y) = -y\hat{x} + x\hat{y} = -R\sin\theta\,\hat{x} + R\cos\theta\,\hat{y} = R(-\sin\theta\,\hat{x} + \cos\theta\,\hat{y}) = r\,\hat{\phi}$$



See Vector Graph Attached


10
5
0
−5
−10
−10
−5
0
5
10

Question 2: Let $\vec{E} = \frac{B_0}{2\tau}(-y\hat{x} + x\hat{y}) \iff \vec{E}(r) = \frac{B_0}{2\tau} r(-\sin\phi\,\hat{x} + \cos\phi\,\hat{y}) = \frac{B_0 R}{2\tau}\hat{\phi}$

a) $[B_0] = \frac{[E][\tau]}{[x]} = \frac{N\cdot s}{C\cdot m} = \boxed{\frac{kg}{C\cdot s}}$

b) $\oint \vec{E}\cdot dl = \frac{B_0 R}{2\tau}\int_0^{2\pi} \hat{\phi}\cdot R\hat{\phi}\, d\phi = \frac{2\pi B_0 R^2}{2\tau} = \boxed{\frac{B_0}{\tau}(\pi R^2)}$

c) $\oint \vec{E}\cdot d\vec{l} = \int_{-a/2}^{a/2}\left(\vec{E}(\tfrac{b}{2},y) - \vec{E}(\tfrac{-b}{2},y)\right)\cdot\hat{y}\,dy + \int_{-b/2}^{b/2}\left(\vec{E}(x,-\tfrac{a}{2}) - \vec{E}(x,\tfrac{a}{2})\right)\cdot\hat{x}\,dx$

$= \frac{B_0}{2\tau}\left(\frac{b\cdot a}{2} + \frac{b\cdot a}{2} + \frac{a\cdot b}{2} + \frac{a\cdot b}{2}\right) = \frac{4B_0}{4\tau}(ab) = \boxed{\frac{B_0}{\tau}(ab)}$

d) $\vec{\nabla}\times\vec{E} = -\begin{vmatrix} i & j & k \\ E_x & E_y & E_z \\ \partial_x & \partial_y & \partial_z \end{vmatrix} = -\left(\frac{\partial(-y)}{\partial y} - \frac{\partial(x)}{\partial x}\right)\hat{z} = \boxed{2\hat{z}}$

The curl is a uniform vector field pointing 2 up everywhere.

$\oint_{\partial A} \vec{E}\cdot dl = \int(\vec{\nabla}\times\vec{E})d\vec{A} = \frac{B_0}{2\tau}(2\hat{z})\int d\vec{A} = \frac{B_0}{\tau}\cdot A\cdot\hat{z}^2 = \boxed{\frac{B_0}{\tau}(A)}$ ✓

e) There is no $\phi \mid \vec{E} = -\vec{\nabla}\phi$, since $\vec{\nabla}\times\vec{E} \neq 0$.

We could create infinite energy with a field like this.
Any loop we have a charge move around will gain kinetic energy, so infinite power $\Rightarrow$ Loads of \$.

$\vec{\nabla}\times\vec{E} = -\mu_0\vec{J}_m - \frac{\partial\vec{B}}{\partial t}$ Assuming no changing $\vec{B}$ fields, it would require magnetic monopole current.

Statically, nothing is being violated.
However, it would need something like magnetic monopoles, so technically $\vec{\nabla}\cdot\vec{B} = 0$ is violated.

Question 3:

a) $V = \int_{-d/2}^{d/2} \frac{k\,dq}{r} = \int_{-d/2}^{d/2} \frac{k\lambda}{(x^2+y^2)^{1/2}} \cdot dy = 2k\lambda \int_{0}^{d/2} \frac{dy}{(x^2+y^2)^{1/2}}$

$= 2k\lambda \tanh^{-1}\left(\frac{y}{\sqrt{x^2+y^2}}\right)\Big|_0^{d/2} = 2k\lambda \tanh^{-1}\left(\frac{d/2}{\sqrt{x^2+\left(\frac{d}{2}\right)^2}}\right)$

$$\boxed{V(x) = 2k\lambda \tanh^{-1}\left(\frac{d}{2\sqrt{x^2+(d/2)^2}}\right)}$$

b) $E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x} V(x) = -2k\lambda \frac{\partial}{\partial x}\left[\tanh^{-1}\left(\frac{d}{2\sqrt{x^2+(d/2)^2}}\right)\right]$

$= -2k\lambda \cdot \frac{d/2}{x\sqrt{x^2+(d/2)^2}} = \frac{-k\lambda d}{x\sqrt{x^2+(d/2)^2}}$

$$\boxed{E_x(x) = \frac{-k\lambda d}{x\sqrt{x^2+(d/2)^2}}}$$

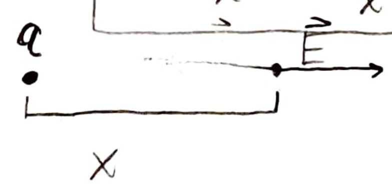
c) $\lim_{x \gg d} E_x(x) = \lim_{x \gg d} \frac{-k\lambda d}{x\sqrt{x^2}} = -\frac{k\lambda d}{x^2}$ $\qquad \boxed{\lim_{x \gg d} E_x(x) = -\frac{k\lambda d}{x^2}}$

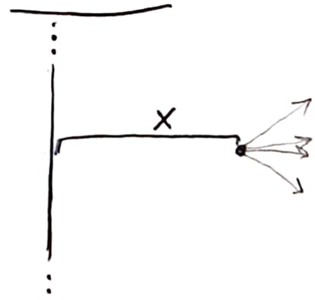
$\lim_{d \gg x} E_x(x) = \lim_{d \gg x} \frac{-k\lambda d}{x\sqrt{\left(\frac{d}{2}\right)^2}} = -\frac{2k\lambda d}{xd}$ $\qquad \boxed{\lim_{d \gg x} E_x(x) = -\frac{2k\lambda}{x}}$

d) $x \gg d$ $\qquad\qquad$ $d \gg x$

$$\boxed{\vec{E} = -\frac{kQ}{x^2} = -\frac{k(\lambda d)}{x^2}}$$

$\sim$

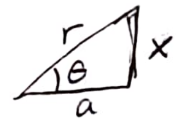




For small x looks like $d \to \infty$.

$$\boxed{\vec{E}_x \propto \frac{\lambda}{x}}$$

Question 4



a) $$\frac{\partial B(x)}{\partial x} = \frac{\mu_0}{4\pi}\int \frac{I\,d\vec{l}\times\hat{r}}{r^2} = \frac{\mu_0 I}{4\pi}\int \frac{dl}{r^2}\cos\theta$$

$$= \frac{\mu_0 I}{4\pi}\int_0^{2\pi} \frac{a\,d\phi}{a^2+x^2}\cdot\frac{a}{\sqrt{a^2+x^2}} = \frac{\mu_0 I}{4\pi}\cdot\frac{a^2}{(a^2+x^2)^{3/2}}\cdot 2\pi$$

$$\frac{\partial B(x)}{\partial x} = \frac{\mu_0 I}{2}\cdot\frac{a^2}{(a^2+x^2)^{3/2}}$$

$$B(x) = \int_{x-L/2}^{x+L/2} \frac{\mu_0 I}{2}\cdot\frac{a^2}{(a^2+x^2)^{3/2}}\,dx = \frac{\mu_0 I}{2}\left(\frac{x}{\sqrt{a^2+x^2}}\right)\Bigg|_{x-L/2}^{x+L/2}$$

$$\boxed{B(x) = \frac{\mu_0 I}{2}\left(\frac{x+L/2}{\sqrt{a^2+(x+L/2)^2}} - \frac{x-\frac{L}{2}}{\sqrt{a^2+(x-L/2)^2}}\right)}$$

$$\lim_{L\to\infty} B(x) = \frac{\mu_0 I}{2}\left(1-(-1)\right) = \frac{2\mu_0 I}{2} \qquad \boxed{\lim_{L\to\infty} B(x) = \mu_0 I}$$

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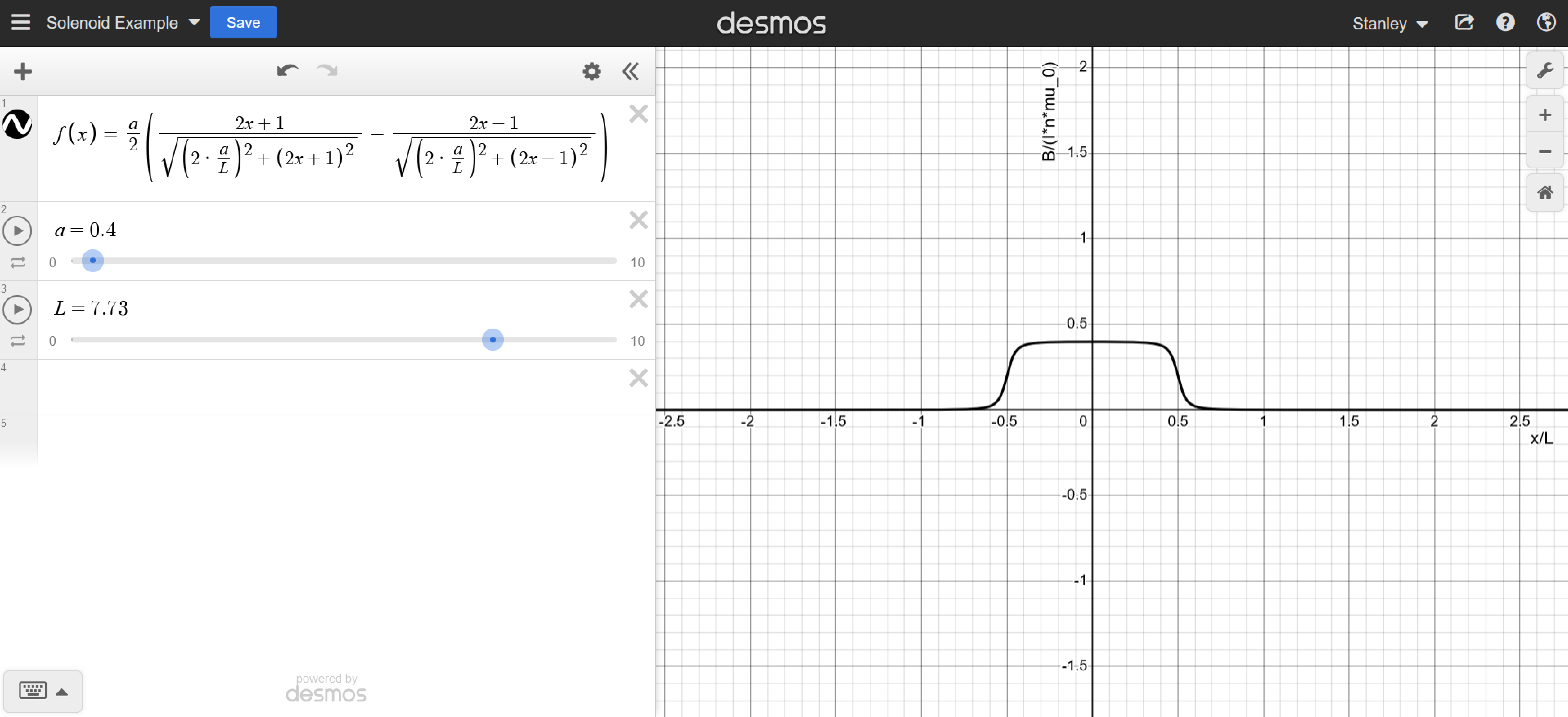
b) Examples attached!


Solenoid Example
Save
desmos
Stanley
$f(x) = \frac{a}{2}\left(\frac{2x+1}{\sqrt{\left(2\cdot\frac{a}{L}\right)^2+(2x+1)^2}} - \frac{2x-1}{\sqrt{\left(2\cdot\frac{a}{L}\right)^2+(2x-1)^2}}\right)$
$a = 0.4$
0
10
$L = 7.73$
0
10
powered by desmos
B/(I*n*mu_0)
x/L
2
1.5
1
0.5
-0.5
-1
-1.5
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5

Solenoid Example Save desmos Stanley

1
$$f(x)=\frac{a}{2}\left(\frac{2x+1}{\sqrt{\left(2\cdot\frac{a}{L}\right)^2+(2x+1)^2}}-\frac{2x-1}{\sqrt{\left(2\cdot\frac{a}{L}\right)^2+(2x-1)^2}}\right)$$

2
$a=7$

0 10

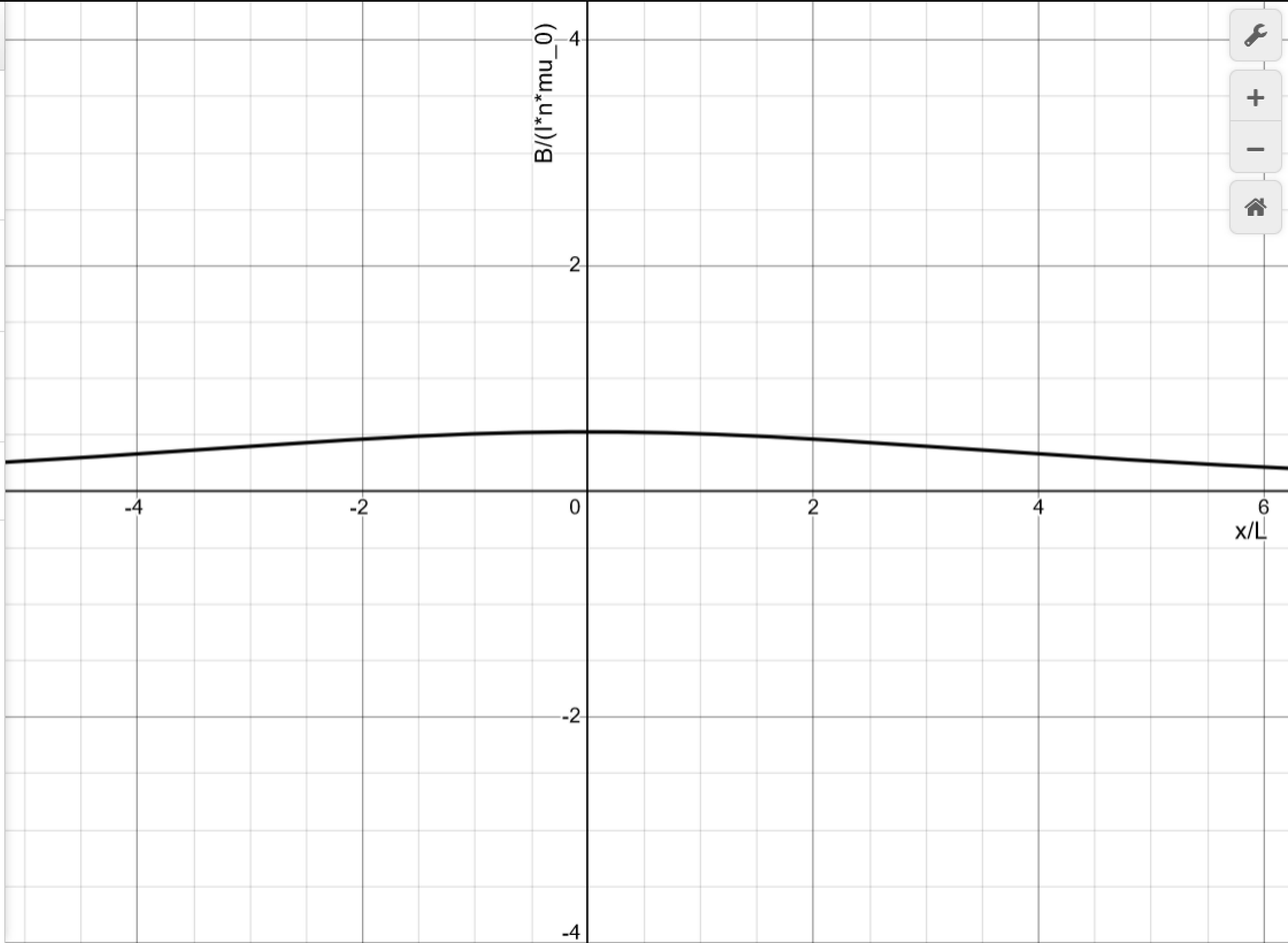
3
$L=1.06$

0 10

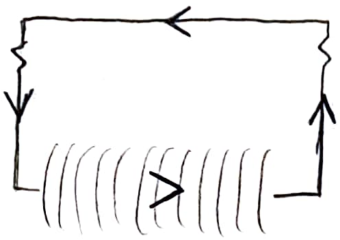
4

5

powered by desmos



c) Show that the solution :



$$\oint_\gamma \vec{B}\cdot d\vec{l} = \int_{-\infty}^{\infty} \vec{B}(x)dx + 0 + 0 + 0$$
$$= \mu_0 I$$

The $\perp$ terms, would equal and cancel.
The line at ∞ is effiecly ∞ distance from the B-field ($1/r$ decay).

d) $|\vec{B}| = 7\,T$. Supposing an infinitly long MRI,

$$\vec{B} = \mu_0 I a \Rightarrow I = \frac{\vec{B}}{\mu_0 a}$$

The current has to be very very large.
Sending so much current through copper wire would definitely melt it in seconds.

Moreover, as T increases, the resistivity of copper increases, and that means it would limit current.

$$I \sim \frac{\overset{|\vec{B}|}{10}}{\underbrace{10^{-6}}_{\mu_0}\cdot\underbrace{10^{0}}_{\text{radius}}} \quad \boxed{\sim 10^7 A}$$

We've gotten around this problem with superconducting coils.
When I was talking to some folks at Boston Memorial Hospital, the technician said it was liquid helium & Niobium-Titanium coils.
There's copper in there somewhere, but it is 100% superconductors.