Math Methods Homework #3

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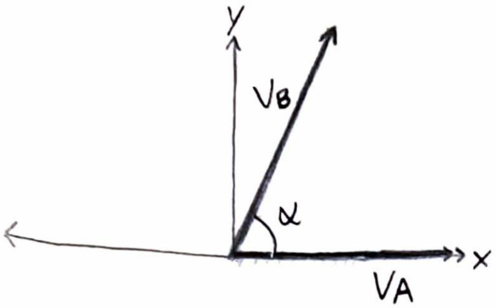


1) Let $f(z) = A\ln(z) + B$. Since $\phi \in [0, \alpha]$ and $0 < \alpha < \pi$,
then $0 \leq \phi < \pi$.

For the choice of $\ln(z)$ is on the principle branch $\alpha \in (-\pi, \pi)$.

Since we don't include $\alpha = \pi$, we make sure the region between the plates don't have any branch-cut discontinuities, and so will be holomorphic on the region between the plates.

2) $f(re^{i\phi}) = A\ln(re^{i\phi}) + B = A\ln r + Ai\phi + B = \overbrace{(A\ln r + B)}^{u(r,\phi)} + i\overbrace{(A\phi)}^{v(r,\phi)}$

Boundary Conditions: $f(r, \alpha) = V_B$ & $f(r, 0) = V_A$

$f(re^{i\alpha}) = (A\ln(r) + B) + i(A\alpha) = V_A + i(A\alpha) = V_B \Rightarrow A = i\left(\frac{V_A - V_B}{\alpha}\right)$

$f(re^{i0}) = A\ln(r) + B = V_A \Rightarrow B = V_A - i\left(\frac{V_A - V_B}{\alpha}\right)\ln(r)$

$$f(z) = i\left(\frac{V_A - V_B}{\alpha}\right)\ln(z) + V_A - i\left(\frac{V_A - V_B}{\alpha}\right)\ln(r)$$

$$= \left(\frac{V_A - V_B}{\alpha}\right)\left(i\ln(r) - \phi - i\ln(r)\right) + V_A$$

$$\boxed{f(re^{i\phi}) = \frac{V_B - V_A}{\alpha}\phi + V_A}$$ ← Independent of r → uniform on conductors

2) $f(re^{i\phi}) = A\ln(re^{i\phi}) + B = A\ln(r) + Ai\phi + B$

Boundary Conditions : $Re(f(r,\alpha)) = V_B$ & $Re(f(r,0)) = V_A$

$Re(f(re^{i\alpha})) = Re(A\ln(r) + Ai\phi + B) = V_A \Rightarrow A = \left(\frac{V_A - V_B}{\alpha}\right)i$

$Re(f(re^{i0})) = Re(A\ln(r) + B) = V_A \Rightarrow Re(B) = V_A$

$\boxed{Re(f(z)) = \frac{V_B - V_A}{\alpha}\phi + V_A}$ & $\boxed{Real(f(z)) = u(x,y)}$

1 $a = 0.85$

0 — π

2 $V_B = 4$

−10 — 10

3 $V_A = -2.4$

−10 — 10

4 $y = \tan(a)x\{y > 0\}$

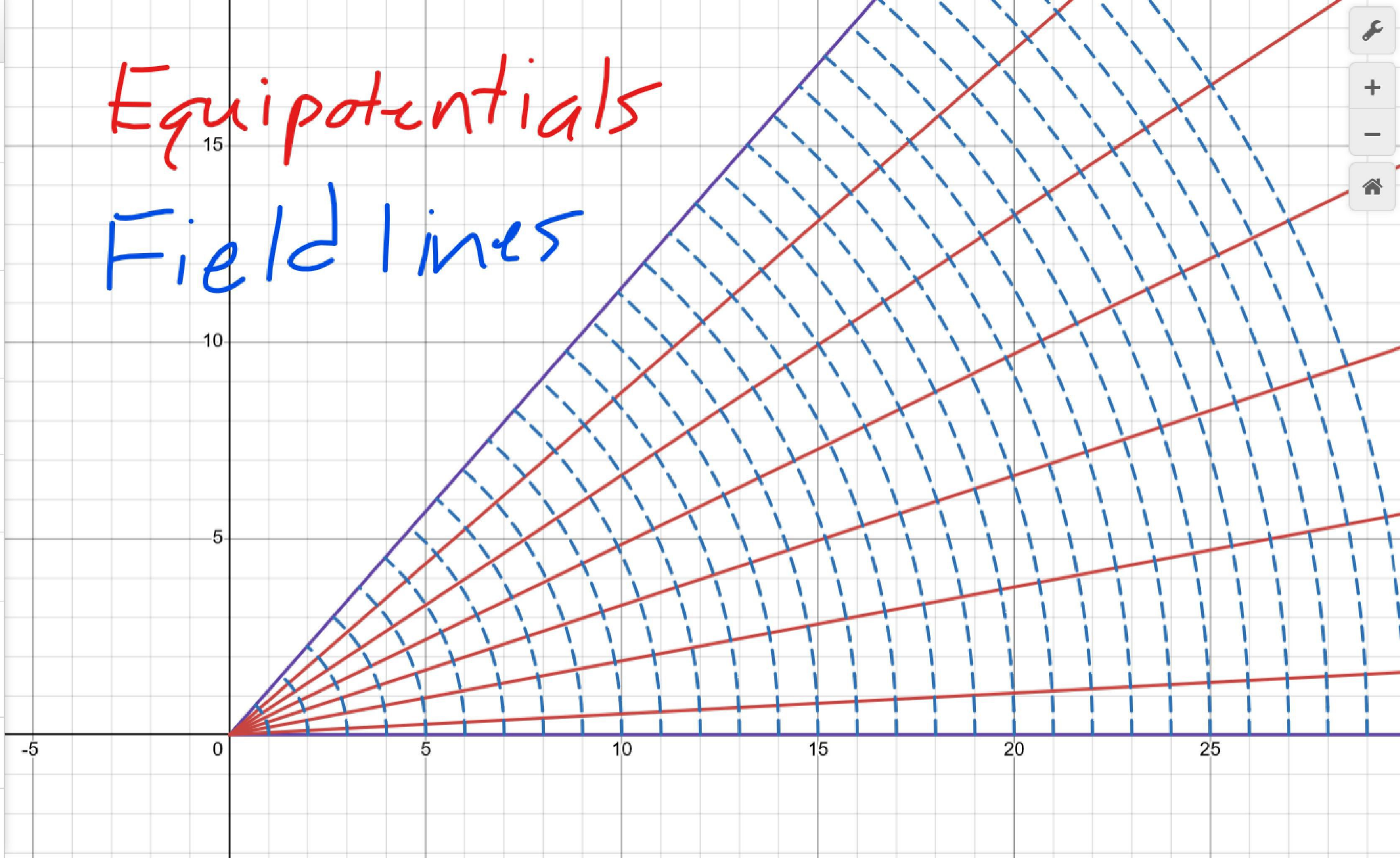
5 $y = 0\{x \geq 0\}$

6 Empty note

7 $y = \tan\left(a\frac{V - V_A}{V_B - V_A}\right)x\{y > 0\}$

8

9 $= [V_B - 1, V_B - 2, \ldots, V_A]$



1 $a = 2.68$

0 — π

2 $V_B = 4$

−10 — 10

3 $V_A = -2.4$

−10 — 10

4 $y = \tan(a)x\{y > 0\}$

5 $y = 0\{x \geq 0\}$

6 Empty note

7 $y = \tan\left(a\frac{V - V_A}{V_B - V_A}\right)x\{y > 0\}$

8

9 $= [V_B - 0.5, V_B - 1, \ldots, V_A + 0.5]$