Graph Theory October 10, 2016

1. Consider an undirected graph $G =< V, E >$ and $L = L(G)$ — the Laplacian matrix of the graph G.

   Sequence $\lambda_1(L) \geq \lambda_2(L) \geq \ldots \geq \lambda_n(L) = 0$ is a specturm of the graph G.

   Find

   - upper bound of $\lambda_1(L)$ (such $\lambda_1(L) \leq f(|V|, |E|)$);
   - is there a graph G such that the obtained upper bound is unimprovable?

2. Formulate the criterion of graph's bipartiteness in terms of a spectrum of Laplacian matrix and/or adjacency matrix.

3. Prove that $\forall x, x^T Lx = tr(Lxx^T) = \sum_{i,j} L_{i,j} x_i x_j$.

4. Consider a clique K_n. Is it true that $\forall n \geq 3, K_n$ — is a Hamiltonian graph.

5. Consider clique $K_n, \forall n \geq 4$. Is it true that if any independent set I is droped from the K_n, then $K_n \setminus I =< V, E \setminus I >$ is a Hamiltonian graph.

6. Assume that G is a graph such that all the edges of G belongs to a certain cut. Prove that maximal cut equals to the optimal value of corresponding semidefinite relaxation.