**The Doomed Dice Challenge - Solution**

**Part A**

1. A dice has 6 outcomes. Having two dice rolled together gives an output of 36 possible combinations since we can multiply 6 possible outcomes of dice A and 6 possible outcomes of dice B. Therefore, sample space = **6 \* 6 = 36**
2. This is the sample space of rolling two dice:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| (1,1) | (1,2) | (1,3) | (1,4) | (1,5) | (1,6) |
| (2,1) | (2,2) | (2,3) | (2,4) | (2,5) | (2,6) |
| (3,1) | (3,2) | (3,3) | (3,4) | (3,5) | (3,6) |
| (4,1) | (4,2) | (4,3) | (4,4) | (4,5) | (4,6) |
| (5,1) | (5,2) | (5,3) | (5,4) | (5,5) | (5,6) |
| (6,1) | (6,2) | (6,3) | (6,4) | (6,5) | (6,6) |

To find the distribution of all the possible combinations, we can use a map to find the total sum existent with its occurrences.

|  |  |
| --- | --- |
| Sum | Frequency |
| 2,12 | 1 |
| 3,11 | 2 |
| 4,10 | 3 |
| 5,9 | 4 |
| 6,8 | 5 |
| 7 | 6 |

1. Based on the above distribution, we can find the probability of all possible sums among the combinations by dividing the occurrences with total number of outcomes i.e., 36.

|  |  |
| --- | --- |
| Sum | Probability |
| 2 | 1/36 |
| 3 | 2/36 |
| 4 | 3/36 |
| 5 | 4/36 |
| 6 | 5/36 |
| 7 | 6/36 |
| 8 | 5/36 |
| 9 | 4/36 |
| 10 | 3/36 |
| 11 | 2/36 |
| 12 | 1/36 |

**Part B**

The goal of this challenge is to ensure that the probabilities are not changed despite the new conditions introduced for dice A and dice B.

* The constraints for dice A – **1 <= x < 4** (multiple elements are allowed)
* The constraints for dice B – **1 <= x < infinity**

We can perform an exhaustive search to find out suitable pairs that contain sums and its probabilities that are equivalent to the standard dices A and B.

**Steps:**

1. Find the sum distribution of the original pairs. Sum distribution is sorted to avoid repeated permutations found.
2. Compute all possible combinations of dices for a 6-sided dice. It needs to loop through each possible value for each of the die, starting from 1 and going up to 12 and do this total 5 iterations to obtain 6 elements within a potential die.
3. Generate all the possible pairs of die where the first die ‘A’ has values less than or equal to the second die ‘B’ to avoid repeated permutations i.e.; (A, B) recorded but not (B, A).
4. Iterate through each pair and ignore potential die\_A pair if it contains value that is greater than 4.
5. If the selected pair’s sum distribution is equal to the standard die’s sum distribution and the pair is not equivalent to the original pair i.e.; not the same pair as the original, then add this pair to the matching pairs list result.
6. Do this for all pairs till completion.
7. Return the matching pairs.

**Analysis:**Since exhaustive search is performed to seek potential pairs and obtain a sum distribution that is equivalent to the standard die, it is facing a O(n^4) + O(n^2) + O(n^2) + O(n^5) time complexity which is ultimately **O(n^5)**.

In terms of space complexity, the generated pairs need to be stored leading to O(n^2) where n is the number of sides of the dice. The product of multiplying sides leads to O(n^2) space complexity produced and finally, the number of generated pairs need to match the original sums distribution, which in the worst case, leads to O(n^2). Therefore, the overall space complexity is O(n^2) + O(n^2) + O(n^2) = **O(n^2)**.