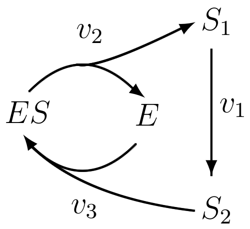
Box 1. Reaction Network Consider the simple reaction network shown on the left below:



$$
\begin{array}{cc}
 & \begin{array}{ccc} v_1 & v_2 & v_3 \end{array} \\
\begin{array}{c} ES \\ S_1 \\ S_2 \\ E \end{array} & \left[\begin{array}{rrr} 0 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{array}\right]
\end{array}
$$

The **stoichiometry matrix** for this network is shown to the right. This network possesses two conserved cycles given by the constraints: $S_1 + S_2 + ES = T_1$ and $E + ES = T_2$. The set of independent species includes: $\{ES, S_1\}$ and the set of dependent species $\{E, S_2\}$.

The $\boldsymbol{L_0}$ matrix can be shown to be:

$$
\boldsymbol{L_0} = \begin{bmatrix} -1 & -1 \\ -1 & 0 \end{bmatrix}
$$

The complete set of equations for this model is therefore:

$$
\begin{bmatrix} S_2 \\ E \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} ES \\ S_1 \end{bmatrix} + \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}
$$

$$
\begin{bmatrix} dES/dt \\ dS_1/dt \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}
$$

Note that even though there appears to be four variables in this system, there are in fact only two independent variables, $\{ES, S_1\}$, and hence only two differential equations and two linear constraints.