Physics 1C

2023-24
Winter 24

Instructor: Jay Hauser
Textbook: Young & Freedman - Mastering Physics
Topics: Magnetism, inductance, RLC circuits, AC circuits; Maxwell's equations, E&M waves, geometrical optics, interference & diffraction; special relativity

Table of Contents

1) Magnetism - 63
   i) Magnetic Force - 63
   ii) Biot-Savart Law - 66
   iii) Ampere's Law - 67
   iv) Magnetic Induction - 71
2) RLC & AC Circuits - 75
   i) Inductors & LC Circuits - 75
   ii) RLC Circuits - 76
   iii) AC Circuits - 78
3) E&M Waves - 81
   i) Maxwell's Equations - 81
   ii) E&M Plane Waves - 82
4) Geometrical Optics - 84
   i) Reflection & Refraction - 84
   ii) Polarization - 86
   iii) Mirrors - 88
   iv) Lenses & Magnification - 89
   v) Applications of Lenses - 91
5) Interference & Diffraction - 92
   i) 2-Slit Interference - 92
   ii) Thin-Film Interference - 93
   iii) Diffraction - 94
6) Special Relativity - 97
   i) Galilean Relativity - 97
   ii) Einsteinian Relativity - 98
   iii) Relativistic Momentum - 102
   iv) Relativistic Energy - 103

Magnetic Forces

1/9/24

Lecture 1

Magnetic Force Law

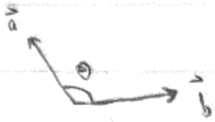
Recall (Electric force law): $\vec{F} = q\vec{E}$

Can define an analogous force law for magnetic fields [denoted $\vec{B}$]: $\boxed{\vec{F} = q\vec{v} \times \vec{B}}$

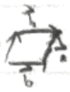
(*) Cross Product

Recall: Given two vectors $\vec{a}, \vec{b}$: i) $(\vec{a}\times\vec{b}) \perp \vec{a}$ and $(\vec{a}\times\vec{b}) \perp \vec{b}$ ← use right-hand rule to find dir.

ii) $|\vec{a}\times\vec{b}| = |\vec{a}|\cdot|\vec{b}|\sin\Theta$

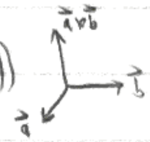


↳ geometrically: $|\vec{a}\times\vec{b}|$ = area



Explicitly: Writing $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ (and similarly for $\vec{b}$)

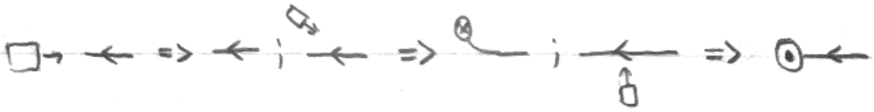
$$\rightarrow \vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} \quad (\text{ex: } (1,0,0)\times(0,1,0) = (0,0,1))$$



(*) Notation: $\odot$ = out of page (arrow "tip"); $\otimes$ = into page (arrow "shaft"); $|\vec{a}| = a$

→ Consequence: A magnetic force is only exerted on moving charges (e.g. in a current)

Ex: Electron beam w/ rightward current ← ; magnet w/ outward field



(*) Demo: Magnet on DC circuit [constant current] → constant force on circuit

Magnet on AC circuit [current oscillates] → oscillating force on circuit

(*) Particle Motion w/ Constant $\vec{B}$

Ex (i): $\vec{B}$ parallel to $\vec{v}$ => $\vec{v}\times\vec{B} = 0$, $\vec{F} = 0$ (motion unaffected)

Ex (ii): $\vec{B}$ perpendicular to $\vec{v}$ => $|\vec{F}| = qvB$ (motion curves)

(*) Ex: $v_0 = \uparrow$, $\vec{B} = \odot$, $q = \oplus$ ↳

[Circular motion]

radius: $\boxed{F = \frac{mv^2}{r}} \rightarrow r = \frac{mv}{|q|B}$

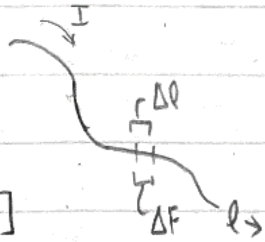
Magnetic Force & Current

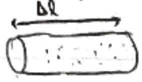
1/17/24

Lecture 2

Magnetic Force on a Current

Applications: Magnetic field effects are typically studied w.r.t. currents



Current:  Δl (n particles w/ velocity $\vec{v}$) → define $\vec{\Delta l} = \Delta l$ in the direction of $\vec{v}$

$$\vec{v} = \frac{dx}{dt} \to \text{define } \Delta t = \frac{\Delta l}{v} \to I = \frac{\text{charge}}{\text{time}} = \frac{nq}{\Delta t} = \frac{nqv}{\Delta l} \quad [\text{current}]$$

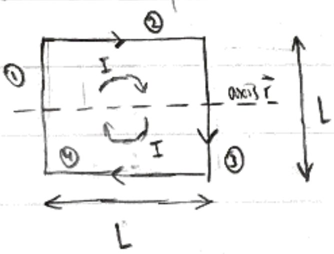
$$\Delta F = n(q\vec{v} \times \vec{B});\ \vec{v} = \left(\frac{\vec{\Delta l}}{\Delta l}\right)v \to \Delta F = nqv\frac{\vec{\Delta l}}{\Delta l} \times \vec{B} \to \Delta F = I(\vec{\Delta l} \times \vec{B})$$

$$\sim \vec{F}_{tot} = \int_l \Delta F \to \boxed{\vec{F} = \int I\, \vec{dl} \times \vec{B}} \quad [\text{integrated along the wire}]$$

(*) I constant → $F = I\int \vec{dl} \times \vec{B}$

(*) Straight wire, constant $\vec{B}$ → $\vec{F} = I\vec{L} \times \vec{B}$

Current Loops



$\vec{\tau} = \vec{r} \times \vec{F}$ (in dir. of rotational axis)

$\sim$ Want to find torque $\vec{\tau}$, force $\vec{F}$ from constant $\vec{B}$

(first: find $\vec{F}$ on individual faces ①, ②, ③, ④)

$\vec{B}$ dir.	$\vec{F}$: ①	②	③	④	$\vec{F}_{net}$	$\vec{\tau}$	
⊗	←	↑	→	↓	∅	∅	~ ($\vec{B}$ acts as an expansive force)
↑	∅	⊙	∅	⊗	∅	LF[·↑]	

Define magnetic moment: $\boxed{\mu = IA}$ ~ A = area of loop [n loops → $\mu = nIA$]

→ $\boxed{\vec{\tau} = \vec{\mu} \times \vec{B}}$ ~ find dir. of $\vec{\mu}$ via right-hand rule: curl fingers around loop in dir. of current

(*) Permanent Magnets

1/11/24
Lecture 2
(Cont.)

Permanent Magnets

Solids composed of atoms; atoms contain electrons orbiting nuclei.

→ View electrons as small charges moving in a loop

→ each $\bar{e}$ has a magnetic moment $\vec{\mu}_{atom}$.

Usual case: $\vec{\mu}_{atom}$'s inside a substance are unaligned, cancel out

(!) Special case: rotations of electrons are aligned → $\vec{\mu}_{tot} = \#\text{ atoms} \cdot \vec{\mu}_{atom}$ [adds up]

→ entire substance gains a (potentially significant) magnetic moment $\vec{\mu}_{tot}$

↳ Occurs in permanent magnets

(*) Magnetic field alignment can also be induced, e.g. via high temperatures
[μ alignment is a high-energy state; need thermal E > atomic E to create]

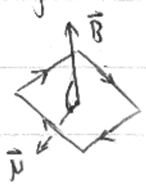
(*) Ferromagnets, once aligned, can be made to stay aligned [magnetized]

↳ magnetization places ferromagnet into a crystal shape / lattice

(*) Motors [Preview]

The torque generated by constant $\vec{B}$ on a current loop works to bring $\vec{\mu}$ to align with $\vec{B}$

Usual case: μ goes partway (halfway, e.g.), then position stabilizes



→ Motors contain a commutator: on reaching the halfway, current flips direction [$\vec{\mu}$ flips]

→ torque stays in same direction, loop continues moving in same dir. (past halfway)

Conversely: spinning a motor w/o current creates a current [generators - Faraday's Law]

Biot-Savart Law

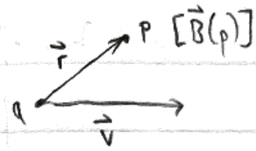
1/16/24

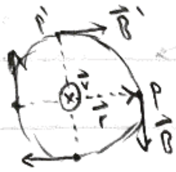
Lecture 3

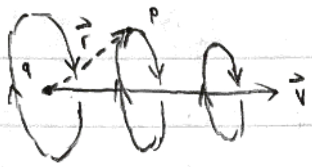
(*) Potential energy from a magnetic field: $\boxed{U = -\vec{\mu} \cdot \vec{B}}$

Biot-Savart Law (Moving Charge)

Moving charges create magnetic fields → Biot-Savart Law (i): $\boxed{\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}}$







(*) $\vec{v}, \hat{r}$ closer to parallel → smaller $\vec{B}$

Constant: $\boxed{\mu_0 = 4\pi \cdot 10^{-7}\ T \cdot m/A}$ $\longrightarrow \frac{\mu_0}{4\pi} = 10^{-7}\ T \cdot m/A$ (*) Obs.: μ_0 is very small

[Consequence: $|B| \ll |E|$, typically]

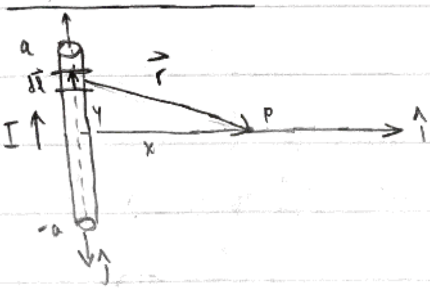
Biot-Savart Law (Current)

Similar to Mag. Force Law:

→ Biot-Savart Law (ii): $\boxed{d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{\ell} \times \hat{r}}{r^2}}$ $\rightarrow \vec{B} = \int_{wire} d\vec{B}$

(*) μ_0 small ⇒ requires large current

(*) Ex (Finite Wire)



$\vec{r} = \langle x, y \rangle = x\hat{i} - y\hat{j}$, $r = (x^2+y^2)^{1/2}$, $\hat{r} = \vec{r}/r$; $d\vec{\ell} = dy \cdot \hat{j}$

$$\rightarrow d\vec{B} = \frac{\mu_0}{4\pi} I \frac{(dy \cdot \hat{j}) \times (x\hat{i} - y\hat{j})}{(x^2+y^2)^{3/2}} = \frac{\mu_0}{4\pi} I \frac{(x\,dy(-\hat{k}))}{(x^2+y^2)^{3/2}}$$

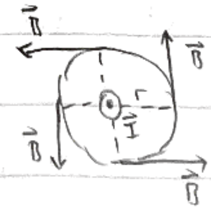
$$\rightarrow B = \int_{-a}^{+a} d\vec{B} = \int_{-a}^{a} -\frac{\mu_0 I x}{4\pi} \hat{k} \int_{-a}^{a} \frac{dy}{(x^2+y^2)^{3/2}}$$; replace "x" with "r"

[$y = r\tan\theta$]

$$\rightarrow \boxed{\vec{B} = -\hat{k} \frac{\mu_0 I}{4\pi} \left[\frac{2a}{r\sqrt{r^2+a^2}} \right]}$$ [Finite Wire]

(*) Ex (Infinite Wire)

$a \to \infty\ [r \ll a] \Rightarrow \frac{2a}{\sqrt{r^2+a^2}} \to 1 \Rightarrow \boxed{\vec{B} = -\hat{k} \frac{\mu_0 I}{2\pi r}}$



Ampere's Law

1/16/24
Lecture 3 (cont.)

(*) Ex: Force Between Parallel Wires

Force between 2 parallel wires w/ currents I_1, I_2: $I_1 \odot \cdots \odot I_2$ ↖ (*) I_2 has no force on itself

Force of I_1 on I_2: $F_2 = I_2 \vec{L} \times \vec{B} = I_2 L \cdot \frac{\mu_0 I_1}{2\pi r} \rightarrow \boxed{F = \left(\frac{\mu_0}{2\pi}\right) \frac{I_1 I_2}{r} L}$ [small]

↳ same dir. → attractive
opposite dir. → repulsive

(*) Can define 1 ampere as requisite I to produce a certain F/L (given fixed r)

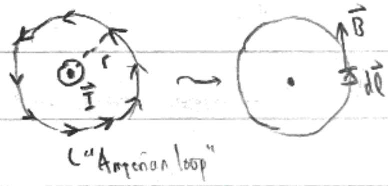
Ampere's Law

Recall (Electricity): $\vec{E} = \frac{q}{4\pi\varepsilon_0} \frac{\hat{r}}{r^2}$ [E. field] $\longleftrightarrow$ $\oiint \vec{E} \cdot d\vec{A} = \frac{q_{encl}}{4\pi\varepsilon_0}$ [Gauss's Law]

Can define similar analogue for magnetism:

$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times \hat{r}}{r^2}$ [Biot-Savart] $\longleftrightarrow$ Ampere's Law: $\boxed{\oint_{loop} \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}}$ ↖ (*) #3 of Maxwell's equations

(*) Ex: Long Wire



$\oint \vec{B} \cdot d\vec{l} \overset{\vec{B} \parallel d\vec{l}}{=} \oint B \cdot dl \overset{B \text{ const.}}{=} B \oint dl = 2\pi r B$

$\rightarrow 2\pi r B = \mu_0 I_{encl} \Rightarrow \underline{\vec{B} = \frac{\mu_0 I}{2\pi r}}$. [same result as before]

(*) Ex: Finite Thickness Wire

Wire w/ thickness R:

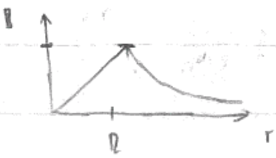
↖ (*) Unlike static conductors, current flows through all parts of the wire (incl. inside), not just the surface

$\vec{B}$ for $r < R$:

$\oint \vec{B} \cdot d\vec{l} = 2\pi r B = \mu_0 I_{encl}$. Current density: $\boxed{J = I/A} = \frac{I}{\pi R^2}$.

$\rightarrow 2\pi r B = \mu_0 J \pi r^2 \rightarrow \boxed{B = \frac{\mu_0 I}{2\pi} \frac{r}{R^2}}$ $[r < R]$

$r > R$: $I_{encl} = I \rightarrow \overbrace{2\pi r B = \mu_0 I}^{\text{as above}} \longrightarrow \boxed{B = \frac{\mu_0 I}{2\pi r}}$ $[r > R]$



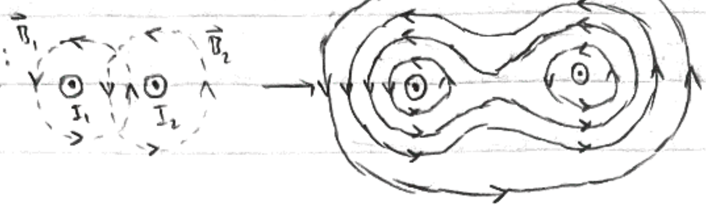
Ampere's Law (cont.)

1/18/24

Lecture 4

Ampere's Law

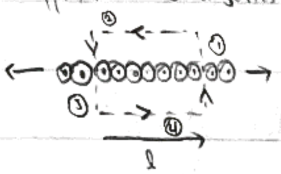
(*) Parallel Wires :



$\vec{B}$ [mag. fields combine]

(*) Infinite Plane of Current/Charged Sheet

Can approximate as a series of conductors : ← ▭ → [I ⊙] ⟹ ← ⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙ →

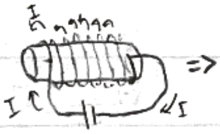


$\Rightarrow \oint \vec{B}\cdot d\vec{l} = \int_1 + \int_2 + \int_3 + \int_4 = 2\int_4 = 2\vec{B}l \rightsquigarrow \underline{\vec{B} = \frac{\mu_0 NI}{2l}}$ [N = # I's]

Let $\underline{n = \frac{N}{l}} \rightarrow \underline{\vec{B} = \frac{\mu_0 In}{2}}$.

Ampere's Law for Solenoids

Solenoid - wire coiled into a helix shape ; converts electrical energy to mech. energy [via mag. fields] (used for switches, e.g.)

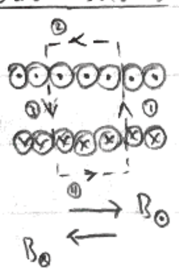


⟹

; $N = \#$ coils → define winding density $n = N/L$

└ ideal solenoid - assume infinite length

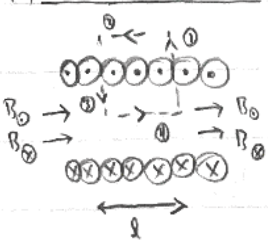
i) $\vec{B}$ Outside Solenoid



$\rightarrow \oint \vec{B}\cdot d\vec{l} = \int_1 + \int_2 + \int_3 + \int_4 = \int_2 + \int_4$; $\begin{cases} \int_2 = \int_2 (\vec{B}_{\otimes} - \vec{B}_{\odot})\cdot d\vec{l} = 0 \\ \int_4 = \int_4 (\vec{B}_{\odot} - \vec{B}_{\otimes})\cdot d\vec{l} = 0 \end{cases}$ (since L assumed to be infinite)

$\rightarrow \oint \vec{B}\cdot d\vec{l} = 0$; $\mu_0 I_{encl} = \mu_0 I_{net} = \mu_0 (0) = 0$ $\boxed{B = 0}$

ii) $\vec{B}$ Inside Solenoid



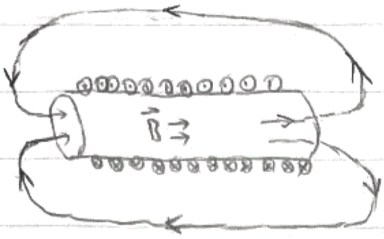
$\rightarrow \oint \vec{B}\cdot d\vec{l} = \int_1 + \underbrace{\int_2}_{} + \int_3 + \int_4 = \int_4 \vec{B}\cdot d\vec{l} = Bl$; $\mu_0 I_{encl} = \mu_0 NI$ (outside $\vec{B} = 0$)

$$\boxed{B = \mu_0 \frac{NI}{l} = \mu_0 I n}$$

Magnetism in Matter

1/18/24
Lecture 4
(cont.)

(*) Real Solenoids:



Typically: length >> width
→ strong $\vec{B}$ inside; very weak $\vec{B}$ outside

Atomic Magnetism

Electrons drive magnetism: $\vec{B}$, $\vec{\mu}$, I, e^- $\quad U = -\vec{\mu} \cdot \vec{B} \rightarrow U$ minimized when $\vec{\mu} \parallel \vec{B}$: $\vec{B}$, $\vec{\mu}$

Recall (Electricity): $\vec{E}$ $\quad U = -\vec{p} \cdot \vec{E}$; aligned: $\vec{E}$, $\vec{E}'$ ↳ $\vec{E}'$ counteracts $\vec{E}$; $E_{tot} \downarrow$
[dielectrics]

Magnetism: $\vec{B}$, $\vec{B}'$ ↳ $B_{tot} = B_0 + B'$ → $\vec{B}$ increases by aligning

→ Ampere's Law: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} \Rightarrow \oint \vec{B} \cdot d\vec{l} = \mu I_{encl}$; define permeability: $\boxed{K = \frac{\mu}{\mu_0}}$
(after alignment)
└ material constant
[T-dependent]

Alt: define susceptibility: $\boxed{\chi_m = K_m - 1}$ [rep. deviation from 1]

3 Types of Magnetic Materials

i) Ferromagnetic: $K_m >> 1$ [ex: iron, cobalt, nickel + alloys]
- Materials naturally already have internal small areas of alignment [magnetic domains], due to alignment promoting "better fit" into crystal structure; domains align easily on external $\vec{B}$

ii) Paramagnetic: $\chi_m \sim 10^{-4}$ [most common]
- In most materials, U gained from alignment << thermal energy; heat causes collisions, preventing alignment

iii) Diamagnetic: $K_m = 1 - \chi_m$; $\chi_m \sim 10^{-5}$ [rare: "antimagnetism"]
- Due to material properties (ex: graphite); causes repulsion rather than attraction

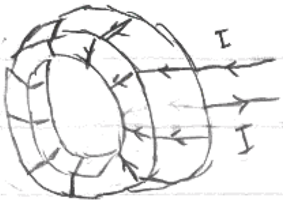
(*) Can cause levitation

(*) Ampere's Law

1/18/24

Disc 2

(*) Ex: Toroidal Solenoid



→ Want to find $\vec{B}$: i) at $r \le r_1$

ii) at $r_1 < r \le r_2$

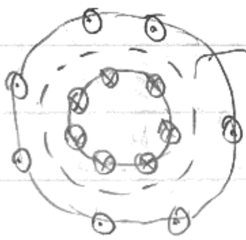
iii) at $r_2 < r$

i) $r \le r_1$

$$\oint_{loop} \vec{B} \cdot \vec{dl} = \mu_0 I_{encl} \; ; \; I_{encl} = 0 \rightarrow \underline{\vec{B} = 0}$$

(Ampere's Law)

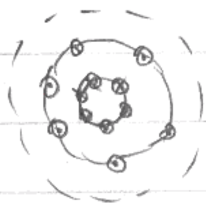
ii) $r_1 < r \le r_2$



$$\oint_{loop} \vec{B} \cdot \vec{dl} = \mu_0 I_{encl} = \mu_0 N I$$

$$\rightarrow \vec{B} \cdot 2\pi r = \mu_0 N I \longrightarrow \boxed{\vec{B} = \frac{\mu_0 N I}{2\pi r}}$$

iii) $r_2 < r$



$$\oint_{loop} \vec{B} \cdot \vec{dl} = \mu_0 I_{encl} = \mu_0 (NI - NI) = 0 \rightarrow \underline{\vec{B} = 0}$$

Magnetic Induction & Faraday's Law

1/23/24

Lecture 5

Magnetic Induction

Induction: We can use magnetic fields to generate electrical current (ex: generators)

Magnet Into/Out of Loop

Assume $\vec{B}$ uniform, and that $|B| = \alpha t$ $\left[\frac{dB}{dt} = \alpha > 0\right]$

→ change in $\vec{B}$ causes current; creates voltage difference $\varepsilon = \Delta V_{ab}$ ← called an electromotive force (EMF)

Properties: i) ε is larger when dB/dt is larger (higher rate of change)

ii) ε is larger when the loop is larger

→ Formula: $\boxed{\varepsilon = \pm A \frac{dB}{dt}}$

Magnetic flux: $\boxed{\Phi_B = \iint \vec{B} \cdot \vec{dA}}$

→ General case (Faraday's Law): $\boxed{\varepsilon = -\frac{d}{dt} \iint \vec{B} \cdot \vec{dA}}$ [over surface bounded by loop]

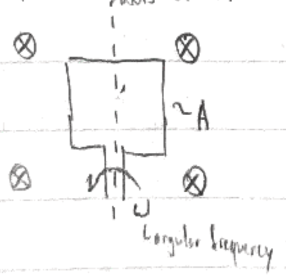
equivalently: $\boxed{\varepsilon = -\frac{d\Phi_B}{dt}}$

Generating ε (4 ways)

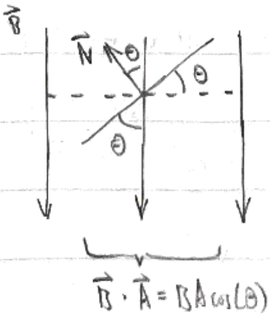
1) Change $\vec{B}$ over time (ex: circuits / transformers)
2) Change the area A of the loop
3) Change the angle between $\vec{B}$, $\vec{A}$ (ex: generators)
4) [Non-uniform $\vec{B}$] Move the loop - motional EMF

(*) Rotating Loop, Constant $\vec{B}$

top view:



side view:



$\theta = \omega t$

→ $\Phi_B = \vec{B} \cdot \vec{A} = BA\cos(\theta) = \pm BA\cos(\omega t)$

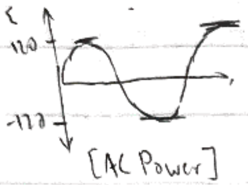
→ (Faraday's): $\varepsilon = BA\sin(\omega t)$

N loops: $\boxed{\varepsilon = NBA\sin(\omega t)}$

EMF & Lenz's Law

1/23/24
Lecture 5
(cont.)

(*) Ex: A.C. Power



U.S. AC: $f = 60\ Hz \rightsquigarrow \omega = 2\pi f = 377$; want $\varepsilon_{max} = 120V$

Let $\vec{B} = 1.5T$, coil $10\ cm \times 10\ cm$; how many loops are needed?

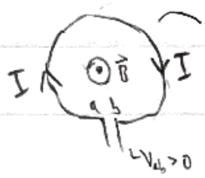
A: $120 = NBA\sin(\omega t) \rightarrow$ $N \approx 21$ loops

ε: (+) or (-)?

Faraday's Law: $\varepsilon = -\frac{d}{dt}\Phi_B$ $\rightsquigarrow$ finding dir. of ε (steps):

1) Choose dir. for $\vec{A}$, normal to loop
2) Evaluate ε using Faraday's Law
3) RHR: align thumb $\parallel \vec{A}$; curl fingers w/ loop

$\rightarrow$ dir. of fingers = dir. of ε if $\varepsilon > 0$



i) Set $\vec{A} = \odot$

ii) $\vec{A} = \odot \rightarrow \varepsilon < 0$

iii) dir: $\circlearrowright$

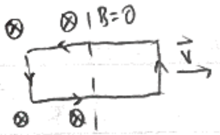
Lenz's Law	"Induced currents & forces act in directions opposing changes to magnetic flux Φ_B."

Lenz's Law

(*) Ex:

$\rightsquigarrow$ $\vec{B}'$ from $\vec{I} = \otimes$ [opposes original $\vec{B}$]

(*) Ex:



Loop leaving B-field => Φ_B decreasing

$\rightarrow$ creates $\vec{I}$ with $\vec{B}' = \otimes$, $\vec{F} = \leftarrow$ [opposing $\vec{v}$]

Induced Electric Fields

Recall: With Coulomb's Law, $\Delta V_{ab} = -\int_a^b \vec{E} \cdot d\vec{l}$ path-independent ($\vec{E}$ from charges) $\rightsquigarrow \oint_{loop} \vec{E} \cdot d\vec{l} = 0$

Now: Changing $\vec{B}$ generates $\vec{E}$ loops $\rightsquigarrow$ Faraday's: $\boxed{\oint_{loop} \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\Phi_B}$ $\rightsquigarrow \oint_{loop} \vec{E} \cdot d\vec{l} \neq 0$ if B changing

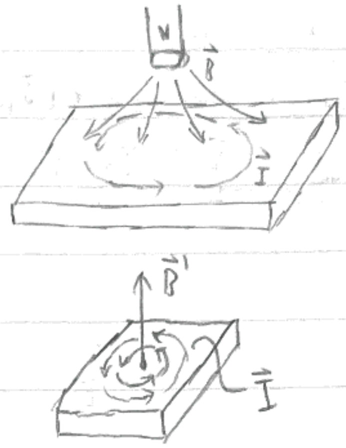
(*) Differential form: $\vec{\nabla} \times \vec{E} = -\frac{d}{dt}\vec{B}$

Eddy Currents

1/25/24
Lecture 6

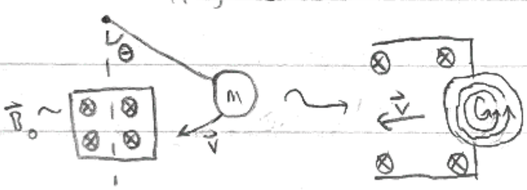
Eddy Currents

When a magnet is brought near a conductor, generates current $\vec{I}$ as loops inside conductor; I creates $\vec{B}'$ opposing $\vec{B}$ [Lenz's Law]. ⌊ called eddy currents



⌊ If $R>0$, some I dissipated as heat; remainder creates $\vec{B}'$ creates $\vec{F}$. (various applications)

(*) Ex: Stopping a Pendulum



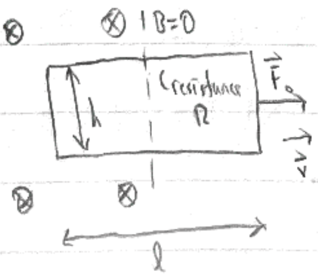
$\rightarrow \vec{B}'$: ⊙ [against $\vec{B}_0$]

$\vec{F}_{\vec{B}_0 \text{ on } \vec{I}}$: $\rightarrow$ [against $\vec{v}$]

(*) Superconductivity

Superconductors: materials with exactly 0 electrical resistance
- Eddy currents, once generated, persist forever
- Try to "avoid"/"exclude" mag. fields & changes in mag. field within interior ⇝ strange interactions w/ magnets (e.g. levitation)
- First superconductors found at ~4°K (requires liquid He); later: "high-temp" @ ~77°K (liquid N)

(*) Ex: Motional EMF



$\Phi_B = |\vec{B}|\cdot(A - h\cdot vt) \rightsquigarrow \varepsilon = -\frac{d\Phi_B}{dt} = |\vec{B}|vh$ [Faraday's Law]

Ohm's Law: $I = V/R \rightsquigarrow I = \varepsilon/R = \frac{|B|vh}{R}$

$\rightarrow \vec{F}_B = \int I\, d\vec{l} \times \vec{B} = \frac{|B|^2 v h^2}{R}$

$\rightsquigarrow F_{tot} = F_0 - \vec{F}_B = \vec{F}_0 - \frac{|B|^2 v h^2}{R}$

Displacement Current

1/25/24
Lecture 6
(cont.)

Displacement Current

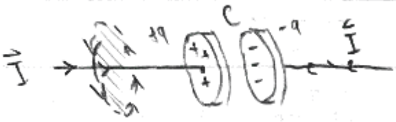
formally: $I_{encl} = \iint \vec{J} \cdot d\vec{A}$ [over surface enclosed by loop]

Recall (Ampere's Law): $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$; works for any loop shape, not just planar loops

$\rightsquigarrow \vec{B} = \frac{\mu_0 I}{2\pi r} \leftsquigarrow$

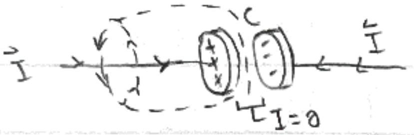
[gives same answer]

$\rightsquigarrow$ Paradox:



$\rightsquigarrow \vec{B} = \frac{\mu_0 I}{2\pi r}$

$\neq$



$\rightsquigarrow \vec{B} = 0$

$I_{encl} = 0$

Observation: as $I \to C$, $\vec{E}_{capacitor} \uparrow$

Recall: $q = CV$, $C = \frac{\varepsilon_0 A}{d}$, $V = \vec{E} \cdot d$

$\rightsquigarrow q = CV = \varepsilon_0 \vec{E} \cdot A = \varepsilon_0 \iint \vec{E} \cdot d\vec{A} = \varepsilon_0 \Phi_E$

$\rightsquigarrow I = \frac{dq}{dt} = \varepsilon_0 \frac{d\Phi_E}{dt}$

Maxwell: Define displacement current $\boxed{i_D = \varepsilon_0 \frac{d\Phi_E}{dt}}$, vs conduction current i_C [I from before]

$\rightsquigarrow$ Ampere's Law (ii): $\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 (i_C + i_D) = \mu_0 I_{encl} + \mu_0 \varepsilon_0 \frac{d}{dt} \iint \vec{E} \cdot d\vec{A}}$

Consequence: $\vec{B} > 0$ between capacitor plates due to displacement current

(*) Maxwell's Equations

Maxwell's Equations:

i) Gauss's Law: $\Phi_E = \frac{q_{encl}}{\varepsilon_0}$

ii) Gauss's Law for Magnetism: $\oint\!\!\oint \vec{B} \cdot d\vec{S} = 0$

iii) Faraday's Law: $\mathcal{E} = -\frac{d}{dt} \Phi_B$ $\sim$ changing $\vec{B}$ gives $\vec{E}$

iv) Ampere's Law: $\oint \vec{B} \cdot d\vec{l} = \mu_0 (i_C + i_D)$ $\sim$ changing $\vec{E}$ gives $\vec{B}$

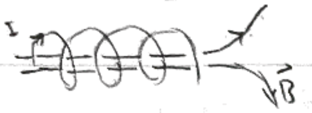
electromagnetic radiation [light]: electric, magnetic fields propagate each other

LC Circuits

1/30/24

Lecture 7

Self-Inductance in a Solenoid



Recall: $B = \mu n I \rightsquigarrow \Phi_B = B \cdot A \cdot N = (\mu n I) AN$

$\rightarrow \mathcal{E} = -\frac{d}{dt}\Phi_B = -\underbrace{(\mu n A N)}_{\text{call this self-inductance } L} \frac{dI}{dt}$ $\quad \boxed{L_{solenoid} = \mu n A N}$

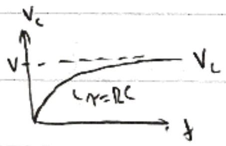
[units: Henries]

LC Circuits

3 characterizations of ΔV:

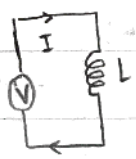
i) Capacitors	ii) Resistors	iii) Inductors
C	$I \rightarrow R$	$I \rightarrow L$
$\Delta V = -Q/C$	$\Delta V = IR = -R\frac{dQ}{dt}$	$\Delta V = -L\frac{dI}{dt} = -L\frac{d^2Q}{dt^2}$

Recall RC circuits: 

$\Delta V = V_c$; capacitor builds up charge $\rightsquigarrow$ 

$\rightsquigarrow$ Inductors: 2^{nd} Q derivative gives oscillatory behavior (useful for wireless signals, e.g.)

Magnetic Field Energy

power supply 

Assume V, I start at 0; V increasing over time

Recall: $P = IV = I(-\mathcal{E}) = I(L\frac{dI}{dt})$ [into inductor]

note: energy stored in inductor as current; released when $I\downarrow$

$\rightsquigarrow \Delta U = P\Delta t \rightarrow U = \int P\,dt = \int LI\frac{dI}{dt}dt = L\int_0^I I'\,dI' \rightarrow \boxed{U = \frac{1}{2}LI^2}$ [Energy in magnetic field]

Recall: $L = \mu n N A = \frac{\mu N^2 A}{\ell}$ [solenoid]; $B = \frac{\mu n I}{\ell} \rightarrow I = \frac{\ell B}{\mu N} \longrightarrow U = \frac{1}{2}LI^2 = \frac{1}{2\mu}(A\ell)B^2 = \frac{1}{2\mu}(\text{Volume})B^2$

$\rightsquigarrow u = U/\text{Volume} \rightarrow \boxed{u = \frac{1}{2\mu}B^2}$ [E in magnetic field B]

(*) Vs capacitors: $U = \frac{1}{2C}Q^2 \rightarrow u = \frac{1}{2}\varepsilon_0|\vec{E}|^2$ [E in electric field]

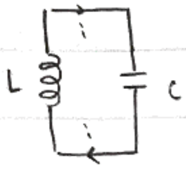
LC & RLC Circuits

1/30/24

Lecture 7

(cont.)

LC Circuits



Loop Rule

$\sum \Delta V = -L\frac{d^2q}{dt^2} - \frac{q}{C} = 0$ (*) Recall: $m\frac{d^2x}{dt^2} + kx = 0$ (mass on spring)

$\boxed{\omega = \sqrt{1/LC}}$ $x(t) = A\cos(\omega t + \phi)$ $[\omega = \sqrt{k/m}]$

$\boxed{q(t) = A\cos(\omega t + \phi)}$

$\xrightarrow{dq/dt}$ $\boxed{i(t) = -A\omega\sin(\omega t + \phi)} = A\omega\cos(\omega t + \phi + \pi/2)$

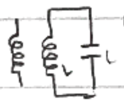
Observation: when charge q maximal, $i = 0$ [& vice versa]

$E_{cap} = \frac{1}{2C}q^2 \propto q^2$

$E_{ind} = \frac{1}{2}Li^2 \propto i^2$

Energy moving between C, L; current flipping direction

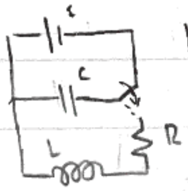
(*) Where does energy come from? i) AC power supply; ii) ω_{ext} radio waves



antenna creates $\vec{B}$ from ω, energy → L

[only works if $\omega_{ext} = \sqrt{1/LC}$]

RLC Circuits

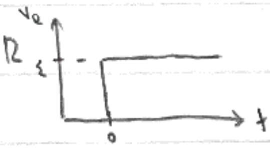


More "realistic" circuit: i) All circuits have some R [have some (small) R]

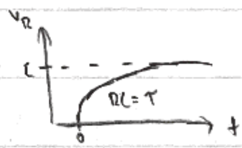
ii) All circuits store some charge on wire [have some C]

iii) All current loops create $\vec{B}$, creates L [have some L]

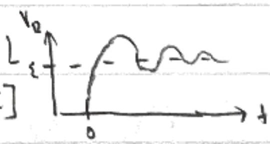
Behavior: i) just R



ii) add C [RC]



iii) add L [RLC]

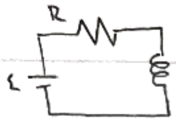


- (iii) akin to damping; can critically damp
- (ii) & (iii): more R → more time to reach ε

2/1/24
Disc 4
+ Lecture 8

LR & RLC Circuits

(*) LR Circuits

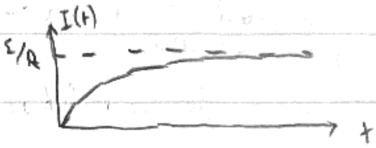


$\sum_{loop} \Delta V = \varepsilon - IR - L\frac{dI}{dt} = 0$ [Loop Rule]

$\rightarrow \varepsilon = IR + L\frac{dI}{dt} \rightarrow \frac{dI}{dt} = \varepsilon/L - RI/L \rightarrow \int_{I(0)}^{I(t)} \frac{dI}{\varepsilon/L - RI/L} = \int_0^t dt$

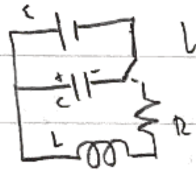
$\rightarrow -\frac{L}{R}\ln\left(\frac{\varepsilon/L - R/L\, I(t)}{\varepsilon/L}\right)\Big|_{I(0)}^{I(t)} = t$

solve for I $\longrightarrow$ $\boxed{I(t) = \frac{\varepsilon}{R}\left(1 - e^{-R/L\, t}\right)}$



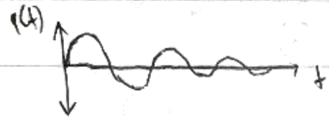
$\boxed{\tau = L/R}$ [LR]

RLC Circuits



Look at bottom loop: $\sum_{loop} \Delta V = 0 = -L\frac{d^2q}{dt^2} - R\frac{dq}{dt} - \frac{q}{C}$ [Damped harmonic oscillator]

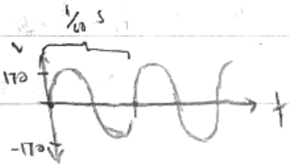
$\rightarrow$ Solution: $\boxed{q(t) = q_0 e^{-t/\tau}\cos(\omega t + \phi)}$; $\boxed{\tau = 2L/R}$, $\boxed{\omega = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}}$



(*) "Critical damping": $\omega = 0$

(*) AC Power

US: 120V, 60 Hz



(*) 170 → 120 : take RMS of V [$\sqrt{\text{avg.}\, V^2}$]
[fun fact]

Power P in a resistor: $P = V^2/R$ [varying w/ rate 60 Hz]

$\rightarrow$ average power $\langle P \rangle = \langle V^2/R \rangle = \frac{1}{R}\langle V_0^2 \sin^2\omega t\rangle = \frac{V_0^2}{R}\langle \sin^2\omega t\rangle$; $\langle \sin^2 \rangle = 1/2$

$\rightarrow \boxed{\langle P \rangle = \frac{(V_0/\sqrt{2})^2}{R} = \frac{V_{RMS}^2}{R}}$ [$V = 170 \rightarrow V_{RMS} = 120$]

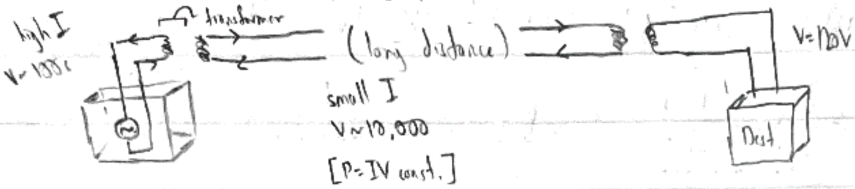
Define $I_{RMS} = I_0/\sqrt{2}$; $P = I^2R$ $\rightarrow$ $\boxed{\langle P \rangle = I_{RMS}^2 R}$

AC Circuits

2/1/24
Lecture 8

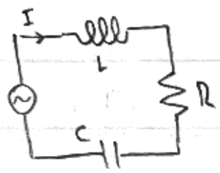
(*) AC Power

AC power good for transporting power over long distances



low I in middle
→ less P lost due to resistance in the wire

AC-driven RLC Circuits



Recall (RLC Circuit): $\mathcal{E} = -L\frac{d^2q}{dt^2} - R\frac{dq}{dt} - \frac{1}{C}q$

⤷ AC Power: $\boxed{-L\frac{d^2q}{dt^2} - R\frac{dq}{dt} - \frac{1}{C}q = V_0\sin(\omega t)}$

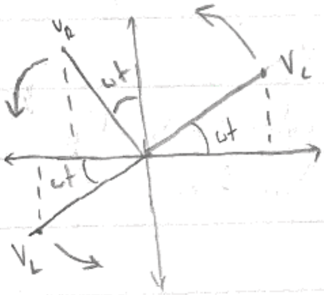
→ 2 solutions: 1) Transitory: (already discussed)

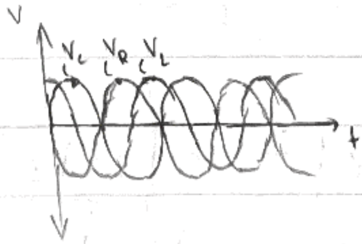
2) Steady state:

$\underbrace{}$ add together to obtain final soln.

Choose phase: $\boxed{q = A\sin(\omega t)}$

- $V_C = q/C = \frac{A}{C}\sin(\omega t) \rightarrow V_C = A\cdot\frac{1}{C}\sin(\omega t)$
- $V_R = R\frac{dq}{dt} = AR\omega\cos(\omega t) \rightarrow V_R = AR\omega\sin(\omega t + \pi/2)$
- $V_L = L\frac{d^2q}{dt^2} = -AL\omega^2\sin(\omega t) \rightarrow V_L = -AL\omega^2\sin(\omega t + \pi)$





(*) Observe: $V_R = IR = R\cdot I_0\sin(\omega t + \pi/2)$

→ V_R carries phase of current vs V_{cap} (behind), V_L (ahead)

Reactance & Impedance

2/6/24
Lecture 9

Reactance

Want to find effect of current on voltage: $\frac{|V_C|}{|I|}$ and $\frac{|V_L|}{|I|}$; called reactance (Ohms)

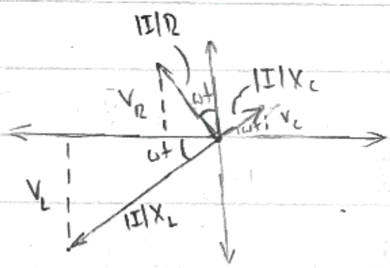
Recall: $q = A\sin(\omega t) \rightarrow i = A\omega\sin(\omega t + \pi/2)$; $|V_C| = \frac{|q|}{C} = A/C$, $|V_L| = L\left|\frac{dI}{dt}\right| = LA\omega^2$

$\rightarrow$ Capacitive reactance: $\frac{|V_C|}{|I|} = \boxed{X_C = \frac{1}{\omega C}}$; Inductive reactance: $\frac{|V_L|}{|I|} = \boxed{X_L = \omega L}$

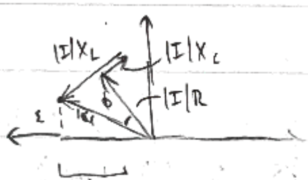
notice: as $\omega \rightarrow 0$ [DC], $X_C \rightarrow \infty$

Phasors & Impedance

Recall: $V = IR$ $\rightarrow$ $\boxed{\varepsilon(t) = |I|\left[X_C\sin(\omega t) + R\sin(\omega t + \pi/2) + X_L\sin(\omega t + \pi)\right]}$



sum to find ε $\rightarrow$



ε also rotates around in a circle: $\varepsilon = \sim\sim \rightarrow$

In particular:

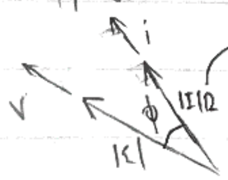
$\rightarrow |\varepsilon| = \sqrt{|I|^2R^2 + (|I|X_C - |I|X_L)^2}$

$\rightarrow \boxed{|\varepsilon| = |I|\sqrt{R^2 + (X_C - X_L)^2}}$

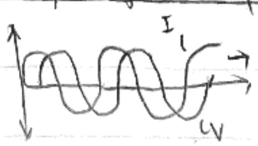
Define impedance: $\boxed{Z = \sqrt{R^2 + (X_C - X_L)^2}}$ $\longrightarrow$ $\boxed{V = IZ}$ — works for both V_{max}/I_{max}, V_{RMS}/I_{RMS}

[Ohms] (*) LR, C circuits: just remove other terms

Additionally, observe:



Current, voltage out of phase; ratio: $\tan\phi = \frac{|I|(X_C - X_L)}{|I|R}$



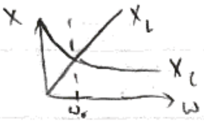
$\longrightarrow \boxed{\tan\phi = \frac{X_C - X_L}{R}}$

Resonance & Transformers

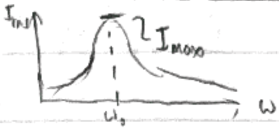
2/6/24
Lecture 9
(cont.)

Resonance

$X_C = \frac{1}{\omega C}$, $X_L = \omega L$ ⤳ can vary circuit ω to obtain $X_C = X_L$ at resonant frequency $\boxed{\omega_0 = \sqrt{1/LC}}$

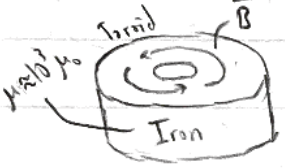


⤳ Z_{min} occurs at $\omega_0 \to I_{rms} = \frac{\varepsilon_{rms}}{Z}$ max at ω_0: $\boxed{I_{max} = \frac{\varepsilon_{rms}}{R}}$

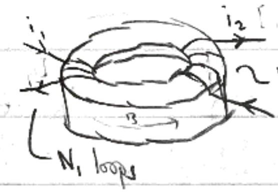


Transformers

Transformers used to transfer energy between circuits, change voltage levels



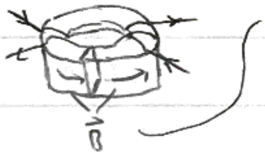
→



~ N_2 loops [i_1 = input, generates i_2 = output]
(primary) (secondary)

N_1 loops

Look at cross-section:



$\Phi_B = \iint \vec{B} \cdot d\vec{A}$ everywhere inside toroid; AC current → alternating $\vec{B}$ ⤳ $\exists \frac{d\Phi_B}{dt}$

⟶ for each loop: $\varepsilon = -\frac{d\Phi_B}{dt}$ ⤳ $\varepsilon_1 = N_1 \frac{d\Phi_B}{dt}$; $\varepsilon_2 = N_2 \frac{d\Phi_B}{dt}$

⤳ Observe: $\boxed{\frac{\varepsilon_2}{\varepsilon_1} = \frac{N_2}{N_1}}$; alt: $\boxed{\varepsilon_2 = \frac{N_2}{N_1} \varepsilon_1}$ ~ $\frac{N_2}{N_1}$ called "turns ratio"

→ Consequence: more turns N_2 than N_1 → ε_2 larger [increases voltage]

(*) Note: Overall power constant ($\underbrace{P_{in} = VI = \varepsilon_1 i_1 = P_{out} = \varepsilon_2 i_2}_{\text{Assumes no power lost (due to eddy currents, e.g.)}}$) ⤳ i_2 lower to compensate

Complete equation: $\boxed{\frac{i_1}{i_2} = \frac{\varepsilon_2}{\varepsilon_1} = \frac{N_2}{N_1}}$

(*) Power in AC Circuits

Recall (Power): $P = VI$

In AC circuits: V, I out of phase by Φ ⤳ new equation: $\boxed{P_{avg} = V_{RMS} I_{RMS} \cos(\Phi)}$

Planar Waves

2/6/24
Lecture 9
+Lecture 10

Planar Waves

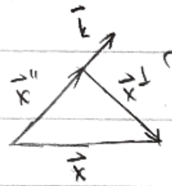
k: "wave number"; wavelength $k\lambda = 2\pi$

Recall (1D Waves): $f(x) = A\sin(kx)$, $f(x-vt)$ = traveling wave [for each input, produce 1 value]

~> 3D Waves [EM Waves]: output vectors for $\vec{E}$, $\vec{B}$ field (6 numbers)

$\boxed{\omega = \vec{k}\cdot\vec{v}}$

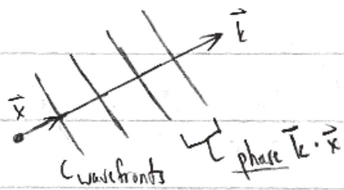
Specify position vector $\vec{x}$, wave vector $\vec{k}$ → new eqn.: $\boxed{f(\vec{x},t) = A\sin(\vec{k}\cdot\vec{x} - \omega t)}$



$\{\ \vec{x} = \vec{x}^{\parallel} + \vec{x}^{\perp} \Rightarrow \vec{k}\cdot\vec{x} = \vec{k}\cdot(\vec{x}^{\parallel} + \vec{x}^{\perp}) = \vec{k}\cdot\vec{x}^{\parallel}$; independent of $\vec{x}^{\perp}$

→ f constant at all points along lines/planes $\perp$ to $\hat{k}$

→ creates wavefronts of planar wave, $\perp$ to dir. of travel



Maxwell's Equations in Vacuum

Vacuum: no charge Q, current I; but can still have $\vec{E}$, $\vec{B}$ fields

~> Maxwell's Eqns: [in vacuum]

i) $\oint E\cdot dA = 0$

ii) $\oint B\cdot dA = 0$

iii) $\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$

iv) $\oint \vec{B}\cdot d\vec{l} = \mu_0\varepsilon_0\frac{d\Phi_E}{dt}$

} Symmetry between $\vec{E}$, $\vec{B}$

(iii) & (iv): Changing $\vec{E}$ gives $\vec{B}$, & vice versa ~> can configure to create each other

~> $\vec{E}$ loop from $\frac{d\Phi_B}{dt}$ from $\frac{d^2\Phi_E}{dt}$ gives 2nd order diff. eq (electromagnetic wave eqn.)

~> $\vec{E}$, $\vec{B}$ propagate like waves: define EM waves / EM radiation to describe

81

EM Plane Waves

2/8/24
Lecture 10
(cont.)

EM Plane Waves

(EM waves are <u>transverse waves</u>)

From Maxwell's eqns., derive results: i) $\vec{E} \perp \vec{B} \perp$ dir. of wave $\hat{k}$, velocity $\vec{v} = \vec{c}$

ii) $E = cB$

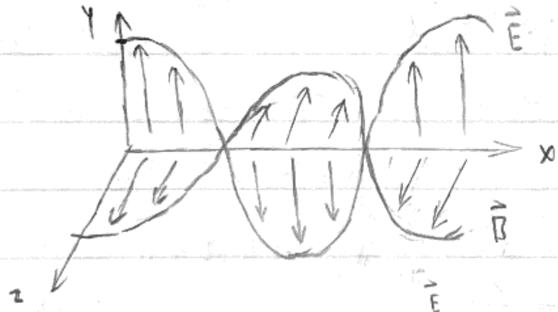
iii) $c = 1/\sqrt{\mu_0 \varepsilon_0}$ $\boxed{c \approx 2.997 \cdot 10^8}$

iv) Waves can travel in vacuum

Wave equation: $\boxed{\frac{\partial^2 \vec{E}_y}{\partial x^2} = \varepsilon_0 \mu_0 \frac{\partial^2 \vec{E}_y}{\partial t^2}}$ $(\varepsilon_0 \mu_0 = 1/v^2)$

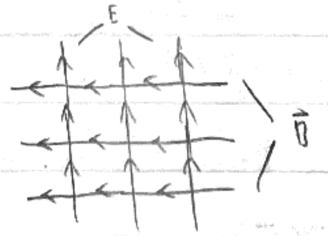
$\rightarrow \vec{E}_y(\vec{x}, t) = \vec{E}_y(x, t) = \vec{E}_y(x - vt)$; model with/assume sine wave
(x along $\hat{k}$)

Visualizing EM Plane Waves



wave dir: $\boxed{\hat{k} \parallel \vec{E} \times \vec{B}}$ [in $\vec{x}$ dir.]

$\vec{E}, \vec{B}$ perpendicular:

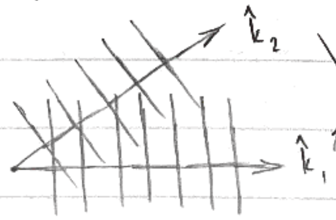


(*) planar wave: value of $\vec{E}, \vec{B}$ constant across wavefront

(*) Superposition

Maxwell's equations linear [can add together diff. solns. for new soln.]

$\sim$ can superimpose EM waves:



"ignore" each other, just add up
((*) not true of all waves (e.g. water waves))

EM Plane Waves (cont.)

2/13/24

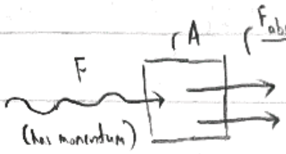
Lecture 11

Intensity & Radiation Pressure

corollary: $\boxed{E_{max} = \sqrt{\frac{2I}{\varepsilon_0 c}}\ ;\ B_{max} = \frac{E_{max}}{c}}$

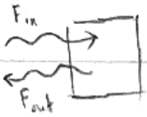
Recall: intensity = $\frac{\text{power}}{\text{area}}$ $\rightsquigarrow$ intensity of an EM wave: $\boxed{I = \frac{1}{2} c\varepsilon_0 E_{max}^2}$

Intensity & pressure coincide:



Force / Area $\rightsquigarrow$ $\boxed{P_{rad} = I/c}$ [Pa / $\frac{N}{m}$]

(*) Some materials: F reflected

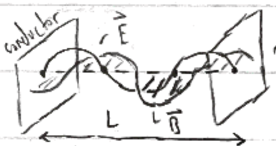


$\rightsquigarrow$ $\boxed{P_{rad} = 2I/c}$ $\rightarrow$ Generally quite small [1 billionth of Patm]

$\rightsquigarrow$ more consequential in space

Standing EM Waves

Recall: $\vec{E} = 0$ inside conductors $\rightsquigarrow$ EM waves between conductors:



conductors are nodes for wave

(*) L, B have opposite nodes / anti-nodes

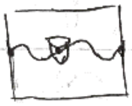
$\rightsquigarrow$ when $\boxed{L = n\frac{\lambda}{2}}$, obtain standing EM wave:

[same frequency, opposite direction]

[standing wave: locations of peak amplitude constant]

(*) Ex (Microwaves): $\lambda = 12.2$ cm good for heating water $\rightsquigarrow$ all dimensions multiples of 6.1 cm

$\rightsquigarrow$ Consequence: heating cup w/ $d < 6.1$ $\rightsquigarrow$



[$\vec{E}$ node $\rightarrow$ no heating]

EM Waves in Matter

(!) $v \leq c$ always

In vacuum: $c = 1/\sqrt{\mu_0\varepsilon_0}$; in matter, $\mu = K_m\mu_0$ (generally small: $K_m \sim 1+10^{-4}$) & $\varepsilon = K_E\varepsilon_0$ (water, glass: $K_E \approx 2$) $\rightarrow$ in matter: $\boxed{v = \frac{1}{\sqrt{\mu\varepsilon}}}$

$\rightsquigarrow$ Define refractive index: $\boxed{n = \frac{c}{v} = \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}}}$ [≥ 1] $\rightsquigarrow$ (*) Water: $n \sim 1.33$

Glass: $n \sim 1.4 - 1.7$ [diff. types]

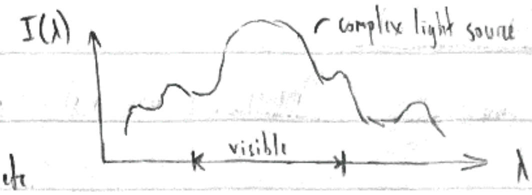
Intro to Optics

2/13/24
Lecture 11
(cont.)

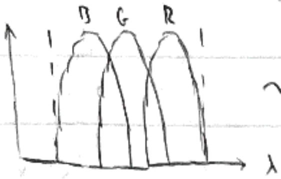
Types of Light

Many answers for "types of light": 1, 7, ∞, etc.

└ Visible light: 7 (rainbow), 4 (CMYK), 3 (RGB), etc.



Eye sensitivity:

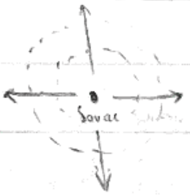


$\rightsquigarrow$ brain gets B, G, R signals → 3 nos.: luminance (total)

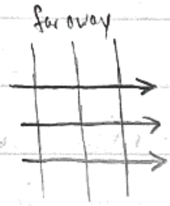
+ 2 chromaticity: ratios $\frac{\text{blue}}{\text{total}}$, $\frac{\text{green}}{\text{total}}$

(*) tetrachromats: 4 signals

Wavefronts & Rays

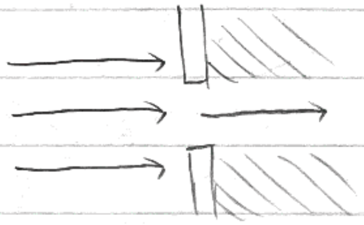


. . .



$\rightsquigarrow$ can abstract as rays:

[no wavefronts]



→ Geometrical optics: study of light in terms of rays; for mirrors, lenses, etc.

(*) Ignores wavelength: $\lambda_{light} = 500\,nm$ (small) $\rightsquigarrow$ within $10\,mm$ lens: 20,000 λs [a lot]

Reflection & Refraction

Reflection law for smooth surfaces: $\boxed{\theta_i = \theta_r}$

[Specular reflection]

(*) metal conductor, e.g.

Reflection on non-flat surfaces:

[curved mirror],

no single angle/dir.
[Diffuse reflection]

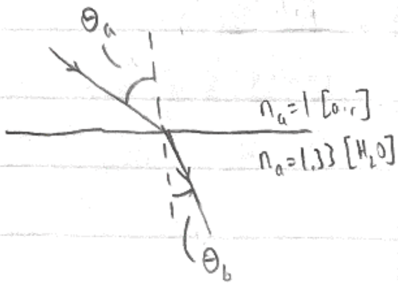
Refraction law for smooth surfaces: $\boxed{n_a \sin\theta_a = n_b \sin\theta_b}$

[Snell's Law]

Alt.: $\boxed{\sin\theta_a = \frac{n_b}{n_a}\sin\theta_b}$

(*) all θ's $\in (0, \pi/2)$

(*) Critical angle: value of θ_a when $\theta_b = 90°$



Intro to Optics (cont.)

2/15/24

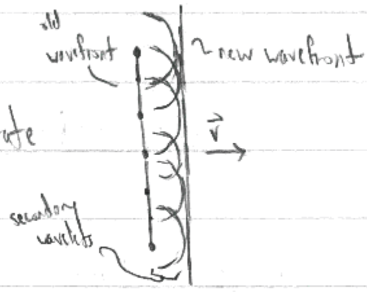
Lecture 12

(*) Refraction (Prisms)

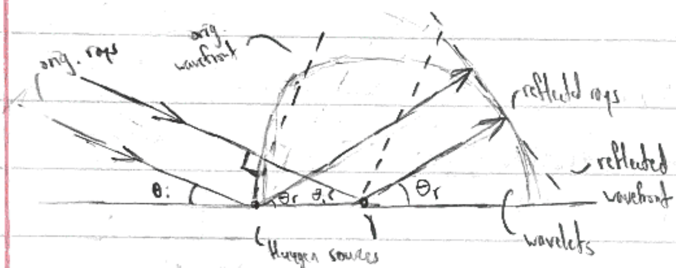
Principle: n varies slightly with λ [recall: diff. colors = diff. λs]

$\rightsquigarrow$ colors bent at slightly different angles (dispersion)

Huygens's Principle



Huygens's Principle: Every point on a wavefront will itself generate secondary wavelets radially outward.

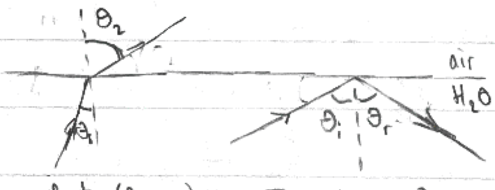


Applied to reflection: wavefronts hitting reflector creates Huygen sources generating wavefronts in in dir. of reflected rays $(\theta_i = \theta_r)$.

Total Internal Reflection (TIR)

Recall (Snell's Law): $n_1 \sin\theta_1 = n_2 \sin\theta_2 \rightsquigarrow \sin\theta_2 = \left(\frac{n_1}{n_2}\sin\theta_1\right)$

$\rightarrow$ If $\frac{n_1}{n_2} > 1$, for some angles θ_1: no solution $\rightarrow$ no refraction, just reflection (TIR)

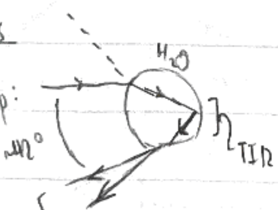


refraction (θ_i small) vs TIR (θ_i large)

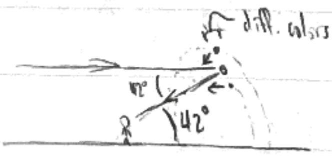
(*) Most reflectors (e.g. mirrors) have only partial reflection (~80%); TIR $\rightarrow$ 100%.

(*) Ex: Rainbows

Sunlight into raindrop:



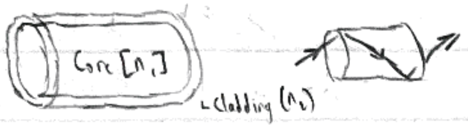
$\rightsquigarrow$ if sun low:



Polarization

2/15/24
Lecture 12
(cont)

(*) Fiber Optics

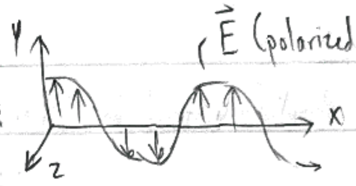


- Transmits information using light (for communication, e.g.)

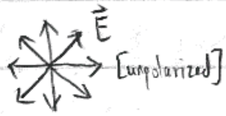
Long distance → needs lots of reflections; uses TIR to prevent decay

Polarization

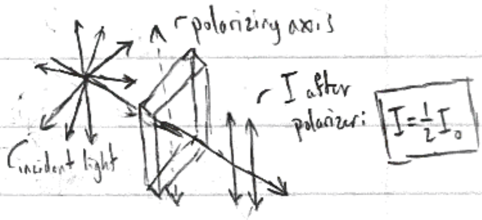
Polarized light travels with $\vec{E}$ displaced along only 1 axis:



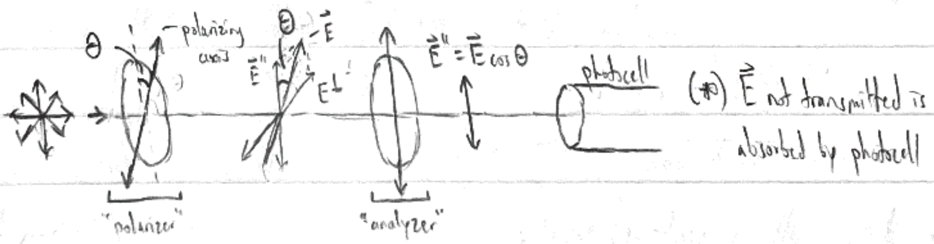
Most light (e.g. natural light) unpolarized: comprised of many waves going in all directions randomly (i.e. polarizations in all directions)



~> Use polarizers to create polarized light: given unpolarized light, only lets specific polarizations [parts of $\vec{E}$ field] pass through



Multiple polarizers:



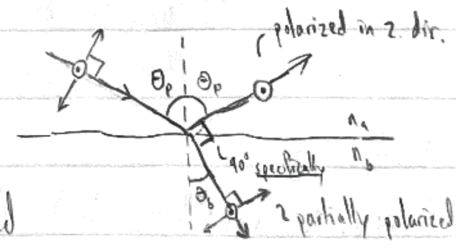
(*) $\vec{E}$ not transmitted is absorbed by photocell

$\vec{E}_{analyzer} = \vec{E}_{polarizer}\cos\Theta$ ~> Intensity: $I \propto E^2$ => Malus's Law: $\boxed{I = I_{max}\cos^2\Theta}$

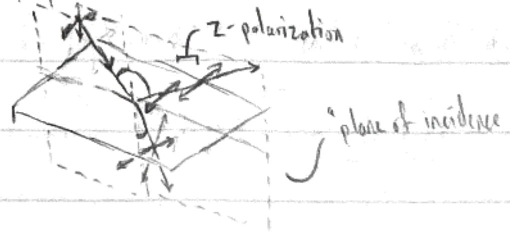
(0 if $\Theta = 90°$; max if $\Theta = 0°$)

- rel. to "original" incident I_0: $I = \frac{1}{2} I_0 \cos^2\Theta$

Polarization by reflection: at a specific angle of incidence, any $\vec{E} \perp$ to plane of incidence [plane containing ray, reflected + refracted rays] only refracted → reflected ray completely polarized



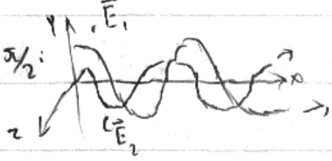
→ Occurs at Brewster's Angle: $\boxed{\tan\Theta_p = \frac{n_b}{n_a}}$

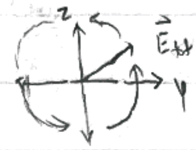


2/20/24
Lecture 13

Scattering of Light

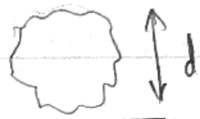
(*) Circular Polarization

Can combine 2 waves w/ phase shift $\pi/2$: 

$\rightsquigarrow$ 

$\leftarrow$ (*) Used in 3D glasses, e.g.

Scattering of Light

Light can be [elastically] scattered upon colliding with particles [scatterers]: 

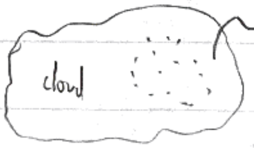
$\rightsquigarrow$ If $d \ll \lambda$ [e.g. $N_2 = 0.3$ nm vs $\lambda_{visible} = 400-750$], Rayleigh scattering: $\boxed{I_{scattered} \propto \lambda^{-4}}$

(*) Ex: Smaller λs [e.g. violet] cause more scattering $\rightsquigarrow$ sky is blue

(*) Sunsets

During sunset, light has to travel further through atmosphere [is scattered more]
$\rightarrow$ blues totally scattered, only higher-λ colors (e.g. reds, yellows) remain [sunset]

If $d > \lambda$, all λs scattered equally
$\rightsquigarrow$ appears white/dark (dep. on scatterer density).
(*) Exception: Dust absorbs more blue than red
$\rightsquigarrow$ sandstorms, Mars atmosphere appear reddish



water droplets [less dense $\rightarrow$ white; more dense $\rightarrow$ dark]

(*) Antennas

Receive EM waves (ex: radio) $\rightsquigarrow$ want find good antenna length
(λ too small $\rightarrow$ little signal [energy transferred]; λ too big also undesirable)

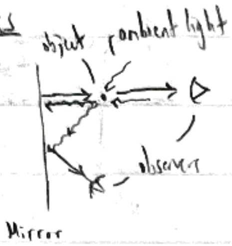
$\rightsquigarrow$ Optimal: $\boxed{L = \lambda/4}$

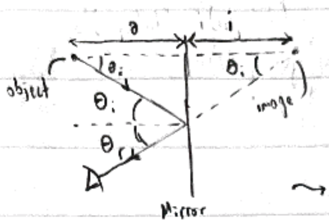
Plane & Concave Mirrors

2/20/24
Lecture 13
(cont.)

Geometrical Optics

Plane Mirrors:



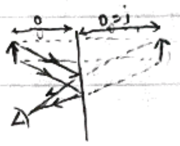


o: object distance

i: image distance

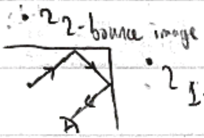
→ Observe: $\boxed{o = i}$

Use rays to determine image direction:



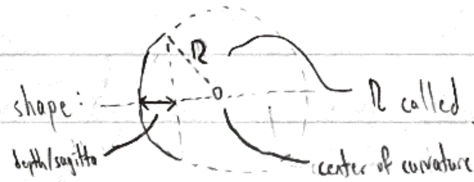
~ planar mirrors preserve direction

(*) Multiple mirrors:



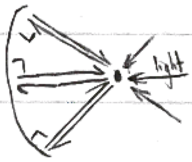
Concave Mirrors:

Concave mirrors spherical in shape:



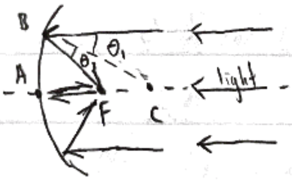
R called radius of curvature

Case (i) - Light from center of curvature:

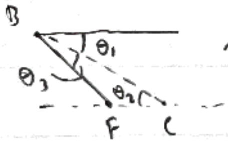


(Image at same point as object)

Case (ii) - Light from far away (dist ≈ ∞):



light parallel to central axis → all reflected toward focal point F



– See: $\theta_1 = \theta_2$ [interior angles], $\theta_1 = \theta_3$ [law of reflection]

→ $\theta_2 = \theta_3$ → $FB = CF$

Small-angle approximation: If $AB \ll CA$ (i.e. small depth), then $FA \approx FB$

→ $CF \approx FA$ ⟶ FA [focal length] = $\boxed{f = R/2}$

Focal Length

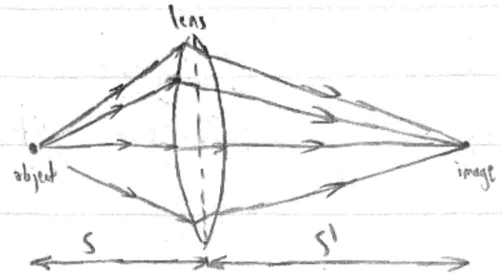
2/20/24
Lecture 13
+ Disc 7

Focal Length

Given a lens w/ focal length f (given as a point) & an object, define

(i) S = dist. from lens center to object

(ii) S' = dist. from lens center to ~~object~~ image

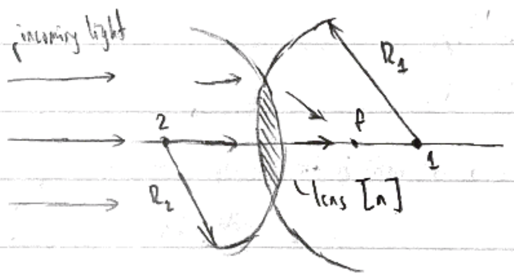


→ Focal length equation: $\boxed{\frac{1}{f} = \frac{1}{S} + \frac{1}{S'}}$ ~ (*) $1/f$ called "bending power"

(*) For eyeglasses: use power $P = 1/f$ [units: Diopters $= \frac{1}{m}$]

Lensmaker's Equation

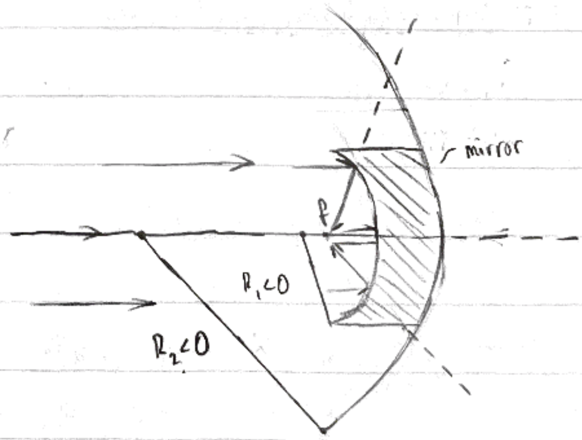
Represent a lens as intersection of two spheres with radius R_1, R_2 & assign refractive index n



~ Lensmaker's Equation: $\boxed{\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$

(*) Sign Conventions

- Side of incoming light has $R < 0$
- Side of outgoing light has $R > 0$
- If image on opposite side as object, $f > 0$
- If image on same side as object, $f < 0$
  (i.e. side of incoming light)



(*) Plane Mirror: $R_1, R_2 = \infty \rightarrow 1/f = 0,\ S = -S'$

(*) Concave Mirror: $f = R/2$

(*) Convex Mirror: $f = -R/2$

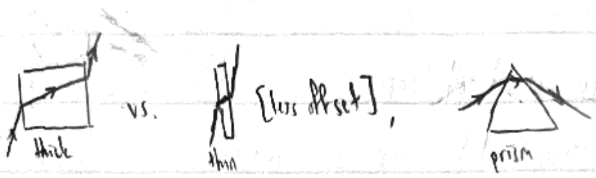
Lenses & Magnification

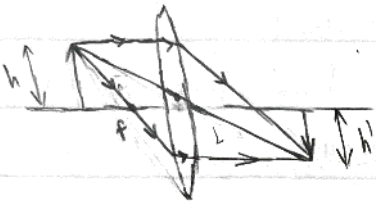
2/22/24

Lecture 14

Lenses

Lenses offset rays:





2 observations: (i) Image across lens is flipped

(ii) Image, object need not be same size

→ Define magnification: $\boxed{M = h'/h = -s'/s}$

(*) Sign Rules

Convention for signs in equations:

(i) Object distance - $S > 0$ if object on same side as incoming light; $S < 0$ otherwise

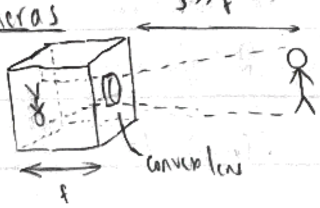
(ii) Image distance - $S' > 0$ if image on same side as outgoing light; $S' < 0$ otherwise

(*) Focal Length Signs

(i) Convex lens: () $f > 0$ (ii) Concave lens:)($f < 0$

(ii) Plano-convex: |) $f > 0$ (iv) Plano-concave: |($f < 0$

Cameras



– If S large, $1/f = 1/s + 1/s' \rightarrow s' = f$, image recorded by film/digital sensor

If S small:

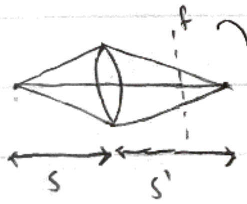


image blurry at distance f

→ 2 solutions: (i) Move lens

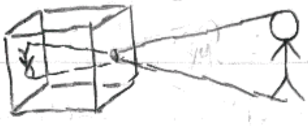
(ii) Adjust f of lens [focusing]

(*) Modern lenses composed of 5-13 indiv. lenses

Applications of Lenses

2/22/24
Lecture 14
(cont.)

(*) Pinhole Cameras



(*) very small hole: called an aperture

Pinhole cameras use small holes instead of lenses:

- Pros: Takes rays directly → no need to focus, as in lenses
- Cons: Small hole → low light received [$A\downarrow \Rightarrow I\downarrow$]; requires long exposures

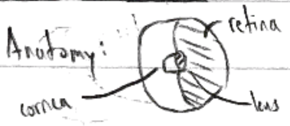
D: diameter of lens

For fast exposures, need sensitive sensors; sensitivity given by f-number/f-stop $\boxed{f_{num} = f/D}$

[Small f_{num} → more light; typically $f_{num} \approx 2$, "very good" ≈ 1.4]

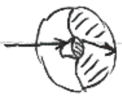
(*) Human Eyes

Anatomy:



exact # age-dependent

→ for $s > 25$ cm:

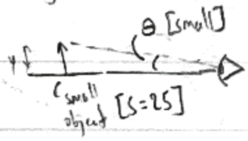


Eye uses muscles to control lens [accommodation]:

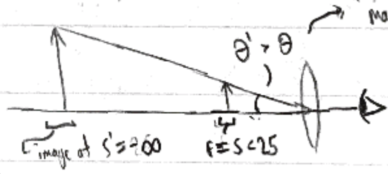
20 Diopters (lens) + 40 (cornea) → total: $P_{eye} = 60$ Diopters

(*) Magnifying Lenses

Unaided:



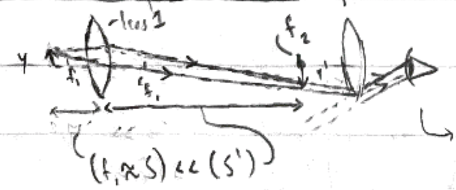
vs. with magnifier:



"magnifies angles"

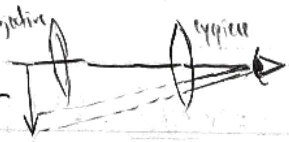
$s' = -\infty > 25 \to s \approx f_{lens}$; if θ small, $\theta \approx \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{y}{25\text{ cm}}$, $\theta' \approx \tan\theta' = y/f$ → Magnification factor [angular]: $\boxed{M = \frac{\theta'}{\theta} = \frac{25}{f}}$ f in cm

(*) Microscopes (2 Lenses)



$M_1 = -\frac{s'}{s} \approx -\frac{s'}{f_1}$, $M_2 = \frac{25}{f_2}$ ⤳ $\boxed{M_{tot} = M_1 \cdot M_2}$

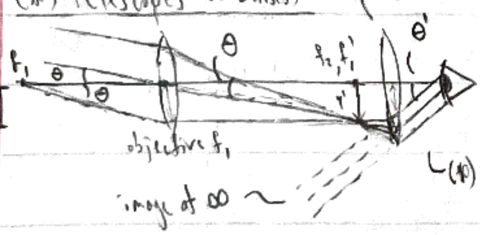
↳ result:



M_1: lateral magnification

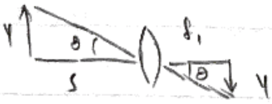
M_{tot}, M_2: angular magnification

(*) Telescopes (2 Lenses)



→ $\theta = -\frac{y'}{f_1}$, $\theta' = \frac{y'}{f_2}$ → angular magnification: $\boxed{M = \frac{\theta'}{\theta} = -\frac{f_1}{f_2}}$

└(*) image size:



(*) For taking pictures: put image sensor at f_1 directly

2-Slit Interference

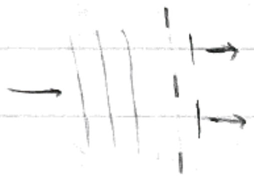
2/27/24
Lecture 15

<u>Physical Optics</u>: Field studying optical phenomena resulting from wave nature of light [not rays]

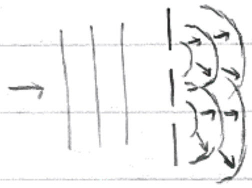
- <u>Interference</u> caused by two waves overlapping at a point [two forms: <u>constructive</u> & <u>destructive</u>]
- <u>Diffraction</u> caused by multiple waves overlapping

2-Slit Interference

2 slits: (i) <u>Large slits (width >> λ)</u> (ii) <u>Narrow slits (width not >> λ)</u>

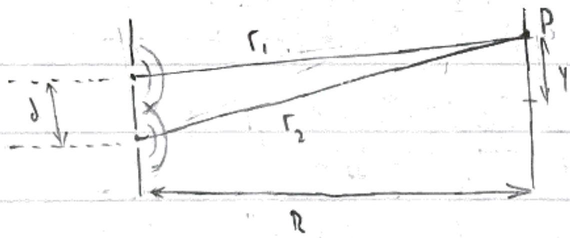


[like rays - geo. optics]

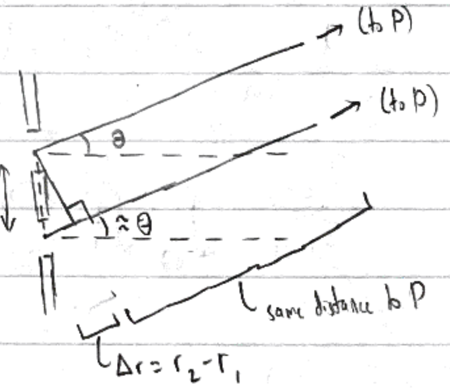


← radial wavefronts (due to Huygens' Principle) [physical optics]

Assume slits narrow:



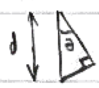
if $R >> d$:



(*) Why interested in Δr? Want to find $\vec{E}_{tot}$ at P

$\rightsquigarrow E_{tot}(P) = E_1(P) + E_2(P) \propto \sin(kr_1 - \omega t) + \sin(kr_2 - \omega t)$ $\left[k = \frac{2\pi}{\lambda}\right]$

$\rightarrow E_{tot} \propto \sin(kr_1 - \omega t) + \sin(kr_1 - \omega t + \underline{k[r_2 - r_1]})$



$$\boxed{\Delta r = d\sin(\theta)}$$

$\rightsquigarrow$ Define $\underline{\Delta\phi = k[r_2 - r_1] = k\Delta r}$ $\longrightarrow$ $\boxed{\Delta\phi = \frac{2\pi}{\lambda} d\sin(\theta)}$

$\rightsquigarrow$ <u>Observations</u>: (i) If $\underline{\Delta\phi = 2\pi m}$ $\underline{[m = 0, 1, 2, \ldots]}$, waves add (<u>constructive interference</u>)

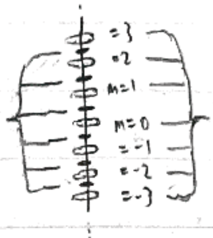
(ii) If $\underline{\Delta\phi = 2\pi(m + \tfrac{1}{2})}$, waves subtract (<u>destructive interference</u>)

2-Slit & Thin-Film Interference

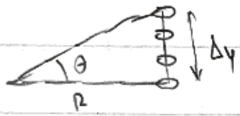
2/27/24
Lecture 15
+ Disc 8

2-Slit Interference (cont.)

On screen, observe alternating bright & dark spots:

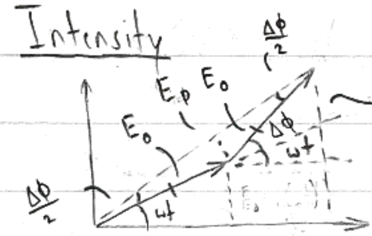


If d, θ known, can find λ_{light}: $\Delta\phi = \frac{2\pi}{\lambda} d\sin\theta$ + $\Delta\phi = m \cdot 2\pi$ $\rangle$ $\boxed{d\sin\theta = m\lambda}$



$\Delta y = R\tan\theta \approx R\sin(\theta)$ [if θ small] $= R\frac{m\lambda}{d}$ $\rightarrow$ $\boxed{\lambda = \frac{\Delta y \cdot d}{R \cdot m}}$

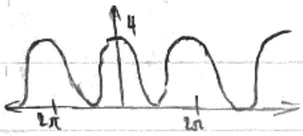
Intensity



$\rightarrow E_{tot} = E_0\sin(\omega t) + E_0\sin(\omega t + \Delta\phi)$

$\boxed{E_p = 2E_0\cos\left(\frac{\Delta\phi}{2}\right)}$; $I \propto E^2 \rightarrow$ $\boxed{I = I_0\cos^2(\Delta\phi/2)}$

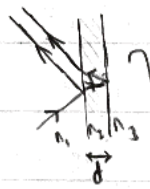
$\frac{I_{2\,slits}}{I_{1\,slit}}$:



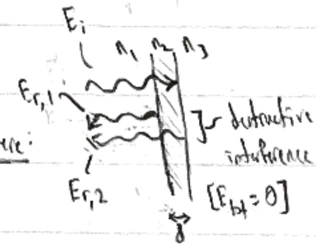
$\sim$ avg.: $\frac{1}{2}$

Thin-Film Interference

In thin films ($d \sim \lambda$), have two reflected rays:



May interfere:



$\left(k = \frac{2\pi}{\lambda}\right.$, [since λ changes in medium]

Reflected ray: $E_{r,tot} = E_{r,1}\sin(\omega t + k_1 x) + E_{r,2}\sin(\omega t + k_1 x + 2k_2 d)$

$\sim$ whether $E_{r,1}$, $E_{r,2}$ in-phase [constructive interf.] or out-of phase [destructive] depends on d

Amplitudes: $E_{r,1} = \frac{n_1 - n_2}{n_1 + n_2}E_i$, $E_{r,2} = \frac{n_2 - n_3}{n_2 + n_3}E_i$ (λ in medium: $\lambda_2 = \frac{n_1}{n_2}\lambda_1$)

$\sim$ if $n_1 > n_2 > n_3$, $n_1 < n_2 < n_3$: (i) Constructive interference if $2k_2 d = 2\pi m \rightarrow 2d = m\lambda_2$

[else, flip] (ii) Destructive interference if $2k_2 d = 2\pi(m + \frac{1}{2}) \rightarrow 2d = (m + \frac{1}{2})\lambda_2$

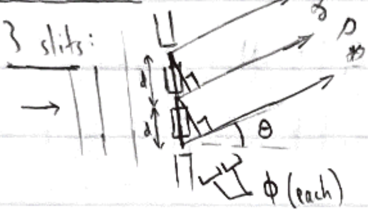
Diffraction

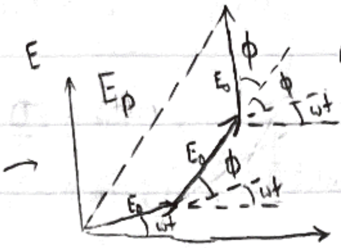
2/29/24

Lecture 16

Diffraction

3 slits:



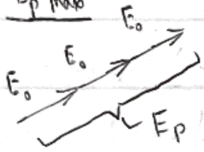


(i) if $\phi = m(2\pi)$,

→ achieves $E_{p_{max}}$: $E_p = 3E_0$

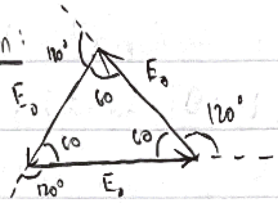
& $I_{p_{max}} \propto E_{p_{max}}^2$: $I_{3\,slit} = 9 I_{1\,slit}$

E_p max:



vs.

E_p min:



(ii) if $\phi = m(2\pi) + 120°$, $m(2\pi) + 240°$

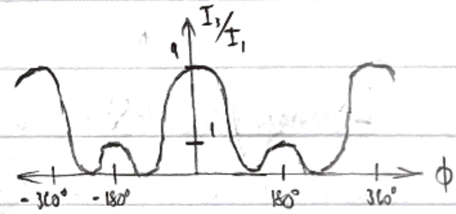
→ achieves $E_{p_{min}}$: $E_p = 0$, $I = 0$

[no $\vec{E}_p$]

(iii) if $\phi = m(2\pi) + 180°$,

→ $E_p = E_0$; $I_{1\,slit} = I_{1\,slit}$

~ Intensity:



For $N > 3$ slits: (i) $\phi = m(2\pi)$ → constructive interference

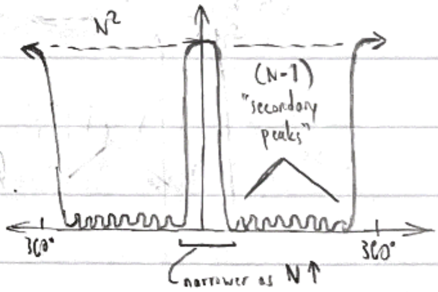
[$E_p = NE_0$, $I_N = N^2 I_1$]

(ii) $\phi = n\left(\frac{2\pi}{N}\right)$ [for $n = 1, 2, \dots, (N-1)$]

→ destructive interference [$E_p = I = 0$]

Max: $\boxed{\phi = m(2\pi)}$

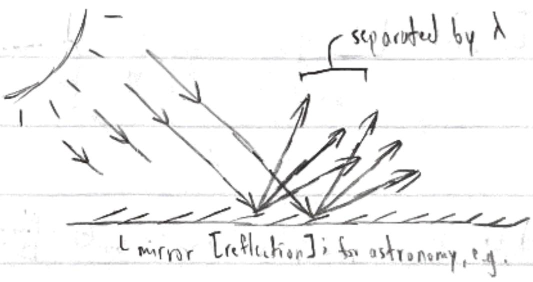
Min: $\boxed{\phi = n\left(\frac{2\pi}{N}\right)}$



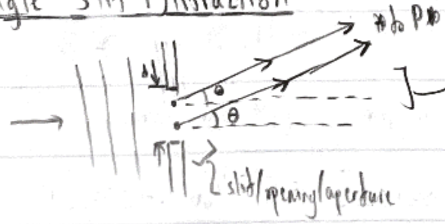
Diffraction Grating

"Peaks" narrower as N↑ ; recall: $\sin\theta = \frac{m\lambda}{d}$

~ for small d [$1/d = 300/mm$, e.g.]; $\sin\theta$ changes more as λ changes → can obtain separation of light



Single-Slit Diffraction

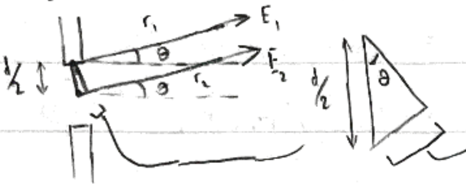


In case of single slit, look at rays from individual points

Diffraction (cont.)

2/29/24
Lecture 16
(cont.)

Single-Slit Diffraction (cont.)



$\Delta r = \frac{d}{2}\sin\theta$
(i) if $\theta = 0$, E_1 & E_2 add [constructive]
(ii) if $\Delta r = \lambda/2$, E_1 & E_2 subtract [destructive]

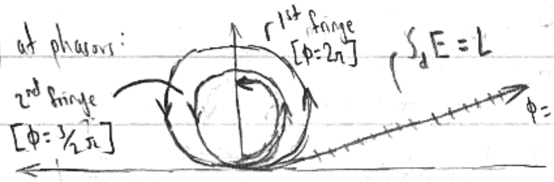
(*) "$\frac{d}{2}\sin\theta = m\lambda/2$" → "$d\sin\theta = m\lambda$"

~ Holds across entire slit: (i) If $\theta = 0$, E adds up along entire slit [constructive]

(ii) If $d\sin\theta = m\lambda$ [$m \neq 0$], for every point in slit, can find a point $d/2$ away (e.g.) to cancel it around ~ total: $E = 0$ [destructive]
↳ dark fringe

Intensity of Single-Slit Diffraction

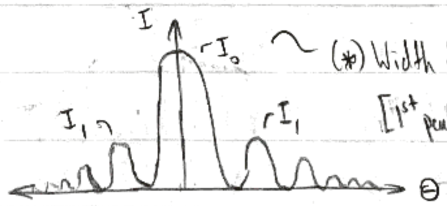
Looking at phasors:



Fringe every ½ circles; $L = N \cdot \pi D \rightarrow D = \frac{L}{N\pi}$ (diameter)

~ $I \propto D^2$ => $\boxed{I_N / I_0 = \left[\frac{1}{N\pi}\right]^2}$

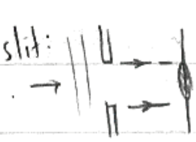
=> I peaks decrease for larger θ:



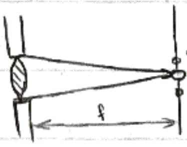
~ (*) Width of central max: $\sin\theta = \frac{2\lambda}{d}$
[1st peak: $\theta = \lambda/d$]; less diffraction as $\lambda\downarrow$, $d\uparrow$

(*) 2D Resolution Limits

Lens on slit:



[unfocused] vs:



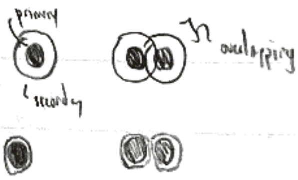
secondary peaks from diffraction [bad]
~ primary peak [I_0]

(*) Ex: $d = 1$ mm, $\lambda = 500$ nm → $\sin\theta = \frac{\pm 1}{2000}$ rad. $\approx \theta$; human eye resolution limit: $\sim \frac{1}{2000}$ rad.

lens

minima: $\boxed{\theta \approx \sin\theta = 1.22\,\lambda/d}$ ~ in telescopes:
[Rayleigh criterion – θ = angle of 1st minimum]

(i) bad lens [low d]:
(ii) good lens [high d]:



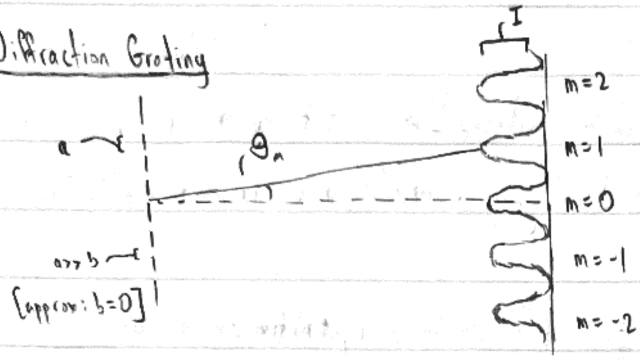
(*) Astronomers deal with small angles: $1° = \frac{2\pi}{360}\Omega$ → $1' = 1°/60$ [arc-minute] → $1'' = 1'/60$ [arc-second]
(limit of human eye, approx.) (good amateur telescope)

(*) Diffraction Review

3/7/24

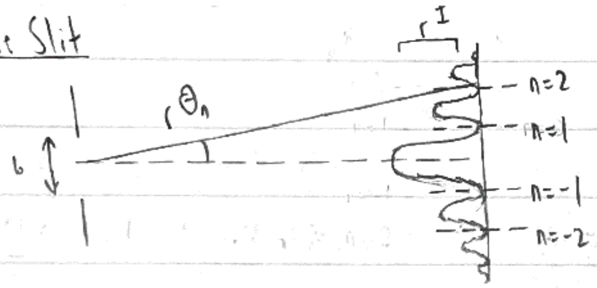
Disc 9

Diffraction Grating



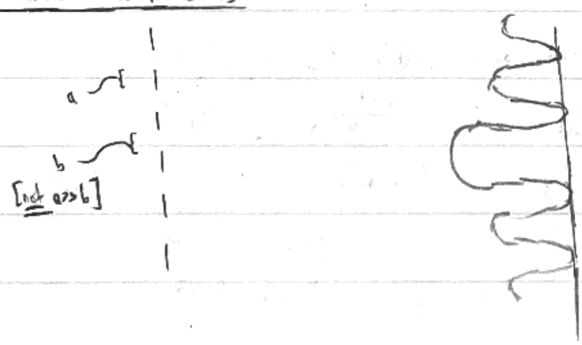
Maxima occur at $\boxed{a\sin(\theta_m) = m\lambda}$

Single Slit



Minima occur at $\boxed{b\sin(\theta_n) = n\lambda}$

Finite-Width Slits



In general: <u>both single slit, diffraction grating equations apply</u>

- As $b \to 0$, closer to diffraction grating
- As $n \to 1$, closer to single slit

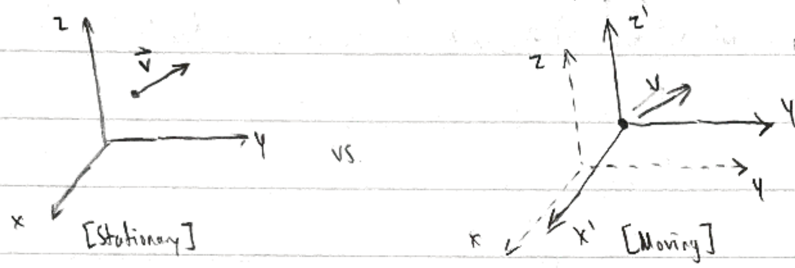
Galilean Relativity

3/5/24
Lecture 17

Galilean Relativity

Galileo: An object stays in motion until stopped by friction [later: Newton's Laws]

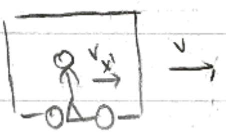
$\rightsquigarrow$ Galilean relativity: the motion of a moving object can be described either in a stationary plane (x, y, z) or moving plane (x', y', z')



(*) Ex: $x = x' + vt$

$$\left[\frac{d}{dt}x = \frac{d}{dt}x' + \frac{d}{dt}vt\right]$$

$\rightsquigarrow$ Velocity addition: $\underline{v_x = v_{x'} + v}$



velocity in stationary frame

$v_x = v_{x'} + v$ ~ velocity of moving frame

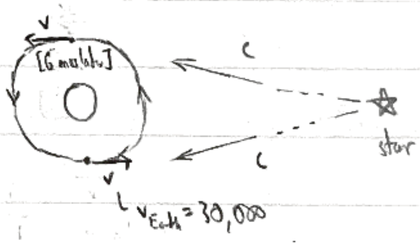
velocity in moving frame

Michelson-Morley Experiment

Per Maxwell's equations, $c = 3 \cdot 10^8$ $\rightsquigarrow$ Q: in what reference frame?

Michelson-Morley - measure speed of light from distant star at two points in Earth's orbit: (i) while $\vec{v}$ towards star

(ii) while $\vec{v}$ away from star



Expected (Galilean relativity): Light has $v_{light} = c + 30{,}000$ in case (i); $v_{light} = c - 30{,}000$ in case (ii) [relative to moving frame] if $v_{light} = c$ in stationary frame

Observed: $v_{light} = c$ in both instances, (i) & (ii) $\rightsquigarrow$ velocity addition fails (?)

Initial explanation: A "luminiferous aether" around Earth results in $v_{light} = c$ within aether

Later (Einstein, 1905): What if the E&M laws [$v_{light} = c$] are true everywhere, and the idea of "reference frames" was incorrect?

Einsteinian Relativity

3/5/24
Lecture 17
(cont.)

Einsteinian Relativity

(*) "Einstein's 1st Postulate"

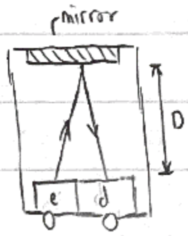
Per E&M laws, $v_{light} = c$ [within vacuum] is a universal result, but inconsistent with Galilean relativity

~ what needs to be changed about "reference frames" to accommodate E&M laws?

Answer: In "reference frames" assumption, time [t] assumed universal (i.e. same for all objects); in actuality, not true

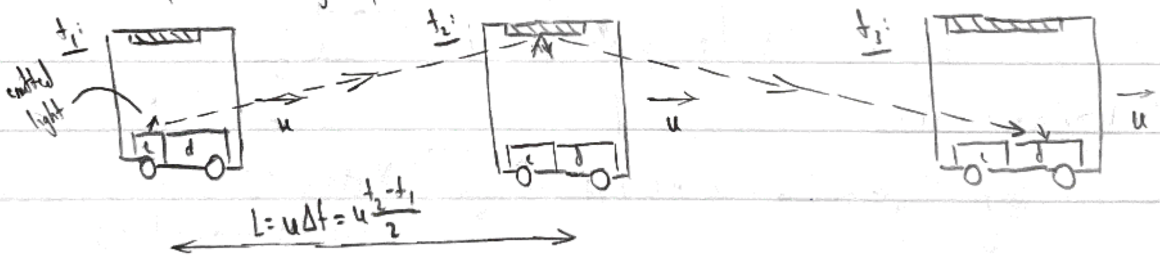
(*) "Light Clock"

Setup: Emitter of light e + detector of light d; one tick [of time] occurs when light from e reaches d [after reflecting off a mirror]



one tick takes $\Delta\tau \approx \frac{2D}{c}$ time [stationary]

~ Same setup, but moving very fast:



$s = \frac{c\Delta t}{2}$; $s^2 = D^2 + L^2 = \left(\frac{c\Delta\tau}{2}\right)^2 + \left(\frac{u\Delta t}{2}\right)^2 \Rightarrow (c^2 - u^2)\Delta t^2 = c^2\Delta\tau^2$

$\rightarrow \Delta t^2 = \frac{c^2}{c^2 - u^2}\Delta\tau^2$

path taken (moving frame)

path taken (stationary frame)

Define $\boxed{\beta = u/c} \rightarrow \Delta t^2 = \frac{1}{1-\beta^2}\Delta\tau^2 \Rightarrow \boxed{\Delta t = \frac{1}{\sqrt{1-\beta^2}}\Delta\tau}$

$\Delta\tau$ = time to tick in stationary frame; Δt = time to tick in moving frame; $\underline{\Delta\tau \neq \Delta t}$

=>

Special Relativity

3/7/24

Lecture 18 "Modern physics" built on 2 pillars: relativity & quantum mechanics

Special Relativity

Special relativity derived from 2 postulates:

(i) 1st Postulate: The laws of physics are the same in any inertial (i.e. non-accelerating) reference frame

(ii) 2nd Postulate: The speed of light in vacuum is universal [constant for all observers]

(*) Time Dilation

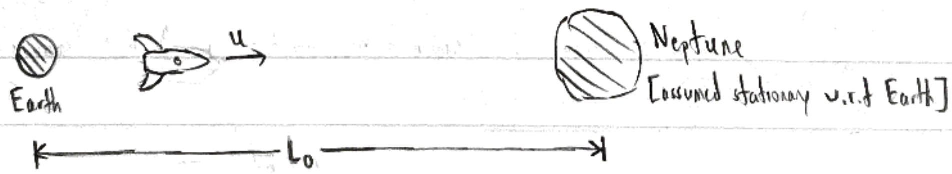
Recall (Light Clock): $\beta = u/c \Rightarrow \Delta t = \frac{1}{\sqrt{1-\beta^2}}\Delta\tau$; define $\boxed{\gamma = \frac{1}{\sqrt{1-\beta^2}}}$ [Lorentz factor]

(*) $0 \le \beta < 1$; $\gamma \ge 1$

(*) For small u: $\gamma = (1-(u/c)^2)^{-1/2}$ ⇝ use $(1-\varepsilon)^\alpha \approx 1-\alpha\varepsilon \Rightarrow \gamma \approx 1+\frac{1}{2}\left(\frac{u}{c}\right)^2$

(*) Length Contraction

Setup:



Spaceship moving w/ speed u relative to E-N frame [stationary]

⇝ By symmetry, E-N moving w/ speed u relative to spaceship

From stationary POV: $L_0 = ut$

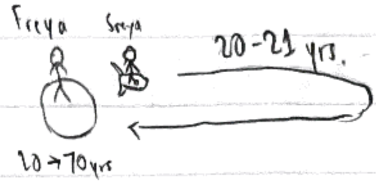
From moving POV: $L = u\tau = u(t/\gamma) \Rightarrow \boxed{L = L_0/\gamma} \Rightarrow$ distance is shorter in dir. of travel

(*) By symmetry, from stationary POV:  spaceship "shortened" by γ factor

Special Relativity (cont.)

3/7/24
Lecture 18
(cont.)

(*) Time Travel



Setup: Sreya takes a trip w/ $\gamma = 50$ for 1 yr

Their twin, Freya, stays on Earth

~ When Sreya returns @ 21 y.o. (before: 20 y.o.), Freya will be 70 [one-way time travel]

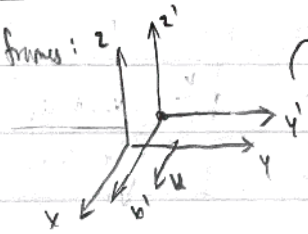
Twin Paradox: If Sreya has $\gamma = 50$ w.r.t. Freya, doesn't Freya have $\gamma = 50$ w.r.t. Sreya? Why isn't Freya the older one?

~ Soln: "Turning around" involves acceleration; no longer an inertial reference frame (spec. relativity doesn't apply)

(*) General relativity: Time runs slower in accelerating reference frames

The Lorentz Transform

Two reference frames:



Galilean relativity:

$x' = x - ut \iff v_x = v_{x'} + u \iff v_{x'} = v_x - u$

$[y' = y;\ z' = z;\ (t' = t)]$

Lorentz transforms - set of equations formalizing special relativity; depict 4D "spacetime"

$$\boxed{\begin{array}{l}\underline{\text{Lorentz Equations}}\\ \text{(i) } \underline{x' = \gamma(x - ut)} \\ \text{(ii) } \underline{y' = y;\ z' = z} \qquad \iff \quad (\vec{x}', t') = \begin{pmatrix} \gamma & 0 & 0 & -\gamma u \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -u^2/c & 0 & 0 & \gamma \end{pmatrix} (\vec{x}, t) \\ \text{(iii) } \underline{t' = \gamma\left(t - \frac{ux}{c^2}\right)}\end{array}}$$

↳ • Inverse equations: $x = \gamma(x' + ut)$, $t = \gamma\left(t' + \frac{ux'}{c^2}\right)$

• (i) gives time dilation: given clock stationary in S', $\Delta t' = t_2' - t_1' \Rightarrow \Delta t = t_2 - t_1 \rightarrow \underline{\Delta t = \gamma \Delta t'}$

• (iii) gives length contraction: given obj. stationary in S', $L' = x_2' - x_1' \Rightarrow L = x_2 - x_1 = \frac{1}{\gamma}(x_2' - x_1') \rightarrow \underline{L = L'/\gamma}$

Special Relativity (cont.)

3/7/24
Lecture 18

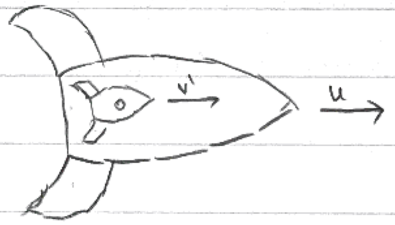
(*) Simultaneity

Assume two events happen at same time t' [in S'] at different positions x_1' & x_2'
$\sim$ what is Δt in S? ($\Delta t' = 0$)

Soln: $\Delta t = t_2 - t_1 = \gamma\left(t' + \frac{ux_2'}{c^2}\right) - \gamma\left(t' + \frac{ux_1'}{c^2}\right) \sim \underline{\Delta t = \gamma \frac{u}{c^2}(x_2' - x_1') \neq 0}$

Consequence: Two events that occur simultaneously in one reference frame S' may not be simultaneous in another reference frame S [if they occur in different locations].

Velocity Addition



Velocity addition (Galilean): $\underline{v = u + v'}$
$\sim$ if $u + v' > c$, claims $v > c$

Lorentz: $v_x = \frac{dx}{dt} = \frac{\gamma(dx' + u\,dt')}{\gamma\left(dt' + \frac{u\,dx'}{c^2}\right)} \xrightarrow{\frac{1/dt'}{1/dt'}} v_x = \frac{\frac{dx'}{dt'} + u}{1 + \frac{u}{c^2}\frac{dx'}{dt'}} \Rightarrow \boxed{v_x = \frac{v_x' + u}{1 + \frac{u}{c^2}v_x'}}$

Special relativity: nothing can travel faster than the speed of light
(*) Tachyon - hypothetical particles w/ $v > c$ [would break many physics laws if extant, e.g. causality]

Relativistic Momentum

3/12/24
Lecture 19

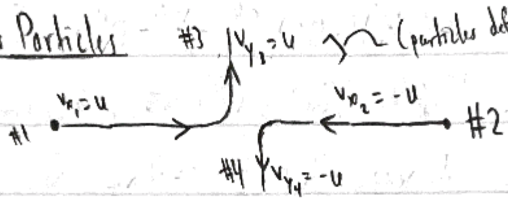
Relativistic Momentum

Recall (Non-Relativistic Momentum): $\vec{p} = m\vec{v}$ [Newton], is constant [conservation of momentum]

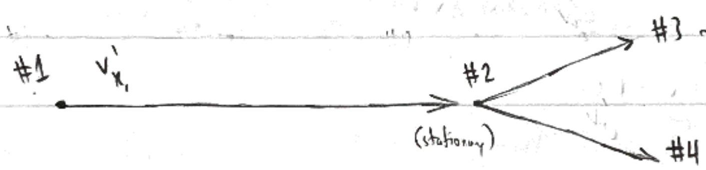
~ formula fails in relativity

(*) Ex: Equal Mass Particles (particles deflected)

Stationary frame:



~ construct moving frame S', moving w/ velocity u from position of #2:



Observations
(i) $v'_{y_3} = -v'_{y_4}$
(ii) $v'_{x_3} = v'_{x_4} = v'_{x}$ (unknown)

Looking at momentum before/after (using $\vec{p} = m\vec{v}$):

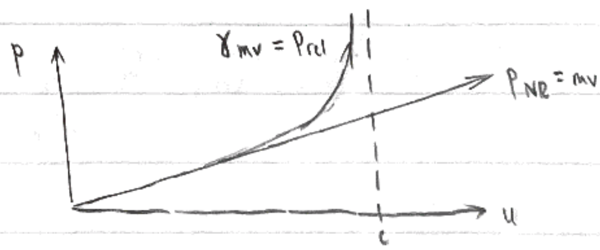
Initial total momentum: $\vec{p}'_{x_{tot}} = \vec{p}'_{x_1} = mv'_{x_1} = m\left(\frac{u+u}{1+u^2/c^2}\right) = \frac{2mu}{1+\beta^2}$ [velocity addition]

Final total momentum: $v'_x = \frac{0+u}{1+\frac{u(0)}{c^2}} = u \longrightarrow p'_{tot} = 2mv'_x = 2mu \neq \frac{2mu}{1+\beta^2}$

$\Rightarrow \vec{p}'_{initial} \neq \vec{p}'_{final}$ [if define $\vec{p} = m\vec{v}$, conservation of momentum fails]

~ Define relativistic momentum $\boxed{\vec{p} = \gamma m\vec{v}}$

preserves conservation of momentum in relativistic settings



Relativistic Energy

3/12/24
Lecture 19
(cont)

Relativistic Kinetic Energy

KE (non-relativistic) $= \frac{1}{2}mv^2 \rightarrow$ define relativistic kinetic energy: $\boxed{K = (\gamma - 1)mc^2}$

(*) To show $K \approx K_{NR}$ for $v \ll c$, use $(1+\varepsilon)^n = 1 + n\varepsilon + \frac{n(n-1)}{2}\varepsilon^2 + \ldots$:

$\gamma = (1-\beta^2)^{-1/2} \Rightarrow \gamma \approx 1 + (-\tfrac{1}{2})(-\beta^2) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2}\beta^4 + \ldots$ $[\varepsilon = \beta^2;\ n = -\tfrac{1}{2}]$

$\Rightarrow K = (\gamma - 1)mc^2 = \left(\beta^2/2 + 3\beta^4/8 + \ldots\right)mc^2 = \left(\frac{v^2}{2c^2} + \frac{3}{8}\frac{v^4}{c^4} + \ldots\right)mc^2 = \frac{mv^2}{2} + \frac{3}{8}\frac{mv^4}{c^2} + \ldots$

$\rightsquigarrow = \frac{1}{2}mv^2\left(1 + \frac{3}{4}\frac{v^2}{c^2} + \ldots\right)$; if $v \ll c$, $K \approx \frac{1}{2}mv^2 = K_{NR}$. ☒

Total & Rest Energy

Looking at KE expression: $K = \underbrace{\gamma mc^2}_{\text{dependent on } v \text{ [motion]}} - \overbrace{mc^2}^{\text{not dependent on } v \text{ [motion]}}$

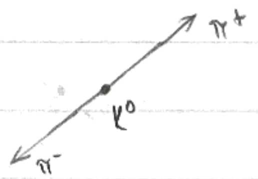
Rest energy also called "mass-equivalent energy"

$\rightsquigarrow$ Define total energy as sum of kinetic and rest energies:

$\boxed{E_{tot} = \gamma mc^2 = K + mc^2}$ $\Longleftrightarrow$ $\boxed{K = \gamma mc^2\ [E_{tot}] - mc^2\ [E_{rest}]}$

(*) Ex: Energy of Kaon Decay

A kaon is a particle that decays into 2 pions: $K^0 \rightarrow \pi^+\pi^-$.



Looking at rest energies: $E_{rest_0} = m_{K^0}c^2 = 493$ MeV

$E_{rest_f} = 2m_\pi c^2 = 2 \cdot 135 = 270$ MeV

$\rightsquigarrow E_0 = E_f \Rightarrow E_{rest_0} = K_f + E_{rest_f} \Rightarrow 493 = K + 270 \Rightarrow K_f = 223$ MeV

(!) Observation: Mass-equivalent energy decreased $\Rightarrow$ mass was lost! [converted to energy]

Relativistic Energy (cont.)

3/12/24
Lecture 19
(cont.)

Conservation Laws (Relativistic): (i) Energy IS conserved
(ii) Momentum is conserved
(iii) Mass is not conserved

Energy, Momentum, and Mass

Energy, momentum, and mass all related via relativity.

(*) Derivation: $(pc)^2 = (\gamma m v c)^2 = \gamma^2 m^2 v^2 c^2$

$\rightarrow (mc^2)^2 + (pc)^2 = m^2 c^4 (1 + \gamma^2 v^2/c^2) = m^2 c^4 (1 + \gamma^2 \beta^2)$

Looking at γ term: $1 + \gamma^2\beta^2 = 1 + \frac{\beta^2}{1-\beta^2} = \frac{1}{1-\beta^2} = \gamma^2$

$\Rightarrow (mc^2)^2 + (pc)^2 = \gamma^2 m^2 c^4 = (\gamma m c^2)^2 = E^2$

~ Final equation: $\boxed{E^2 = (mc^2)^2 + (pc)^2}$

(*) Equations for energy: (i) $E = \gamma m c^2$
(ii) $E_0 = mc^2$
(iii) $E = mc^2 + K$
(iv) $E = \sqrt{(mc^2)^2 + (pc)^2}$

(*) If in units w/ $c=1$, $E = \sqrt{m^2 + p^2}$ [theoretical physics]

~ Notice: None of these are exactly $E = mc^2$!

(*) Newton's 2nd Law

$F_{NR} = \frac{d}{dt} P_{NR} \Rightarrow$ relativistic generalization (using $\vec{p}_{rel}$): $\boxed{F = \gamma^3 m a} \Leftrightarrow \boxed{a = \frac{F}{\gamma^3 m}}$

(*) Relativistic Doppler Effect

3/12/24

Disc 10

+Lecture 19

(cont.)

Doppler Shift for E&M Waves

Setup: 

(*) Signs: $v > 0$ if towards source

→ Non-relativistic: $T_{source} = \frac{1}{f_{source}} = \lambda/c$, $T_{observed} = \lambda/(c+v)$ → $f_{obs} = \frac{c+v}{c} f_{source}$

$\underbrace{T_{observed}}_{\text{time between crests}}$

→ Relativity: $T_{source} = \gamma(\lambda/c)$ [from observer's frame - time dilation]

$$\Rightarrow f_{observed} = \frac{c+v}{\frac{1}{\gamma}c} f_s = \gamma \frac{c+v}{c} f_s = \gamma(1+v/c) f_s = \frac{1+v/c}{\sqrt{1-v^2/c^2}} f_s = \frac{1+v/c}{\sqrt{(1+v/c)(1-v/c)}} f_s = \sqrt{\frac{1+v/c}{1-v/c}} f_s$$

→ Relativistic Doppler effect: $\boxed{f_{obs} = \sqrt{\frac{c+v}{c-v}} f_{source}}$ [v: observer towards source]

(*) Nuclear Energy

Background:

- 1911 - Most of an atom's mass (> 99.95%) located in the nucleus (radius ≈ 10^{-15} m)
- 1932 - Neutron discovered; nuclei made up of protons & neutrons

  [Neutron: ~same mass as proton, no charge]

→ There must be some "new" strong & attractive force keeping nuclei together
+ force must be short-range (unlike Coulomb force) to have escaped prior detection

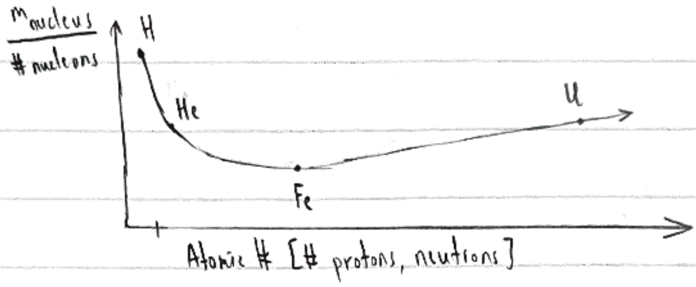
[Modern day: protons and neutrons each composed of 3 quarks; nuclear mass arises from forces between quarks]

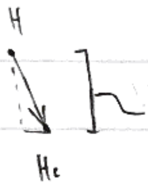
(*) Nuclear Energy (cont.)

3/12/24
Lecture 19
(cont.)

(*) Nuclear Energy (cont.)

Measuring nuclear mass:

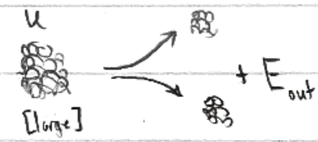




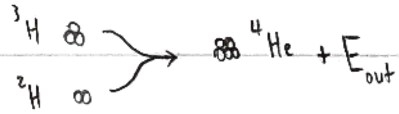
call change a decrease in "binding energy" $[\Delta E = \Delta m \cdot c^2]$

~ can interpret graph as an energy diagram - atoms want to go from higher E to lower E (Fe)

2 ways to reach lower energy: (i) Fission:



(ii) Fusion:



Benefit: Nuclear [mass-equivalent] energies much larger than chemical reactions ~ potentially more E

($m_{proton}c^2 = 938$ MeV $\rightarrow E_{nuclear} \approx$ MeV $= 10^6$ eV; vs $E_{chemical}$ = a few eV)

(*) Ex: 1 kg uranium fuel → 24,000,000 kWh <=> 2500 tons coal, 1600 tons of oil

(*) Nuclear Challenges: Hard to obtain fuel, generates radioactive byproducts

(*) Fast fission reactions used in atomic bombs

Fusion reactions (H → He, e.g.) drive Sun & stars in their cores [fuse until Fe → no more energy]

(*) Potential Upsides: More E than fission (10-100x), easier to acquire $^2H/^3H$, no radioactive waste

(*) Used in hydrogen bombs