

Physics 1C

2023-24

Winter 24

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Textbook: Young & Freedman - Mastering Physics

Topics: Magnetism, inductance, RLC circuits, AC circuits; Maxwell's equations, E&M waves, geometrical optics, interference & diffraction; special relativity

Table of Contents

1) Magnetism - 63

- i) Magnetic Force - 63
- ii) Biot-Savart Law - 66
- iii) Ampere's Law - 67
- iv) Magnetic Induction - 71

2) RLC & AC Circuits - 75

- i) Inductors & LC Circuits - 75
- ii) RLC Circuits - 78
- iii) AC Circuits - 78

3) E&M Waves - 81

- i) Maxwell's Equations - 81
- ii) E&M Plane Waves - 82

4) Geometrical Optics - 84

- i) Reflection & Refraction - 84
- ii) Polarization - 86
- iii) Mirrors - 88
- iv) Lenses & Magnification - 89
- v) Applications of Lenses - 91

5) Interference & Diffraction - 92

- i) 2-Slit Interference - 92
- ii) Thin-Film Interference - 93
- iii) Diffraction - 94

6) Special Relativity - 97

- i) Galilean Relativity - 97
- ii) Einsteinian Relativity - 98
- iii) Relativistic Momentum - 102
- iv) Relativistic Energy - 103

Magnetic Forces

1/9/24

Lecture 1

Magnetic Force Law

$$\text{D'Aloli (Electric force law): } \vec{F} = q\vec{E}$$

[Can define an analogous force law for magnetic fields [denoted \vec{B}]:

$$\boxed{\vec{F} = q\vec{v} \times \vec{B}}$$

(*) Cross Product

use right-hand rule to find dir.

D'Aloli: Given two vectors \vec{a}, \vec{b} : i) $(\vec{a} \times \vec{b}) \perp \vec{a}$ and $(\vec{a} \times \vec{b}) \perp \vec{b}$

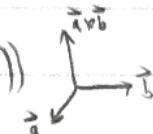


$$\text{ii) } |\vec{a} \times \vec{b}| = |\vec{a}| \cdot |\vec{b}| \sin \theta$$

$$\text{geometrically: } |\vec{a} \times \vec{b}| = \text{area of } \triangle \frac{1}{2} |\vec{a}| |\vec{b}|$$

Explicitly: Writing $\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$ (and similarly for \vec{b})

$$\rightarrow \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} \quad (\text{e.g. } (1, 0, 0) \times (0, 1, 0) = (0, 0, 1))$$



(*) Notation: $\odot = \text{at this page}$; $\otimes = \text{into page}$; $|\vec{a}| = a$

→ Consequence: A magnetic force is only exerted on moving charges (e.g. in a current)

Ex: Electron beam w/ rightward current \leftarrow ; magnet w/ outward field \Rightarrow



(*) Demo: Magnet on DC circuit [constant current] → constant force on circuit

Magnet on AC circuit [current oscillates] → oscillating force on circuit

(*) Particle Motion w/ Constant \vec{B}

Ex(i): \vec{B} parallel to $\vec{v} \Rightarrow \vec{v} \times \vec{B} = 0, \vec{F} = 0$ (motion unaffected)

Ex(ii): \vec{B} perpendicular to $\vec{v} \Rightarrow |\vec{F}| = qvB$ (motion curves)

(*) Ex: $v_i = \uparrow, \vec{B} = \odot, q = 0 \Rightarrow$



$$r \text{ radius: } \boxed{F = \frac{mv^2}{r}} \rightarrow r = \frac{mv}{qB}$$

[Circular motion]

Magnetic Force & Current

1/11/24

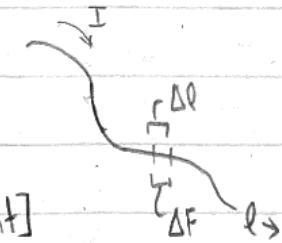
Magnetic Force on a Current

Applications: Magnetic field effects are typically studied w.r.t. currents

Current: 

Cn particles w/ velocity \vec{v}

Constitutive relation \vec{v} → define $\overline{\Delta l} = \Delta l$ in the direction of \vec{v}



$$\vec{v} = \frac{dx}{dt} \rightarrow \text{hence } \underline{\Delta t = \frac{\Delta l}{v}} \rightarrow I = \frac{\text{charge}}{\text{time}} = \frac{nq}{\Delta t} = \frac{nqv}{\Delta l} \text{ [current]}$$

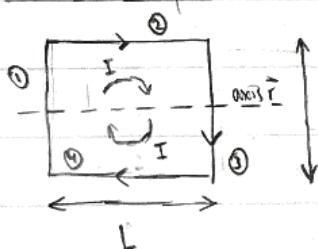
$$\Delta F = n(q\vec{v} \times \vec{B}) ; \vec{v} = \left(\frac{\vec{\Delta l}}{\Delta t}\right) v \rightarrow \Delta F = nv \frac{\vec{\Delta l}}{\Delta t} \times \vec{B} \rightarrow \underline{\Delta F = I(\vec{\Delta l} \times \vec{B})}$$

$$\text{Ansatz: } \vec{F}_{\text{ext}} = \int \Delta F \rightarrow \boxed{\vec{F} = \int I \vec{dl} \times \vec{B}} \quad [\text{integrated along the wire}]$$

$$(*) \text{ I constant} \rightarrow F = I \vec{S} \vec{j} \times \vec{B}$$

(*) Straight wire, constant $\vec{B} \rightarrow \vec{F} = I\vec{L} \times \vec{B}$

Current Loops



$$\vec{\tau} = \vec{r} \times \vec{F} \text{ (in dir. of rotational axis)}$$

Want to find torque $\vec{\tau}$, force \vec{F} from constant \vec{B}

(first: find \vec{F} on individual faces ①, ②, ③, ④)

\vec{B} dir.	$\vec{F}:$ ① ② ③ ④	\vec{F}_{Nt}	\vec{T}
⊗	↖ ↗ ↘ ↙	↗ ↘ ↗ ↖	↖ ↗ ↘ ↙
↑	↗ ↗ ↗ ↗	↗ ↗ ↗ ↗	LF[↑]

Define magnetic moment: $\mu = IA$ ~ A = area of loop [n loops $\rightarrow \mu = nIA$]

$$\rightarrow \vec{\tau} = \vec{\mu} \times \vec{B}$$

2. Find dir. of $\vec{\mu}$ via right-hand rule: curl fingers around loop in dir. of current

(*) Permanent Magnets

1/11/24

Lecture 2

Permanent Magnets

(cont.)

Solids composed of atoms; atoms contain electrons orbiting nuclei.

→ View electrons as small charges moving in a loop

→ each \vec{e} has a magnetic moment $\vec{\mu}_{\text{atom}}$.



Usual case: $\vec{\mu}_{\text{atom}}$'s inside a substance are unaligned, cancel out

(!) Special case: rotations of electrons are aligned $\rightarrow \vec{\mu}_{\text{tot}} = \# \text{ atoms} \cdot \vec{\mu}_{\text{atom}}$ [adds up]
→ entire substance gains a (potentially significant) magnetic moment $\vec{\mu}_{\text{tot}}$



Occurs in permanent magnets

(*) Magnetic field alignment can also be induced, e.g. via high temperatures

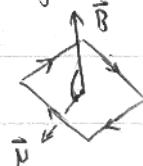
[$\vec{\mu}$ alignment is a high-energy state; need thermal $E >$ atomic E to create]

(*) Ferromagnets, once aligned, can be made to stay aligned [magnetized]

↳ magnetization places ferromagnet into a crystal shape / lattice

(*) Motors [Preview]

The torque generated by constant \vec{B} on a current loop works to bring $\vec{\mu}$ to align with \vec{B}



Usual case: $\vec{\mu}$ goes halfway (halfway, e.g.), then position stabilizes

→ Motors contain a commutator: on reaching the halfway, current flips direction [$\vec{\mu}$ flips]

→ torque stays in same direction, loop continues moving in same dir. (past halfway)

Conversely: spinning a motor w/o current creates a current [generators - Faraday's Law]

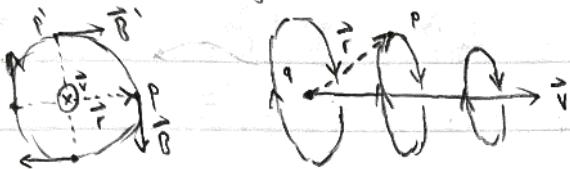
Biot-Savart Law

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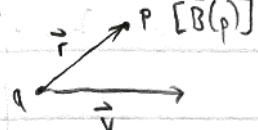
(*) Potential energy from a magnetic field: $U = -\vec{p} \cdot \vec{B}$

Biot-Savart Law (Moving Charge)

Moving charges create magnetic fields \rightarrow Biot-Savart Law (i):



$$\vec{B} = \frac{\mu_0 q v \times \hat{r}}{4\pi r^2}$$



(*) v, \hat{r} closer to parallel \rightarrow smaller \vec{B}

Constant: $\mu_0 = 4\pi \cdot 10^{-7} \text{ T} \cdot \text{m/A}$ $\rightarrow \frac{\mu_0}{4\pi} = 10^{-7} \text{ T} \cdot \text{m/A}$ (*) Obs.: μ_0 is very small

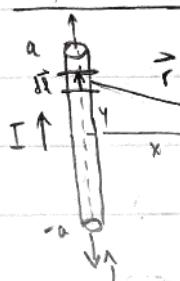
[Consequence: $|\vec{B}| \ll |\vec{E}|$, typically]

Biot-Savart Law (Current)

Similar to Mag. Force Law: $\text{Current } \xrightarrow{\text{length}} \text{Magnetic Dipole}$ \rightarrow Biot-Savart Law (ii): $d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2} \rightarrow \vec{B} = \int \vec{d}\vec{B}$

(*) μ_0 small \Rightarrow requires large current

(*) Ex (Finite Wire)



$$\vec{r} = \langle x, y \rangle = x\hat{i} - y\hat{j}, \quad r = \sqrt{x^2 + y^2}, \quad \hat{r} = \vec{r}/r; \quad d\vec{l} = dy\hat{j}$$

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{(dy\hat{j}) \times (x\hat{i} - y\hat{j})}{(x^2 + y^2)^{3/2}} = \frac{\mu_0}{4\pi} I \frac{(xdy)(-\hat{k})}{(x^2 + y^2)^{3/2}}$$

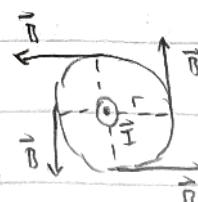
$$\vec{B} = \int_{-a}^{+a} d\vec{B} = \int_{-a}^{+a} \frac{-\mu_0 I x}{4\pi} \hat{k} \int_{-a}^{+a} \frac{dy}{(x^2 + y^2)^{3/2}} \quad ; \text{ replace "x" with "r"} \\ [y = r \tan \theta] \quad \mu_0 I = k$$

$$\rightarrow \vec{B} = -\hat{k} \frac{\mu_0 I}{4\pi} \left[\frac{2a}{r\sqrt{r^2 + a^2}} \right] \quad [\text{Finite Wire}]$$

(*) Ex (Infinite Wire)

$$a \rightarrow \infty \quad [r \ll a] \Rightarrow \frac{2a}{r^2 + a^2} \rightarrow 1 \Rightarrow$$

$$\vec{B} = -\hat{k} \frac{\mu_0 I}{2\pi r}$$



Ampere's Law

1/16/24

Lecture 3

(*) Ex: Force Between Parallel Wires

(cont.) Force between 2 parallel wires w/ currents I_1, I_2 : $I_1 \odot \dots \odot I_2 \sim (*)$ I_2 has no force on itself

$$\text{Force b/w } I_1 \text{ and } I_2: F_2 = I_2 \vec{L} \times \vec{B} = I_2 L \cdot \frac{\mu_0 I_1}{2\pi r} \rightarrow \boxed{F = \left(\frac{\mu_0}{2\pi}\right) I_1 I_2 L} \quad \begin{array}{l} [\text{small}] \\ (\text{same dir.} \rightarrow \text{attractive}) \\ (\text{opp dir.} \rightarrow \text{repulsive}) \end{array}$$

(*) Can define 1 ampere as requisite I to produce a certain F/L (given fixed r)

Ampere's Law

$$\text{Recall (Electricity): } \vec{E} = \frac{1}{4\pi\epsilon_0} \frac{\vec{I}}{r^2} \quad [\text{E. field}] \leftrightarrow \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{ext}}}{4\pi\epsilon_0} \quad [\text{Gauss's Law}]$$

Can define similar analogue for magnetism:

$$\oint \vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times \hat{r}}{r^2} \quad [\text{Biota-Savart}] \leftrightarrow \text{Ampere's Law: } \oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}} \quad \begin{array}{l} (\#3 \text{ of} \\ \text{Maxwell's} \\ \text{equations}) \end{array}$$

(*) Ex: Long Wire



$$\vec{B} \parallel d\vec{l} \quad B \text{ const.}$$

$$\oint \vec{B} \cdot d\vec{l} = \oint B \cdot dl = B \oint dl = 2\pi r B$$

$$2\pi r B = \mu_0 I_{\text{enc}} \Rightarrow \boxed{B = \frac{\mu_0 I}{2\pi r}} \quad [\text{some results as before}]$$

"Amperean loop"

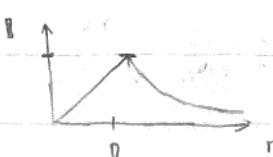
(*) Ex: Finite Thickness Wire

Wire w/ thickness R : \rightarrow (*) Unlike static conductors, current flows through all parts of the wire (incl. inside), not just the surface

$$\vec{B} \text{ for } r < R: \quad \oint \vec{B} \cdot d\vec{l} = 2\pi r B = \mu_0 I_{\text{enc}} \quad (\text{current density: } J = \frac{I}{A} = \frac{I}{\pi R^2})$$

$$\rightarrow 2\pi r B = \mu_0 J \pi r^2 \rightarrow \boxed{B = \frac{\mu_0 I}{2\pi r} \frac{r}{R^2}} \quad [r < R]$$

$$r > R: I_{\text{enc}} = I \rightarrow 2\pi r B = \mu_0 I \rightarrow \boxed{B = \frac{\mu_0 I}{2\pi r}} \quad [r > R]$$



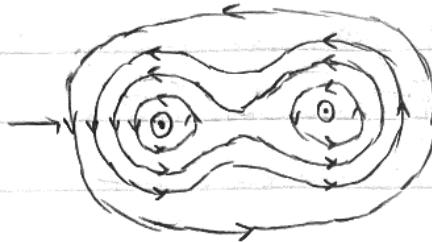
Ampere's Law (cont.)

1/18/24

Lecture 4

Ampere's Law

(*) Parallel Wires:



\vec{B} [mag. fields combine]

(*) Infinite Plane of Current/Charged Sheet

(can approximate as a series of conductors: $\leftarrow \boxed{\text{---}} \rightarrow [I \odot] \Rightarrow \leftarrow \text{----} \rightarrow \text{----} \rightarrow$)

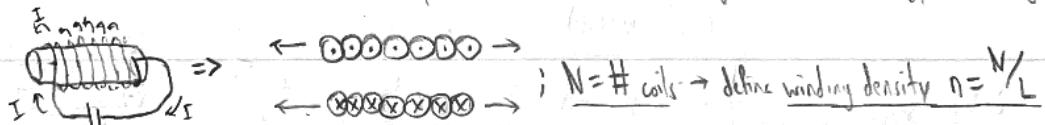
$$\leftarrow \text{----} \rightarrow \Rightarrow \oint \vec{B} \cdot d\vec{l} = \sum_1^4 S + \sum_2^3 S + \sum_3^4 S = 2S = 2Bl \sim \vec{B} = \frac{\mu_0 NI}{2\pi} \quad [N = \# \text{ I's}]$$

let $n = \frac{N}{l} \rightarrow \vec{B} = \frac{\mu_0 In}{2}$.

Ampere's Law for Solenoids

(used for switches, e.g.)

Solenoid - wire coiled into a helix shape; converts electrical energy to mech. energy [via mag. fields]



ideal solenoid - assume infinite length

i) \vec{B} Outside Solenoid

$$\leftarrow \text{----} \rightarrow \Rightarrow \oint \vec{B} \cdot d\vec{l} = \sum_1^4 S + \sum_2^3 S + \sum_3^4 S = S + S ; \begin{cases} S_1 = S(\vec{B}_0 - \vec{B}_0) \cdot d\vec{l} = 0 \\ S_2 = S(\vec{B}_0 - \vec{B}_0) \cdot d\vec{l} = 0 \end{cases} \text{ since L assumed to be infinite}$$

$\vec{B}_0 \rightarrow \vec{B}_0$

$$\rightarrow \oint \vec{B} \cdot d\vec{l} = 0 ; \mu_0 I_{ext} = \mu_0 I_{net} = \mu_0 (0) = 0 \quad \boxed{B=0}$$

ii) \vec{B} Inside Solenoid

$$\leftarrow \text{----} \rightarrow \Rightarrow \oint \vec{B} \cdot d\vec{l} = \sum_1^4 S + \sum_2^3 S + \sum_3^4 S = S \vec{B} \cdot d\vec{l} = Bl ; \mu_0 I_{ext} = \mu_0 NI$$

$\vec{B}_0 \rightarrow \vec{B}_0 \rightarrow \vec{B}_0 \rightarrow \vec{B}_0$

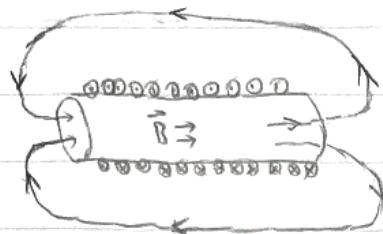
$\vec{B} = \frac{\mu_0 NI}{l} = \mu_0 I n$

Magnetism in Matter

1/18/24

Lecture 4
(cont.)

(* Real Solenoids:



Typically: length >> width

→ strong \vec{B} inside; very weak \vec{B} outside

Atomic Magnetism

Electrons drive magnetism: $\vec{B} \leftarrow \vec{p}$ (e^-) $V = -\vec{p} \cdot \vec{B} \rightarrow V$ minimized when $\vec{p} \parallel \vec{B}$: $\vec{B} \rightarrow \vec{p}$

Recall (Electricity): \vec{E} $V = -\vec{p} \cdot \vec{E}$; aligned: $\vec{E} \leftarrow \vec{p}$ \vec{E} unchanged; $\vec{E}_{ext} \downarrow$
[dielectrics]

Magnetism: $\vec{B} \rightarrow \vec{p} \cdot \vec{B}'$ $\rightarrow B_{tot} = B_0 + B' \rightarrow \vec{B}$ increases by aligning

→ Ampere's Law: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} \Rightarrow \oint \vec{B} \cdot d\vec{l} = \mu I_{enc}$; define permeability: $K = \frac{\mu}{\mu_0}$
(after alignment)

Alt: define susceptibility: $X_m = K_m - 1$ [rep. deviation from 1] material constant
[T-dependent]

3 Types of Magnetic Materials

i) Ferromagnetic: $K_m \gg 1$ [ex: iron, cobalt, nickel + alloys]

- Materials naturally already have internal small areas of alignment [magnetic domains], due to alignment promoting "better fit" into crystal structure; domains align easily on external \vec{B}

ii) Paramagnetic: $X_m \sim 10^{-4}$ [most common]

- In most materials, U gained from alignment \approx thermal energy; heat causes collisions, preventing alignment

iii) Diamagnetic: $K_m = 1 - X_m$; $X_m \sim 10^{-5}$ [rare: "antimagnetism"]

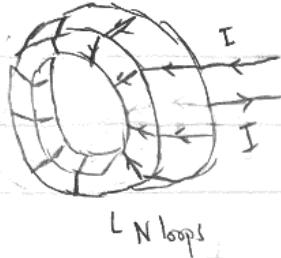
- Due to material properties (ex: graphite); causes repulsion rather than attraction

(* Can cause levitation

(*) Ampere's Law

1/18/24

(*) Ex: Toroidal Solenoid



- Want to find \vec{B} : i) at $r \leq r_1$
ii) at $r_1 < r \leq r_2$
iii) at $r_2 < r$

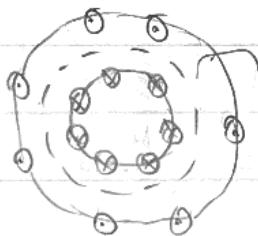
i) $r \leq r_1$



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}} ; I_{\text{enc}} = 0 \rightarrow \vec{B} = 0$$

(Ampere's Law)

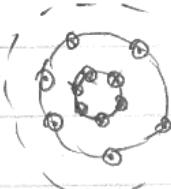
ii) $r_1 < r \leq r_2$



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}} = \mu_0 NI$$

$$\rightarrow \vec{B} \cdot 2\pi r = \mu_0 NI \rightarrow \boxed{\vec{B} = \frac{\mu_0 NI}{2\pi r}}$$

iii) $r_2 < r$



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}} = \mu_0 (NI - NI) = 0 \rightarrow \vec{B} = 0$$

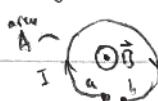
Magnetic Induction & Faraday's Law

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Lecture 5 Magnetic Induction

Induction: We can use magnetic fields to generate electrical current (ex: generators)

Magnet Into/Out of Loop



Assume \vec{B} uniform, and that $|B| = \text{const}$ $\left[\frac{dB}{dt} = \alpha > 0 \right]$ ↳ called an electromotive force (EMF)

→ change in \vec{B} causes current; creates voltage difference $\varepsilon = \Delta V_{ab}$

Properties: i) ε is larger when $\frac{dB}{dt}$ is larger (higher rate of change)

ii) ε is larger when the loop is larger

$$\rightarrow \text{Formula: } \varepsilon = \pm A \frac{dB}{dt}$$

$$\text{Magnetic flux: } \Phi_B = \int \int \vec{B} \cdot d\vec{A}$$

→ General case (Faraday's Law): $\varepsilon = - \frac{d}{dt} \int \int \vec{B} \cdot d\vec{A}$ [over surface bounded by loop]

equivalently: $\varepsilon = - \frac{d\Phi_B}{dt}$

Generating ε (4 ways)

1) Change \vec{B} over time (ex: circuits / transformers)

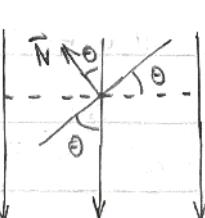
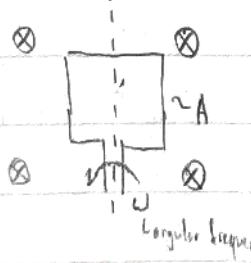
2) Change the area A of the loop

3) Change the angle between \vec{B}, \vec{A} (ex: generators)

4) [Non-uniform \vec{B}] Move the loop - motional EMF

(*) Rotating Loop, Constant \vec{B}

Top view: axis of rot., side view:



$$\theta = \omega t$$

$$\rightarrow \Phi_B = \vec{B} \cdot \vec{A} = BA_{\text{cos}(\theta)} = \pm BA_{\text{cos}(\omega t)}$$

$$\rightarrow (\text{Faraday's:}) \quad \varepsilon = BA_{\text{sin}(\omega t)}$$

$$\vec{B} \cdot \vec{A} = BA_{\text{cos}(\theta)}$$

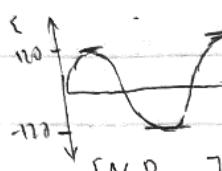
N loops: $\varepsilon = NBA_{\text{sin}(\omega t)}$

EMF & Lenz's Law

1/27/24

Lecture 5

(*) Ex: A.C. Power



U.S. AC: $f=60\text{ Hz} \rightarrow \omega = 2\pi f = 377$; want $\epsilon_{max} = 120\text{ V}$

(cont.)

Let $\vec{B} = 1.5\text{ T}$, coil $10\text{ cm} \times 10\text{ cm}$; how many loops are needed?

$$A: 120 = N B A \sin(\omega t) \rightarrow N \approx 21 \text{ loops}$$

ϵ : (+) or (-)?

Faraday's Law: $\epsilon = -\frac{d}{dt} \Phi_B \rightsquigarrow$ finding dir. of ϵ (steps): 1) Choose dir. for \vec{A} , normal to loop
 i) $\vec{S} \cdot \vec{A} = \odot$ ii) $\vec{A} = \odot \rightarrow \epsilon = 0$ iii) $\text{dir. of } \epsilon = \odot$
 2) Evaluate ϵ using Faraday's Law
 3) RHD: align thumb $\parallel \vec{A}$; curl fingers w/ loop
 $\rightarrow \text{dir. of fingers} = \text{dir. of } \epsilon \text{ if } \epsilon > 0$

Lenz's Law

"Induced currents & forces act in directions opposing changes to magnetic flux Φ_B ."

Lenz's Law

(*) Ex: $\sim \vec{B}'$ from $\vec{I} = \odot \times \vec{r}$ [Opposes original \vec{B}]

(*) Ex: Loop leaving B -field $\Rightarrow \Phi_B$ decreasing
 \rightarrow creates \vec{I} with $\vec{B}' = \odot \times \vec{r}$, $\vec{F} = \leftarrow$ [Opposing \vec{v}]

Induced Electric Fields

Recall: With Coulomb's Law, $\Delta V_{ab} = -\oint_C \vec{E} \cdot d\vec{l}$ path-independent (\vec{E} from charges) $\rightsquigarrow \oint_C \vec{E} \cdot d\vec{l} = 0$

Now: Changing \vec{B} generates \vec{E} loops \rightsquigarrow Faraday's: $\oint_C \vec{E} \cdot d\vec{l} = -\frac{1}{H} \Phi_B$ $\rightsquigarrow \oint_C \vec{E} \cdot d\vec{l} \neq 0$ if B changing



(*) Differential form: $\nabla \times \vec{E} = -\frac{1}{H} \vec{B}$

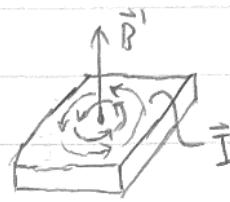
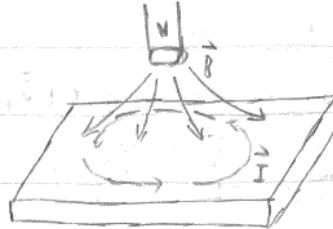
Eddy Currents

1/25/24

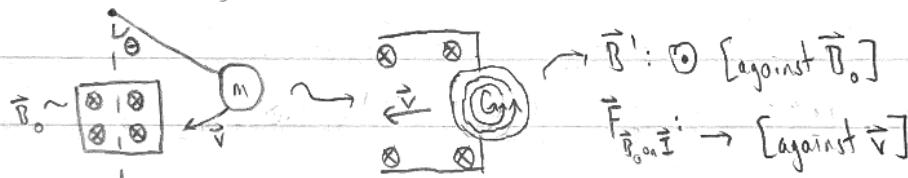
Lecture 6 Eddy Currents

When a magnet is brought near a conductor, generates current \vec{I} as loops inside conductor; \vec{I} creates \vec{B}' opposing \vec{B} [Lenz's Law], (called eddy currents)

If $R > 0$, some \vec{I} dissipated as heat; remainder creates \vec{B}' creates \vec{F} . (various applications)



(+) Ex: Stopping a Pendulum



(*) Superconductivity

Superconductors: materials with exactly 0 electrical resistance

- Eddy currents, once generated, persist forever
- Try to "avoid" / "exclude" mag. fields & changes in mag. field within interior \rightarrow strange interactions w/ magnets (e.g. levitation)
- First superconductors found at -4°K (requires liquid He); later: "high-temp" @ $\sim 77^{\circ}\text{K}$ (liquid N)

(+) Ex: Motional EMF

$$\textcircled{1} \quad \vec{B} = 0 \quad \Phi_B = |\vec{B}| \cdot (A - h \cdot v t) \rightarrow \epsilon = -\frac{d\Phi_B}{dt} = |\vec{B}| v h \quad [\text{Faraday's Law}]$$

$$\text{Diagram: A rectangular loop of wire with height } h \text{ and width } l \text{ is moving with velocity } v \text{ to the right. The loop has resistance } R \text{ and is connected to an open terminal pair. The magnetic field is zero. The induced emf is } \epsilon = \frac{Blv}{R}.$$

$$\text{Ohm's Law: } I = \frac{V}{R} \rightarrow I = \frac{\epsilon}{R} = \frac{Blv}{R}$$

$$\rightarrow \vec{F}_B = SI \vec{A} \times \vec{B} = \frac{IB^2 v h^2}{R}$$

$$\text{Diagram: A rectangular loop of wire with height } h \text{ and width } l \text{ is moving with velocity } v \text{ to the left. The loop has resistance } R \text{ and is connected to an open terminal pair. The magnetic field is zero. The induced emf is } \epsilon = \frac{Blv}{R}.$$

$$\rightarrow F_{\text{tot}} = \vec{F}_0 - \vec{F}_D = \vec{F}_0 - \frac{IB^2 v h^2}{R}$$

Displacement Current

1/25/24

Displacement Current

(formally: $I_{\text{end}} = \oint \vec{J} \cdot d\vec{A}$ [over surface enclosed by loop])

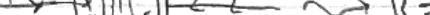
Lettres

Recall (Ampere's Law): $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$ works for any loop shape, not just planar loops.

$$\rightarrow \frac{I}{I} \sim \bar{B} = \frac{\mu_0 I}{2\pi r} \quad \leftarrow \rightarrow \frac{I}{I} \quad [\text{gives same answer}]$$

→ Paradox:

 $\vec{B} = \frac{\mu_0 I}{2\pi r}$

 $\vec{B} = 0$

Observation: as $I \rightarrow C$, $\vec{E}_{\text{capacitor}} \uparrow$
 Recall: $q = CV$, $C = \frac{\epsilon_0 A}{d}$, $V = \vec{E} \cdot \vec{d}$

$\rightarrow q = CV = \epsilon_0 \vec{E} \cdot A = \epsilon_0 \oint \vec{E} \cdot d\vec{A} = \epsilon_0 \Phi_E$

$\rightarrow I = \frac{dq}{dt} = \epsilon_0 \frac{d\Phi_E}{dt}$

Maxwell: Define displacement current $i_0 = \epsilon_0 \frac{\partial E}{\partial t}$, vs conduction current i_c [I from before]

$$\sim \text{Amper's Law (ii)}: \oint \vec{B} \cdot d\vec{l} = \mu_0 (i_c + i_o) = \mu_0 I_{\text{end}} + \mu_0 \epsilon \frac{d}{dt} (\oint \vec{E} \cdot d\vec{A})$$

Consequence: $\vec{B} > 0$ between capacitor plates due to displacement current

(*) Maxwell's Equations

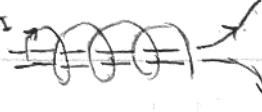
- Maxwell's Equations $\left\{ \begin{array}{l} \text{i) Gauss's Law: } \Phi_E = \frac{q_{\text{enc}}}{\epsilon_0} \\ \text{ii) Gauss's Law for Magnetism: } \nabla \cdot \vec{B} = 0 \\ \text{iii) Faraday's Law: } \epsilon = -\frac{d}{dt} \Phi_B \quad \sim \text{charging } \vec{D} \text{ gives } \vec{E} \quad \text{fields propagate each other} \\ \text{iv) Ampere's Law: } \nabla \cdot \vec{B} = \mu_0 (i + i_0) \quad \sim \text{charging } \vec{E} \text{ gives } \vec{B} \end{array} \right.$ → electromagnetic radiation
[light]: electric, magnetic

LC Circuits

1/30/24

Lecture 7

Self-Inductance in a Solenoid



Recall: $B = \mu_0 I \rightarrow \Phi_B = B \cdot A \cdot N = (\mu_0 I) AN$

$$\rightarrow \epsilon = -\frac{1}{R} \Phi_B = -(\mu_0 AN) \frac{dI}{dt}$$

$$L_{\text{solenoid}} = \mu_0 AN$$

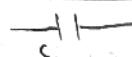
call this self-inductance L

[units: Henries]

LC Circuits

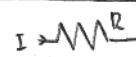
3 characterizations of ΔV :

i) Capacitors



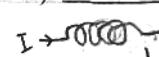
$$\Delta V = \frac{Q}{C}$$

ii) Resistors



$$\Delta V = IR = R \frac{dQ}{dt}$$

iii) Inductors

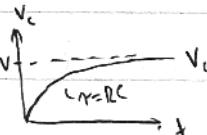


$$\Delta V = -L \frac{dI}{dt} = -L \frac{dQ}{dt^2}$$

Recall RC circuits:

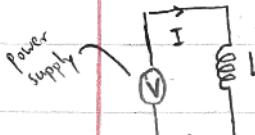


$\Delta V = V_C$; capacitor builds up charge



→ Inductors: 2nd Q derivative gives oscillatory behavior (useful for wireless signals, e.g.)

Magnetic Field Energy



Assume V, I start at 0; V increasing over time
 Recall: $P = IV = I(-\epsilon) = I(L \frac{dI}{dt})$ [into inductor] note: energy stored in inductor as current released when $I \downarrow$

$$\sim \Delta U = P \Delta t \rightarrow U = \int P dt = \int I(L \frac{dI}{dt}) dt = L \int I dI \rightarrow U = \frac{1}{2} LI^2$$

[Energy in magnetic field]

Recall: $L = \mu_0 NA = \frac{\mu_0 A}{l} [solenoid]$; $B = \frac{\mu_0 I}{l} \rightarrow I = \frac{lB}{\mu_0 N} \rightarrow U = \frac{1}{2} LI^2 = \frac{1}{2} \mu_0 (Al) B^2 = \frac{1}{2} \mu_0 (Volume) B^2$.

$$\sim U = \frac{1}{2} \mu_0 Volume B^2$$

[E in magnetic field B]

(*) Vs capacitors: $U = \frac{1}{2} C Q^2 \rightarrow U = \frac{1}{2} \epsilon_0 |E|^2$ [E in electric field]

LC & RLC Circuits

1/30/24

Lecture 7

LC Circuits

$$\text{Loop Rule: } \sum \Delta V = -L \frac{di}{dt} - \frac{q}{C} = 0$$

$q(t) = A \cos(\omega t + \phi)$

$\omega = \sqrt{\frac{1}{LC}}$

$x(t) = A \cos(\omega t + \phi) \leftrightarrow [x = \sqrt{V_m}]$

$i(t) = -A \omega \sin(\omega t + \phi) = A \omega \cos(\omega t + \phi + \pi/2)$

Observation: when charge q maximal, $i=0$ [B vice versa]

$$E_{cap} = \frac{1}{2} C q^2 \propto q^2$$

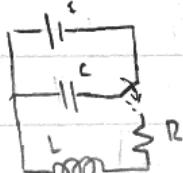
$$E_{ind} = \frac{1}{2} L i^2 \propto i^2$$

Energy moving between C, L ; current flipping direction

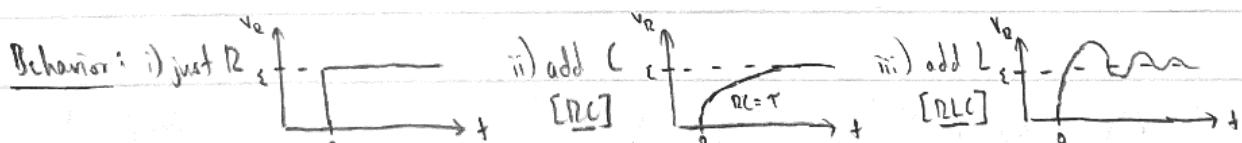
(*) Where does energy come from? i) AC power supply \oplus ii) $\overset{\text{radio waves}}{\text{antenna creates } \vec{B} \text{ from } \omega}$

$$[\text{only works if } \omega_{\text{ext}} = \sqrt{\frac{1}{LC}}]$$

RLC Circuits



- More "realistic" circuit: i) All circuits have some R [have some R]
 ii) All circuits store some charge on wire [have some C]
 iii) All current loops create \vec{B} , creates L [have some L]



- (iii): skin depth damping; can critically damp

- (ii) & (iii): more $R \rightarrow$ more time to reach 0

LR & RLC Circuits

2/1/24

Disc 4

+ lecture 8

(*) LR Circuits

$$\sum \Delta V = E - IR - L \frac{dI}{dt} = 0 \quad [\text{Loop Rule}]$$

$$\rightarrow E = IR + L \frac{dI}{dt} \rightarrow \frac{dI}{dt} = \frac{E}{L} - \frac{RI}{L} \rightarrow \int_{I(0)}^{I(t)} \frac{dI}{\frac{E}{L} - \frac{RI}{L}} = \int_0^t dt$$

$$\rightarrow -\frac{L}{R} \ln \left(\frac{I(t) - \frac{E}{R}}{\frac{E}{R}} \right) \Big|_{I(0)}^{I(t)} = t$$

solve for I

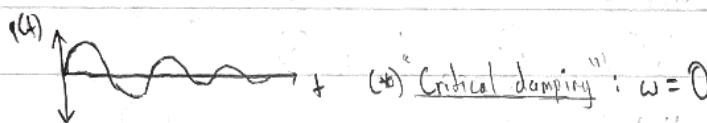
$$I(t) = \frac{E}{R} \left(1 - e^{-\frac{Rt}{L}} \right)$$

$$\tau = L/R \quad [\text{LR}]$$

RLC Circuits

Look at bottom loop: $\sum \Delta V = 0 = -L \frac{dq}{dt} - R \frac{dq}{dt} - \frac{q}{C}$ [Damped harmonic oscillator]

$$q(t) = q_0 e^{-\frac{Rt}{2L}} \cos(\omega t + \phi) ; \quad \gamma = \frac{R}{2L}, \quad \omega = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$



(*) AC Power

VS: 120V, 60Hz

(*) $170 \rightarrow 120$: take RMS & V $[\sqrt{\text{avg. } V^2}]$

[fun fact]

Power P in a resistor: $P = V^2/R$ [varying w/ rate 60 Hz]

\rightarrow average power $\langle P \rangle = \langle V^2/R \rangle = \frac{1}{2} \langle V_0^2 \sin^2 \omega t \rangle = \frac{V_0^2}{2} \langle \sin^2 \omega t \rangle$

$$\rightarrow \langle P \rangle = \frac{(V_0/\sqrt{2})^2}{R} = \frac{V_{\text{RMS}}^2}{R} \quad [V=170 \rightarrow V_{\text{RMS}}=120]$$

Define $I_{\text{RMS}} = I_0/\sqrt{2}$; $P = I^2 R$ \rightarrow

$$\langle P \rangle = I_{\text{RMS}}^2 R$$

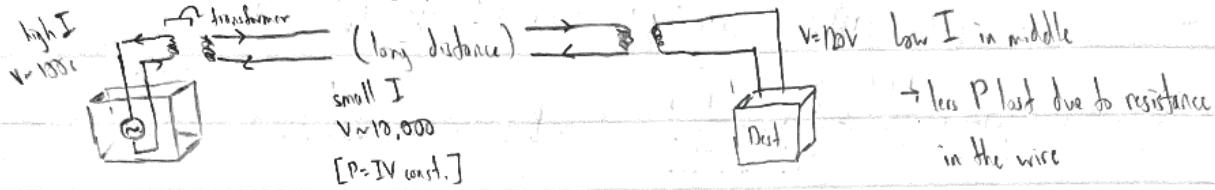
AC Circuits

2/1/24

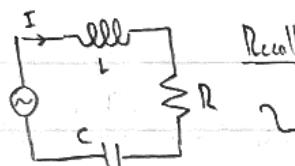
Lecture 8

(*) AC Power

AC power good for transporting power over long distances



AC-driven RLC Circuits



$$\text{Recall (RLC Circuits): } \ddot{q} = -L\frac{d^2q}{dt^2} - R\frac{dq}{dt} - \frac{1}{C}q$$

$$\rightarrow \text{AC Power: } \boxed{-L\frac{d^2q}{dt^2} - R\frac{dq}{dt} - \frac{1}{C}q = V_s \sin(\omega t)}$$

→ 2 solutions: 1) Transitory: (already discussed)

2) Steady state:

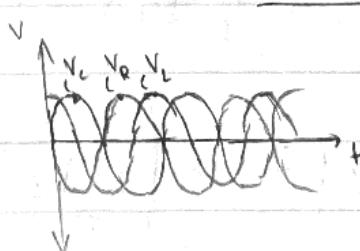
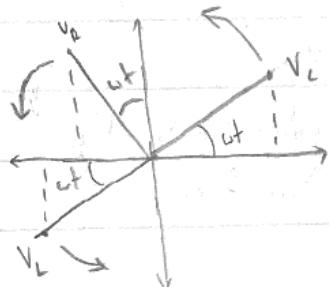
add together to obtain final soln.

Choose phase: $q = A \sin(\omega t)$

$$V_C = \frac{q}{C} = \frac{A}{C} \sin(\omega t) \rightarrow V_C = A \cdot \frac{1}{C} \sin(\omega t)$$

$$V_R = R \frac{dq}{dt} = AR \omega \cos(\omega t) \rightarrow V_R = AR \omega \sin(\omega t + \pi/2)$$

$$V_L = L \frac{dq}{dt} = -AL \omega^2 \sin(\omega t) \rightarrow V_L = -AL \omega^2 \sin(\omega t + \pi)$$



(*) Observe: $V_R = IR = R \cdot I_0 \sin(\omega t + \pi/2)$

$\rightarrow V_R$ carries phase of current
vs V_C (behind), V_L (ahead)

Reactance & Impedance

2/6/24

Lecture 9

Reactance

Want to find effect of current on voltage: $\frac{|V_L|}{|I|}$ and $\frac{|V_C|}{|I|}$; called reactance (ohms)

$$\text{Recall: } q = A \sin(\omega t) \rightarrow i = A \omega \sin(\omega t + \frac{\pi}{2}) ; |V_L| = \frac{|q|}{C} = A/C, |V_L| = L \left| \frac{dI}{dt} \right| = LA\omega^2$$

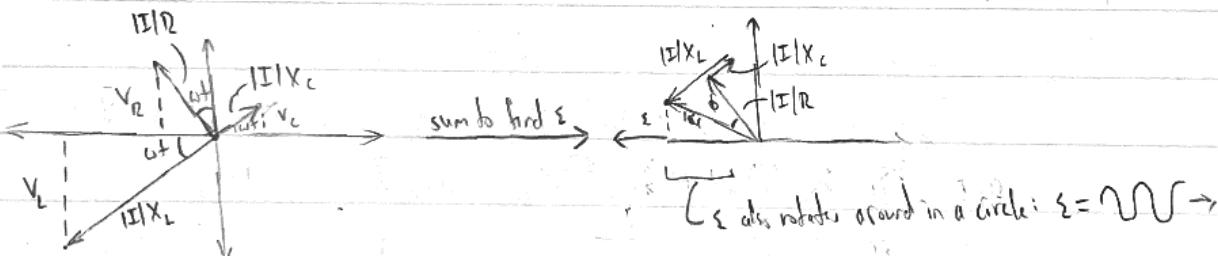
$$\rightarrow \text{Capacitive reactance: } \frac{|V_C|}{|I|} = X_C = \frac{1}{\omega C} ; \text{ Inductive reactance: } \frac{|V_L|}{|I|} = X_L = \omega L$$

(note: as $\omega \rightarrow 0$ [DC], $X_C \rightarrow \infty$)

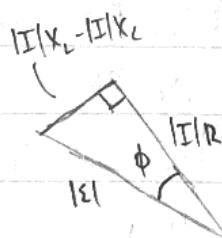
Phasors & Impedance

$$\text{Recall: } V = IR$$

$$e(t) = |I| [X_C \sin(\omega t) + R \sin(\omega t + \frac{\pi}{2}) + X_L \sin(\omega t + \pi)]$$



In particular:



$$|e| = \sqrt{|I|^2 R^2 + (|I|X_C - |I|X_L)^2}$$

$$|e| = |I| \sqrt{R^2 + (X_C - X_L)^2}$$

Define impedance:

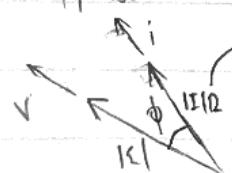
$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$V = IZ$$

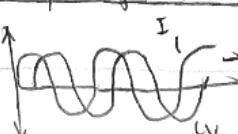
works for both V_{rms}/I_{rms} , V_{dc}/I_{dc}

[Ohms] (DC) LR, C circuits: just remove other terms

Additionally, observe:



Current, voltage out of phase i ratio: $\tan \phi = \frac{|I|(X_C - X_L)}{|I|R}$



$$\tan \phi = \frac{X_C - X_L}{R}$$

Resonance & Transformers

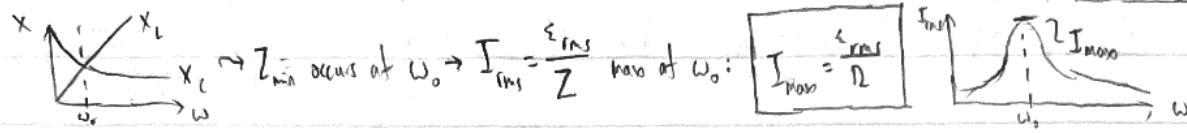
2/6/24

Lecture 9

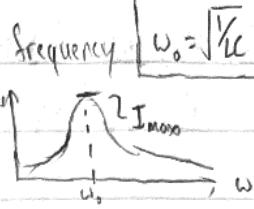
(cont.)

Resonance

$X_C = \frac{1}{\omega C}$, $X_L = \omega L \rightarrow$ can vary circuit ω to obtain $X_C = X_L$ at resonant frequency $\omega_0 = \sqrt{\frac{1}{LC}}$

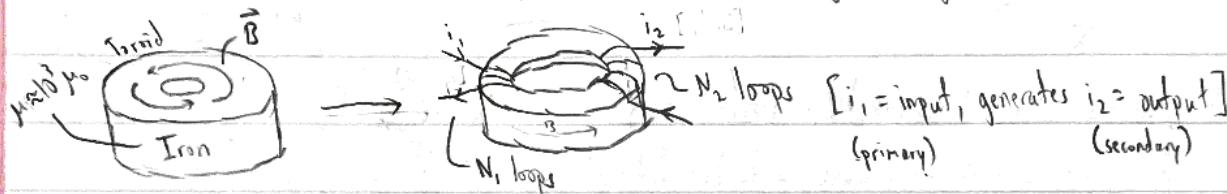


$$I_{\text{max}} = \frac{V_{\text{rms}}}{Z}$$



Transformers

Transformers used to transfer energy between circuits, change voltage levels



Look at cross-section:

$\Phi_B = \int \vec{B} \cdot d\vec{A}$ everywhere inside toroid; AC current \rightarrow alternating $\vec{B} \sim \vec{E} \frac{d\vec{B}}{dt}$

\rightarrow for each loop: $\epsilon = -\frac{d\Phi_B}{dt} \sim \epsilon_1 = N_1 \frac{d\Phi_B}{dt}; \epsilon_2 = N_2 \frac{d\Phi_B}{dt}$

\sim Observe: $\frac{\epsilon_2}{\epsilon_1} = \frac{N_2}{N_1}$; alt: $\frac{\epsilon_2}{\epsilon_1} = \frac{N_2}{N_1} \epsilon_1 \sim \frac{N_2}{N_1}$ called "turns ratio"

\rightarrow Consequence: more turns N_2 than $N_1 \rightarrow \epsilon_2$ larger [increases voltage]

(*) Note: Overall power constant ($P_{in} = VI = \epsilon_1 i_1 = P_{out} = \epsilon_2 i_2$) $\rightarrow i_2$ lower to compensate

[Assumes no power lost (due to eddy currents, e.g.)]

Complete equation:

$$\frac{i_1}{i_2} = \frac{\epsilon_2}{\epsilon_1} = \frac{N_2}{N_1}$$

(*) Power in AC Circuits

Recall (Power): $P = VI$

In AC circuits: V, I out of phase by $\phi \rightarrow$ new equation:

$$P_{\text{avg}} = V_{\text{rms}} I_{\text{rms}} \cos(\phi)$$

Planar Waves

2/6/24

Lecture 9

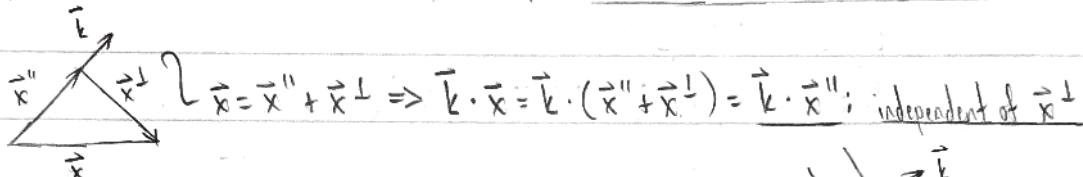
Planar Waves

(k : "wave number"; wavelength $\lambda = 2\pi/k$)

+ Lecture 10
Recall (1D Waves): $f(x) = A \sin(kx)$, $f(x-vt) = \text{traveling wave}$ [for each input, produces 1 value]
 ~ 3D Waves [EM Waves]: output vectors for \vec{E} , \vec{B} field (6 numbers)

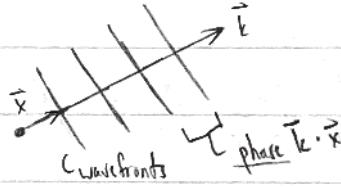
$$\omega = \vec{k} \cdot \vec{v}$$

Specify position vector \vec{x} , wave vector \vec{k} → new eqn.: $f(\vec{x}, t) = A \sin(\vec{k} \cdot \vec{x} - \omega t)$



→ f constant at all points along lines/planes \perp to \vec{k}

→ creates wavefronts of planar wave, \perp to dir. of travel



Maxwell's Equations in Vacuum

Vacuum: no charge Q , current I ; but can still have \vec{E} , \vec{B} fields

~ Maxwell's Eqs. [in vacuum]:

- i) $\oint \vec{E} \cdot d\vec{A} = 0$
- ii) $\oint \vec{B} \cdot d\vec{A} = 0$
- iii) $\oint \vec{E} \cdot d\vec{l} = - \frac{\partial \vec{B}}{\partial t}$
- iv) $\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

Symmetry between \vec{E} , \vec{B}

(iii) & (iv): Changing \vec{E} gives \vec{B} , & vice versa ~ can configure to create each other

~ \vec{E} loop from $\frac{\partial \vec{B}}{\partial t}$ from $\frac{\partial^2 \vec{E}}{\partial t^2}$ gives 2nd order diff. eq. (electromagnetic wave eqn.)

~ \vec{E} , \vec{B} propagate like waves: define EM waves (EM radiation to describe)

EM Plane Waves

2/8/24

EM Plane Waves

(EM waves are transverse waves)

Lecture 10

From Maxwell's eqns., derive results: i) $\vec{E} \perp \vec{B} \perp$ dir. of wave \hat{k} , velocity $\vec{v} = \vec{c}$ (cont.)

ii) $E = cB$

iii) $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

$$c \approx 2.997 \cdot 10^8$$

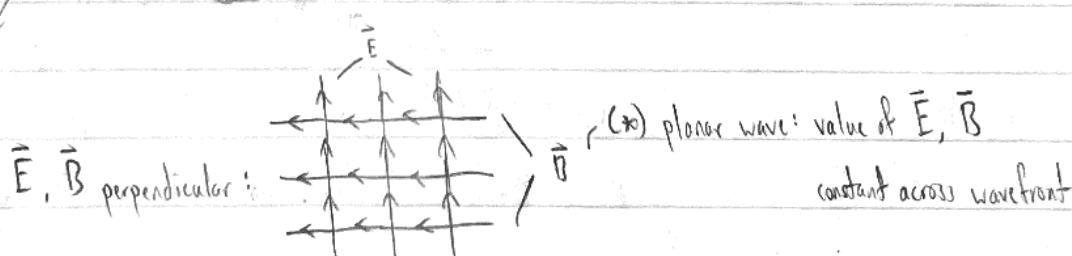
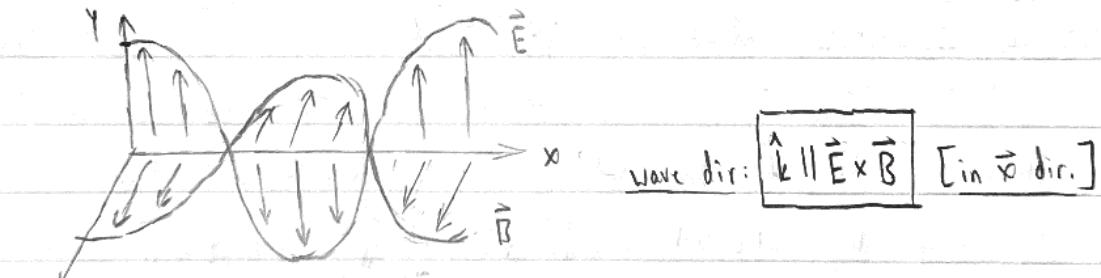
iv) Waves can travel in vacuum

Wave equation:

$$\frac{\partial^2 \vec{E}_y}{\partial x^2} = \epsilon_0 \mu_0 \frac{\partial^2 \vec{E}_y}{\partial t^2} \quad (\epsilon_0 \mu_0 = \nu^2)$$

$\rightarrow \vec{E}_y(\vec{x}, t) = \vec{E}_y(x, t) = \vec{E}_y(x - vt)$; model with/assume sine wave
 (x along \hat{k})

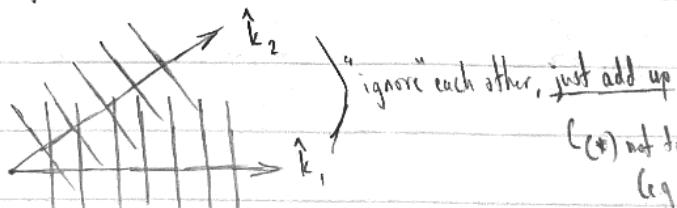
Visualizing EM Plane Waves



(*) Superposition

Maxwell's equations linear [can add together diff. solns. for new soln.]

~ can superimpose EM waves:



(*) not true of all waves
(e.g. water waves)

EM Plane Waves (cont.)

2/13/24

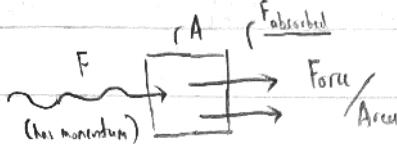
Lecture 11

Intensity & Radiation Pressure

Recall: intensity = $\frac{\text{power}}{\text{area}}$ \rightarrow intensity of an EM wave: $I = \frac{1}{2} \epsilon_0 E_{\max}^2$

$$\text{corollary: } E_{\max} = \sqrt{\frac{2I}{\epsilon_0 c}} ; B_{\max} = \frac{E_{\max}}{c}$$

Intensity & pressure coincide:



$$P_{\text{rad}} = \frac{I}{c} [\text{Pa} / \frac{\text{W}}{\text{m}^2}]$$

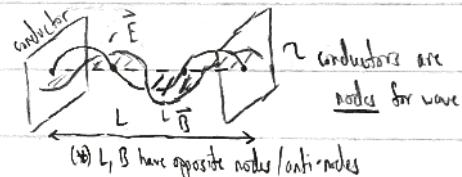
(*) Some materials: F reflected:

$$\begin{array}{c} F_{\text{in}} \\ \curvearrowleft \\ F_{\text{out}} \end{array} \rightarrow P_{\text{rad}} = \frac{2I}{c}$$

Generally quite small [1 billionth of Pa]
 \rightarrow more consequential in space

Standing EM Waves

Recall: $\vec{E} = 0$ inside conductors \rightarrow EM waves between conductors:



\sim when $L = n\frac{\lambda}{2}$, obtain standing EM wave:

[standing wave: locations of peak amplitude constant]

\sim [some frequency, opposite direction]

(*) Ex (Microwaves): $\lambda = 12.2$ good for heating water \sim all dimensions multiples of 6.1 cm

\sim consequence: heating cup w/ $d \approx 6.1$



\vec{E} node \rightarrow no heating

(!) $v \leq c$ always

EM Waves in Matter

In vacuum: $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$; in matter, $\mu = K_m \mu_0$ & $\epsilon = K_E \epsilon_0$ \rightarrow in matter:

$$v = \frac{1}{\sqrt{\mu \epsilon}}$$

(generally small; $K_m \approx 10^{-4}$) water, glass: $K_E \approx 2$

\sim Define refractive index:

$$n = \frac{c}{v} = \sqrt{\frac{\mu \epsilon}{\mu_0 \epsilon_0}}$$

$$\geq 1$$

\sim (*) Water: $n \approx 1.33$

Glass: $n \approx 1.4 - 1.7$ [diff. types]

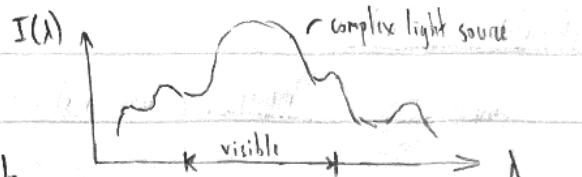
Intro to Optics

2/13/24

Types of Light

Many answers for "types of light": 1, 7, ∞ , etc.

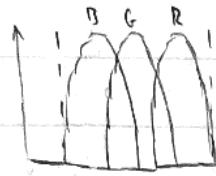
Visible light: 7 (rainbow), 4 (CMYK), 3 (RGB), etc.



lecture 11

(cont.)

Eye sensitivity:

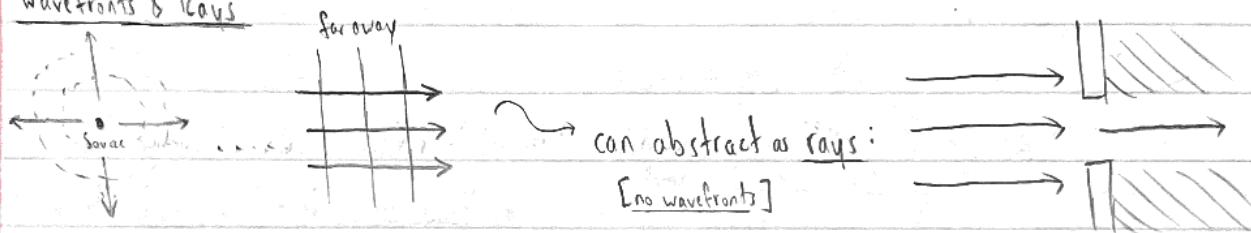


→ brain gets B, G, R signals → 3 ms: luminance (total)

(*) tetrachromats: 4 signals

+ 2 chromaticity: ratios $\frac{\text{blue}}{\text{total}}$, $\frac{\text{green}}{\text{total}}$

Wavefronts & Rays



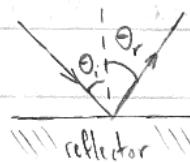
→ Geometrical optics: study of light in terms of rays; for mirrors, lenses, etc.

(*) Ignored wavelength: $\lambda_{\text{light}} = 500 \text{ nm}$ (small) \sim within 10 mm lens: 20,000 λ s [a lot]

Reflection & Refraction

Reflection law for smooth surfaces:

$$\theta_i = \theta_r$$

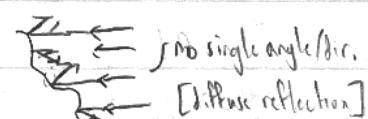


[Specular reflection]

(*) metal conductor, e.g.



[curved mirror]

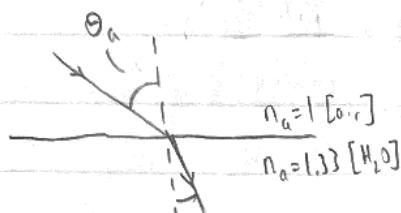


) no single angle/dir.
[diffuse reflection]

Refraction law for smooth surfaces:

[Snell's Law]

$$n_a \sin \theta_a = n_b \sin \theta_b$$



(*) Critical angle: value

if θ_a when $\theta_b = 90^\circ$

Alt.:

$$\sin \theta_a = \frac{n_b}{n_a} \sin \theta_b$$

(*) all θ 's $\in (0, \pi/2)$

Intro to Optics (cont.)

2/15/24

Lecture 12

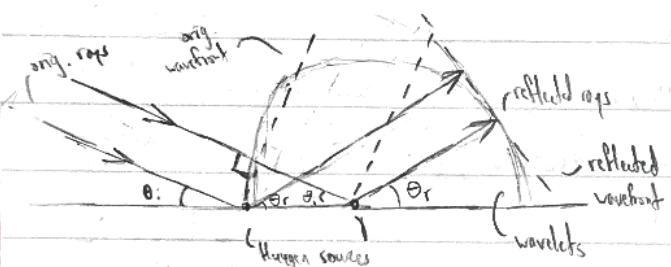
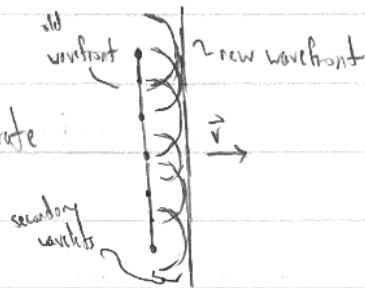
(*) Refraction (Prisms)



Principle: n varies slightly with λ [recall: diff. colors = diff. λ_s] ~ colors bent at slightly different angles (dispersion)

Huygen's Principle

Huygen's Principle: Every point on a wavefront will itself generate secondary wavelets radially outward.

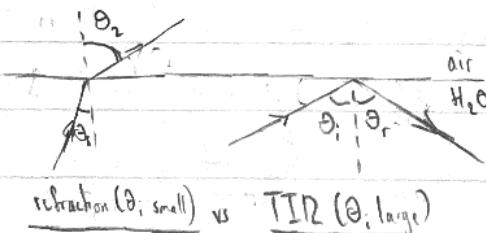


Applied to reflection: wavefronts hitting reflector creates Huygen sources generating wavefronts in dir. of reflected rays ($\theta_i = \theta_r$).

Total Internal Reflection (TIR)

Recall (Snell's Law): $n_1 \sin \theta_1 = n_2 \sin \theta_2 \rightsquigarrow \sin \theta_2 = \left(\frac{n_1}{n_2}\right) \sin \theta_1$

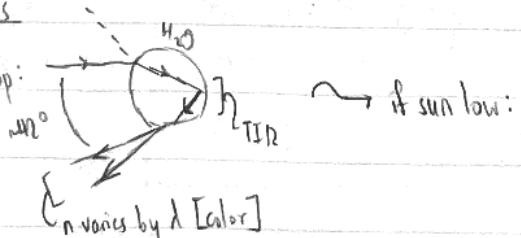
→ If $\frac{n_1}{n_2} > 1$, for some angle θ_1 : no solution → no refraction, just reflection (TIR)



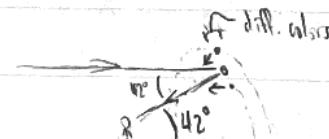
(*) Most reflectors (e.g. mirrors) have only partial reflection (~80%); TIR → 100%.

(*) Ex: Rainbows

Sunlight into raindrop:



if sun low:

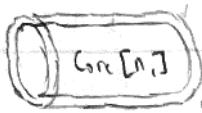


Polarization

2/15/24

Lecture 12

(*) Fiber Optics



-cladding (n₂)



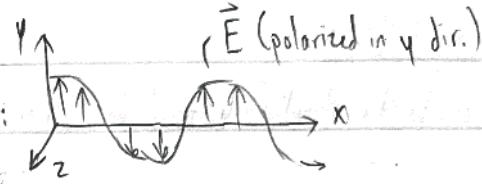
Transmits information using light (for communication, e.g.)

long distance → needs lots of reflections; uses TIR to prevent decay

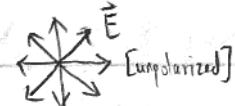
(cont)

Polarization

Polarized light travels with \vec{E} displaced along only 1 axis:

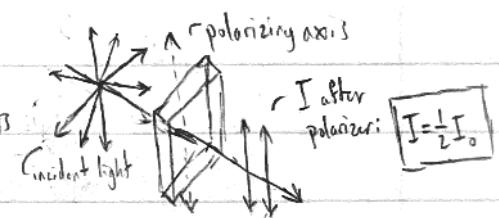


Most light (e.g., natural light) unpolarized: composed of many waves going in all directions randomly (i.e., polarizations in all directions)

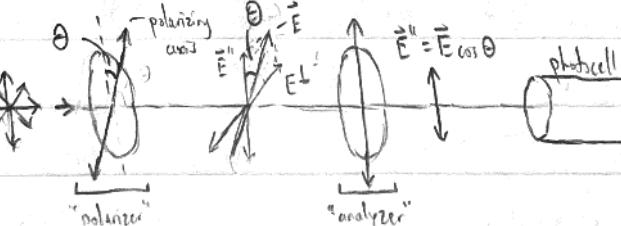


$$I_{\text{after polarizer}} = \frac{1}{2} I_0$$

→ Use polarizers to create polarized light: given unpolarized light, only lets specific polarizations [parts of \vec{E} field] pass through



Multiple polarizers:



(*) \vec{E} not transmitted is absorbed by photocell

$$\vec{E}_{\text{analyzer}} = \vec{E}_{\text{polarizer}} \cos \theta$$

(0 if $\theta = 90^\circ$; max if $\theta = 0^\circ$)

Intensity: $I \propto E^2 \Rightarrow$

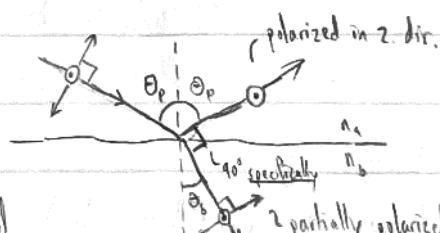
$$I = I_{\max} \cos^2 \theta \quad (\text{rel. to "original" incident})$$

$$I_0: I = \frac{1}{2} I_0 \cos^2 \theta$$

Polarization by reflection: at a specific angle of incidence,

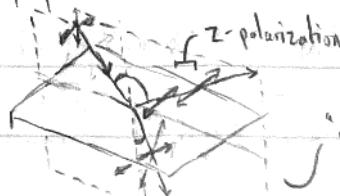
any $\vec{E} \perp$ to plane of incidence [plane containing ray, reflected

+ refracted rays] only reflected → reflected ray completely polarized



→ Occurs at Brewster's Angle:

$$\tan \theta_p = \frac{n_b}{n_a}$$



"plane of incidence"

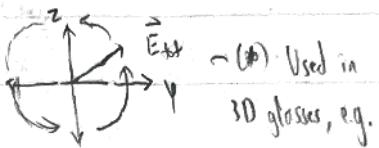
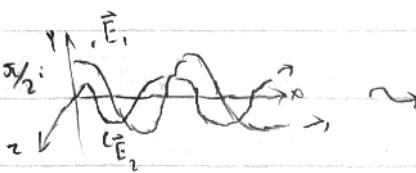
Scattering of Light

2/20/24

Lecture 13

(*) Circular Polarization

Can combine 2 waves w/ phase shift $\pi/2$:



~ (*) Used in
3D glasses, e.g.

Scattering of Light

Light can be [elastically] scattered upon colliding with particles [scatters]:



~ If $\delta \ll \lambda$ [e.g. $N_2 = 0.3 \text{ nm}$ vs $\lambda_{\text{visible}} = 400-750$], Rayleigh scattering:

$$I_{\text{scattered}} \propto \lambda^{-4}$$

(*) Ex: Smaller δ [e.g. violet] cause more scattering ~ sky is blue

(*) Sunsets



During sunset, light has to travel further through atmosphere [is scattered more]

→ blues totally scattered; only higher- λ colors (e.g. reds, yellows) remain [sunset]

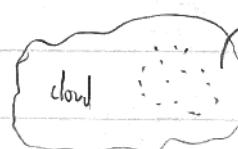
atmosphere

If $\delta > \lambda$, all δ s scattered equally

~ appears white/dark (dep. on scatterer density).

(*) Exception: Dust absorbs more blue than red

~ sandstorms, Mars atmosphere appear reddish



water droplets

cloud

[less dense → white;
more dense → dark]

(*) Antennas

Receive EM waves (e.g. radio) ~ want find good antenna length

(λ too small → little signal [energy transferred]; λ too big also undesirable)

~ Optimal: $L = \lambda/4$

Plane & Concave Mirrors

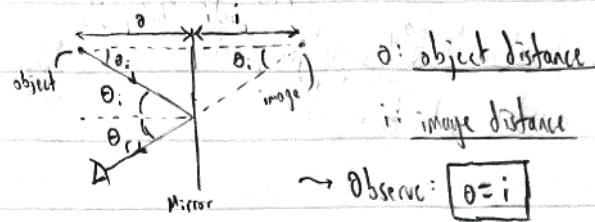
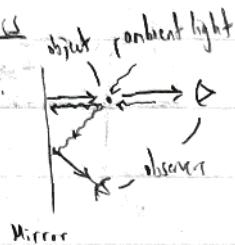
2/20/24

Lecture 13

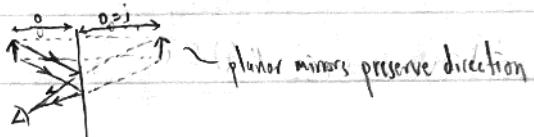
(cont.)

Geometrical Optics

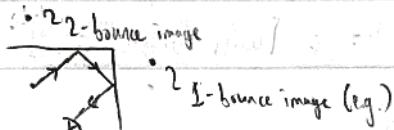
Plane Mirrors:



Use rays to determine image direction

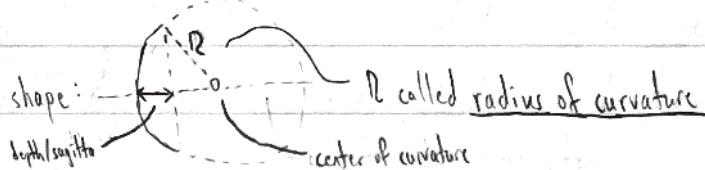


(*) Multiple mirrors:

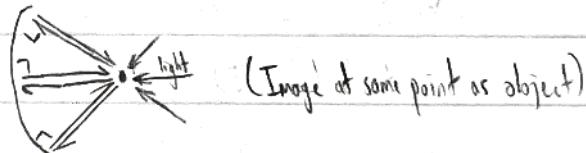


Concave Mirrors

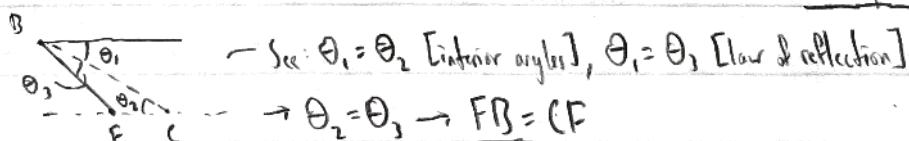
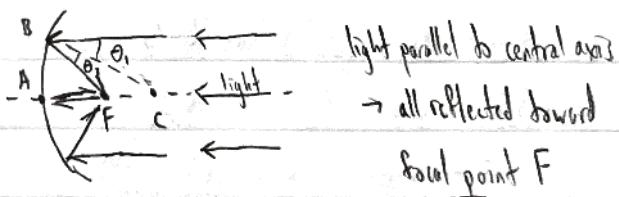
Concave mirrors spherical in shape:



Case (i) - Light from center of curvature:



Case (ii) - Light from far away (dist. ∞):



Small-angle approximation: If AB \parallel CA (i.e. small depth), then FA \approx FB

$$\rightarrow CF \approx FA \rightarrow FA \text{ [focal length]} = f = R/2$$

Focal Length

2/20/24

Lecture 13
+ Disc 7

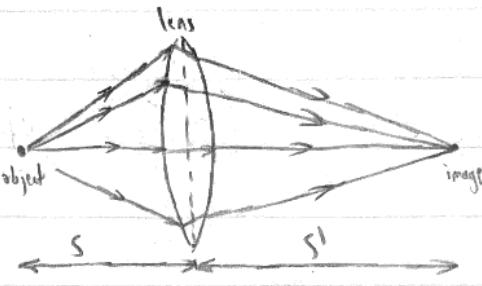
Focal Length

(given as a point)

Given a lens w/ focal length f & an object, define

(i) $s =$ dist. from lens center to object

(ii) $s' =$ dist. from lens center to object image

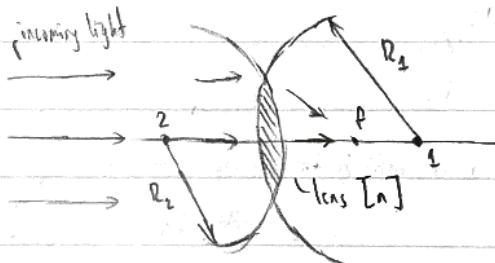


→ Focal length equation: $\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$ ~ (☞ $1/f$ called "bending power")

(*) For eyeglasses i. use power $P = 1/f$ [units: Diopters = $\frac{1}{m}$]

Lensmaker's Equation

Represent a lens as intersection of two spheres with radii R_1, R_2 & assign refractive index n



→ Lensmaker's Equation: $\frac{1}{f} = (n-1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$

(*) Sign Conventions

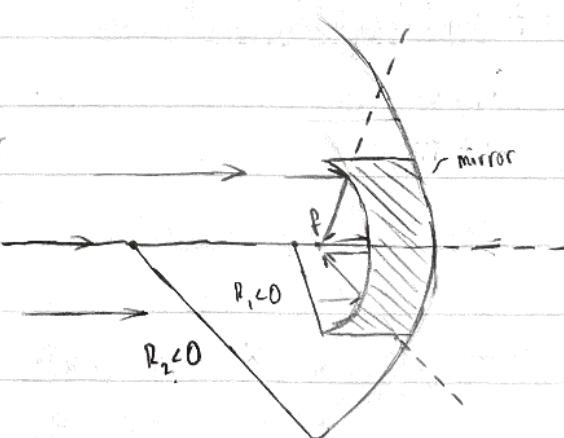
• Side of incoming light has $R < 0$

• Side of outgoing light has $R > 0$

• If image on opposite side of object, $f > 0$

• If image on same side as object, $f < 0$

(i.e. side of incoming light)



(*) Plane Mirror: $R_1, R_2 = \infty \rightarrow 1/f = 0, s = -s'$

(*) Concave Mirror: $f = R/2$

(*) Convex Mirror: $f = -R/2$

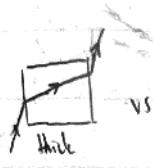
Lenses & Magnification

2/22/24

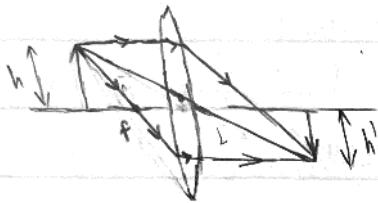
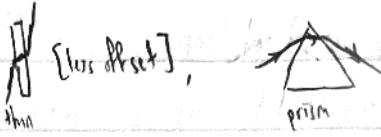
Lecture 14

Lenses

Lenses offset rays:



vs.



2 observations: (i) Image across lens is flipped

(ii) Image, object need not be same size

→ Define magnification:

$$M = \frac{h'}{h} = -\frac{s'}{s}$$

(*) Sign Rules

Convention for signs in equations:

(i) Object distance - $s > 0$ if object on same side as incoming light; $s < 0$ otherwise

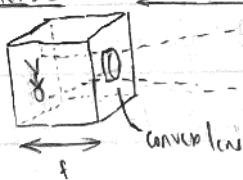
(ii) Image distance - $s' > 0$ if image on same side as outgoing light; $s' < 0$ otherwise

(*) Focal Length Signs

(i) Convex lens: (1) $f > 0$ (ii) Concave lens: (2) $f < 0$

(iii) Plano-convex: (3) $f > 0$ (iv) Plano-concave: (4) $f < 0$

Cameras



$s > f$

- If s large, $\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \rightarrow s' = f$, image \times recorded by film/digital sensor

If s small:

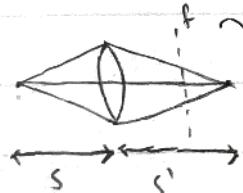


image blurry at distance f

→ 2 solutions: (i) Move lens

(ii) Adjust f of lens [focusing]

(*) Modern lenses

composed of 5-13
indiv. lenses

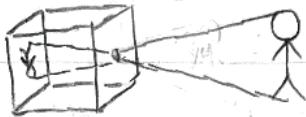
Applications of Lenses

2/22/24

(*) Pinhole Cameras

(cont.)

Pinhole cameras use small holes instead of lenses:



(*) very small hole; called an aperture

- Pros: Takes rays directly \rightarrow no need to focus, as in lenses.

- Cons: Small hole \rightarrow low light received [$A \downarrow \Rightarrow I \downarrow$]; requires long exposures

For fast exposures, need sensitive sensors; sensitivity given by f-number/f-stop

[Small f-num \rightarrow more light; typically from ≈ 2 , "very good" ≈ 1.4]

$$f_{\text{num}} = f/D$$

(*) Human Eyes



Anatomy: \rightarrow for $s > 75 \text{ cm}$: exact # age-dependent

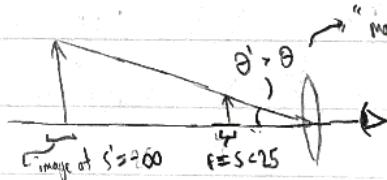
Eye uses muscles to control lens [accommodation]:

20 Diopters (lens) + 40 (cornea) \rightarrow total: $P_{\text{eye}} = 60 \text{ Diopters}$

(*) Magnifying Lenses

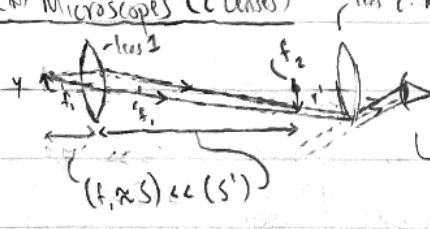
Uncorrected: $\frac{1}{f} = \frac{1}{s} - \frac{1}{s'} \quad (\theta \text{ small})$

vs. with magnifier:



$$s = -25 \rightarrow s' \approx f_{\text{lens}}; \text{ if } \theta \text{ small, } \theta \approx \tan \theta = \frac{s}{s'} = \frac{1}{f} \Rightarrow M = \frac{s'}{s} = \frac{1}{f} \quad [\text{angular}]$$

(*) Microscopes (2 lenses)



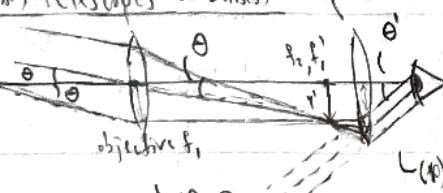
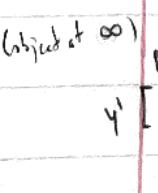
lens 2: magnifying glass

$$M_1 = -\frac{s'}{s} \approx \frac{1}{f_1}, M_2 = \frac{1}{f_2} \approx \frac{1}{s'}$$

$$M_{\text{tot}} = M_1 \cdot M_2$$

M_1 : lateral magnification
 M_1, M_2 : angular magnification

(*) Telescopes (2 lenses)



$$\theta = -\frac{1}{f_1}, \theta' = \frac{1}{f_2} \rightarrow \text{angular magnification}$$

$$M = \frac{\theta'}{\theta} = -\frac{f_1}{f_2}$$

(*) image size: $\frac{1}{s} = \frac{1}{f_1} + \frac{1}{f_2}$

(*) For taking pictures: put image sensor at f_2 directly

2-Slit Interference

2/27/24

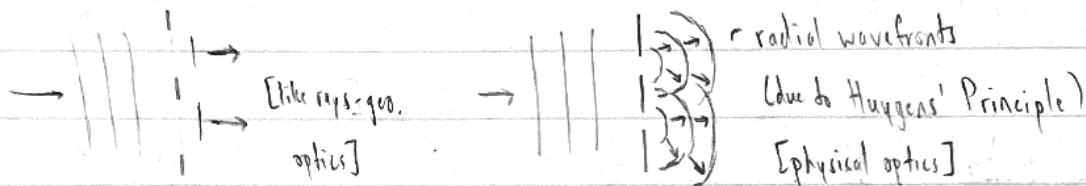
Physical Optics: Field studying optical phenomena resulting from wave nature of light [Ent rays]

- Interference caused by two waves overlapping at a point [transforms: constructive & destructive]

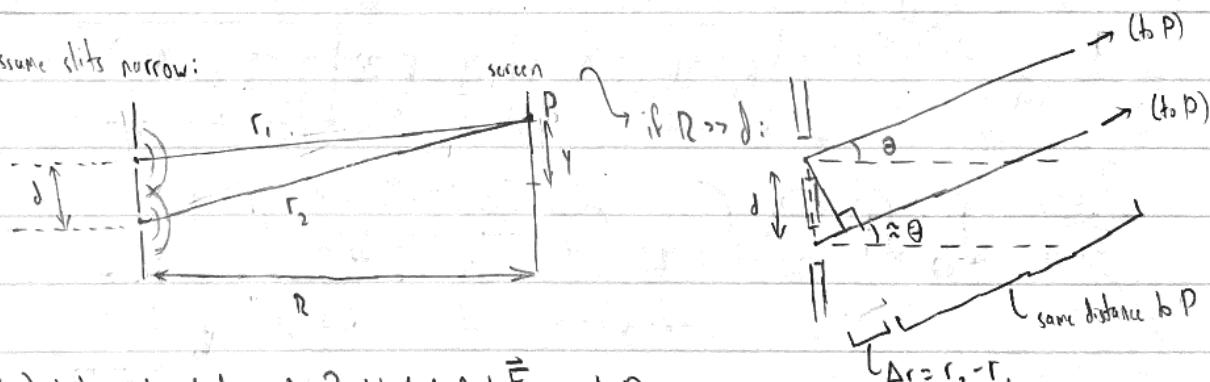
- Diffraction caused by multiple waves overlapping

2-Slit Interference

2 slits: (i) Large slits ($\text{width} \gg \lambda$) (ii) Narrow slits ($\text{width} \text{ not } \gg \lambda$)



Assume slits narrow:



(*) Why interested in Δr ? Want to find \vec{E}_{int} at P

$$\rightarrow E_{\text{int}}(P) = E_1(P) + E_2(P) \propto \sin(kr_1 - wt) + \sin(kr_2 - wt) \quad [k = \frac{2\pi}{\lambda}]$$

$$\rightarrow E_{\text{int}} \propto \underbrace{\sin(kr_1 - wt) + \sin(kr_2 - wt)}_{\sin(k[r_2 - r_1] + k\Delta r)}$$

$$\Delta r = \delta \sin(\theta)$$

$$\rightarrow \text{Define } \Delta\phi = k[r_2 - r_1] = k\Delta r \rightarrow \boxed{\Delta\phi = \frac{2\pi}{\lambda} \delta \sin(\theta)}$$

\rightarrow Observations: (i) If $\Delta\phi = 2\pi m$ [$m=0, 1, 2, \dots$], waves add (constructive interference)

(ii) If $\Delta\phi = 2\pi(m + \frac{1}{2})$, waves subtract (destructive interference)

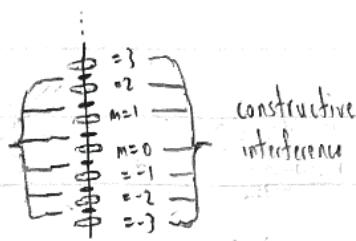
2-Slit & Thin-Film Interference

2/27/24

Lecture 15 2-Slit Interference (Cont.)

+ Disc 8

On screen, observe alternating bright & dark spots: destructive interference



$$\text{If } d, \theta \text{ known, can find } \lambda_{\text{light}}: \frac{\Delta\phi}{\lambda} = \frac{2\pi}{\lambda} d \sin\theta \rightarrow \boxed{d \sin\theta = m\lambda}$$

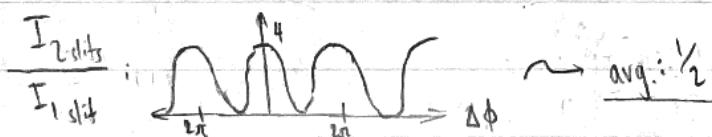
$$+ \Delta\phi = m \cdot 2\pi$$

$$\text{Diagram: A right-angled triangle with hypotenuse } R, \text{ angle } \theta, \text{ and vertical leg } \Delta y. \rightarrow \Delta y = R \tan\theta \approx R \sin(\theta) [\text{if } \theta \text{ small}] = \frac{R \sin\theta}{d} \rightarrow \boxed{\lambda = \frac{\Delta y \cdot d}{R \cdot m}}$$

$$\text{Intensity}$$

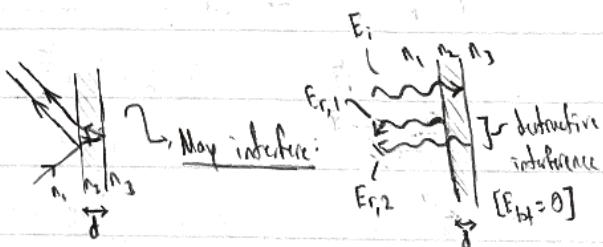
Two waves, E_p and E_o , are shown originating from different points on a horizontal axis. They interfere to form a resultant wave $E_{\text{tot}} = E_o \sin(\omega t) + E_p \sin(\omega t + \Delta\phi)$. The phase difference $\Delta\phi$ is indicated between the two waves.

$$E_p = 2E_o \cos\left(\frac{\Delta\phi}{2}\right) ; I \propto F^2 \rightarrow I = I_0 \cos^2\left(\frac{\Delta\phi}{2}\right)$$



Thin-Film Interference

In thin films ($\delta \sim \lambda$), have two reflected rays:



$E_{r1} = E_i \sin(\omega t + k_1 x) + E_{r2} \sin(\omega t + k_2 x + 2k_2 \delta)$ [since λ changes in medium]

$$\text{Reflected ray: } E_{r1,2} = E_{r1} \sin(\omega t + k_1 x) + E_{r2} \sin(\omega t + k_2 x + 2k_2 \delta)$$

~ Whether E_{r1}, E_{r2} in-phase [constructive interf.] or out-of-phase [destructive] depends on δ

$$\text{Amplitudes: } E_{r1} = \frac{n_1 - n_2}{n_1 + n_2} E_i, E_{r2} = \frac{n_2 - n_3}{n_2 + n_3} E_i \quad (\lambda \text{ in medium: } \lambda_2 = \frac{n_1}{n_2} \lambda_1)$$

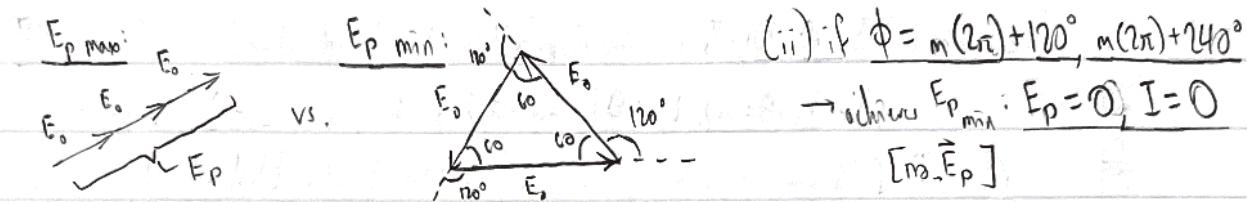
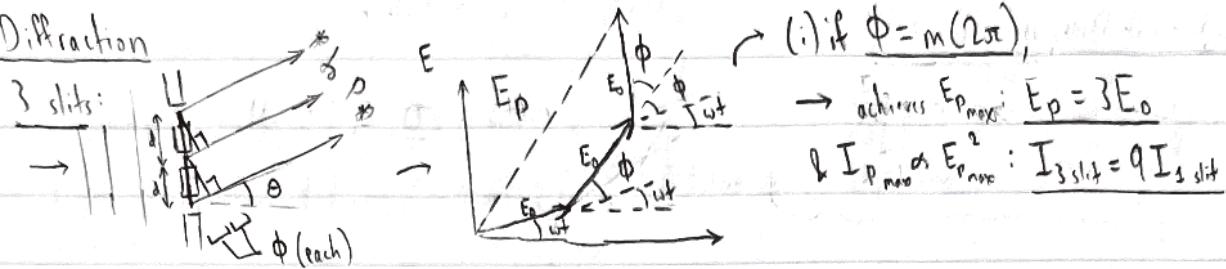
~ if $n_1 > n_2 > n_3, n_1 < n_2 < n_3$: (i) Constructive interference if $2k_2 \delta = 2\pi m \rightarrow 2\delta = m\lambda_2$

[else, flip] (ii) Destructive interference if $2k_2 \delta = 2\pi(m + \frac{1}{2}) \rightarrow 2\delta = (m + \frac{1}{2})\lambda_2$

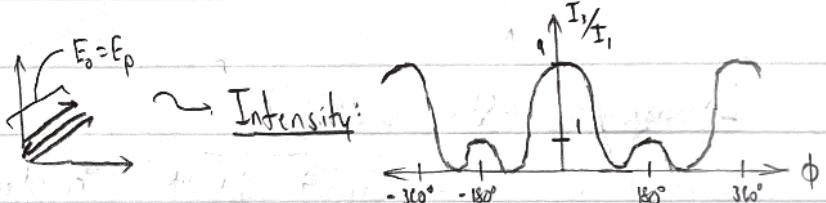
Diffraction

2/29/24

Diffraction



(iii) if $\phi = m(2\pi) + 180^\circ$,
 $\rightarrow E_p = E_0$; $I_{1\text{slit}} = I_{1\text{slit}}$

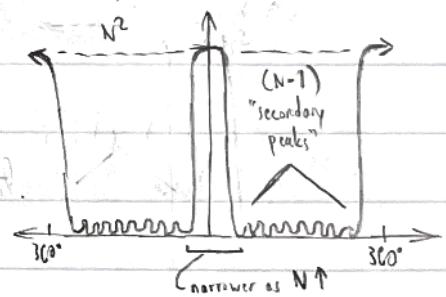


For $N > 3$ slits: (i) $\phi = m(2\pi) \rightarrow$ constructive interference

$$\text{Max: } \boxed{\phi = m(2\pi)} \quad [E_p = NE_0, I_N = N^2 I_1]$$

$$\text{(ii) } \phi = n\left(\frac{2\pi}{N}\right) \quad [\text{for } n=1, 2, \dots, (N-1)]$$

$$\text{Min: } \boxed{\phi = n\left(\frac{2\pi}{N}\right)} \quad \rightarrow \text{destructive interference } [E_p = I = 0]$$

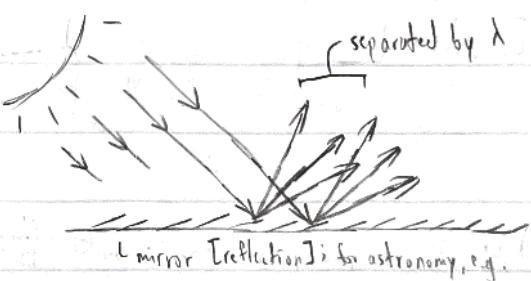


Diffraction Grating

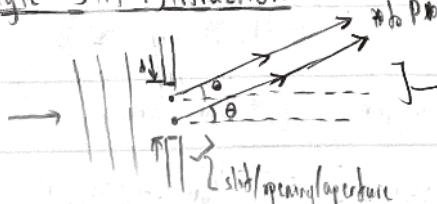
"Peaks" narrower as $N \uparrow$; recall: $\sin\theta = \frac{m\lambda}{d}$

\rightarrow for small d [$\lambda = 300\text{nm}$, e.g.]; $\sin\theta$ changes more

as λ changes \rightarrow can obtain separation of light



Single-Slit Diffraction



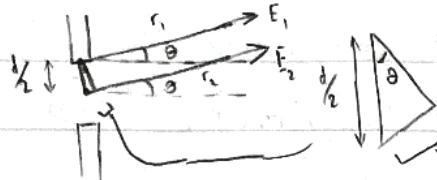
In case of single slit, look at rays from individual points

Diffraction (cont.)

2/29/24

Lecture 16
(cont.)

Single-Slit Diffraction (cont.)



$$(*) \frac{d}{2} \sin \theta = m\lambda/2 \rightarrow d \sin \theta = m\lambda$$

- $$\Delta r = \frac{d}{2} \sin \theta$$
- (i) if $\theta = 0$, E_1 & E_2 add [constructive]
 - (ii) if $\Delta r = \lambda/2$, E_1 & E_2 subtract [destructive]

→ Holds across entire slit: (i) If $\theta = 0$, E adds up along entire slit [constructive]

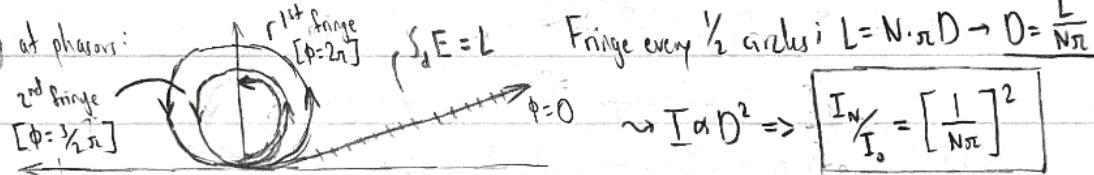


(ii) If $d \sin \theta = m\lambda$ [$m \neq 0$], for every point in slit, can find a point $\frac{1}{2}$ away (leg.) to cancel it around \sim total: $E = 0$ [destructive]

↳ dark fringe

Intensity of Single-Slit Diffraction

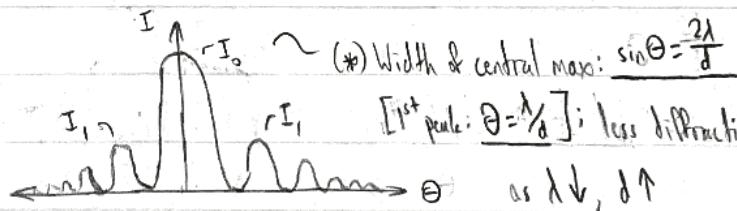
Looking at phasors:



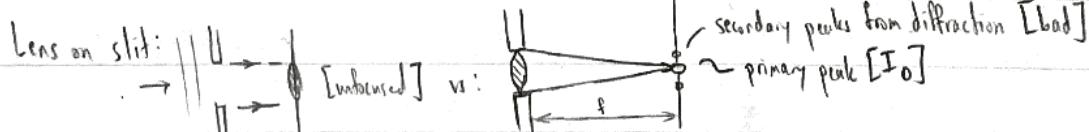
Fringe every $\frac{1}{2}$ arcus: $L = N \cdot \pi D \rightarrow D = \frac{L}{N\pi}$

$$\sim I \propto D^2 \Rightarrow \frac{I}{I_0} = \left[\frac{1}{N\pi} \right]^2$$

$\Rightarrow I$ peaks decrease for larger θ : $I_{1,2}, I_1, I_0$ \sim (*) Width & central max: $\sin \theta = \frac{\lambda}{d}$
as $\lambda \downarrow, \delta \uparrow$



(*) 2D Resolution Limits



(*) Ex: $\delta = 1 \text{ mm}$, $\lambda = 500 \text{ nm} \rightarrow \sin \theta = \frac{\pm 1}{2000} \text{ rad.} \approx \theta$; human eye resolution limit: $\sim \frac{1}{2000} \text{ rad.}$



$$\text{minima: } \theta \approx \sin \theta = 1.22 \frac{\lambda}{\delta}$$

[Rayleigh criterion: $\theta = \text{angle of } 1^\text{st} \text{ minimum}$]

- in telescopes:
- (i) bad lens [low δ]:
 - (ii) good lens [high δ]:

limit of human eye, telescope

good amateur telescope

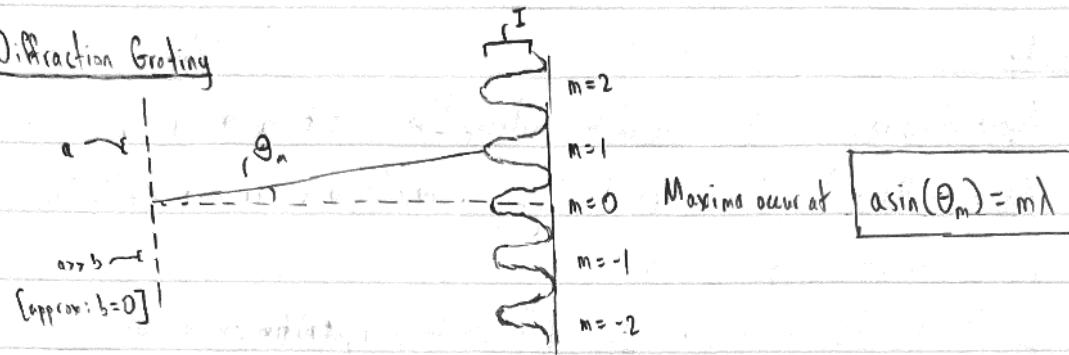
(*) Astronomers deal with small angles: $1^\circ = \frac{2\pi}{360} R \rightarrow 1^\circ = \frac{1}{60} [\text{arc-minute}] \rightarrow 1'' = \frac{1}{60} [\text{arc-second}]$

(*) Diffraction Review

3/7/24

Disc 9

Diffraction Grating



$m=2$

$m=1$

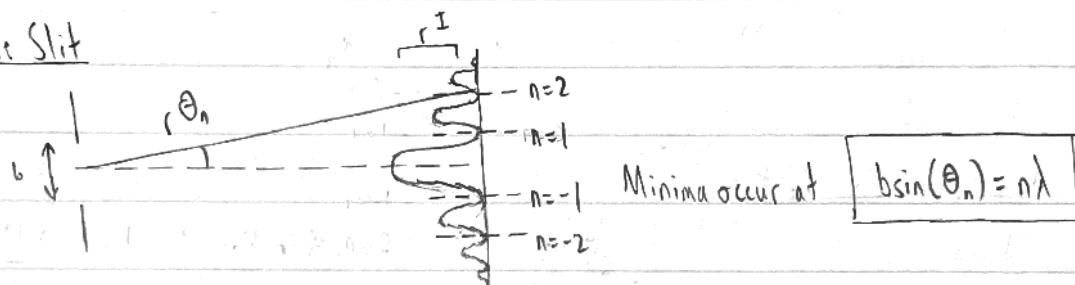
$m=0$ Maxima occur at

$$a \sin(\theta_m) = m\lambda$$

$m=-1$

$m=-2$

Single Slit



$n=2$

$n=1$

$n=0$

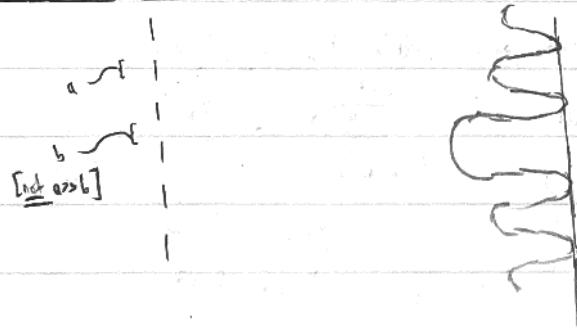
$n=-1$

$n=-2$

Minima occur at

$$b \sin(\theta_n) = n\lambda$$

Finite-Width Slits



$a \gg b$

In general: both single slit, diffraction grating equations apply

- As $b \rightarrow 0$, closer to diffraction grating
- As $n \rightarrow 1$, closer to single slit

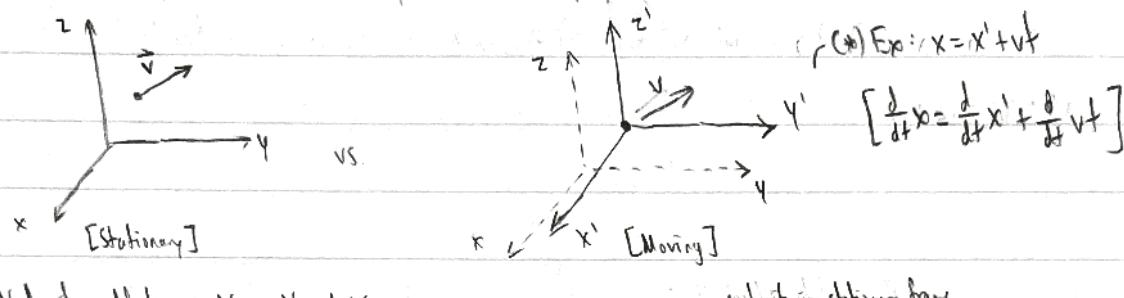
Galilean Relativity

3/5/24

Lecture 17 Galilean Relativity

Galileo: An object stays in motion until stopped by friction [Later: Newton's Laws]

→ Galilean relativity: the motion of a moving object can be described either in a stationary plane (x, y, z) or moving plane (x', y', z')



→ Velocity addition: $v_x = v_{x'} + v$

$$\boxed{\text{velocity in stationary frame}} \rightarrow \begin{aligned} &v_x = v_{x'} + v \sim \text{velocity in moving frame} \\ &\text{velocity in moving frame} \end{aligned}$$

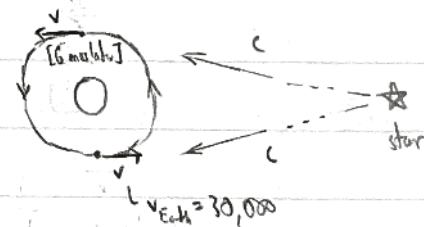
Michelson-Morley Experiment

Per Maxwell's equations, $c = 3 \cdot 10^8$ → Q: in what reference frame?

Michelson-Morley - measure speed of light from distant star

at two points in Earth's orbit: (i) while \vec{v} towards star

(ii) while \vec{v} away from star



Expected (Galilean relativity): light has $v_{\text{light}} = c + 30,000$ in case (i); $v_{\text{light}} = c - 30,000$ in case (ii) [relative to moving frame] if $v_{\text{light}} = c$ in stationary frame

Observed: $v_{\text{light}} = c$ in both instances, (i) & (ii) → velocity addition fails (?)

Initial explanation: A "luminiferous aether" around Earth results in $v_{\text{light}} = c$ within aether

Later (Einstein, 1905): What if the EBM laws [$v_{\text{light}} = c$] are true everywhere, and the idea of "reference frames" was incorrect?

Einsteinian Relativity

3/5/24

Einsteinian Relativity

(*) "Einstein's 1st Postulate"

Lecture 17

Per E&M laws, $v_{light} = c$ [within vacuum] is a universal result, but inconsistent with Galilean relativity (cont.)

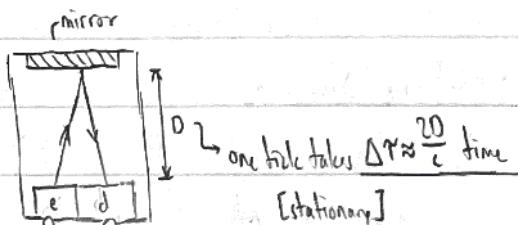
→ what needs to be changed about "reference frames" to accommodate E&M laws?

Answer: In "reference frames" assumption, time [t] assumed universal (i.e. same for all objects); in actuality, not true

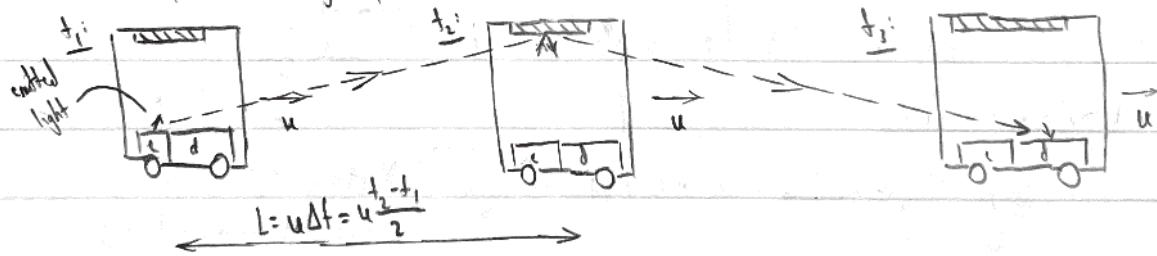
(*) "Light Clock"

Setup: Emitter of light (e) + detector of light (d)

one tick [of time] occurs when light from e reaches d [after reflecting off a mirror]



→ Same setup, but moving very fast:



$$s = \frac{ct}{2}, s^2 = D^2 + L^2 = \left(\frac{ct}{2}\right)^2 + \left(\frac{u\Delta t}{2}\right)^2 \Rightarrow (c^2 - u^2)\Delta t^2 = \frac{c^2}{4} \Delta t'^2$$

$$\rightarrow \Delta t^2 = \frac{c^2}{c^2 - u^2} \Delta t'^2$$

↑ path taken (moving frame)

$$\text{Define } \beta = \frac{u}{c} \rightarrow \Delta t^2 = \frac{1}{1-\beta^2} \Delta t'^2 \Rightarrow \boxed{\Delta t = \frac{1}{\sqrt{1-\beta^2}} \Delta t'}$$

↑ path taken (stationary frame)

$\Delta t'$ = time to tick in stationary frame; Δt = time to tick in moving frame; $\Delta t' \neq \Delta t$

⇒

Special Relativity

3/7/24

Lecture 18: "Modern physics" built on 2 pillars: relativity & quantum mechanics

Special Relativity

Special relativity derived from 2 postulates:

(i) 1st Postulate: The laws of physics are the same in any inertial (i.e. non-accelerating) reference frame

(ii) 2nd Postulate: The speed of light in vacuum is universal [constant for all observers]

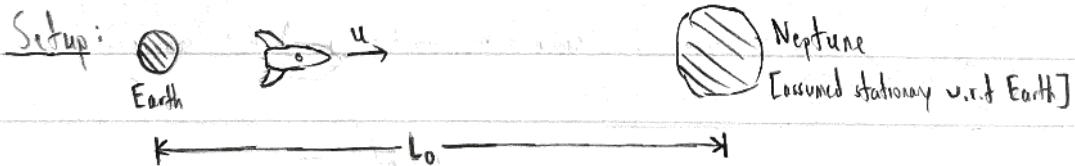
(*) Time Dilation

Recall (Light Clock): $\beta = u/c \Rightarrow \Delta t = \frac{1}{\sqrt{1-\beta^2}} \Delta \tau$; define $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ [Lorentz factor]

(*) $0 \leq \beta < 1$, $\gamma \geq 1$

(*) For small u : $\gamma = (1 - (u/c)^2)^{-1/2} \approx_{\text{use } (1-s)^a \approx 1 - as} 1 - \alpha s \Rightarrow \gamma \approx 1 + \frac{1}{2} \left(\frac{u}{c}\right)^2$

(*) Length Contraction

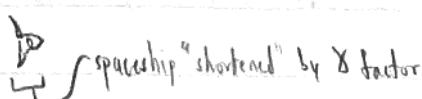


Spaceship moving w/ speed u relative to E-N frame [stationary]

~ By symmetry, E-N moving w/ speed u relative to spaceship

From stationary POV: $L_0 = ut$

From moving POV: $L = u\tau = u(1/\gamma) \Rightarrow L = \frac{L_0}{\gamma}$ \Rightarrow distance is shorter in dir. of travel

(*) By symmetry, from stationary POV:  spaceship "shortened" by γ factor

Special Relativity (cont.)

3/7/24

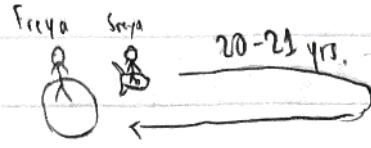
Lecture 18

(cont.)

(*) Time Travel

Setup: Sreya takes a trip w/ $\gamma = 50$ for 1 yr

Their twin, Freya, stays on Earth



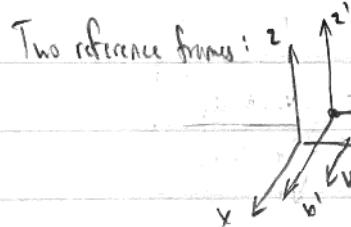
→ When Sreya returns @ 21 yrs. (before: 20 yrs.), Freya will be 70 [one-way time travel]

Twin Paradox: If Sreya has $\gamma = 50$ v.r.t. Freya, doesn't Freya have $\gamma = 58$ v.r.t. Sreya? Why isn't Freya the older one?

→ Soln: "Turning around" involves acceleration; no longer an inertial reference frame (spec. relativity doesn't apply)

(*) General relativity: Time runs slower in accelerating reference frames

The Lorentz Transform



→ Galilean relativity:

$$x' = x - ut \Leftrightarrow v_x' = v_x + u \Leftrightarrow v_x' = v_x - u$$

$$[y' = y; z' = z; (t' = t)]$$

Lorentz transforms - set of equations formalizing special relativity; depict 4D "spacetime"

Lorentz Equations

$$(i) x' = \gamma(x - ut)$$

$$(ii) y' = y; z' = z$$

$$(iii) t' = \gamma(t - \frac{ux}{c^2})$$

$$\Leftrightarrow (\vec{x}, t) = \begin{pmatrix} \gamma & 0 & 0 & -\gamma u \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{u}{c^2} & 0 & 0 & \gamma \end{pmatrix} (\vec{x}', t')$$

→ Inverse equations: $x = \gamma(x' + ut')$, $t = \gamma(t' + \frac{ux'}{c^2})$

• (i) gives time dilation: given obj. stationary in S, $\Delta t = t_2 - t_1 \Rightarrow \Delta t' = t'_2 - t'_1 \rightarrow \Delta t' = \gamma \Delta t$

• (iii) gives length contraction: given obj. stationary in S', $L = x_2 - x_1 \Rightarrow L = x'_2 - x'_1 = \frac{1}{\gamma}(x'_2 - x'_1) \rightarrow L' = \frac{L}{\gamma}$

Special Relativity (cont.)

3/7/24

Lecture 18 (\star) Simultaneity

Assume two events happen at same time t' [in S'] at different positions x'_1 & x'_2

\rightarrow what is Δt in S ? ($\Delta t' = 0$)

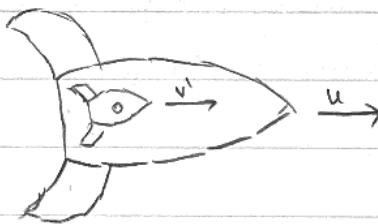
$$\text{Soln: } \Delta t = t_2 - t_1 = \gamma\left(t' + \frac{u x'_2}{c^2}\right) - \gamma\left(t' + \frac{u x'_1}{c^2}\right) \rightarrow \Delta t = \gamma \frac{u}{c^2} (x'_2 - x'_1) \neq 0$$

Consequence: Two events that occur simultaneously in one reference frame S' may not be simultaneous in another reference frame S [if they occur in different locations].

Velocity Addition

Velocity addition (Galilean): $v = u + v'$

\rightarrow if $u + v' > c$, claims $v > c$



$$\text{Lorentz: } v_x = \frac{dx}{dt} = \frac{\gamma(dx' + u dt')}{\gamma(dt' + \frac{u dx'}{c^2})} \xrightarrow{\gamma \frac{dx'}{dt'}} v_x = \frac{\frac{dx'}{dt'} + u}{1 + \frac{u}{c^2} \frac{dx'}{dt'}} \Rightarrow v_x = \frac{v'_x + u}{1 + \frac{u}{c^2} v'_x}$$

Special relativity: nothing can travel faster than the speed of light

(\star) Tachyon - hypothetical particles w/ $v > c$ [would break many physics laws if existent, e.g. causality]

Relativistic Momentum

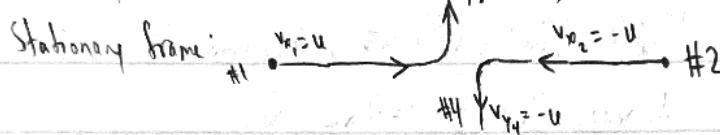
3/12/24

Relativistic Momentum

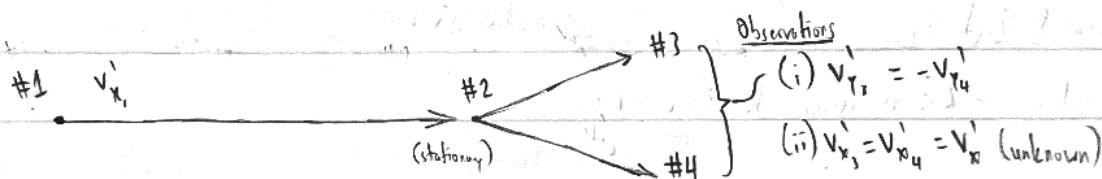
Recall (Non-Relativistic Momentum): $\vec{p} = m\vec{v}$ [Newton], is constant [conservation of momentum]

→ formula fails in relativity

(*) Ex: Equal Mass Particles → #3 $v_{x_3}=u$ ↗ (particle deflected)



→ construct moving frame s' , moving w/ velocity $-u$ from position of #2:



Looking at momentum before/after (using $\vec{p} = m\vec{v}$):

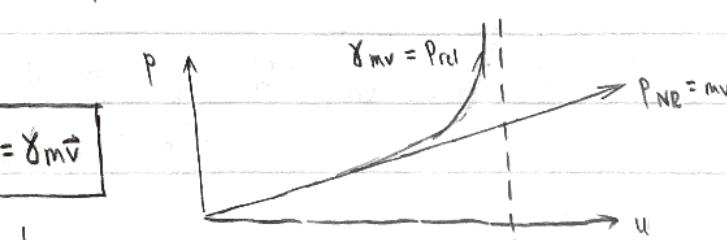
$$\text{Initial total momentum: } \vec{p}_{\text{tot}}^{\text{initial}} = \vec{p}_{x_1}^{\text{initial}} = m\vec{v}_{x_1} = m \left(\frac{u+u}{1+u^2/c^2} \right) = \frac{2mu}{1+\beta^2}$$

$$\text{Final total momentum: } \vec{p}_{\text{tot}}^{\text{final}} = \vec{p}_{x_3}^{\text{final}} = \frac{0+u}{1+u^2/c^2} = u \rightarrow \vec{p}_{\text{tot}}^{\text{final}} = 2m\vec{v}_{x_3} = 2mu \neq \frac{2mu}{1+\beta^2}$$

$\Rightarrow \vec{p}_{\text{initial}} \neq \vec{p}_{\text{final}}$ [If define $\vec{p} = m\vec{v}$, conservation of momentum fails]

→ Define relativistic momentum

$$\vec{p} = \gamma m\vec{v}$$



preserves conservation of momentum in relativistic settings

Relativistic Energy

3/12/24

Lecture 19 Relativistic Kinetic Energy

(cont) $\text{KE}(\text{non-relativistic}) = \frac{1}{2}mv^2 \rightarrow \text{define relativistic kinetic energy: } K = (\gamma - 1)mc^2$

(*) To show $K \approx K_{\text{NR}}$ for $v \ll c$, use $(1+\epsilon)^n = 1+n\epsilon + \frac{n(n-1)}{2}\epsilon^2 + \dots$:

$$\gamma = (1-\beta^2)^{-\frac{1}{2}} \Rightarrow \gamma \approx 1 + (-\frac{1}{2})(-\beta^2) + \frac{(-\frac{1}{2})(-\frac{1}{2})}{2}\beta^4 + \dots [\epsilon = \beta^2; n = -\frac{1}{2}]$$

$$\Rightarrow K = (\gamma - 1)mc^2 = (\frac{\gamma^2}{2} + \frac{3}{8}\frac{\gamma^4}{4!} + \dots)mc^2 = (\frac{m^2}{2} + \frac{3}{8}\frac{m^4}{c^2} + \dots)mc^2 = \frac{1}{2}mv^2(1 + \frac{3}{8}\frac{v^2}{c^2} + \dots)$$

$$\approx \frac{1}{2}mv^2(1 + \frac{3}{8}\frac{v^2}{c^2} + \dots); \text{ if } v \ll c, K \approx \frac{1}{2}mv^2 = K_{\text{NR}}.$$

Total & Rest Energy

Looking at KE expression: $K = \gamma mc^2 - mc^2$

γ dependent on v [motion]

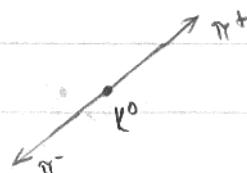
→ Define total energy as sum of kinetic and rest energies:

Rest energy also called
"mass-equivalent energy"

$$E_{\text{tot}} = \gamma mc^2 = K + mc^2 \Leftrightarrow K = \gamma mc^2 [E_{\text{tot}}] - mc^2 [E_{\text{rest}}]$$

(*) Ex: Energy of Kaon Decay

A kaon is a particle that decays into 2 pions: $K^0 \rightarrow \pi^+ \pi^-$



Looking at rest energies: $E_{\text{rest}_0} = m_{K^0}c^2 = 493 \text{ MeV}$

$$E_{\text{rest}_\pi} = 2m_{\pi}c^2 = 2 \cdot 135 = 270 \text{ MeV}$$

$$\approx E_0 = E_p \Rightarrow E_{\text{rest}_0} = K_p + E_{\text{rest}_\pi} \Rightarrow 493 = K_p + 270 \Rightarrow K_p = 223 \text{ MeV}$$

(!) Observation: Mass-equivalent energy decreased \Rightarrow mass was lost! [converted to energy]

Relativistic Energy (cont.)

3/12/24

Lecture 19

(cont.)

- Conservation Laws (Relativistic):
- (i) Energy is conserved
 - (ii) Momentum is conserved
 - (iii) Mass is not conserved

Energy, Momentum, and Mass

Energy, momentum, and mass all related via relativity.

(*) Derivation: $(pc)^2 = (\gamma mvc)^2 = \gamma^2 m^2 v^2 c^2$
 $\rightarrow (mc^2)^2 + (pc)^2 = m^2 c^4 (1 + \gamma^2 v^2/c^2) = m^2 c^4 (1 + \gamma^2 \beta^2)$

Looking at γ term: $1 + \gamma^2 \beta^2 = 1 + \frac{\beta^2}{1 - \beta^2} = \frac{1}{1 - \beta^2} = \gamma^2$
 $\Rightarrow (mc^2)^2 + (pc)^2 = \gamma^2 m^2 c^4 = (\gamma m c^2)^2 = E^2$

~ Final equation: $E^2 = (mc^2)^2 + (pc)^2$

(*) Equations for energy: (i) $E = \gamma mc^2$

(ii) $E_0 = mc^2$

(iii) $E = mc^2 + K$

(iv) $E = \sqrt{(mc^2)^2 + (pc)^2}$

(*) If in units $v/c=1$, $E = \sqrt{m^2 + p^2}$ [Theoretical physics]

~ Notice: None of these are exactly $E=mc^2$!

(*) Newton's 2nd Law

$$F_{NN} = \frac{d}{dt} P_{NN} \Rightarrow \text{relativistic generalization (using } \vec{p}_{NN}) : F = \gamma^3 ma \Leftrightarrow a = \frac{F}{\gamma^3 m}$$

(*) Relativistic Doppler Effect

3/12/24

Disc 10 Doppler Shift for E&M Waves

+ Lecture 19
(cont.)

Setup:  Non-relativistic: $T_{\text{source}} = \frac{1}{f_s} = \frac{\lambda}{c}$ $f_{\text{obs}} = \frac{c+v}{c} f_s$

(*) Signs: $v > 0$ if towards source.

→ Relativity: $T_{\text{source}} = \gamma(\frac{\lambda}{c})$ [from observer's frame - time dilation]

$$\Rightarrow f_{\text{observed}} = \frac{c+v}{\frac{1}{\gamma} c} f_s = \gamma \frac{c+v}{c} f_s = \gamma(1+\frac{v}{c}) f_s = \frac{1+\frac{v}{c}}{\sqrt{1-\frac{v^2}{c^2}}} f_s = \frac{1+\frac{v}{c}}{\sqrt{(1+\frac{v}{c})(1-\frac{v}{c})}} f_s = \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} f_s$$

→ Relativistic Doppler effect:
$$f_{\text{obs}} = \boxed{\sqrt{\frac{c+v}{c-v}} f_{\text{source}}} \quad [v: \text{observer } \underline{\text{towards}} \text{ source}]$$

(*) Nuclear Energy

Background:

- 1911 - Most of an atom's mass ($> 99.95\%$) located in the nucleus (radius $\approx 10^{-15} \text{ m}$)
- 1932 - Neutron discovered; nuclei made up of protons & neutrons
 - [Neutron: \sim same mass as proton, no charge]

→ There must be some "new" strong & attractive force keeping nuclei together
+ force must be short-range (unlike Coulomb force) to have escaped prior detection

[Modern day: protons and neutrons each composed of 3 quarks; nuclear mass arises from forces between quarks]

(*) Nuclear Energy (cont.)

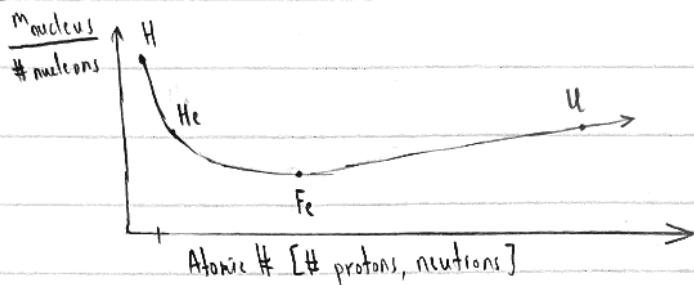
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Lecture 19

(cont.)

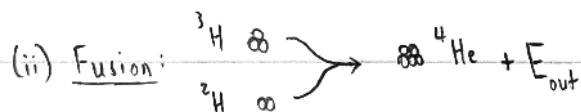
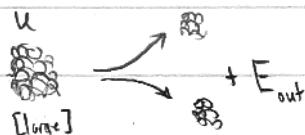
(*) Nuclear Energy (cont.)

Measuring nuclear mass:



H
He
↓
call change a decrease in "binding energy" [$\Delta E = \Delta m \cdot c^2$]
→ can interpret graph as an energy diagram - atoms want to go from higher E to lower E (Fe)

2 ways to reach lower energy: (i) Fission:



Benefit: Nuclear [mass-equivalent] energies much larger than chemical reactions \leadsto potentially more E
($m_{\text{proton}}c^2 = 938 \text{ MeV} \rightarrow E_{\text{nuclear}} \approx \text{MeV} = 10^6 \text{ eV}$; vs $E_{\text{chemical}} = \text{a few eV}$)

(*) Ex: 1 kg uranium fuel \rightarrow 24,000,000 kWh \Leftrightarrow 2500 tons coal, 1600 tons of oil

(*) Nuclear Challenges: Hard to obtain fuel, generates radioactive byproducts

(*) Fast fission reactions used in atomic bombs

Fusion reactions ($\text{H} \rightarrow \text{He}$, e.g.) drive Sun & stars in their cores [fuse until Fe \rightarrow no more energy]

(*) Potential Upsides: More E than fission (10-100x), easier to acquire ^2H / ^3H , no radioactive waste

(*) Used in hydrogen bombs